# ECE 322L
# Electronics 2

04/9/20 - Lecture 20

Power considerations

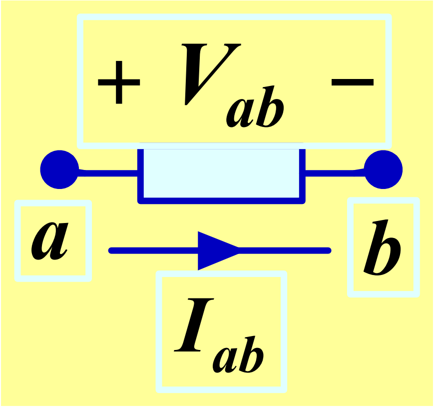
# Overview

Lecture 20:

- Electrical power (Review)
- Power gain
- Power dissipation
- Power efficiency and maximum undistorted power

(Neamen 6.10, S&S 1.4.4-1.4.6, Handout on UNM-Learn)

# Electrical Power (Review)




**Electrical power is the rate at which electrical energy is supplied or received by a component.**

$$\boxed{P = dw/dt = VI \ \ [\mathrm{W}]}$$

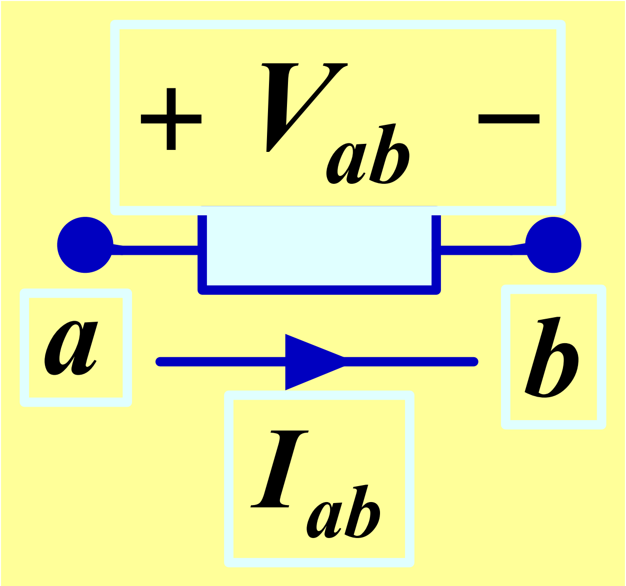
**One watt of power is required to transfer one Joule of energy (to/from a given component) in one second**

$$1\,\mathrm{W} = 1\,\frac{\mathrm{J}}{\mathrm{s}}$$

**One watt of power is required to drive one ampere of current against an electromotive difference of one volt.**

$$1\,\mathrm{W} = 1\,\mathrm{V}\cdot\mathrm{A}$$

# Power in DC




**In DC the electrical power supplied or received by a circuit element is simply**

$$\boxed{P_{dc} = V_{ab} I_{ab} = V_{dc} I_{dc}}$$

# Power in AC

$$\boxed{p(t) = v_{ab}(t)i_{ab}(t)}$$

**Instantaneous power**
**(Definition is valid for any ac signal)**

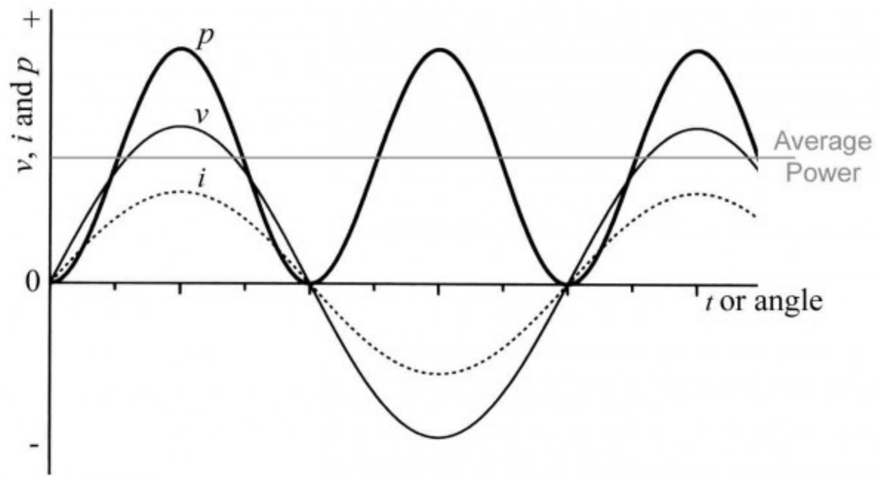
$$\boxed{P = \frac{1}{T}\int_{t_0}^{t_0+T} p(t)dt = \frac{1}{T}\int_{t_0}^{t_0+T} v(t)_{ab} i_{ab}(t)dt}$$

**Average power**
**(Definition is valid for any periodic signal with period T)**

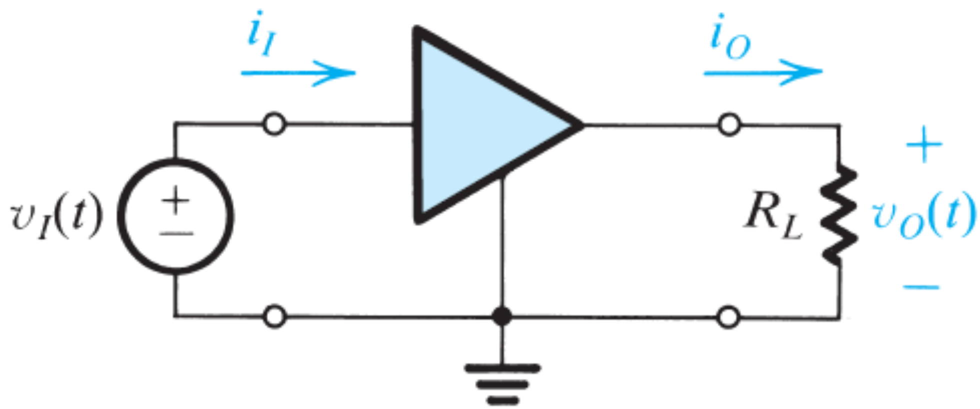
**Definition of average power in terms of rms value of voltage and current**

$$\boxed{P = V_{rms} I_{rms}}$$

$$\boxed{V_{rms} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} v^2(t)dt} \quad I_{rms} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} i^2(t)dt}}$$



