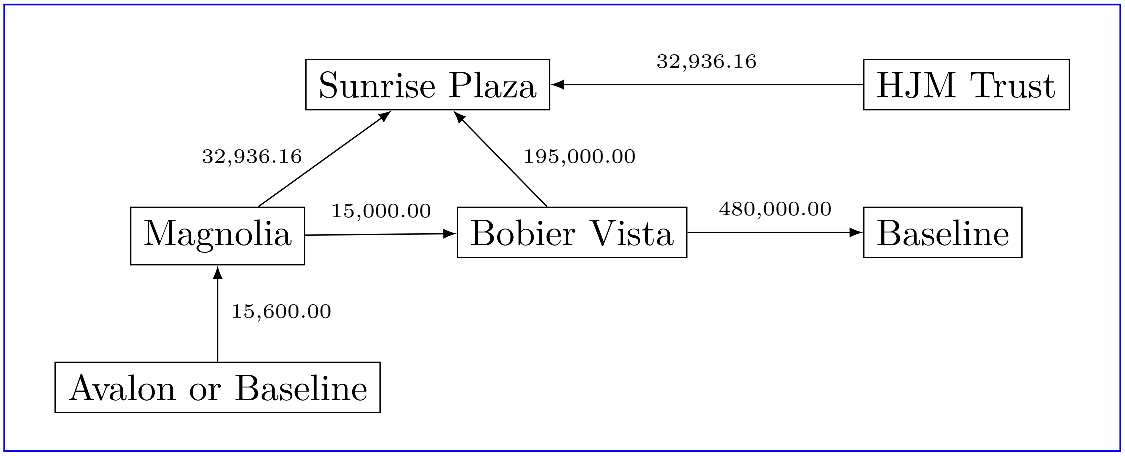

Figure 1: HJM Trust Loan Graph

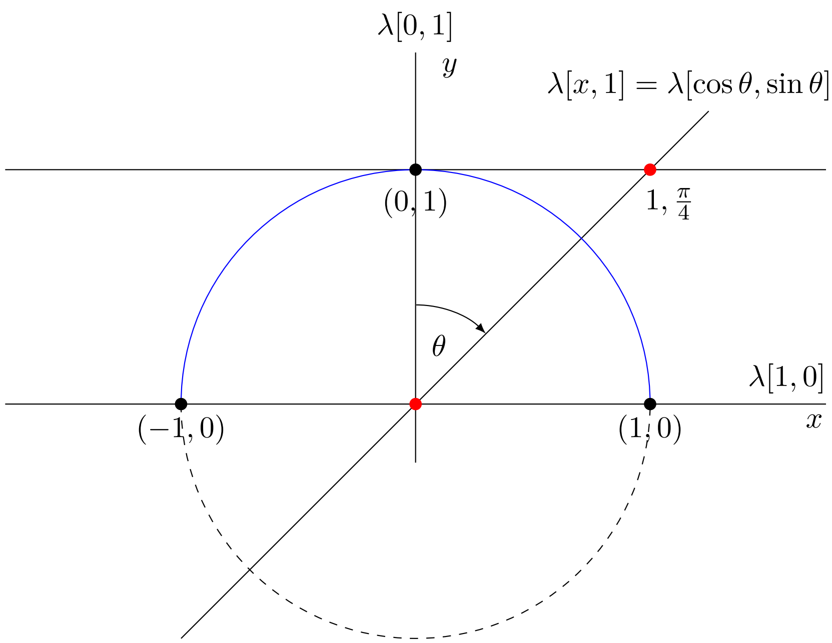

Figure 2: Real Projective Line Setup

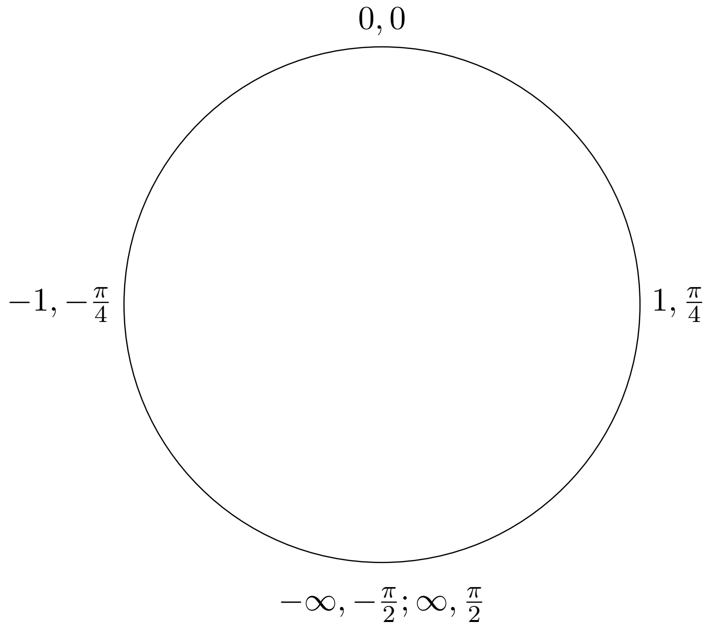

Figure 3: The Real Projective Line is a Circle: (x, θ)

What	Linear Algebra	Fourier Series
Vector Space	$\mathbb{R}^n$	Piecewise smooth 2π-periodic functions on $\mathbb{R}$
Inner Product	$\langle \mathbf{u}, \mathbf{v} \rangle = \sum_{i=1}^{n} u_i v_i$	$\langle f(t), g(t) \rangle = \frac{1}{\pi} \int_{-\pi}^{\pi} f(t)g(t)\,dt$
Orthonormal Basis $\left(\mathbb{R}^3\right)$	$\{(1,0,0),(0,1,0),(0,0,1)\}$	$\{1, \cos mt, \sin nt\},\ n, m \in \mathbb{N}\backslash\{0\}$
Representation of a Vector in the Basis	$\mathbf{x} = \sum_{i=1}^{n} a_i \mathbf{e}_i$	$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nt + \sum_{n=1}^{\infty} b_n \sin nt$
Coefficients are Projections	$a_i = \langle \mathbf{x}, \mathbf{e}_i \rangle$	$a_0 = \langle f(t), 1 \rangle$ $a_m = \langle f(t), \cos mt \rangle$ $b_m = \langle f(t), \sin mt \rangle$

Table 1: Vector Spaces: Linear Algebra vs. Fourier Series

