How Did Price's Metonic Cycle Gear Train Work?

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1 Introduction

The advent of new insight into the structure and function of the Antikythera Mechanism [3] made me wonder exactly how Derek J. de Solla Price's [4] proposed Metonic Cycle gear train in the Mechanism works[1]. I decided to look at the Metonic gear train first since it's a simple train (neither compound nor epicyclic).

These notes briefly investigate how and why Price's Metonic Cycle gear train works.

2 First, What Is the Metonic Cycle?

The Metonic cycle or enneadecaeteris (from Ancient Greek enneakaidekaeteris meaning "nineteen") is a period of approximately 19 years after which the phases of the moon recur on the same day of the year. The recurrence is not perfect, and by precise observation the Metonic cycle is defined as 235 synodic lunar months, a period which is just 1h27m33s longer than 19 tropical years. Learning from the Babylonian and Hebrew lunisolar calendars in which the years 3, 6, 8, 11, 14, 17, and 19 are the long (13-month) years, the 5^{th}century BC Greek mathematician, astronomer, geometer, and engineer Meton of Athens judged the cycle to be a whole number of days, specifically 6,940 days. Using these integer numbers facilitates the construction of a luni-solar calendar.

The currently accepted values are for the Metonic Cycle are:

One Metonic Cycle is defined to be 19 *tropical years* which is 235 *synodic months* (lunar phases) = 6,939.688 days. Note that 19 tropical years = 6,939.602 days; the difference of 0.086 days for a cycle which mean that after a dozen returns there will be a full day of

[1] Freeth agrees that Price's Metonic Cycle gear train is essentially correct [2].

delay between the astronomical data and calculations. The error is actually one day every 219 years, or 12.4 parts per million.

However, the Metonic cycle turns out to be very close to two other important lunar periods:

- 254 sidereal months (lunar orbits) = 6,939.702 days
- 255 draconic months (lunar nodes) = 6,939.1161 days

In summary:

- One Metonic Cycle $\approx$ 19 years
- $\approx$ 235 synodic (lunar) months
- $\approx$ 254 sidereal months
- $\approx$ 255 draconic months

Interestingly, $\frac{254}{19} = 13.36842$, an important astronomical constant.

All very interesting but the Metonic cycle seems to be a coincidence. The periods of the Moon's orbit around the Earth and the Earth's orbit around the Sun are believed to be independent, and not to have any known physical resonance. An example of a non-coincidental cycle is the orbit of Mercury, with its 3:2 spin-orbit resonance.

3 Price's Metonic Gearing Scheme

The objective of the Metonic gearing scheme(which Price didn't know at the time) is to turn the output pointer Moon such that it, along with the Sun pointer, follows the Metonic Cycle. Price's Metonic gearing scheme, described in his classic work "Gears from the Greeks" [1], is shown in Figure 1.

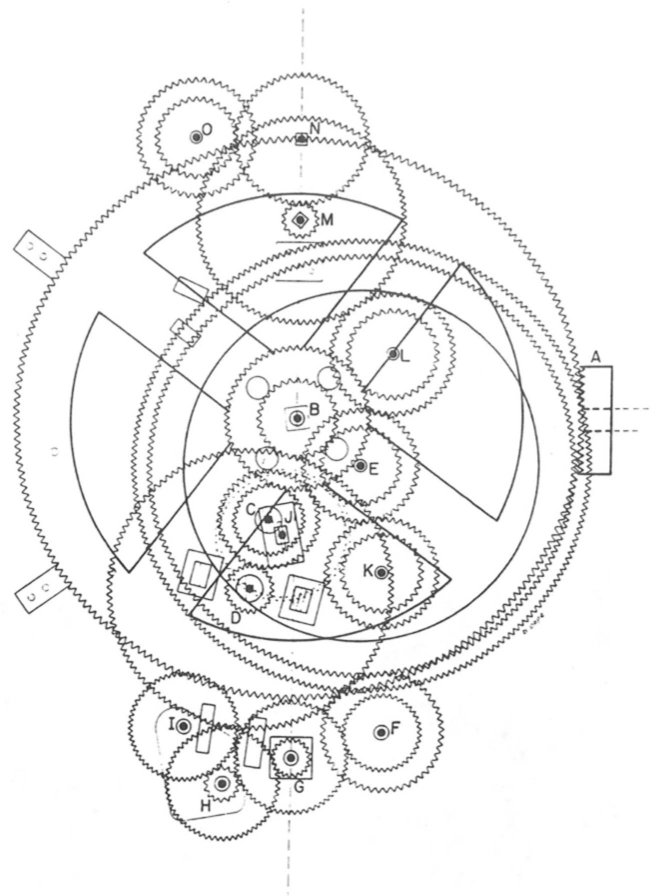

Figure 6: General plan of the complete gearing system [Price,10; Reproduced by permission of the American Philosophical Society].

Figure 1: Price's General Gearing Plan [1]

For calculating gear ratios, Price's sectional gearing diagram Figure 2 is more useful. As we can see from Figure 2, the gears of interest are B2, C1, C2, D1, D2, and E2, with the following tooth counts[2]:

[2]Tooth counts were controversial in the 1950s when Price did much of his work.

