ECE 322L
Electronics 2

02/25/20 - Lecture 10

Cascode Amplifiers

The Bipolar Junction Transistor

Updates and overview

- I have posted two handouts within Lecture 10 folder:
  - Handout about p-n junctions
  - Handout about the cascode amplifier.
- Cascode amplifiers (Neamen 4.8.2)
- The Bipolar Junction Transistor (BJT):

  Structure, Operating regions,DC analysis,

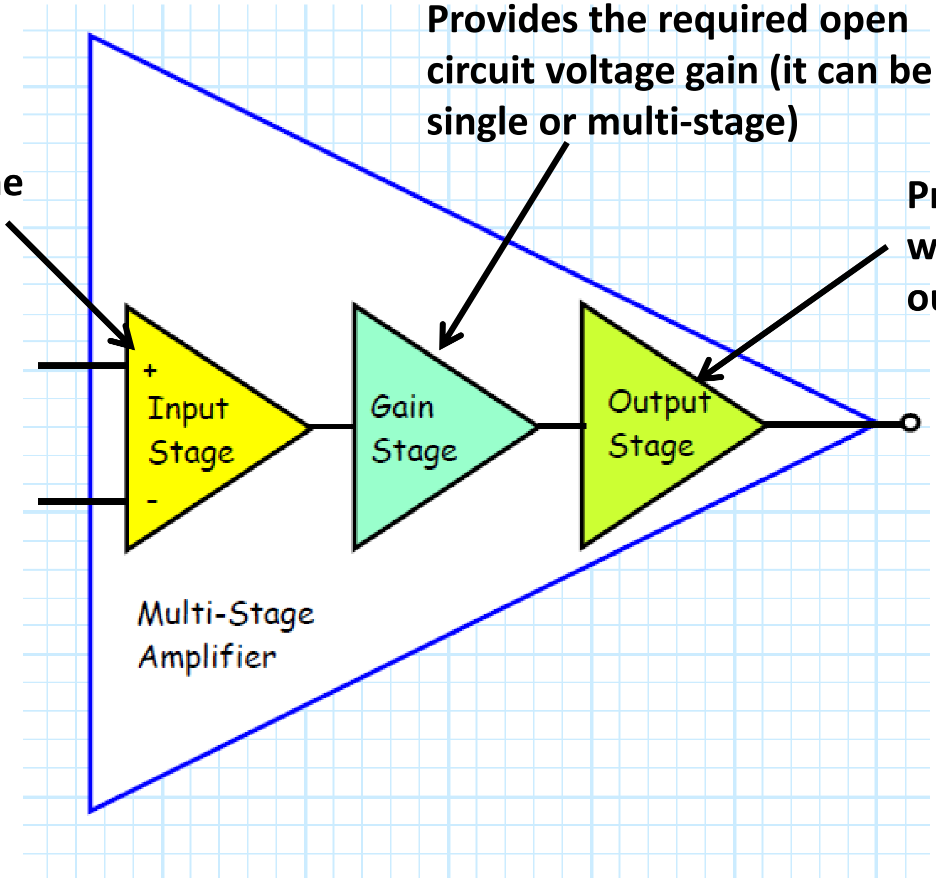
  Load lines (Neamen-From 5.1.1 to 5.1.5)

Multi-stage amplifiers

Provides the amplifier with the optimum input resistance

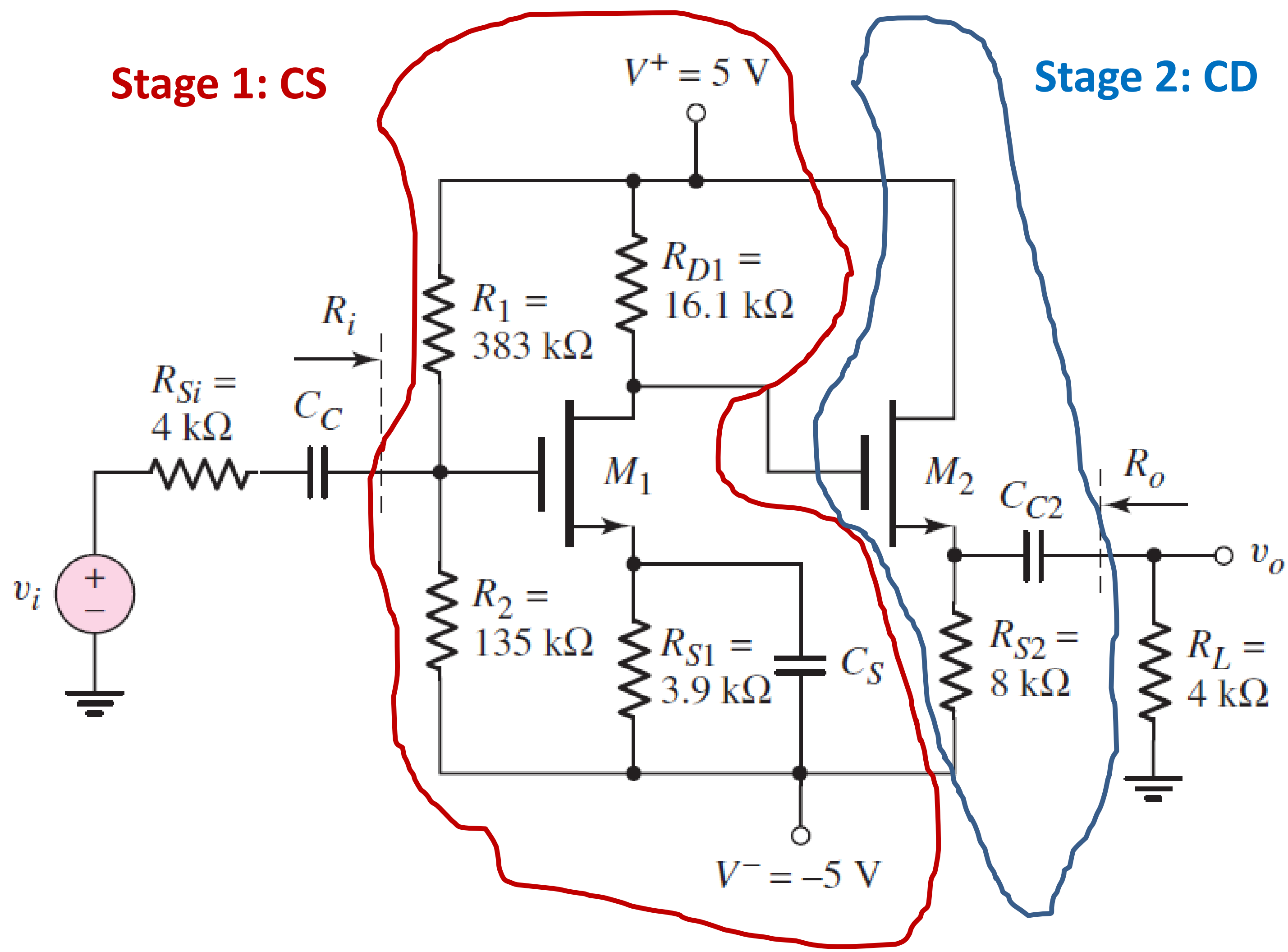
Provides the required open circuit voltage gain (it can be single or multi-stage)

Provides the amplifier with the optimum output resistance

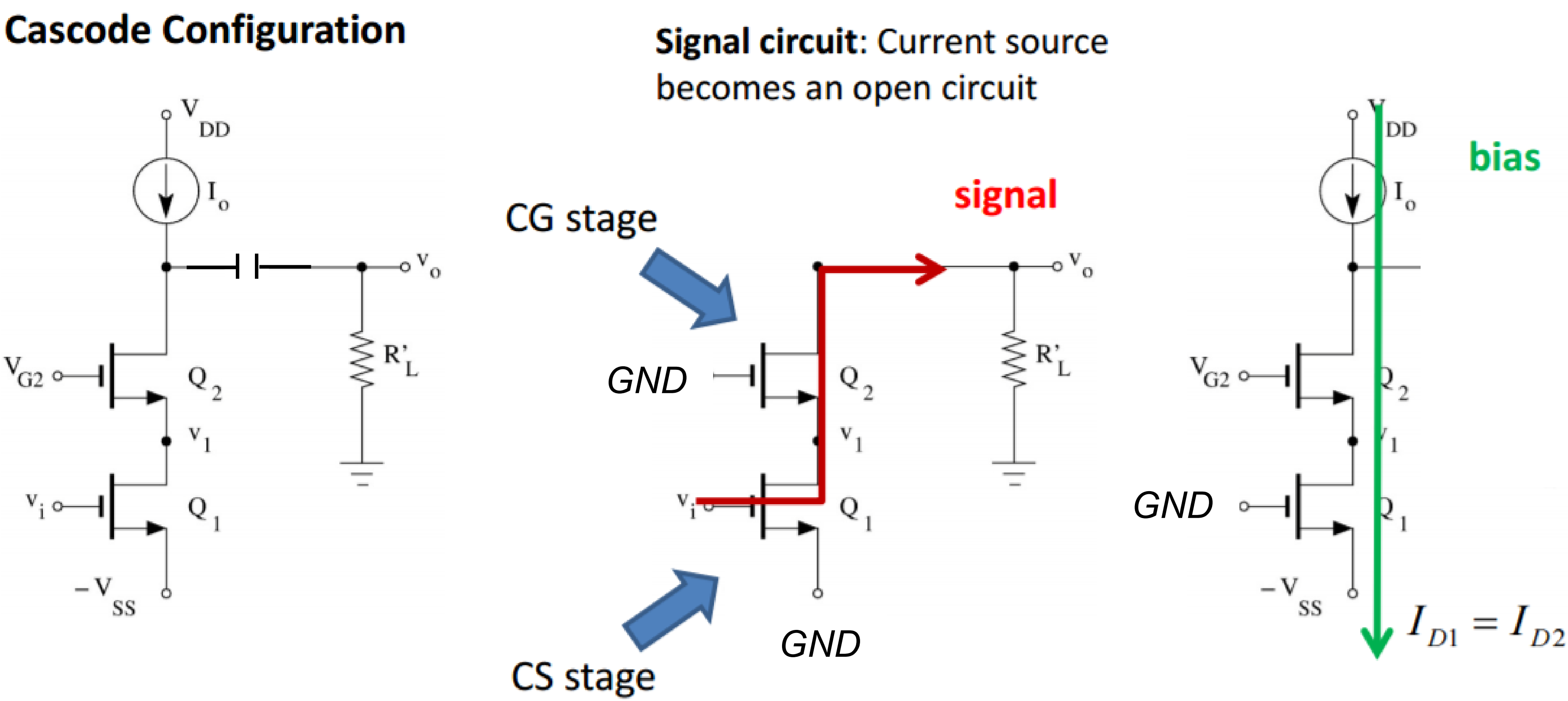


It is important to understand which basic configurations to select per each stage, and how placing two basic configurations in series affects their performance