Note: Some people think that the orthonormal basis for the Fourier series should be $\{\mathbf{1}, \sqrt{2}\cos mt, \sqrt{2}\sin nt\}$, since

$$\langle \mathbf{1}, \mathbf{1} \rangle = \frac{1}{\pi}\int_{-\pi}^{\pi} \mathbf{1}^2\,dt = \frac{1}{\pi}\int_{-\pi}^{\pi} dt = \frac{1}{\pi}\,t\Big|_{-\pi}^{\pi} = \frac{1}{\pi}\left(\pi - (-\pi)\right) = \frac{1}{\pi}2\pi = 2$$

OTOH, $\langle \cos nt, \cos nt \rangle = \frac{1}{\pi}\int_{-\pi}^{\pi} \cos^2 nt\,dt = \frac{1}{\pi}\pi = 1$. In the same way, $\langle \sin nt, \sin nt \rangle = 1$.

One of the points here (or perhaps, the point here) is that $\langle \hat{\mathbf{u}}, \hat{\mathbf{u}} \rangle = 1$ for all unit vectors $\hat{\mathbf{u}}$, so what about $\langle \mathbf{1}, \mathbf{1} \rangle$ if $\mathbf{1}$ is part of an orthonormal basis (again, since $\langle \mathbf{1}, \mathbf{1} \rangle = 2$)?

Note also that here $\langle \mathbf{1}, \sin nt \rangle = \frac{1}{\pi}\int_{-\pi}^{\pi} \mathbf{1}\sin nt\,dt = \frac{1}{\pi}\int_{-\pi}^{\pi} \sin nt\,dt = 0$. Similarly, $\langle \mathbf{1}, \cos nt \rangle = \langle \cos nt, \sin mt \rangle = 0$.

