Notice that $\sum_{n=1}^{\infty} \frac{1}{3^k} = \frac{1}{2}$. But why? Consider

$$
\begin{array}{rcll}
S & = & \sum_{n=1}^{\infty} \frac{1}{3^k} & \text{\# define } S \\
& = & \frac{1}{3^1} + \frac{1}{3^2} + \frac{1}{3^3} + \cdots & \text{\# expand terms} \\
& \Rightarrow & 3 \cdot S = 3 \cdot [\frac{1}{3^1} + \frac{1}{3^2} + \frac{1}{3^3} + \cdots] & \text{\# multiply both sides by 3} \\
& \Rightarrow & 3 \cdot S = 1 + \frac{1}{3^1} + \frac{1}{3^2} + \frac{1}{3^3} + \cdots & \text{\# multiply through on right side} \\
& \Rightarrow & 3 \cdot S = 1 + S & \text{\# definition of } S \\
& \Rightarrow & 3 \cdot S - S = 1 & \text{\# subtract } S \text{ from both sides} \\
& \Rightarrow & 2 \cdot S = 1 & \text{\# } 3 \cdot S - S = 2 \cdot S \\
& \Rightarrow & S = \frac{1}{2} & \text{\# divide both sides by 2}
\end{array}
$$