4.3

(a) $n_i^2 = N_c N_\upsilon \exp\left(\frac{-E_g}{kT}\right)$

$$\left(5\times10^{11}\right)^2 = \left(2.8\times10^{19}\right)\left(1.04\times10^{19}\right)\left(\frac{T}{300}\right)^3$$

$$\times\exp\left[\frac{-1.12}{(0.0259)(T/300)}\right]$$

$$2.5\times10^{23} = \left(2.912\times10^{38}\right)\left(\frac{T}{300}\right)^3$$

$$\times\exp\left[\frac{-(1.12)(300)}{(0.0259)(T)}\right]$$

By trial and error, $T \cong 367.5\ \text{K}$

(b)

$$n_i^2 = \left(5\times10^{12}\right)^2 = 2.5\times10^{25}$$

$$= \left(2.912\times10^{38}\right)\left(\frac{T}{300}\right)^3 \exp\left[\frac{-(1.12)(300)}{(0.0259)(T)}\right]$$

By trial and error, $T \cong 417.5\ \text{K}$

4.5

$$\frac{n_i(B)}{n_i(A)} = \frac{\exp\left(\frac{-1.10}{kT}\right)}{\exp\left(\frac{-0.90}{kT}\right)} = \exp\left(\frac{-0.20}{kT}\right)$$

For $T = 200\ \text{K}$, $kT = 0.017267\ \text{eV}$
For $T = 300\ \text{K}$, $kT = 0.0259\ \text{eV}$
For $T = 400\ \text{K}$, $kT = 0.034533\ \text{eV}$

(a) For $T = 200\ \text{K}$,

$$\frac{n_i(B)}{n_i(A)} = \exp\left(\frac{-0.20}{0.017267}\right) = 9.325\times10^{-6}$$

(b) For $T = 300\ \text{K}$,

$$\frac{n_i(B)}{n_i(A)} = \exp\left(\frac{-0.20}{0.0259}\right) = 4.43\times10^{-4}$$

(c) For $T = 400\ \text{K}$,

$$\frac{n_i(B)}{n_i(A)} = \exp\left(\frac{-0.20}{0.034533}\right) = 3.05\times10^{-3}$$

4.6

(a) $g_c f_F \propto \sqrt{E-E_c}\,\exp\left[\frac{-(E-E_F)}{kT}\right]$

$$\propto \sqrt{E-E_c}\,\exp\left[\frac{-(E-E_c)}{kT}\right]$$

$$\times\exp\left[\frac{-(E_c-E_F)}{kT}\right]$$

Let $E - E_c = x$

Then $g_c f_F \propto \sqrt{x}\,\exp\left(\frac{-x}{kT}\right)$

To find the maximum value:

$$\frac{d(g_c f_F)}{dx} \propto \frac{1}{2}x^{-1/2}\exp\left(\frac{-x}{kT}\right)$$

$$-\frac{1}{kT}\cdot x^{1/2}\exp\left(\frac{-x}{kT}\right) = 0$$

which yields

$$\frac{1}{2x^{1/2}} = \frac{x^{1/2}}{kT} \Rightarrow x = \frac{kT}{2}$$

The maximum value occurs at

$$E = E_c + \frac{kT}{2}$$

(b)

$$g_\upsilon(1-f_F) \propto \sqrt{E_\upsilon - E}\,\exp\left[\frac{-(E_F-E)}{kT}\right]$$

$$\propto \sqrt{E_\upsilon - E}\,\exp\left[\frac{-(E_\upsilon-E)}{kT}\right]$$

$$\times\exp\left[\frac{-(E_F-E_\upsilon)}{kT}\right]$$

Let $E_\upsilon - E = x$

Then $g_\upsilon(1-f_F) \propto \sqrt{x}\,\exp\left(\frac{-x}{kT}\right)$

To find the maximum value

$$\frac{d\left[g_\upsilon(1-f_F)\right]}{dx} \propto \frac{d}{dx}\left[\sqrt{x}\,\exp\left(\frac{-x}{kT}\right)\right] = 0$$

