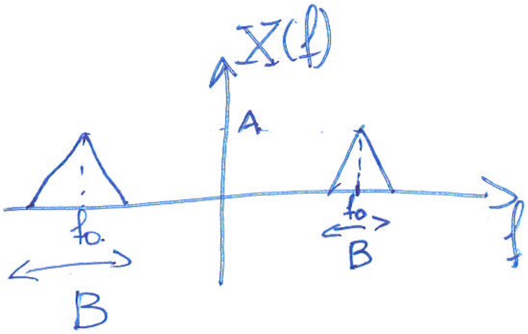
...andpass signals

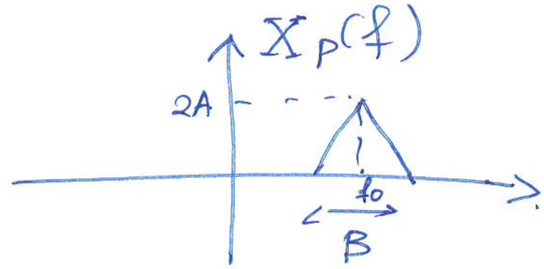
represent.

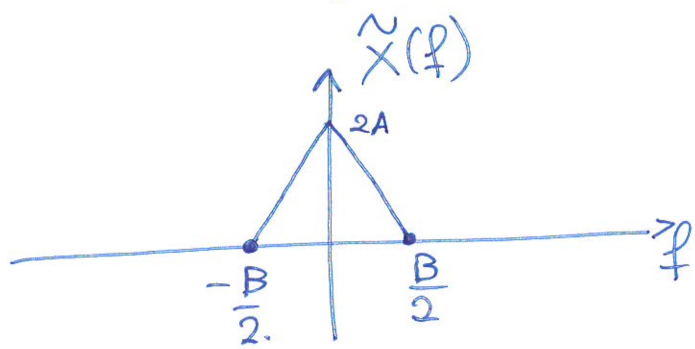
$\boxed{\text{Convex Envelope}}$

$$x_p(t) = \tilde{x}(t) \cdot e^{j2\pi f_0 t}$$

$$\tilde{x}(t) = x_p(t) \cdot e^{-j2\pi f_0 t}$$







$$x_p(t) = \tilde{x}(t) \cdot e^{j2\pi f_0 t} = x(t) + j\hat{x}(t)$$

$$x(t) = \mathrm{Re}\left[\tilde{x}(t) \cdot e^{j2\pi f_0 t}\right]$$

$$\hat{x}(t) = \mathrm{Im}\left[\tilde{x}(t) \cdot e^{j2\pi f_0 t}\right]$$

$$x(t) = \mathrm{Re}[\tilde{x}(t) \cdot e^{j2\pi f_0 t}]$$

$$= \underbrace{\mathrm{Re}[\tilde{x}(t)]}_{x_R(t)} \cdot \cos(2\pi f_0 t) - \underbrace{\mathrm{Im}[\tilde{x}(t)]}_{x_I(t)} \cdot \sin(2\pi f_0 t)$$

$$\boxed{\tilde{x}(t) = \underbrace{x_R(t)}_{\text{inphase component of } x(t)} + j\underbrace{x_I(t)}_{\text{quadrature component of } x(t)}}$$

Example:

$$x(t) = \cos(22\pi t)$$

Hilbert transform

$$\hat{x}(t) = \sin(22\pi t)$$

Analytic Signal

$$x_p(t) = x(t) + j\hat{x}(t)$$

$$= \cos(22\pi t) + j\sin(22\pi t)$$

$$= e^{j22\pi t}$$

③

$$\boxed{f_0 = 10\,H}$$

$$\tilde{x}(t) = x_p(t) \cdot e^{-j2\pi f_0 t}$$
$$= e^{j22\pi t} \cdot e^{-j20\pi t}$$
$$= e^{j2\pi t} = \cos(2\pi t) + j\sin(2\pi t)$$

$$x(t) = x_R(t) \cdot \cos(2\pi f_0 t) - x_I(t) \cdot \sin(2\pi f_0 t)$$
$$= \cos(2\pi t) \cdot \cos(20\pi t) - \sin(2\pi t) \cdot \sin(20\pi t)$$
$$= \cos(22\pi t)$$

