

How Did Price's Metonic Cycle Gear Train Work?

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1 Introduction

The advent of new insight into the structure and function of the Antikythera Mechanism [5] made me wonder exactly how Derek J. de Solla Price's [7] proposed Metonic Cycle gear train in the Mechanism works.¹ I decided to look at the Metonic gear train first since it is a simple gear train; specifically this gear train has no epicyclic gears [6] or pin-and-slot mechanisms [2].

These notes briefly investigate how and why Price's Metonic Cycle gear train works.

2 First, what is the Metonic Cycle?

The Metonic Cycle is *a period of approximately 19 years after which the phases of the moon recur on the same day of the year*. The recurrence is not perfect, and by precise observation the Metonic cycle is defined as 235 synodic lunar months, a period which is just 1h27m33s longer than 19 tropical years. Learning from the Babylonian and Hebrew lunisolar calendars in which the years 3, 6, 8, 11, 14, 17, and 19 are the long (13-month) years, the 5th century BC Greek mathematician, astronomer, geometer, and engineer Meton of Athens [8] judged the cycle to be a whole number of days, specifically 6,940 days. Using these integer values facilitated the construction of a lunisolar calendar.

One Metonic Cycle is defined to be 19 tropical years which is 235 synodic months (lunar phases) which in turn equals 6,939.688 days. Note that 19 tropical years = 6,939.602 days; the difference of 0.086 days per cycle means that after a dozen returns there will be a full day of delay between the astronomical data and calculations. The error is one day every 219 year or 12.4 parts per million.

¹Price's Metonic Cycle gear train is generally considered to be correct [3].

The Metonic Cycle also turns out to be very close to integer multiples of two other important lunar periods:

- 254 sidereal months (lunar orbits) = 6,939.702 days
- 255 draconic months (lunar nodes) = 6,939.116 days

In summary:

| | | |
|-------------------|-----------------------|------------------|
| One Metonic Cycle | = 19 years | # 6,939.602 days |
| | ≈ 235 synodic months | # 6,939.688 days |
| | ≈ 254 sidereal months | # 6,939.702 days |
| | ≈ 255 draconic months | # 6,939.116 days |

Interestingly $\frac{254}{19} \approx 13.36842$, which is said to be an important astronomical constant.²

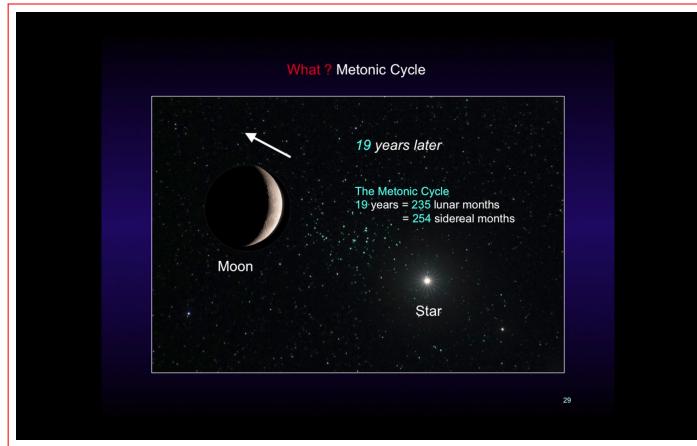


Figure 1: The Metonic Cycle [4]

All very interesting but the Metonic Cycle seems to be a coincidence. The periods of the Moon's orbit around the Earth and the Earth's orbit around the Sun are believed to be independent, and not to have any known physical resonance. An example of a non-coincidental cycle is the orbit of Mercury, with its 3:2 spin-orbit resonance.

²Why exactly this constant is considered to be "important" is something I have not been able to learn.

3 Price's Metonic Gearing Scheme

The purpose of the Metonic Cycle gearing scheme (which Price discovered) is to turn the (theorized) Moon output pointer such that it follows the Metonic Cycle. Price's Metonic gearing scheme, described in his classic work "Gears from the Greeks. The Antikythera Mechanism: A Calendar Computer from ca. 80 B. C. T" [1], is shown in Figure 2.