Cascade amplifier

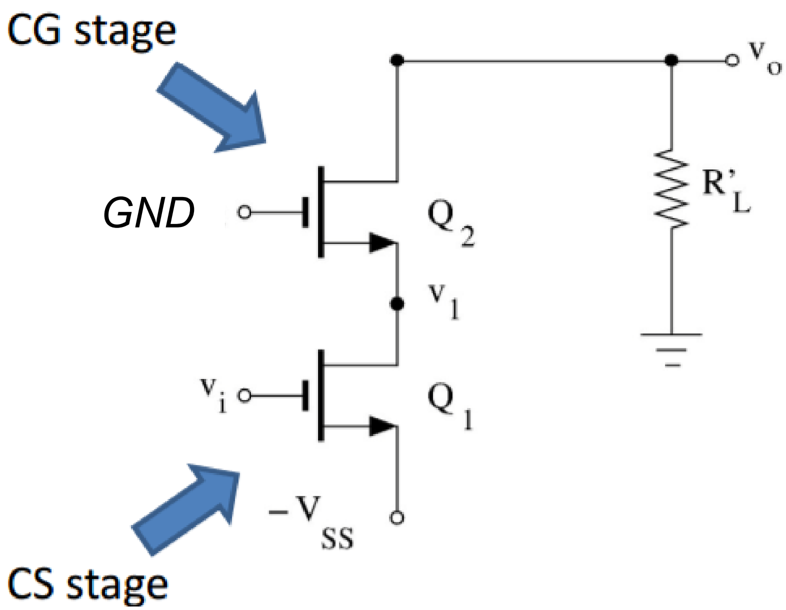


Cascode amplifier: conceptual circuit

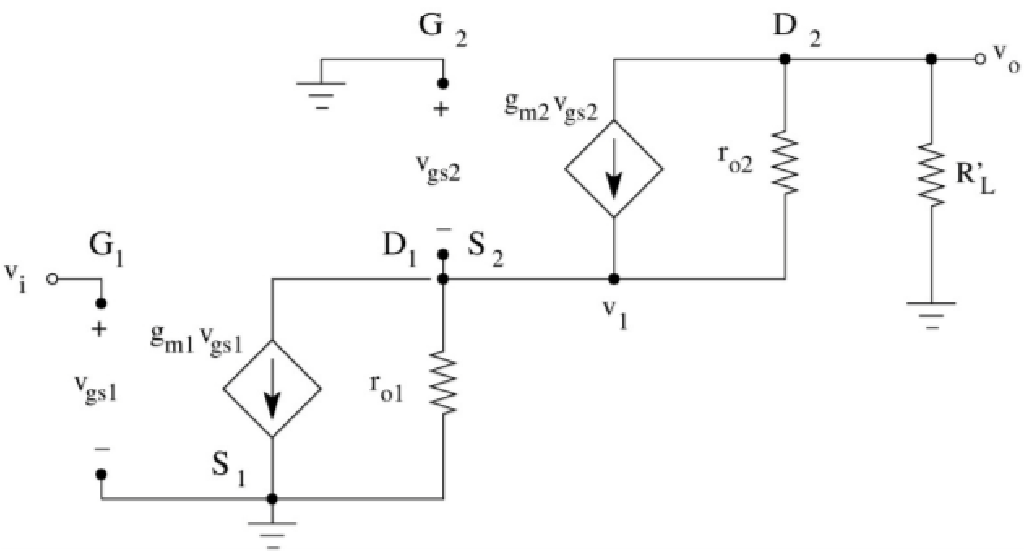


Cascode amplifier is a two-stage, CS-CG configuration

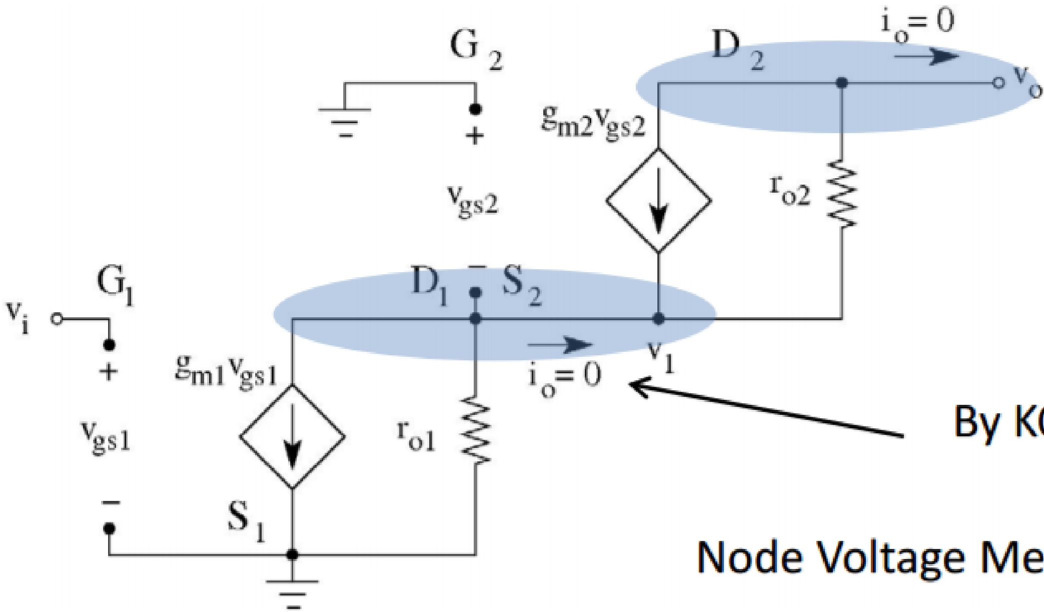
Cascode amplifier



Small Signal Model



Cascode amplifier



Open Loop Gain ($R'_L \to \infty$, $i_o = 0$):

By KCL around Q2

Node Voltage Method:

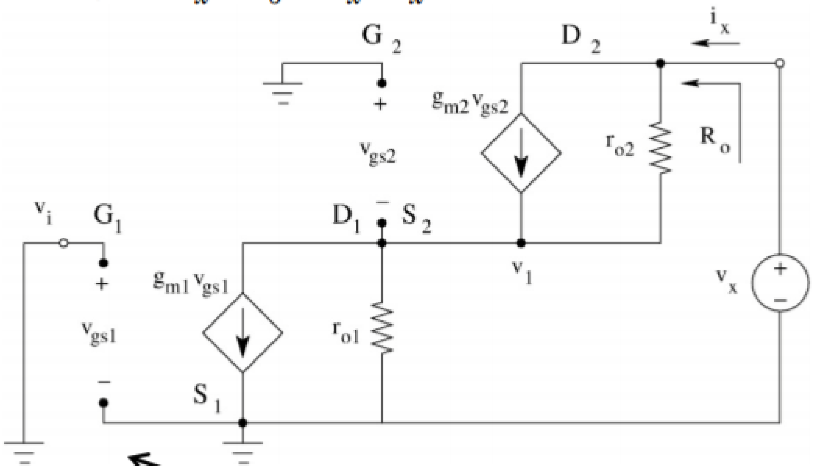
Node v_o: $$\frac{v_o - v_1}{r_{o2}} - g_{m2}v_1 = 0 \quad \Rightarrow \quad v_o = (1 + g_{m2}r_{o2})\, v_1$$

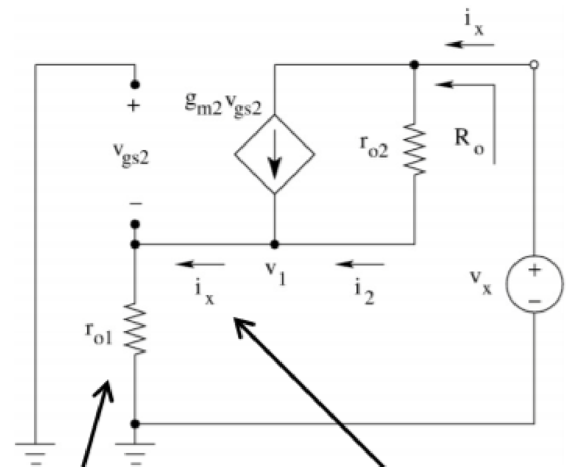
Node v_1: $$\frac{v_1}{r_{o1}} + g_{m1}v_i + 0 = 0 \quad \Rightarrow \quad v_1 = -g_{m1}r_{o1}\, v_i$$

$$\boxed{A_{vo} = \frac{v_o}{v_i} = -g_{m1}r_{o1} \times (1 + g_{m2}r_{o2}) \approx -g_{m1}r_{o1}g_{m2}r_{o2}}$$

Cascode amplifier

Set $v_i = 0$, attach a voltage source v_x,
compute i_x, $R_o = v_x / i_x$





$v_i = v_{gs1} = 0 \rightarrow g_{m1}\, v_{gs1}$ current source becomes open circuit

By KCL around Q2

KVL : $v_{gs2} = -i_x r_{o1}$

KCL : $i_2 = i_x - g_{m2} v_{gs2} = i_x + i_x g_{m2} r_{o1} = i_x (1 + g_{m2} r_{o1})$

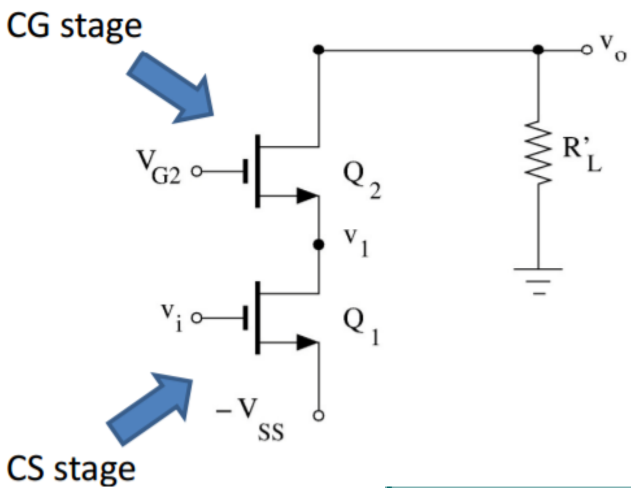
KVL : $v_x = i_2 r_{o2} + i_x r_{o1} = i_x (1 + g_{m2} r_{o1}) r_{o2} + i_x r_{o1}$

$v_x = i_x [(1 + g_{m2} r_{o1}) r_{o2} + r_{o1}]$

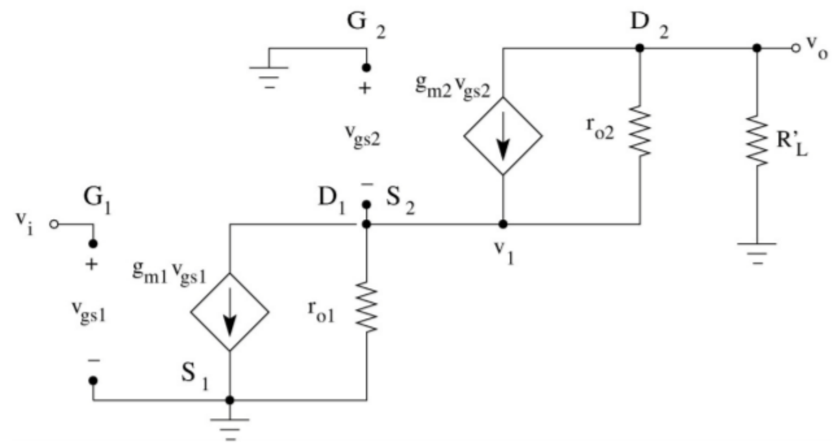
$$\boxed{R_o = \frac{v_x}{i_x} = r_{o1} + r_{o2} + g_{m2}\, r_{o1}\, r_{o2}}$$

Note: $A_v = A_{vo} \times \dfrac{R_L' + R_o}{R_L'}$

Cascode amplifier



Small Signal Model



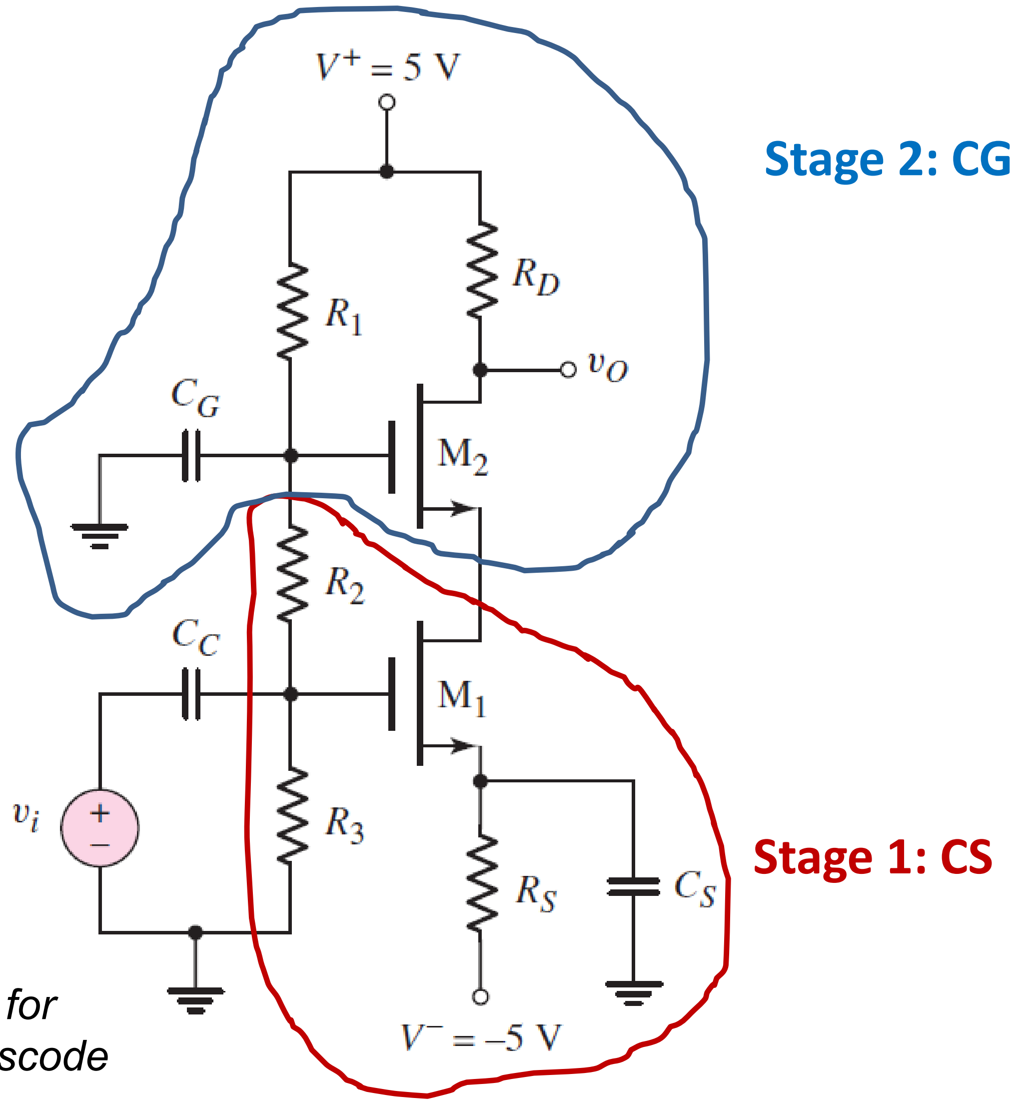
$\boxed{A_{v2} = g_{m2}(r_{o2} \parallel R'_L)}$ $\boxed{R_{L1} = R_{i2} = \dfrac{r_{o2} + R'_L}{1 + g_{m2} r_{o2}}}$ $\boxed{A_{v1} = -g_{m1}(r_{o1} \parallel R_{i2})}$ $\boxed{R_o \approx g_{m2}\, r_{o1}\, r_{o2}}$

