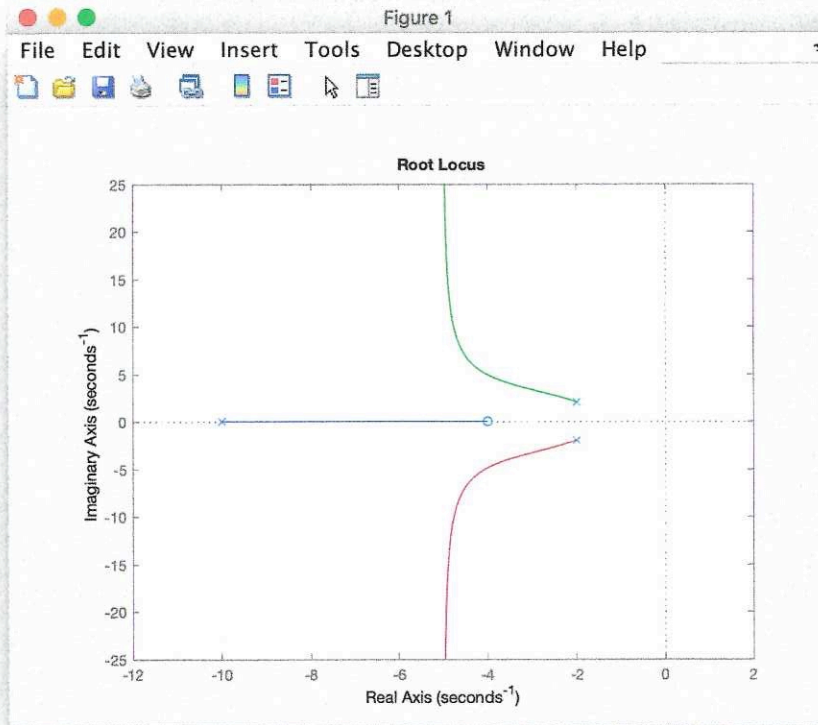


PS #4 SSins
ECE 345 / ME 380

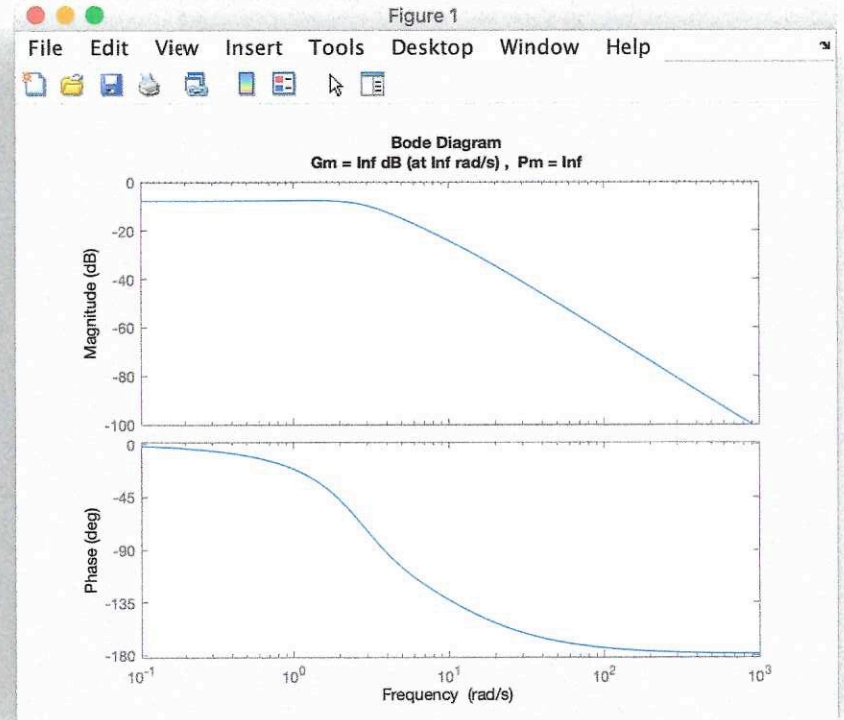
Fall 2020
M. Ogh

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a)



c) Stable w/ $k=1$ because $\zeta_M > 0$ + $G_M > 0$ dB.



