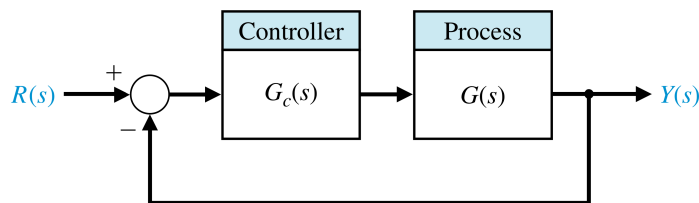# ECE 345 / ME 380: Introduction to Control Systems
Problem Set #4

David Kirby

Due Tuesday, November 24, 2020 at 3:30pm

Consider the system in the figure below with $G(s) = \dfrac{8}{s^2 + 4s + 8}$.

This assignment will investigate the use of three different controllers $Gc(s)$ under negative unity feedback: (1) Lead $Gc(s) = K\frac{s+4}{s+10}$, (2) Lag $Gc(s) = K\frac{s+10}{s+4}$, and (3) PID $Gc(s) = K\frac{(s+4)(s+10)}{s}$.

1. (+10 points) Consider the lead controller $G(s) = \dfrac{s+4}{s+10}$.

   (a) Plot (via Matlab) or sketch (by hand) the root locus for this system. In Matlab, use `GcG = tf(8*[1 4],conv([1 10],[1 4 8]))` to represent the open-loop system $G_c(s)G(s)$, then use `rlocus(GcG)`.

   Please see Matlab plot below.

   (b) Using the Hurwitz conditions, find the values of $K > 0$, if any, that will make the closed-loop system asymptotically stable.

   $$\Delta_{CL}(s) = D(s) + KN(s)$$
   $$= (s+10)(s^2 + 4s + 8) + 8K(s+4)$$
   $$= s^3 + 14s^2 + (8K + 48)s + (32K + 80)$$

   $$a_0 = 32K + 80, \qquad a_1 = 8K + 48, \qquad a_2 = 14, \qquad a_3 = 1$$

   $$a_1 a_2 - a_0 > 0, \qquad\qquad a_2 > 0, \quad a_1 > 0, \quad a_0 > 0$$
   $$(8K + 48)(14) - (32K + 80) > 0 \qquad 14 > 0, \quad 8K + 48 > 0, \quad 32K + 80 > 0$$
   $$K > -\frac{37}{5} \qquad\qquad\qquad K > -6, \quad K > -\frac{5}{2}$$

   Closed-loop system will be stable for all values $K > 0$.

(c) Use the Matlab command `margin(GcG)` to compute the phase margin and gain margin with $K = 1$. Is the system stable with $K = 1$?

Please see Matlab plot below. $P_m = \infty, G_m = \infty$, therefore the system is stable with $K = 1$.

2. (+15 points) Consider the lag controller $G(s) = \dfrac{s + 10}{s + 4}$.

(a) Plot (via Matlab) or sketch (by hand) the root locus for this system. In Matlab, use `tf` to represent the open-loop system $G_c(s)G(s)$, then use `rlocus(GcG)`.

Please see Matlab plot below.

(b) Using the Hurwitz conditions, find the values of $K > 0$, if any, that will make the closed-loop system asymptotically stable.

$$\Delta_{CL}(s) = D(s) + KN(s)$$
$$= (s + 4)(s^2 + 4s + 8) + 8K(s + 10)$$
$$= s^3 + 8s^2 + (8K + 24)s + (80K + 32)$$

$$a_0 = 80K + 32, \qquad a_1 = 8K + 24, \qquad a_2 = 8, \qquad a_3 = 1$$

$$a_1 a_2 - a_0 > 0, \qquad\qquad a_2 > 0, \quad a_1 > 0, \quad a_0 > 0$$
$$(8K + 24)(8) - (80K + 32) > 0 \qquad 8 > 0, \quad 8K + 24 > 0, \quad 80K + 32 > 0$$
$$K < 10 \qquad\qquad\qquad K > -3, \quad K > -\frac{2}{5}$$

Closed-loop system will be stable for values $K < 10$.

