

$\sum_{n=1}^{\infty} \frac{1}{3^n} = \frac{1}{2}$. But why? Consider

S	$=$	$\sum_{n=1}^{\infty} \frac{1}{3^n}$	# definition of S
	$=$	$\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \cdots$	# expand terms
	\Rightarrow	$3 \cdot S = 3 \cdot [\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \cdots]$	# multiply both sides by 3
	\Rightarrow	$3 \cdot S = 1 + \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \cdots$	# multiply through on right side
	\Rightarrow	$3 \cdot S = 1 + S$	# definition of S
	\Rightarrow	$3 \cdot S - S = 1$	# subtract S from both sides
	\Rightarrow	$2 \cdot S = 1$	# $3 \cdot S - S = 2 \cdot S$
	\Rightarrow	$S = \frac{1}{2}$	# divide both sides by 2