ECE 371
Materials and Devices
HW #2
Due: 09/12/19 at the beginning of class

*All problems from Neamen 4th Edition Ch. 2

2.10 (*a*) The de Broglie wavelength of an electron is 85 Å. Determine the electron energy (eV), momentum, and velocity. (*b*) An electron is moving with a velocity of 8×10^5 cm/s. Determine the electron energy (eV), momentum, and de Broglie wavelength (in Å).

2.11 It is desired to produce x-ray radiation with a wavelength of 1 Å. (*a*) Through what potential voltage difference must the electron be accelerated in vacuum so that it can, upon colliding with a target, generate such a photon? (Assume that all of the electron's energy is transferred to the photon.) (*b*) What is the de Broglie wavelength of the electron in part (*a*) just before it hits the target?

2.15 (*a*) The electron's energy is measured with an uncertainty no greater than 0.8 eV. Determine the minimum uncertainty in the time over which the measurement is made. (*b*) The uncertainty in the position of an electron is no greater than 1.5 Å. Determine the minimum uncertainty in its momentum.

2.17 Consider the wave function $\Psi(x, t) = A\left(\cos\left(\frac{\pi x}{2}\right)\right) e^{-j\omega t}$ for $-1 \leq x \leq +3$. Determine A so that $\int_{-1}^{+3} |\Psi(x, t)|^2 dx = 1$.

2.20 An electron is described by a wave function given by $\psi(x) = \sqrt{\frac{2}{a}}\cos\left(\frac{\pi x}{a}\right)$ for $\frac{-a}{2} < x < \frac{+a}{2}$. The wave function is zero elsewhere. Calculate the probability of finding the electron between (*a*) $0 < x < \frac{a}{4}$, (*b*) $\frac{a}{4} < x < \frac{a}{2}$, and (*c*) $\frac{-a}{2} < x < \frac{+a}{2}$.

2.26 An electron is bound in a one-dimensional infinite potential well with a width of 10 Å. (*a*) Calculate the first three energy levels that the electron may occupy. (*b*) If the electron drops from the third to the second energy level, what is the wavelength of a photon that might be emitted?

2.29 Consider the particle in the infinite potential well as shown in Figure P2.29. Derive and sketch the wave functions corresponding to the four lowest energy levels. (Do not normalize the wave functions.)

***2.30** Consider a three-dimensional infinite potential well. The potential function is given by $V(x) = 0$ for $0 < x < a$, $0 < y < a$, $0 < z < a$, and $V(x) = \infty$ elsewhere. Start with Schrodinger's wave equation, use the separation of variables technique, and show that the energy is quantized and is given by

$$E_{n_x n_y n_z} = \frac{\hbar^2 \pi^2}{2ma^2}\left(n_x^2 + n_y^2 + n_z^2\right)$$

where $n_x = 1, 2, 3, \ldots$, $n_y = 1, 2, 3, \ldots$, $n_z = 1, 2, 3, \ldots$.

2.32 Consider a proton in a one-dimensional infinite potential well shown in Figure 2.6. (*a*) Derive the expression for the allowed energy states of the proton. (*b*) Calculate the energy difference (in units of eV) between the lowest possible energy and the next higher energy state for (*i*) $a = 4$ Å, and (*ii*) $a = 0.5$ cm.

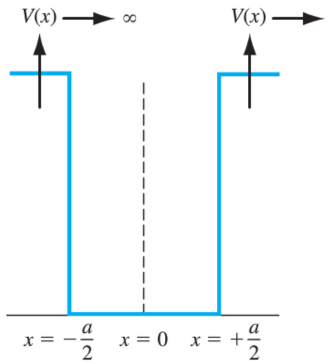


Figure P2.29 | Potential function for Problem 2.29.

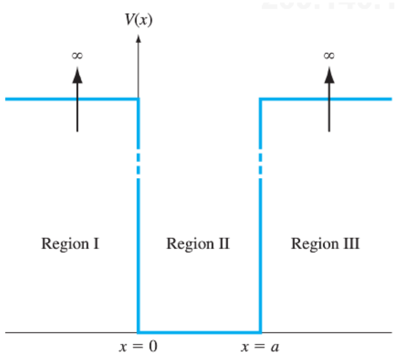


Figure 2.6 | Potential function of the infinite potential well.

