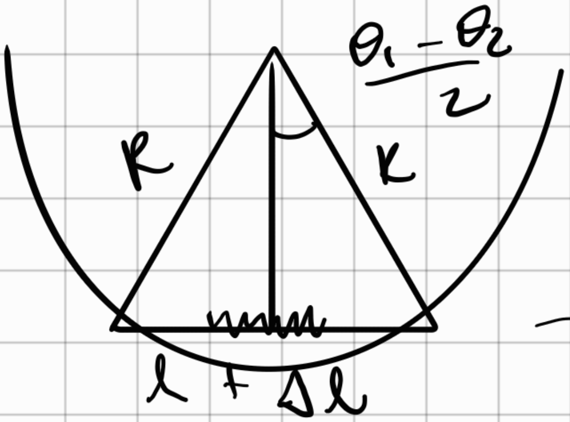
1ro parte

3)



$$2R\sin\left(\frac{\theta_1-\theta_2}{2}\right)=l+\Delta l$$

$$V=-\frac{1}{2}k\left(2R\sin\left(\frac{\theta_1-\theta_2}{2}\right)-l\right)^2$$
$$-m_1 g R\sin\theta_1-m_2 g R\sin\theta_2$$

$$T=\frac{1}{2}m_1\left(R^2\dot{\theta}_1^2\right)+\frac{1}{2}m_2\left(R^2\dot{\theta}_2^2\right)$$

$$\mathcal{L}=\frac{1}{2}R^2\left(m_1\dot{\theta}_1^2+m_2\dot{\theta}_2^2\right)$$
$$+\frac{1}{2}k\left(2R\sin\left(\frac{\theta_1-\theta_2}{2}\right)-l\right)^2$$
$$+gR\left(m_1\sin\theta_1-m_2\sin\theta_2\right)$$

en este caso las coordenadas generalizadas son θ_1 y θ_2 a)

b.) $$\frac{d}{dt}\left(\frac{\partial\mathcal{L}}{\partial\dot{\theta}_1}\right)=\frac{\partial\mathcal{L}}{\partial\theta_1}$$ para θ_1

$$\rightarrow R^2 m_1\ddot{\theta}_1=k\left(2R\sin\left(\frac{\theta_1-\theta_2}{2}\right)-l\right)\cdot$$

$$R\cos\left(\frac{\theta_1-\theta_2}{2}\right)$$

$$+\,gR\,m_1\cos\theta_1$$

para θ_2

$$R^2 m_2 \ddot{\theta}_2 = K\left(2R\sin\left(\frac{\theta_1-\theta_2}{2}\right) - \ell\right)$$

$$\cdot\,\ell - R\cos\left(\frac{\theta_1-\theta_2}{2}\right)$$

$$+\,gr\,m_2\cos\theta_2$$

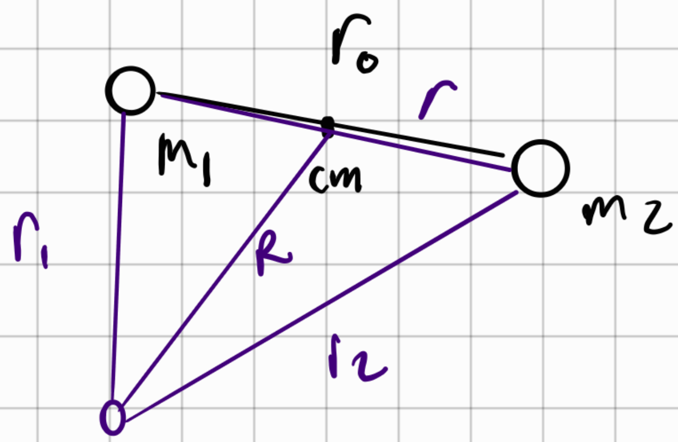
C

d.)

$$\frac{dV}{d\theta_1} = K\left[\left(2R\sin\left(\frac{\theta_1-\theta_2}{2}\right)-\ell\right)\left(R\cos\left(\frac{\theta_1-\theta_2}{2}\right)\right)\right] + m_1 g R\cos\theta_1 = 0$$

$$\frac{dV}{d\theta_2} = K\left[\left(2R\sin\left(\frac{\theta_1-\theta_2}{2}\right)-\ell\right)\left(-R\cos\left(\frac{\theta_1-\theta_2}{2}\right)\right)\right] + m_2 g R\cos\theta_2 = 0$$