# Power Gain in Amplifiers



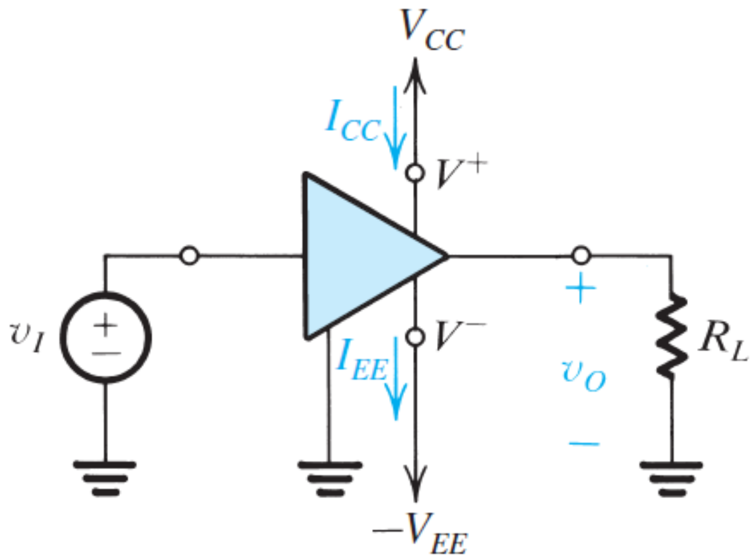

$$\text{Power gain } (A_p) \equiv \frac{\text{load power } (P_L)}{\text{input power } (P_I)}$$

$$= \frac{v_O i_O}{v_I i_I}$$

$$A_p = A_v A_i$$

**Where is this extra power coming from? Isn't energy supposed to be conserved?**

# Power Gain in Amplifiers



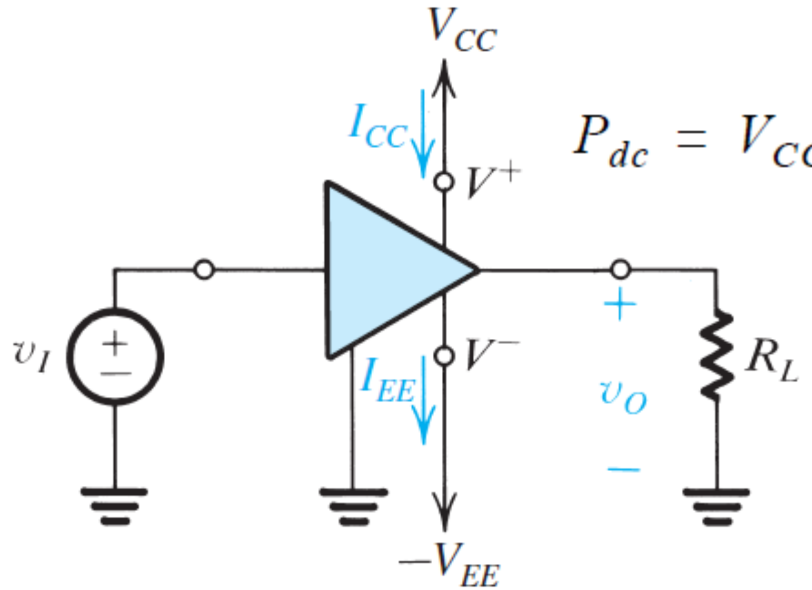

$$\text{Power gain } (A_p) \equiv \frac{\text{load power } (P_L)}{\text{input power } (P_I)}$$

$$= \frac{v_O i_O}{v_I i_I}$$

$$A_p = A_v A_i$$

**The DC sources used to bias the transistors in the amplifier supply the "extra" power to the load**

# Power Balance Equation

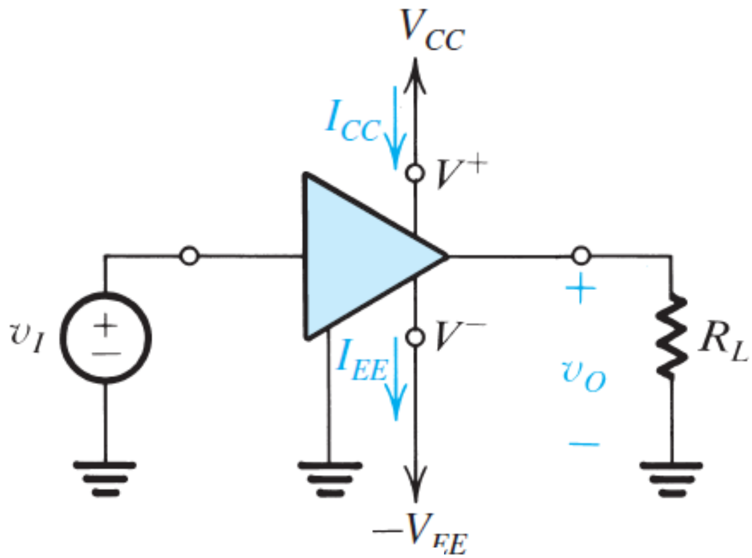



$$P_{dc} = V_{CC}I_{CC} + V_{EE}I_{EE}$$

$$\boxed{P_{\mathrm{dc}} + P_I = P_L + P_{\mathrm{dissipated}}}$$

- ➢ Power drawn from the DC voltage/current sources ($P_{dc}$)
- ➢ Power drawn from the signal source ($P_I$)
- ➢ Power transferred to the load ($P_L$)
- ➢ Power dissipated in the circuit (by the transistor and the biasing network) ($P_{dissipated}$)

