ECE 371
Materials and Devices
HW #3
Due: Tuesday 10/01/19 at the beginning of class

*All problems from Neamen 4th Edition Ch. 2 and Ch. 3

2.33 For the step potential function shown in Figure P2.33, assume that $E > V_0$ and that particles are incident from the $+x$ direction traveling in the $-x$ direction. (*a*) Write the wave solutions for each region. (*b*) Derive expressions for the transmission and reflection coefficients.

2.34 Consider an electron with a kinetic energy of 2.8 eV incident on a step potential function of height 3.5 eV. Determine the relative probability of finding the electron at a distance (*a*) 5 Å beyond the barrier, (*b*) 15 Å beyond the barrier, and (*c*) 40 Å beyond the barrier compared with the probability of finding the incident particle at the barrier edge.

2.35 (*a*) Calculate the transmission coefficient of an electron with a kinetic energy of 0.1 eV impinging on a potential barrier of height 1.0 eV and a width of 4 Å (*b*) Repeat part (*a*) for a barrier width of 12 Å. (*c*) Using the results of part (*a*), determine the density of electrons per second that impinge the barrier if the tunneling current density is 1.2 mA/cm^2.

***2.40** Consider the one-dimensional potential function shown in Figure P2.40. Assume the total energy of an electron is $E < V_0$. (*a*) Write the wave solutions that apply in each region. (*b*) Write the set of equations that result from applying the boundary conditions. (*c*) Show explicitly why, or why not, the energy levels of the electron are quantized.

*Hint for 2.40: from the boundary conditions, solve to find an equation involving the tangent function, some constants, and E and V_0. To determine if the energy states are quantized, you can plot both sides of this equation as a function of E to see where they intersect. The intersections of the two sides give the allowed solutions.

3.1 Consider Figure 3.4b, which shows the energy-band splitting of silicon. If the equilibrium lattice spacing were to change by a small amount, discuss how you would expect the electrical properties of silicon to change. Determine at what point the material would behave like an insulator or like a metal.

3.5 (*a*) Plot the function $f(\alpha a) = 12(\sin \alpha a)/\alpha a + \cos \alpha a$ for $0 \leq \alpha a \leq 4\pi$. Also, given the function $f(\alpha a) = \cos ka$, indicate the allowed values of αa that will satisfy this equation. (*b*) Determine the values of αa at (*i*) $ka = \pi$ and (*ii*) $ka = 2\pi$.

3.8 Using the parameters of Problem 3.5 for a free electron and letting $a = 4.2$ Å, determine the width (in eV) of the forbidden energy bands that exist at (*a*) $ka = \pi$ and (*b*) $ka = 2\pi$. (Refer to Figure 3.8c).

3.9 Using the parameters in Problem 3.5 for a free electron and letting $a = 4.2$ Å, determine the width (in eV) of the allowed energy bands that exist for (*a*) $0 < ka < \pi$ and (*b*) $\pi < ka < 2\pi$.

2.33 For the step potential function shown in Figure P2.33, assume that $E > V_0$ and that particles are incident from the $+x$ direction traveling in the $-x$ direction. (*a*) Write the wave solutions for each region. (*b*) Derive expressions for the transmission and reflection coefficients.

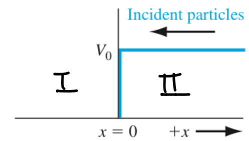


Figure P2.33 | Potential function for Problem 2.33.

no reflected wave

(a) Region I:

$$\psi_1(x) = \cancelto{0}{A_1 e^{jk_1 x}} + B_1 e^{-jk_1 x}, \quad k_1 = \sqrt{\frac{2mE}{\hbar^2}}$$

Region II:

$$\psi_2(x) = A_2 e^{jk_2 x} + B_2 e^{-jk_2 x}, \quad k_2 = \sqrt{\frac{2m(E-V_0)}{\hbar^2}}$$

