

A Few Notes on Special Relativity

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1 Introduction

TBD

2 The Postulates of Special Relativity

1. The laws of physics are the same for all observers in uniform motion relative to each other (i.e., in inertial reference frames). This means that no inertial observer can detect a preferred or absolute state of motion. There is no absolute "rest" frame; all motion is relative.
2. The speed of light in a vacuum is always constant, regardless of the motion of the observer or the light source. This speed is approximately $c = 299,792,458$ m/s.

3 Time Dilation

Time dilation is an unusual phenomenon predicted by Einstein's special theory of relativity. It refers to the effect where time appears to pass more slowly for an observer in motion relative to a stationary observer. In simpler terms, a moving clock ticks more slowly compared to a clock that is at rest, as observed by someone who is not moving with the clock [3].

The key idea is that when an object moves at a speed close to the speed of light relative to an observer, time for the moving object (as seen by the observer) slows down. This effect becomes more pronounced as the speed of the object approaches the speed of light.

Said another way, special relativity tells us that for an observer O in an inertial frame of reference, a clock that is moving relative to the observer will be measured to tick more slowly than a clock at rest in the observer's frame of reference [5].

To see why this happens, consider the following thought experiment. The experimental setup is shown in Figure 1. ¹ Here the observer, light source and detector are at rest in inertial frame of reference I (even though the train car is moving with velocity v).

¹Assume here that the thickness of the source and the detector are negligible so that the distance light travels is h .

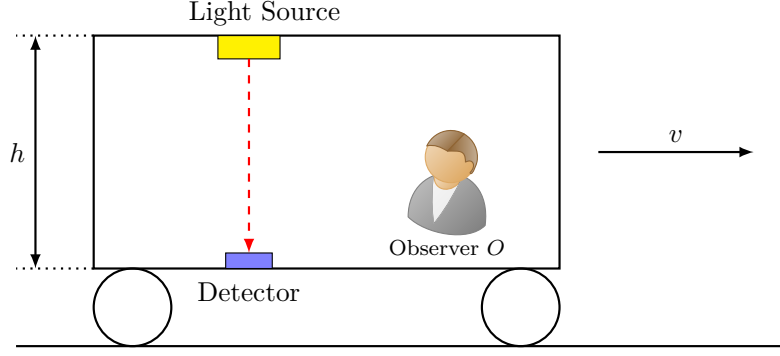


Figure 1: Observer O at rest with respect to a light source and detector

For observer at rest with respect to an inertial frame of reference, the time taken for a beam of light to travel from the light source to the detector is

$$\begin{aligned}
 h &= ct && \# t \text{ is called the proper time [4]} \\
 \Rightarrow t &= \frac{h}{c} && \# \text{ solve for } t
 \end{aligned}$$

Now consider the same experimental setup as in Figure 1 except this time the observer is on the train landing. That is, the observer's frame of reference is at rest with respect to the train car's frame of reference (which is moving with velocity v).²