# Power Gain in Amplifiers




$$\text{Power gain } (A_p) \equiv \frac{\text{load power } (P_L)}{\text{input power } (P_I)}$$

$$P_{\text{dc}} + P_I = P_L + P_{\text{dissipated}}$$

$$\text{Power gain } (A_p) \equiv \frac{P_{\text{dc}} + P_I - P_{\text{dissipated}}}{\text{input power } (P_I)}$$

$$= \frac{v_O i_O}{v_I i_I}$$

# Power Efficiency

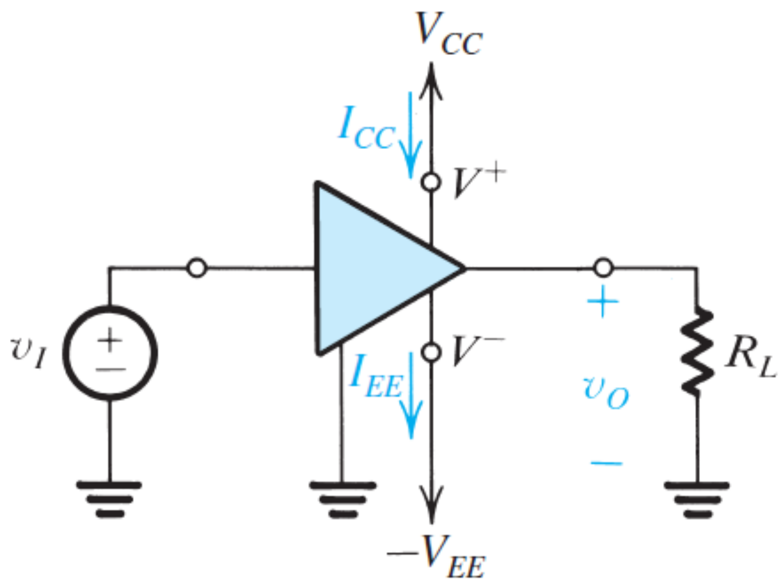
It quantifies the efficiency of the power transfer from the TWO inputs (DC power supplies and signal source) to the load

$$\eta(\%) = \frac{P_L}{P_{dc} + P_I} \times 100 \quad P_{dc} \gg P_I$$

$$\boxed{\eta(\%) \cong \frac{P_L}{P_{dc}} \times 100}$$

# Take-home problem

Consider an amplifier operating from ±10-V power supplies. It is fed with a sinusoidal voltage having 1 V peak and delivers a sinusoidal voltage output of 9 V peak to a 1-kΩ load. The amplifier draws a current of 9.5 mA from each of its two power supplies. The input current of the amplifier is found to be sinusoidal with 0.1 mA peak. Find the voltage gain, the current gain, the power gain, the power drawn from the dc supplies, the power dissipated in the amplifier, and the amplifier efficiency.




$$P = V_{rms} I_{rms}$$

$$V_{rms} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} v^2(t)dt} \quad I_{rms} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} i^2(t)dt}$$

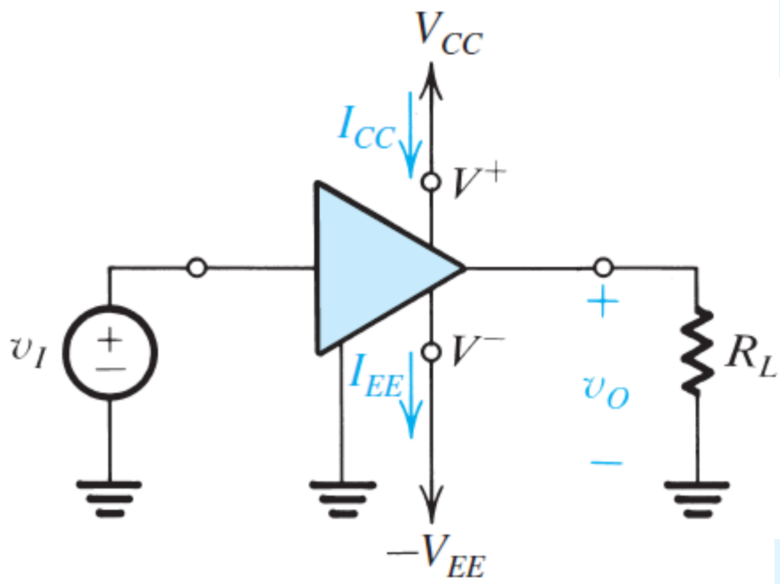
$$\text{Power gain } (A_p) \equiv \frac{\text{load power } (P_L)}{\text{input power } (P_I)}$$

$$= \frac{v_O i_O}{v_I i_I} \qquad A_p = A_v A_i$$

$$\eta(\%) \cong \frac{P_L}{P_{dc}} \times 100$$

# Take-home problem, solution

Consider an amplifier operating from ±10-V power supplies. It is fed with a sinusoidal voltage having 1 V peak and delivers a sinusoidal voltage output of 9 V peak to a 1-kΩ load. The amplifier draws a current of 9.5 mA from each of its two power supplies. The input current of the amplifier is found to be sinusoidal with 0.1 mA peak. Find the voltage gain, the current gain, the power gain, the power drawn from the dc supplies, the power dissipated in the amplifier, and the amplifier efficiency.




