ECE 345 / ME 380: Introduction to Control Systems
Midterm #2

Dr. Oishi

Due November 13, 2020, by 11:59am

This midterm is open note, open book, and Matlab and electronic resources are allowed. **No communication of any sort regarding the content of the exam is allowed with anyone other than Dr. Oishi.**

For full credit, show all your work.

Please provide your written response on the exam .pdf if possible, adding additional sheets as necessary.

Academic dishonesty is a violation of the UNM Student Code of Conduct. Students suspected of academic dishonesty will be referred for disciplinary action in accordance with University procedures.

By signing below, I affirm that I have completed the midterm independently, under the conditions stated above.

SOLUTIONS
_____

Student Name   Student ID #

Problem #	Actual points	Possible points
1		20
2		40
3		30
Total:		90

1 BIBO stability (20 points)

Consider the transfer function $G(s) = \dfrac{s}{(s+1)(s^2+3s+a)}$, where a is a real-valued number.

1. (10 points) Use a Routh table to assess asymptotic stability of $G(s)$. Which of the following is most correct?

   (a) The closed-loop system is unstable for $a < -4$ since there are ~~two~~ poles in the RHP.

   (b) The closed-loop system is unstable for $a > 4$ since there are *two* poles in the RHP.

   (c) The closed-loop system is unstable for $0 > a > -4$ since there is *one* pole in the RHP.

   (d) The closed-loop system is stable for $a < 0$ since there are *no* poles in the RHP.

$$\Delta(s) = (s+1)(s^2+3s+a) = s^3 + 3s^2 + as + s^2 + 3s + a$$
$$= s^3 + 4s^2 + (3+a)s + a$$

Routh table

s^3: $\quad 1 \qquad\qquad\qquad 3+a$

s^2: $\quad 4 \qquad\qquad\qquad a$

s^1: $\quad \dfrac{-\begin{vmatrix} 1 & 3+a \\ 4 & a \end{vmatrix}}{4} = \dfrac{12+3a}{4} \qquad 0$

$-\begin{vmatrix} 1 & 3+a \\ 4 & a \end{vmatrix} = -(a - 4(3+a))$
$= -(a - 12 - 4a)$
$= 12 + 3a$

s^0: $\quad \dfrac{-\begin{vmatrix} 4 & a \\ \frac{12+3a}{4} & 0 \end{vmatrix}}{\left(\frac{12+3a}{4}\right)} = a \qquad 0$

$-\begin{vmatrix} 4 & a \\ \frac{12+3a}{4} & 0 \end{vmatrix} = -\left(0 - \left(\frac{12+3a}{4}\right)\cdot a\right)$
$= \frac{12+3a}{4} \cdot a$

For all coeff in 1st column to be positive, need

$\dfrac{12+3a}{4} > 0 \implies 12 > -3a$
$\qquad\qquad\qquad -4 < a$

and $\qquad a > 0$

$\therefore \quad a > 0$ for stability. $\implies 0$ poles in RHP

$0 > a > -4 \qquad\qquad\qquad \implies 1$ pole in RHP

$-4 > a \qquad\qquad\qquad\qquad \implies 1$ pole in RHP.

$\therefore$ (c) is correct.

Recall $G(s) = \dfrac{s}{(s+1)(s^2+3s+a)}$.

2. (10 points) Presume $a = 2$. Which *one* of the following is correct?

(a) Since $G(s)$ is marginally stable, it is also BIBO stable.

(b) Since $G(s)$ is BIBO stable, it is also asymptotically stable.

(c) Since $G(s)$ is asymptotically stable, it is also BIBO stable.

(d) Since $G(s)$ is asymptotically stable, there may be bounded input trajectories that generate unbounded output trajectories, hence more work is needed to assess BIBO stability.

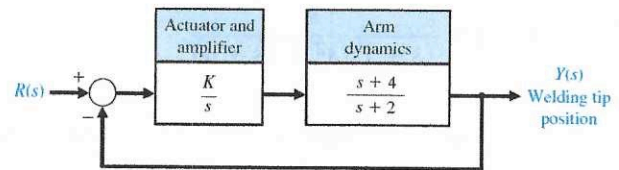(e) Since $G(s)$ is asymptotically unstable, it is also BIBO unstable.

