# What is Rejection Sampling?

David Meyer

dmm@{1-4-5.net,uoregon.edu,brocade.com,...}

September 14, 2016

## 1 Introduction

Suppose that we want to sample from a distribution $f(x)$ that is difficult or impossible to sample from directly. Instead of trying to sample from $f(x)$, can we use a simpler distribution $q(x)$ from which sampling is easier? Perhaps, and the idea behind Rejection Sampling (aka Acceptance-rejection sampling) is to sample from $q(x)$ and apply some rejection/acceptance criterion such that the samples that are accepted are distributed according to $f(x)$.

### 1.1 Envelope distribution and rejection criterion

In order to be able to reject samples from $q(x)$ that aren't (approximately) sampled from $f(x)$, $q(x)$ must cover or envelop the distribution $f(x)$. This is generally done by choosing a constant $c > 1$ such that $cq(x) > f(x)$, for all $x$. For this reason $cq(x)$ is often called the *envelope distribution*. A common criterion for accepting samples from $x \sim q(x)$ is based on the ratio of the target distribution to that of the envelope distribution. The samples are accepted if

$$\frac{f(x)}{cq(x)} > u \tag{1}$$

where $u \sim Unif(0, 1)$, and rejected otherwise. If the ratio is close to one, then $f(x)$ must have a large amount of probability mass around $x$ and that sample should be more likely accepted. If the ratio is small, then it means that $f(x)$ has low probability mass around $x$ and we should be less likely to accept the sample. See Figure 1. I think...

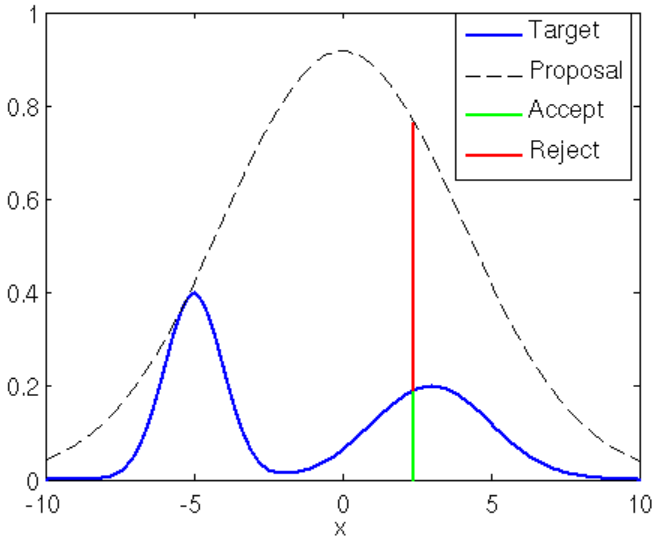


Figure 1: Rejection Sampling with a Normal proposal distribution