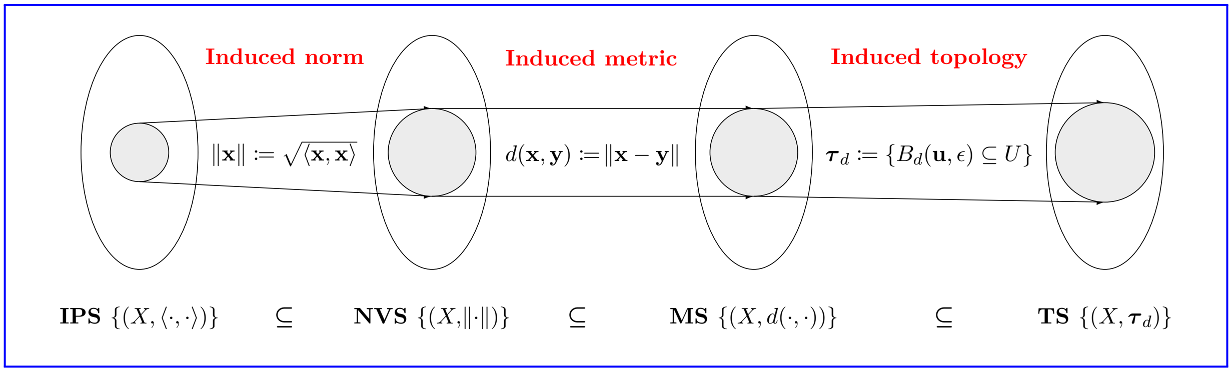

Figure 4: Induced Norm, Metric, and Topology

IPS	$--$	Inner Product Spaces	$(X, \langle \mathbf{x}, \mathbf{y} \rangle)$
NVS	$--$	Normed Vector Spaces	$(X, \|\mathbf{x}\|) = (X, \sqrt{\langle \mathbf{x}, \mathbf{x} \rangle})$
MS	$--$	Metric Spaces	$(X, d(\mathbf{x}, \mathbf{y})) = (X, \|\mathbf{x} - \mathbf{y}\|) = (X, \sqrt{\langle \mathbf{x} - \mathbf{y}, \mathbf{x} - \mathbf{y} \rangle})$
TS	$--$	Topological Spaces	$(X, \boldsymbol{\tau}_d)$, where $\boldsymbol{\tau}_d = \{\text{open subsets of } (X, d)\}$

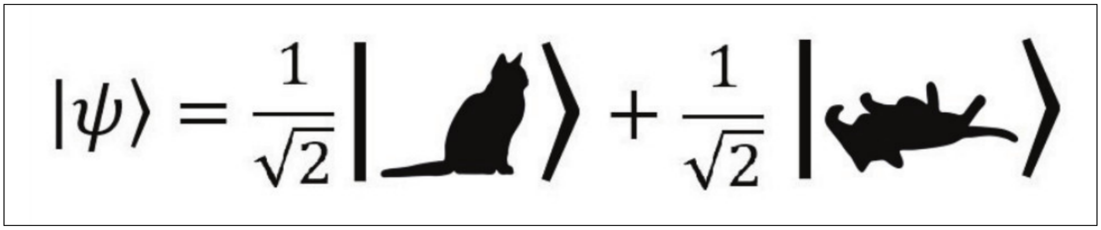

Figure 5: Quantum superposition and Schrödinger's cat

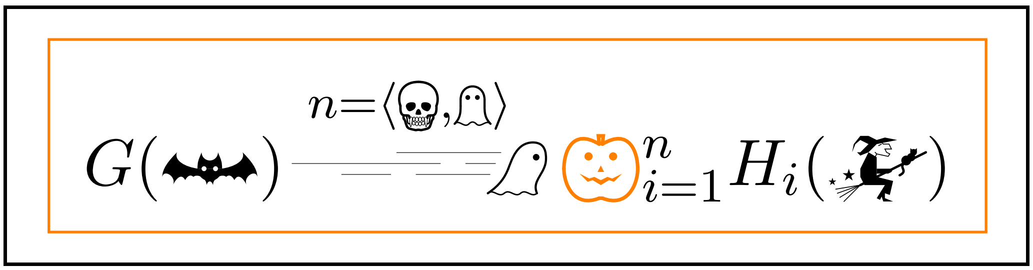

Figure 6: Happy Halloween!

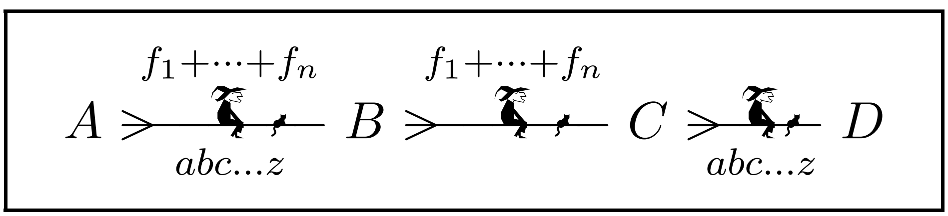

Figure 7: Happy Halloween!

