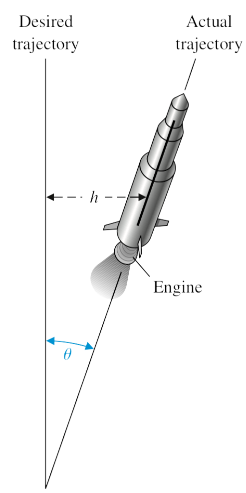
ECE 345 / ME 380: Introduction to Control Systems
Problem Set #1

Dr. Oishi

Due Thursday, September 3, 2020 at 3:30pm

This homework is open note and open book. You are welcome to discuss the problems with other students, but your solutions *must be written independently.* Copying will not be tolerated.

1. (+5 points) Consider lateral control of a rocket with a gimbaled engine. The equations of



motion describe the relationship between the pitch, $\theta(t)$, the lateral deviation, $h(t)$, and the torque, $f(t)$, from the engine,

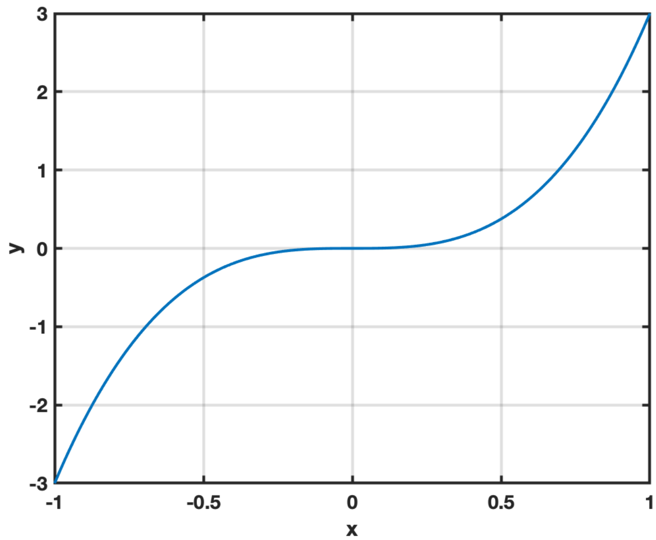
$$\begin{aligned} J\ddot{\theta}(t) &= -k_1\dot{\theta}(t) - k_0\theta(t) + f(t) \\ \dot{h}(t) &= V\theta(t) \end{aligned} \tag{1}$$

with constant rocket speed V and positive, constant values k_1, k_0. Presume all initial conditions are zero.

   (a) Find the transfer function representation for the system with input $F(s)$ and output $H(s)$.

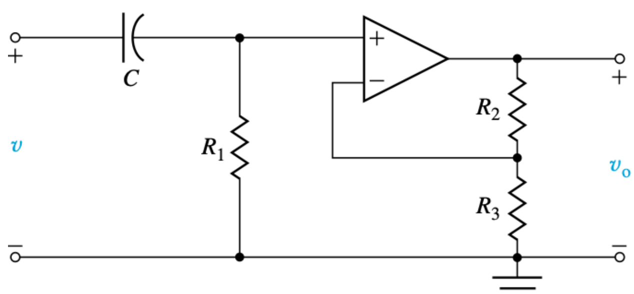
2. (+10 points) Amplifiers often have a deadband region in which amplification has little effect. This may be modeled as a cubic equation $y = 3x^3$, in which the input to the amplifier is x, and the output, i.e., the amplified signal, is y.

   (a) Create a linear approximation for the spring around $x_0 = 0.6$.



(b) On the diagram above, sketch the linear approximation. Hand this in with your completed assignment.

3. (+10 points) Consider the linear op-amp circuit shown below.



(a) Assuming all initial conditions are zero, find the transfer function $G(s) = V_0(s)/V(s)$. *For full credit, write the transfer function in standard polynomial form (i.e., no fractions in s in the numerator or denominator).*

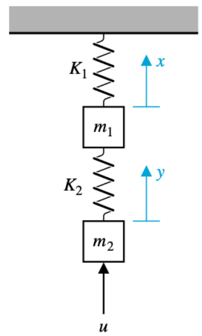
Now assume that $C = 1$, $R_1 = 1$, $R_2 = 2$, $R_3 = 1$ (unitless quantities).

(b) What is the system response $v_0(t)$ to a step $v(t) = \mathbf{1}(t)$?

4. (+10 points) Consider the spring mass damper system shown on the next page with $m_1 = m_2 = 1$ kg, $k_1 = k_2 = 1$ N/m. Note that you can neglect gravity by considering deviations from the system at rest.

(a) Draw a free-body diagram for each mass. Write the equations of motion in terms of $x(t)$ and $y(t)$ and their derivatives. Presume zero initial conditions.

(b) What is the transfer function with input $U(s)$ and output $X(s)$?



5. (+10 points) For this exercise, you will hand in a history of Matlab command-line inputs and outputs (so do not silence the output with '`;`'). Use `diary` or `publish` to record your session. To use the commands below, note that it is almost always helpful to refer to Matlab's `help` command or to the online documentation.

   Consider the following transfer function:

$$G(s) = \frac{4}{(s+1)(s+4)} \tag{2}$$

   (a) Find the coefficients of the polynomial in the denominator using the `conv` command. Your answer should be of the form `[1 x y]`, with x and y representing numerical values.

   (b) Find the roots of the denominator using the `roots` command on your answer to the previous question.

   (c) Construct the transfer function $G(s)$ via the `tf` command.

   (d) Calculate the step response of $G(s)$ via the `step` command. Compare your step response to the one below, which is computed for the system $G_2(s) = \frac{4}{s^2+s+4}$. What are the significant similarities and differences between the two responses and transfer functions? Do the step responses appear to have any oscillatory component? (You do not need to hand in the plot of the step response, but may do so if it aids in your discussion.)