DFT - Discrete Fourier Transform

$$X_K = \sum_{n=0}^{N-1} x_n \cdot e^{-j\frac{2\pi n K}{N}}, \quad K = 0, 1, \dots, N-1$$

Frequencies that I am taking samples of the signal

$$\rightarrow \frac{K}{N \cdot T_s}, \quad K = 0, 1, \dots, N-1$$

Sample sequence $\{x_n\}$

$$x_n = \frac{1}{N} \sum_{K=0}^{N-1} X_K \cdot e^{j\frac{2\pi n K}{N}},$$

$$K = 0, 1, \dots, N-1$$

Linear Modulation Techniques

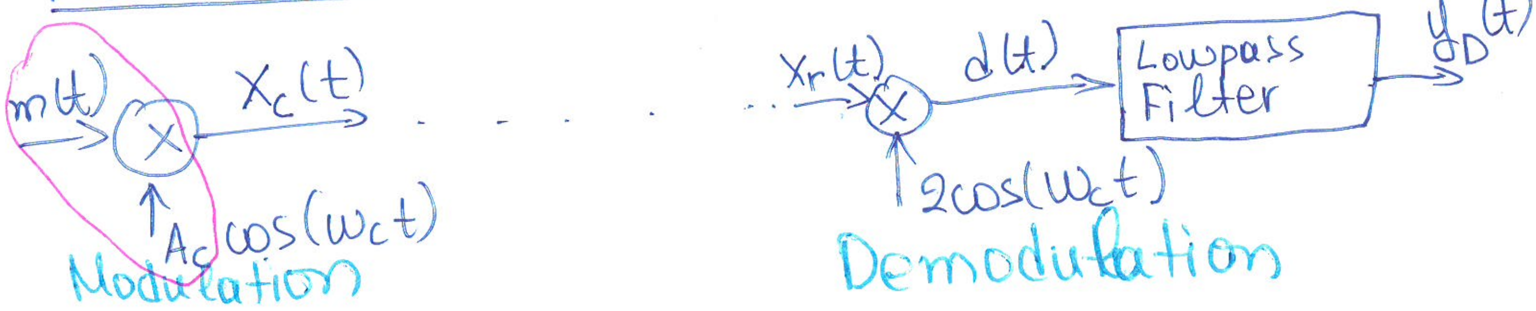
message signal $\xrightarrow[\text{process.}]{\text{modulation}}$ Demodulator extracts the information from the original signal

Analog Modulation

- continuous-wave modulation
  - $A(t)$ — Amplitude Modulation (AM) — Linear Modulation
  - $\phi(t)$ — Phase Modulation (PM)
  - derivative of $\phi(t)$ — Frequency Modulation (FM)
  - Phase Modulation and Frequency Modulation: Angle Modulation
- pulse modulation

Double-SideBand Modulation (DSB)

use both sides of freq.



