PS#3 Sol'ns

ECE 345/ME 380
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$\boxed{1}$

a) Type 2.

b). Any $K>0$ generates $e_{ss}=0$.

c) Any $K>0$ generates $e_{ss}=0$

d) $e_{ss}=\frac{1}{K_a}$, $K_a=\lim_{s\to 0} s^2 K G(s)$

$$= \frac{1\cdot 2}{3}\cdot K$$

$$= \frac{3}{2K} \le \frac{1}{10}$$

$$15 \le K$$

$\boxed{2}$

a) $$G_D(s) = \frac{G_2}{1+G_1G_2} = \frac{10K_2/(s+2)}{1+\frac{K_1K_2\cdot 10}{(s+5)(s+2)}}\cdot\frac{(s+5)(s+2)}{(s+5)(s+2)}$$

$$= \frac{10K_2(s+\ )}{(s+5)(s+2)+K_1K_2\cdot 10}$$

$$= \frac{10K_2(s+5)}{s^2+7s+10(1+K_1K_2)}$$

$$G_R(s) = \frac{G_1 G_2}{1+G_1G_2} = \frac{10k_1k_2}{(s+5)(s+2)+10k_1k_2}$$

$$= \frac{10k_1k_2}{s^2+7s+10(1+k_1k_2)}$$

b) $\Delta_R(s) = \Delta_D(s)$ (same char. eqn in both $G_R(s)$ & $G_D(s)$)

c) i) $y_{ss} = \lim_{s\to 0} \cancel{s} \cdot G_D(s) \cdot \frac{1}{\cancel{s}} \le \frac{2}{100}$

$$\frac{\cancel{10} \cdot k_2 \cdot 5}{\cancel{10}(1+k_1k_2)} \le \frac{2}{100}$$

$$500k_2 \le 2 + 2k_1k_2$$

$$250k_2 \le 1 + k_1k_2$$

ii) $e_{ss} = \frac{1}{1+K_p}$, $K_p = \lim_{s\to 0} KG(s)$, for $KG(s) = \frac{10k_1k_2}{(s+5)(s+2)}$

$$= \frac{10k_1k_2}{10}$$

$$= k_1k_2$$

$$e_{ss} = \frac{1}{1+k_1k_2} \le \frac{\cancel{5}}{\cancel{100}} \frac{1}{20}$$

$$20 \le 1 + k_1k_2 \Rightarrow k_1k_2 \ge 19.$$

For $k_1 = 250$, $k_2 = \frac{1}{10}$

i): $250 \cdot \frac{1}{10} \leq 1 + 250 \cdot \frac{1}{10}$

$25 \leq 26$

ii) $19 \leq k_1 k_2$

$\leq 250 \cdot \frac{1}{10}$

$\leq 25$

Both conditions are satisfied.

$\boxed{13}$ a) Type 0.

b) $e_{ss} = \frac{1}{1+K_p}$, $K_p = \lim_{s \to 0} G_c(s) G(s)$

$= \lim_{s \to 0} \frac{s+2}{s^2+2s+3} \cdot \frac{k}{s+1} = \frac{2}{3}k$

$= \frac{1}{1 + 2/3 k}$

$= \frac{3}{3+2k} \leq \frac{1}{100}$

$300 \leq 3 + 2k$

$\frac{297}{2} \leq k$

c) Type 1

d) $e_{ss} = 0$ for any value $K > 0$.

e) $K = 150$ will meet steady-state error criterion for both controllers. However, w/ $G_c(s) = \frac{K}{s+1}$, there is steady-state error, but with $G_c(s) = \frac{K}{s(s+1)}$, error is 0 (i.e., perfect tracking is possible). So $G_c(s) = \frac{K}{s(s+1)}$ has better steady state performance.