2.10 (*a*) The de Broglie wavelength of an electron is 85 Å. Determine the electron energy (eV), momentum, and velocity. (*b*) An electron is moving with a velocity of 8×10^5 cm/s. Determine the electron energy (eV), momentum, and de Broglie wavelength (in Å).

$\lambda = 85 \text{Å} = 85 \times 10^{-10}$ m

$E = T = \frac{1}{2} m v^2 \quad \lambda = \frac{h}{p}$

$p = mv$

$v = \frac{p}{m} \quad h = 6.625 \times 10^{-34}$ J-s

$m = 9.11 \times 10^{-31}$ kg

(a)

$p = \frac{h}{\lambda} = \boxed{7.7954 \times 10^{-26} \text{ kg-m/s}}$

$v = \frac{p}{m} = \boxed{8.5575 \times 10^{4} \text{ m/s}}$

$E = \frac{1}{2} m v^2 = 3.3355 \times 10^{-21}$ J

$= \boxed{2.0818 \times 10^{-2} \text{ eV}}$

(b) $v = 8 \times 10^5$ cm/s $= 8 \times 10$ m/s

$E = \frac{1}{2} m v^2 = 2.9150 \times 10^{-23}$ J

$\boxed{= 1.8194 \times 10^{-4} \text{ eV}}$

$p = mv = \boxed{7.2875 \times 10^{-27} \text{ kg-m/s}}$

$\lambda = \frac{h}{p} = 9.0924 \times 10^{-8}$ m

$= \boxed{9.0924 \times 10^{2} \text{ Å}}$

2.11 It is desired to produce x-ray radiation with a wavelength of 1 Å. (*a*) Through what potential voltage difference must the electron be accelerated in vacuum so that it can, upon colliding with a target, generate such a photon? (Assume that all of the electron's energy is transferred to the photon.) (*b*) What is the de Broglie wavelength of the electron in part (*a*) just before it hits the target?

$$\lambda = 1\ \text{Å} = 1 \times 10^{-10}\ \text{m} \qquad \lambda = \frac{h}{p} = \frac{hc}{E}$$

(a)

$$E = \frac{hc}{\lambda} = 1.9864 \times 10^{-15}\ \text{J}$$

$$V = \frac{E}{e^-} = \boxed{1.2398 \times 10^{4}\ \text{V}}$$

(b)

$$p = \sqrt{2mE} = 6.0159 \times 10^{-23}\ \text{kg-m/s}$$

$$\lambda = \frac{h}{p} = \boxed{1.1014 \times 10^{-11}\ \text{m}}$$

2.15 (*a*) The electron's energy is measured with an uncertainty no greater than 0.8 eV. Determine the minimum uncertainty in the time over which the measurement is made. (*b*) The uncertainty in the position of an electron is no greater than 1.5 Å. Determine the minimum uncertainty in its momentum.

(a) $\Delta E \leq 0.8 \, eV$
$\leq 1.2817 \times 10^{-19} \, J$ $\longrightarrow$ $\Delta t \geq \dfrac{\hbar}{\Delta E}$

$$\boxed{\Delta t \geq 8.2276 \times 10^{-16} \, s}$$

(b) $\Delta x \leq 1.5 \, Å$
$\leq 1.5 \times 10^{-10} \, m$ $\longrightarrow$ $\Delta p \geq \dfrac{\hbar}{\Delta x}$

$$\boxed{\Delta p \geq 7.0305 \times 10^{-25} \, kg\text{-}\frac{m}{s}}$$

2.17 Consider the wave function $\Psi(x, t) = A\left(\cos\left(\frac{\pi x}{2}\right)\right) e^{-j\omega t}$ for $-1 \le x \le +3$.