0.6249999121401877

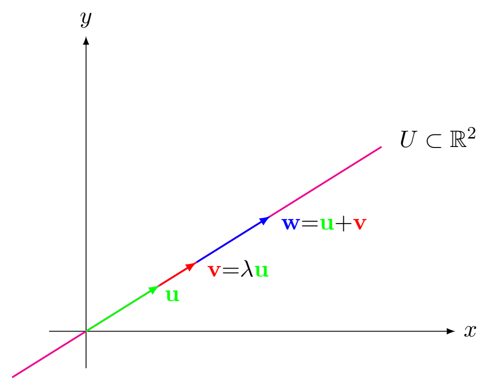

Figure 8: The line U is a linear subspace of $\mathbb{R}^2$

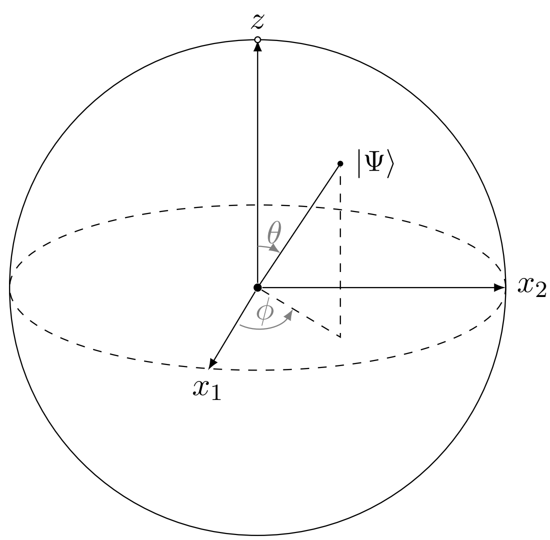

Figure 9: The Bloch Sphere

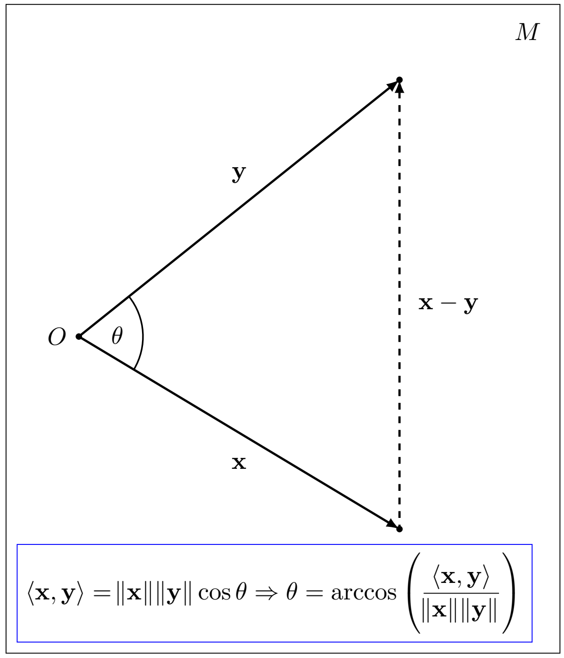

Figure 10: Geometric interpretation of $\langle \mathbf{x}, \mathbf{y} \rangle$

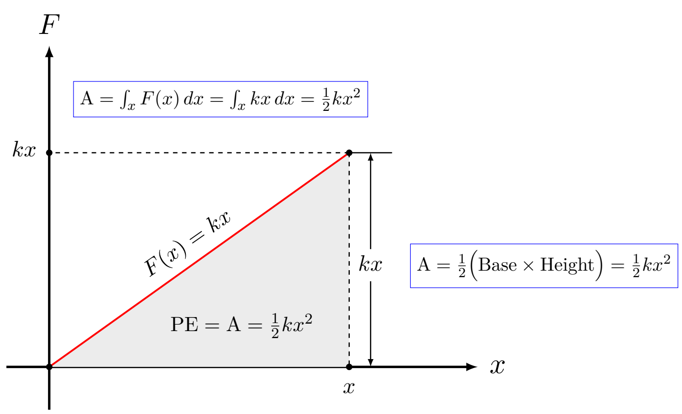

Figure 11: Potential Energy of a Simple Harmonic Oscillator

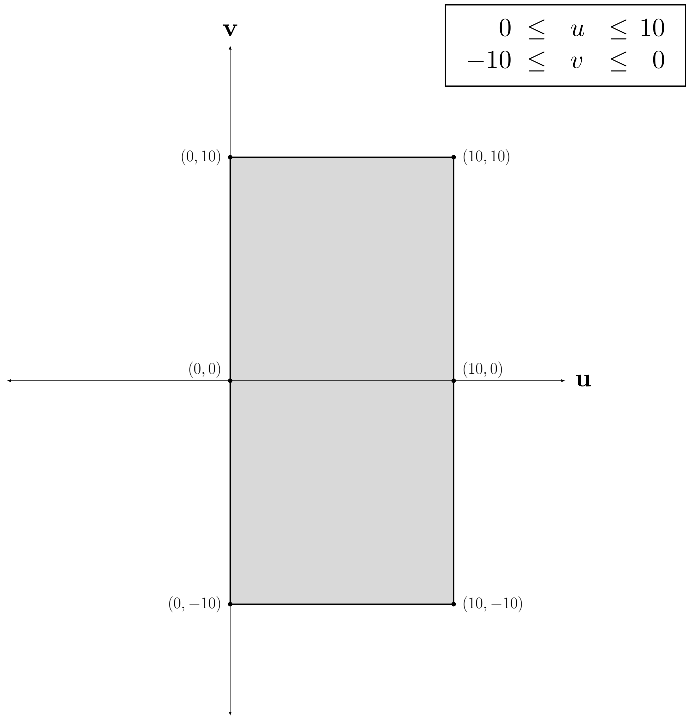

