A Few Notes on Algebraic Structures (WIP)

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1 Introduction

Structure	**ABO**[1]	**Identity**	**Inverse**	**Distributive**[2]	**Commutative**[3]	**Comments**
Semigroup	✓	no	no	N/A	no	$(S, \circ)$
Monoid	✓	✓	no	N/A	no	Semigroup plus idenity $\in S$
Group	✓	✓	✓	N/A	no	Monoid plus inverse $\in S$
Abelian Group	✓	✓	✓	N/A	✓	Commutative group
Ring_{+}	✓	✓	✓	✓	✓	Abelian group under $+$
Ring_{*}	✓	✓	no	✓	no	Monoid under $*$
$\text{Field}_{(+,*)}$	✓	✓$(+,*)$	✓$(+,*)$	✓	✓	Abelian group under $+$ and $*$
Vector Space	✓	✓$(+,*)$	✓$(+)$	✓	✓	Abelian group under $+$

Table 1: A Few Algebraic Structures and Their Features

Abbreviations:

1. **ABO:** Associative Binary Operation
   - $(x \circ y) \circ z = x \circ (y \circ z)$ for all $x, y, z \in S$
   - $x \circ y \in S$ (closure under $\circ$)
2. **Distributive:** Distributive Property
   - Left Distributive Property: $x * (y + z) = (x * y) + (x * z)$ for all $x, y, z \in S$
   - Right Distributive Property: $(y + z) * x = (y * x) + (z * x)$ for all $x, y, z \in S$
   - $*$ is *distributive* over $+$ if $*$ is left and right distributive
3. **Commutative:** Commutative Property

- $x \circ y = y \circ x$ for all $x, y \in S$

Notes:

- Table 1 implies that F $\subset$ R $\subset$ G $\subset$ M $\subset$ SG
- VS $\subset$ G_+ (vector spaces are Abelian groups under +)
- F $\subset$ VS since the field axioms require a multiplicative inverse (a^{-1}) while vector spaces do not

2 Acknowledgements