Determine A so that $\int_{-1}^{+3} |\Psi(x, t)|^2 dx = 1$.

$$\cos^2\left(\frac{\pi}{2}x\right) = \frac{1}{2} + \frac{1}{2}\cos\left(\frac{\cancel{2}\pi}{\cancel{2}}x\right)$$

$$\int_{-1}^{3} A^2 \cos^2\left(\frac{\pi}{2}x\right)dx = 1$$

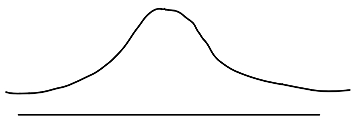
$$|A|^2 \int_{-1}^{3} \cos^2\left(\frac{\pi}{2}x\right)dx = 1$$

$$|A|^2(2) = 1$$

$$|A|^2 = \frac{1}{2}$$

$$\boxed{A = \frac{1}{\sqrt{2}}}$$

2.20 An electron is described by a wave function given by $\psi(x) = \sqrt{\frac{2}{a}}\cos\left(\frac{\pi x}{a}\right)$ for $\frac{-a}{2} < x < \frac{+a}{2}$. The wave function is zero elsewhere. Calculate the probability of finding the electron between (*a*) $0 < x < \frac{a}{4}$, (*b*) $\frac{a}{4} < x < \frac{a}{2}$, and (*c*) $\frac{-a}{2} < x < \frac{+a}{2}$.



(a) $$\int_0^{a/4} \left|\sqrt{\frac{2}{a}}\cos\left(\frac{\pi}{a}x\right)\right|^2 dx$$

$$= \frac{2}{a}\int_0^{a/4} \cos^2\left(\frac{\pi}{a}x\right) dx$$

$$= \frac{2}{a}\int_0^{a/4} \frac{1}{2} + \frac{1}{2}\cos\left(\frac{2\pi}{a}x\right) dx$$

$$= \frac{2}{a}\left[\frac{1}{2}x + \frac{1}{2}\frac{a}{2\pi}\sin\left(\frac{2\pi}{a}x\right)\Big|_0^{a/4}\right]$$

$$= \frac{2}{a}\left[\frac{1}{2}\left(\frac{a}{4}\right) + \frac{a}{4\pi}\sin\left(\frac{\cancel{2}\pi}{\cancel{a}}\left(\frac{\cancel{a}}{\cancel{4}_2}\right)\right)\right]$$

$$= \frac{2}{a}\left[\frac{a}{8} + \frac{a}{4\pi}\overset{1}{\cancel{\sin\left(\frac{\pi}{2}\right)}}\right]$$

$$= \frac{2}{a}\left[\frac{\pi a + 2a}{8\pi}\right] = \frac{\cancel{2}}{\cancel{a}}\left[\frac{\cancel{a}(\pi+2)}{4\cancel{8}\pi}\right] = \frac{\pi+2}{4\pi} = \frac{\cancel{\pi}}{4\cancel{\pi}} + \frac{\cancel{2}}{2\cancel{4}\pi}$$

$$= \frac{1}{4} + \frac{1}{2\pi} \approx 0.4092 = \boxed{40.92\%}$$

(b) $$\int_{a/4}^{a/2} \left|\sqrt{\frac{2}{a}}\cos\left(\frac{\pi}{a}x\right)\right|^2 dx = \frac{2}{a}\left[\frac{1}{2}x + \frac{1}{2}\frac{a}{2\pi}\sin\left(\frac{2\pi}{a}x\right)\Big|_{a/4}^{a/2}\right]$$

$$= \frac{2}{a}\left[\left[\frac{1}{2}\left(\frac{a}{2}\right) + \frac{1}{2}\frac{a}{2\pi}\sin\left(\frac{\cancel{2}\pi}{\cancel{a}}\left(\frac{\cancel{a}}{\cancel{2}}\right)\right)\right] - \left[\frac{1}{2}\left(\frac{a}{4}\right) + \frac{1}{2}\frac{a}{2\pi}\sin\left(\frac{\cancel{2}\pi}{\cancel{a}}\left(\frac{\cancel{a}}{\cancel{4}_2}\right)\right)\right]\right]$$

$$= \frac{2}{a}\left[\left[\frac{a}{4} + \cancel{\frac{a}{4\pi}\sin(\pi)}^{\,0}\right] - \left[\frac{a}{8} + \frac{a}{4\pi}\cancel{\sin\left(\frac{\pi}{2}\right)}^{\,1}\right]\right]$$

$$= \frac{2}{a}\left[\frac{a}{4} - \left(\frac{a}{8} + \frac{a}{4\pi}\right)\right] = \frac{2}{a}\left[\frac{a}{4} - \frac{a\pi + a2}{8\pi}\right]$$

