Example Exercises

1. Sketch the single-sided and double-sided amplitude and phase spectra of the following signal:

$$x(t) = 2\sin\left(6\pi t + \frac{\pi}{3}\right) \cdot \cos\left(4\pi t + \frac{\pi}{4}\right)$$

$$\sin u \cdot \cos v = \frac{1}{2}\sin(u-v) + \frac{1}{2}\sin(u+v)$$

$$\Rightarrow x(t) = \sin\left(6\pi t + \frac{\pi}{3} - 4\pi t - \frac{\pi}{4}\right) + \sin\left(10\pi t + \frac{\pi}{3} + \frac{\pi}{4}\right)$$

$$= \sin\left(2\pi t + \frac{\pi}{12}\right) + \sin\left(10\pi t + \frac{7\pi}{12}\right)$$

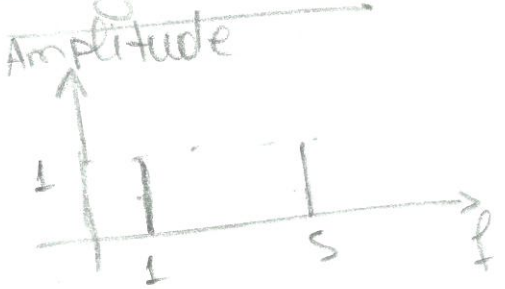
$$= \cos\left(2\pi t + \frac{\pi}{12} - \frac{6\pi}{12}\right) + \cos\left(10\pi t + \frac{7\pi}{12} - \frac{6\pi}{2}\right)$$

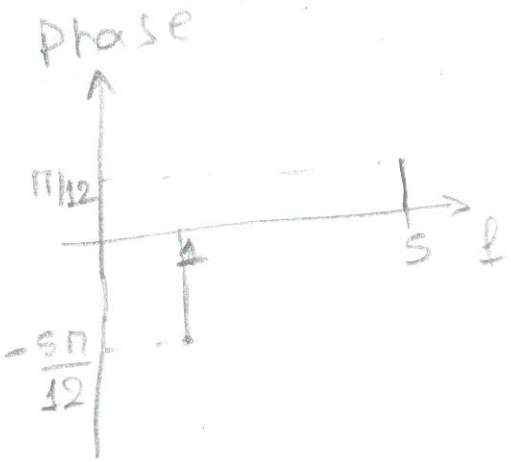
$$= \cos\left(2\pi t - \frac{5\pi}{12}\right) + \cos\left(10\pi t + \frac{\pi}{12}\right)$$

$\downarrow$ $f_1 = 1$ Hz

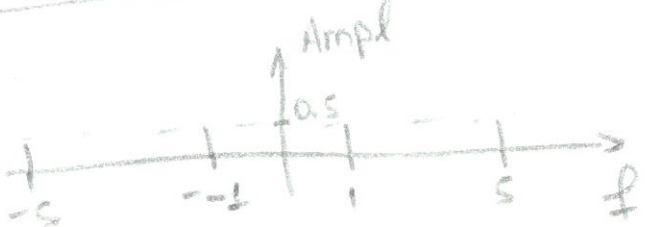
$\downarrow$ $f_2 = 5$ Hz

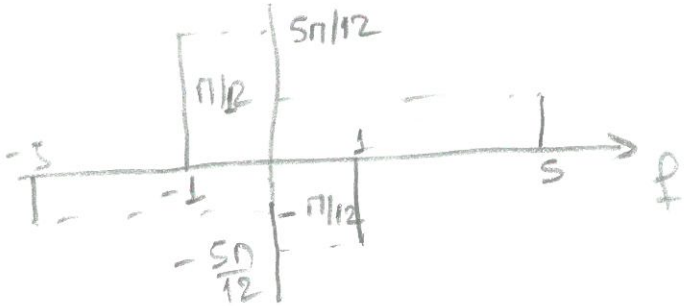
Single-sided:





Double-sided:





