5.)

$$\mathcal{L} = \frac{m^2 \dot{x}^4}{12} + m \dot{x}^2 f(x) - f^2(x)$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}}\right) - \frac{\partial \mathcal{L}}{\partial x} = 0$$

$$\frac{d}{dt}\left[\frac{1}{3} m^2 \dot{x}^3 + 2m \dot{x} f(x)\right] - m \dot{x}^2 f'(x) - 2 f(x) f'(x)$$

$$m^2 \dot{x}^2 \ddot{x} + 2m f(x) \ddot{x} + \cancel{2}^{1} m \dot{x}^2 f'(x) - \cancel{m \dot{x}^2 f'(x)} - 2 f(x) f'(x) = 0$$

$$m^2 \dot{x}^2 \ddot{x} + 2m f(x) \ddot{x} + m \dot{x}^2 f'(x) - 2 f(x) f'(x) = 0$$

$$\underbrace{m^2 \dot{x}^2 \ddot{x} + 2m f(x) \ddot{x} + (m \dot{x}^2 - 2 f(x)) f'(x) = 0}_{\text{EC de movimiento}}$$

6.) $$\mathcal{L} = \frac{1}{2} g_{ab}(q_c) \dot{q}^a \dot{q}^b$$

$$\underbrace{\frac{d}{dT}\left(\frac{d\mathcal{L}}{d\dot{q}_i}\right)} - \underbrace{\frac{\partial \mathcal{L}}{\partial q_i}} = 0$$

$$\frac{d}{dt}\left[\frac{d}{d\dot{q}^a}\left(\frac{1}{2}g_{ab}(q_c)\dot{q}^a\dot{q}^b\right)\right]$$

$$= \frac{d}{dt}\left[\frac{1}{2}g_{ab}\left(\dot{q}^b + \underbrace{\dot{q}^a\frac{d\dot{q}^b}{d\dot{q}^a}}_{\dot{q}^a\delta^b_a = \dot{q}^b}\right)\right] =$$

$$= \frac{d}{dt}\left[\frac{1}{2}g_{ab}\left(2\dot{q}^b\right)\right] = \frac{d}{dt}g_{ab}\dot{q}^b$$

$$= \dot{g}_{ab}\dot{q}^b + g_{ab}\ddot{q}^b$$

$$= \frac{\partial g_{ab}}{\partial q^c}\dot{q}^c\dot{q}^b + g_{ab}\ddot{q}^b$$

$$\frac{d}{dq^a}\left[\frac{1}{2}g_{bc}(q_a)\dot{q}^b\dot{q}^c\right] = \frac{1}{2}\frac{\partial g_{bc}}{\partial q^a}\dot{q}^b\dot{q}^c$$

$$\frac{\partial g_{ab}}{\partial q^c}\dot{q}^c\dot{q}^b + g_{ab}\ddot{q}^b - \frac{1}{2}\frac{\partial g_{bc}}{\partial q^a}\dot{q}^b\dot{q}^c = 0$$

$$g_{ab}\ddot{q}^b + \left[\underbrace{\frac{\partial g_{ab}}{\partial q^c}} - \frac{1}{2}\frac{\partial g_{bc}}{\partial q^a}\right]\dot{q}^c\dot{q}^b = 0$$

ya que g_{ab} es simetrico

$$\frac{\partial g_{ab}}{\partial q^c} = \frac{1}{2}\frac{\partial g_{ab}}{\partial q^c} + \frac{1}{2}\frac{\partial g_{ac}}{\partial q^b}$$

$$g_{ab}\ddot{q}^b + \frac{1}{2}\left[\frac{\partial g_{ab}}{\partial q^c} + \frac{\partial g_{ac}}{\partial q^b} - \frac{\partial g_{bc}}{\partial q^a}\right]\dot{q}^c\dot{q}^b$$

multipicamos por g^{ad}

$$\underbrace{\cancel{g^{ad} g_{ab}}}_{\delta^d_b}\ddot{q}^b + \underbrace{\frac{1}{2} g^{ad}\left[\frac{\partial g_{ab}}{\partial q^c} + \frac{\partial g_{ac}}{\partial q^b} - \frac{\partial g_{bc}}{\partial q^a}\right]}_{\Gamma^d_{bc}}\dot{q}^c\dot{q}^b$$

$$\ddot{q}^d + \Gamma^d_{bc}\dot{q}^b\dot{q}^c = 0$$