ECE 371
Materials and Devices

10/03/19 - Lecture 12

Thermal Equilibrium and Intrinsic Semiconductors

General Information

- Homework 4 assigned, due Tuesday 10/15
- Please do not plot by hand – use Matlab or Excel
- Midterm solutions posted
- Reading for next time: 4.2

Fermi-Dirac Distribution

Describes the probability that an available state is filled at a given energy and temperature

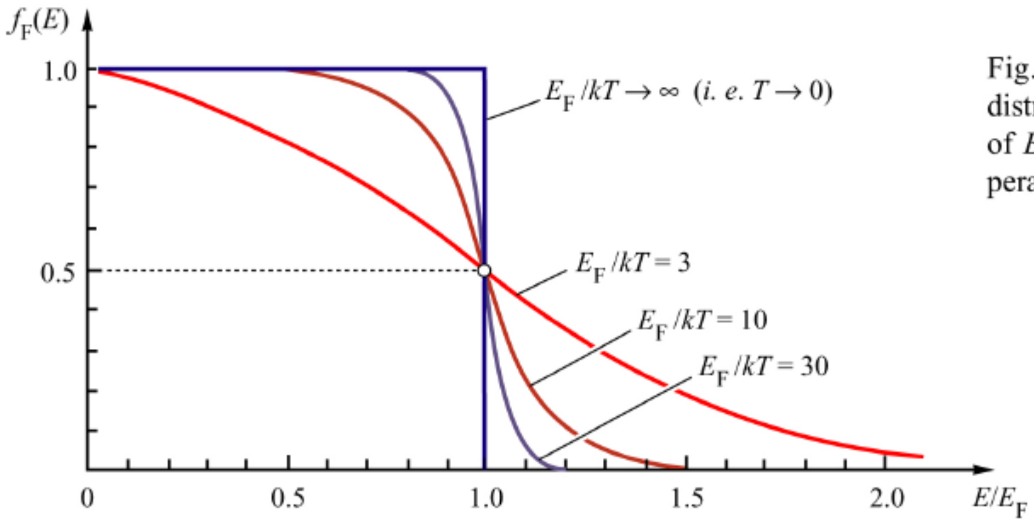


Fig. 13.4. Fermi–Dirac distribution as a function of E/E_F for different temperatures.

$$f_F(E) = \frac{1}{e^{(E-E_F)/kT} + 1}$$

*E_F is the Fermi energy

Probability of Non-Occupation

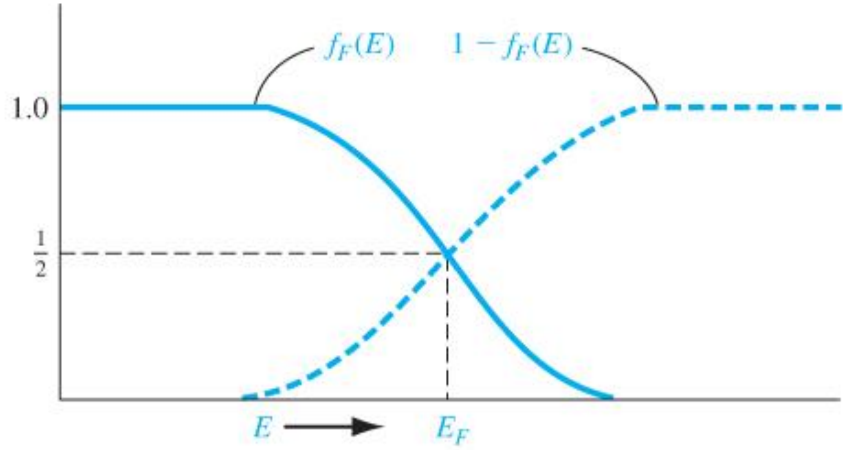


Figure 3.34 | The probability of a state being occupied, $f_F(E)$, and the probability of a state being empty, $1 - f_F(E)$.