Figure 12: The transformed parallelogram is a rectangle in the uv-plane

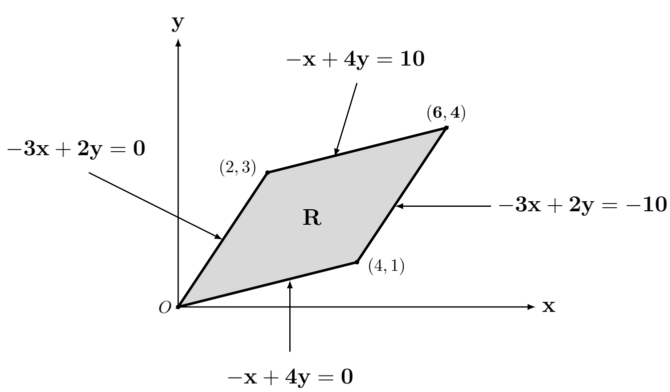

Figure 13: Equations of the Sides of the Parallelogram

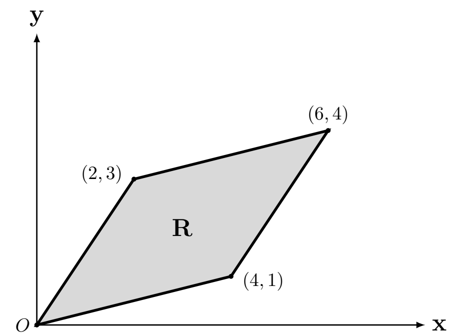

Figure 14: Parallelogram in the xy-plane

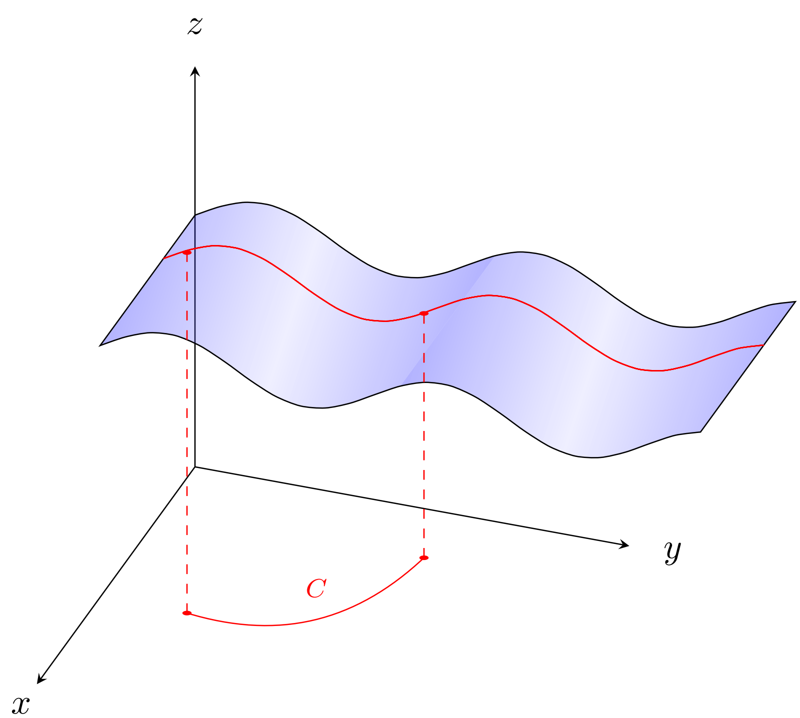

Figure 15: 3D

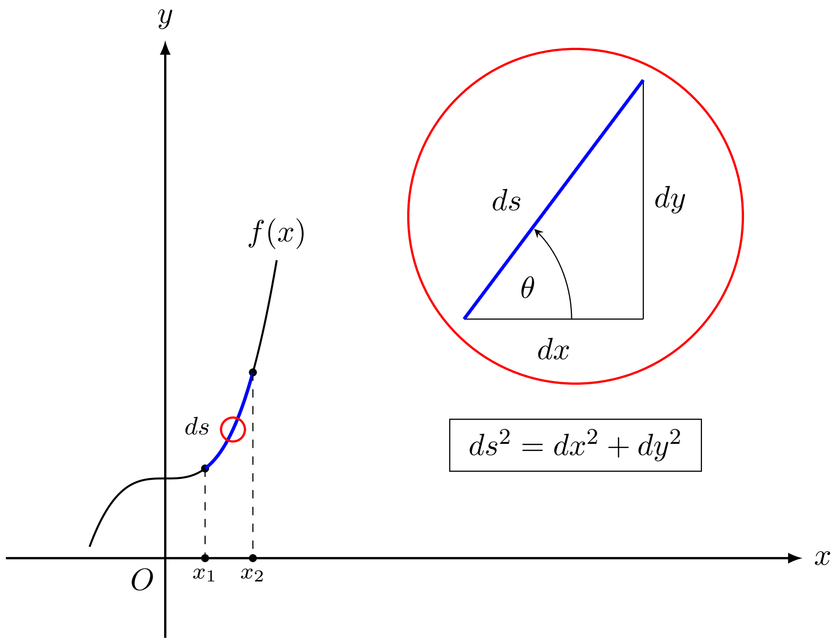

Figure 16: $f(x)$, ds and the Pythagorean Theorem

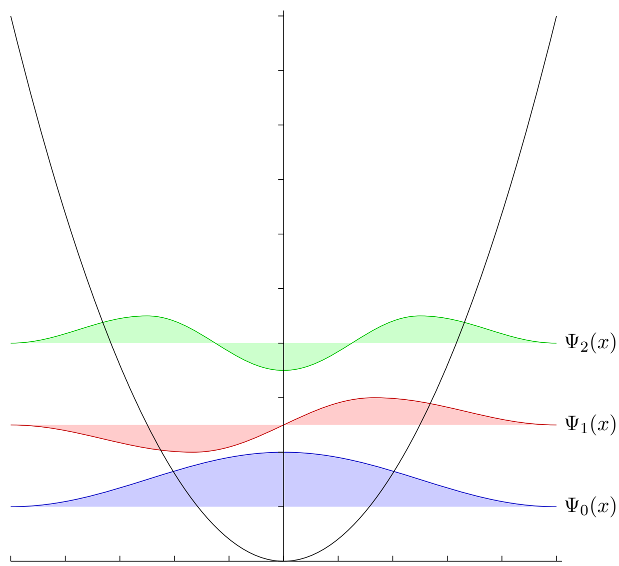

Figure 17: Wave Functions

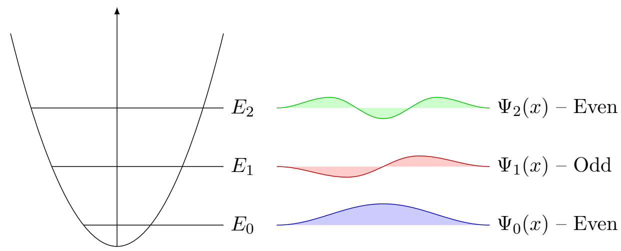

Figure 18: Wave Functions

$$
\boxed{
\begin{array}{rcrcrcr}
9^2 & = & 81 & \longrightarrow & 8+1 & = & 9 \\
45^2 & = & 2025 & \longrightarrow & 20+25 & = & 45 \\
703^2 & = & 494209 & \longrightarrow & 494+209 & = & 703 \\
7777^2 & = & 60481729 & \longrightarrow & 6048+1729 & = & 7777 \\
857143^2 & = & 734694122449 & \longrightarrow & 734694+122449 & = & 857143
\end{array}
}
$$