(c) Use the Matlab command `margin(GcG)` to compute the phase margin and gain margin with $K = 1$. Is the system stable with $K = 1$?

Please see Matlab plot below. $P_m = 47.1°, G_m = 20$ dB, therefore the system is stable with $K = 1$.

(d) What is the gain margin in magnitude (not dB)? Compare this to your answer in Question 2(b).

$G_m = 20 \log x = 20 \quad \rightarrow \quad G_m = 10$. We get the same results as in 2(b).

3. (+10 points) Now compare the lead and lag controllers.

(a) Compare the order, number of asymptotes, and location of the centroid for the two systems. What is the primary effect of reversing the location of the controller pole and zero?

The order and number of asymptotes are the same for both the lead and the lag controllers; however, the centroids are different. If the pole is greater than the zero, the centroid will stay in the left half plane. Reversing the location, as the zero grows larger than the pole, the centroid is pushed further right and eventually into the right-half plane.

(b) Which of the two systems (under lead or lag control) has *more* relative stability? Justify your answer in a single sentence.

The lead controller is stable for all $K > 0$, therefore it has more relative stability.

4. (+10 points) Lastly, consider the effect of a Proportional-Integral-Derivative (PID) controller $Gc(s) = K\dfrac{(s+4)(s+10)}{s} = 14K + \dfrac{40K}{s} + Ks$.

   (a) Plot (via Matlab) the root locus for this system. Use `tf` to represent the open-loop system $G_c(s)G(s)$, then use `rlocus(GcG)`.

   Please see Matlab plot below.

   (b) Use `rlocfind` to find the value of K that results in a critically damped system.

   $K = 5.1213$

   (c) Based on your root locus plot, is it possible to destabilize the system by making K sufficiently large? Why or why not?

   The root locus plot shows roots in the left-half plane for $K > 0$, therefore the system will be asymptotically stable for all $K > 0$.