For simplicity assume $r_{o1} = r_{o2} = r_o$ and $g_{m1} = g_{m2} = g_m$

R'_L	A_{v2}(CG)	$R_{i2} = R_{L1}$	A_{v1} (CS)	$A_v = A_{v1}\, A_{v2}$	
∞	$g_m r_o$	∞	$-g_m r_o$	$-(g_m r_o)^2$	**Max. Gain**

**The cascode provides a high gain to large loads (i.e., loads that are comparable to or larger than to r_o). Later on we'll see that the cascode configuration provides a high bandwidth.*

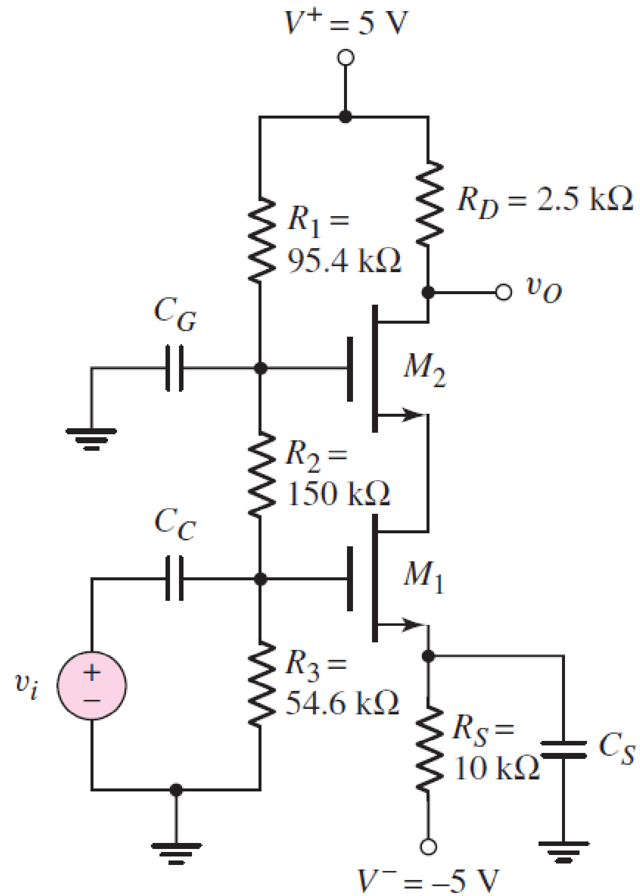
Cascode amplifier: practical circuit



**See handout in Lec 10 folder for additional details about the cascode circuit analysis.*

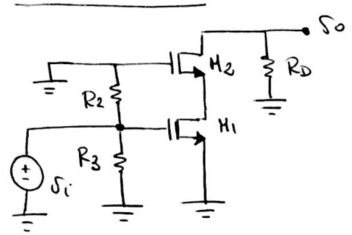
Take-home problem 1

The transistor parameters of the NMOS cascode circuit below are $V_{TN1}=V_{TN2}=0.8$ V, $K_{n1}=K_{n2}=3$ mA/V^2, $\lambda_1=\lambda_2=0.02$ V^{-1}. The coordinates of the Q point are the following: $I_{DQ}=0.471$ mA, $V_{DSQ1}=2.5$ V, $V_{DSQ2}=1.61$ V. Calculate the small-signal voltage gain of stage 1 and 2 when isolated and when connected.

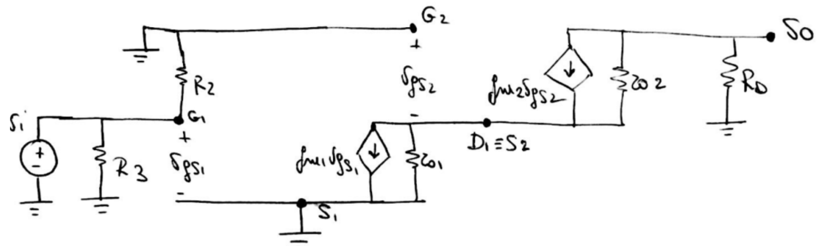


Take-home problem 1, Sol

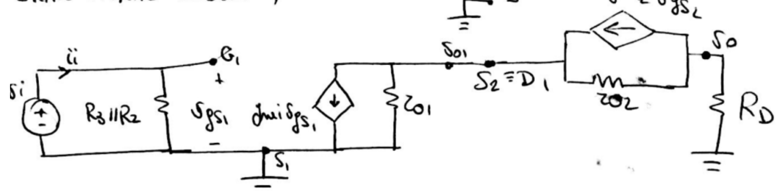
AC CIRCUIT



SMALL-SIGNAL MODEL (I)



SMALL-SIGNAL MODEL (II)



$R_i = \frac{v_i}{i_i} = R_3 // R_2 = 40K\Omega$

To calculate the gain we can regard RD as a load and model the amplifier as a generic voltage amplifier

$v_o = A_{vo} v_i \frac{R_D}{R_o + R_D}$

$A_v = \frac{v_o}{v_i} = A_{vo} \cdot \frac{R_D}{R_o + R_D}$

Take-home problem 1, Sol

$$A_{vo} = \frac{v_o}{v_i}\Big|_{R_D=\infty}$$

Node v_o: $\frac{v_{ro2}}{r_{o2}} + g_{m2} v_{gs2} = 0 \qquad v_{gs2} = -v_{o1}$

$$\frac{v_o - v_{o1}}{r_{o2}} - g_{m2} v_{o1} = 0 \Rightarrow v_o = (1+g_{m2} r_{o2})\, v_{o1} \quad (1)$$

Node v_{o1}: $\frac{v_{ro1}}{r_{o1}} + g_{m1} v_{gs1} = 0$

$$\frac{v_{o1}}{r_{o1}} + g_{m1} v_i = 0 \Rightarrow v_{o1} = -g_{m1} r_{o1} v_i \quad (2)$$

Combining (1) and (2) I obtain

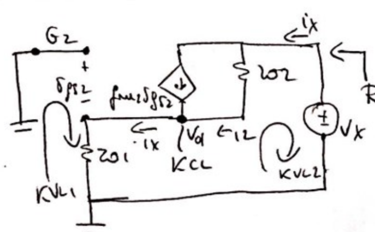
$$v_o = -g_{m1} r_{o1} \cdot (1+g_{m2} r_{o2})\, v_i$$

Hence

$$A_{vo} = \frac{v_o}{v_i}\Big|_{R_D=\infty} = -g_{m1} r_{o1}(1+g_{m2} r_{o2}) \cong -g_{m1} g_{m2} r_{o1} r_{o2} =$$

$$= -g_m^2 r_o^2$$

To calculate the output resistance we set $v_i = 0 \Rightarrow v_{gs1} = 0 \Rightarrow g_{m1} v_{gs1} = 0$



$R_o = \frac{v_x}{i_x}$

KVL1: $v_{gs2} = -i_x r_{o1}$

KCL: $i_2 = i_x - g_{m2} v_{gs2} =$

$= i_x + i_x g_{m2} r_{o1} = i_x(1+g_{m2} r_{o1})$

KVL2: $v_x = i_2 r_{o2} + i_x r_{o1} =$

$= i_x(1+g_{m2} r_o) r_{o2} + i_x r_{o1}$

$v_x = i_x[(1+g_{m2} r_o) r_{o2} + r_{o1}]$

Take-home problem 1, Sol

$$R_o = \frac{v_x}{i_x} = r_o + r_{o2} + g_{m2} r_{o1} r_{o2} \simeq g_{m2} r_o^2$$

$$A_v = -g_m^2 r_o^2 \frac{R_D}{R_D + g_{m2} r_o^2}$$

$$g_m = 2\sqrt{K_{n1,2} I_{DQ}} = 2\sqrt{3m \cdot 0.471m} = 2.3\,mA/V$$

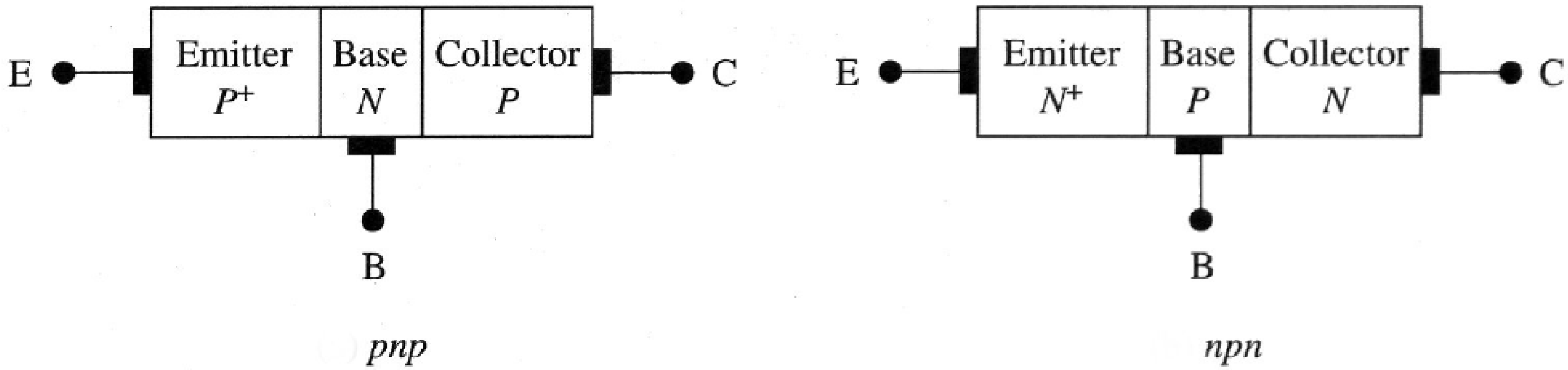
$$r_o = \frac{1}{\lambda I_{DQ}} = \frac{1}{0.02m \cdot 0.471m} = 106\,k\Omega$$

$$A_v = -(2.3m)^2 (106k)^2 \cdot \frac{2.5K}{2.5K + (2.3m)(106k)^2} \simeq$$

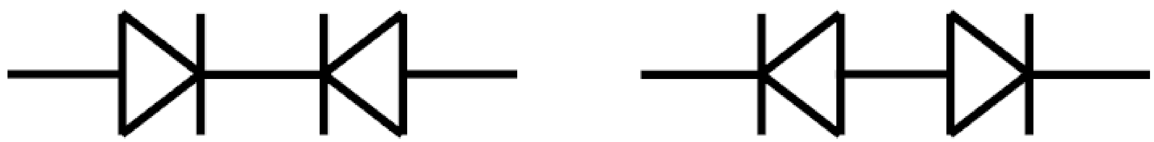
$$\simeq -5.75$$

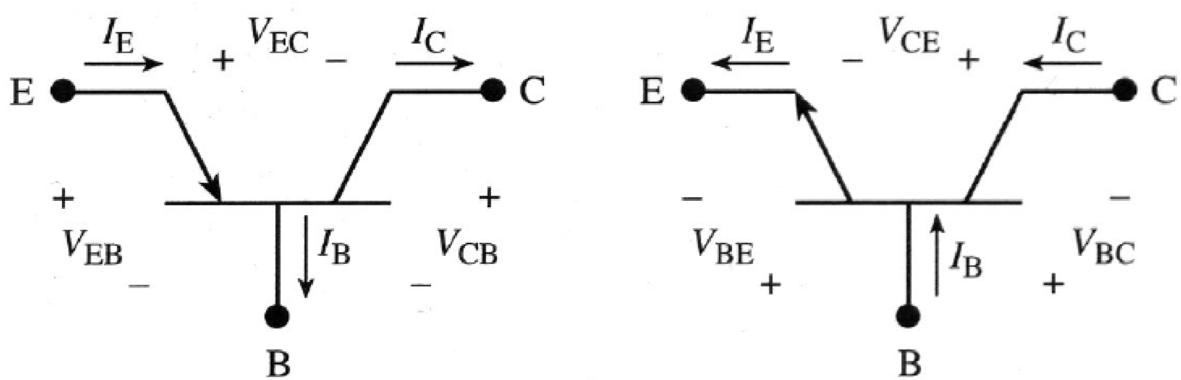
$$R_o = 25.8\,K\Omega$$

Bipolar junction transistor: structure

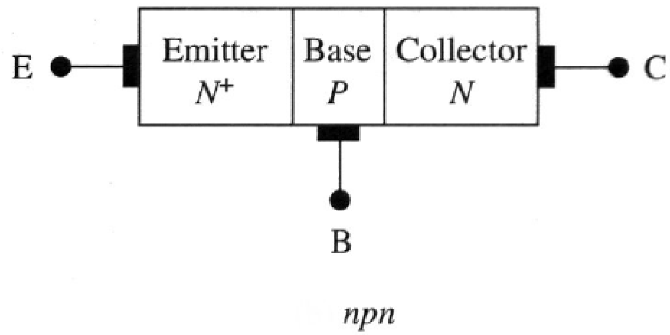


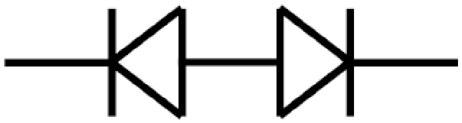
Looks sort of like two diodes back to back

