TRIG AND EXPONENTIAL FUNCTIONS — Math 1110

February 01, 2017 — Spring 2017

1 TRIGONOMETRY

(1) Fill in the following table with *exact* values.

θ **(degrees)**	θ **(radians** $0 \leq \theta \leq 2\pi$)	$\sin\theta$	$\cos\theta$	$\tan\theta$
0°				
30°				
45°				
60°				
90°				
120°				
135°				
150°				
180°				
210°				
225°				
240°				
270°				
300°				
315°				
330°				

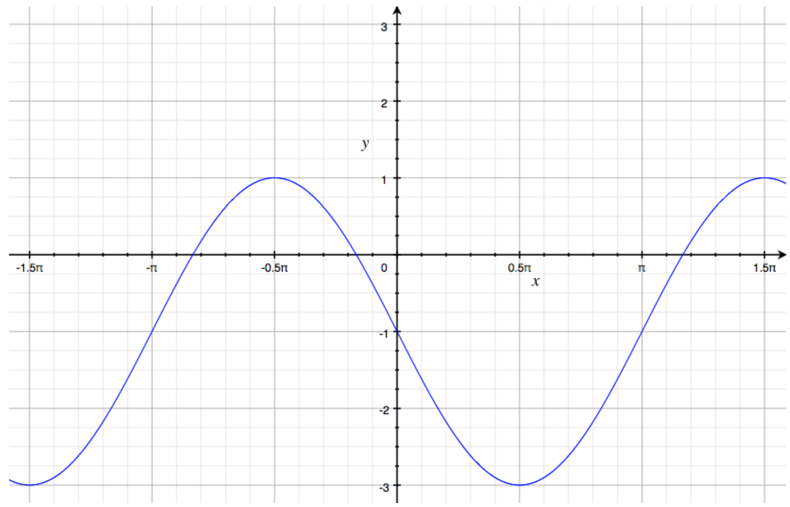
(2) What are the period and amplitude of each of the following functions? Sketch a graph.

(a) $\cos\left(x - \frac{\pi}{2}\right)$

SOLUTION: The period is 2π, and the amplitude is 1. The graph is the same as the graph of $y = \sin(x)$. (!!)

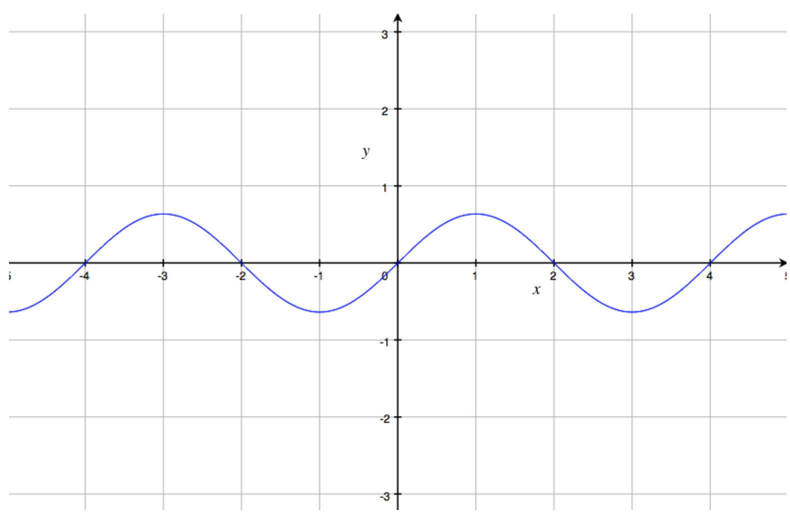
(b) $2\sin(x + \pi) - 1$

SOLUTION: The period is 2π, and the amplitude is 2.



(c) $-\frac{2}{\pi}\sin\left(\frac{\pi}{2}x\right)$

SOLUTION: The period is 4 and the amplitude is $\frac{2}{\pi}$.



(3) Match the left-hand side of each of the following trigonometric identities with the correct right-hand side.

