QUIZ 2

Name: ____________________

Math 1110, Section 2
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(1) What is the definition of the derivative of a function $f(x)$ at the point $x = a$? Give a precise mathematical statement.

SOLUTION: The derivative of $f(x)$ at the point $x = a$ is

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

(2) Find an equation for the tangent line to the curve $y = \frac{1}{x^2}$ at the point $(-1, 1)$.

SOLUTION: The slope of the tangent line is the derivative of the function at $x = -1$. Set $f(x) = \frac{1}{x^2}$. Then

$$\begin{aligned}
f'(-1) &= \lim_{h \to 0} \frac{f(-1+h) - f(-1)}{h} \\
&= \lim_{h \to 0} \frac{\frac{-1}{(h-1)^2} - \frac{1}{(-1)^2}}{h} \\
&= \lim_{h \to 0} \frac{1}{h} \left(\frac{-1}{(h-1)^2} - 1 \right) \\
&= \lim_{h \to 0} \frac{1}{h} \left(\frac{-1 - (h-1)^2}{(h-1)^2} \right) \\
&= \lim_{h \to 0} \frac{1}{h} \left(\frac{-1 - (h^2 - 2h + 1)}{(h-1)^2} \right) \\
&= \lim_{h \to 0} \frac{1}{h} \left(\frac{-h^2 + 2h}{(h-1)^2} \right) \\
&= \lim_{h \to 0} \frac{-h + 2}{(h-1)^2} \\
&= 2
\end{aligned}$$

So the slope of the tangent line is $m = 2$. The line passes through the point $(-1, 1)$, so we have a slope and a point. This determines the equation of a line. The answer is

$$y = 2(x+1) + 1 = 2x + 3.$$