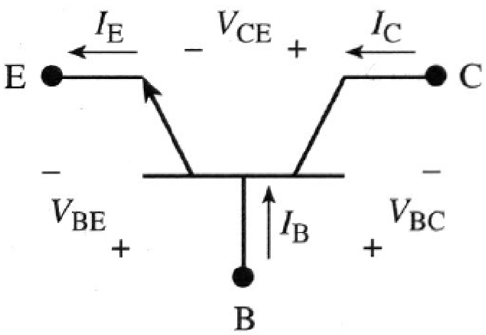


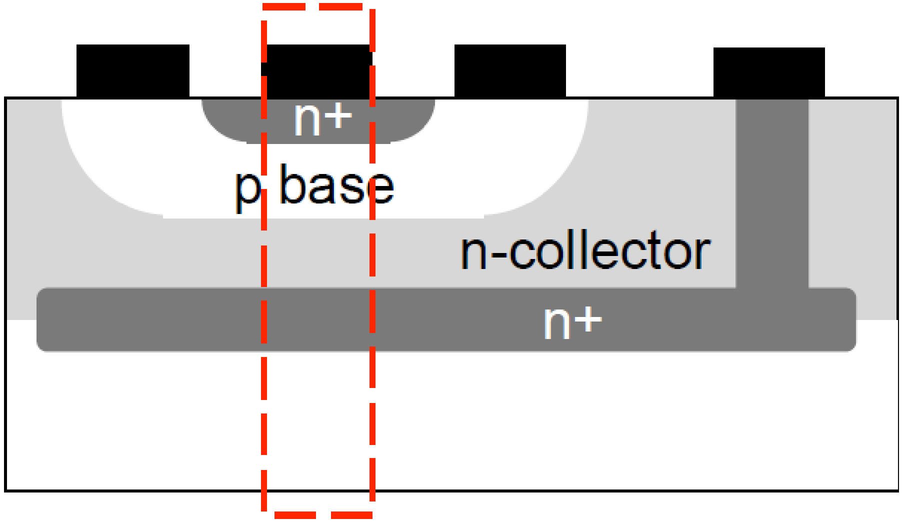


Bipolar junction transistor: structure







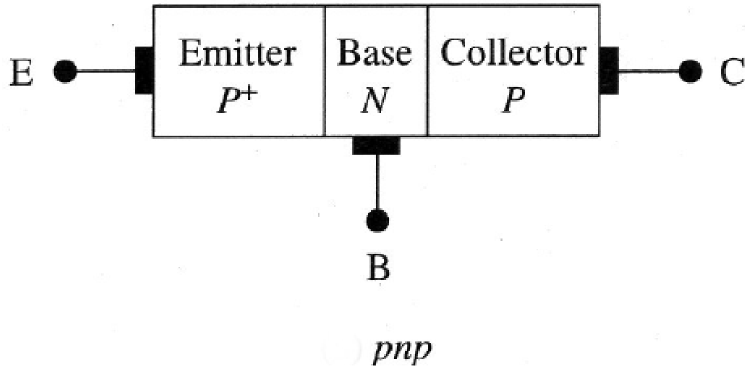


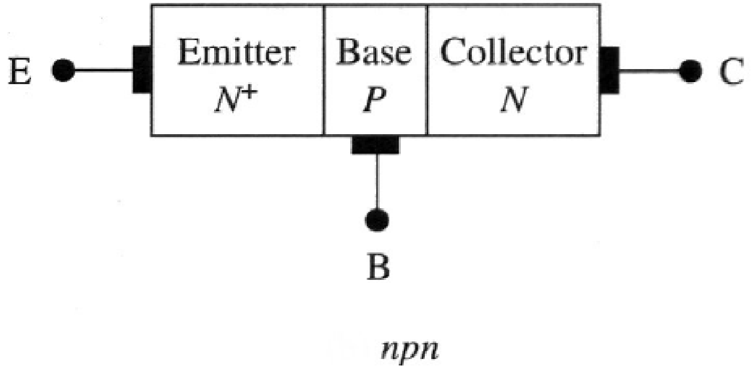
- Base is narrow (<10s μm) so the two p-n junctions can interact (W_B<< L_n).
- Doping decreases from the emitter to the collector so that switching the polarity of the two ends will lead to a drastically different behavior

W_B: Width of the quasi-neutral region in the base
L_n: Diffusion length of minority carriers in the base

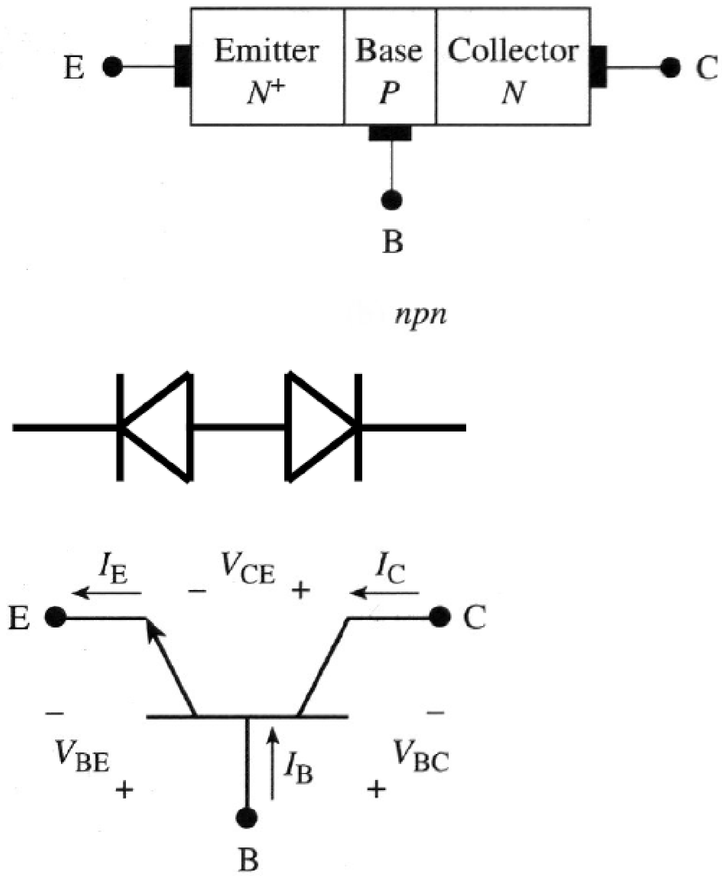
Bipolar junction transistor: operation

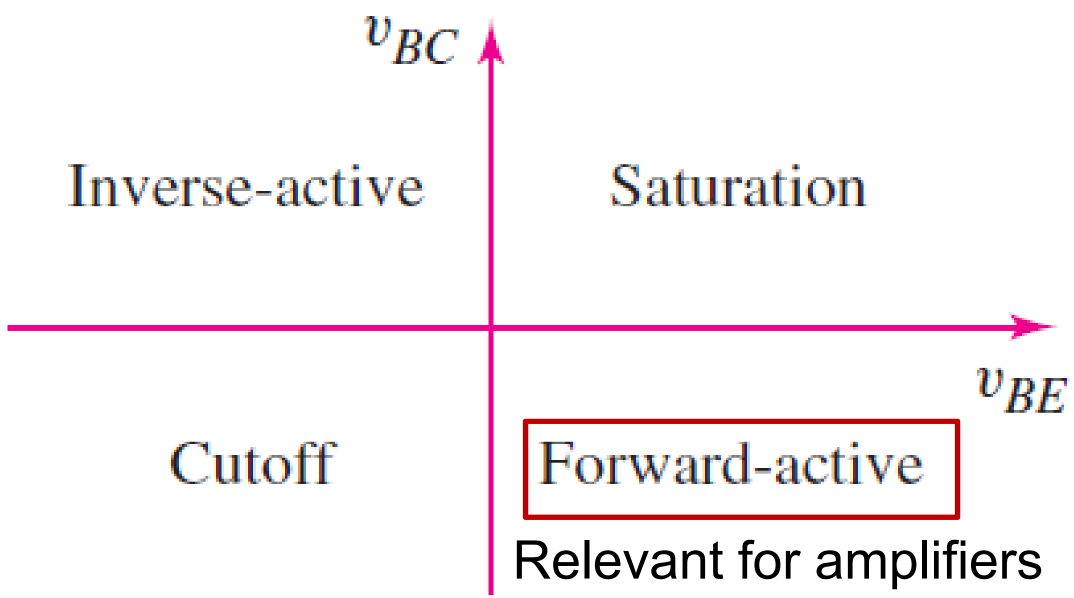
Biasing Mode	*Biasing Polarity E–B Junction*	*Biasing Polarity C–B Junction*
Saturation	Forward	Forward
Active	**Forward**	**Reverse**
Inverted	Reverse	Forward
Cutoff	Reverse	Reverse