2.) $m_1 + m_2 = M$



$$r = r_1 - r_2$$

$$R = \frac{m_1 r_1 + m_2 r_2}{M}$$

$$r_1' = r_1 - R = -\frac{m_2}{M} r$$

$$r_2' = r_2 - R = -\frac{m_1}{M} r$$

$$v_1' = -\frac{m_2}{M}\dot{r}$$

$$v_2' = \frac{m_1}{M}\dot{r}$$

$$\mathcal{L} = \underbrace{\frac{1}{2}(m_1+m_2)\dot{R}^2 + \frac{1}{2}\mu\dot{r}^2}_{T} + \underbrace{G\frac{m_1 m_2}{r}}_{V(r)}$$

ya que $\dot{R}$ = cte ya que R es cordenada ciclica podemos tomar el cm como origen del sistema $\rightarrow R = 0$

$$\mathcal{L} = \frac{1}{2}\mu\dot{r}_2 + \frac{G m_1 m_2}{r}$$

la energía se conserva ya que t es cordenada cíclica.

$$E_0 = E$$

$$-\frac{Gm_1m_2}{r_0} = \frac{1}{2}\mu\dot{r}^2 - \frac{Gm_1m_2}{r}$$

$$\frac{1}{2}\mu\dot{r}^2 = Gm_1m_2\left(\frac{1}{r} - \frac{1}{r_0}\right)$$

$$\dot{r} = \sqrt{2G\underbrace{\frac{m_1m_2}{\mu}}_{M}\left(\frac{1}{r} - \frac{1}{r_0}\right)}$$

$$\grave{v}_1 = m_2\sqrt{2GM\left(\frac{1}{r} - \frac{1}{r_0}\right)}$$

$$\grave{v}_1 = m_1\sqrt{2GM\left(\frac{1}{r} - \frac{1}{r_0}\right)}$$

$$V(r) = mgR_{cm} + \frac{1}{2}k(r-l)^2$$

2.)

$$F = -\frac{\partial V}{\partial r}$$

$$\mathcal{L} = \frac{1}{2}\mu(\dot{r}^2 + r^2\dot{\theta}^2) - V(r)$$

θ es cíclico

$$L^2 = \mu^2 r^4 \dot{\theta}^2$$

$$\frac{\partial \mathcal{L}}{\partial \dot{\theta}} = \mu r^2 \dot{\theta} = L = cte$$

$$V(r) = \int_0^r \left[\frac{K}{r^2} + \frac{\lambda}{r^3} \right) dr$$

$$V(r) = -\frac{K}{r} - \frac{\lambda}{2r^2}$$

Lo E se conserva $E = T + V = cte$

$$\frac{1}{2}\mu\dot{r}^2 + \underbrace{\frac{L^2}{2\mu r^2} - \frac{K}{r} - \frac{\lambda}{2r^2}}_{V_{ef}} = cte$$

2do parte:

1. $V(r) = -\frac{k}{r} + \frac{\epsilon}{r^2} = -ku + \varepsilon u^2$

$\frac{dV}{du} = -k + 2\varepsilon u$ $\quad \varphi = \mu$ masa reducida

sustituyendolo en la ecuacion de binet

$$\frac{d^2u}{d\theta^2} + u = -\frac{\varphi}{l^2}(-k + 2\varepsilon u)$$

$$\frac{d^2u}{d\theta^2} + \left(1 + \frac{2\varepsilon\varphi}{l^2}\right)u = \frac{\varphi k}{l^2}$$

la sol a la ecuacion

$$u(\theta) = A\cos\left(\underbrace{\sqrt{1 + \frac{2\varepsilon\varphi}{l^2}}}_{\alpha}\theta + \phi\right) + \frac{k\varphi}{l^2\left(1 + \frac{2\varepsilon\varphi}{l^2}\right)}$$

el termino α indica que la orbita no es cerrada el angulo de precesion

$$\Delta\theta = 2\pi\left(\frac{1}{\alpha} - 1\right)$$ cuando ε es pequeño $\alpha \simeq 1 + \frac{\varepsilon \varphi}{l^2}$

$$\Delta\theta \simeq -\frac{2\pi\varepsilon\varphi}{l^2}$$

2. $F(r) = -\frac{\partial V}{\partial r} \rightarrow V = -\frac{K}{r} - \frac{\lambda}{2r^2}$