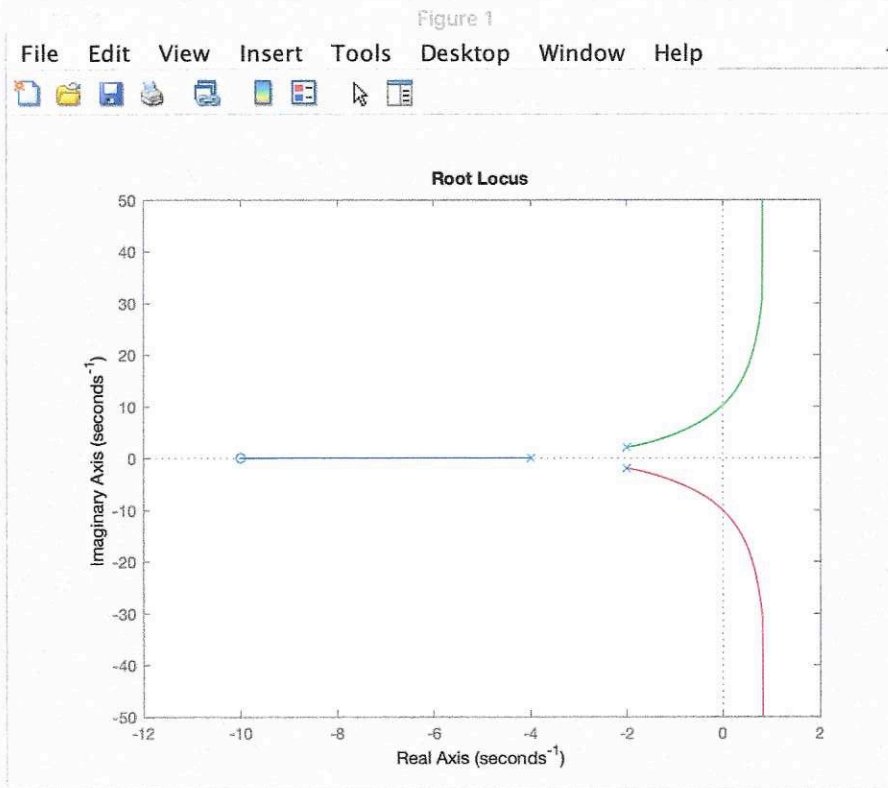
$$\begin{aligned}
 b) \Delta(s) &= D(s) + kN(s) \\
 &= (s^2 + 4s + 8)(s + 10) + 8k(s + 4) \\
 &= s^3 + 14s^2 + (48 + 8k)s + (80 + 32k)
 \end{aligned}$$

Hurwitz criterion: $a_0, a_1, a_2 > 0$ for $k > 0$.

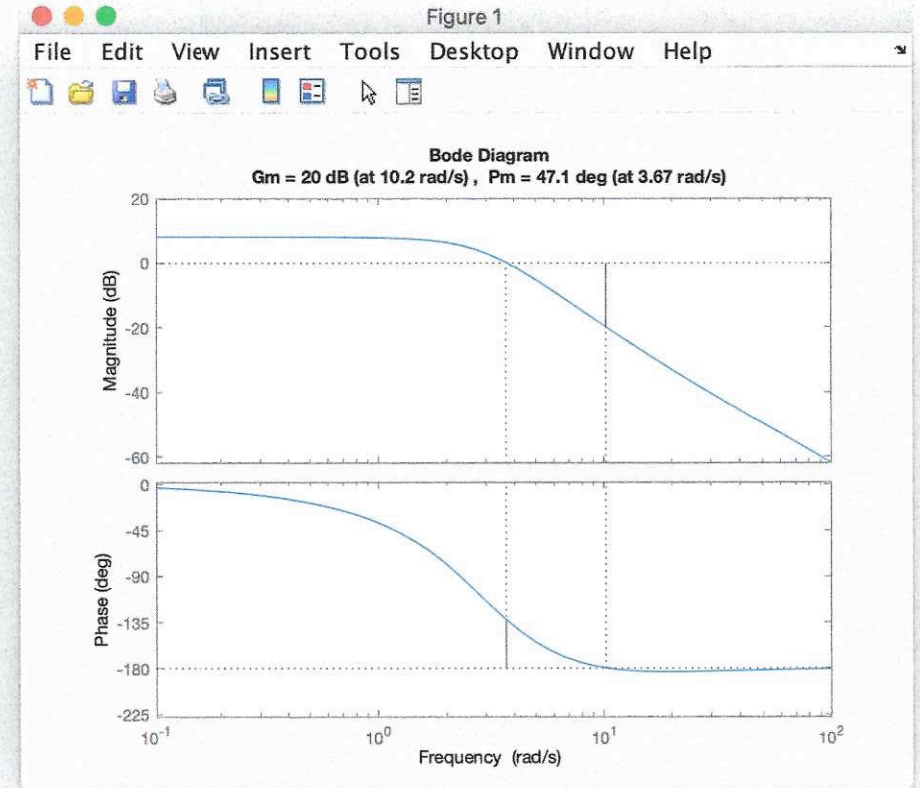
$$\begin{aligned}
 a_1 a_2 - a_3 &> 0 \\
 (48 + 8k)14 - (80 + 32k) &> 0 \\
 (6 + k)14 - (10 + 4k) &> 0 \\
 74 + 10k &> 0 \\
 k &> -7.4 \Rightarrow \boxed{k > 0}
 \end{aligned}$$