$$A_v = \frac{9}{1} = 9 \text{ V/V}$$

$$\hat{I}_o = \frac{9 \text{ V}}{1 \text{ k}\Omega} = 9 \text{ mA}$$

$$A_i = \frac{\hat{I}_o}{\hat{I}_i} = \frac{9}{0.1} = 90 \text{ A/A}$$

$$P_L = V_{o_{\text{rms}}} I_{o_{\text{rms}}} = \frac{9}{\sqrt{2}}\frac{9}{\sqrt{2}} = 40.5 \text{ mW}$$

$$P_I = V_{i_{\text{rms}}} I_{i_{\text{rms}}} = \frac{1}{\sqrt{2}}\frac{0.1}{\sqrt{2}} = 0.05 \text{ mW}$$

$$A_p = \frac{P_L}{P_I} = \frac{40.5}{0.05} = 810 \text{ W/W}$$

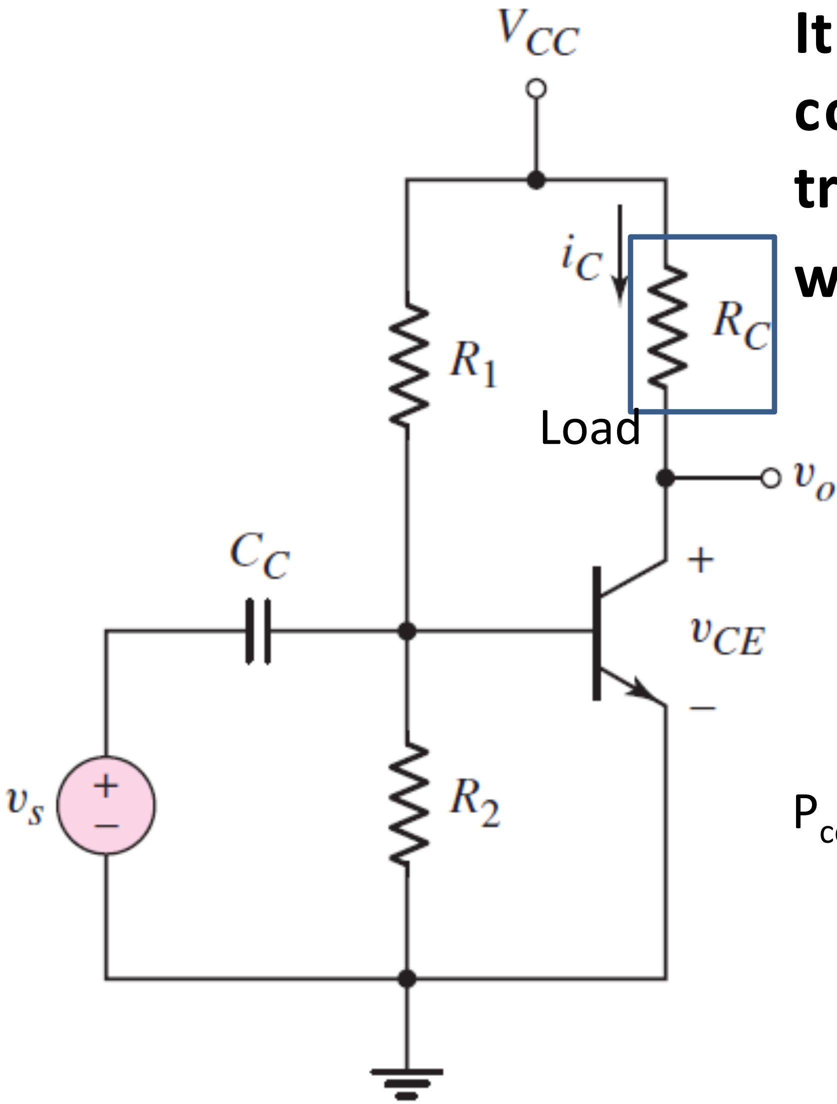
$$P_{\text{dc}} = 10 \times 9.5 + 10 \times 9.5 = 190 \text{ mW}$$

$$P_{\text{dissipated}} = P_{\text{dc}} + P_I - P_L$$

$$= 190 + 0.05 - 40.5 = 149.6 \text{ mW}$$

$$\eta = \frac{P_L}{P_{\text{dc}}} \times 100 = 21.3\%$$

# Power in Transistor Amplifiers




**It is relevant to determine the various components of the power in a transistor amplifier with ($v_s \neq 0$) and without an applied signal ($v_s = 0$).**

**Supplied power=**

**Power dissipated in the amplifier**

**+**

**Power transferred to the load**

$$P_{cc}+P_s=P_Q+P_{Bias}+P_{Rc}$$

$P_{cc}$:Power supplied by the DC voltage source (Vcc)

