

# Is $i^i$ a real number?

David Meyer

dmm@{1-4-5.net,uoregon.edu}

Last update: July 25, 2018

$e^{ix}$	$=$	$\cos x + i \sin x$	# Euler's formula
$\Rightarrow$	$e^{i\frac{\pi}{2}}$	$= \cos \frac{\pi}{2} + i \sin \frac{\pi}{2}$	# set $x = \frac{\pi}{2}$
$\Rightarrow$	$e^{i\frac{\pi}{2}}$	$= 0 + i \cdot 1$	# $\cos \frac{\pi}{2} = 0$ and $\sin \frac{\pi}{2} = 1$
$\Rightarrow$	$e^{i\frac{\pi}{2}}$	$= i$	# simplify
$\Rightarrow$	$(e^{i\frac{\pi}{2}})^i$	$= i^i$	# raise both sides to $i$
$\Rightarrow$	$e^{\frac{i^2\pi}{2}}$	$= i^i$	# $(x^m)^n = x^{mn}$
$\Rightarrow$	$e^{-\frac{\pi}{2}}$	$= i^i$	# $i^2 = -1$
$\Rightarrow$	$e^{-\frac{\pi}{2}} \in \mathbb{R}$	$\Rightarrow i^i \in \mathbb{R}$	# $i^i$ is a real number