The Square Root of a Positive Integer is the Largest Factor to Check for Primality

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Theorem For any number $n \in \mathbb{Z}^+, n > 2$, the largest factor required for prime determination is $\sqrt{n}$

Proof By definition, a prime number n is only divisible by 1 and n.

$$n = a \cdot b$$

where:

$$a, b \in \mathbb{Z}^+$$

let:

$$a = c_1 \cdot \sqrt{n}$$
$$b = c_2 \cdot \sqrt{n}$$

assume:

$$a, b > \sqrt{n}$$

so

$$c_1, c_2 \in \mathbb{R}, c_1, c_2 > 1$$

then:

$$n = (c_1 \cdot \sqrt{n})(c_2 \cdot \sqrt{n})$$
$$n = c_1 \cdot c_2 \cdot n$$
$$1 = c_1 \cdot c_2$$
$$c_1 = \frac{1}{c_2}$$

substituting into our assumptions:

$$\frac{1}{c_2} > 1$$
$$c_2 < 1$$

$$\therefore\ b < \sqrt{n}, \blacksquare$$