CHROMATIC HOMOTOPY NAME: ____________________

These problems are not due and will not be graded.

Reading: [Bea19] or [Rav23].

(1) If R is a commutative ring spectrum and $x \in \pi_n R$, let $R[x^{-1}]$ be the homotopy colimit of

$$R \xrightarrow{x} \Sigma^{-n} R \xrightarrow{x} \Sigma^{-2n} R \xrightarrow{x} \cdots,$$

where by abuse of notation we write $x \colon \Sigma^n R \to R$ for any suspension/desuspension of the map

$$\Sigma^n R = \mathbb{S}^n \wedge R \xrightarrow{x \wedge \mathrm{id}_R} R \wedge R \xrightarrow{\mu} R.$$

Show that $\pi_*(R[x^{-1}]) = (\pi_* R)[x^{-1}]$.

(2) If A is a commutative ring, a regular sequence $a_1, a_2, \ldots$ is a sequence of elements of A such that multiplication by a_n is injective on $A/(a_1, \ldots, a_{n-1})$.

If R is a commutative ring spectrum and $x_1, x_2, \ldots$ is a regular sequence in $\pi_* R$, show that

$$\pi_*(R/(x_1, x_2, \ldots)) \cong (\pi_* R)/(x_1, x_2, \ldots),$$

where $R/(x_1, x_2, \ldots)$ is the homotopy colimit of the spectra $R/(x_1, x_2, \ldots, x_n)$. Recall that R/x_1 is the homotopy cofiber of $x_1 \colon \Sigma^{k_1} R \to R$, and $R/(x_1, x_2)$ is the homotopy cofiber of $x_2 \colon \Sigma^{k_2}(R/x_1) \to R/x_1$, etc.

(3) Recall that the spectra $MU_{(p)}$ and BP with

$$\begin{aligned} \pi_* MU_{(p)} &= \mathbb{Z}_{(p)}[x_1, x_2, \ldots], & |x_i| &= 2i, \\ \pi_* BP &= \mathbb{Z}_{(p)}[v_1, v_2, \ldots], & |v_i| &= 2(p^i - 1). \end{aligned}$$

Assuming that $MU_{(p)}$ splits as a wedge sum of copies of BP, show that there must be infinitely many copies of BP in this wedge sum.

(4) Use the chromatic fracture square to show that if X is $E(n)$-local, then X/p is $K(n)$-local.

REFERENCES

[Bea19] Agnes Beaudry. An introduction to chromatic homotopy theory. eCHT Minicourse, https://s.wayne.edu/echt/echt-minicourses/, 2019.

[Rav23] Doug Ravenel. The background and motivation for the telescope conjecture. eCHT Minicourse, https://s.wayne.edu/echt/echt-minicourses/, 2023.

Credit for all problems to Bert Guillou.