- The probability of non-occupation (i.e. – of finding an empty state) is $1 - f_F(E)$
- Applicable to holes

Boltzmann Approximation

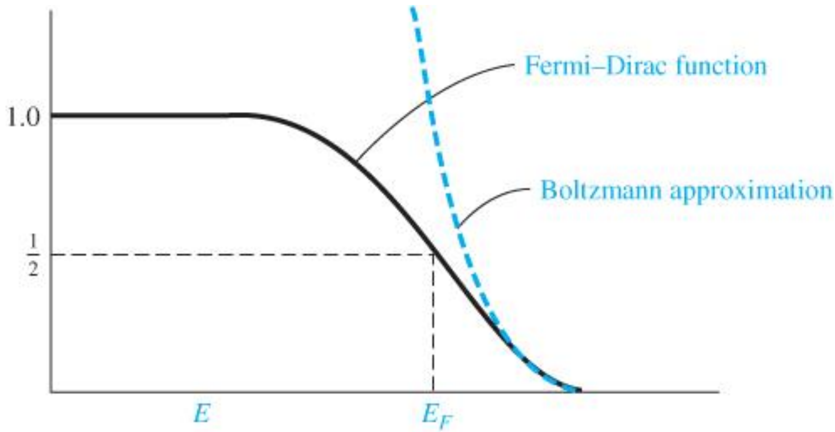


Figure 3.35 | The Fermi–Dirac probability function and the Maxwell–Boltzmann approximation.

$$\boxed{f_F(E) \approx e^{-(E-E_F)/kT}}$$

valid when $E - E_F \gg kT$

- Approximation to the Fermi-Dirac distribution when the energy of interest (E) is far above the Fermi energy
- Useful to simplify expressions for carrier density
- Applicable to LEDs under low-level injection and some electronic devices. Not typically applicable to diode lasers.

Carrier Density vs. Energy

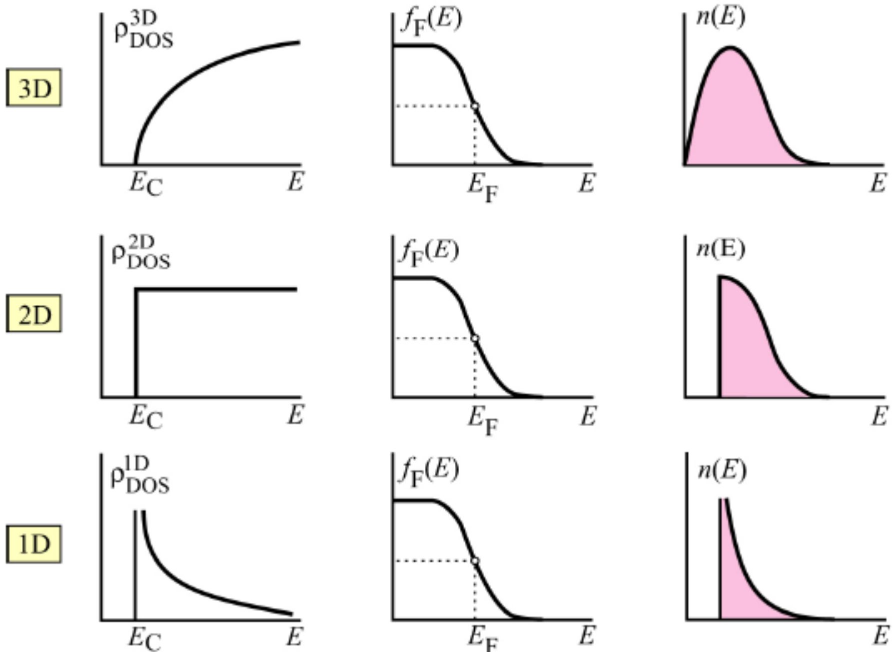


Fig. 13.5. Density of states (ρ_{DOS}), Fermi-Dirac distribution function (f_F) and carrier concentration (n) as a function of energy for a 3D, 2D, and 1D system. The shaded areas represent the total carrier concentration in the conduction band.

