CS 355 Homework #5:
3D Rendering Geometry

DO NOT DISTRIBUTE

1. A camera is located at position $(25, 20, 5)$ in the 3D world and is looking at the point $(25, 40, 25)$ so that the direction $[0, 1, 0]$ points (roughly!) up.

   (a) Use the process we covered in class (a 3D variant of Gram-Schmidt orthonormalization using cross products) to calculate the camera's x, y, and z axis directions.

   ANSWER:

   Subtracting the "look from" point from the "look at" point gives the vector $[0, 20, 20]$*, which when normalized gives the viewing direction vector* $\frac{1}{\sqrt{2}}[0, 1, 1]$*. Taking the cross product between this and the rough "up" vector gives the* $\mathbf{x}$ *direction, and then taking the cross product between that and the viewing direction gives the* $\mathbf{y}$ *direction:*

$$\mathbf{x} = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} \qquad \mathbf{y} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} \qquad \mathbf{z} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

   (b) Write this camera's world-to-camera transformation as the composition of a rotation matrix and translation matrix. (You do not have to multiply out this matrix.)

   ANSWER:

$$\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -25 \\ 0 & 1 & 0 & -20 \\ 0 & 0 & 1 & -5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

   (c) What are the camera-space coordinates of the point $\mathbf{p}_w = (5, 6, 7)$?

   ANSWER:

$$\mathbf{p}_c = (20, -8\sqrt{2}, -6\sqrt{2}) \approx (20.00, -11.31, -8.49)$$

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2. A camera is located at position $(20, 5, -40)$ and oriented so that it is pointing parallel to the x-z plane at an angle of 30 degrees off the z axis. (This is the basic setup for Labs #4 and #5.)

   (a) Write this camera's world-to-camera transformation using the composition of a 3D rotation matrix (around the y axis) and a translation matrix. (You also do not have to multiply out this matrix. You may also leave your answer in terms of trig functions.)
   ANSWER:
   $$\begin{bmatrix} \cos\frac{\pi}{6} & 0 & \sin\frac{\pi}{6} & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\frac{\pi}{6} & 0 & \cos\frac{\pi}{6} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -20 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 40 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

   (b) What are the camera-space coordinates of the point $\mathbf{p}_w = (5, 6, 7)$?
   ANSWER:
   $$\mathbf{p}_c \approx (-36.49, 1, 33.20)$$
   or, depending on which way you rotate off the z-axis,
   $$\mathbf{p}_c \approx (10.51, 1, 48.20)$$

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3. A virtual camera has the following parameters:

   - vertical field of view of 60 degrees
   - aspect ratio of 16:9 (horizontal to vertical)
   - near plane $n = 10$
   - far plane $f = 1000$

   (a) What is the clip matrix for this camera?

   ANSWER:

$$\begin{bmatrix} \frac{9}{16}\sqrt{3} & 0 & 0 & 0 \\ 0 & \sqrt{3} & 0 & 0 \\ 0 & 0 & \frac{101}{99} & \frac{-2000}{99} \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

   (b) What are the clip-space coordinates of the camera-space point $\mathbf{p}_c = (5, -5, 50)$?

   ANSWER:

$$\mathbf{p}_{clip} = (\frac{45}{16}\sqrt{3}, -5\sqrt{3}, \frac{3050}{99}, 50) \approx (4.87, -8.66, 30.80, 50)$$

   (c) Is this point $\mathbf{p}_c = (5, -5, 50)$ within the view frustum of this camera? How can you tell without doing a devision?

   ANSWER: *Yes. The first three elements of the previous answer are within plus or minus the last one.*

   (d) What are the canonical coordinates of this point $\mathbf{p}_c = (5, -5, 50)$?

   ANSWER:

   Normalizing the clip-space coordinate and keeping only the x and y coordinates:

$$\mathbf{p}_{canonical} = (\frac{9\sqrt{3}}{160}, \frac{-\sqrt{3}}{10}) \approx (0.097, -0.173)$$

   (e) If rendered to a high-definition display (1920×1080), what are the screen coordinates of this point?

   ANSWER:

$$\mathbf{p}_{screen} = (960 + 54\sqrt{3}, 540 + 54\sqrt{3}) \approx (1053.53, 633.53)$$