# Is $i^i$ a real number?

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\begin{array}{rcll}
e^{ix} & = & \cos x + i \sin x & \text{\# Euler's formula} \\
& \Rightarrow & e^{i\frac{\pi}{2}} = \cos\frac{\pi}{2} + i \sin\frac{\pi}{2} & \text{\# set } x = \frac{\pi}{2} \\
& \Rightarrow & e^{i\frac{\pi}{2}} = 0 + i \cdot 1 & \text{\# } \cos\frac{\pi}{2} = 0 \text{ and } \sin\frac{\pi}{2} = 1 \\
& \Rightarrow & e^{i\frac{\pi}{2}} = i & \text{\# simplify} \\
& \Rightarrow & (e^{i\frac{\pi}{2}})^i = i^i & \text{\# raise both sides to } i \\
& \Rightarrow & e^{\frac{i^2\pi}{2}} = i^i & \text{\# } (x^m)^n = x^{mn} \\
& \Rightarrow & e^{-\frac{\pi}{2}} = i^i & \text{\# } i^2 = -1 \\
& \Rightarrow & e^{-\frac{\pi}{2}} \in \mathbb{R} \Rightarrow i^i \in \mathbb{R} & \text{\# } i^i \text{ is a real number}
\end{array}
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