How Did Price's Metonic Cycle Gear Train Work?

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1 Introduction

The advent of new insight into the structure and function of the Antikythera Mechanism [5] made me wonder exactly how Derek J. de Solla Price's [8] proposed Metonic Cycle gear train in the Mechanism works.[1] I decided to look at the Metonic gear train first since it is a simple gear train; specifically this gear train has no epicyclic gears [6] or pin-and-slot mechanisms [2].

These notes briefly investigate how and why Price's Metonic Cycle gear train works.

2 First, what is the Metonic Cycle?

The Metonic Cycle is *a period of approximately 19 years after which the phases of the moon recur on the same day of the year*. It is defined by observation to be 235 synodic (lunar) months, just 1h27m33s longer than 19 tropical years. Learning from the Babylonian and Hebrew lunisolar calendars in which the years 3, 6, 8, 11, 14, 17, and 19 are the long (13-month) years, the 5^{th}century BC Greek mathematician, astronomer, geometer, and engineer Meton of Athens [9] judged the cycle to be a whole number of days, specifically 6,940 days. Using these integer values facilitated the construction of a lunisolar calendar.

One Metonic Cycle is defined to be 19 tropical years, which is 235 synodic months (lunar phases), which in turn equals 6,939.688 days. Since 19 tropical years equals 6,939.602 days the difference of $6,939.688 - 6,939.602 = 0.086$ days/cycle means that after twelve cycles there will be a 1.032 day difference between observation and calculation (since $0.086 \text{ days/cycle} * 12 \text{ cycles} = 1.032 \text{ days}$).

[1] Price's Metonic Cycle gear train is generally considered to be correct [3].

The Metonic Cycle also turns out to be very close to integer multiples of two other important lunar periods:

- 254 sidereal months (lunar orbits) = 6,939.702 days
- 255 draconic months (lunar nodes) = 6,939.116 days

So in summary:

$$\begin{aligned} \text{One Metonic Cycle} &= \text{19 tropical years} && \#\ \text{6,939.602 days} \\ &\approx \text{235 synodic months} && \#\ \text{6,939.688 days} \\ &\approx \text{254 sidereal months} && \#\ \text{6,939.702 days} \\ &\approx \text{255 draconic months} && \#\ \text{6,939.116 days} \end{aligned}$$

Interestingly $\frac{254}{19} \approx 13.36842$, which is said to be an important astronomical constant.[2]

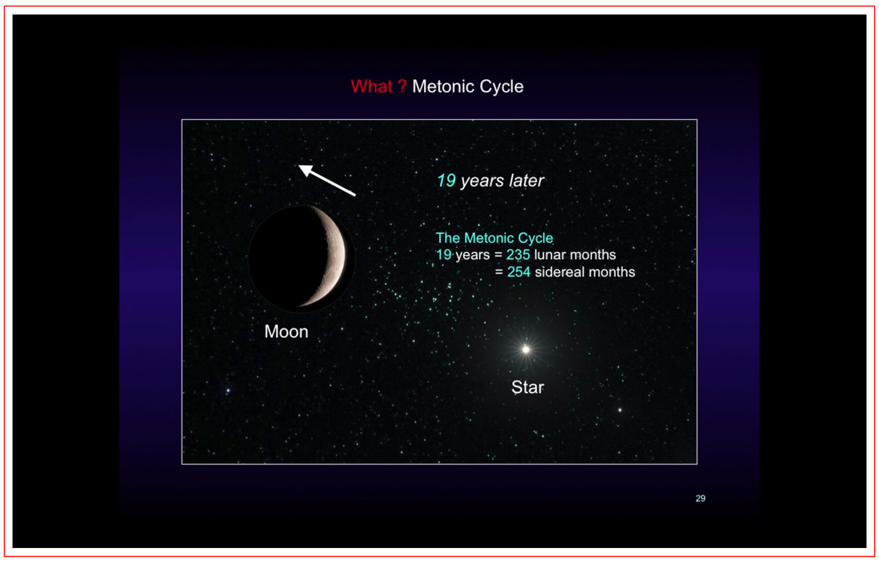

Figure 1: The Metonic Cycle [4]

This is all very interesting. However, the Metonic Cycle seems to be a coincidence. The periods of the Moon's orbit around the Earth and the Earth's orbit around the Sun are believed to be independent, and not to have any known physical resonance. An example of a non-coincidental cycle is the orbit of Mercury, with its 3:2 spin-orbit resonance.

[2] Why exactly this constant is considered to be "important" is something I have not been able to learn.

3 Price's Metonic Gearing Scheme

The purpose of Price's Metonic Cycle gearing scheme is to turn the pointer on the Metonic Dial, the main upper dial on the back of the Mechanism. Price's Metonic gearing scheme, described in his classic work "Gears from the Greeks. The Antikythera Mechanism: A Calendar Computer from ca. 80 B. C. T" [1], is shown in Figure 2.

