How Did Price's Metonic Cycle Gear Train Work?

David Meyer

dmm@{1-4-5.net,uoregon.edu}

Last update: March 23, 2021

1 Introduction

The advent of new insight into the structure and function of the Antikythera Mechanism [6] made me wonder exactly how Derek J. de Solla Price's [9] proposed Metonic Cycle gear train in the Mechanism works.[1] I decided to look at the Metonic gear train first since it is a simple gear train; specifically this gear train has no epicyclic gears [7] or pin-and-slot mechanisms [2].

These notes briefly investigate how and why Price's Metonic Cycle gear train works.

2 First, what is the Metonic Cycle?

The Metonic Cycle is *a period of approximately nineteen years after which the phases of the moon recur on the same day of the year*. It is defined by observation to be 235 synodic (lunar) months, just 1h27m33s longer than nineteen tropical years. Learning from the Babylonian and Hebrew lunisolar calendars in which the years 3, 6, 8, 11, 14, 17, and 19 are the long (13-month) years, the 5^{th}century BC Greek mathematician, astronomer, geometer, and engineer Meton of Athens [10] judged the cycle to be a whole number of days, specifically 6,940 days. Using these integer values facilitated the construction of a lunisolar calendar.

One Metonic Cycle is defined to be 19 tropical years, which is 235 synodic months (lunar phases), which in turn equals 6,939.688 days. Since 19 tropical years equals 6,939.602 days the difference of $6,939.688 - 6,939.602 = 0.086$ days/cycle means that after twelve cycles there will be a 1.032 day difference between observation and calculation (since $0.086 \text{ days/cycle} * 12 \text{ cycles} = 1.032 \text{ days}$).

[1] Price's Metonic Cycle gear train is generally considered to be correct [4].

The Metonic Cycle also turns out to be very close to integer multiples of two other important lunar periods:

- 254 sidereal months (lunar orbits) = 6,939.702 days
- 255 draconic months (lunar nodes) = 6,939.116 days

So in summary:

$$\begin{aligned}
\text{One Metonic Cycle} &= 19 \text{ tropical years} && \# \text{ 6,939.602 days} \\
&\approx 235 \text{ synodic months} && \# \text{ 6,939.688 days} \\
&\approx 254 \text{ sidereal months} && \# \text{ 6,939.702 days} \\
&\approx 255 \text{ draconic months} && \# \text{ 6,939.116 days}
\end{aligned}$$

Interestingly $\frac{254}{19} \approx 13.36842$, which is said to be an important astronomical constant.[2]

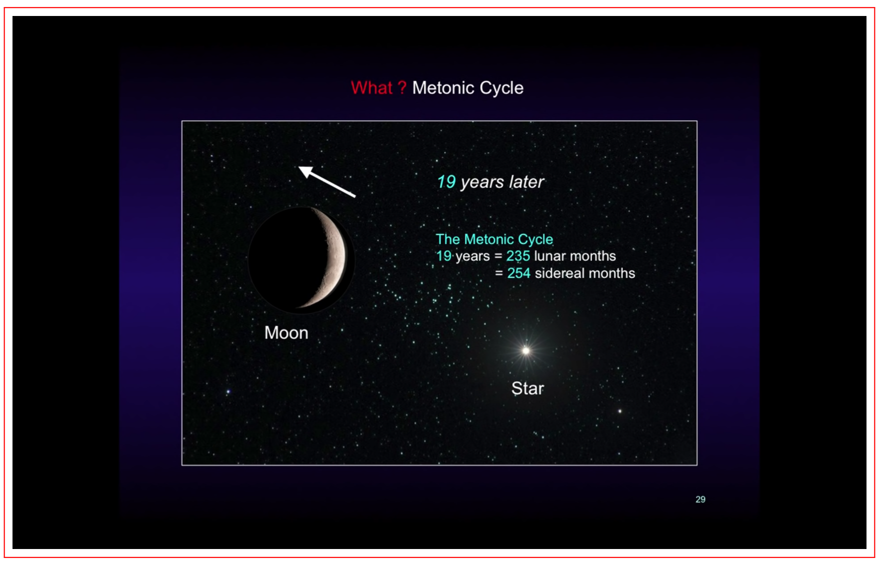

Figure 1: The Metonic Cycle [5]

This is all very interesting. However, the Metonic Cycle seems to be a coincidence. The periods of the Moon's orbit around the Earth and the Earth's orbit around the Sun are believed to be independent, and not to have any known physical resonance. An example of a non-coincidental cycle is the orbit of Mercury, with its 3:2 spin-orbit resonance.