$g_c(E) = \frac{4\pi}{h^3}(2m_n^*)^{3/2}\sqrt{E-E_c}$

electron density

$$\boxed{n = \int g_c(E)\, f_F(E)\, dE}$$

hole density

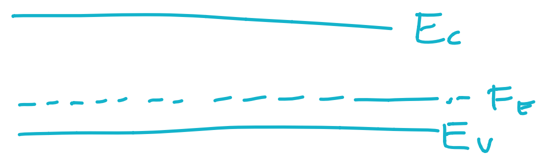
$$\boxed{p = \int g_v(E)[1 - f_F(E)]\, dE}$$

Example

TEST YOUR UNDERSTANDING

TYU 3.5 Assume that the Fermi energy level is 0.35 eV above the valence band energy. Let $T = 300$ K. (a) Determine the probability of a state being empty of an electron at $E = E_v - kT/2$. (b) Repeat part (a) for an energy state at $E = E_v - 3kT/2$.
[Ans. (a) 8.20×10^{-7}; (b) 3.02×10^{-7}]

$E_F = 0.35eV$



(a)

$$\boxed{f_F(E) \approx e^{-(E-E_F)/kT}}$$

Goals in Chapter 4

- *Thermal equilibrium:* no external forces such as voltages, electric fields, magnetic fields, or temperature gradients
- Derive thermal equilibrium concentrations for electrons (n_0) and holes (p_0)
- Discuss intrinsic (pure) semiconductors
- Discuss how impurities (dopants) change the properties of semiconductors
- Determine n_0 and p_0 as a function of dopant concentration
- Determine E_F as a function of dopant concentration