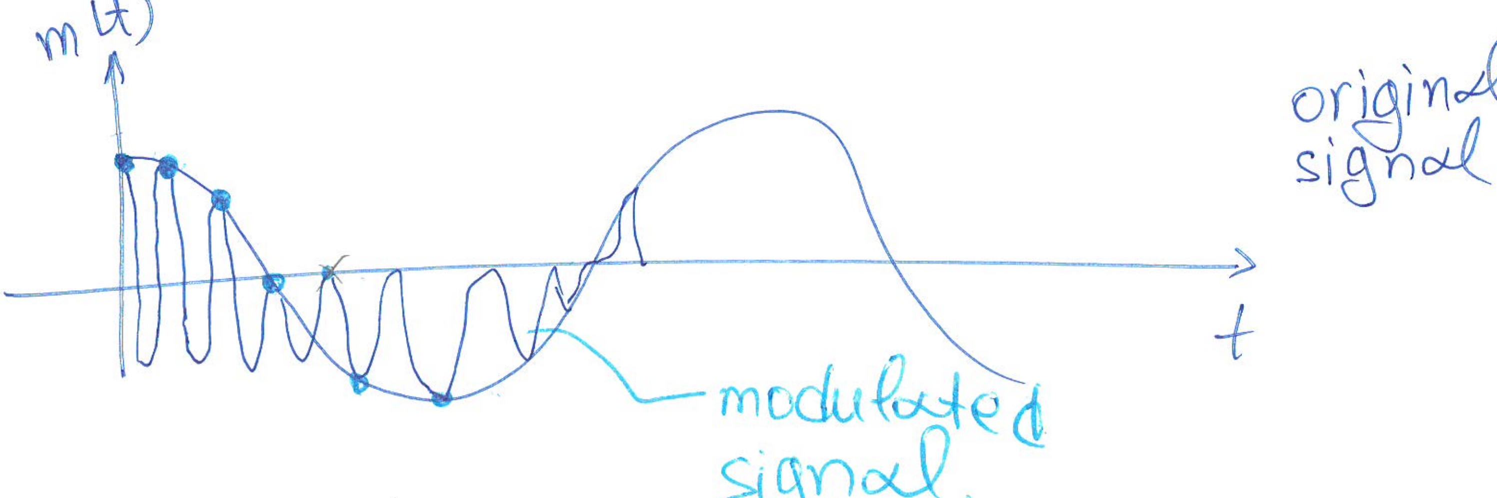
$$x_c(t) = A(t)\cos(\omega_c t + \phi(t))$$

Linear Modulation || $x_c(t) = \boxed{A(t)}\cos(\omega_c t)$

DSB || $x_c(t) = A_c \cdot m(t) \cdot \cos(\omega_c t)$

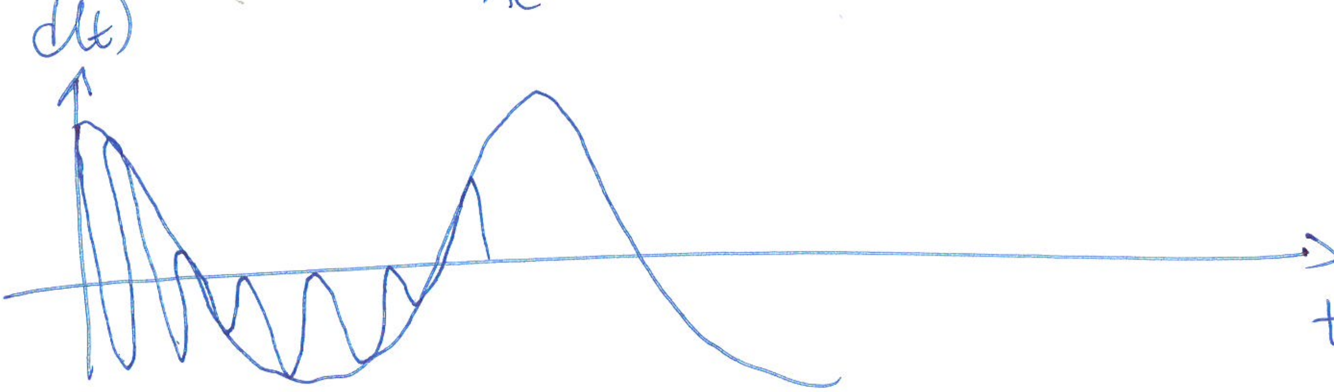
$$\Updownarrow$$

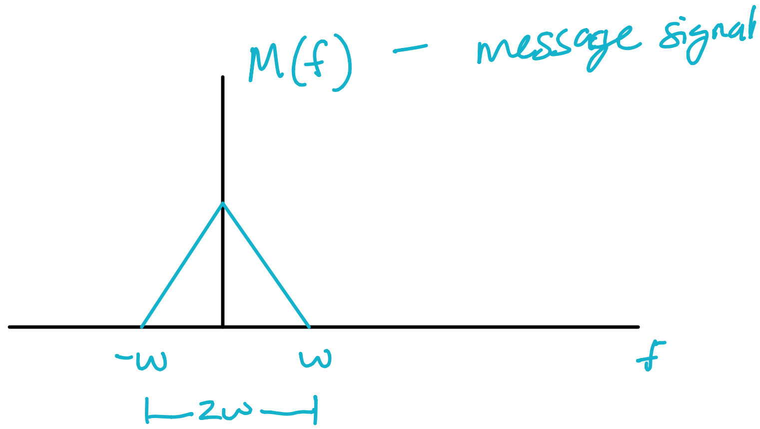
$$X_c(f) = \frac{1}{2}A_c M(f - f_c) + \frac{1}{2}A_c M(f + f_c)$$