[2]Why exactly this constant is considered to be "important" is something I have not been able to learn.

3 Price's Metonic Gearing Scheme

The purpose of Price's Metonic Cycle gearing scheme is to turn the pointer on the Metonic Dial, the main upper dial on the back of the Mechanism. Price's Metonic gearing scheme, described in his classic work "Gears from the Greeks. The Antikythera Mechanism: A Calendar Computer from ca. 80 B. C. T" [1], is shown in Figure 2.

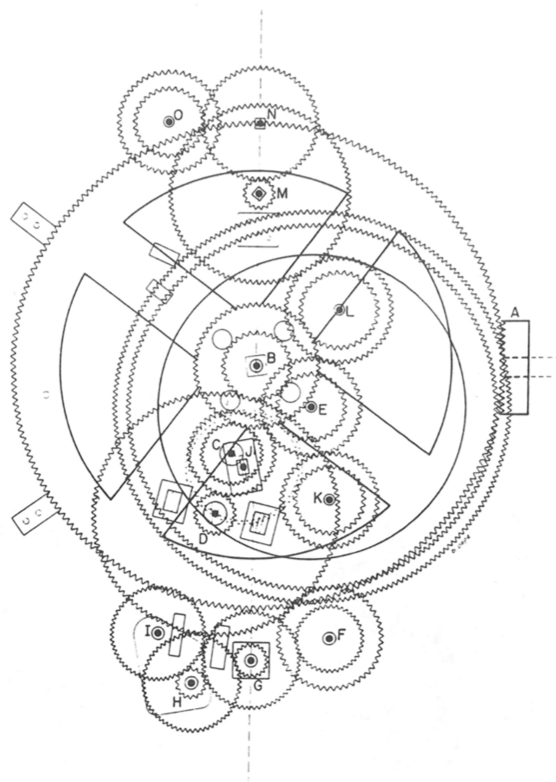

Figure 6: General plan of the complete gearing system [Price,10; Reproduced by permission of the American Philosophical Society].

Figure 2: Price's General Gearing Plan [1]

For calculating gear ratios, Price's sectional gearing diagram Figure 3 is more useful. As we can see from Figure 3, the gears of interest are B2, C1, C2, D1, D2, and E2, with the following tooth counts[3]:

[3]The tooth counts were controversial in the 1950s when Price did much of his work.

