

OPRE 3360, Spring 2023, Exam #1 Practice Questions

The table below lists data collected on seven customers arriving to a bank over an hour. The data listed for each customer are: customer ID, gender, savings account ownership (Y:yes, N:no), and annual income.

Table 1: Data on customers

Customer	Customer ID	Gender	Savings account ownership	Annual Income
1	5223	F	Y	\$40000
2	1478	M	Y	\$85000
3	7483	M	N	\$60000
4	1800	F	Y	\$40000
5	7396	F	Y	\$120000
6	9619	F	Y	\$30000
7	3541	M	Y	\$50000

Considering all the data available in Table 1, respond to Questions 1 through 4.

Question 1

How many elements are there in the data set?

- A. 35
- B. 8
- C. 28
- D. 4
- E. 7

Solution: (E)

Customers are the entities on which data are collected. Therefore, there are 7 elements.

Question 2

How many variables are there in the data set?

- A. 6
- B. 4
- C. 7

- D. 5
- E. 13

Solution: (B)

The variables are: customer ID, gender, savings account ownership (Y:yes, N:no), and annual income. There are 4 variables.

Question 3

How many observations are there in the data set?

- A. 28
- B. 4
- C. 7
- D. 60
- E. 35

Solution: (C)

This is a complete data set with 7 elements. Therefore, there are 7 observations.

Question 4

What is the scale of measurement for variable "gender"?

- A. Nominal
- B. Ordinal
- C. Ratio
- D. Interval
- E. None of the above

Solution: (A)

Gender is a categorical type of data in which ordering is irrelevant. Therefore, it is "nominal".

The table below includes data on two characteristics of interest (calories and protein) in one serving of different protein bars.

Table 2: Data on protein bars

Protein bar	Calories	Protein (g)
Quest Bar	200	20
Power Crunch	205	14
Detour	150	15
MET-Rx	310	30
Think Thin	230	20
KIND	180	5
Larabar	230	6
Nature Valley	190	10

Considering all the data available in Table 2, respond to Questions 5 through 7

Question 5

What is the percentage frequency of protein bars with less than 200 calories?

- A. 50%
- B. 0.5
- C. 0.375
- D. 37.5%
- E. 3/10

Solution: (D)

There are 8 protein bars listed in Table 2. Three of them have their calories less than 200. Therefore, the required percentage frequency is $(3/8) \times 100\% = 37.5\%$.

Question 6

Of all the protein bars in Table 2 with at least 200 calories, what percentage have at least 20 g. of protein?

- A. 50%
- B. 30%

- C. 40%
- D. 60%
- E. 100%

Solution: (D)

There are five protein bars in Table 2 with at least 200 calories. Out of these five protein bars, three of them have at least 20 g. of protein. That is, $(3/5) \times 100\% = 60\%$ of the protein bars with at least 200 calories have a protein amount of at least 20 g.

Question 7

What is the relative frequency of the protein bars in Table 2 that have at least 15 g of protein?

- A. 50
- B. 50%
- C. 0.5
- D. 80%
- E. 0.8

Solution: (C)

Out of 8 protein bars listed, 4 of them have at least 15 g of protein. The relative frequency of the bars that satisfy the given condition is then $4/8 = 0.5$.

The table below includes data regarding the waiting times (in minutes) of five randomly selected customers in the line of a call center.

Table 3: Customer waiting times in minutes

6	10	5	7	2
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Considering all the data available in Table 3, respond to Questions 8 through 12.

Question 8

What is the average waiting time?

- A. 6 seconds

- B. 360 seconds
- C. 9.6 minutes
- D. 7 minutes
- E. None of the above

Solution: (B)

$(6+10+5+7+2)/5=6$ minutes=360 seconds

Question 9

What is the median of the waiting times?

- A. 0.5
- B. 6 seconds
- C. 6 minutes
- D. 5 minutes
- E. 360 minutes

Solution: (C)

Numbers sorted: 2, 5, 6, 7, 10

The mid value is 6 minutes.

Question 10

What is the 25th percentile of the waiting times in Table 3?

- A. 1.75
- B. 1.5 minutes
- C. 3.5 seconds
- D. 5.25 minutes
- E. 3.5 minutes

Solution: (E)

Numbers sorted: 2, 5, 6, 7, 10

The location of the 25th percentile is $(25/100) \times (5+1) = 1.5$

The value that occupies this location in the sorted sequence is $2 + (5 - 2) \times 0.5 = 3.5$ minutes

Question 11

What is the interquartile range of the waiting times in Table 3?

