

These problems are not due and will not be graded.

Reading: [Bea19] or [Rav23].

- (1) If R is a commutative monoid in $\mathrm{ho}(\mathcal{S}p)$ and $x \in \pi_n R$, let $R[x^{-1}]$ be the homotopy colimit of

$$R \xrightarrow{x} \Sigma^{-n} R \xrightarrow{x} \Sigma^{-2n} R \xrightarrow{x} \dots,$$

where by abuse of notation we write $x: \Sigma^n R \rightarrow R$ for any suspension/desuspension of the map

$$\Sigma^n R = S^n \wedge R \xrightarrow{x \wedge \mathrm{id}_R} R \wedge R \xrightarrow{\mu} R.$$

Show that $\pi_*(R[x^{-1}]) = (\pi_* R)[x^{-1}]$.

- (2) If A is a commutative ring, a regular sequence a_1, a_2, \dots is a sequence of elements of A such that multiplication by a_n is injective on $A/(a_1, \dots, a_{n-1})$.

If R is a commutative monoid in $\mathrm{ho}(\mathcal{S}p)$ and x_1, x_2, \dots is a regular sequence in $\pi_* R$, show that

$$\pi_*(R/(x_1, x_2, \dots)) \cong (\pi_* R)/(x_1, x_2, \dots),$$

where $R/(x_1, x_2, \dots)$ is the homotopy colimit of the spectra $R/(x_1, x_2, \dots, x_n)$.

- (3) Assuming that $MU_{(p)}$ splits as a wedge sum of copies of BP , show that there must be infinitely many copies of BP in this wedge sum.
- (4) Use the chromatic fracture square to show that if X is $E(n)$ -local, then X/p is $K(n)$ -local.

REFERENCES

- [Bea19] Agnes Beaudry. An introduction to chromatic homotopy theory. eCHT Minicourse, <https://s.wayne.edu/echt/echt-minicourses/>, 2019.
- [Rav23] Doug Ravenel. The background and motivation for the telescope conjecture. eCHT Minicourse, <https://s.wayne.edu/echt/echt-minicourses/>, 2023.

Credit for all problems to Bert Guillou.