(b) $\psi_1(0) = \psi_2(0) \quad \rightarrow \quad A_2 + B_2 = B_1$

$\psi_1'(x)\big|_{x=0} = \psi_2'(x)\big|_{x=0} \quad \rightarrow \quad k_2 A_2 - k_2 B_2 = -k_1 B_1$

$$k_2 A_2 = -k_1 B_1 + k_2 B_2$$

$$k_2 A_2 = -k_1 A_2 - k_1 B_2 + k_2 B_2$$

$$k_2 A_2 + k_1 A_2 = (-k_1 + k_2) B_2$$

$$A_2 (k_1 + k_2) = (-k_1 + k_2) B_2$$

$$A_2 = \frac{-k_1 + k_2}{k_1 + k_2} B_2$$

$$B_1 = -k_2 A_2 - k_2 B_2$$

$$= -k_2 \left(\frac{-k_1 + k_2}{k_1 + k_2} \right) - k_2 B_2$$

$$= \frac{2k_2}{k_1 + k_2} B_2$$

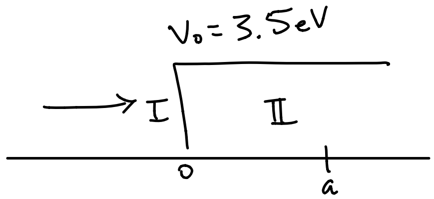
$$R = \frac{A_2 A_2^*}{B_2 B_2^*} = \frac{\frac{-k_1+k_2}{k_1+k_2} \cancel{B_2} \; \frac{-k_1+k_2}{k_1+k_2} \cancel{B_2}}{\cancel{B_2 B_2}}$$

$$= \left(\frac{-k_1 + k_2}{k_1 + k_2} \right)^2 = \boxed{\frac{k_1^2 - 2k_1 k_2 + k_2^2}{k_1^2 + 2k_1 k_2 + k_2^2}}$$

$$T = 1 - R = \boxed{\frac{4k_1 k_2}{k_1^2 + 2k_1 k_2 + k_2^2}}$$

2.34 Consider an electron with a kinetic energy of 2.8 eV incident on a step potential function of height 3.5 eV. Determine the relative probability of finding the electron at a distance (*a*) 5 Å beyond the barrier, (*b*) 15 Å beyond the barrier, and (*c*) 40 Å beyond the barrier compared with the probability of finding the incident particle at the barrier edge.

$E < V_0$



$|\psi(x)|^2$ $\qquad E = 2.8 eV$

$$\psi_2(x) = A_2 e^{-k_2 x} + \cancel{B_2 e^{k_2 x}} \rightarrow 0$$ no reflected wave, $k_2 = \sqrt{\frac{2m(V_0 - E)}{\hbar^2}}$

$$k_2 = 4.2863 \times 10^9$$

$$P(x) = \frac{|\psi(x)|^2}{|\psi(0)|^2} = \frac{\cancel{A_2^2} e^{-2k_2 x}}{\cancel{A_2^2} \cancel{e^{-2k_2 0}} \rightarrow 1} = e^{-2k_2 x}$$

$$p(5Å) = e^{-2(4.2863 \times 10^9)(5 \times 10^{-10})} = 13.755 \times 10^{-3} = \boxed{0.0138\%}$$

$$p(15Å) = e^{-2(4.2863 \times 10^9)(15 \times 10^{-10})} = 2.6025 \times 10^{-6} = \boxed{2.60 \times 10^{-4}\%}$$

$$p(40Å) = e^{-2(4.2863 \times 10^9)(40 \times 10^{-10})} = 1.28 \times 10^{-15} = \boxed{1.28 \times 10^{-13}\%}$$

2.35 (*a*) Calculate the transmission coefficient of an electron with a kinetic energy of 0.1 eV impinging on a potential barrier of height 1.0 eV and a width of 4 Å (*b*) Repeat part (*a*) for a barrier width of 12 Å. (*c*) Using the results of part (*a*), determine the density of electrons per second that impinge the barrier if the tunneling current density is 1.2 mA/cm².