$P_s$: Power supplied by the ac voltage source

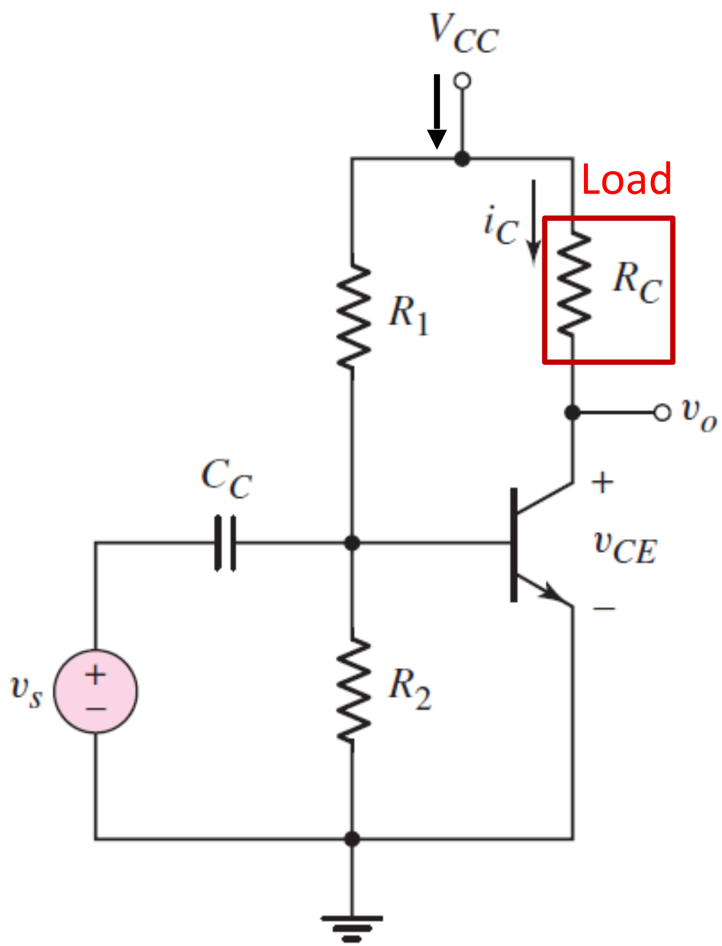
$P_Q$: Power dissipated by the BJT

$P_{Bias}$: Power dissipated by the biasing resistors

$P_{Rc}$: Power dissipated by the load

# Power in Transistor Amplifiers ($v_s$=0)

$P_{supplied} = P_{dissipated} + P_{transferred\ to\ the\ load}$




$P_{supplied} = P_{CC} + P_s = P_{CC}$

$P_{CC} = V_{CC} I_{CC} = V_{CC}(I_{CQ} - I_{R_1,R_2}) \approx V_{CC} I_{CQ}$

$P_{dissipated} = P_{bias} + P_Q$

$P_Q = I_{CQ} V_{CEQ} + I_{BQ} V_{BEQ} \approx I_{CQ} V_{CEQ}$

$(I_{CQ} >> I_{BQ})$

$P_{bias} = f(I_{BQ})$, $P_Q = f(I_{CQ})$, $I_{CQ} >> I_{BQ}$

$P_{dissipated} = P_{bias} + P_Q \approx P_Q$

$P_{transferred\ to\ the\ load} = P_{R_C} = I_{CQ}{}^2 R_C$

DC only

# Power in Transistor Amplifiers ($v_s \neq 0$)

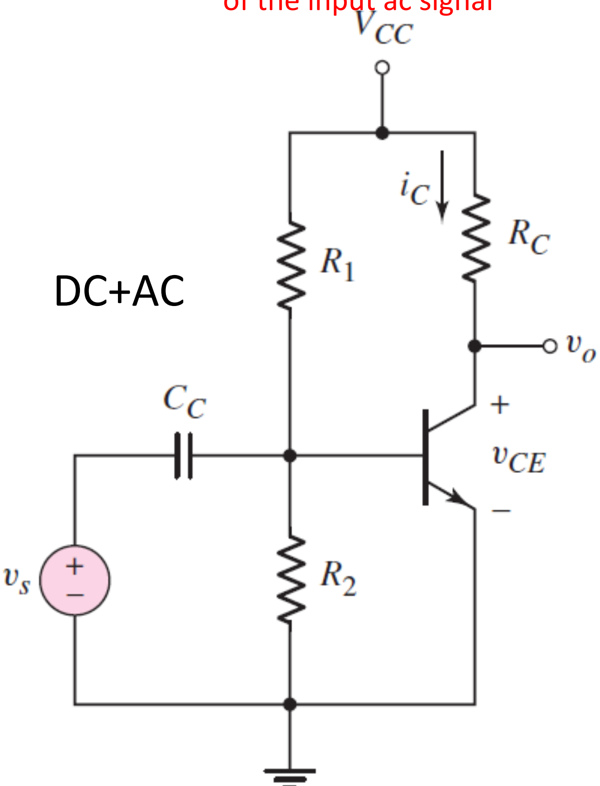
$\overline{p}_{supplied} = \overline{p}_{dissipated} + \overline{p}_{transferred\ to\ the\ load}$

Average supplied power over one period of the input ac signal

Average supplied power over one period of the input ac signal.

Average power supplied to the load over one period of the input ac signal.

DC+AC




$\overline{p}_{supplied} = \overline{p}_{cc} + \overline{p}_s \approx \overline{p}_{cc}$

$\overline{p}_{cc} >> \overline{p}_s$ as the input signal is small w.r. t. the DC voltage source (Vcc)

$\overline{p}_{dissipated} = \overline{p}_Q + \overline{p}_{bias} \approx \overline{p}_Q$