Bipolar junction transistor: operation

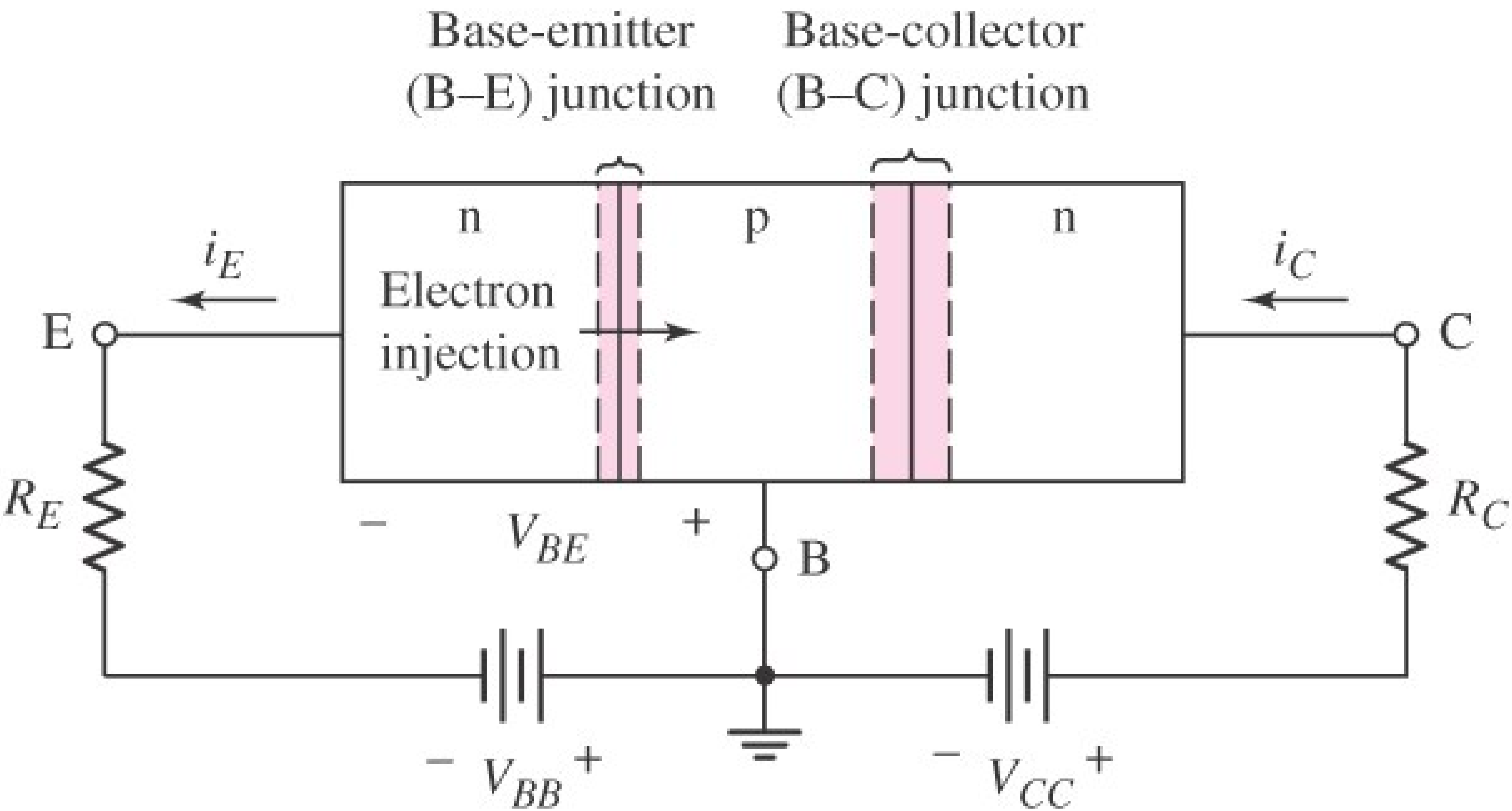




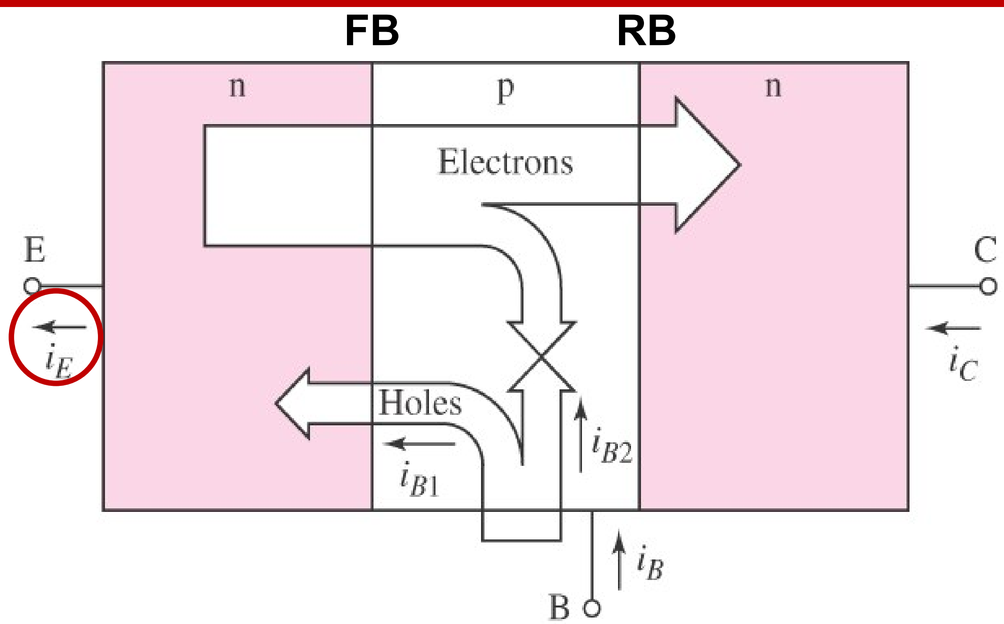
Note: Replace v_{BC} by v_{CB} and v_{BE} by v_{EB} for pnp transistor

BJT in forward-active mode

Operation in forward-active region or mode



BJT in forward active mode: currents



The direction of the current follows the flow of positive charges

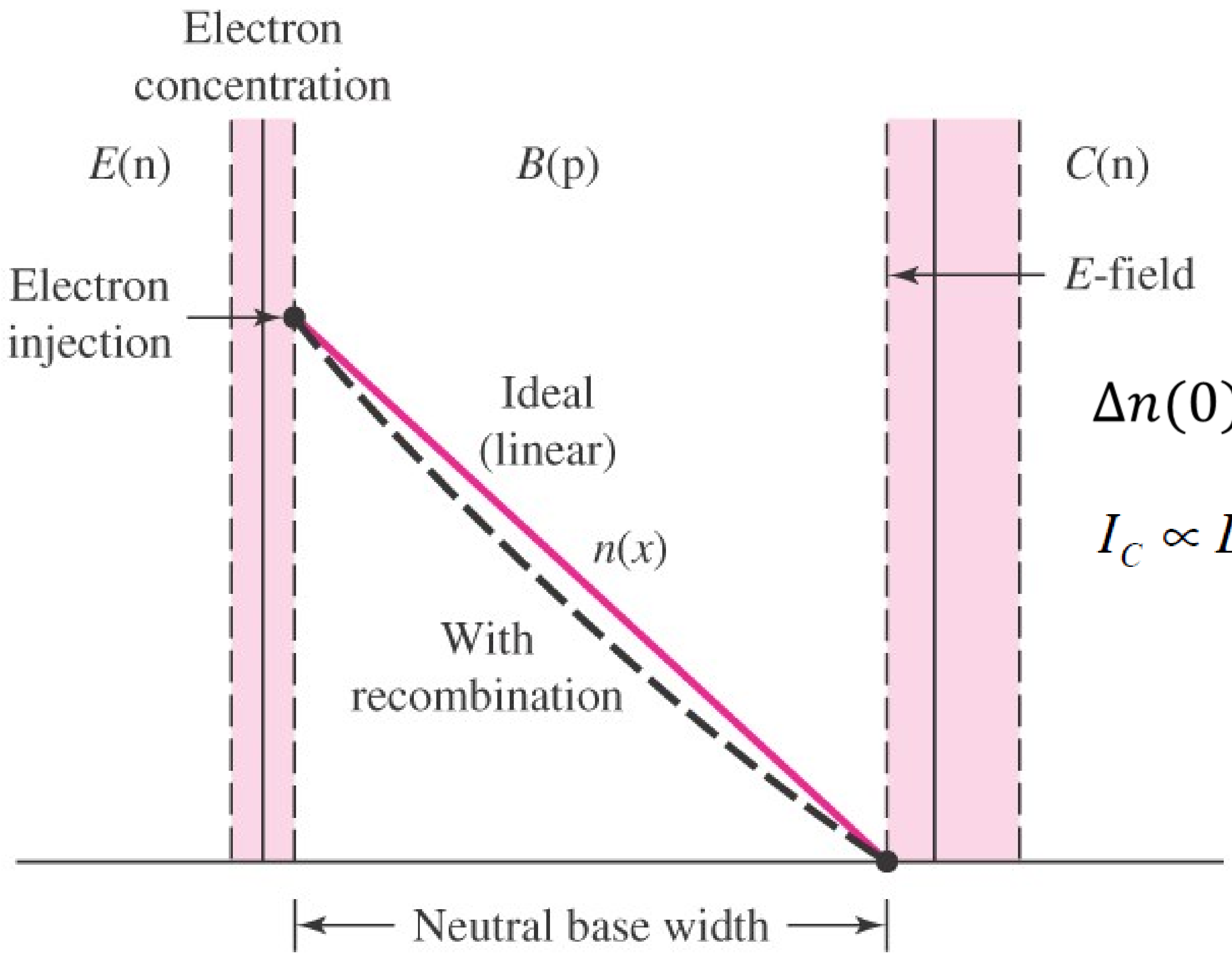
Emitter current :
Holes injected from B to E +Electrons injected from E to B. The latter is dominant as the emitter is more highly doped than the base

Base current :
Holes injected from B to E+Holes recombining with electrons injected from E to B

Collector current:
Holes drifting from C to B+Electrons injected from E to B, diffusing across the base and swept towards C by the high electric field across the B-C junction. The latter is dominant as the collector is lightly doped.

The Bipolar Junction Transistor (BJT)

Operation in forward active region or mode

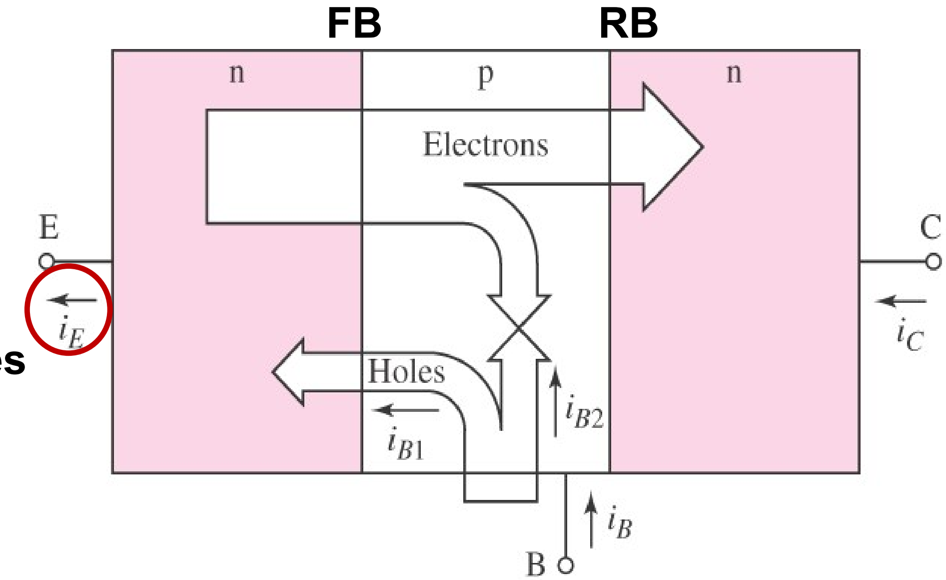


$$\Delta n(0) \propto e^{v_{BE}/V_t}$$

$$I_C \propto D_n \frac{\Delta n(0)}{W_B}$$

BJT in forward active mode: currents

Current flow indicates flow of positive charges



Notes:

- The base current is much smaller than the emitter and the collector current.
- I_{EO}, I_{BO}, and I_s depend on doping levels, device geometry, and temperature.

Emitter current :

$$i_E = I_{EO}[e^{v_{BE}/V_T} - 1] \cong I_{EO}e^{v_{BE}/V_T} \qquad v_{BE} \gg V_T$$

Base current :

$$i_B = i_{B1} + i_{B2}; \quad i_{B1} \propto e^{v_{BE}/V_T} \quad i_{B2} \propto e^{v_{BE}/V_T} \quad i_B = I_{BO}e^{v_{BE}/V_T}$$

Collector current:

$$i_C = I_S e^{v_{BE}/V_T}$$

BJT in forward-active mode: currents

As all terminal currents in a BJT have an exponential dependence on V_{BE}, they will be linearly related.

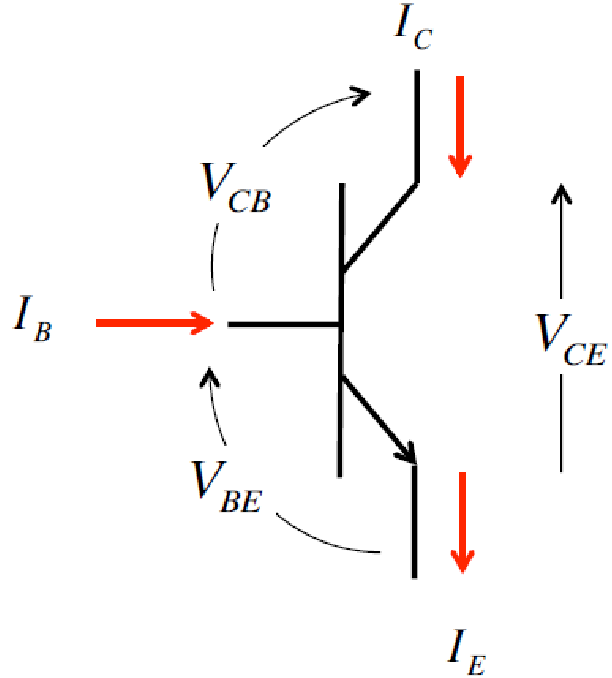
$\frac{i_C}{i_B} = \beta$ **($\beta = I_s/I_{BO}$ varies with transistor parameters and temperature)**

$$i_E = i_C + i_B$$

$$i_E = (1+\beta) i_B$$

$$i_C = \left(\frac{\beta}{1+\beta}\right) i_E \quad i_C = \alpha i_E$$

$$\alpha = \frac{\beta}{1+\beta}$$



β: Common-emitter current gain- $50 < \beta < 300$

α: Common-base current gain- $\alpha \approx$ **0.99**

BJT in forward-active mode

Current-voltage relationships in the forward-active operating region*

Table 5.1 Summary of the bipolar current–voltage relationships in the active region

npn	pnp
$i_C = I_S e^{v_{BE}/V_T}$	$i_C = I_S e^{v_{EB}/V_T}$
$i_E = \frac{i_C}{\alpha} = \frac{I_S}{\alpha} e^{v_{BE}/V_T}$	$i_E = \frac{i_C}{\alpha} = \frac{I_S}{\alpha} e^{v_{EB}/V_T}$
$i_B = \frac{i_C}{\beta} = \frac{I_S}{\beta} e^{v_{BE}/V_T}$	$i_B = \frac{i_C}{\beta} = \frac{I_S}{\beta} e^{v_{EB}/V_T}$

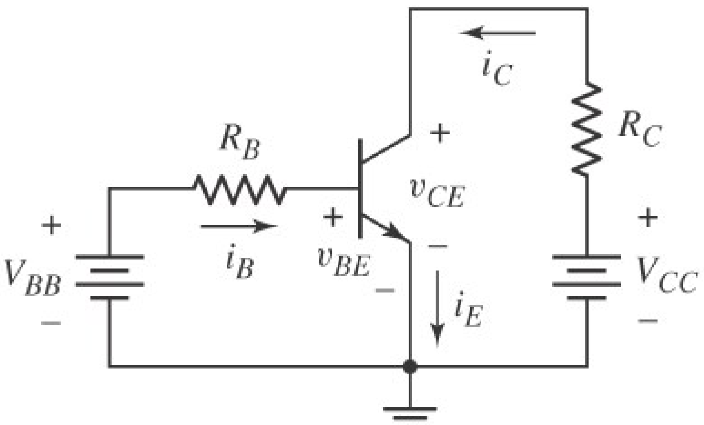
For both transistors

$i_E = i_C + i_B$	$i_C = \beta i_B$
$i_E = (1+\beta) i_B$	$i_C = \alpha i_E = \left(\frac{\beta}{1+\beta}\right) i_E$
$\alpha = \frac{\beta}{1+\beta}$	$\beta = \frac{\alpha}{1-\alpha}$

