ECE 345 / ME 380: Introduction to Control Systems
Collaborative Quiz #5

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Researchers recently deployed a team of quadrotors to autonomously survey a large penguin colony in Antarctica [1]. Planning algorithms were designed to optimally synthesize the routes that the quadrotors fly, to minimize the time needed to cover a moving colony of 300,000 Adelie penguins in extremely challenging conditions. This is important to avoid disrupting the penguins and to accommodate battery limitations in cold environments. The aerial imagery gathered helps ecologists and biologists determine the size of the colony, the number of nests, and other information relevant to assessing colony health and behavior.

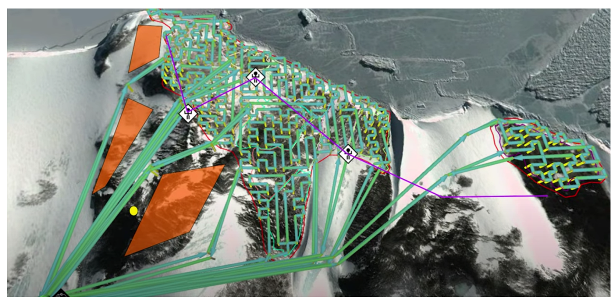
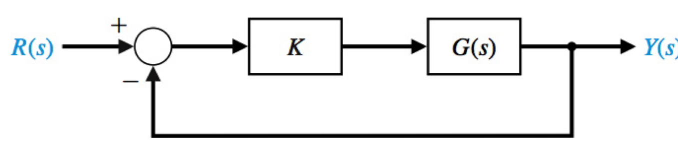


Figure 1: Path planning for quadrotor-based automated penguin habitat surveying in Antarctica. Images from Stanford University, at `https://youtu.be/w4d7MPYgNA4`.

We model the dynamics of a single quadrotor as the negative unity feedback system with positive gain K, with plant described by the transfer function



$$G(s) = \frac{80}{(s+2)(s^2+8s+20)}$$

The input $r(t)$ to the system is the position dictated by the planning algorithm. The output $y(t)$ of the system is the actual position of the quadrotor.

The objective of this exercise is to analyze the stability of the system through root locus, Bode, and Nyquist diagrams, and to select a control gain to meet relative stability as well as transient performance criteria.

1 Questions to be completed before class

1. Find the characteristic equation $\Delta(s)$ of the closed-loop system $\frac{Y(s)}{R(s)}$.

2. Use the Hurwitz criterion to show that the system is stable for $K < 4$. *Note: An additional Routh table can be constructed to show that with $K = 4$, the poles on the imaginary axis are at a location $\pm 6j$.*

3. Plot or sketch a root locus diagram of the open-loop transfer function $G(s)$.

4. Plot the Bode diagram of $G(s)$, and identify gain margin and phase margin. *You are welcome to check your result via Matlab's `margin` function, but you should be able to find these on your own, as well, as numerical errors in Matlab are possible.*

2 Questions to be completed in class, in groups

1. Consider your root locus diagram from Question 1.3. What happens to the poles of the closed-loop system as K is increased? *More than one answer may be correct.*

   (a) One pole on the negative real line moves farther into the LHP.

   (b) A complex conjugate pair of poles moves from the LHP to the RHP.

   (c) A complex conjugate pair of LHP poles follows two asymptotes further into the LHP.

   (d) The pair of poles closest to the origin is overdamped for all $K > 0$.

2. Consider your responses to Questions 1.2 and 1.4. Which *one* of the following most correctly describes the relationship between gain K (magnitude) and gain margin G_M (in dB)?

   (a) They are related by the fact that $G_M = K$.

   (b) They are the same value, since $20 \log G_M$ is equal to the value of K required to place a pair of poles of the closed-loop system $\frac{Y(s)}{R(s)}$ on the imaginary axis.

   (c) They are the same value, since $G_M = 20 \log K$, where K is the value of gain required to place a pair of poles of the closed-loop system $\frac{Y(s)}{R(s)}$ on the imaginary axis.

   (d) They are different values, since $G_M < 0$ and $K > 0$.

3. Using the Nyquist diagram of $G(s)$ on the answer sheet, answer the following: What is the phase margin and gain margin of the closed-loop system? For full credit, mark the locations on the Nyquist diagram where you measure phase margin and gain margin.

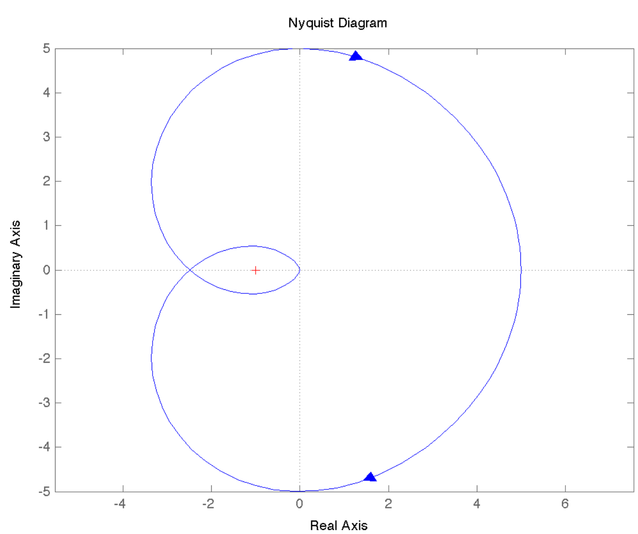
4. Consider your Bode diagram of $G(s)$ from Question 1.4. A phase margin between 20° and 30° is often desirable, to counter modeling errors, disturbance forces, or measurement uncertainties that might inadvertently destabilize the system. What value of K will achieve a phase margin of 25°? *Provide your result in unitless magnitude, NOT in decibels.* For full credit, hand in your Bode diagram with the relevant measurements clearly marked.

5. Imagine that as a roboticist, you are tasked with selecting an appropriate control gain to ensure both relative stability as well as desired transient response characteristics for the closed-loop system. A gain of $K \geq 3$ provides acceptable transient response characteristics, because of the short peak time, but results in a phase margin of $\Phi_M \leq 10.6°$. Is $K = 3$ an acceptable control gain? Why or why not? In your response, describe the implications of inadequate relative stability – how does it impact quadrotor tracking capabilities?

   Recall that `log10` is used to take a base 10 log in Matlab, whereas `log` calculates the natural log (i.e., base e).

If your group finishes early

- Describe the benefit of Nyquist / Bode diagrams in assessing stability. What additional information about stability is provided in the Nyquist or Bode diagrams that is not provided in the root locus?
- Consider the Nyquist diagram below, corresponding to a transfer function $G(s)$ with three poles in the LHP.
  - By what factor should the gain be *reduced* to make the closed-loop system stable?
  - Will increasing the gain change the stability of the closed-loop system?
- Consider the value of ω solved for Question 1.2. What significant feature on the Bode diagram does this frequency correspond to? *Hint: What is the phase condition of all points on the root locus?*
- If you were given a Bode diagram of an open-loop system based on experimental data, would you be able to assess the stability of the closed-loop system? Why or why not?



References

[1] K. Shah, G. Ballard, A. Schmidt, and M. Schwager, "Multidrone aerial surveys of penguin colonies in antarctica," *Science Robotics*, vol. 5, no. 47, 2020.