- A. 5 minutes
- B. 4.5 minutes
- C. 6.5 minutes
- D. 6 minutes
- E. None of the above

Solution: (A)

Interquartile range (IQR) is the difference between third quartile (Q_3) and first quartile (Q_1). We found $Q_1=3.5$ in Question 10. The location of Q_3 is $(75/100) \times (5+1) = 4.5$. Therefore, $Q_3 = 7 + 0.5 \times (10-7) = 8.5$. It turns out that $IQR = Q_3 - Q_1 = 8.5 - 3.5 = 5$ minutes.

Question 12

What is the standard deviation of the customer waiting times in Table 3?

- A. 2.16 (minutes)^2
- B. 2.92 minutes
- C. 8.5 (minutes)^2
- D. 8.5 minutes
- E. 3.56 (minutes)^2

Solution: (B)

Sample standard deviation is given by

$$\sqrt{\frac{(x_i - \bar{x})^2}{n-1}} = \sqrt{\frac{(2-6)^2 + (5-6)^2 + (6-6)^2 + (7-6)^2 + (10-6)^2}{5-1}} = \sqrt{\frac{16+1+0+1+16}{4}} = \sqrt{\frac{34}{4}} = \sqrt{8.5} = 2.92 \text{ min}$$

Question 13

If A and B are mutually exclusive events with $P(A) = 0.1$ and $P(B) = 0.6$, then what is $P(A \cup B)$?

- A. 1
- B. 0.9
- C. Impossible to determine without knowing $P(A \cap B)$.
- D. 0.7
- E. 0.02

Solution: (D)

For mutually exclusive events, we have $P(A \cup B) = P(A) + P(B)$. Then,
 $P(A \cup B) = 0.1 + 0.6 = 0.7$

Question 14

If $P(A) = 0.5$, $P(B) = 0.8$, and $P(A \cup B) = 0.9$, then what is $P(A \cap B)$?

- A. 0.5
- B. 0.4
- C. 0.3
- D. 0.1
- E. 0.7

Solution: (B)

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$. Therefore, $0.9 = 0.5 + 0.8 - P(A \cap B)$, which implies $P(A \cap B) = 0.4$.

Question 15

If $P(A) = 0.12$, $P(B) = 0.3$, and $P(B|A) = 0.6$, then what is $P(A|B)$?

- A. 0.09
- B. 0.6
- C. 0.24
- D. 0.45
- E. 0.35

Solution: (C)

$P(B|A) = P(A \cap B) / P(A)$. If we plug in the given info, we have $0.6 = P(A \cap B) / 0.12$. This implies $P(A \cap B) = 0.072$.

Now, in order to compute $P(A|B)$, we will use the definition of conditional probabilities once again. That is, $P(A|B) = P(A \cap B) / P(B)$. Since $P(A \cap B) = 0.072$ and $P(B) = 0.3$, we have $P(A|B) = 0.072 / 0.3 = 0.24$.

Question 16

A six-sided die is rolled twice. Compute the probability of observing 4 in the first roll, and observing 2 in the second roll.

- A. 1/216
- B. 1/30

- C. $1/36$
- D. $1/4$
- E. $1/23$

Solution: (C)

This is a two-stage experiment. In each stage, there are 6 outcomes. Overall there are $6 \times 6 = 36$ outcomes. (4,2) is just one of these equally likely outcomes. Therefore, by the classical method of assigning probabilities, we know that (4,2) has a probability of $1/36$.

Question 17

Suppose that two letters are to be selected from X, Y, and Z, and arranged in order. How many permutations are possible?

- A. 5
- B. 25
- C. 6
- D. 20
- E. None of the above

Solution: (C)

$$P_2^3 = \frac{3!}{(3-2)!} = \frac{3 \times 2 \times 1}{1} = 6$$

Question 18

A student club has 7 candidates to fill 2 similar positions. What is the total number of possible ways this selection can be made?

- A. 42
- B. 14
- C. 21
- D. 8
- E. 4

Solution: (C)

$$C_2^7 = \frac{7!}{(7-2)!2!} = \frac{7 \times 6 \times 5!}{5! \times 2 \times 1} = 21$$