Electron and Hole Concentrations

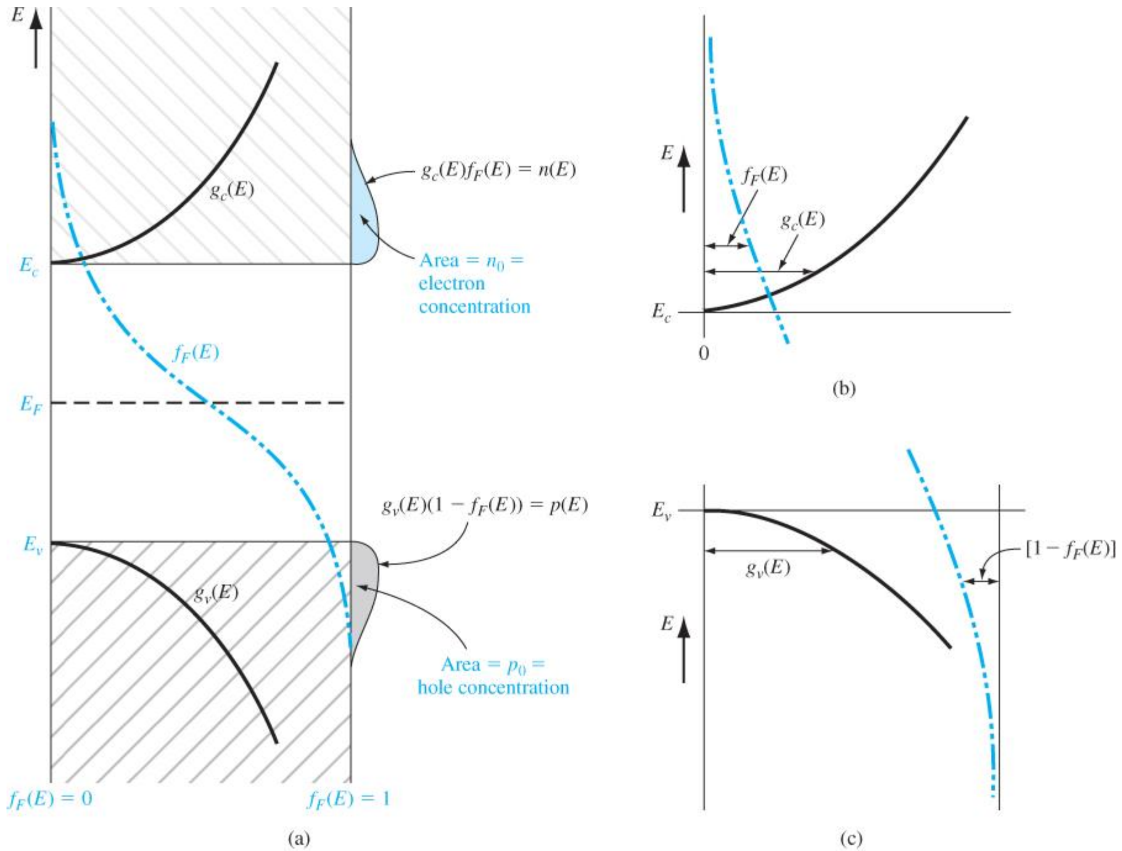


Figure 4.1 | (a) Density of states functions, Fermi–Dirac probability function, and areas representing electron and hole concentrations for the case when E_F is near the midgap energy; (b) expanded view near the conduction-band energy; and (c) expanded view near the valence-band energy.

Equilibrium Carrier Concentrations

Thermal Equilibrium: no external forces (e.g., voltages, electric fields, magnetic fields, temperature gradients, act on the semiconductor. Properties are time independent.

Equilibrium electron and hole concentrations:

$$\boxed{n_0 = N_c \, exp\left[\frac{-(E_c - E_F)}{kT}\right]}$$

$$\boxed{p_0 = N_v \, exp\left[\frac{-(E_F - E_v)}{kT}\right]}$$

Effective density of states:

$$\boxed{N_c = 2\left(\frac{2\pi m_n^* kT}{h^2}\right)^{\frac{3}{2}}}$$

$$\boxed{N_v = 2\left(\frac{2\pi m_p^* kT}{h^2}\right)^{\frac{3}{2}}}$$

	N_c (cm^{-3})	**N_v (cm^{-3})**	**m_n^*/m_0**	**m_p^*/m_0**
Si	2.8e19	1.04e19	1.08	0.56
GaAs	4.7e17	7.0e18	0.067	0.48
Ge	1.04e19	6.0e18	0.55	0.37

Exercise 4.1

■ EXERCISE PROBLEM

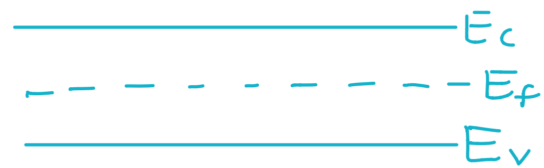
Ex 4.1 Determine the probability that a quantum state at energy $E = E_c + kT$ is occupied by an electron, and calculate the electron concentration in GaAs at $T = 300$ K if the Fermi energy level is 0.25 eV below E_c.

[Ans. $f_F(E) = 2.36 \times 10^{-5}$, $n_0 = 3.02 \times 10^{13}$ cm^{-3}]