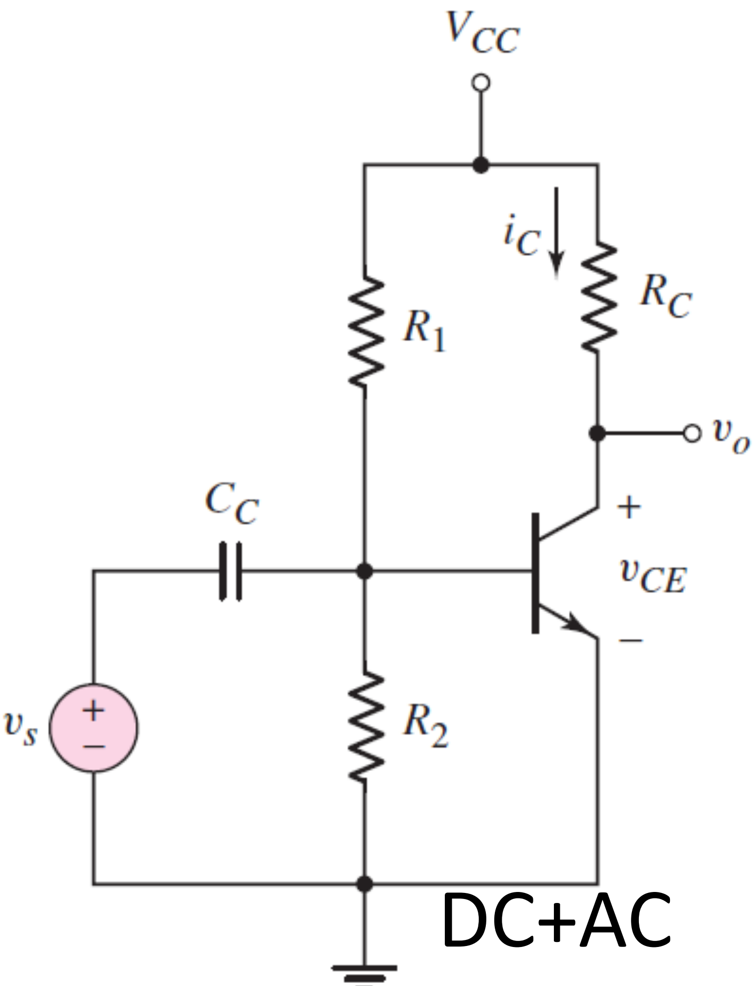
$\overline{p}_Q >> \overline{p}_{bias}$ because $i_C >> i_B$ and the base current/the collector current determines $\overline{p}_{bias} / \overline{p}_Q$.

$\overline{p}_{transferred\ to\ the\ load} = \overline{p}_{Rc}$

$\overline{p}_{cc} \approx \overline{p}_Q + \overline{p}_{Rc}$

# Power in Transistor Amplifiers ($v_s \neq 0$)

$\bar{p}_{supplied} = \bar{p}_{dissipated} + \bar{p}_{transferred\ to\ the\ load}$




DC+AC

$\bar{p}_{supplied} = \bar{p}_{cc} + \bar{p}_s \approx \bar{p}_{cc}$ $\quad \bar{p}_{cc} >> \bar{p}_s$

$\bar{p}_{dissipated} = \bar{p}_Q + \bar{p}_{bias} \approx \bar{p}_Q$ $\quad \bar{p}_Q >> \bar{p}_{bias}$

$\bar{p}_{transferred\ to\ the\ load} = \bar{p}_{Rc}$

$\bar{p}_{cc} \approx \bar{p}_Q + \bar{p}_{Rc}$

The next few slides will show how to calculate

$$v_s = V_p \cos \omega t$$

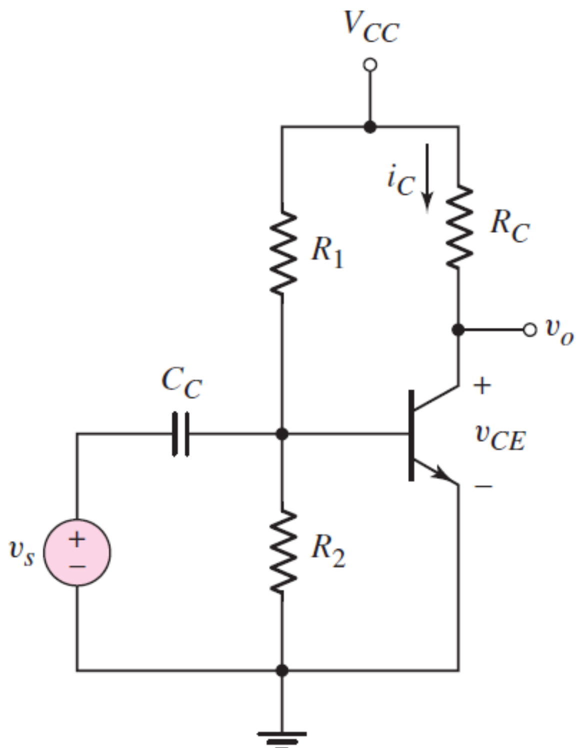
$$\bar{p}_{cc} = \frac{1}{T}\int_0^T V_{CC} \cdot i_C \, dt$$

$$\bar{p}_Q = \frac{1}{T}\int_0^T i_C \cdot v_{CE} \, dt$$

$$\bar{p}_{RC} = \frac{1}{T}\int_0^T i_C^2 R_C \, dt$$

# Power in Transistor Amplifiers ($v_s \neq 0$)

$$\bar{p}_{cc} = \frac{1}{T}\int_0^T V_{CC} \cdot i_C \, dt$$




$$v_s = V_p \cos \omega t$$

$$i_B = I_{BQ} + \frac{V_p}{r_\pi}\cos \omega t = I_{BQ} + I_b \cos \omega t$$

$$i_C = I_{CQ} + \beta I_b \cos \omega t = I_{CQ} + I_c \cos \omega t$$

$$\bar{p}_{cc} = \frac{1}{T}\int_0^T V_{CC} \cdot i_C \, dt =$$

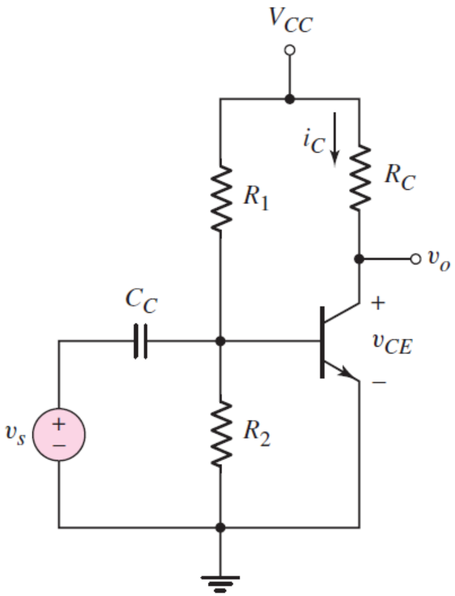
$$= \frac{1}{T}\int_0^T V_{CC} \cdot [I_{CQ} + I_c \cos \omega t]\, dt =$$