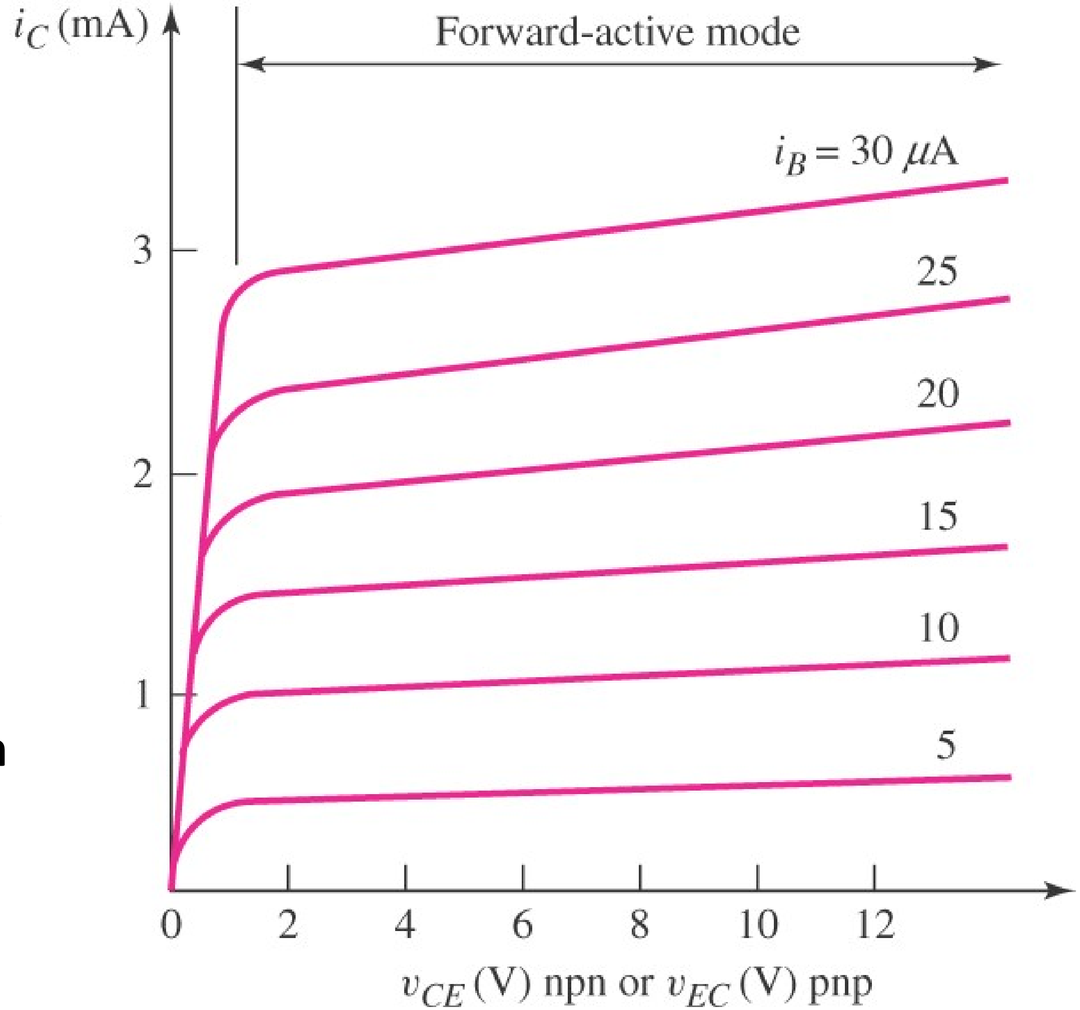
* **The Early effect is neglected here**

BJT in forward active mode

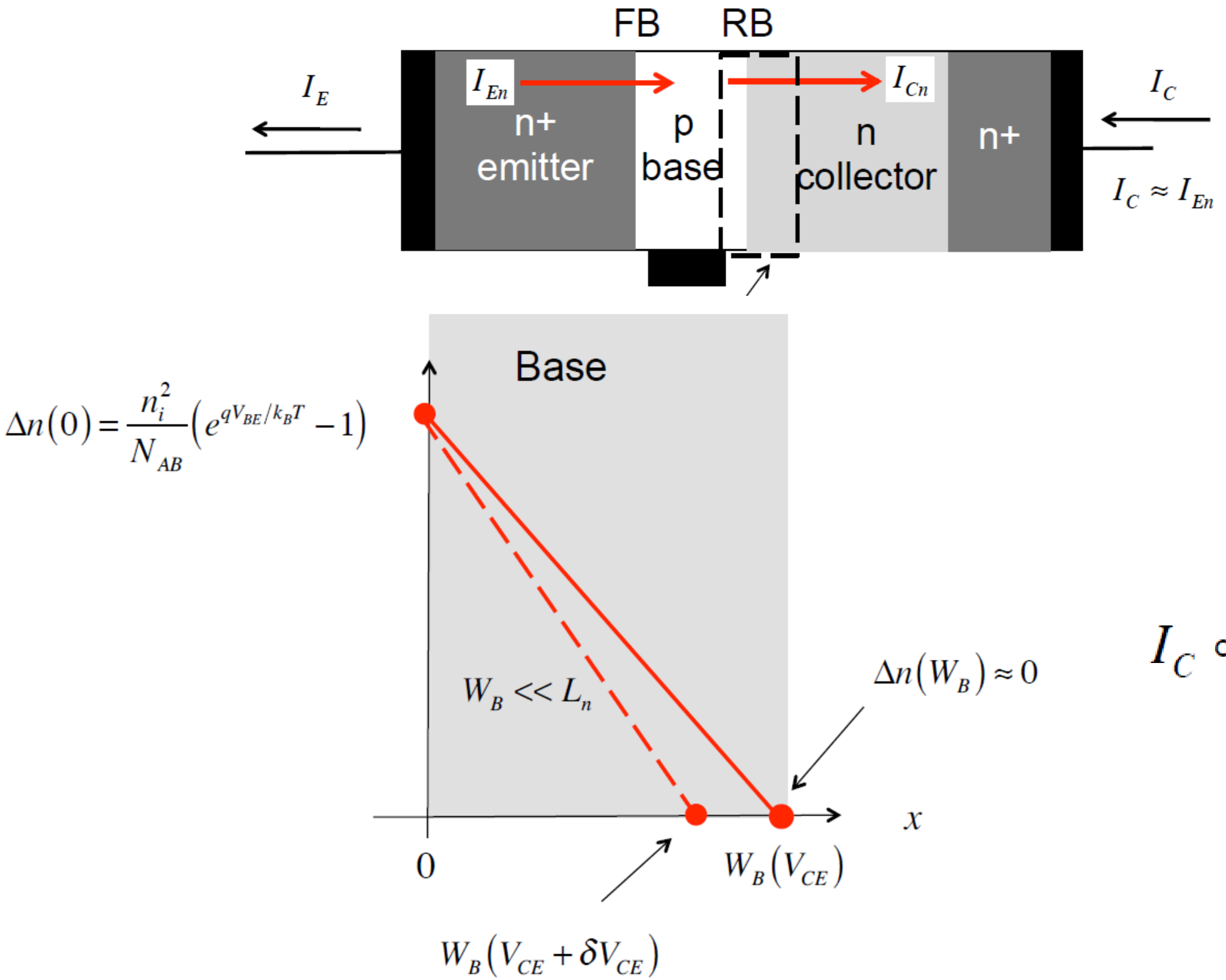
Current-voltage relationships in the forward-active operating region



Common emitter configuration



BJT current-voltage characteristics (Active mode): Early effect



$$I_C \propto D_n \frac{\Delta n(0)}{W_B}$$

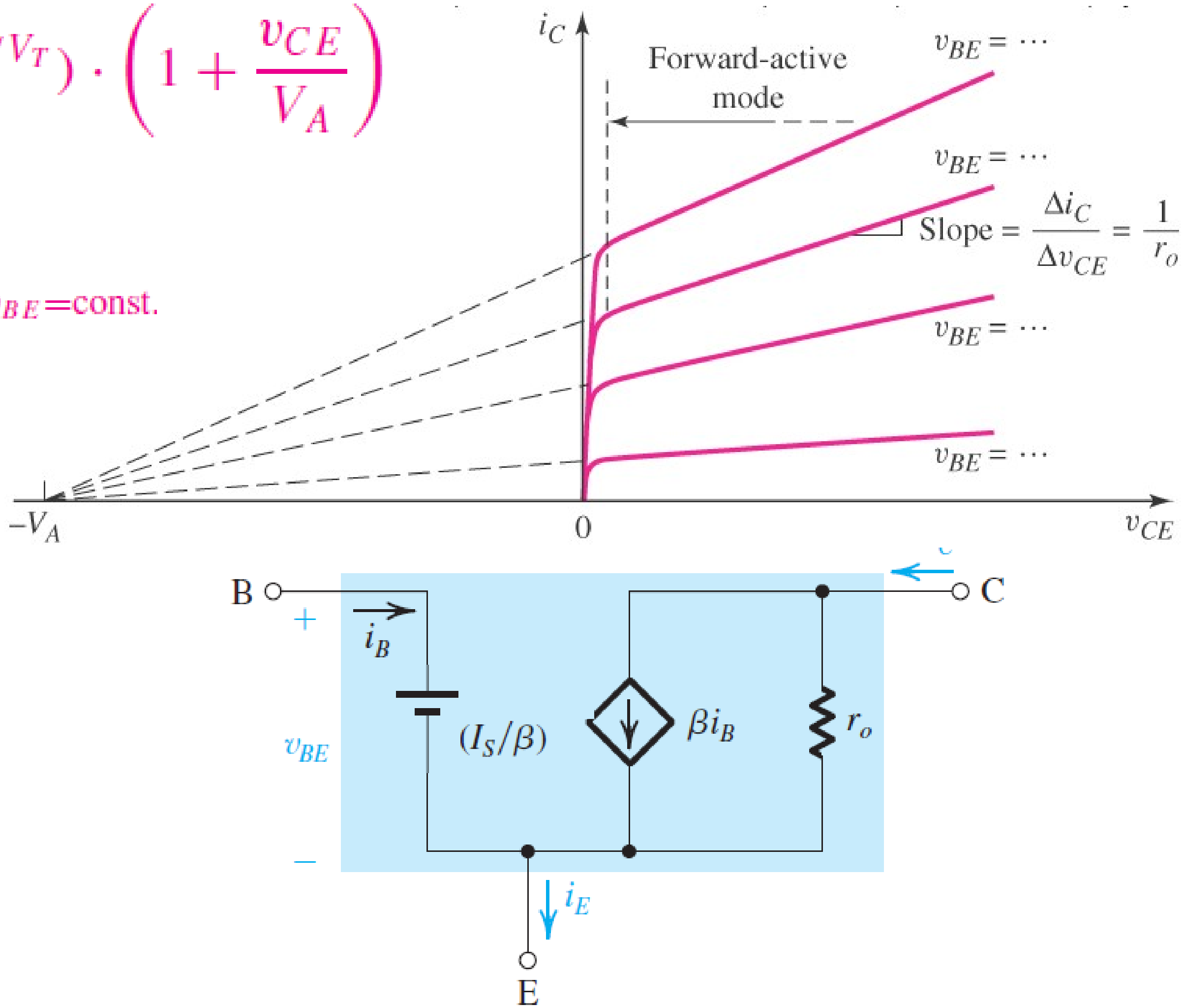
BJT in forward active mode: DC model

Common emitter configuration (npn) (w/ Early effect)

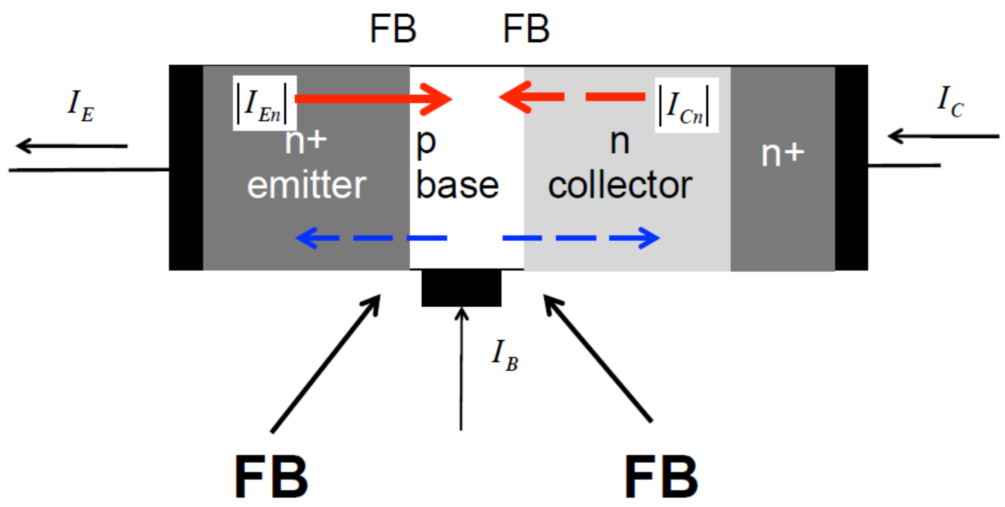
$$i_C = I_S(e^{v_{BE}/V_T}) \cdot \left(1 + \frac{v_{CE}}{V_A}\right)$$