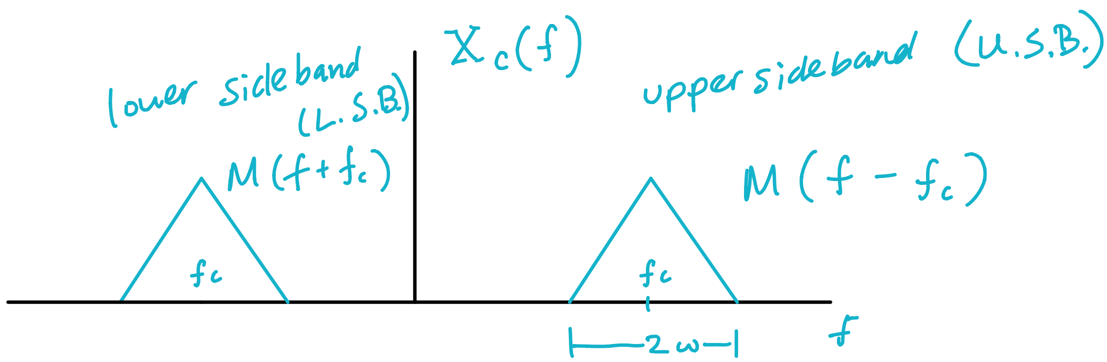


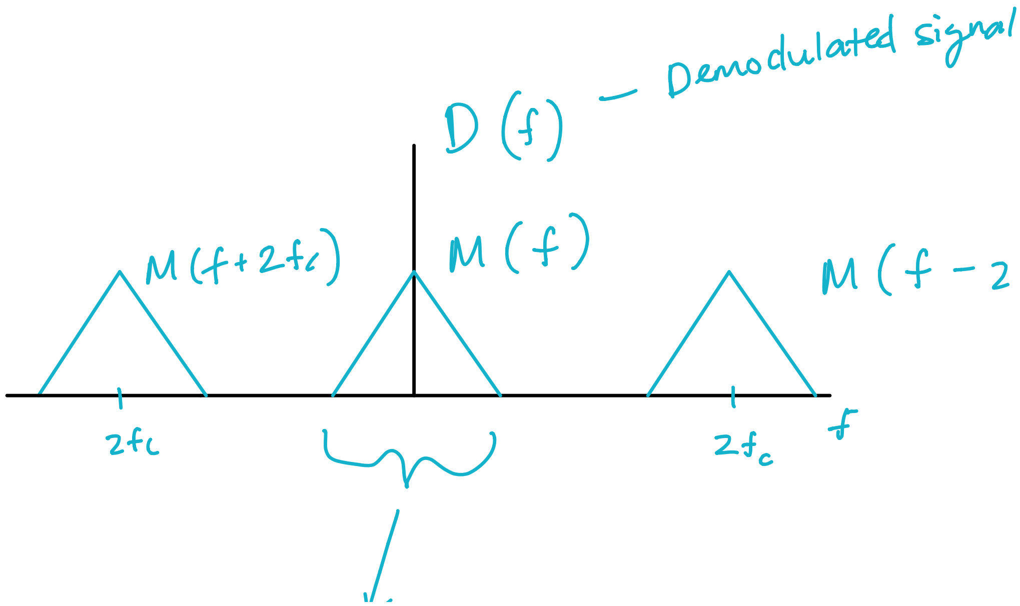
Demodulated Signal || $d(t) = 2A_c m(t)\cdot\cos(\omega_c t)\cdot\cos(\omega_c t)$

$$= A_c m(t) + A_c m(t)\cos(4\pi f_c t)$$




M(f) - message signal
-W
W
2W
f


Xc(f)
lower sideband (L.S.B)
upper sideband (U.S.B.)
M(f+fc)
M(f - fc)
fc
fc
2w
f