$$= V_{CC} I_{CQ} + \frac{V_{CC} I_c}{T}\int_0^T \cos \omega t \, dt = V_{CC} I_{CQ}$$

# Power in Transistor Amplifiers ($v_s \neq 0$)

$$\bar{p}_Q = \frac{1}{T}\int_0^T i_C \cdot v_{CE}\, dt$$

$$i_C = I_{CQ} + \beta I_b \cos \omega t = I_{CQ} + I_c \cos \omega t$$




$$v_s = V_p \cos \omega t$$

$$v_{CE} = V_{CC} - i_C R_C = V_{CC} - (I_{CQ} + I_c \cos \omega t) R_C =$$

$$= V_{CEQ} - I_c R_C \cos \omega t$$

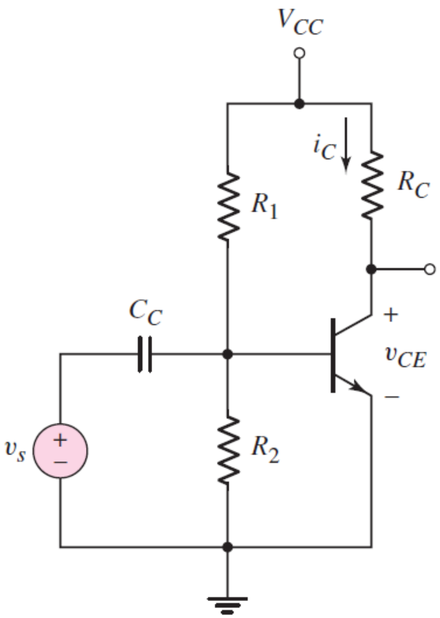
$$\bar{p}_Q = \frac{1}{T}\int_0^T i_C \cdot v_{CE}\, dt$$

$$= \frac{1}{T}\int_0^T [I_{CQ} + I_c \cos \omega t] \cdot [V_{CEQ} - I_c R_C \cos \omega t]\, dt$$

$$\bar{p}_Q = I_{CQ} V_{CEQ} - \frac{I_c^2 R_C}{T}\int_0^T \cos^2 \omega t\, dt$$

$$\bar{p}_Q = I_{CQ} V_{CEQ} - \tfrac{1}{2} I_c^2 R_C$$

# Power in Transistor Amplifiers ($v_s \neq 0$)



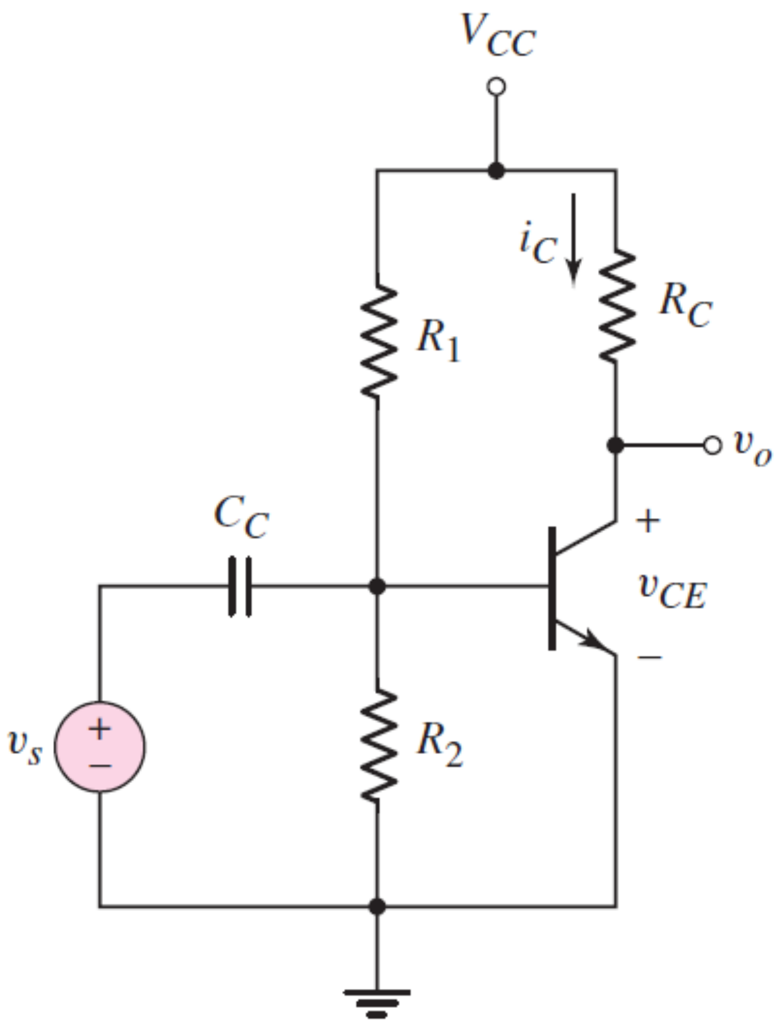

$$\bar{p}_{RC} = \frac{1}{T}\int_0^T i_C^2 R_C\,dt = \frac{R_C}{T}\int_0^T [I_{CQ} + I_c \cos\omega t]^2\,dt$$