$$V = -Ku - \frac{\lambda}{2}u^2$$

de la ec de Binet $\varphi = \mu$ masa reducida

$$\frac{\partial^2 u}{\partial \theta^2} + u = -\frac{\varphi}{l^2}\frac{\partial V}{\partial u}$$

$$\frac{d^2 u}{d\theta^2} + u = \frac{\varphi}{l^2}(K + \lambda u)$$

$$\frac{d^2 u}{d\theta^2} + \underbrace{\left(1 - \frac{\lambda\varphi}{l^2}\right)}_{\alpha} u = \underbrace{\frac{\varphi}{l^2}K}_{b}$$

$$u(\theta) = \frac{b}{\alpha} + c_2 \sin(\sqrt{a}\,\theta) + c_3 \cos(\sqrt{a}\,\theta)$$

$$\underbrace{\frac{1}{r} = \frac{b}{\alpha} + c_2 \sin(\sqrt{a}\,\theta) + c_3 \cos(\sqrt{a}\,\theta)}_{\text{elipse}}$$

si $\lambda = \frac{\ell}{l^2}$ $\quad \alpha = 0$ no hay precesion

si $\lambda > \frac{\ell}{l^2}$ $\quad \alpha < 0$ orbita es inestable

si $\lambda < \frac{\ell}{l^2}$ $\quad \alpha > 0$ elipse con precesion

3. $$\mathcal{L} = \frac{1}{2}\mu \dot{r}^2 + \frac{K}{r} - \frac{1}{2}\frac{\ell^2}{\mu r^2}$$

E y ℓ se conserva

en el punto r_{min} $\quad \dot{r} = 0 \rightarrow E = V_{ef}$

y la E total en el infinito $= \frac{1}{2} m v_0^2$

Sabemos que el $\ell = r \times p = b \mu v_0$

$$E_i = E_f$$

$$\frac{1}{2}\mu v_0^2 = -\frac{k}{r_{min}} - \frac{1}{2}\frac{(\mu v_0 b)^2}{\mu r_{min}^2}$$

$$\frac{1}{2}\mu v_0^2 r_{min}^2 = -k\, r_{min} - \frac{1}{2}\mu v_0^2 b^2$$

$$\mu v_0^2 r_{min}^2 + 2k\, r_{min} + \mu v_0^2 b^2 = 0$$

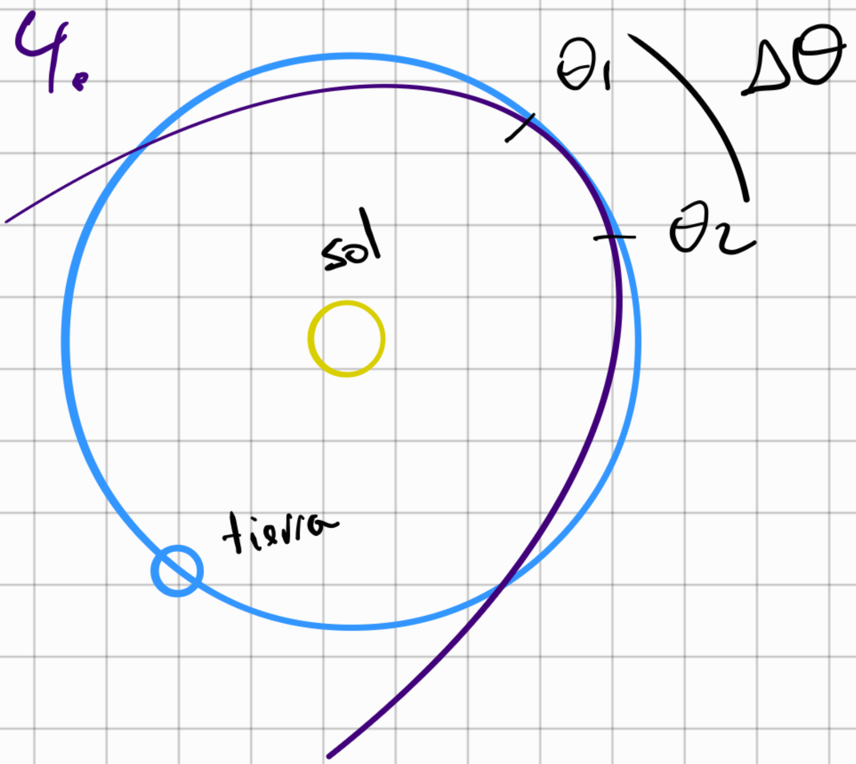
$$r_{min} = \frac{-k \pm \sqrt{k^2 + \mu^2 v_0^4 b^2}}{\mu v_0^2}$$

el angulo de de dispersion χ

$$\chi = \pi - 2\arcsen\left(\frac{1}{e}\right)$$

$$\chi = \pi - 2\arcsen\left(\frac{1}{\sqrt{1 + \left(\frac{v_0^2 b}{k}\right)^2}}\right)$$