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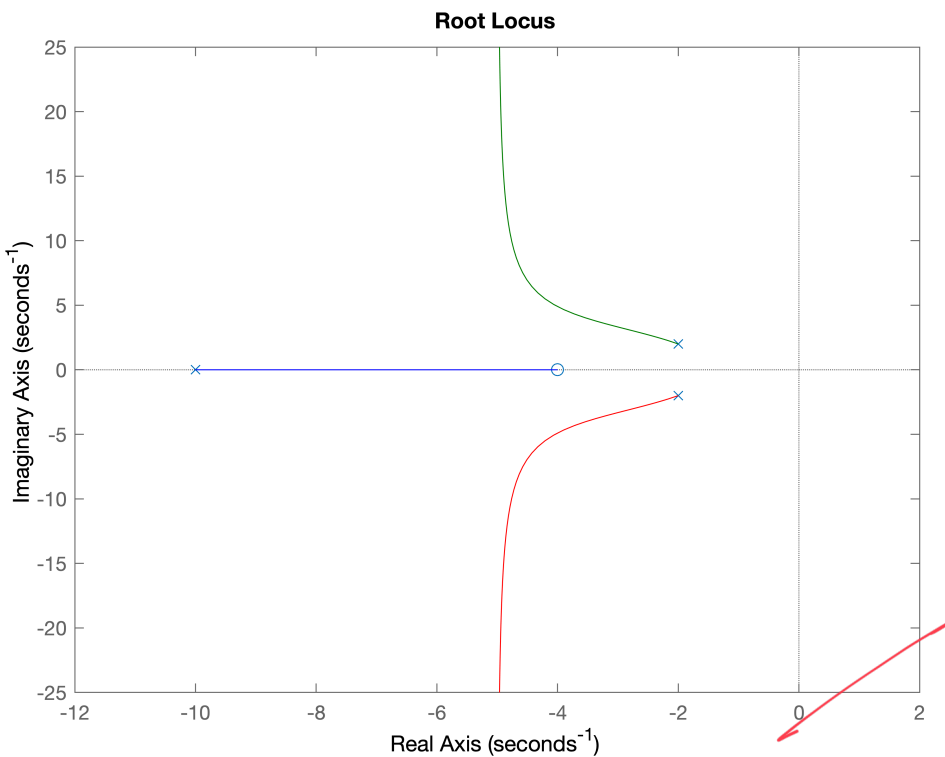**1.** **Consider the lead controller** $G(s) = \dfrac{s+4}{s+10}$.
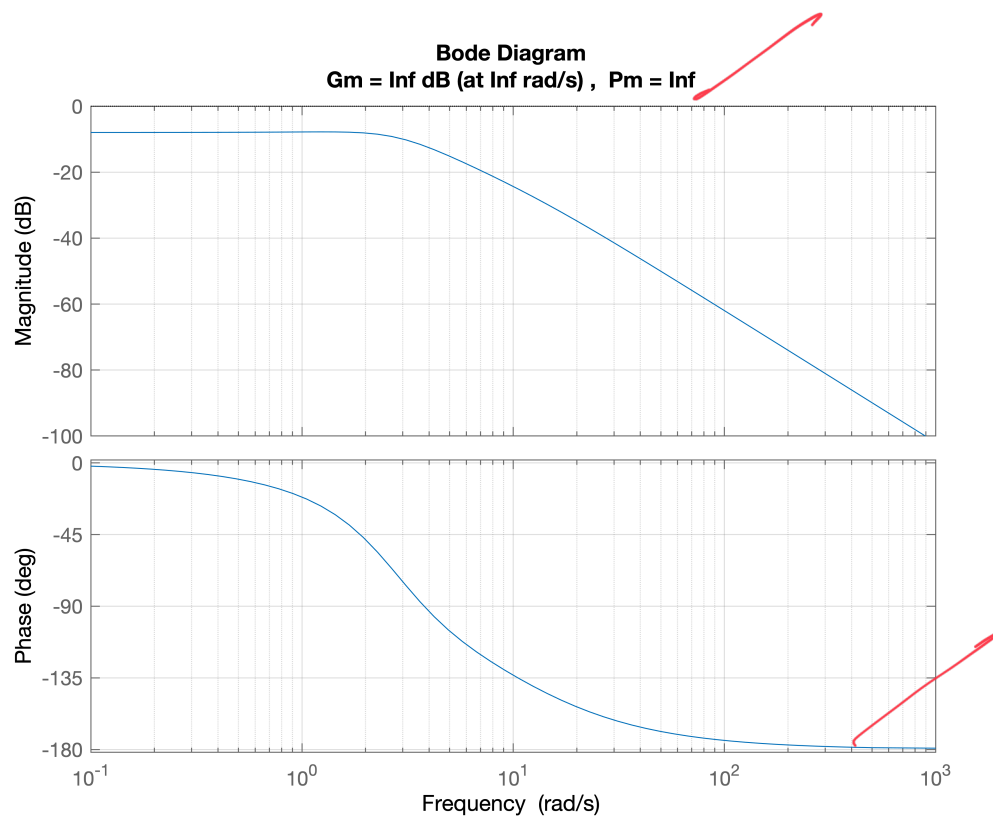
```
GcG1 = tf(8*[1 4],conv([1 10],[1 4 8]))

GcG1 =

           8 s + 32
  ------------------------
  s^3 + 14 s^2 + 48 s + 80

Continuous-time transfer function.
```

```
rlocus(GcG1)
```



```
margin(GcG1);grid on
```