$E = E_c + kT$

$T = 300\text{K}$

$f_F(E) =$

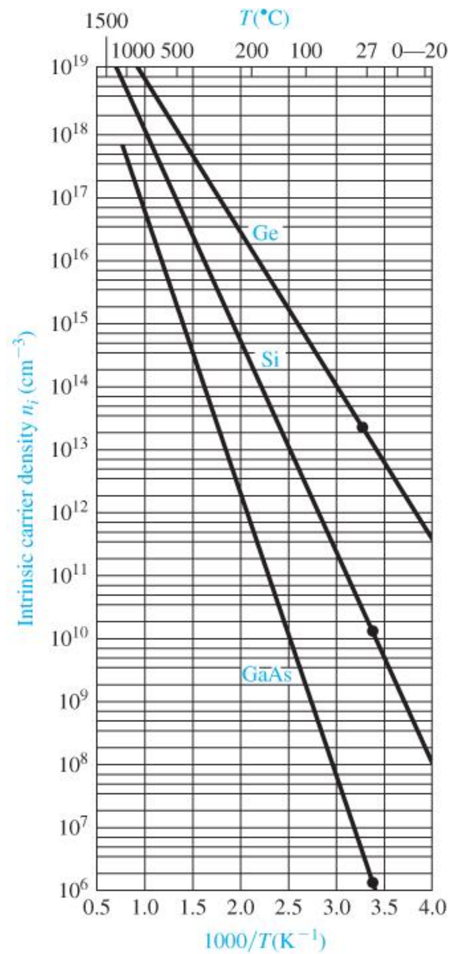


Intrinsic Carrier Concentration

- Intrinsic semiconductor: no impurities
- # electrons in conduction band = # holes in valence band
- n_i is the intrinsic carrier concentration
- E_{Fi} is the intrinsic Fermi level
- E_g is the band gap energy

$$\boxed{n_i^2 = N_c N_v \, exp\left[-\frac{E_g}{kT}\right]}$$

T = 300 K	E_g (eV)	n_i (cm^{-3})
Si	1.12	1.5e10
GaAs	1.42	1.8e6
Ge	0.66	2.4e13

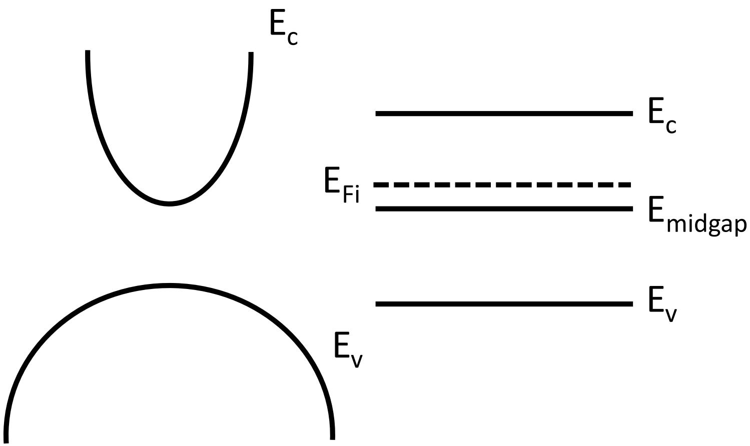


Intrinsic Fermi Level (E_{Fi})

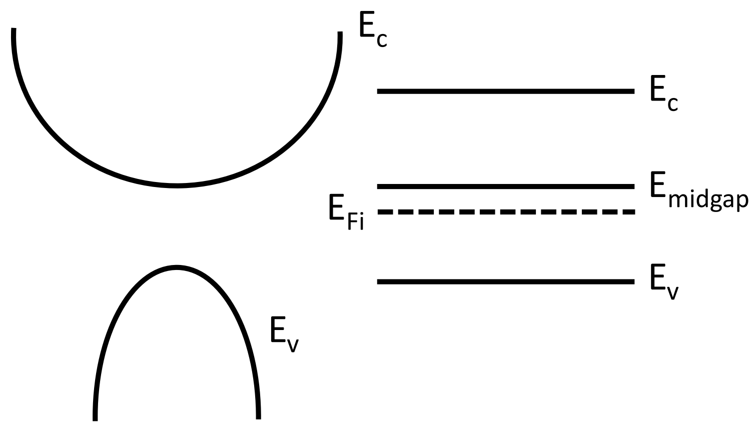
$$E_{Fi} - E_{midgap} = \frac{3}{4} kT\, ln\left(\frac{m_p^*}{m_n^*}\right)$$

$m_n^* = m_p^* \Rightarrow E_{Fi}$ is at midgap

$m_n^* < m_p^* \Rightarrow E_{Fi}$ is above midgap



$m_n^* > m_p^* \Rightarrow E_{Fi}$ is below midgap



E_{Fi} must shift away from the band with the higher DOS (m^*) to maintain $n_0 = p_0$