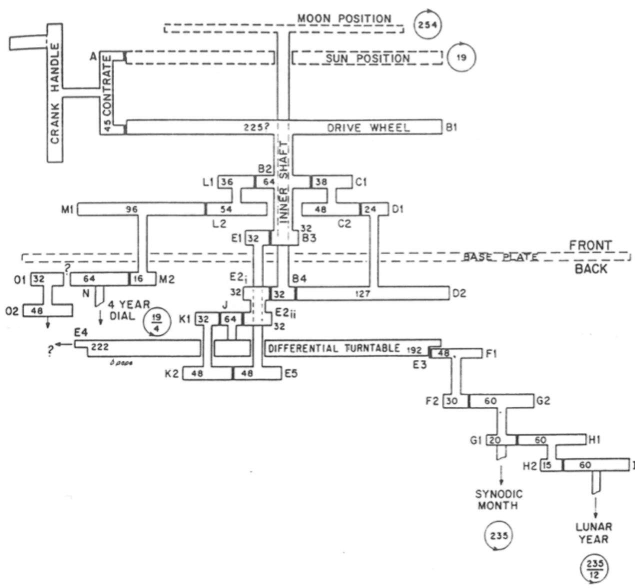

Figure 7: Sectional diagram of the complete gearing system [Price,10; Reproduced by permission of the American Philosophical Society].

Figure 2: Price's Sectional Gearing Diagram [1]

- B2: 64 teeth
- C1: 38 teeth
- C2: 48 teeth
- D1: 24 teeth
- D2: 127 teeth
- E2: 32 teeth

We know that in simple gear trains, we can calculate the Gear Ratio (GR) as

$$\mathrm{GR} = \frac{\text{Number of Teeth on the Driven Gear}}{\text{Number of Teeth on the Driver Gear}}$$

and we know that the driven gear rotates in the opposite direction of the driver gear.

With this information we can start to calculate what Price's Metonic gear train does.

Specifically:

$$\frac{\mathrm{B2}}{\mathrm{C1}} = -\frac{64}{38} = -\frac{32}{19} \quad \text{\# driver \& driven gears turn in opposite directions}$$

$$\frac{\mathrm{B2}}{\mathrm{C1}} \times \frac{\mathrm{C2}}{\mathrm{D1}} = -\frac{32}{19} \times -\frac{48}{24} = -\frac{32}{19} \times -\frac{2}{1} = \frac{64}{19} \quad \text{\# } \frac{\mathrm{C2}}{\mathrm{D1}} \text{ multiplies } \frac{\mathrm{B2}}{\mathrm{C1}} \text{ by 2}$$

$$\frac{\mathrm{B2}}{\mathrm{C1}} \times \frac{\mathrm{C2}}{\mathrm{D1}} \times -\frac{\mathrm{D2}}{\mathrm{E1}} = -\frac{32}{19} \times -\frac{48}{24} \times -\frac{127}{32} = -\frac{254}{19} \quad \text{\# } \frac{254}{19} \approx 13.36842$$

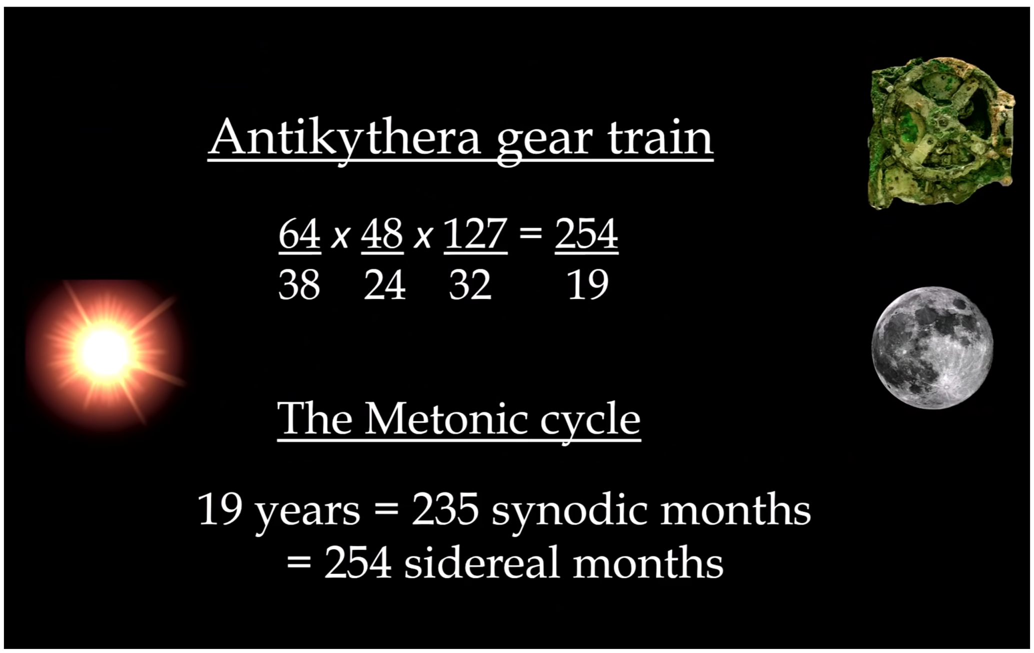

Figure 3: Price's Metonic Gear Train Ratios and the Metonic Cycle

4 Acknowledgements

References

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