$$= \frac{I_{CQ}^2 R_C}{T}\int_0^T dt + \frac{2I_{CQ}I_c}{T}\int_0^T \cos\omega t\,dt + \frac{I_c^2 R_C}{T}\int_0^T \cos^2\omega t\,dt$$

$$\bar{p}_{RC} = I_{CQ}^2 R_C + \tfrac{1}{2} I_c^2 R_C$$

$$v_s = V_p \cos\omega t$$

# Power in Transistor Amplifiers




| $v_s = 0$ | $v_s \neq 0$ |
|---|---|
| $P_{CC} = I_{CQ} V_{CC}$ | $\bar{p}_{cc} = V_{CC} I_{CQ}$ |
| $P_{RC} = I_{CQ}^2 R_C$ | $\bar{p}_{RC} = I_{CQ}^2 R_C + \frac{1}{2} I_c^2 R_C$ |
| $P_Q = I_{CQ} V_{CEQ}$ | $\bar{p}_Q = I_{CQ} V_{CEQ} - \frac{1}{2} I_c^2 R_C$ |

# Power in Transistor Amplifiers

| $v_s=0$ | $v_s \neq 0$ |
|---|---|
| $P_{CC} = I_{CQ}V_{CC}$ | $\bar{p}_{cc} = V_{CC}I_{CQ}$ |
| $P_{RC} = I_{CQ}^2 R_C$ | $\bar{p}_{RC} = I_{CQ}^2 R_C + \frac{1}{2} I_c^2 R_C$ |
| $P_Q = I_{CQ}V_{CEQ}$ | $\bar{p}_Q = I_{CQ}V_{CEQ} - \frac{1}{2} I_c^2 R_C$ |

- ➢ The DC source supplies the same amount of power whether the signal is applied or not
- ➢ The relative distribution of the supplied power between the load and transistor changes when the input signal is applied
- ➢ When no signal is applied the transistor dissipates the max power
- ➢ When signal is applied the transistor transfers a portion of the power supplied from the DC sources to the load, via the input the signal

# Power in Transistor Amplifiers

| $v_s=0$ | $v_s \neq 0$ |
|---|---|
| $P_{CC} = I_{CQ}V_{CC}$ | $\bar{p}_{cc} = V_{CC}I_{CQ}$ |
| $P_{RC} = I_{CQ}^2 R_C$ | $\bar{p}_{RC} = I_{CQ}^2 R_C + \frac{1}{2} I_c^2 R_C$ |
| $P_Q = I_{CQ}V_{CEQ}$ | $\bar{p}_Q = I_{CQ}V_{CEQ} - \frac{1}{2} I_c^2 R_C$ |

- **The power rating for a transistor must be higher than the dissipated power when no input signal is applied.**
- A maximum amount of power is transferred to the load when the amplifier produces the maximum output current.

# Maximum Undistorted Power-Example

***This slide include an example to illustrate the concept of maximum undistorted power***

For the circuit shown in Figure 6.57, the transistor parameters are $\beta = 100$ and $V_A = 100$ V, and the source resistor is $R_S = 0$. Determine the maximum undistorted signal power that can be delivered to $R_L$ if: (a) $R_L = 1\ \text{k}\Omega$, and (b) $R_L = 10\ \text{k}\Omega$.

*Assume a sinusoidal input signal*

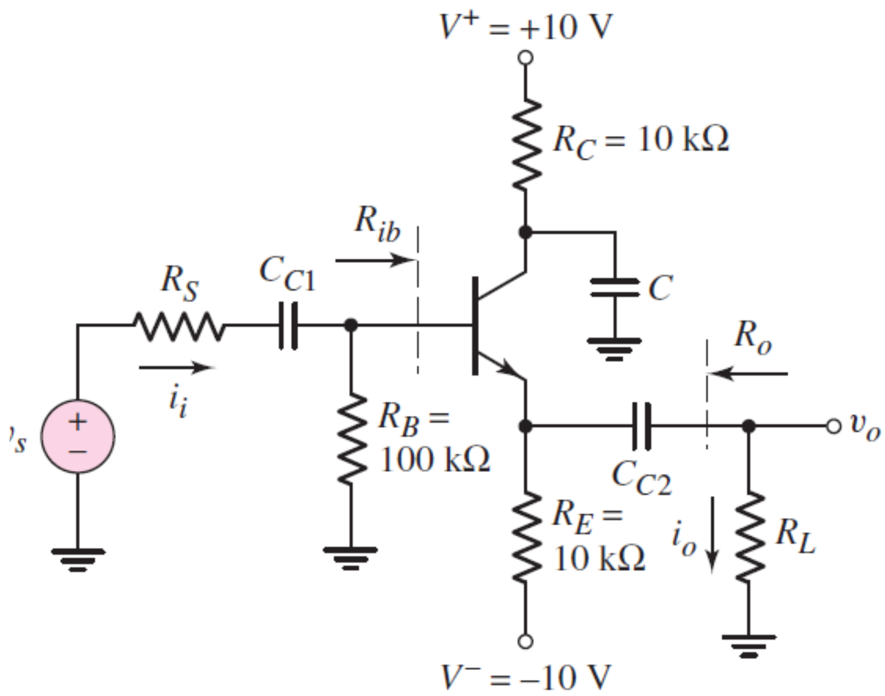



Figure 6.57 Figure for Exercise TYU 6.10

**See notes within lecture 20 folder*