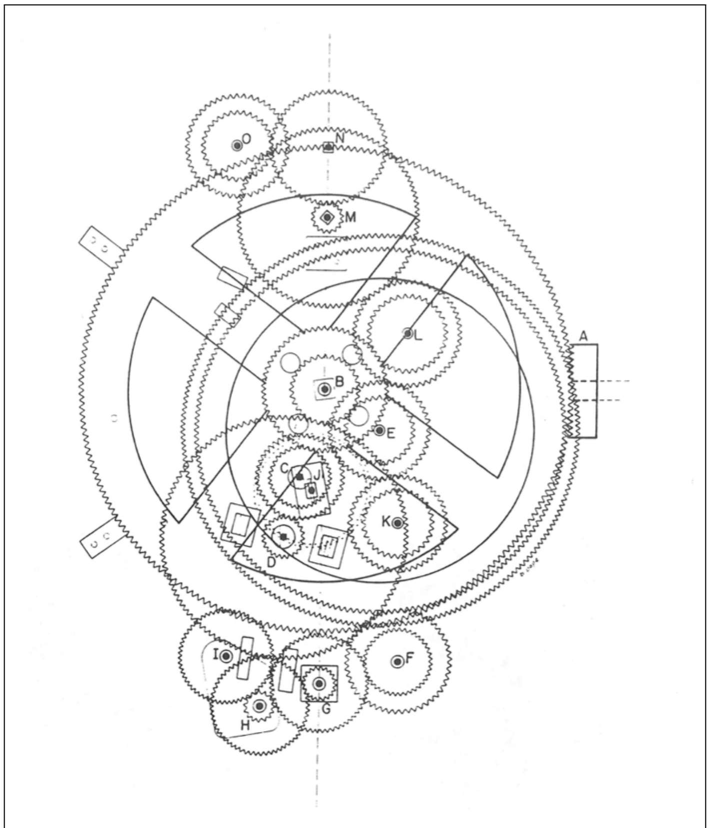

Figure 6: General plan of the complete gearing system [Price,10; Reproduced by permission of the American Philosophical Society].

Figure 2: Price's General Gearing Plan [1]

For calculating gear ratios, Price's sectional gearing diagram Figure 3 is more useful. As we can see from Figure 3, the gears of interest are B2, C1, C2, D1, D2, and E2, with the following tooth counts[3]:

[3] The tooth counts were controversial in the 1950s when Price did much of his work.

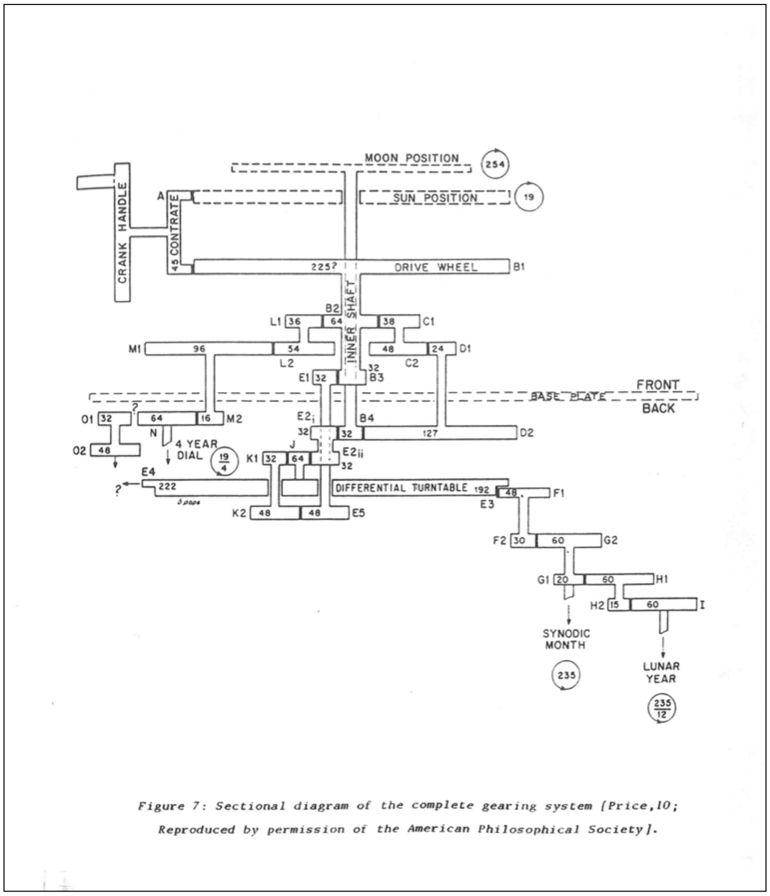

Figure 3: Price's Sectional Gearing Diagram [1]

