The Spiral of Theodorus

David Meyer

dmm613@gmail.com

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1 A Bit About The Spiral

The Spiral of Theodorus is a fantastic object that is composed of right triangles placed edge-to-edge, where the sides of n^{th} triangle are of length 1 and $\sqrt{n}$ and the hypotenuse is of length $\sqrt{n+1}$.

The spiral is named for Theodorus of Cyrene, the ancient Greek mathematician who lived during the 5^{th} century BC and who discovered the spiral [3]. It is thought that Theodorus used the spiral to prove that all of the square roots of the non-square integers from 3 to 17 are irrational [2]. Theodorus' original spiral stopped at $n = 16$, apparently because $\sqrt{16+1} = \sqrt{17}$ is the length of the hypotenuse of the last triangle in the spiral that doesn't overlap; this is shown in Figure 1.

Interestingly, in 1958 Erich Teuffel showed that no two hypotenuses of the spiral will coincide regardless of how far the spiral is continued [1]. Said another way, no two hypotenuses of the spiral will coincide, no matter how big n gets.

In any event, the spiral is an incredible object.

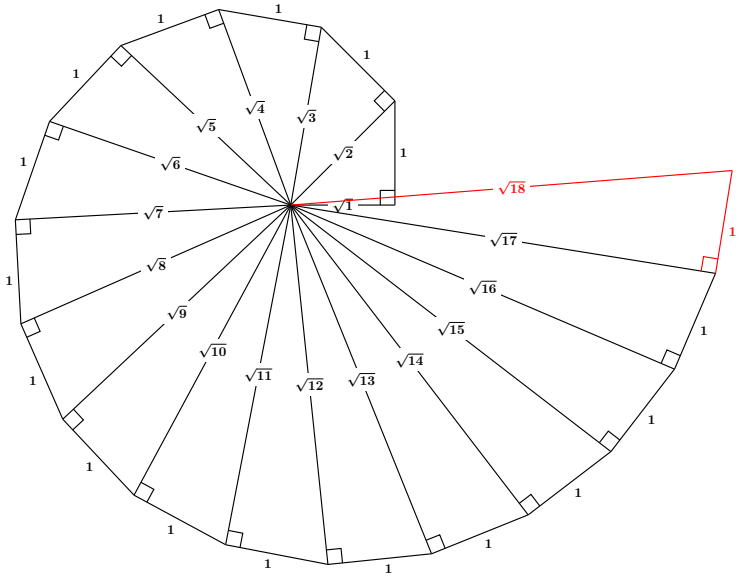

Figure 1: The Spiral of Theodorus ($n = 17$)

2 A Spiral of Theodorus with $n = 128$

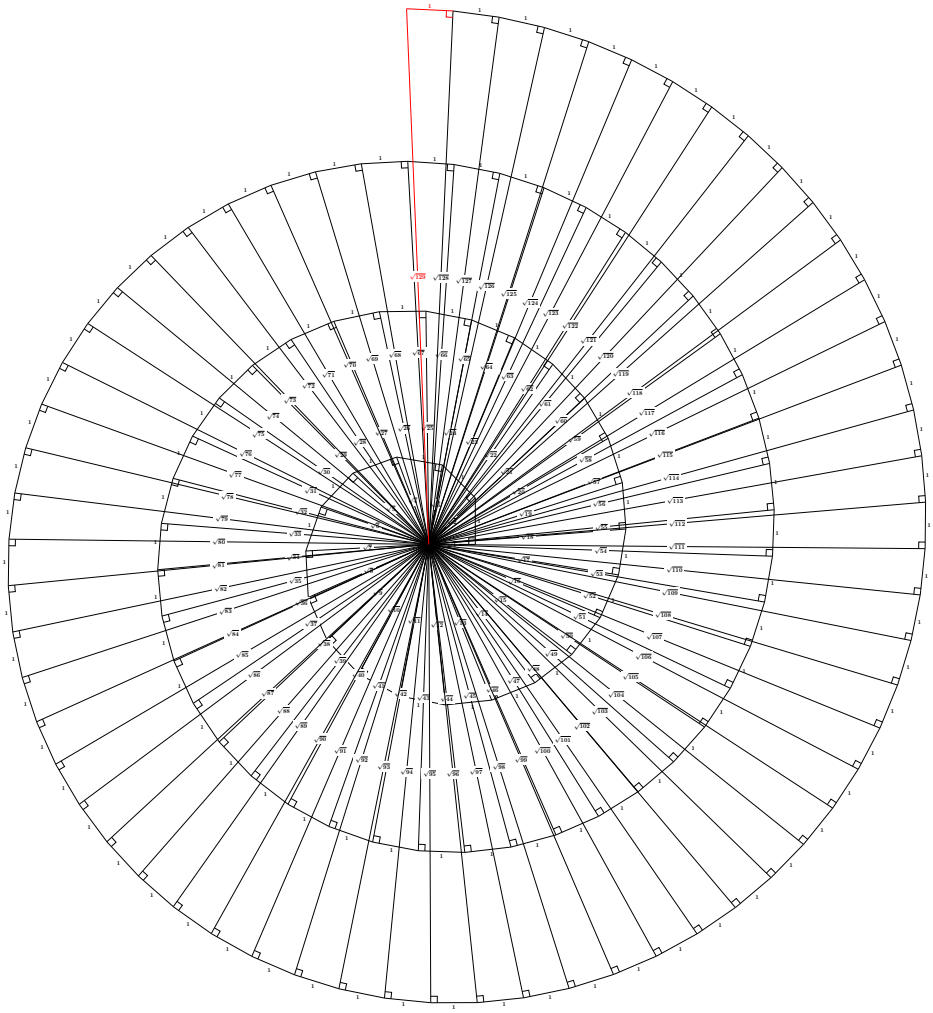

Figure 2: A spiral with $n = 128$

Acknowledgements

LaTeX Source

`https://www.overleaf.com/read/scsptwzqxzdt`

References

[1] https://dewiki.de. Wurzelschnecke. `https://dewiki.de/Lexikon/Wurzelschnecke`, 2023. [Online; accessed 21-August-2023].

[2] Wikipedia Contributors. Spiral of Theodorus — Wikipedia, The Free Encyclopedia. `https://en.wikipedia.org/w/index.php?title=Spiral_of_Theodorus&oldid=1091008882`, 2022. [Online; accessed 17-August-2022].

[3] Wikipedia Contributors. Theodorus of Cyrene — Wikipedia, The Free Encyclopedia. https://en.wikipedia.org/w/index.php?title=Theodorus_of_Cyrene&oldid=1082503450, 2022. [Online; accessed 17-August-2022].