ECE 345 / ME 380: Introduction to Control Systems
Problem Set #2

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Due Thursday, September 17, 2020 at 3:30pm

1. (+10 points) Consider the system described by

$$G(s) = \frac{2(s+2)}{s(s+4)(s+10)} \tag{1}$$

(a) Find the poles and the zeros of $G(s)$.

By inspection:

$$\text{poles: } s = 0, -4, -10$$
$$\text{zeros: } s = -2$$

(b) Put the transfer function $G(s)$ in proper form, with one polynomial in the numerator and one polynomial in the denominator.

Expanding equation (1):

$$G(s) = \frac{2s+4}{s^3 + 14s^2 + 40s}$$

(c) Find the characteristic equation of $G(s)$.

$$\begin{aligned} \Delta(s) &= \text{nullified denominator of } G(s)\text{:} \\ &= s^3 + 14s^2 + 40s = 0 \end{aligned}$$

2. (+10 points) The longitudinal dynamics of a vertical take-off and landing aircraft that is hovering are described by:

$$\begin{aligned} \dot{x} &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -5 & -2 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix} u \\ y &= \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x \end{aligned} \tag{2}$$

(a) Find the characteristic equation of this system.

$$\begin{aligned} \Delta(s) &= |sI - A| \\ &= \left| \begin{bmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 0 & 5 & s+2 \end{bmatrix} \right| \\ &= s^3 + 2s^2 + 5s \end{aligned}$$

(b) Where are the poles of the system located?

$$\begin{aligned}\text{poles} &= \text{Eigenvalues}(A)\\ &= 0, -1 \pm 2i\end{aligned}$$

3. (+10 points) State-space representations are not unique. A single system can be represented in several possible ways. Consider the following two systems:

$$\text{System 1:} \quad \begin{cases} \begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} = \begin{bmatrix} -3 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 3 \\ -2 \end{bmatrix} u \\ y = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \end{cases}$$

$$\text{System 2:} \quad \begin{cases} \begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \\ y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \end{cases}$$

(a) Find the transfer function $G_1(s) = C_1(sI - A_1)^{-1}B_1 + D_1$ for System 1.

$$\begin{aligned} G_1(s) &= \begin{bmatrix} 1 & 1 \end{bmatrix} \left(\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -3 & 0 \\ 0 & -2 \end{bmatrix} \right)^{-1} \begin{bmatrix} 3 \\ -2 \end{bmatrix} \\ &= -\frac{2}{s+2} + \frac{3}{s+3} \\ &= \frac{s}{s^2 + 5s + 6} \end{aligned}$$

(b) Find the transfer function $G_2(s) = C_2(sI - A_2)^{-1}B_2 + D_2$ for System 2.

$$\begin{aligned} G_2(s) &= \begin{bmatrix} 0 & 1 \end{bmatrix} \left(\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix} \right)^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\ &= \frac{s}{s^2 + 5s + 6} \end{aligned}$$

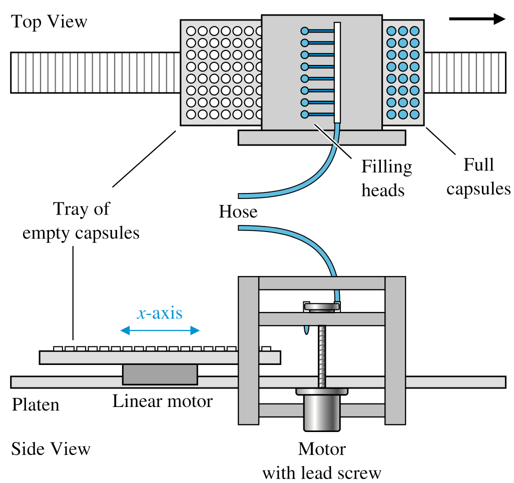
(c) Describe the relationship between G_1 and G_2. What zeros and/or poles do they have in common?