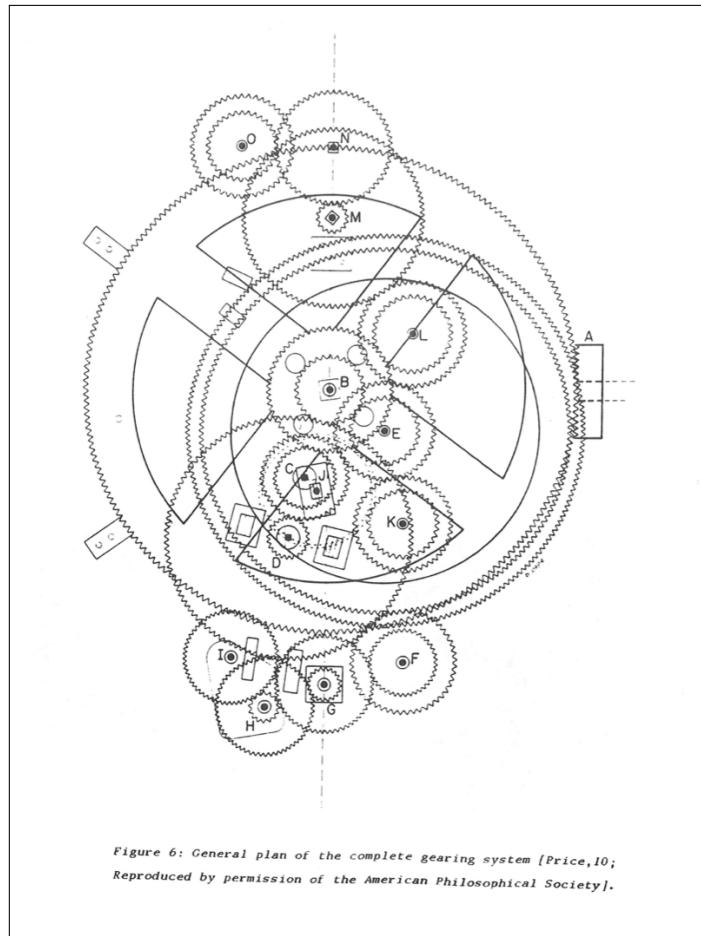


Figure 2: Price's General Gearing Plan [1]

For calculating gear ratios, Price's sectional gearing diagram Figure 3 is more useful. As we can see from Figure 3, the gears of interest are B2, C1, C2, D1, D2, and E2, with the following tooth counts³:

³The tooth counts were controversial in the 1950s when Price did much of his work.



Figure 3: Price's Sectional Gearing Diagram [1]

- B2: 64 teeth
- C1: 38 teeth
- C2: 48 teeth
- D1: 24 teeth
- D2: 127 teeth
- E2: 32 teeth



Figure 4: The Metonic Gear Train [4]

We know that in simple gear trains we can calculate the Gear Ratio (GR) as

$$GR = \frac{\text{Number of Teeth on the Driven Gear}}{\text{Number of Teeth on the Driver Gear}}$$

and we know that the driven gear rotates in the opposite direction of the driver gear.

With this information we can start to calculate what Price's Metonic gear train does.

Specifically:

$$\frac{B_2}{C_1} = -\frac{64}{38} = -\frac{32}{19} \quad \# \text{ driver \& driven gears turn in opposite directions}$$

$$\frac{B_2}{C_1} \times \frac{C_2}{D_1} = -\frac{64}{38} \times -\frac{48}{24} = -\frac{32}{19} \times -\frac{2}{1} = \frac{64}{19} \quad \# \frac{C_2}{D_1} \text{ multiplies } \frac{B_2}{C_1} \text{ by 2}$$

$$\frac{B_2}{C_1} \times \frac{C_2}{D_1} \times -\frac{D_2}{E_1} = -\frac{64}{38} \times -\frac{48}{24} \times -\frac{127}{32} = -\frac{254}{19} \quad \# \frac{254}{19} \approx 13.36842$$



Figure 5: Metonic Gear Train Ratios and the Metonic Cycle

4 Acknowledgements

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