Computação Paralela / Computação Avançada

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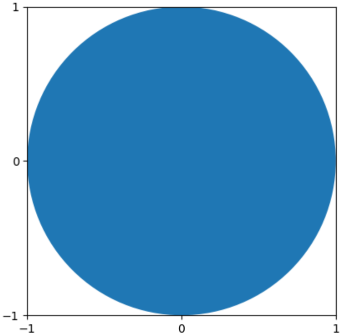
cap2 — 2022-09-28

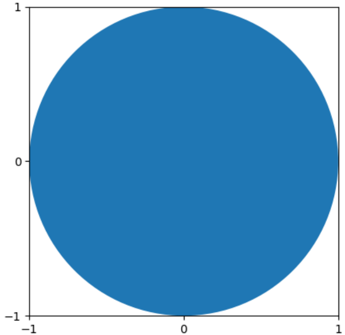
Calculating π

- Draw a circle in a square of size 2×2:



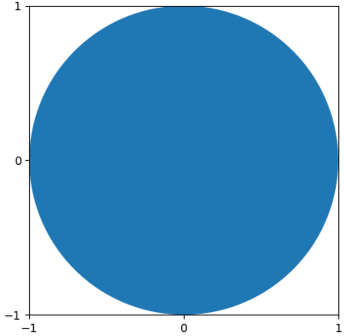
- The area of the square is 4
- The area of the circle is π

Calculating π – Monte Carlo Approach



- Create n random points (x, y) with $x, y \in [-1, 1]$
- Verify for each point if it falls inside the circle
- Be m the number of points in the circle
- $\Rightarrow \pi \approx 4\frac{m}{n}$

Parallelizing



- Straight forward approach: the individual processes of creating one point are independent from each other.
- So if we have p processors, give each processor the task to determine $\frac{n}{p}$ points

Error estimation

- n independent samples, ratio $r = \frac{m}{n}$
- Estimated error Δr:

$$\Delta r \quad \approx \quad z_c \sqrt{\frac{r(1-r)}{n}}$$

- z_c is the *critical value* of the Gaussian distribution:

$$P \quad = \quad \int_{-z_c}^{z_c} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} dz$$

P is the probability that the correct value is located in the interval $[r - \Delta r, r + \Delta r]$

Error estimation

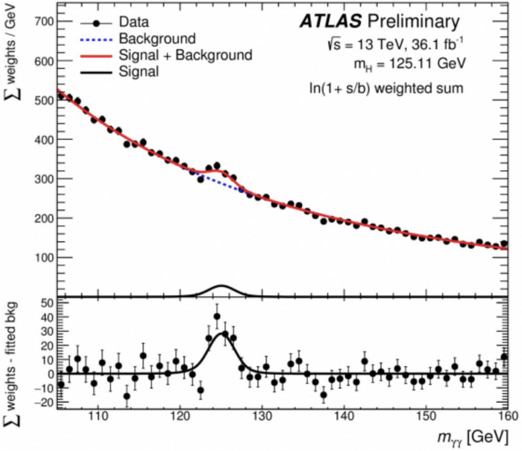
- $z_c = 1$: "one standard deviation" — classical error bars — $P \approx 68.3\%$
- $z_c = 2$: "two standard deviations" — $P \approx 95.5\%$
- $z_c = 3$: "three standard deviations" — $P \approx 99.7\%$

Examples for $z_c = 1$:

- $n = 1000$: $\Delta r \approx 0.013$, $\Delta \pi \approx 0.05$
- $n = 10^6$: $\Delta r \approx 0.0004$, $\Delta \pi \approx 0.0016$
- $n = 10^9$: $\Delta r \approx 0.000013$, $\Delta \pi \approx 0.00005$

Particle Physics: 5σ

"ATLAS and CMS observe an excess of events at a mass of approximately 125 GeV with a statistical significance of five standard deviations (5σ) above background expectations."



Particle Physics: 5σ

- "The chosen significance value corresponds to a p-value of 0.00003% (1 in 3,500,000)."
- This is the probability that this observation was caused by an accidental accumulation of data at this energy.
- This is extraordinarily small, and with good reason.
- Scientists in Particle Physics do thousands of analyses of all kind of filtered data. Every once in a while you will observe accidental artefacts in your data. 99.7% is not enough ...
- Editors of particle physics journals generally require significance levels of 5σ to claim a detection.