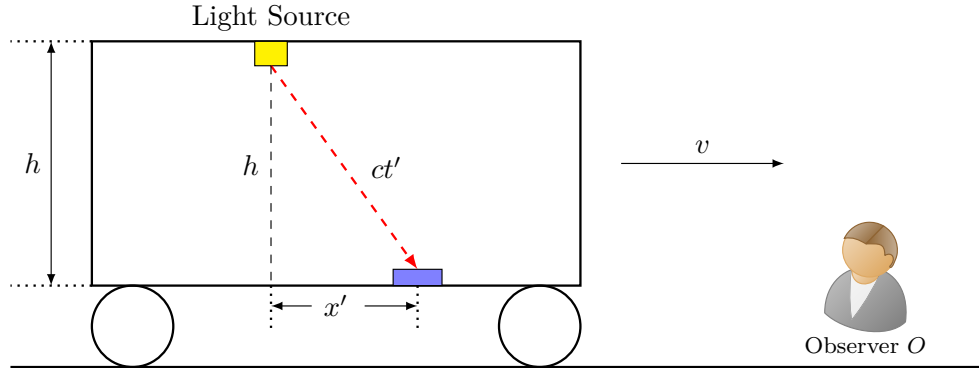


Figure 2: Observer O at rest with respect to another inertial frame of reference

The question now is how much time, call it t' , passes from the point of view of observer O when the light travels from the source to the detector. x' is the distance the train moves forward (from the origin) in time t' . We also know that the speed of light in a vacuum is c for all observers, so the distance that light travels (from the source to the detector) in this case is ct' (the hypotenuse of the triangle shown in Figure 2).

Remark 3.1. Note that in general, the coordinates in the "stationary" frame of reference (call it S) are referred to as (t, x, y, z) , while the corresponding coordinates in the "moving" frame of reference (call it S') are denoted (t', x', y', z') . Coordinates in S' are related to coordinates in S by the Lorentz transformations [1, 2].

²Note that the velocity v here is a the real constant representing a velocity confined to the x-direction.

To compute t' , consider the $h - x' - ct'$ triangle shown in Figure 2, which is abstracted in Figure 3.

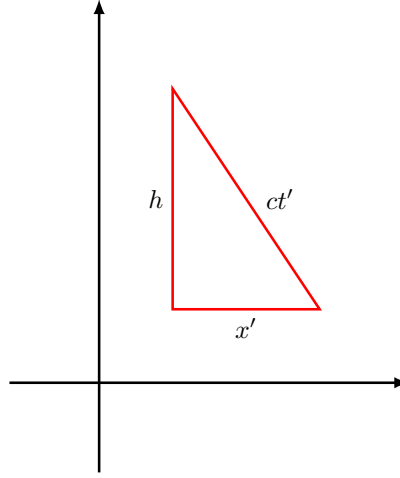


Figure 3: Triangle formed by the motion of the train with respect to the observer O

Using this triangle we can calculate t' using the Pythagorean theorem:

$$\begin{aligned}
(ct')^2 &= h^2 + x'^2 && \# \text{ by the Pythagorean theorem} \\
\Rightarrow (ct')^2 &= (ct)^2 + x'^2 && \# h = ct \\
\Rightarrow (ct')^2 &= (ct)^2 + (vt')^2 && \# x' = vt' \\
\Rightarrow c^2 t'^2 &= c^2 t^2 + v^2 t'^2 && \# \text{ square terms} \\
\Rightarrow t'^2 &= t^2 + \frac{v^2}{c^2} t'^2 && \# \text{ divide both sides by } c^2 \\
\Rightarrow t'^2 - \frac{v^2}{c^2} t'^2 &= t^2 && \# \text{ subtract } \frac{v^2}{c^2} t'^2 \text{ from both sides} \\
\Rightarrow t'^2 \left(1 - \frac{v^2}{c^2} \right) &= t^2 && \# \text{ factor out } t'^2 \\
\Rightarrow t'^2 &= \frac{t^2}{\left(1 - \frac{v^2}{c^2} \right)} && \# \text{ divide both sides by } \left(1 - \frac{v^2}{c^2} \right) \\
\Rightarrow t' &= \frac{t}{\sqrt{1 - \frac{v^2}{c^2}}} && \# \text{ take the square root of both sides}
\end{aligned}$$

So for observer O the passage of time on the moving train slows down by a factor of $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$.

γ is called the Lorentz factor (or sometimes the gamma factor). Note also that t' can be written using the Lorentz factor: $t' = \gamma t$. The Lorentz factor is sometimes written $\gamma(v)$ or γ_v to emphasize v .

4 Length Contraction

5 The Lorentz Transformations

6 The Andromeda Paradox

7 Spacetime Invariance

8 Conclusions

Acknowledgements

L^AT_EX Source

<https://www.overleaf.com/read/xfghcdxjyzhg#675841>

References

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