

Can You Spot The Error?

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$$\begin{aligned} S &:= \sum_{n=0}^{\infty} 2^n && \# \text{ define } S \\ &= 2^0 + 2^1 + 2^2 + 2^3 + 2^4 + \cdots && \# \text{ expand the sum} \\ &= 1 + 2 + 4 + 8 + 16 + \cdots && \# \text{ simplify} \\ &= 1 + 2 \cdot (1 + 2 + 4 + 8 + 16 + \cdots) && \# \text{ factor out } 2 \\ &= 1 + 2S && \# S = 1 + 2 + 4 + 8 + 16 + \cdots \\ \Rightarrow S - 2S &= 1 && \# \text{ solve for } S \\ \Rightarrow -S &= 1 && \# S - 2S = -S \\ \Rightarrow \textcolor{red}{S} &= \textcolor{red}{-1} && \# \text{ what went wrong?} \end{aligned}$$