Figure 19: A Few Example Kaprekar Numbers

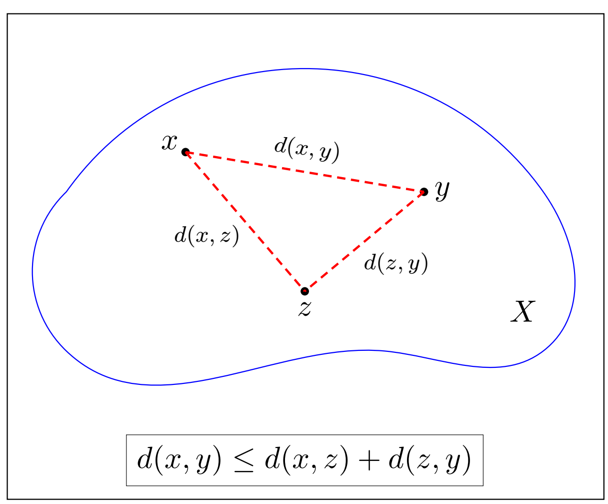

Figure 20: The Triangle Inequality

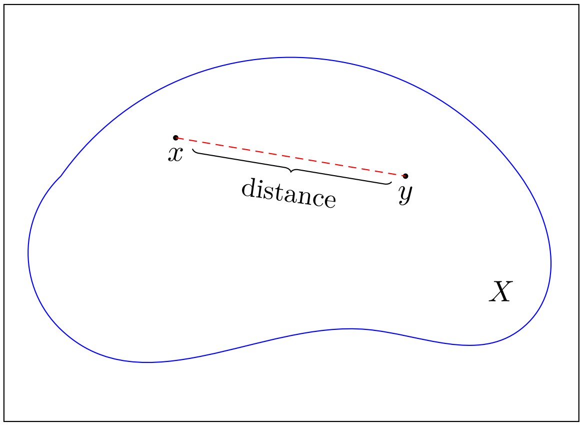

Figure 21: X, x and y

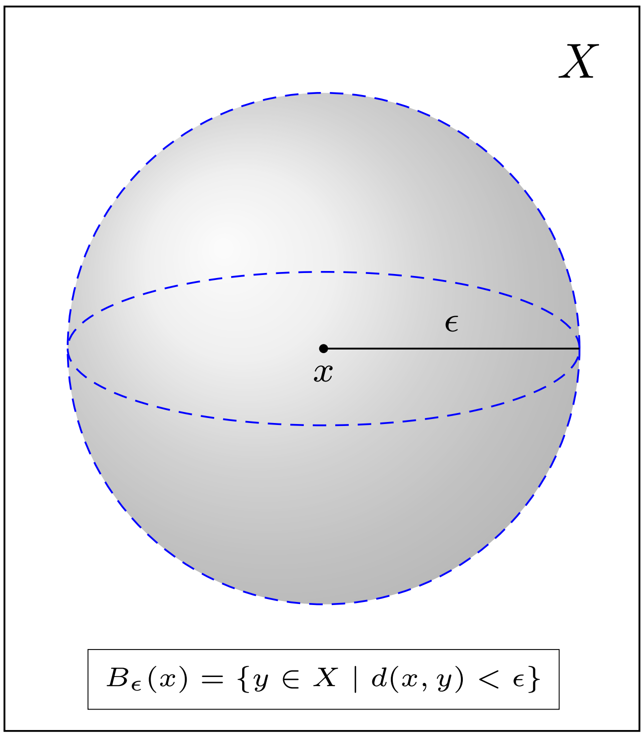

Figure 22: $B_\epsilon(x)$ is an open epsilon ball centered at x with radius ϵ