- B2: 64 teeth
- C1: 38 teeth
- C2: 48 teeth
- D1: 24 teeth
- D2: 127 teeth
- E2: 32 teeth

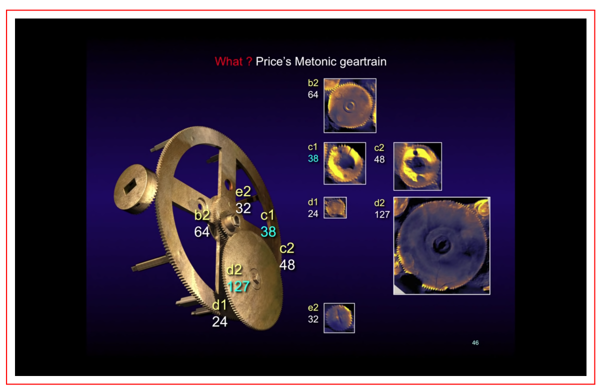

Figure 4: The Metonic Gear Train [4]

We know that in simple gear trains we can calculate the Gear Ratio (GR) as

$$\mathrm{GR} = \frac{\text{Number of Teeth on the Driven Gear}}{\text{Number of Teeth on the Driver Gear}}$$

and we know that the driven gear rotates in the opposite direction of the driver gear.

With this information we can start to calculate what Price's Metonic gear train does.

Specifically:

$$\frac{\mathrm{B2}}{\mathrm{C1}} = -\frac{64}{38} = -\frac{32}{19} \qquad \text{\# driver \& driven gears turn in opposite directions}$$

$$\frac{\mathrm{B2}}{\mathrm{C1}} \times \frac{\mathrm{C2}}{\mathrm{D1}} = -\frac{64}{38} \times -\frac{48}{24} = -\frac{32}{19} \times -\frac{2}{1} = \frac{64}{19} \qquad \text{\# } \frac{\mathrm{C2}}{\mathrm{D1}} \text{ multiplies } \frac{\mathrm{B2}}{\mathrm{C1}} \text{ by 2}$$

$$\frac{\mathrm{B2}}{\mathrm{C1}} \times \frac{\mathrm{C2}}{\mathrm{D1}} \times -\frac{\mathrm{D2}}{\mathrm{E1}} = -\frac{64}{38} \times -\frac{48}{24} \times -\frac{127}{32} = -\frac{254}{19} \qquad \text{\# } \frac{254}{19} \approx 13.36842$$

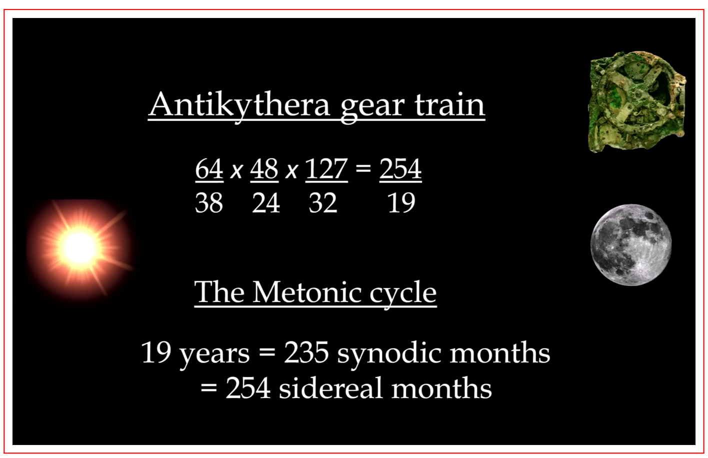

Figure 5: Metonic Gear Train Ratios and the Metonic Cycle

4 Putting it All Together

The Antikythera Mechanism is thought to have been operated by a knob or crank on the side of the device. This knob (or crank) was connected to a crown gear that meshed with

B1, the main drive gear. B1 is the large, four spoked gear seen in Fragment A (see Figure 6), and one revolution of B1 represents one year. Since B2 is planted on B1 to form a compound gear (B1 and B2 are connected to the same axle; see Figures 3 and 6), one revolution of B2 also represents one year.

Figure 6: Fragment A of the Antikythera Mechanism [7]

This configuration of gears means that one revolution of B2 (or B1) moves the Metonic pointer by one nineteenth of the Metonic Cycle, or 13.36842 sidereal months. Thus 19 revolutions of the main drive gear results in one revolution of the Metonic pointer or one Metonic Cycle, just as required.

Next up: Epicyclic gears and pin-and-slot mechanisms.

5 Acknowledgements

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