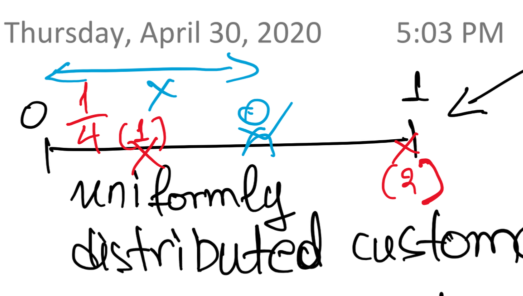
Thursday, April 30, 2020 5:03 PM

Hotelling Model

1) 

uniformly distributed customers

transportation cost : 2d (distance)

rational players

$\boxed{\text{production cost} = 0}$

a) Calculate demand functions

b) Same price charging ?

Solution: Company 1 → P_1
Company 2 → P_2

Customers indifferent from where they buy:

$$P_1 + 2\left(x - \frac{1}{4}\right) = P_2 + 2(1-x)$$

$$P_1 + 2x - \frac{1}{2} = P_2 + 2 - 2x$$

$$4x = P_2 - P_1 + \frac{5}{2}$$

$$\boxed{x = \frac{P_2 - P_1}{4} + \frac{5}{8}}$$

$$D_1(P_1,P_2) = x = \frac{P_2 - P_1}{4} + \frac{5}{8}$$

$$D_2(P_1,P_2) = 1 - x = \frac{3}{8} + \frac{P_1 - P_2}{4}$$

Demand Functions

b) Find Nash Equilibrium ⇒ max{profit}

Profit Functions:

$$\Pi_1(P_1,P_2) = P_1 \cdot D_1 = P_1 \cdot \frac{P_2 - P_1}{4} + \frac{5}{8}P_1 = \frac{P_1P_2}{4} - \frac{P_1^2}{4} + \frac{5}{8}P_1$$

$$\Pi_2(P_1,P_2) = P_2 \cdot D_2 = P_2 \cdot \frac{3}{8} + \frac{P_2P_1}{4} - \frac{P_2^2}{4}$$

$$\frac{\partial \Pi_1}{\partial P_1} = \frac{P_2}{4} - \frac{P_1}{2} + \frac{5}{8} = 0 \Rightarrow 2P_2 - 4P_1 + 5 = 0 \quad (\times 2)$$

$$\frac{\partial \Pi_2}{\partial P_2} = \frac{3}{8} + \frac{P_1}{4} - \frac{P_2}{2} = 0 \Rightarrow 2P_1 - 4P_2 + 3 = 0$$

⇒

$$\cancel{4P_2} - 8P_1 + 10 = 0$$

$$2P_1 - \cancel{4P_2} + 3 = 0$$

(+)

$$-6P_1 + 13 = 0$$

$$\boxed{P_1^* = \frac{13}{6}}$$

$$\frac{13}{3} - 4P_2 + 3 = 0$$

$$13 - 12P_2 + 9 = 0$$

$$\boxed{P_2^* = \frac{22}{12} = \frac{11}{6}}$$

$P_1^* \neq P_2^*$ $\boxed{\text{NO}}$

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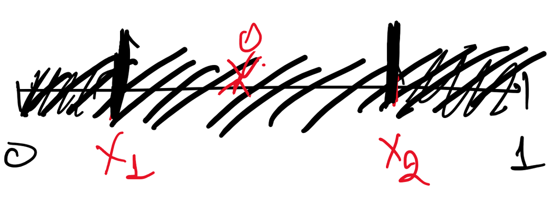
2) linear transportation cost
2 companies
$P_1 = P_2 = P$
production cost = 0
NE? Many NE?
↓
Nash Equilibrium

Company 1 → x_1
Company 2 → x_2.



