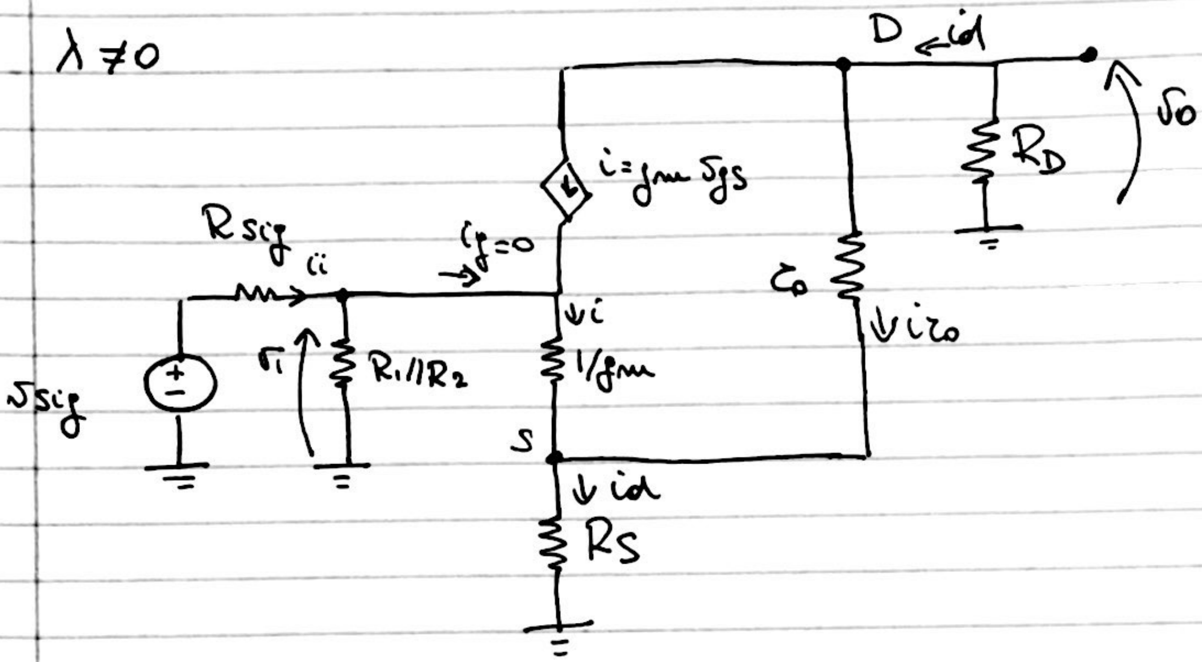
ECE 322 L - Lecture 8

Take home problem 1 - Solution

$\lambda \neq 0$



Input resistance

As $i_g = 0$, $R_i = \frac{v_i}{i_i} = R_1 // R_2$

Gain

As $R_L = \infty$, $A_{vo} = A_v = \frac{v_o}{v_i}$

$v_o = -i_d R_D$

KCL @ S (Source)

$i_d = i + i_{r_o}$

$i = g_m v_{gs}$; $i_{r_o} = \frac{v_o - v_s}{r_o}$

$$i_d = g_m v_{gs} + \frac{v_o - v_s}{r_o}$$

$$v_{gs} = v_i - v_s$$

$$v_s = i_d R_s$$

$$i_d = g_m (v_i - i_d R_s) + \frac{v_o - i_d R_s}{r_o}$$

$$i_d r_o = g_m r_o (v_i - i_d R_s) \quad i_d R_D - i_d R_s$$

$$i_d (r_o + g_m r_o R_s + R_D + R_s) = g_m r_o v_i$$

$$i_d = \frac{g_m r_o v_i}{r_o + R_D + R_s(1 + g_m r_o)}$$

$$v_o = -\frac{g_m r_o R_D v_i}{r_o + R_D + R_s (1 + g_m r_o)}$$

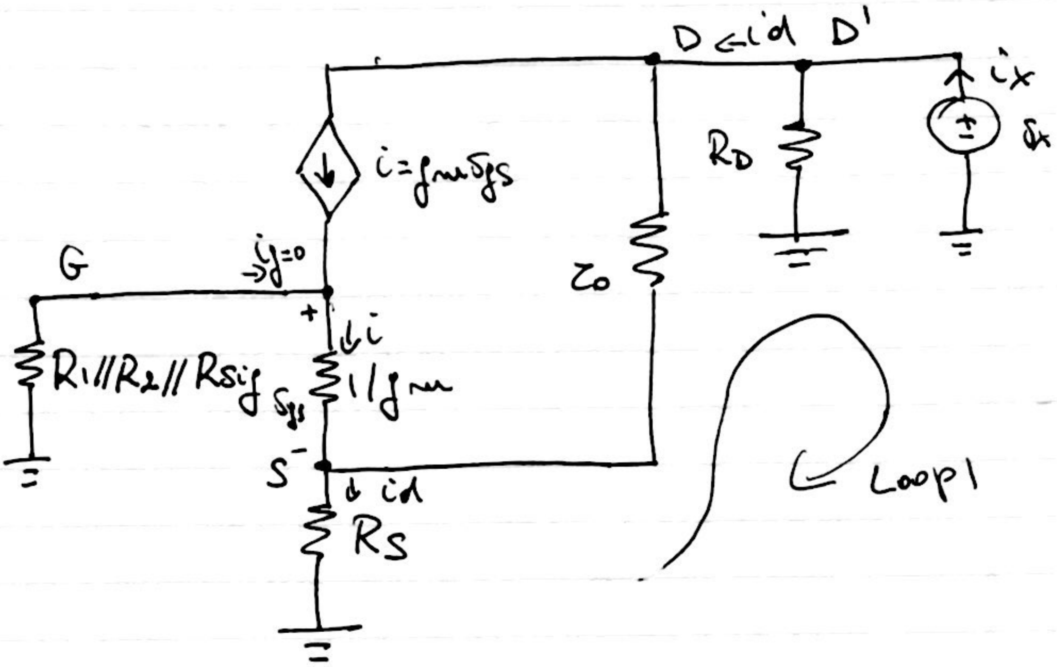
$$A_{vo} = A_v = -\frac{g_m r_o R_D}{r_o + R_D + R_s (1 + g_m r_o)}$$

$$G_v = \frac{v_o}{v_{sig}} = \frac{v_o}{v_i} \cdot \frac{v_i}{v_{sig}} = A_v \cdot \frac{R_i}{R_i + R_{sig}}$$