(A)	$\sec^2\theta$	(I)	$\frac{1-\cos 2\theta}{2}$
(B)	$\csc^2\theta$	(II)	$2\sin\theta\cos\theta$
(C)	$\cos(\theta+\phi)$	(III)	$\cos\theta\cos\phi - \sin\theta\sin\phi$
(D)	$\sin(\theta+\phi)$	(IV)	$\sin\theta\cos\phi + \cos\theta\sin\phi$
(E)	$\cos(2\theta)$	(V)	$1+\tan^2\theta$
(F)	$\sin(2\theta)$	(VI)	$\cos^2\theta - \sin^2\theta$
(G)	$\cos^2\theta$	(VII)	$1+\cot^2\theta$
(H)	$\sin^2\theta$	(VIII)	$\frac{1+\cos 2\theta}{2}$

SOLUTION:

(A) (VII)

(B) (V)

(C) (III)

(D) (IV)

(E) (VI)

(F) (II)

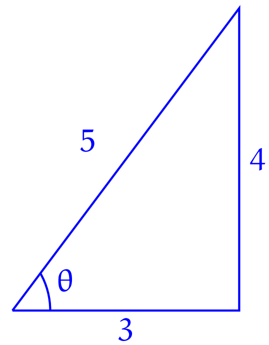
(G) (VIII)

(H) (I)

(4) Find exact values for the following expressions:

(a) $\tan\theta$ when $\sin\theta = 4/5$.

SOLUTION: Draw a triangle!



Then $\tan\theta = \text{opposite}/\text{adjacent} = 4/3$.

(b) $\sin\left(\frac{\pi}{12}\right)$

SOLUTION: Use some trig identities! Notice that

$$\frac{\pi}{3} - \frac{\pi}{4} = \frac{4\pi}{12} - \frac{3\pi}{12} = \frac{1\pi}{12}$$

Then

$$\begin{aligned}
\sin\left(\frac{\pi}{12}\right) &= \sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right) \\
&= \sin\left(\frac{\pi}{3}\right)\cos\left(-\frac{\pi}{4}\right) + \cos\left(\frac{\pi}{3}\right)\sin\left(-\frac{\pi}{4}\right) \\
&= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \frac{1}{2}\left(-\frac{\sqrt{2}}{2}\right) \\
&= \boxed{\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}}
\end{aligned}$$

2 EXPONENTIAL FUNCTIONS

(5) Simplify the following expressions.

(a) $\dfrac{x^2(x^3)^4}{x^4}$ SOLUTION: x^{10}

(b) $9^{\frac{1}{3}} \cdot 9^{\frac{1}{6}}$ SOLUTION: $9^{\frac{1}{3}+\frac{1}{6}}$

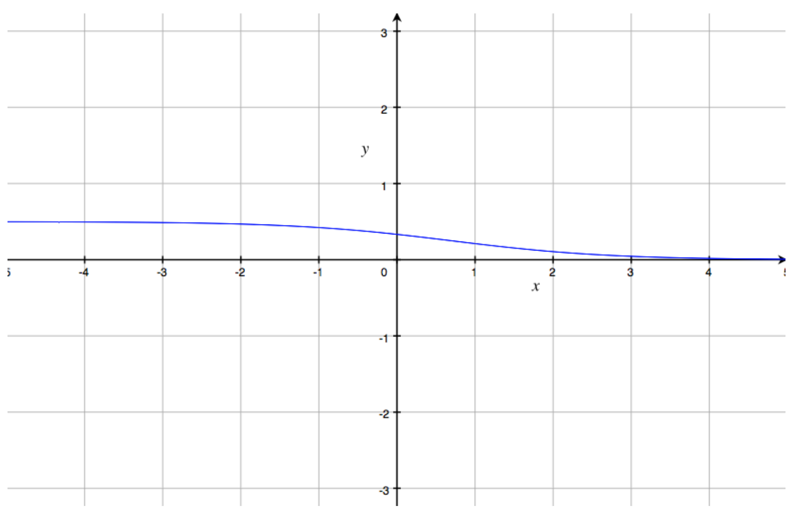
(c) $\left(\sqrt{3}\right)^{\frac{1}{2}} \cdot \left(\sqrt{12}\right)^{\frac{1}{2}}$ SOLUTION: $\sqrt{3} \cdot \sqrt[4]{4}$

(6) What are the domain and range of each of the following functions?