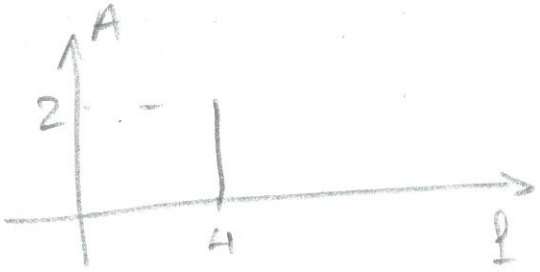
2. Signal: $x(t) = 2\cos(6\pi t + \frac{\pi}{4}) \cdot \cos(2\pi t + \frac{\pi}{3}) - 2\sin(6\pi t + \frac{\pi}{4}) \cdot \sin(2\pi t + \frac{\pi}{3})$ write: a) the real part of a sum of rotating phasors, b) a sum of rotating phasors plus their complex conjugates, and c) sketch the single-sided and double-sided amplitude and phase spectra.

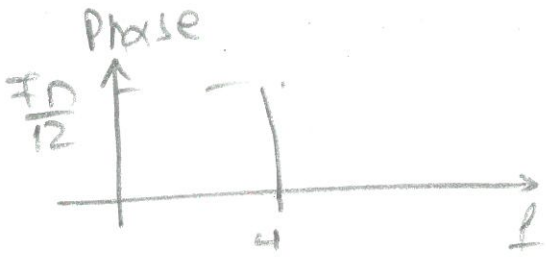
$\cos U \cdot \cos V - \sin U \cdot \sin V = \cos(U+V)$

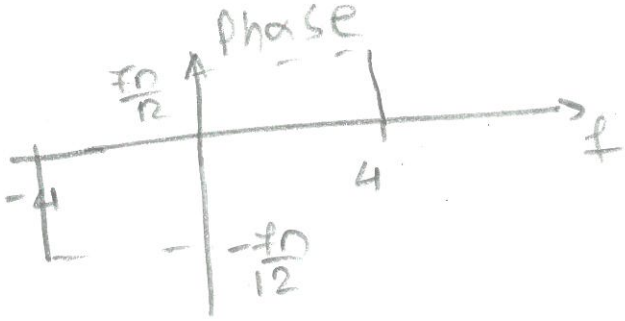
$x(t) = 2\cos(8\pi t + \frac{7\pi}{12})$

$\downarrow$

$f = 4\,Hz$







3. is it power or energy signal? ⇒ Find power or energy (3)

$$x(t) = 3\sin\left(2\pi t + \frac{\pi}{4}\right)$$

$f = 1\,Hz \rightarrow T_0 = 1\,s$

$$P = \frac{1}{T_0}\int_0^T x^2(t)\,dt = 9\int_0^1 \sin^2\left(2\pi t + \frac{\pi}{4}\right)dt = 9\int_0^1 \left(\frac{1}{2} - \frac{1}{2}\cos\left(4\pi t + \frac{\pi}{2}\right)\right)dt$$

$$= 9\left[\left[\frac{1}{2}t\right]_0^1 - \frac{9}{2}\frac{1}{4\pi}\left[\sin\left(4\pi t + \frac{\pi}{2}\right)\right]_0^1\right.$$

$$= 9\left(\frac{1}{2} - 0\right) - \frac{9}{8\pi}\left[\sin\left(4\pi + \frac{\pi}{2}\right) - \sin\frac{\pi}{2}\right]$$

($\sin\left(4\pi + \frac{\pi}{2}\right) > 0$)

$$= \frac{9}{2}\,W \rightarrow \text{power signal}$$

4. Obtain the time-average autocorrelation functio of the signal: $x(t) = \frac{1}{2} + \cos^2\left(4\pi t + \frac{\pi}{6}\right)$ and the powe spectral density and the total power

$$x(t) = \frac{1}{2} + \frac{1}{2} + \frac{1}{2}\cdot\cos\left(8\pi t + \frac{\pi}{3}\right) = 4 + \frac{1}{2}\cdot\cos\left(8\pi t + \frac{\pi}{3}\right)$$

($f = 4\,Hz$, $T = \frac{1}{4}\,sec$)

$$R_x(\tau) = \frac{1}{\frac{1}{4}}\int_0^{1/4}\left[4 + \frac{1}{2}\cos\left(8\pi t + \frac{\pi}{3}\right)\right]\left[4 + \frac{1}{2}\cos\left(8\pi(t+\tau) + \frac{\pi}{3}\right)\right]dt$$

$$= 4\int_0^{1/4}\left[16 + 2\cos\left(8\pi(t+\tau) + \frac{\pi}{3}\right) + 2\cdot\cos\left(8\pi t + \frac{\pi}{3}\right) + \frac{1}{4}\cos\left(8\pi t + \frac{\pi}{3}\right)\cdot\cos\left(8\pi(t+\tau) + \frac{\pi}{3}\right)\right]dt$$