Same as part (a). Maximum occurs at

$$x = \frac{kT}{2}$$

or

$$E = E_\upsilon - \frac{kT}{2}$$

4.10

$$E_{Fi} - E_{midgap} = \frac{3}{4} kT \ln\left(\frac{m_p^*}{m_n^*}\right)$$

Silicon: $m_p^* = 0.56m_o$, $m_n^* = 1.08m_o$

$$E_{Fi} - E_{midgap} = -0.0128 \text{ eV}$$

Germanium: $m_p^* = 0.37m_o$, $m_n^* = 0.55m_o$

$$E_{Fi} - E_{midgap} = -0.0077 \text{ eV}$$

Gallium Arsenide: $m_p^* = 0.48m_o$,

$$m_n^* = 0.067m_o$$

$$E_{Fi} - E_{midgap} = +0.0382 \text{ eV}$$

4.19

(a) $E_c - E_F = kT \ln\left(\frac{N_c}{n_o}\right)$

$$= (0.0259)\ln\left(\frac{2.8\times10^{19}}{2\times10^5}\right)$$

$$= 0.8436 \text{ eV}$$

$$E_F - E_\upsilon = E_g - (E_c - E_F)$$

$$= 1.12 - 0.8436$$

$$E_F - E_\upsilon = 0.2764 \text{ eV}$$

(b) $p_o = (1.04\times10^{19})\exp\left(\frac{-0.27637}{0.0259}\right)$

$$= 2.414\times10^{14} \text{ cm}^{-3}$$

(c) p-type

4.20

(a) $kT = (0.0259)\left(\frac{375}{300}\right) = 0.032375 \text{ eV}$

$$n_o = (4.7\times10^{17})\left(\frac{375}{300}\right)^{3/2} \exp\left[\frac{-0.28}{0.032375}\right]$$

$$= 1.15\times10^{14} \text{ cm}^{-3}$$

$$E_F - E_\upsilon = E_g - (E_c - E_F) = 1.42 - 0.28$$

$$= 1.14 \text{ eV}$$

$$p_o = (7\times10^{18})\left(\frac{375}{300}\right)^{3/2} \exp\left[\frac{-1.14}{0.032375}\right]$$

$$= 4.99\times10^3 \text{ cm}^{-3}$$

(b) $E_c - E_F = (0.0259)\ln\left(\frac{4.7\times10^{17}}{1.15\times10^{14}}\right)$

$$= 0.2154 \text{ eV}$$

$$E_F - E_\upsilon = E_g - (E_c - E_F) = 1.42 - 0.2154$$

$$= 1.2046 \text{ eV}$$

$$p_o = (7\times10^{18})\exp\left[\frac{-1.2046}{0.0259}\right]$$

$$= 4.42\times10^{-2} \text{ cm}^{-3}$$

4.22

(a) p-type

(b) $E_F - E_\upsilon = \frac{E_g}{4} = \frac{1.12}{4} = 0.28 \text{ eV}$

$$p_o = N_\upsilon \exp\left[\frac{-(E_F - E_\upsilon)}{kT}\right]$$

$$= (1.04\times10^{19})\exp\left[\frac{-0.28}{0.0259}\right]$$

$$= 2.10\times10^{14} \text{ cm}^{-3}$$

$$E_c - E_F = E_g - (E_F - E_\upsilon)$$

$$= 1.12 - 0.28 = 0.84 \text{ eV}$$

$$n_o = N_c \exp\left[\frac{-(E_c - E_F)}{kT}\right]$$

$$= (2.8\times10^{19})\exp\left[\frac{-0.84}{0.0259}\right]$$

$$= 2.30\times10^5 \text{ cm}^{-3}$$

4.37

(a) For the donor level

$$\frac{n_d}{N_d} = \frac{1}{1 + \frac{1}{2}\exp\left(\frac{E_d - E_F}{kT}\right)}$$

$$= \frac{1}{1 + \frac{1}{2}\exp\left(\frac{0.20}{0.0259}\right)}$$

or

$$\frac{n_d}{N_d} = 8.85\times10^{-4}$$

(b) We have

$$f_F(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)}$$

Now

$$E - E_F = (E - E_c) + (E_c - E_F)$$

or

$$E - E_F = kT + 0.245$$

Then

$$f_F(E) = \frac{1}{1 + \exp\left(1 + \frac{0.245}{0.0259}\right)}$$

or $\quad f_F(E) = 2.87\times10^{-5}$

4.39

(a) $N_d > N_a \Rightarrow$ n-type

(b) $n_o \cong N_d - N_a = 2\times10^{15} - 1.2\times10^{15}$

$= 8\times10^{14}\ \text{cm}^{-3}$

$$p_o = \frac{n_i^2}{n_o} = \frac{\left(1.5\times10^{10}\right)^2}{8\times10^{14}} = 2.81\times10^5\ \text{cm}^{-3}$$

