One-Page Review

§6.1, §6.2, §6.3 (Areas, Volumes, Revolution)

MATH 1910 Recitation
September 20, 2016

(1) The graph of $x = f(y)$ is the graph of $y = f(x)$ reflected across the line ________ (1).

(2) The area between $f(x)$ and $g(x)$ from a to b is ________ (2) .

(3) The ________ (3) of $f(x)$ over the interval $[a, b]$ is ________ (4) .

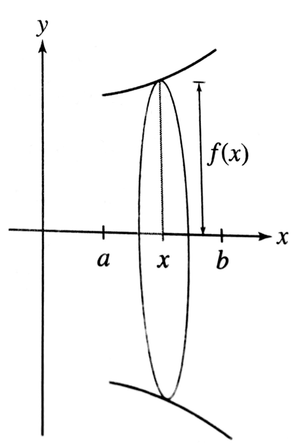
(4) The **Mean Value Theorem for Integrals** says that if f is continuous on $[a, b]$ with mean value M, then there is some $c \in [a, b]$ such that ________ (5) .

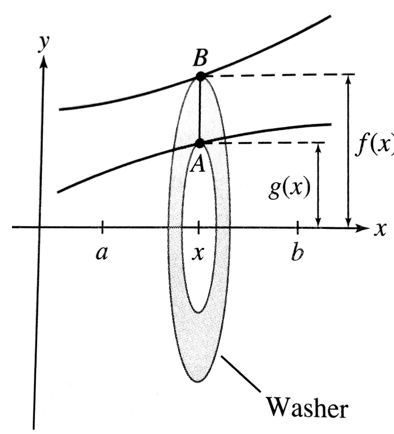
(5) If a shape has cross-sectional area $A(y)$ and height extends from $y = a$ to $y = b$, then it's volume is ________ (6) .

(6) **Cavilieri's Principle** says if two solids have equal ________ (7), then they also have equal ________ (8) .

(7) **The Disk Method:** If $f(x) \geq 0$ on $[a, b]$, then the solid obtained by rotating the region under the graph around the x-axis has volume ________ (9) .

(8) **The Washer Method:** If $f(x) \geq g(x) \geq 0$ on $[a, b]$, then the solid obtained by rotating the region between $f(x)$ and $g(x)$ around the x-axis has volume ________ (10) .





PRACTICE PROBLEMS

§6.1, §6.2, §6.3 (Areas, Volumes, Revolution)

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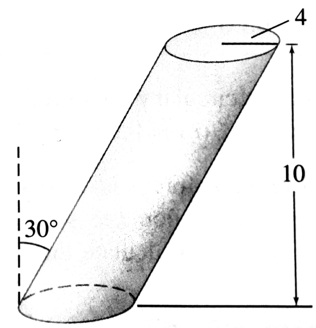
(1) Sketch the region enclosed by the curves and set up an integral to compute it's area, but do not evaluate.

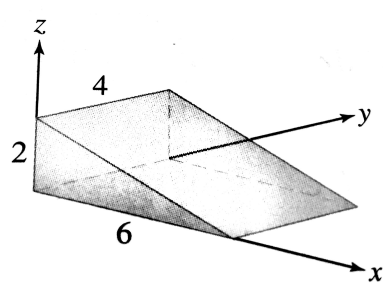
   (a) $y = 4 - x^2$, $y = x^2 - 4$
   (b) $y = x^2 - 6$, $y = 6 - x^3$, $x = 0$
   (c) $y = x\sqrt{x-2}$, $y = -x\sqrt{x-2}$, $x = 4$
   (d) $x = 2y$, $x + 1 = (y-1)^2$
   (e) $y = \cos x$, $y = \cos(2x)$, $x = 0$, $x = \frac{2\pi}{3}$

(2) Calculate the volume of a cylinder inclined at an angle $\theta = \frac{\pi}{6}$ with height 10 and base of radius 4.



(3) Calculate the volume of the ramp in the figure below in three ways by integrating the area of the cross sections:

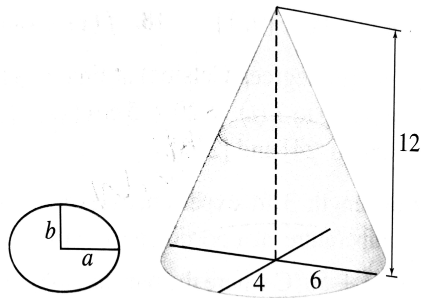
   (a) perpendicular to the x-axis.
   (b) perpendicular to the y-axis.
   (c) perpendicular to the z-axis.



(4) Let M be the average value of $f(x) = 2x^2$ on $[0, 2]$. Find a value c such that $f(c) = M$.

(5) Find the flow rate through a tube of radius 2 meters, if it's fluid velocity at distance r meters from the center is $v(r) = 4 - r^2$.

(6) Compute the volume of a cone of height 12 whose base is an ellipse with semimajor axis $a = 6$ and semiminor axis $b = 4$.



(7) Sketch the region enclosed by the curves, and determine the cross section perpendicular to the x-axis. Set up an integral for the volume of revolution obtained by rotating the region around the x-axis, but do not evaluate.

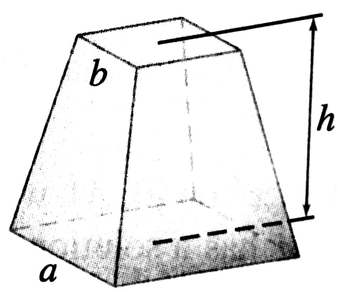
(a) $y = x^2 + 2$, $y = 10 - x^2$.

(b) $y = 16 - x$, $y = 3x + 12$, $x = -1$.

(c) $y = \frac{1}{x}$, $y = \frac{5}{2} - x$.

(d) $y = \sec x$, $y = 0$, $x = -\frac{\pi}{4}$, $x = \frac{\pi}{4}$.

(8) A frustrum of a pyramid is a pyramid with it's top cut off. Let V be the volume of a frustrum of height h whose base is a square of side a and whose top is a square of side b with $a > b > 0$.



(a) Show that if the frustrum were continued to a full pyramid (i.e. the top wasn't cut off), it would have height $ha/(a-b)$.

(b) Show that the cross sectional area at height x is a square of side $(1/h)(a(h-x)+bx)$.

(c) Show that $V = \frac{1}{3}h(a^2 + ab + b^2)$.