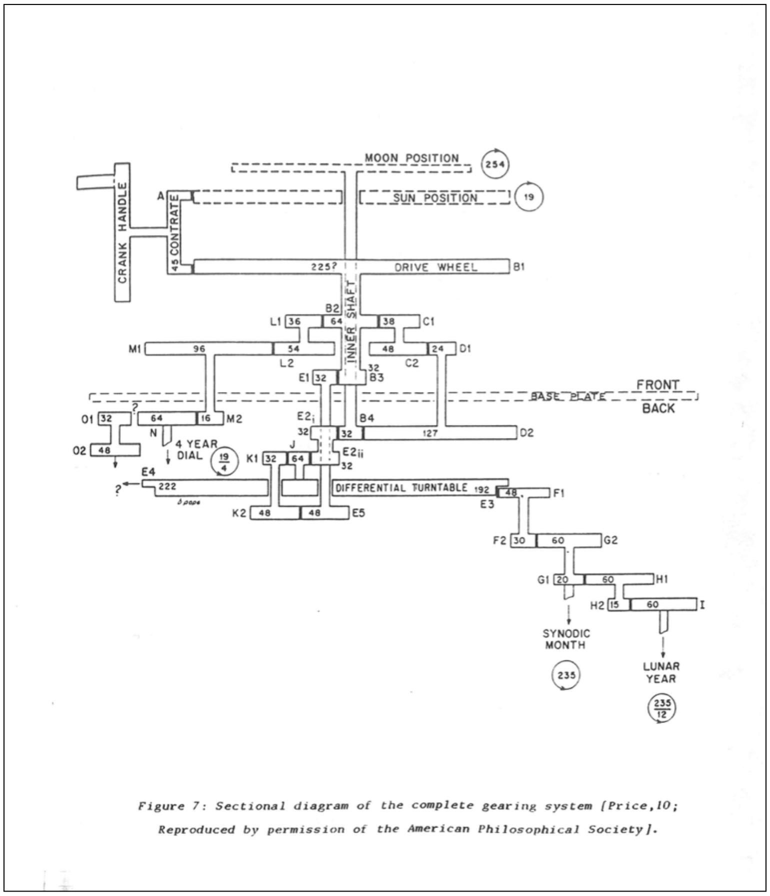

Figure 3: Price's Sectional Gearing Diagram [1]

- B2: 64 teeth
- C1: 38 teeth
- C2: 48 teeth
- D1: 24 teeth
- D2: 127 teeth
- E2: 32 teeth

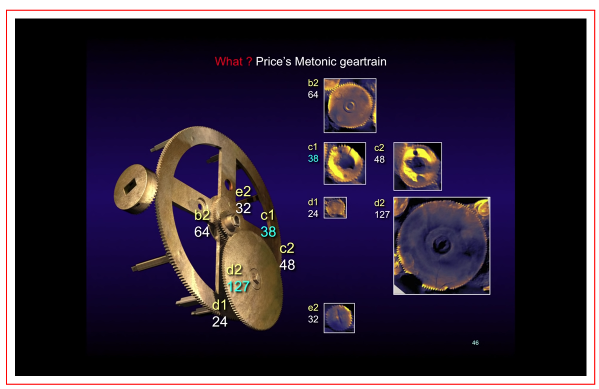

Figure 4: The Metonic Gear Train [5]

We know that in simple gear trains we can calculate the Gear Ratio (GR) as

$$\text{GR} = \frac{\text{Number of Teeth on the Driven Gear}}{\text{Number of Teeth on the Driver Gear}}$$

and we know that the driven gear rotates in the opposite direction of the driver gear.

With this information we can start to calculate what Price's Metonic gear train does.

Specifically:

$$\frac{\text{B2}}{\text{C1}} = -\frac{64}{38} = -\frac{32}{19} \quad \text{\# driver \& driven gears turn in opposite directions}$$

$$\frac{\text{B2}}{\text{C1}} \times \frac{\text{C2}}{\text{D1}} = -\frac{64}{38} \times -\frac{48}{24} = -\frac{32}{19} \times -\frac{2}{1} = \frac{64}{19} \quad \text{\# } \frac{\text{C2}}{\text{D1}} \text{ multiplies } \frac{\text{B2}}{\text{C1}} \text{ by 2}$$

$$\frac{\text{B2}}{\text{C1}} \times \frac{\text{C2}}{\text{D1}} \times -\frac{\text{D2}}{\text{E1}} = -\frac{64}{38} \times -\frac{48}{24} \times -\frac{127}{32} = -\frac{254}{19} \quad \text{\# } \frac{254}{19} \approx 13.36842$$

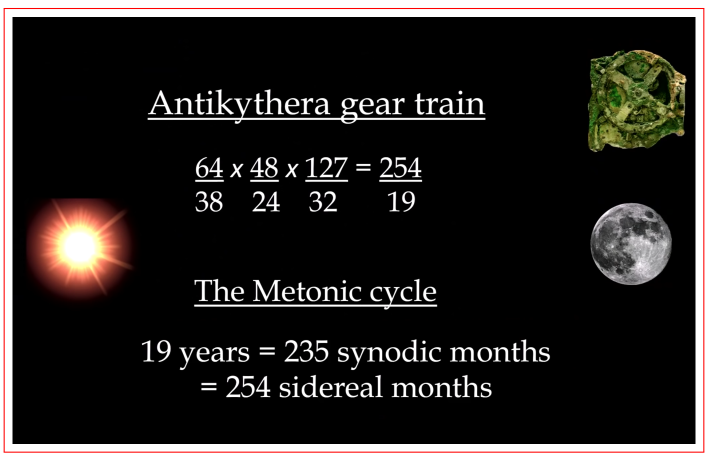

Figure 5: Metonic Gear Train Ratios and the Metonic Cycle

4 Putting it All Together

The Antikythera Mechanism is thought to have been operated by a knob or crank on the side of the device. This knob (or crank) was connected to a crown gear that meshed with