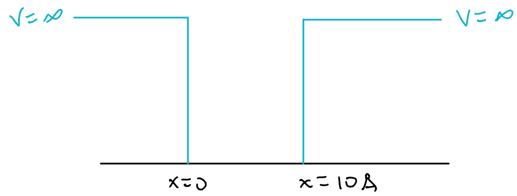
$$= \frac{\cancel{2}}{\cancel{a}}\left[\frac{\cancel{a}^{1}}{\cancel{4}_{2}} - \frac{\cancel{a}(\pi+2)}{4\cancel{8}\pi}\right] = \frac{1}{2} - \frac{(\pi+2)}{4\pi}$$

$$= \frac{1}{2} - \frac{\cancel{\pi}}{4\cancel{\pi}} - \frac{\cancel{2}^{1}}{2\cancel{4}\pi} = \frac{1}{2} - \frac{1}{4} - \frac{1}{2\pi}$$

$$= \frac{\pi - 2}{4\pi} \approx 9.0845 \times 10^{-2} = \boxed{9.0845\%}$$

(c) This wave function is only defined from $-\frac{a}{2} \leq x \leq \frac{a}{2}$ $\therefore$ the probability of it existing between those bounds is $\boxed{100\%}$

2.26 An electron is bound in a one-dimensional infinite potential well with a width of 10 Å. (*a*) Calculate the first three energy levels that the electron may occupy. (*b*) If the electron drops from the third to the second energy level, what is the wavelength of a photon that might be emitted?



(a) $E_n = \dfrac{\hbar^2 n^2 \pi^2}{2ma^2}$, $a = 10 \times 10^{-10}$ m

$m = 9.11 \times 10^{-31}$ kg

$\hbar = 1.055 \times 10^{-34}$

$n = 1 : 6.0247 \times 10^{-20}\ J = \boxed{0.3760\ eV}$

$n = 2 : 2.4099 \times 10^{-19}\ J = \boxed{1.5041\ eV}$

$n = 3 : 5.4222 \times 10^{-19}\ J = \boxed{3.3843\ eV}$

(b) $n_3 - n_2 = 3.0123 \times 10^{-19}\ J = 1.8802\ eV$

$\lambda = \dfrac{hc}{E} = \boxed{659.44\ nm}$

2.29 Consider the particle in the infinite potential well as shown in Figure P2.29. Derive and sketch the wave functions corresponding to the four lowest energy levels. (Do not normalize the wave functions.)

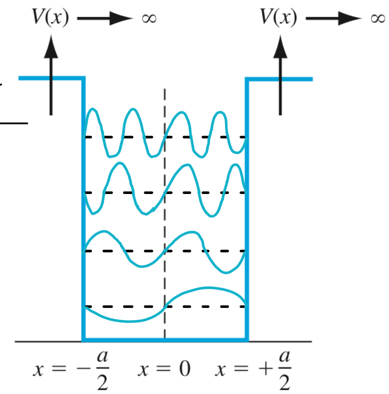


Figure P2.29 | Potential function for Problem 2.29.

$$\frac{\partial^2 \psi(x)}{\partial x^2} + \frac{2m}{\hbar}\left(E - V(x)\right)\psi(x) = 0 \qquad E_n = \frac{\hbar^2 n^2 \pi^2}{2ma^2}$$

Particle cannot exist outside of the well & the wave equation must be continuous, $\therefore$ $|\psi(x)|^2 = 0$ at the boundaries $\left(\psi(\pm a/2) = 0\right)$. Also $V(x)$ is 0 inside the well.

$$\frac{\partial^2 \psi(x)}{\partial x^2} + \frac{2mE}{\hbar}\psi(x) = 0, \quad -\frac{a}{2} \le x \le \frac{a}{2}$$

Solution: $\psi(x) = A\cos(kx) + B\sin(kx)$

$$k = \sqrt{\frac{2mE}{\hbar^2}}$$

$$\psi_1(x) = A\cos(kx) \qquad E_1 = \frac{\hbar^2 \pi^2}{2ma^2}$$

Sketched above ↑

$$\psi_2(x) = B\sin(kx) \qquad E_2 = \frac{\hbar^2 4\pi^2}{2ma^2}$$

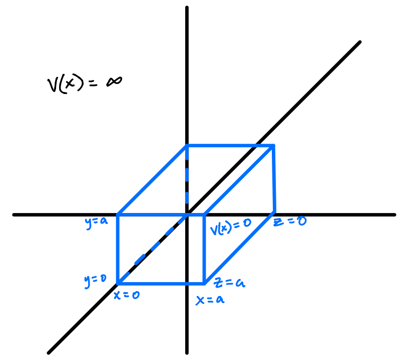
$$\psi_3(x) = A\cos(kx) \qquad E_3 = \frac{\hbar^2 9\pi^2}{2ma^2}$$