(c) $p_o \cong \left(N'_a + N_a\right) - N_d$

$4\times10^{15} = N'_a + 1.2\times10^{15} - 2\times10^{15}$

$\Rightarrow N'_a = 4.8\times10^{15}\ \text{cm}^{-3}$

$$n_o = \frac{\left(1.5\times10^{10}\right)^2}{4\times10^{15}} = 5.625\times10^4\ \text{cm}^{-3}$$

4.45

$$n_o = \frac{N_d - N_a}{2} + \sqrt{\left(\frac{N_d - N_a}{2}\right)^2 + n_i^2}$$

$$1.1\times10^{14} = \frac{2\times10^{14} - 1.2\times10^{14}}{2} + \sqrt{\left(\frac{2\times10^{14} - 1.2\times10^{14}}{2}\right)^2 + n_i^2}$$

$\left(1.1\times10^{14} - 4\times10^{13}\right)^2 = \left(4\times10^{13}\right)^2 + n_i^2$

$4.9\times10^{27} = 1.6\times10^{27} + n_i^2$

so $n_i = 5.74\times10^{13}\ \text{cm}^{-3}$

$$p_o = \frac{n_i^2}{n_o} = \frac{3.3\times10^{27}}{1.1\times10^{14}} = 3\times10^{13}\ \text{cm}^{-3}$$

4.49

(a) $E_c - E_F = kT\ln\left(\frac{N_c}{N_d}\right)$

$$= (0.0259)\ln\left(\frac{2.8\times10^{19}}{N_d}\right)$$

For $10^{14}\ \text{cm}^{-3}$, $E_c - E_F = 0.3249\ \text{eV}$

$10^{15}\ \text{cm}^{-3}$, $E_c - E_F = 0.2652\ \text{eV}$

$10^{16}\ \text{cm}^{-3}$, $E_c - E_F = 0.2056\ \text{eV}$

$10^{17}\ \text{cm}^{-3}$, $E_c - E_F = 0.1459\ \text{eV}$

(b) $E_F - E_{Fi} = kT\ln\left(\frac{N_d}{n_i}\right)$

$$= (0.0259)\ln\left(\frac{N_d}{1.5\times10^{10}}\right)$$

For $10^{14}\ \text{cm}^{-3}$, $E_F - E_{Fi} = 0.2280\ \text{eV}$

$10^{15}\ \text{cm}^{-3}$, $E_F - E_{Fi} = 0.2877\ \text{eV}$

$10^{16}\ \text{cm}^{-3}$, $E_F - E_{Fi} = 0.3473\ \text{eV}$

$10^{17}\ \text{cm}^{-3}$, $E_F - E_{Fi} = 0.4070\ \text{eV}$

4.50

(a) $n_o = \frac{N_d}{2} + \sqrt{\left(\frac{N_d}{2}\right)^2 + n_i^2}$

$n_o = 1.05N_d = 1.05\times10^{15}\ \text{cm}^{-3}$

$$\left(1.05\times10^{15} - 0.5\times10^{15}\right)^2 = \left(0.5\times10^{15}\right)^2 + n_i^2$$

so $n_i^2 = 5.25\times10^{28}$

Now

$$n_i^2 = \left(2.8\times10^{19}\right)\left(1.04\times10^{19}\right)\left(\frac{T}{300}\right)^3 \times\exp\left[\frac{-1.12}{(0.0259)(T/300)}\right]$$

$$5.25\times10^{28} = \left(2.912\times10^{38}\right)\left(\frac{T}{300}\right)^3 \times\exp\left[\frac{-12972.973}{T}\right]$$

By trial and error, $T = 536.5\ \text{K}$

(b) At $T = 300\ \text{K}$,

$$E_c - E_F = kT\ln\left(\frac{N_c}{n_o}\right)$$

$$E_c - E_F = (0.0259)\ln\left(\frac{2.8\times10^{19}}{10^{15}}\right)$$

$= 0.2652\ \text{eV}$

At $T = 536.5\ \text{K}$,

$$kT = (0.0259)\left(\frac{536.5}{300}\right) = 0.046318\ \text{eV}$$

$$N_c = \left(2.8\times10^{19}\right)\left(\frac{536.5}{300}\right)^{3/2}$$

$= 6.696\times10^{19}\ \text{cm}^{-3}$

$$E_c - E_F = kT\ln\left(\frac{N_c}{n_o}\right)$$

$$E_c - E_F = (0.046318)\ln\left(\frac{6.696\times10^{19}}{1.05\times10^{15}}\right)$$

$= 0.5124\ \text{eV}$

then $\Delta\left(E_c - E_F\right) = 0.2472\ \text{eV}$

(c) Closer to the intrinsic energy level.