B1, the main drive gear. B1 is the large, four spoked gear seen in Fragment A (see Figure 6), and one revolution of B1 represents one year. Since B2 is planted on B1 to form a compound gear (B1 and B2 are connected to the same axle; see Figures 3 and 6), one revolution of B2 also represents one year.

Figure 6: Fragment A of the Antikythera Mechanism [8]

This configuration of gears means that one revolution of B2 (or B1) moves the Metonic pointer by one nineteenth of the Metonic Cycle, or 13.36842 sidereal months. Thus 19 revolutions of the main drive gear results in one revolution of the Metonic pointer or one Metonic Cycle, just as required.

5 Michael Wright's Metonic Cycle Gearing Scheme

Michael Wright suggested a different scheme for turning the Metonic pointer [12]. Wright noticed that 235 synodic (lunar) months $= 5 \times 47$ lunar months and proposed that the Metonic dial was a five turn spiral dial where each turn represented 47 lunar months (Price's Metonic Dial was a 4 turn dial). He proposed the following gearing scheme to move the Metonic pointer on this five turn dial:

- I1: 38 teeth
- B2: 64 teeth
- I2: 53 teeth
- M1: 96 teeth
- M2: 15 teeth
- N1: 53 teeth

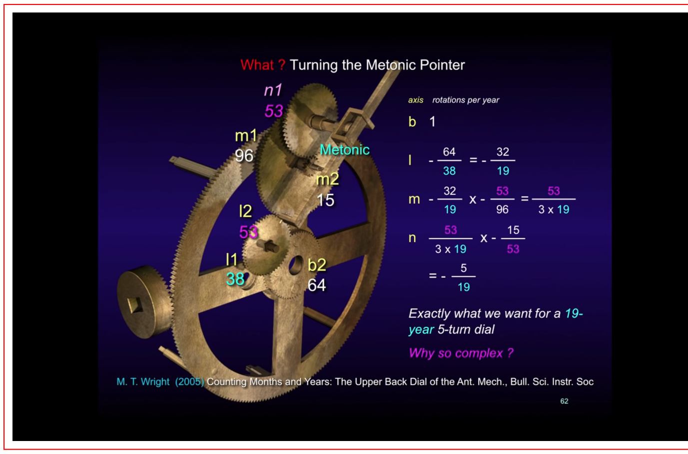

Figure 7: Wright's Metonic Gear Train [5]

Wright's gear ratios work out like this:

$$
\begin{aligned}
-\frac{\mathrm{B2}}{\mathrm{I1}} &= -\frac{64}{38} = -\frac{32}{19} \\
-\frac{\mathrm{B2}}{\mathrm{I1}} \times -\frac{\mathrm{I2}}{\mathrm{M1}} &= -\frac{64}{38} \times -\frac{53}{96} = -\frac{32}{19} \times -\frac{53}{96} = \frac{53}{3\times19} \\
-\frac{\mathrm{B2}}{\mathrm{11}} \times -\frac{\mathrm{I2}}{\mathrm{M1}} \times -\frac{\mathrm{M2}}{\mathrm{N1}} &= -\frac{64}{38} \times -\frac{53}{96} \times -\frac{15}{53} = -\frac{32}{19} \times -\frac{53}{3\times32} \times -\frac{3\times5}{53} = -\frac{5}{19}
\end{aligned}
$$

So one revolution of the main drive gear (B1) moves Wright's Metonic pointer by $\frac{5}{19} \approx 0.2632$, and nineteen revolutions of B1 (one Metonic Cycle) results in five revolutions of Wright's Metonic pointer, consistent with Wright's five turn spiral Metonic dial model.