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a)



c) Stable with $k=1$ since $\Phi_m > 0^\circ$, $G_m > 0 \text{ dB}$



$$\begin{aligned} b) \Delta(s) &= D(s) + kN(s) \\ &= (s^2 + 4s + 8)(s + 4) + 8k(s + 10) \\ &= s^3 + 8s^2 + (24 + 8k)s + (32 + 80k) \end{aligned}$$

$$\begin{aligned} d) G_m &= 20 \text{ dB} = 20 \log x \\ \Rightarrow G_m &= 10, \text{ same as value in 2(b)} \rightarrow \end{aligned}$$

Hurwitz criterion: $a_1, a_2, a_3 > 0$ for $k > 0$

$$\begin{aligned} a_1 a_2 - a_0 a_3 &> 0 \\ (24 + 8k) \cdot 8 - (32 + 80k) &> 0 \\ (3 + k)8 - (4 + 10k) &> 0 \\ 20 - 2k &> 0 \\ 10 &> k \end{aligned}$$

3

a) order = $n - m = 2$ for lead + lag
 # asymptotes = 2 for lead + lag.

$$\sigma = \frac{\sum p_i - \sum z_i}{n - m} \Rightarrow \sigma_{\text{lead}} = \frac{(-2 - 2 - 10) - (-4)}{2} = -5.$$

$$\sigma_{\text{lag}} = \frac{(-2 - 2 - 4) - (-16)}{2} = +1.$$

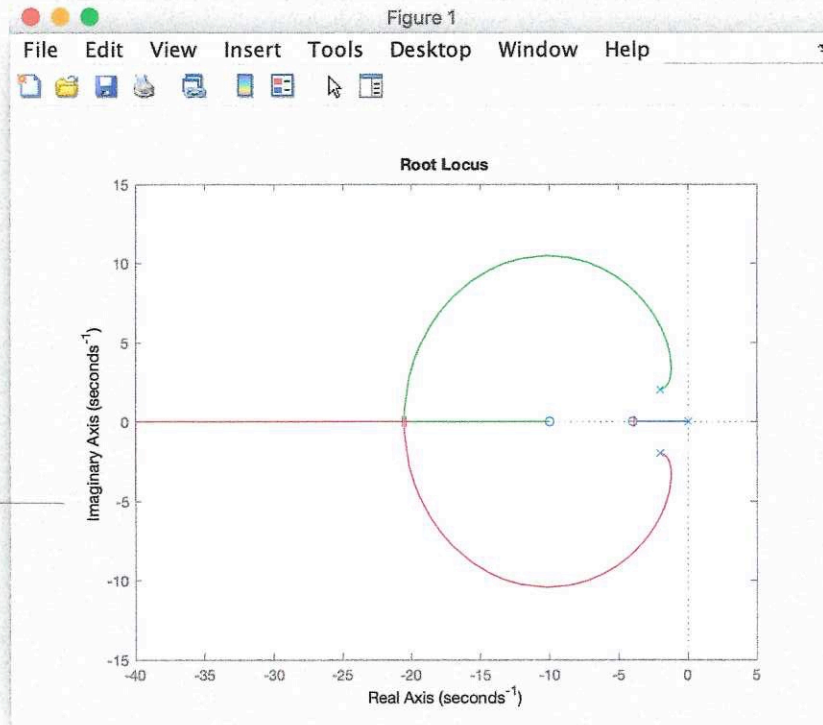
The effect of reversing the pole + zero is the change in the location of the asymptotes (from -5 to +1). Lead control moves the centroid to the left; lag moves the centroid to the right (i.e., compare -2 to -5 & -2 to +1).

b) Lead control provides more relative stability since phase & gain margin are larger than they are for lag control.

It is impossible to destabilize the system under lead control, but it is possible to destabilize the system under lag control.

[4]

a)



c) The locus lies entirely in the open LHP, so it is not possible to de-stabilize the closed loop system w/ any gain $k > 0$.

b) With this point, $k = 5.1215$ to make poles co-located at $s = -20$.


```
>> sysLead = tf(8*[1 4], conv([1 4 8],[1 10]));
>> rlocus(sysLead)
>> margin(sysLead)
>> sysLag = tf(8*[1 10], conv([1 4 8],[1 4]));
>> rlocus(sysLag)
>> margin(sysLag)
>> sysPID = tf(8*conv([1 10],[1 4]), [1 4 8 0]);
>> rlocus(sysPID)
>> rlocfind(sysPID)
Select a point in the graphics window

selected_point =

    -20.4058 - 0.0000i

ans =

    5.1215
```