(a) $f(x) = \dfrac{1}{2+e^x}$

SOLUTION: To find the domain, notice that the denominator is never allowed to be zero. When is the denominator zero? When $2+e^x = 0$. But $2+e^x$ is never zero, since e^x is always positive. Hence, the domain is all real numbers.

The range of this function is all possible values it will output. When x gets larger and larger, the values of $f(x)$ get smaller and smaller as $2+e^x$ grows. But it'll never get to zero, so we have a lower bound of 0 (noninclusive). As x goes into the negatives, the denominator gets close to 2, but never reaches it. So the upper bound is $1/2$ (noninclusive). The range is therefore $(0, \frac{1}{2})$. A graph of this function is below.

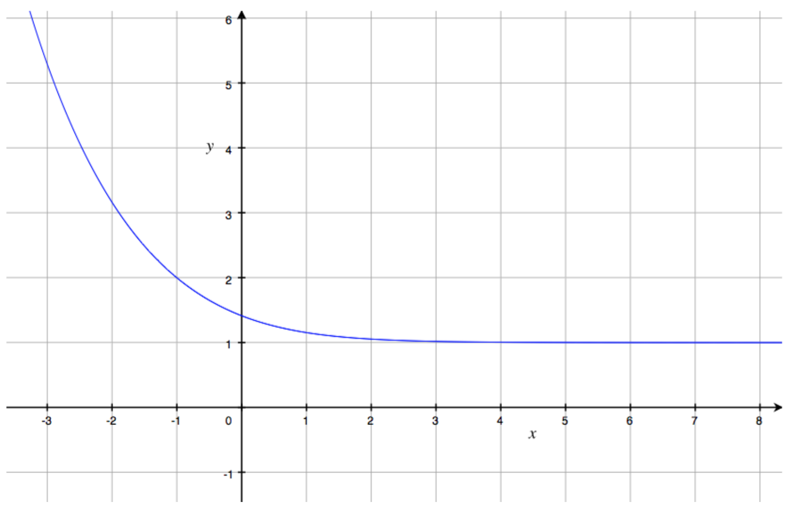


(b) $g(x) = \sqrt{1+3^{-x}}$

SOLUTION: To find the domain, first notice that the stuff under the square root is not allowed to be negative. Hence, we must have $1 + 3^{-x} > 0$. Since $3^{-x} > 0$, this is always true. So the domain is again all real numbers.

To find the range, notice that 3^{-x} is always bigger than zero. Therefore, $1 + 3^{-x}$ is always bigger than 1, so $\sqrt{1 + 3^{-x}}$ is always greater than 1 as well. Hence, the range of g doesn't include any value less than or equal to 1. On the other hand, as x gets far away from zero in the negative direction, $1 + 3^{-x}$ becomes arbitrarily large. Hence, the range of $g(x)$ includes all numbers larger than 1. So the range is $(1, \infty)$.

A graph of this function is below.



(7) The half-life of phosphorus-32 is about 14 days. If there are 6.6 grams present initially, express the amount of phosphorus-32 remaining as a function of time t. When will there be 1 gram remaining?

SOLUTION: The equation for radioactive decay is

$$A(t) = A_0 \left(\frac{1}{2}\right)^{rt}$$

where r is the decay rate and A_0 is the initial amount. The decay rate is $r = \frac{1}{14 \text{ days}}$, and the initial amount is $A_0 = 6.6$ grams. So the equation is

$$A(t) = 6.6 \left(\frac{1}{2}\right)^{t/14}.$$

There will be 1 gram left approximately 38 days later.