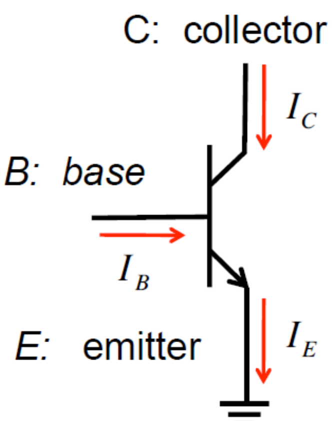
$$\frac{1}{r_o} = \left.\frac{\partial i_C}{\partial v_{CE}}\right|_{v_{BE}=\text{const.}}$$

$$r_o \cong \frac{V_A}{I_C}$$

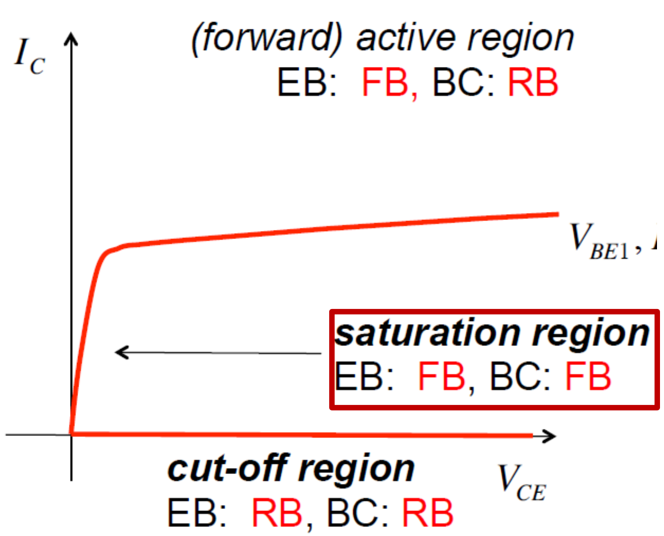


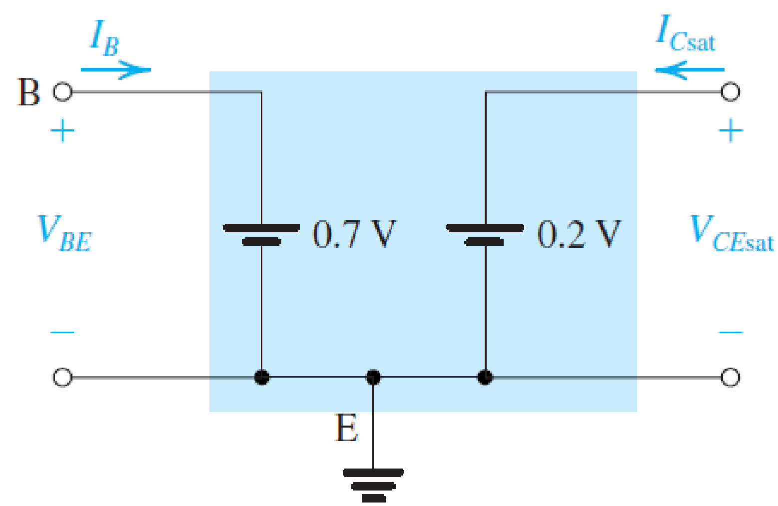
BJT in saturation: characteristics and DC model





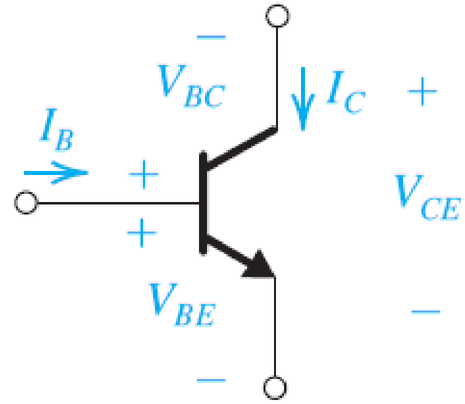
NPN BJT



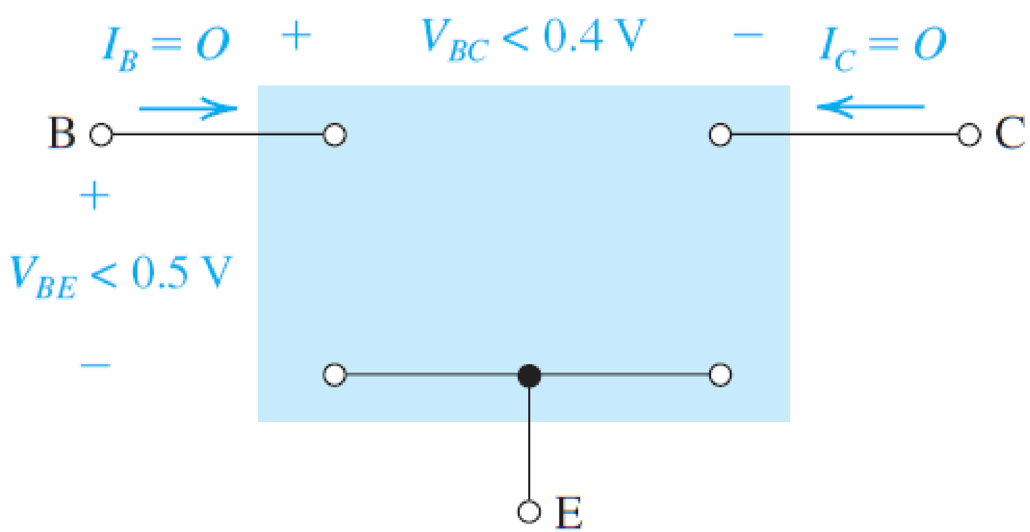


$I_C/I_B<\beta$

BJT in cut-off: DC model



Cutoff
EJB: Reverse Biased
CBJ: Reverse Biased

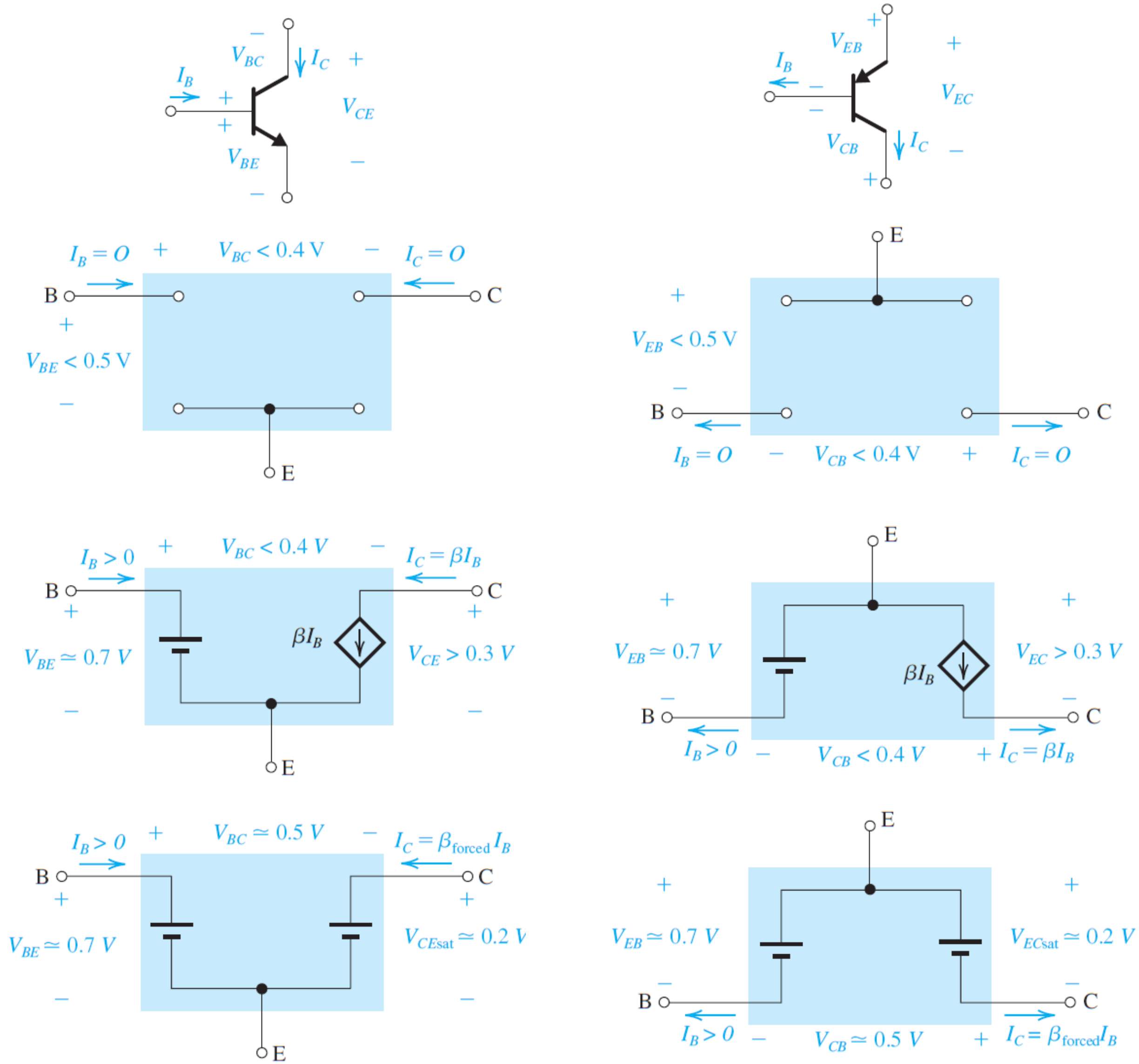


BJT: DC models-Summary

Cutoff
EJB: Reverse Biased
CBJ: Reverse Biased

Active
EBJ: Forward Biased
CBJ: Reverse Biased

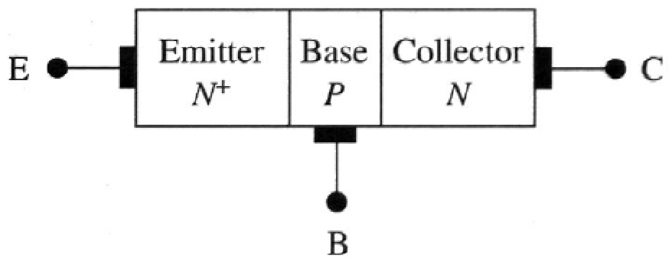
Saturation
EBJ: Forward Biased
CBJ: Forward Biased



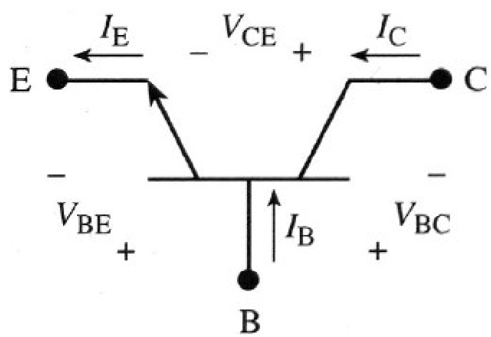
Condition for forward-active mode

B-E junction in forward bias & B-C junction in reverse bias

$v_{BE} > v_{BEon}$, $v_{BC} < v_{BCon}$, $v_{CE} > v_{CESAT} = v_{BEon} - v_{BCon}$

