TRIANGULATED CATEGORIES NAME: ____________________

Due at the beginning of class on 25 March 2025

- *Your answers should be neatly written and logically organized.*
- *You may collaborate on solving the problems, but the solutions you turn in should be your own.*
- *You may use any resource you find online (or elsewhere), but you must cite any resource you use.*

Reading: [Mal23, Section 3.2] and [Wei94, Section 10.2]

(1) A spectrum X is *rational* if each of its homotopy groups is an $\mathbb{Q}$-vector space. Let $\mathrm{ho}(\mathcal{S}\mathrm{p})_{\mathbb{Q}}$ be the full subcategory of $\mathrm{ho}(\mathcal{S}\mathrm{p})$ whose objects are rational spectra. Prove that $\mathrm{ho}(\mathcal{S}\mathrm{p})_{\mathbb{Q}}$ is a triangulated subcategory of $\mathrm{ho}(\mathcal{S}\mathrm{p})$.

(2) Define a triangulated functor $H\colon \mathcal{D}(\mathbb{Q}) \to \mathrm{ho}(\mathcal{S}\mathrm{p})$ such that $\pi_n H(V_\bullet) = H_n(V_\bullet)$ for all $n \in \mathbb{Z}$. *Hint: any chain complex of $\mathbb{Q}$-vector spaces is quasi-isomorphic to its homology.*

(3) (a) Show that in a triangulated category $\mathcal{C}$, all monomorphisms are split monomorphisms.
    (b) Show that the triangulated category $\mathcal{D}(\mathbb{Z})$ is not an abelian category.
    (c) Show that $\mathrm{ho}(\mathcal{S}\mathrm{p})$ is not an abelian category.

REFERENCES

[Mal23] Cary Malkiewich. Spectra and stable homotopy theory. `http://people.math.binghamton.edu/malkiewich/spectra_book_draft.pdf`, October 2023.

[Wei94] Charles A. Weibel. *An introduction to homological algebra*, volume 38 of *Cambridge Studies in Advanced Mathematics*. Cambridge University Press, Cambridge, 1994.