

Notice that  $\sum_{n=1}^{\infty} \frac{1}{3^n} = \frac{1}{2}$ . OK, but why? Consider

$$\begin{array}{ll}
 S &= \sum_{n=1}^{\infty} \frac{1}{3^n} && \# \text{ define } S \\
 &= \frac{1}{3^1} + \frac{1}{3^2} + \frac{1}{3^3} + \cdots && \# \text{ expand terms} \\
 \Rightarrow &3 \cdot S = 3 \cdot \left[ \frac{1}{3^1} + \frac{1}{3^2} + \frac{1}{3^3} + \cdots \right] && \# \text{ multiply both sides by 3} \\
 \Rightarrow &3 \cdot S = 1 + \frac{1}{3^1} + \frac{1}{3^2} + \frac{1}{3^3} + \cdots && \# \text{ multiply through on right side} \\
 \Rightarrow &3 \cdot S = 1 + S && \# \text{ definition of } S \\
 \Rightarrow &3 \cdot S - S = 1 && \# \text{ subtract } S \text{ from both sides} \\
 \Rightarrow &2 \cdot S = 1 && \# 3 \cdot S - S = 2 \cdot S \\
 \Rightarrow &S = \frac{1}{2} && \# \text{ divide both sides by 2}
 \end{array}$$