Bode Diagram
Gm = Inf dB (at Inf rad/s) , Pm = Inf

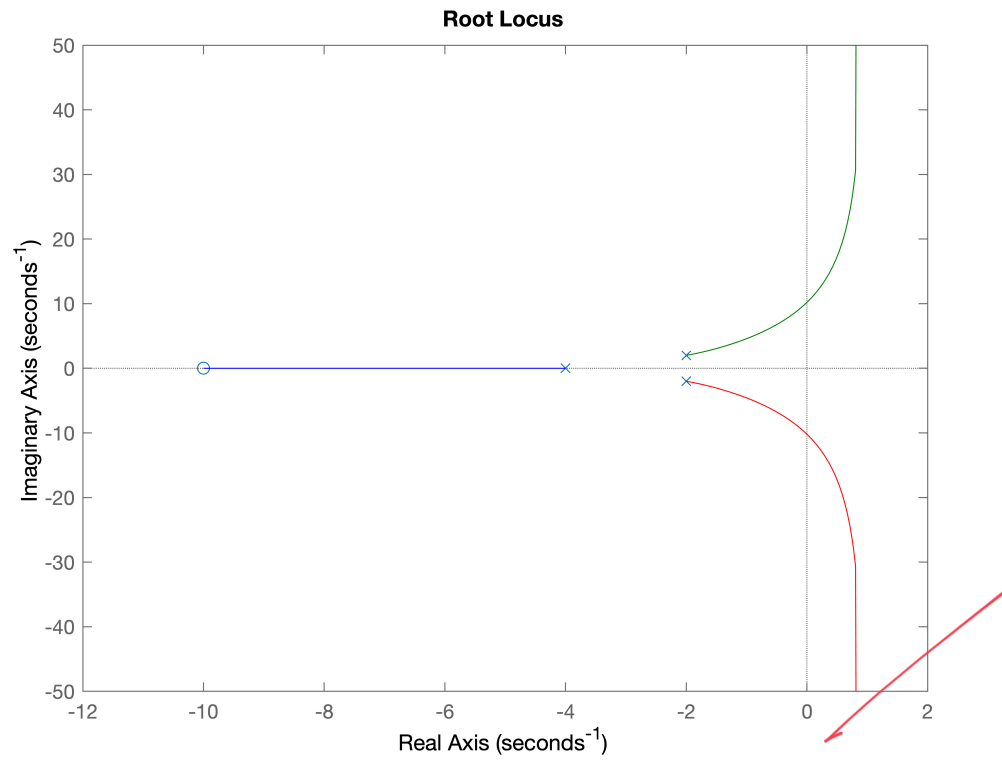## 2. Consider the lag controller $G(s) = \dfrac{s + 10}{s + 4}$.

```
GcG2 = tf(8*[1 10],conv([1 4],[1 4 8]))

GcG2 =

          8 s + 80
  -----------------------
  s^3 + 8 s^2 + 24 s + 32

Continuous-time transfer function.
```
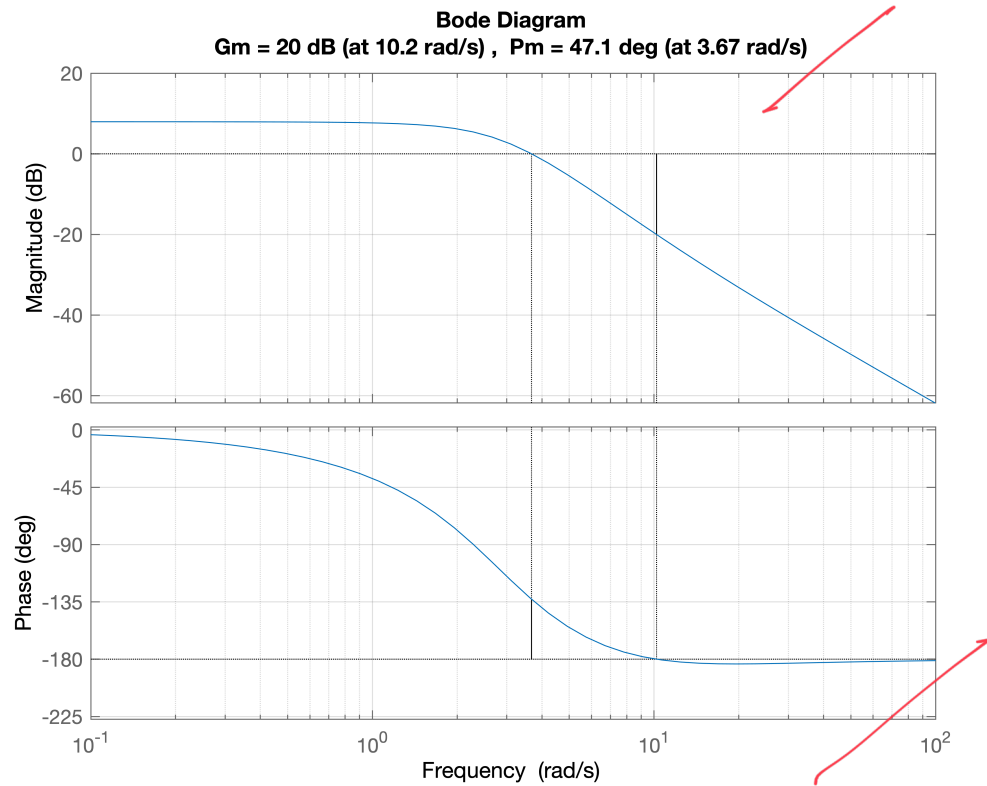
```
rlocus(GcG2)
```