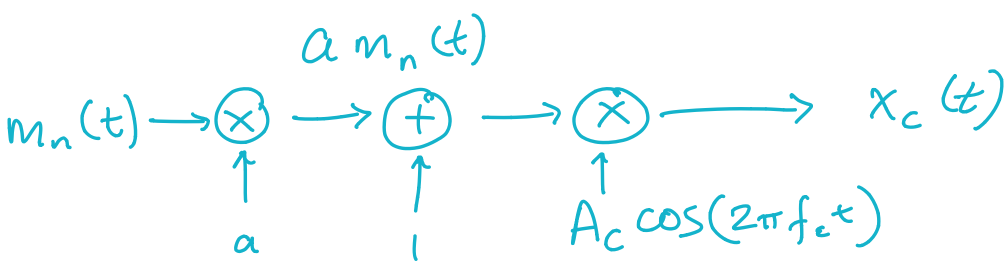

D(f) - Demodulated signal
M(f+2fc)
M(f)
M(f - 2fc)
2fc
2fc
f

Amplitude Modulation (example of linear modulation)

$$x_c(t) = A_c\left[1 + a \cdot m_n(t)\right]\cos(2\pi f_c t)$$

$$m_n(t) = \frac{m(t)}{\left|\min(m(t))\right|}$$

Modulation index



Total Power of the modulated signal

$$\langle x_c^2(t) \rangle = \langle A_c^2 [1 + a\, m_n(t)]^2 \cdot \cos^2(2\pi f_c t) \rangle =$$

$$\langle \tfrac{1}{2} A_c^2 [1 + 2a\, m_n(t) + a^2 m_n^2(t)] \rangle$$

$$= \underbrace{\frac{1}{2} A_c^2}_{\text{carrier}} + \underbrace{\frac{1}{2} A_c^2 a^2 \langle m_n^2(t) \rangle}_{\text{information}}$$

Efficiency of the modulation

$$\frac{\text{information}}{\text{info + carrier}}$$

$$\text{Eff} = \frac{a^2 \langle m_n^2(t) \rangle}{1 + a^2 \langle m_n^2(t) \rangle}$$

Ex

$a = 0.5$

$\frac{1}{2}A_c^2 = 50W$

$\downarrow$

$A_c = 10$

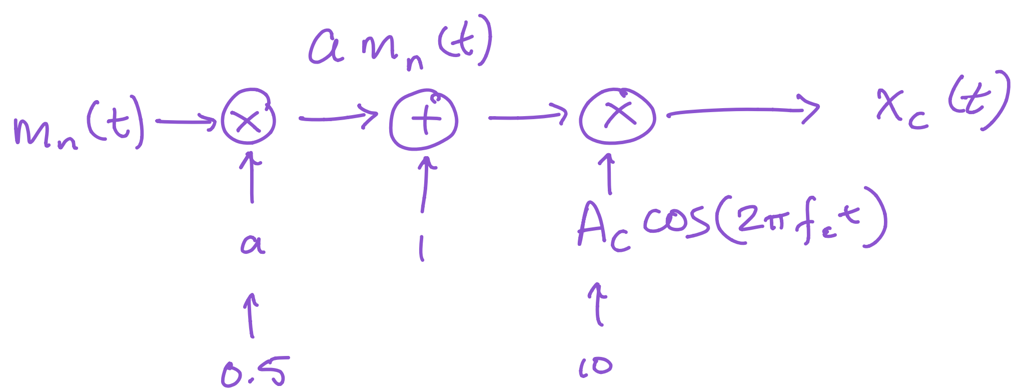
$m(t) = 4\cos\left(2\pi f_m - \frac{\pi}{9}\right) +$

$2\sin(2\pi f_m t)$

$\min(m(t)) = -4.364$ (Hidr)

$$m_n(t) = \frac{m(t)}{|\min(m(t))|} = 0.9166\cos\left(2\pi f_m - \frac{\pi}{9}\right) + 0.483\sin(2\pi f_m t)$$