($\cos\left(8\pi(t+\tau) + \frac{\pi}{3}\right) \rightarrow 0$, $\cos\left(8\pi t + \frac{\pi}{3}\right) \rightarrow 0$)

$$= 4\left[16\right]_0^{1/4} + \int_0^{1/4}\frac{1}{2}\cos(8\pi\tau)\,dt + \frac{1}{2}\int_0^{1/4}\cos\left(16\pi t + 8\pi\tau + \frac{2\pi}{3}\right)dt$$

($\cos\left(16\pi t + 8\pi\tau + \frac{2\pi}{3}\right) \rightarrow 0$)

$$= 4(4-0) + \frac{1}{2}\cos(8\pi\tau)\left(\frac{1}{4} - 0\right) = 16 + \frac{1}{8}\cos(8\pi\tau)$$

$$S(f) = F\{R_x(\tau)\} = 16\delta(f) + \frac{1}{16}\delta(f-4) + \frac{1}{16}\delta(f+4)$$

$P = 16 + \frac{1}{8}\,W$

5. Find the Fourier Series of: $x(t) = 10\sin^4(4\pi f_0 t) - 10\cos^4(4\pi f_0 t)$ (4)

$$x(t) = 10\left[\sin^2(4\pi f_0 t) + \cos^2(4\pi f_0 t)\right]\left[\sin^2(4\pi f_0 t) - \cos^2(4\pi f_0 t)\right]$$

$$= 10\left[1 - 2\cos^2(4\pi f_0 t)\right] = 10 - 20\left[\frac{1}{2} + \frac{1}{2}\cos(8\pi f_0 t)\right]$$

$$= 10 - 10 - 10\cos(8\pi f_0 t) = -10\cos(8\pi f_0 t) = 10\cos(8\pi f_0 t + \pi)$$

$$= 5\cdot e^{j(8\pi f_0 t + \pi)} + e^{-j(8\pi f_0 t + \pi)}$$

$$= 5\, e^{j8\pi f_0 t}\cdot e^{j\pi} + e^{-j8\pi f_0 t}\cdot e^{-j\pi}$$

$$= -5e^{j8\pi f_0 t} - e^{-j8\pi f_0 t}$$

$e^{j\pi} = \cos\pi + j\sin\pi$

$= -1$

$e^{-j\pi} = \cos(-\pi) + j\sin(-\pi)$

$= -1$

$X_4 = -5$

$X_{-4} = -5$

6. Determine the autocorrelation function of the signal with power spectral density:

$$S(f) = \frac{20}{j}\delta(f-10) - \frac{20}{j}\delta(f+f_0) + 3\delta(f)$$

and give the average power

$$S(f) = 40\left(\frac{1}{2j}\delta(f-10) - \frac{1}{2j}\delta(f+f_0)\right) + 3\delta(f)$$

$$R(\tau) = 40\sin(2\pi 10\tau) + 3$$

Average Power $= R(\tau = 0) = 3\,W$

7. For the transfer function: $H(f) = \frac{10}{j2\pi f - j8\pi} + 5\delta(f-4)$ (5)
determine the unit impulse response of the system.

$$H(f) = \frac{10}{j2\pi(f-4)} + \frac{10}{2}\delta(f-4)$$

$$= 10\left[\frac{1}{j2\pi(f-4)} + \frac{1}{2}\delta(f-4)\right] \longleftrightarrow$$

$$h(t) = u(t) \cdot e^{j2\pi 4t}$$

BIBO?

$$\int_{-\infty}^{+\infty} |h(t)|\,dt = \int_{-\infty}^{+\infty} |u(t) \cdot e^{j8\pi t}|\,dt = \int_{0}^{+\infty} |e^{j8\pi t}|\,dt =$$

$$\left[\frac{1}{j8\pi} e^{j8\pi t}\right]_0^{+\infty} \quad \times \quad \text{NO}$$