ECE 371
Materials and Devices

09/10/19 - Lecture 6

Step Potential Function, Potential Barrier, and Tunneling

General Information

- Homework 1 will be returned Thursday 9/12
- Homework #2 assigned and due Thursday 9/12
- First midterm (covers Ch. 1 and Ch. 2) is on Tuesday 9/24. Closed book and notes. Calculator okay.
  - Derive lattice constant
- Reading for next time: 3.1

Step Potential Function

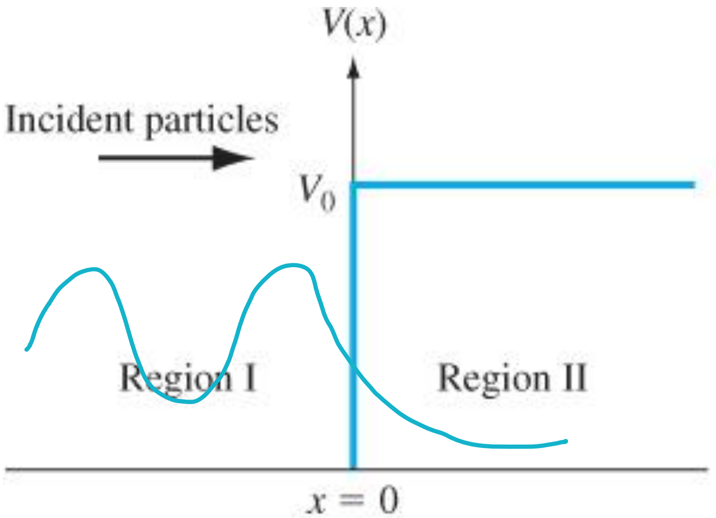


Figure 2.8 | The step potential function.

In region I the TISE is:

$$\boxed{\frac{d^2\psi(x)}{dx^2} + \frac{2mE}{\hbar^2}\psi(x) = 0}$$

In region II the TISE is:

$$\boxed{\frac{d^2\psi(x)}{dx^2} + \frac{2m}{\hbar^2}\big(E - V(x)\big)\psi(x) = 0}$$

evanscent wave in EM

- Find solution in each region separately (for $E<V_0$)
- Match boundary conditions at x=0
- Derive reflection and transmission coefficients

*see in-class derivation

In region II the TISE is:

$$\boxed{\frac{d^2\psi(x)}{dx^2} + \frac{2m}{\hbar^2}(E - V(x))\psi(x) = 0}$$

make minus

reverse $(V(x) - E)$

$p = \hbar k_1$

$\lambda = \frac{2\pi}{k_1}$

Total for Step Potential

$$\psi_I(x) = A_1 e^{-jk_1 x} + B_1 e^{-jk_1 x}$$

incident

reflected

$$\psi_{II}(x) = A_2 e^{-k_2 x}$$

transmitted

Using Boundary Conditions

$$\psi_I(0) = \psi_{II}(0) \longrightarrow A_1 + B_1 = A_2$$

$$\psi_I'(0) = \psi_{II}'(0) \longrightarrow jk_1 A_1 + jk_1 B_1 = -k_2 A_2$$

Step Potential Function

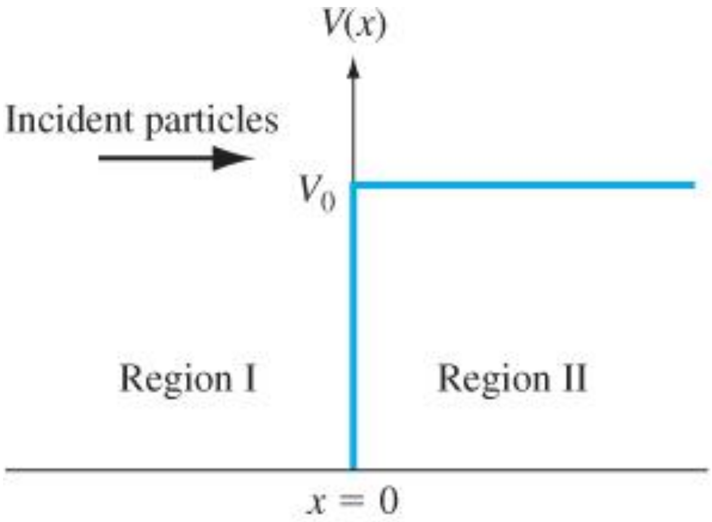


Figure 2.8 | The step potential function.