With $a = 2$, $\Delta(s) = (s+1)(s^2+3s+2)$
$$= (s+1)^2(s+2)$$
$$\Rightarrow \text{poles at } -1, -1, -2.$$

2 Precision welding (40 points)

An automated welding machine must be precise and agile. Consider the welding system on the right.

Block diagram: $R(s) \rightarrow$ + $\bigcirc$ − → [Actuator and amplifier $\frac{K}{s}$] → [Arm dynamics $\frac{s+4}{s+2}$] → $Y(s)$ Welding tip position, with feedback loop.

1. (10 points) Consider the root locus plot of the system, shown below. Is it possible to find a gain K such that the poles of $\frac{Y(s)}{R(s)}$ are located at $-4 \pm 4j$? Why or why not? *You do not need to find the value of K, if it is possible to do so.*
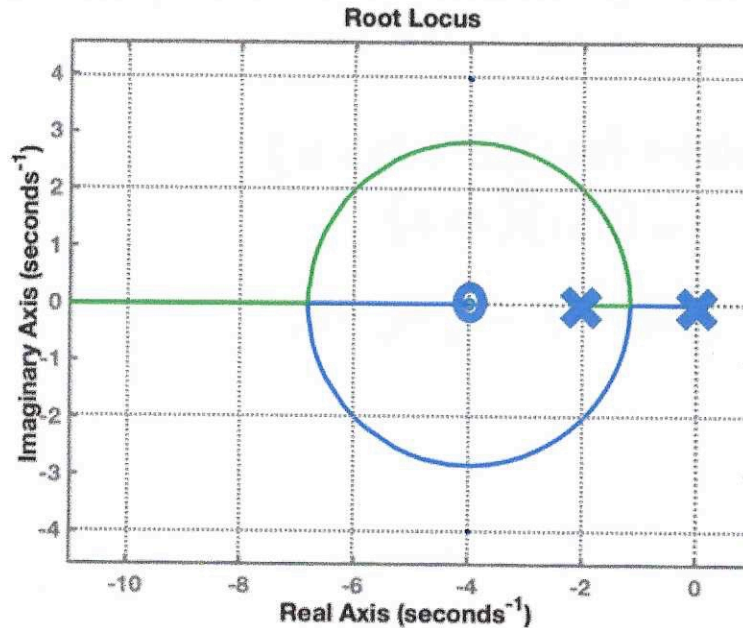
Root Locus

No.

$S = -4 \pm 4j$ is nOt on the locus, therefore no value of gain K can make poles of $\frac{Y(s)}{R(s)}$ take that location.

2. (10 points) Based on the root locus plot above, *one* of the following is correct?

✗ (a) The open-loop system $\frac{K(s+4)}{s(s+2)}$ is asymptotically stable for all $K > 0$ because all of the poles lie in the open left half plane for any $K > 0$.

(b) The closed-loop system $\frac{Y(s)}{R(s)}$ is asymptotically stable for all $K > 0$ because all of the poles lie in the open left-half plane for any $K > 0$.

✗ (c) The closed-loop system $\frac{Y(s)}{R(s)}$ is marginally stable for all $K > 0$ because there is always a pole on the imaginary axis, and all other poles are in the open left half plane.

✗ (d) The stability of the closed-loop system $\frac{Y(s)}{R(s)}$ cannot be inferred from this plot.

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3. (10 points) Which one of the following describes the characteristic equation of the closed-loop system? *Show your work for full credit.*

   (a) $0 = s^2 + 2s$

   (b) $0 = s^2 + (2+K)s + 4K$

   (c) $0 = (K+1)s + 4K + 2$

   (d) $0 = s^2 + 2s + 6 + K$

4. (10 points) What value of gain K is required to make the closed-loop system have damping ratio $\zeta = 1/\sqrt{2}$ and natural frequency $\omega_n = 2\sqrt{2}$? *Hint: Match coefficients of the characteristic equation and $0 = s^2 + 2\zeta\omega_n s + \omega_n^2$.*