4.52

(a)

$$E_{Fi} - E_F = kT\ln\left(\frac{N_a}{n_i}\right) = (0.0259)\ln\left(\frac{N_a}{1.8\times10^6}\right)$$

For $N_a = 10^{14}\ \text{cm}^{-3}$, $E_{Fi} - E_F = 0.4619\ \text{eV}$

$N_a = 10^{15}\ \text{cm}^{-3}$, $E_{Fi} - E_F = 0.5215\ \text{eV}$

$N_a = 10^{16}\ \text{cm}^{-3}$, $E_{Fi} - E_F = 0.5811\ \text{eV}$

$N_a = 10^{17}\ \text{cm}^{-3}$, $E_{Fi} - E_F = 0.6408\ \text{eV}$

(b)

$$E_F - E_\upsilon = kT\ln\left(\frac{N_\upsilon}{N_a}\right) = (0.0259)\ln\left(\frac{7.0\times10^{18}}{N_a}\right)$$

For $N_a = 10^{14}\ \text{cm}^{-3}$, $E_F - E_\upsilon = 0.2889\ \text{eV}$

$N_a = 10^{15}\ \text{cm}^{-3}$, $E_F - E_\upsilon = 0.2293\ \text{eV}$

$N_a = 10^{16}\ \text{cm}^{-3}$, $E_F - E_\upsilon = 0.1697\ \text{eV}$

$N_a = 10^{17}\ \text{cm}^{-3}$, $E_F - E_\upsilon = 0.1100\ \text{eV}$

4.61

$$p_o = \frac{N_a}{2} + \sqrt{\left(\frac{N_a}{2}\right)^2 + n_i^2}$$

$$5.08\times10^{15} = \frac{5\times10^{15}}{2} + \sqrt{\left(\frac{5\times10^{15}}{2}\right)^2 + n_i^2}$$

$$\left(5.08\times10^{15} - 2.5\times10^{15}\right)^2 = \left(2.5\times10^{15}\right)^2 + n_i^2$$

$$6.6564\times10^{30} = 6.25\times10^{30} + n_i^2$$

$$\Rightarrow n_i^2 = 4.064\times10^{29}$$

$$n_i^2 = N_c N_\upsilon \exp\left[\frac{-E_g}{kT}\right]$$

$$kT = (0.0259)\left(\frac{350}{300}\right) = 0.030217\ \text{eV}$$

$$N_c = \left(1.2\times10^{19}\right)\left(\frac{350}{300}\right)^2 = 1.633\times10^{19}\ \text{cm}^{-3}$$

$$N_\upsilon = \left(1.8\times10^{19}\right)\left(\frac{350}{300}\right)^2 = 2.45\times10^{19}\ \text{cm}^{-3}$$

Now

$$4.064\times10^{29} = \left(1.633\times10^{19}\right)\left(2.45\times10^{19}\right)\times\exp\left[\frac{-E_g}{0.030217}\right]$$

So

$$E_g = (0.030217)\ln\left[\frac{\left(1.633\times10^{19}\right)\left(2.45\times10^{19}\right)}{4.064\times10^{29}}\right]$$

$$\Rightarrow E_g = 0.6257\ \text{eV}$$

4.62

(a) Replace Ga atoms $\Rightarrow$ Silicon acts as a donor

$$N_d = (0.05)\left(7\times10^{15}\right) = 3.5\times10^{14}\ \text{cm}^{-3}$$

Replace As atoms $\Rightarrow$ Silicon acts as an acceptor

$$N_a = (0.95)\left(7\times10^{15}\right) = 6.65\times10^{15}\ \text{cm}^{-3}$$

(b) $N_a > N_d \Rightarrow$ p-type

(c) $p_o = N_a - N_d = 6.65\times10^{15} - 3.5\times10^{14}$

$= 6.3\times10^{15}\ \text{cm}^{-3}$

$$n_o = \frac{n_i^2}{p_o} = \frac{\left(1.8\times10^6\right)^2}{6.3\times10^{15}} = 5.14\times10^{-4}\ \text{cm}^{-3}$$

(a) $E_{Fi} - E_F = kT\ln\left(\dfrac{p_o}{n_i}\right)$

$$= (0.0259)\ln\left(\frac{6.3\times10^{15}}{1.8\times10^6}\right) = 0.5692\ \text{eV}$$