Probability Density × velocity

Reflection and Transmission:

$$\boxed{R = \frac{reflected\ particle\ flux}{incident\ particle\ flux} = \frac{B_1 B_1^*}{A_1 A_1^*}}$$

for E < V_0

$$\boxed{\begin{aligned} R &= 1 \\ T &= 0 \end{aligned}}$$

R + T = 1

reflected energy

transmis. energy

- Wave function (particle) may penetrate into region II even for E < V_0
- Particle will eventually be reflected
- The case of E > V_0 is left as a homework problem

*see in-class derivation

Example 2.4

EXAMPLE 2.4

Objective: Calculate the penetration depth of a particle impinging on a potential barrier.

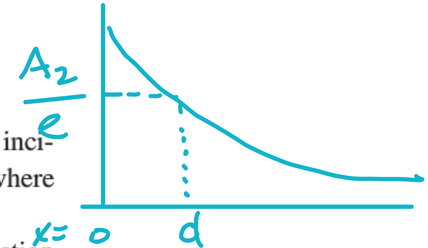
Consider an incident electron that is traveling at a velocity of 1×10^5 m/s in region I.

■ Solution

With $V(x) = 0$, the total energy is also equal to the kinetic energy so that

$$E = T = \frac{1}{2}mv^2 = 4.56 \times 10^{-21}\text{ J} = 2.85 \times 10^{-2}\text{ eV}$$

Now, assume that the potential barrier at $x = 0$ is twice as large as the total energy of the incident particle, or that $V_0 = 2E$. The wave function solution in region II is $\psi_2(x) = A_2 e^{-k_2 x}$, where the constant k_2 is given by $k_2 = \sqrt{2m(V_0 - E)/\hbar^2}$.



In this example, we want to determine the distance $x = d$ at which the wave function magnitude has decayed to e^{-1} of its value at $x = 0$. Then, for this case, we have $k_2 d = 1$ or

$$1 = d\sqrt{\frac{2m(2E - E)}{\hbar^2}} = d\sqrt{\frac{2mE}{\hbar^2}}$$

$$\psi_2(x) = A_2 e^{-k_2 x}$$

The distance is then given by

$$d = \sqrt{\frac{\hbar^2}{2mE}} = \frac{1.054 \times 10^{-34}}{\sqrt{2(9.11 \times 10^{-31})(4.56 \times 10^{-21})}} = 11.6 \times 10^{-10}\text{ m}$$

$$x = d$$

or

$$\frac{\cancel{A_2}}{e} = \psi_2(d) = \cancel{A_2} e^{-k_2 d}$$

$$d = 11.6\ \text{Å}$$

≈ 2 lattice constants in Si

$$\frac{1}{e} = e^{-k_2 d}$$

■ Comment

This penetration distance corresponds to approximately two lattice constants of silicon. The numbers used in this example are rather arbitrary. We used a distance at which the wave function decayed to e^{-1} of its initial value. We could have arbitrarily used e^{-2}, for example, but the results give an indication of the magnitude of penetration depth.

$$\frac{1}{k_2} = d$$

$$k_2 = \sqrt{\frac{2m(V_0 - E)}{\hbar^2}}$$

& $V_0 = 2E$

$$d = \sqrt{\frac{\hbar^2}{2mE}}$$

Potential Barrier

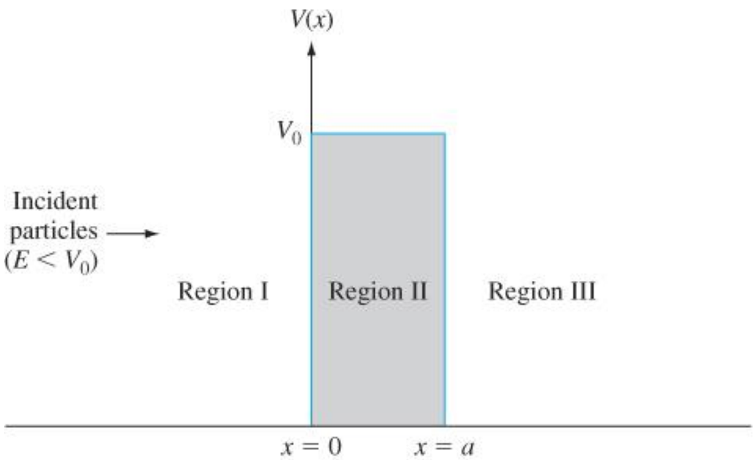


Figure 2.9 | The potential barrier function.