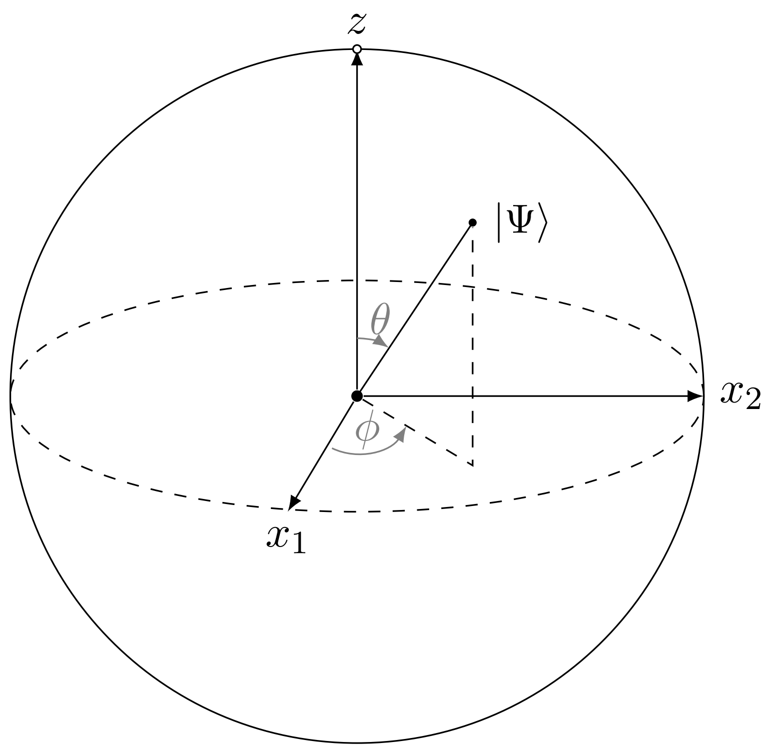

Figure 23: The Bloch Sphere

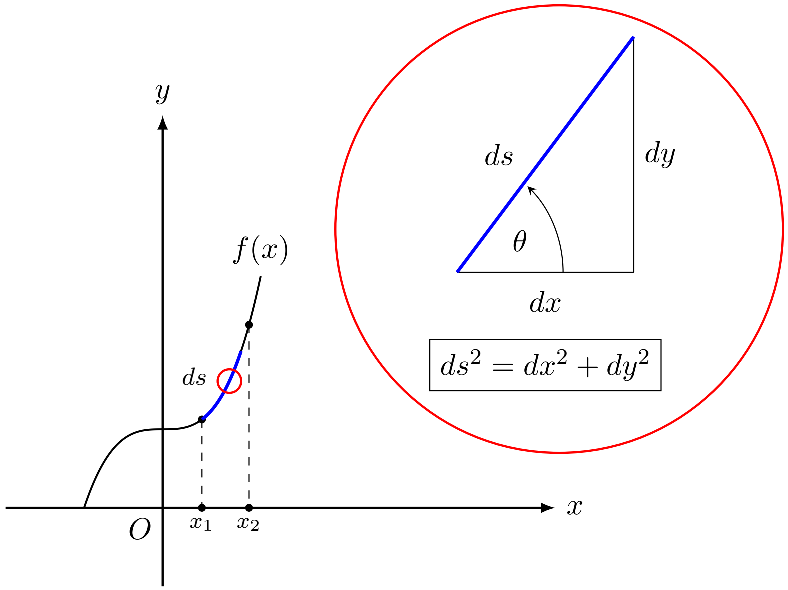

Figure 24: $f(x)$, ds and the Pythagorean Theorem

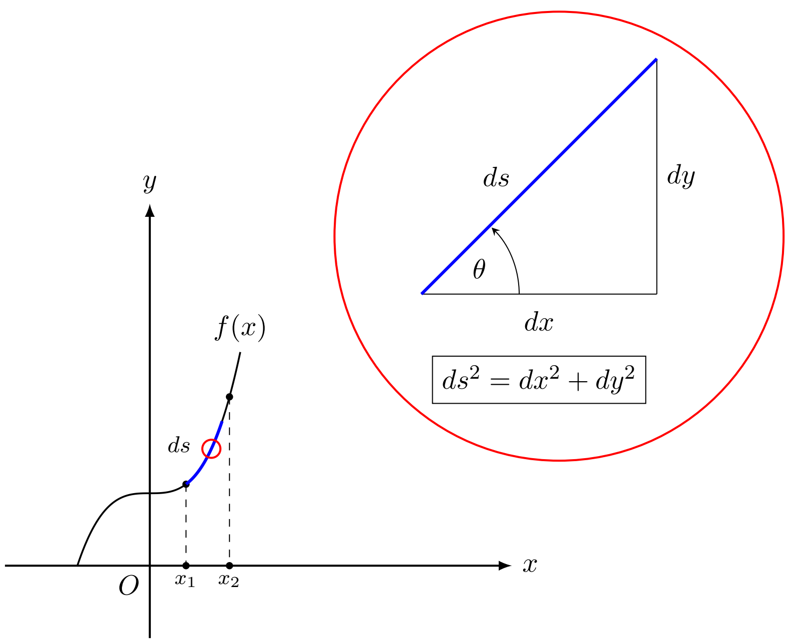

Figure 25: $f(x)$, ds and the Pythagorean Theorem