$$\psi_4(x) = B\sin(kx) \qquad E_4 = \frac{\hbar^2 16\pi^2}{2ma^2}$$

***2.30** Consider a three-dimensional infinite potential well. The potential function is given by $V(x) = 0$ for $0 < x < a$, $0 < y < a$, $0 < z < a$, and $V(x) = \infty$ elsewhere. Start with Schrodinger's wave equation, use the separation of variables technique, and show that the energy is quantized and is given by

$$E_{n_x n_y n_z} = \frac{\hbar^2 \pi^2}{2ma^2}\left(n_x^2 + n_y^2 + n_z^2\right)$$

where $n_x = 1, 2, 3, \ldots$, $n_y = 1, 2, 3, \ldots$, $n_z = 1, 2, 3, \ldots$.



$$\frac{\partial^2 \psi(x)}{\partial x^2} + \frac{2m}{\hbar^2}\left(E - V(x)\right)\psi(x) = 0$$

Everywhere there is an x, replace with the respective dimension (i.e. $\frac{\partial^2 \psi(x,y,z)}{\partial (x,y,z)^2} + \frac{2mE}{\hbar^2}\psi(x,y,z) = 0$

$$= YZ \frac{\partial^2 \psi(x)}{\partial x^2} + \frac{2mE}{\hbar^2}\psi(x) +$$

$$XZ \frac{\partial^2 \psi(Y)}{\partial y^2} + \frac{2mE}{\hbar^2}\psi(Y) +$$

$$XY \frac{\partial^2 \psi(z)}{\partial z^2} + \frac{2mE}{\hbar^2}\psi(z)$$

$$k = \sqrt{\frac{2mE}{\hbar^2}}$$

$$E = \frac{\hbar^2 \pi^2}{2ma^2}\left(n_x^2 + n_y^2 + n_z^2\right) \checkmark$$

2.32 Consider a proton in a one-dimensional infinite potential well shown in Figure 2.6. (*a*) Derive the expression for the allowed energy states of the proton. (*b*) Calculate the energy difference (in units of eV) between the lowest possible energy and the next higher energy state for (*i*) $a = 4$ Å, and (*ii*) $a = 0.5$ cm.

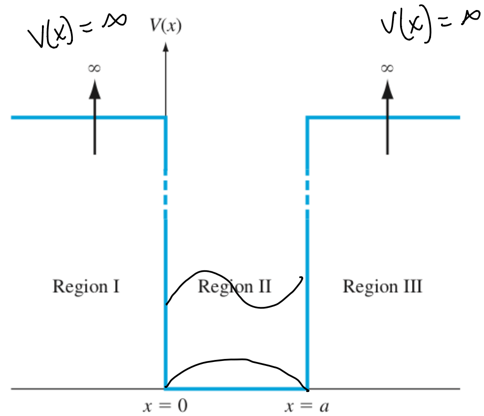


Figure 2.6 | Potential function of the infinite potential well.

(a) only difference is the mass in our k wave #

$$k = \sqrt{\frac{2m_pE}{\hbar^2}}$$

$$\boxed{E_n = \frac{\hbar^2 n^2 \pi^2}{2m_p a^2}}$$

(b) (i) $a = 4$ Å $= 4\times10^{-10}$ m $\qquad n = 1, \; n = 2$

$E_1 = 2.0507\times10^{-22}$ J

$= 1.27995\times10^{-3}$ eV

$E_2 = 8.2028\times10^{-22}$ J

$= 5.1198\times10^{-3}$ eV

$$\boxed{\Delta E = 3.8399\times10^{-3}\text{ eV}}$$

(ii) $a = 0.5$ cm $= 5\times10^{-3}$ m $\qquad n = 1, \; n = 2$

$E_1 = 1.3125\times10^{-36}$ J

$= 8.1917\times10^{-18}$ eV

$E_2 = 5.2498\times10^{-36}$ J

$= 3.27667\times10^{-17}$ eV

$$\boxed{\Delta E = 2.4575\times10^{-17}\text{ eV}}$$