Even though the state-space representations are different, G_1 and G_2 have the exact same transfer functions. This is because, as the problem stated, state-space representations are not unique and a single system can be represented in several possible ways.

$$\begin{aligned} &\text{poles for both: } s = -2, -3 \\ &\text{zeros for both: } s = 0 \end{aligned}$$

4. (+15 points) Consider the following state-space system, that describes the dynamics of a system for automatically dispensing fluid into capsules. A tray of capsules is guided through the dispenser by a linear motor with motor torque $u(t)$ (the input). The tray position is the output, $y(t)$.

$$G(s) = \frac{3}{s^2 + Ks + 3} \tag{3}$$



(a) Using Matlab, follow the steps below for each of $K = 1, 2, 3, 4$. Use the `diary` or `publish` commands to record your code, and hand in the history of Matlab command-line inputs and outputs as well as the *single* plot that you generate. *Note: Please append the Matlab file and plot to your homework, so that you hand in a* ***single*** *pdf. Multiple files will not be accepted.*

i. First create transfer functions $G_1(s), \cdots, G_4(s)$. For example, for the first system,

```
>> G1 = tf(3, [1 1 3];
```

ii. On a single figure, plot step responses for each of these systems using

```
>> step(G1,G2,G3,G4)
>> legend('G1','G2','G3','G4')
```

Please see the final two pages for Matlab code and figure.

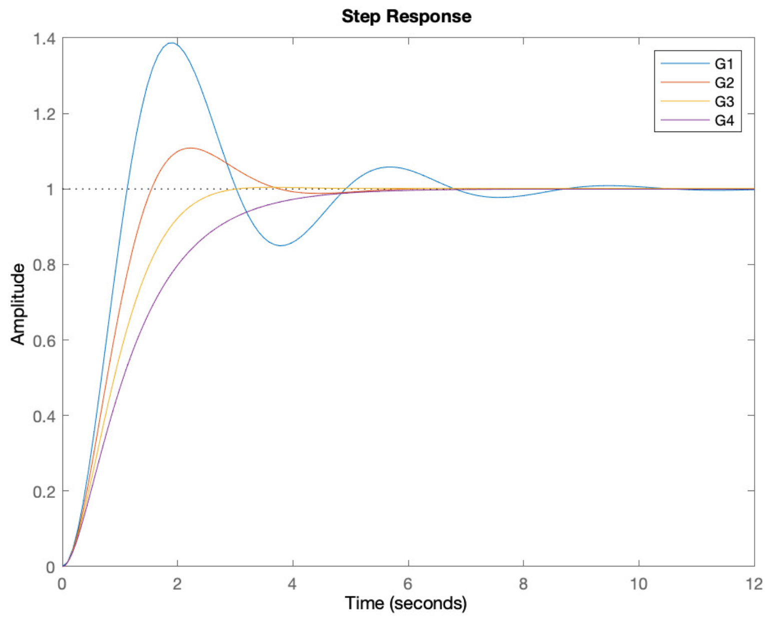
(b) Consider the oscillatory nature of the step responses. What happens as K increases? Which value of K produces the most oscillatory response? Which produces the least?

K has the effect of being a damping agent in our step responses. As K increases, the oscillations decrease; thus, $K = 1$ is the least damped and most oscillatory response, while $K = 4$ is the most damped and exhibits the least oscillatory response.

```
G = cell(4,1);
for K = 1:1:4
   G{K}=tf(3, [1 K 3]);G{K}
end
```

```
ans =

       3
  -----------
  s^2 + s + 3

Continuous-time transfer function.


ans =

        3
  -------------
  s^2 + 2 s + 3

Continuous-time transfer function.


ans =

        3
  -------------
  s^2 + 3 s + 3

Continuous-time transfer function.


ans =

        3
  -------------
  s^2 + 4 s + 3

Continuous-time transfer function.
```

```
step(G{1},G{2},G{3},G{4});
legend('G1','G2','G3','G4')
```


Step Response
Amplitude
Time (seconds)
G1
G2
G3
G4
0
0.2
0.4
0.6
0.8
1
1.2
1.4
0
2
4
6
8
10
12