october 30

6. $S(f) = 8\delta(f-30) + 8\delta(f+30)$
$R(T) = ?$
$P = ? = 16W$
$\mathcal{F}^{-1}[S(f)] = 16\cos(2\pi 30 T) = 16\cos(60\pi t)$

7. $H_1(f) = \dfrac{1}{10 + j2\pi f} = e^{-10t}u(t)$
$H_2(f) = \dfrac{e^{-j20\pi f}}{10 + j2\pi f} = e^{-j20\pi f}\dfrac{1}{10 + j2\pi f}$
$= e^{-10(t-10)}u(t-10)$

8. $h(t) = \cos(40\pi t)\, u(t)$

$$\int_{-\infty}^{\infty} |h(t)|\, dt = \int_0^{\infty} |\cos(40\pi t)|\, dt = \infty$$
Not BIBO stable

9. $x(t) = Ae^{jw_0 t}$ $\quad \hat{x}(t)$

$= A\cos(w_0 t) + Aj\sin(w_0 t)$
$\hat{x}(t) = A\sin(w_0 t) - Aj\cos(w_0 t) = -Aj(\cos(w_0 t) + j\sin(w_0 t))$
$= -jAe^{jw_0 t}$
$$\lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} x(t)\cdot \hat{x}(t)\,dt = \lim_{T\to\infty} \frac{-jA^2}{2T}\int_{-T}^{T} e^{2jw_0 t}\,dt$$
$$= \lim_{T\to\infty} \frac{-jA^2}{2T}\frac{1}{2jw_0}\left[e^{2jw_0 t}\right]_{-T}^{T} = \lim_{T\to\infty}\frac{-A^2 j}{4T w_0}\left[\frac{e^{j2w_0 T} - e^{-j2w_0 T}}{2j}\right]$$
$$= \lim_{T\to\infty} \frac{-A^2 j}{2 w_0}\frac{\sin(2w_0 T)}{T} = \frac{-A^2 j}{2w_0}\cdot 0 = 0$$

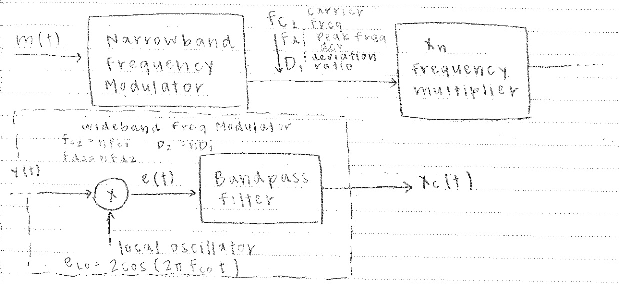
10. $x_c(t) = 50\cos(2\pi 150 t) + 10\cos(2\pi 160 t) + 10\cos(2\pi 140 t)$
$= 50\cos(2\pi 150 t) + 20\cos(2\pi 150 t)\cos(2\pi 10 t)$

$x_c(A) = A_c(1 + a\,m_n(t))\cdot\cos(2\pi f_c t)$

$= 50\cos(2\pi 150 t)\left[1 + \frac{20}{50}\cos(2\pi 10 t)\right]$

$a = \frac{20}{50}$ $\quad P_{car} = \frac{50^2}{2} = \frac{2500}{2} = 1250W$
$P = \frac{10^2}{2} + \frac{10^2}{2} = 100$

Narrow Band → wideband Modulation



$y(t) = A_c\cos(2\pi n f_{c1} t + n\varphi(t))$
$e(t) = A_c\cos(2\pi(nf_{c1} + f_{LO}) + n\varphi(t)] + A_c\cos[2\pi(nf_{c1} - f_{LO})t + n\varphi(t)]$

Bandpass filter : center frequency → $nf_{c1} + f_{LO}$ or $nf_{c1} - f_{LO}$

$\cos(a+b) = \cos a \cos b - \sin a \sin b$

Demodulation of angle-modulated signals

recieved signal $\quad x_r(t) = A_c \cos(2\pi f_c t + \varphi(t))$

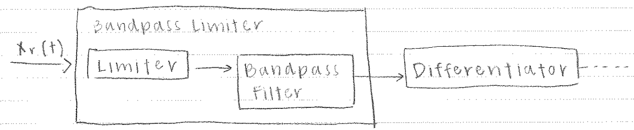
output of an ideal frequency demodulator $\left.\right\}$ $\quad y_D(t) = \dfrac{1}{2\pi} K_D \dfrac{d\varphi}{dt}$ → information of my original message signal

K_D → demodulation constant

For FM $\quad \varphi(t) = 2\pi f_d \int_0^t m(a)\, da$

output of an ideal phase demodulator $\left.\right\}$ $\quad y_d(t) = K_D \cdot f_d\, m(t)$

FM Demodulator



$e(t)$ → Envelope Detector → $y_D(t) = A_c \dfrac{d\varphi}{dt} = A_c 2\pi f_d\, m(t)$ → K_D

$x_r(t) = A_c \cos(2\pi f_c t + \varphi(t))$

$e(t) = \dfrac{d x_r(t)}{dt} = -A_c \underbrace{\left[\underbrace{2\pi f_c}_{\text{constant}} + \dfrac{d\varphi}{dt}\right]}_{\text{Envelope}} \sin(2\pi f_c t + \varphi(t))$

Interference in Angle Modulation

Input to the demodulator $\left.\right\}$ $x_r(t) = A_c \cos(2\pi f_c t) + \underbrace{A_i \cos[2\pi(f_c + f_i)t]}_{\text{interference tone}}$

$= A_c \cos(2\pi f_c t) + A_i \cos(2\pi f_i t)\cos(2\pi f_c t) - A_i \sin(2\pi f_i t)\sin(2\pi f_c t)$

$= R(t)\cos(2\pi f_c t + \Psi(t))$

$R(t) = \sqrt{[A_c + A_i \cos(2\pi f_i t)]^2 + [A_i \sin(2\pi f_i t)]^2}$

$\Psi(t) = \tan^{-1}\left[\dfrac{A_i \sin(2\pi f_i t)}{A_c + A_i \cos(2\pi f_i t)}\right]$

If: $A_c \gg A_i$

$R(t) = A_c + A_i \cos(2\pi f_i t)$

$\Psi(t) = \tan^{-1}\left(\dfrac{A_i}{A_c}\sin(2\pi f_i t)\right)$

$x_r(t) = A_c\left[1 + \dfrac{A_i}{A_c}\cos(2\pi f_i t)\right]\cos(2\pi f_c t + \Psi(t))$

PM demodulation

$y_D = K_D \dfrac{A_i}{A_c}\sin(2\pi f_i t)$

FM demodulation

$y_D(t) = \dfrac{1}{2\pi} K_D \dfrac{d}{dt}\left(\dfrac{A_i}{A_c}\sin(2\pi f_i t)\right)$

Pulse Width Modulation

message signal $= 0$

smaller width negative values of signal ← $\dfrac{T_s}{2}$ → larger width for positive values of the signal