Indifferent Consumer's position ⇒ $x = \frac{x_1 + x_2}{2}$

$D_1(x_1,x_2) = \frac{x_1 + x_2}{2}$

$D_2(x_1,x_2) = 1 - \frac{x_1 + x_2}{2}$

Profit Functions

$$\Pi_1(x_1,x_2) = \underbrace{\frac{x_1 + x_2}{2}}_{\text{Demand}} \cdot \underbrace{P}_{\text{Price}} \quad D_1$$

$$\Pi_2(x_1,x_2) = \left[1 - \frac{x_1 + x_2}{2}\right] \cdot P$$

$D_2 = 1 - D_1$

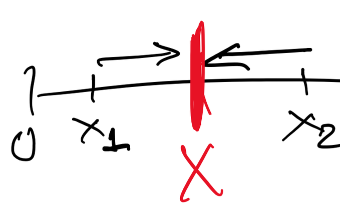
Determine NE:

$\frac{\partial \Pi_1}{\partial x_1} = \frac{1}{2} \cdot P > 0$ ⇒ Company 1 $\xrightarrow{\text{approaches}}$ Company 2 ⇒ profit↑

$\frac{\partial \Pi_2}{\partial x_2} = -\frac{1}{2} P < 0$ ⇒ Company 2 $\xrightarrow{\text{approaches}}$ Company 1 ⇒ profit↑

Company 1 ⇄ Company 2

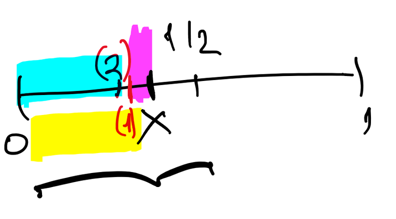
Candidate NE:



$$x_1 = x_2 = X = \frac{1}{2}$$

$$\Pi_1 = \Pi_2 = \frac{P}{2}$$

If $x < \frac{1}{2}$: Company 1 ⇒ profit↑ ⇒ going slightly to the right of Company 2
✗ NE



If $x > \frac{1}{2}$: Company 2 ⇒ profit↑ ⇒ going slightly to the left of Company 1
✗ NE

NE ⇒ $x_1^* = x_2^* = x = \frac{1}{2}$

payoff ↳ $\Pi_1(x_1^*, x_2^*) = \Pi_2(x_1^*, x_2^*) = \frac{P}{2}$

$$\left(\underbrace{G}_{\text{game}} = [N, \{A_i\}_{i \in N}, \underbrace{\{U_i\}_{i \in N}}_{\text{payoff}}] \right)$$

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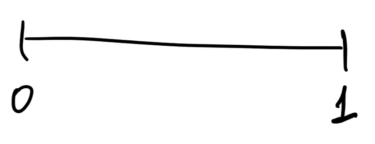
3) Hotelling Model $(\underline{P_1, P_2}, \underline{x_1, x_2})$ P_s, x_s ⇒ calculated simultaneously.



NE?

A) $P_1^* = P_2^* > 0$ $x_1^* = x_2^* = \frac{1}{2}$

∀ company ⇒ profit↑ by changing the location.

Thus, $P_1^* = P_2^*$, $x_1^* = x_2^* = \frac{1}{2}$ ⇒ ~~NE~~ ⇒ because one company can undercut the competing company by a very small amount ⇒ get the whole demand

B) $P_1^* > P_2^* > 0$:

B1) $x_1^* = x_2^*$: Company 1 makes zero profit ⇒ ~~NE~~

$\lim_{P} P_1^* \to P_2^*$ ⇒ Company's 1 profit $\frac{P_2^*}{2}$

B2) $x_1^* \neq x_2^*$: (B2.a) → Company 1: $D_1^* = 0$

(B2.b) → Company 1: $D_1^* > 0$

Company 2: $\Pi_2 = (1 - D_1^*) \cdot P_2^*$

company 2 will increase its profit by setting $x_1^* = x_2^*$.

⇒ B1 ⇒ ~~NE~~

C) $P_2^* > P_1^* > 0$: Exactly as case B.

D) $P_i^* = 0$ at least one company ⇒ ~~NE~~

↳ no profit