BTW, note that the two 53 tooth gears I2 and N1 cancel in this gear train, so why are they there? Part of the answer is to drive the Saros eclipse prediction dial [3], a subject for another note. However, for reference: one Saros Cycle after an eclipse the Sun, Earth, and Moon return to approximately the same relative geometry (close to a straight line) and a nearly identical eclipse will occur in what is referred to as an eclipse cycle [11].

$$
\begin{aligned}
\text{One Saros Cycle} &= \text{6,585.321347 solar days} \\
&\approx \text{18.029 tropical years} \\
&\approx \text{223 synodic months} \\
&\approx \text{241.999 draconic months} \\
&\approx \text{238.992 anomalistic months} \\
&\approx \text{18.999 eclipse years (38 eclipse seasons)}
\end{aligned}
$$

6 Acknowledgements

Thanks to Lars-Johan Liman for all of his help and LaTeX expertise. Thanks also to Dave Neary and Marshall Eubanks for their formatting suggestions.

References

[1] Derek de Solla Price. Gears from the Greeks. The Antikythera Mechanism: A Calendar Computer from ca. 80 B. C. *Transactions of the American Philosophical Society*, 64(7):1–70, 1974.

[2] James Evans, Christián C. Carman, and Alan S. Thorndike. Solar Anomaly and Planetary Displays in the Antikythera Mechanism. *Journal for the History of Astronomy*, 41(1):1–39, Feb 2010.

[3] T. Freeth. Eclipse prediction on the ancient Greek astronomical calculating machine known as the Antikythera Mechanism. *PLoS One*, 9(7):e103275, 2014.

[4] T. Freeth, Y. Bitsakis, X. Moussas, J. H. Seiradakis, A. Tselikas, H. Mangou, M. Zafeiropoulou, R. Hadland, D. Bate, A. Ramsey, M. Allen, A. Crawley, P. Hockley, T. Malzbender, D. Gelb, W. Ambrisco, and M. G. Edmunds. Decoding the ancient Greek astronomical calculator known as the Antikythera Mechanism. *Nature*, 444(7119):587–591, Nov 2006.

[5] Tony Freeth. The Antikythera Mechanism: A Shocking Discovery from Ancient Greece. `https://www.youtube.com/watch?v=xWVA6TeUKYU&t=1295s`, 2021. [Online; accessed 19-March-2021].

[6] Tony Freeth, David Higgon, Aris Dacanalis, Lindsay MacDonald, Myrto Georgakopoulou, and Adam Wojcik. A Model of the Cosmos in the ancient Greek Antikythera Mechanism. *Scientific Reports*, 11(1):5821, Mar 2021.

[7] M.T. Wight. Epicyclic Gearing and the Antikythera Mechanism, parts 1 & 2. *Antiquarian Horology*, 29:54–60, Sep 2005.

[8] Wikipedia contributors. Antikythera mechanism — Wikipedia, the free encyclopedia. `https://en.wikipedia.org/w/index.php?title=Antikythera_mechanism&oldid=1012934141`, 2021. [Online; accessed 19-March-2021].

[9] Wikipedia contributors. "Derek J. de Solla Price — Wikipedia, The Free Encyclopedia". `https://en.wikipedia.org/w/index.php?title=Derek_J._de_Solla_Price&oldid=1007148723`, 2021. [Online; accessed 19-March-2021].

[10] Wikipedia contributors. "Meton of Athens — Wikipedia, The Free Encyclopedia". https://en.wikipedia.org/w/index.php?title=Meton_of_Athens&oldid=1011807751, 2021. [Online; accessed 19-March-2021].

[11] Wikipedia contributors. Saros (astronomy) — Wikipedia, the free encyclopedia. https://en.wikipedia.org/w/index.php?title=Saros_(astronomy)&oldid=1013468074, 2021. [Online; accessed 23-March-2021].

[12] M.T. Wright. Counting Months and Years: The Upper Back Dial of the Antikythera Mechanism 1. *Bulletin of the Scientific Instrument Society*, 87:8–13, Dec 2005.