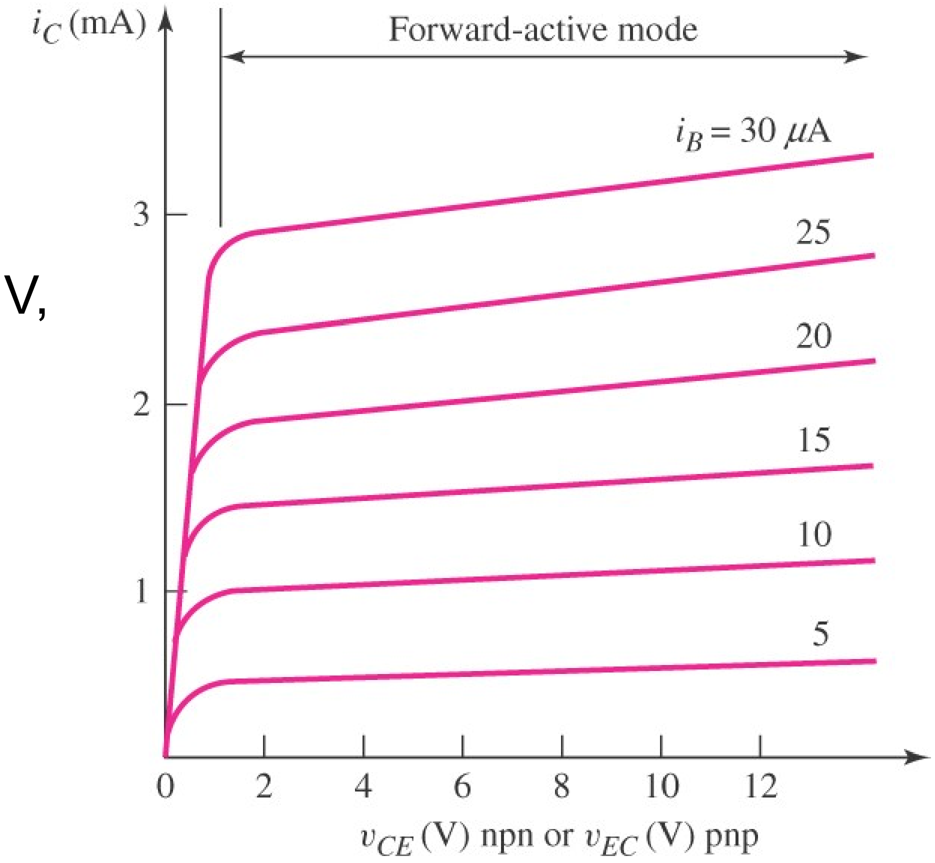


v_{BEon}~0.6-0.7 V, v_{BCon}~0.4-0.5 V, $v_{CE,sat}$ ~0.1-0.3 V



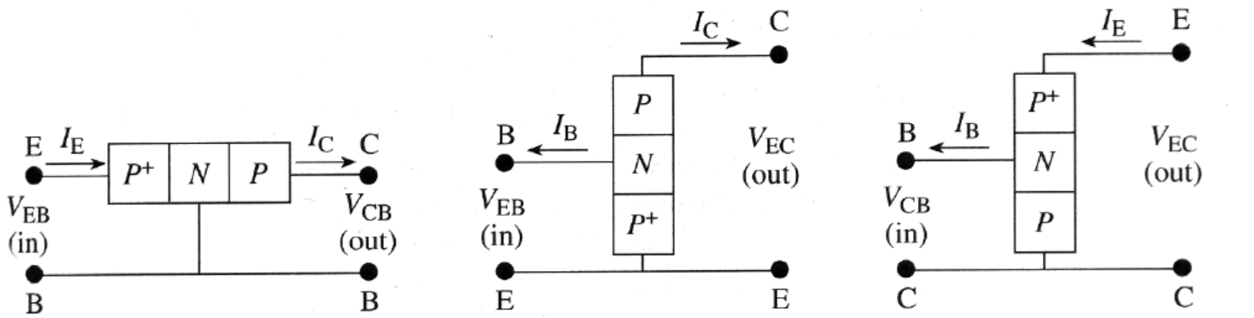
$$I_B > 0, \text{ and } I_C = \beta I_B$$

Additional conditions for active operation



*Practice writing the conditions for FA operation of a pnp

BJTs configurations



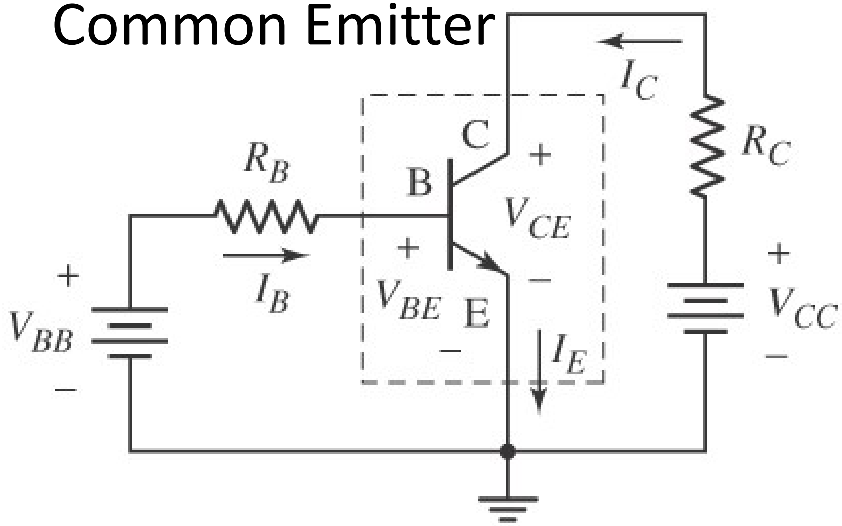
Common base — Both the input and output share the base "in common"

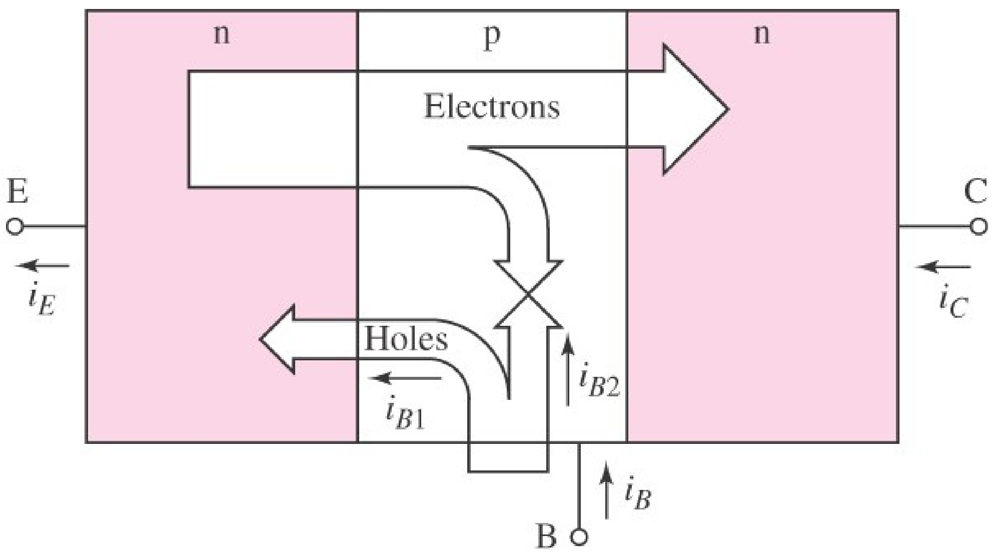
Common emitter — Both the input and output share the emitter "in common"

Common collector — Both the input and output share the Collector "in common"

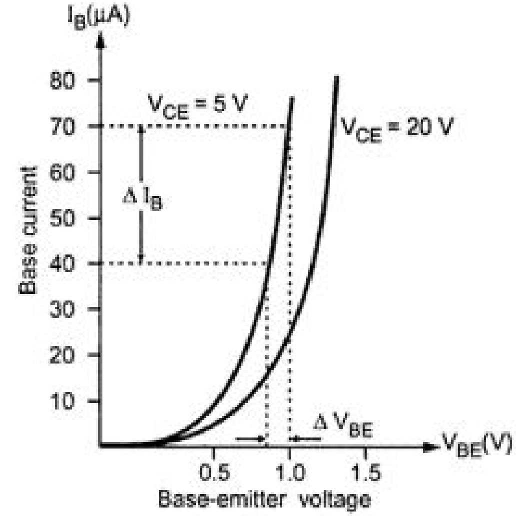
I/O current-voltage characteristics

Common Emitter

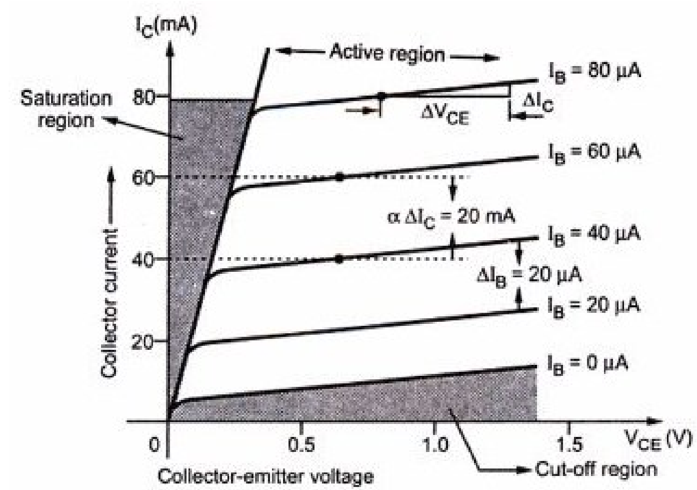




Input characteristics

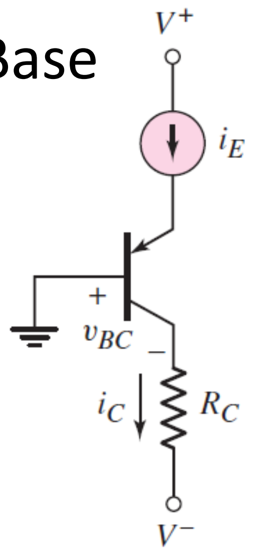


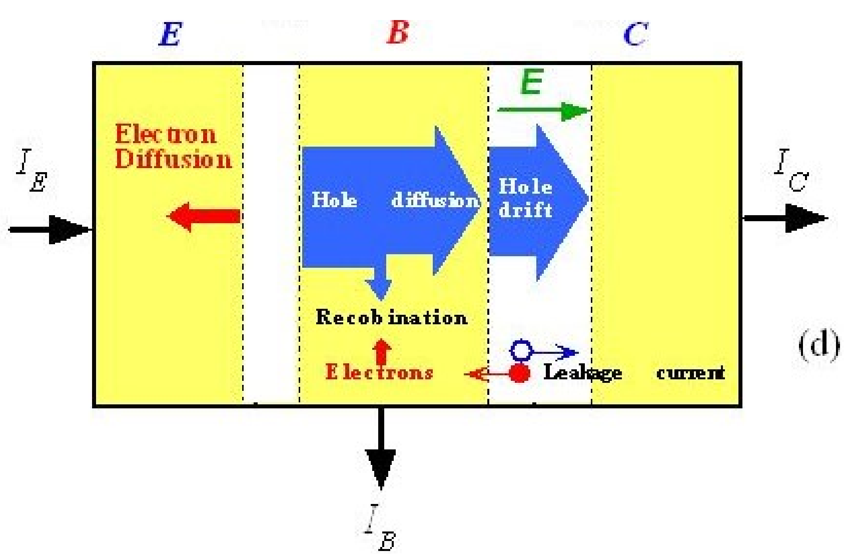
Output characteristics



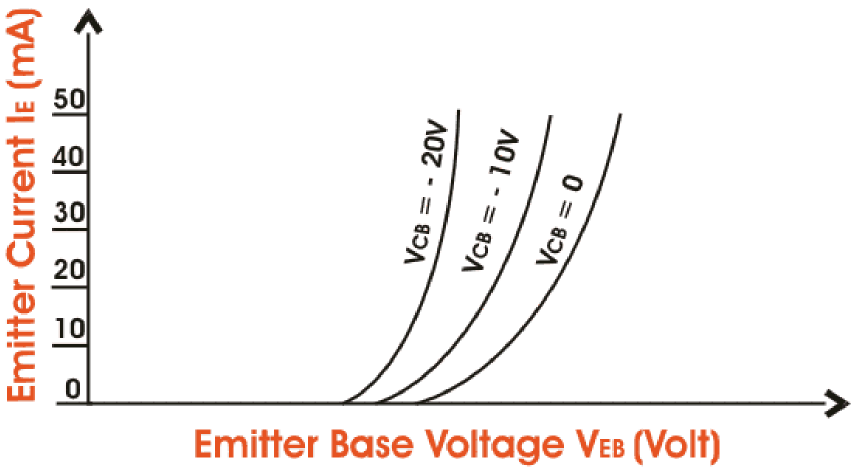
I/O current-voltage characteristics

Common Base

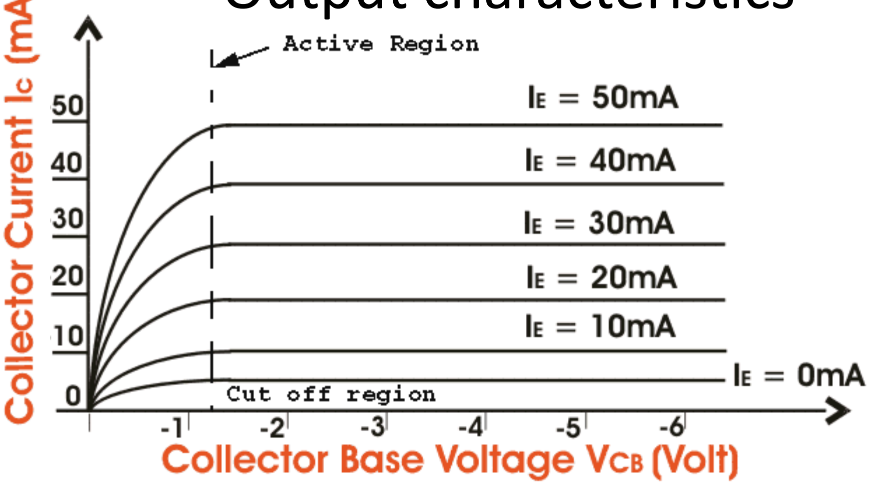




Input characteristics



Output characteristics



Overview of lecture 11

- Lecture 11-
- The Bipolar Junction Transistor (BJT):

  Structure, Operating regions, DC analysis,

  Load lines (Neamen-From 5.1.1 to 5.1.5)