SPECTRAL SEQUENCES NAME: ______________________

These problems are not due and will not be graded.

Reading: Read [Hat17] for an introduction to spectral sequences. Read [BC18] for information on the Adams spectral sequence.

(1) In this problem, we prove the following fact:

$$H^*(K(\mathbb{Z},n);\mathbb{Q}) = \begin{cases} \mathbb{Q}[x_n] & (n \text{ even}) \\ E_{\mathbb{Q}}(x_n) & (n \text{ odd}), \end{cases}$$

where $|x_n| = n$ and $E_{\mathbb{Q}}(x_n) \cong \mathbb{Q}[x_n]/(x_n^2)$ is an exterior $\mathbb{Q}$-algebra on a single generator in degree n.

(a) Compute $H^k(K(\mathbb{Z},n);\mathbb{Q})$ for $k \leq n$ without spectral sequences.

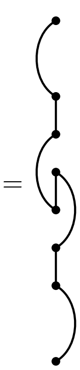
(b) Use induction and the Serre spectral sequence for the fiber sequence

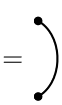
$$K(\mathbb{Z},n-1) \to PK(\mathbb{Z},n) \to K(\mathbb{Z},n)$$

to verify the formula given.

(2) Let $\mathcal{A}_1$ be the subalgebra of the Steenrod algebra $\mathcal{A}$ generated by Sq^1 and Sq^2. A depiction of $\mathcal{A}_1$ as a module over itself appears in [BC18, Figure 3]. For each of the following $\mathcal{A}_1$-modules M, draw the first few stages of a projective resolution of M and write an Adams chart for $\mathrm{Ext}_{\mathcal{A}}^{s,t}(M,\mathbb{Z}/2)$.

(a) $M_0 =$ 

(b) $M_1 =$ 

(c) $\Sigma^{-2}H^*(\mathbb{C}P^\infty;\mathbb{Z}/2) =$ 

REFERENCES

[BC18] Agnès Beaudry and Jonathan A. Campbell. A guide for computing stable homotopy groups. In *Topology and quantum theory in interaction*, volume 718 of *Contemp. Math.*, pages 89–136. Amer. Math. Soc., Providence, RI, 2018.

[Hat17] Allen Hatcher. Spectral Sequences. `https://pi.math.cornell.edu/~hatcher/AT/SSpage.html`, 2017.