$x_c(t) =$



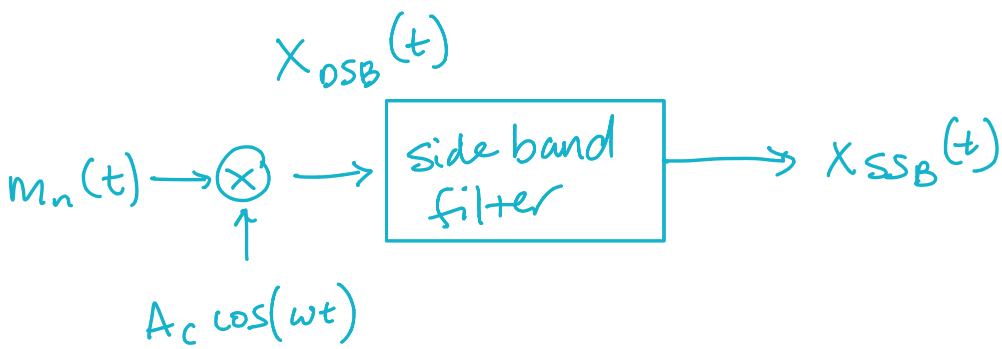
$$10\left\{1 + 0.5\left[0.9166\cos\left(2\pi f_m t - \frac{\pi}{9}\right) + 0.4583\sin(4\pi f_m\right]\right\} \cdot \cos(2\pi f_c t)$$

$$\langle m_n^2(t)\rangle = \frac{1}{2}0.3166^2 + \frac{1}{2}0.4583^2 = 0.5251$$

$$\text{Eff} = \frac{0.5251}{1 + 0.5^2 + 0.5251} = 0.166$$

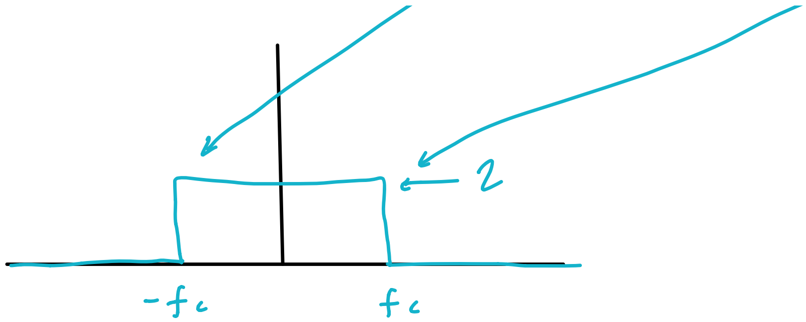
Single-Sided Band Modulation

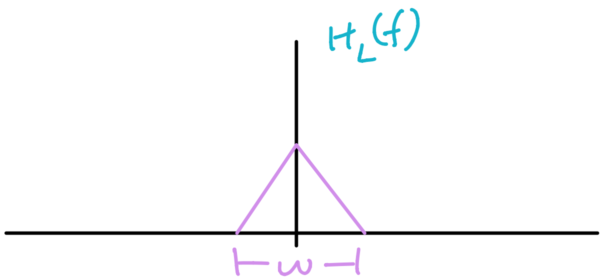
less energy expended than Double

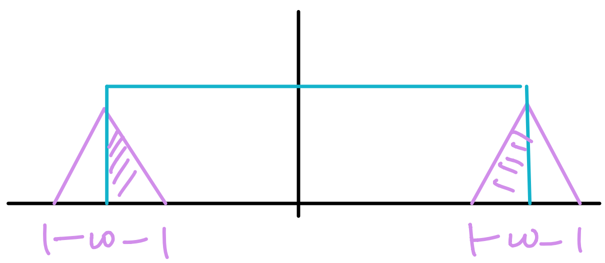


A Transfer Funct. of Filter
(filters DSB into SSB)

$$H_2(f) = \frac{1}{2}\left[\operatorname{sgn}(f+f_c) - \operatorname{sgn}(f-f_c)\right]$$







$$X_{SSB}(f) = X_{DSB}(f)\, H_L(f)$$

$$= \frac{1}{4} A_c \left[M(f+f_c)\, \mathrm{sgn}(f+f_c) + \right.$$