

A Few Notes on Algebraic Structures (WIP)

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1 Introduction

Structure	ABO ¹	Identity	Inverse	Distributive ²	Commutative ³	Comments
Semigroup	✓	no	no	N/A	no	(S, \circ)
Monoid	✓	✓	no	N/A	no	Semigroup plus identity $\in S$
Group	✓	✓	✓	N/A	no	Monoid plus inverse $\in S$
Abelian Group	✓	✓	✓	N/A	✓	Commutative group
Ring ₊	✓	✓	✓	✓	✓	Abelian group under +
Ring _*	✓	✓	no	✓	no	Monoid under *
Field _(+,*)	✓	✓(+,*)	✓(+,*)	✓	✓	Abelian group under + and *
Vector Space	✓	✓(+,*)	✓(+)	✓	✓	Abelian group under +

Table 1: A Few Algebraic Structures and Their Features

Abbreviations:

1. **ABO:** Associative Binary Operation

- $(x \circ y) \circ z = x \circ (y \circ z)$ for all $x, y, z \in S$
- $x \circ y \in S$ (closure under \circ)

2. **Distributive:** Distributive Property

- Left Distributive Property: $x * (y + z) = (x * y) + (x * z)$ for all $x, y, z \in S$
- Right Distributive Property: $(y + z) * x = (y * x) + (z * x)$ for all $x, y, z \in S$
- $*$ is *distributive* over $+$ if $*$ is left and right distributive

3. **Commutative:** Commutative Property

- $x \circ y = y \circ x$ for all $x, y \in S$

Notes:

- Table 1 implies that $F \subset R \subset G \subset M \subset SG$
- $VS \subset G_+$ (vector spaces are Abelian groups under $+$)
- $F \subset VS$ since the field axioms require a multiplicative inverse (a^{-1}) while vector spaces do not

2 Acknowledgements