Transmission:

$$\boxed{T = \frac{A_3 A_3^*}{A_1 A_1^*} = 16\left(\frac{E}{V_0}\right)\left(1 - \frac{E}{V_0}\right)e^{-2k_2 a}}$$

for low tunneling probability (i.e., $k_2 a >> 1$)

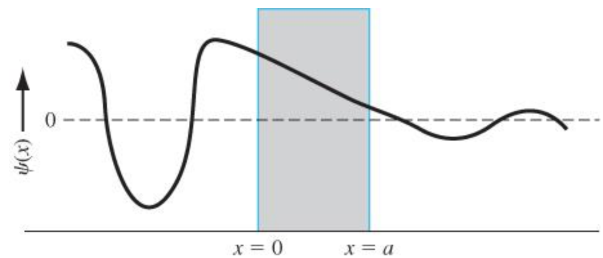
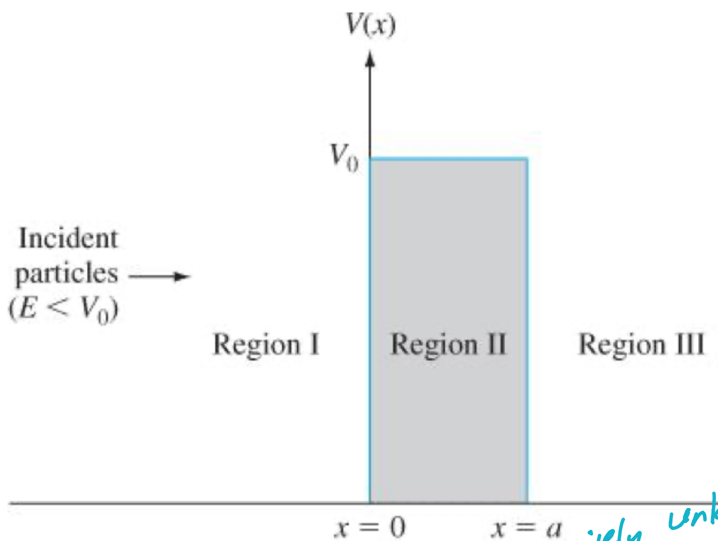


Figure 2.10 | The wave functions through the potential barrier.

- Solve TISE in all three regions
- There are four boundary conditions, two at x = 0 and two at x = a
- We can solve the boundary conditions for the transmission probability
- Finite probability the particle will tunnel through the barrier
- Tunneling probability goes down for thicker barriers

*see in-class notes



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definitely unknown then put all others in terms of this

① Solutions

I: $A_1 e^{jk_1 x} + B_1 e^{-jk_1 x}$

II: $A_2 e^{jk_2 x} + B_2 e^{-jk_2 x}$

III: $A_3 e^{jk_1 x} + \cancel{B_3 e^{-jk_1 x}}$ → 0 reflected

$$k_1 = \sqrt{\frac{2mE}{\hbar^2}} \qquad k_2 = \sqrt{\frac{2m(V_0 - E)}{\hbar^2}}$$

Boundary Conditions

$$\psi_I(0) = \psi_{II}(0) \qquad \psi_{II}(a) = \psi_{III}(a)$$

$$\psi_{I}'(x)\Big|_{x=0} = \psi_{II}'(x)\Big|_{x=0}$$

$$A_1 + B_1 = A_2 + B_2$$

$$\psi_{II}'(x)\Big|_{x=a} = \psi_{III}'(x)\Big|_{x=a}$$

$$A_2 e^{k_2 a} + B_2 e^{-k_2 a} =$$

$$A_3 e^{jk_1 a}$$

Best we can do is Find in terms of A_1

Transmission Coeff.

$$\frac{A_3 \cdot A_3^* \cdot v_t}{A_1 \cdot A_1^* \cdot v_i}$$

$$= 16\left(\frac{E}{V_0}\right)\left(1-\frac{E}{V_0}\right)e^{-2k_2a}$$

Tunneling usually occurs < 10nm