3. $\Delta(s) = D(s) + KN(s)$

   $= s(s+2) + K(s+4)$

   $= s^2 + 2s + ks + 4k$

   $= s^2 + (2+k)s + 4k$

4. $0 = s^2 + 2\zeta\omega_n s + \omega_n^2 = s^2 + 2 \cdot \frac{1}{\sqrt{2}} \cdot 2\sqrt{2} \cdot s + (2\sqrt{2})^2$

   $= s^2 + 4s + 8$

 $\therefore \quad 2+k = 4, \qquad\qquad 4k = 8$

   $\boxed{k = 2} \qquad\qquad k = 2 \quad \checkmark$

3 Aircraft carrier landing (30 points)

Landing on an aircraft carrier is difficult partly because the landing surface constantly moves. Consider an automated landing system, which aids the aircraft in tracking an orientation dictated by the landing surface.

We model the actual orientation of the aircraft as $y(t)$, the desired orientation as $r(t)$, and describe the open-loop aircraft dynamics as $KG(s) = K\frac{s+1}{s(s+3)(s-1)}$, for some $K > 0$. The characteristic equation of the closed-loop system is $\Delta(s) = s^3 + 2s^2 + (K-3)s + K$.
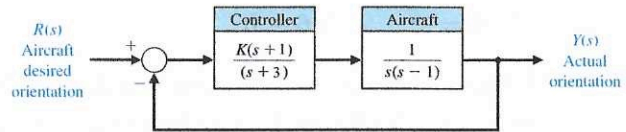
We wish for the closed-loop system $\frac{Y(s)}{R(s)}$ to have steady-state error with magnitude less than or equal to 0.15 in response to a unit ramp, i.e., $|e_{ss}| \le 0.15$.

1. (10 points) Use type number to find all values of K that ensure that the steady-state error criterion is met.

Type number is 1 b/c there is one pole of $KG(s)$ at the origin.

$$\Rightarrow e_{ss} = \frac{1}{K_v}, \quad K_v = \lim_{s \to 0} sKG(s)$$

$$= \lim_{s \to 0} s \cdot K \frac{s+1}{s(s+3)(s-1)}$$

$$= \frac{-K}{3}$$

$$|e_{ss}| \le \frac{15}{100}$$

$$\left| \frac{-3}{K} \right| \le \frac{15}{100}$$

$$300 \le 15 \cdot K$$

$$20 \le K$$

Now consider the case in which an external disturbance affects the system. The output response of the closed-loop system can be written as
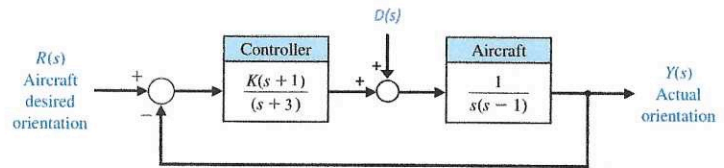
$$Y(s) = G_D(s) \cdot D(s) + G_R(s) \cdot R(s) \qquad (1)$$

where $G_D(s) = \frac{1}{s^3+2s^2+(K-3)s+K}$ and $G_R(s) = \frac{K(s+1)}{s^3+2s^2+(K-3)s+K}$.

2. (10 points) What values of K assure that the steady-state response due to a unit step disturbance, presuming $R(s) = 0$ (i.e., no reference input), is less than or equal to 0.05?

3. (10 points) Which one of the following is true?

   (a) As K increases, the steady state response due to a unit step disturbance *increases*.

   (b) As K increases, the steady state response due to a unit step disturbance *decreases*.

   (c) The steady-state response to a unit step disturbance is 0.05, for any $K > 0$.

   (d) The steady-state response to a unit step disturbance is 0, for any $K > 0$.

2. $y_{ss} = \lim_{s \to 0} s \cdot G_D(s) \cdot \frac{1}{s}$

   $= \lim_{s \to 0} \frac{1}{s^3 + 2s^2 + (k-3)s + k}$

   $= \frac{1}{k}$

   For $y_{ss} \leq \frac{1}{20}$

   $\frac{1}{k} \leq \frac{1}{20}$

   $\underline{20 \leq K}$

Since $y_{ss} = \frac{1}{k}$.

$k\uparrow$ means $\frac{1}{k}\downarrow$.

End of exam.