Output resistance

The output resistance is calculated setting $v_{sig} = 0$ and placing a probe source v_x at the output terminal.

$R_o = \frac{v_x}{i_x}$ (See circuit on the next page).



$$i_x = i_d + \frac{v_x}{R_D} \quad (\text{KCL @ } D')$$

$$i_d = \frac{v_s}{R_s}$$

$$v_s = v_x - r_o(i_d - g_m v_{gs}) \quad (\text{KVL @ Loop 1})$$

$$R_s i_d = v_x - r_o(i_d - g_m v_{gs})$$

$$v_{gs} = -v_s = -R_s i_d$$

$$R_s i_d = v_x - r_o i_d - g_m r_o R_s i_d$$

$$(R_s + r_o + g_m r_o R_s) i_d = v_x$$

$$i_d = \frac{v_x}{r_o + R_s(1 + g_m r_o)}$$

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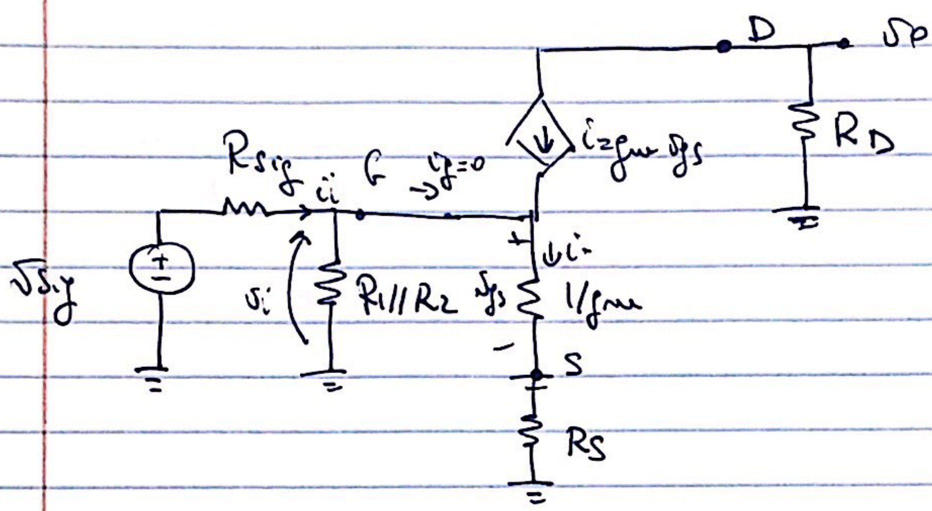
$$i_x = \frac{v_x}{r_o + R_S(1+g_m r_o)} + \frac{v_x}{R_D}$$

$$R_o = \frac{v_x}{i_x} = \frac{\cancel{v_x}}{\frac{\cancel{v_x}}{r_o+R_S(1+g_m r_o)} + \frac{\cancel{v_x}}{R_D}}$$

$$R_o = \frac{R_D(r_o + R_S(1+g_m r_o))}{r_o + R_D + R_S(1+g_m r_o)}$$

$\lambda = 0$



Input resistance

$$R_i = \frac{v_i}{i_i} = R_1 // R_2$$

Gain

$$A_v = A_{vo} = \frac{v_o}{v_i}$$

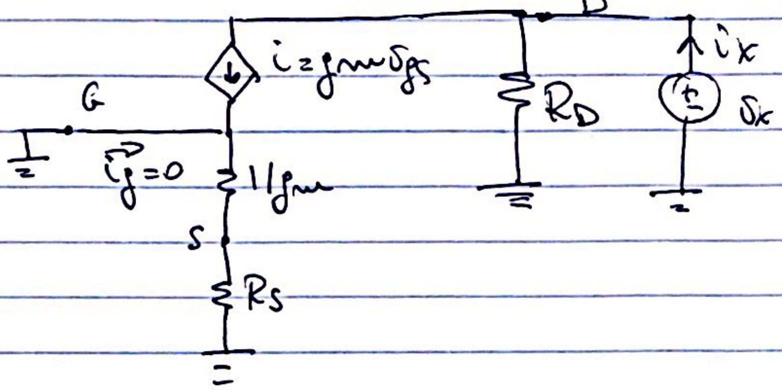
$$v_o = -g_m v_{gs} R_D$$

$$v_{gs} = v_i \frac{1/g_m}{1/g_m + R_S} = v_i \frac{1}{1 + g_m R_S}$$

$$A_v = A_{vo} = \frac{g_m R_D}{1 + g_m R_S}$$

$$G_v = A_{vo} \cdot \frac{R_i}{R_i + R_{sig}}$$

Output resistance ($\lambda = 0$)



$$R_o = \frac{v_x}{i_x}$$

$$i_x = i + \frac{v_x}{R_D}$$

$$i = +g_m v_{gs} = -g_m v_s$$

$$v_s = g_m v_s R_s \Rightarrow v_s = 0 \Rightarrow i = 0 \Rightarrow i_x = \frac{v_x}{R_D} \Rightarrow R_o = R_D$$