2

Root Locus


```
margin(GcG2);grid on
```

Bode Diagram
Gm = 20 dB (at 10.2 rad/s) , Pm = 47.1 deg (at 3.67 rad/s)

4. Consider the effect of a Proportional-Integral-Derivative (PID) controller

$$Gc(s) = K\frac{(s+4)(s+10)}{s} = 14K + \frac{40K}{s} + Ks.$$

```
GcG3 = tf(8*[conv([1 10],[1 4])],conv([1 0],[1 4 8]))

GcG3 =

  8 s^2 + 112 s + 320
  -------------------
   s^3 + 4 s^2 + 8 s

Continuous-time transfer function.
```
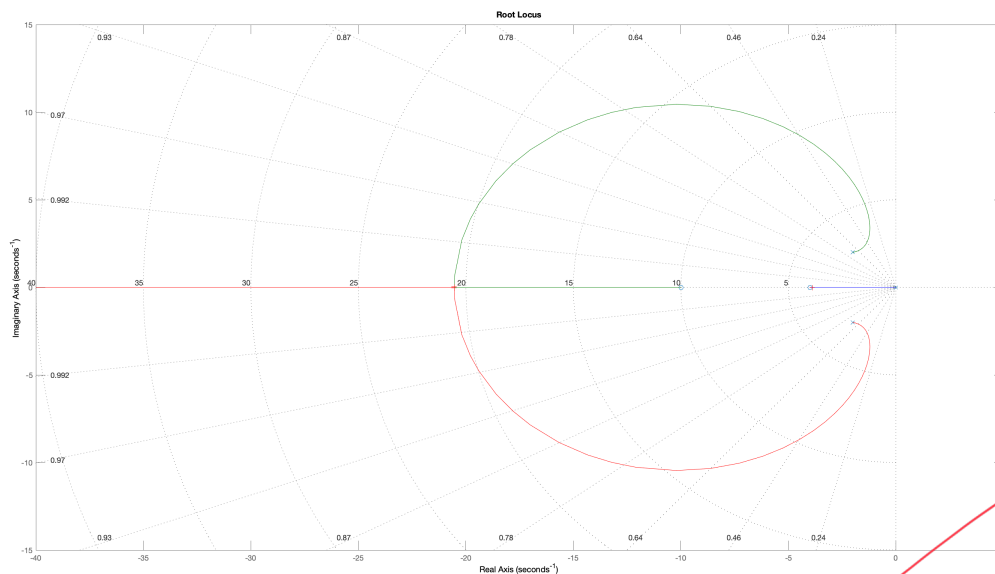
```
rlocus(GcG3);grid on
rlocfind(GcG3)
```

```
Select a point in the graphics window
```


Root Locus

```
selected_point = -20.5262 + 0.0322i
ans = 5.1213
```