4.



igualamos las trayectorias

trayectoria para el cometa

$$\frac{q_c}{r} = 1 + e_c \cos\theta$$

para uno palabra la exentricidad es 1

$$r = q_c + q_c \cos\theta$$

trayectoria de la tierra

$$\frac{q_t}{r} = 1$$ la exentricidad es 0

$$r = \frac{1}{q_t}$$

$$q_c + q_c \cos\theta = \frac{1}{q_t} \rightarrow \cos\theta = \frac{1}{q_t q_c} - 1$$

$$q = \frac{l^2}{\mu K}$$

para el cometa $\mu \approx m$
para la tierra $\mu = m_t$

$$\theta = \operatorname{Arccos}\theta \left[\frac{1}{\left(\frac{l_t^2}{m_t K}\right)\left(\frac{l_c^2}{m K}\right)} \right] - 1$$

$$\theta = \operatorname{Arccos}\left[\frac{m_t m K^2}{l_t^2 l_c^2} \right]$$

el area que barre en la orbita terrestre es:

$$A = \frac{1}{2} r_t^2 \Delta\theta$$

r_t = radio de la orbita de la tierra

$$A = \frac{l_c}{2m} \Delta t$$

$$\rightarrow \Delta t = \frac{m r_t^2}{l_c} \Delta\theta$$

5. a) $r = a(1+\cos\theta)$

$$\mathbb{F} = -\frac{\partial V}{\partial r}$$

entonces de la ec de Binet

$$F(r) = -\frac{l^2}{mr^2}\left(\frac{d^2u}{d\theta^2} + u\right)$$

$$u = \frac{1}{r} = \frac{1}{a(1+\cos\theta)}$$

$$\frac{d^2u}{d\theta^2} = \frac{\cos\theta + \cos^2\theta + 2\sin^2\theta}{a(1+\cos\theta)^3}$$

$$= \frac{\cos\theta + \cos^2\theta + 2 - 2\cos^2\theta)}{a(1+\cos\theta)^3}$$

$$= \frac{\cos\theta - \cos^2\theta + 2}{a(1+\cos\theta)^3}$$

$$F(r) = \left(\frac{\cos\theta - \cos^2\theta + 2}{a(1+\cos\theta)^3} + \frac{1}{a(1+\cos\theta)} \right)$$

$$\cdot -\frac{\ell^2}{mr^2}$$

$$= \left(\frac{2+\cos\theta - \cos^2\theta + \overbrace{(1+\cos\theta)^2}^{\cos^2\theta + 2\cos\theta + 1}}{a(1+\cos\theta)^3} \right) \cdot -\frac{\ell^2}{mr^2}$$

$$= \left(\frac{3+3\cos\theta}{a(1+\cos\theta)^3} \right) \cdot -\frac{\ell^2}{mr^2} = \frac{3(1+\cos\theta)}{a(1+\cos\theta)^3}$$

$$\cdot \frac{\ell^2}{mr^2} = -\frac{\ell^2}{mr^2}\left(\frac{3}{r^2}\right) = -\frac{3\ell^2}{mr^4}$$

b.] el area de la orbita

$$A = \int_0^{2\pi} \frac{1}{2} a^2 (1+\cos\theta)^2 \, d\theta = \frac{3\pi a^2}{2}$$

la tasa de barrido $\dot{A} = \frac{\ell}{2m}$

el periodo $T = \frac{A}{\bar{A}} = \frac{3\pi a^2 m}{\ell}$

$$V(r) = -\int \mathbb{F}(r)\, d_r = -\int \frac{3\ell^3}{\mu r^4} = \frac{-\ell^2}{\mu r^3}$$

para que la particula escape $E = 0$

$$V_{ef} = -\frac{\ell^2}{\mu r^3} + \frac{\ell^2}{\mu r^2}$$