A Few Notes On The Dirac Delta Function

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1 Introduction

These notes began life as some thoughts on the Dirac Delta Function and evolved into notes on several related topics including Laplace Transforms. The Dirac Delta function has all kinds of crazy and interesting properties. More TBD.

2 The Dirac Delta Function

The Dirac Delta Function is defined as shown in Figure 1. In the limit ($\epsilon \to 0$) the Dirac Delta function is written $\delta_a(t)$ or sometimes $\delta(t-a)$. As we will see in a moment, the $\delta_{a,\epsilon}(t)$ form of the delta function is useful when we want to use the Mean Value Theorem for Integrals [1] to evaluate integrals involving the delta function.

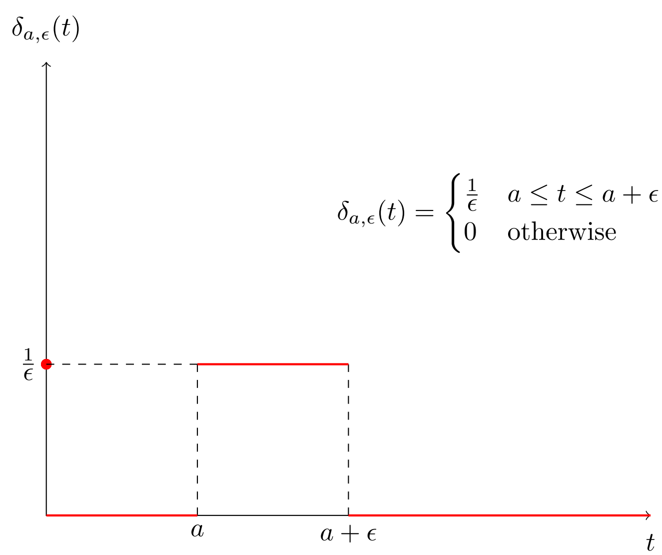

Figure 1: The $\delta_{a,\epsilon}(t)$ function

So $\delta_{a,\epsilon}(t)$ is defined to be

$$\delta_{a,\epsilon}(t) = \begin{cases} \frac{1}{\epsilon} & a \leq t \leq a + \epsilon \\ 0 & \text{otherwise} \end{cases}$$

and has the constraint that

$$\int_0^\infty \delta_{a,\epsilon}(t) = 1$$

That is, $\delta_{a,\epsilon}(t)$ is in some sense a probability density.

Acknowledgements

References

[1] Proof Wiki Contributors. Mean Value Theorem For Integrals. https://proofwiki.org/wiki/Mean_Value_Theorem_for_Integrals, 2020. [Online; accessed 11-May-2021].