$E < V_0$

$$\boxed{T \approx 16\left(\frac{E}{V_0}\right)\left(1-\frac{E}{V_0}\right)\exp(-2k_2 a)} \quad (2.63)$$

(a) $E = 0.1\text{eV} \quad V_0 = 1.0\text{eV} \quad a = 4\text{Å}$

$$k_2 = \sqrt{\frac{2m(V_0 - E)}{\hbar^2}} = 4.8603\times10^{9}$$

$$T \approx 16\left(\frac{0.1}{1.0}\right)\left(1-\frac{0.1}{1.0}\right)e^{-2(4.8603\times10^{9})(4\times10^{-10})}$$

$$\boxed{T \approx 29.494\times10^{-3}}$$

(b) $a = 12\text{Å}$

$$\boxed{T \approx 12.373\times10^{-6}}$$

(c) $E = \frac{1}{2}mv^2 = 0.1\,\text{eV} = 16.022\times10^{-21}\,\text{J} \longrightarrow v = \sqrt{\frac{2E}{m}}$

$$v = 1.8755\times10^{5}\,\frac{\text{m}}{\text{s}} = 1.8755\times10^{7}\,\frac{\text{cm}}{\text{s}}$$

$$J = n_t e v \longrightarrow n_t = \frac{J}{ev} = \frac{1.2\,\text{mA/cm}^2}{(1.6\times10^{-19})(1.8755\times10^{7}\,\text{cm/s})}$$

$$n_t = 3.9934\times10^{8}\,\frac{\text{electrons}}{\text{cm}^3}$$

$$n_i = \frac{n_t}{T} = \boxed{13.540\times10^{9}\,\frac{\text{electrons}}{\text{cm}^3}}$$

***2.40** Consider the one-dimensional potential function shown in Figure P2.40. Assume the total energy of an electron is $E < V_0$. (*a*) Write the wave solutions that apply in each region. (*b*) Write the set of equations that result from applying the boundary conditions. (*c*) Show explicitly why, or why not, the energy levels of the electron are quantized.

*Hint for 2.40: from the boundary conditions, solve to find an equation involving the tangent function, some constants, and E and V_0. To determine if the energy states are quantized, you can plot both sides of this equation as a function of E to see where they intersect. The intersections of the two sides give the allowed solutions.

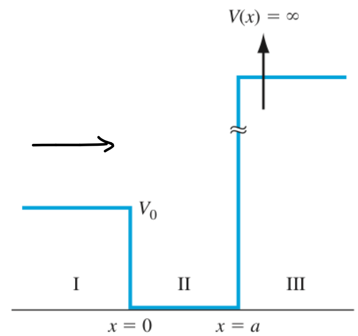


Figure P2.40 | Potential function for Problem 2.40.

(a)

$$\psi_1(x) = B_1 e^{k_1 x}, \quad k_1 = \sqrt{\frac{2m(V_0 - E)}{\hbar^2}},$$

$$\psi_2(x) = A_2 \sin(k_2 x) + B_2 \cos(k_2 x), \quad k_2 = \sqrt{\frac{2mE}{\hbar^2}}$$

$$\psi_3(x) = 0$$

(b)

$$\psi_1(0) = \psi_2(0) \longrightarrow B_1 = B_2$$

$$\psi_1'(0) = \psi_2'(0) \longrightarrow k_1 B_1 = k_2 A_2$$

$$\psi_2(a) = \psi_3(a) \longrightarrow A_2 \sin(k_2 a) + B_2 \cos(k_2 a) = 0$$

$$B_2 = -\frac{A_2 \sin(k_2 a)}{\cos(k_2 a)} = -A_2 \tan(k_2 a)$$

(c)

$$k_1 B_1 = k_2 A_2 \rightarrow k_1 B_2 = k_2 A_2$$

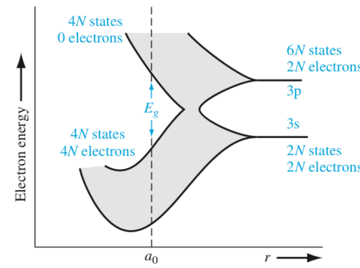
$$-k_1 \cancel{A_2} \tan(k_2 a) = k_2 \cancel{A_2} \rightarrow \tan(k_2 a) = -\frac{k_2}{k_1}$$

$$\tan\left(\sqrt{\frac{2mE}{\hbar^2}}\, a\right) = -\frac{\sqrt{\frac{\cancel{2m}E}{\cancel{\hbar^2}}}}{\sqrt{\frac{\cancel{2m}(V_0 - E)}{\cancel{\hbar^2}}}}$$

$$\tan\left(\sqrt{\frac{2mE}{\hbar^2}}\, a\right) = -\sqrt{\frac{E}{V_0 - E}}$$

only possible for specific values $\rightarrow$ quantized

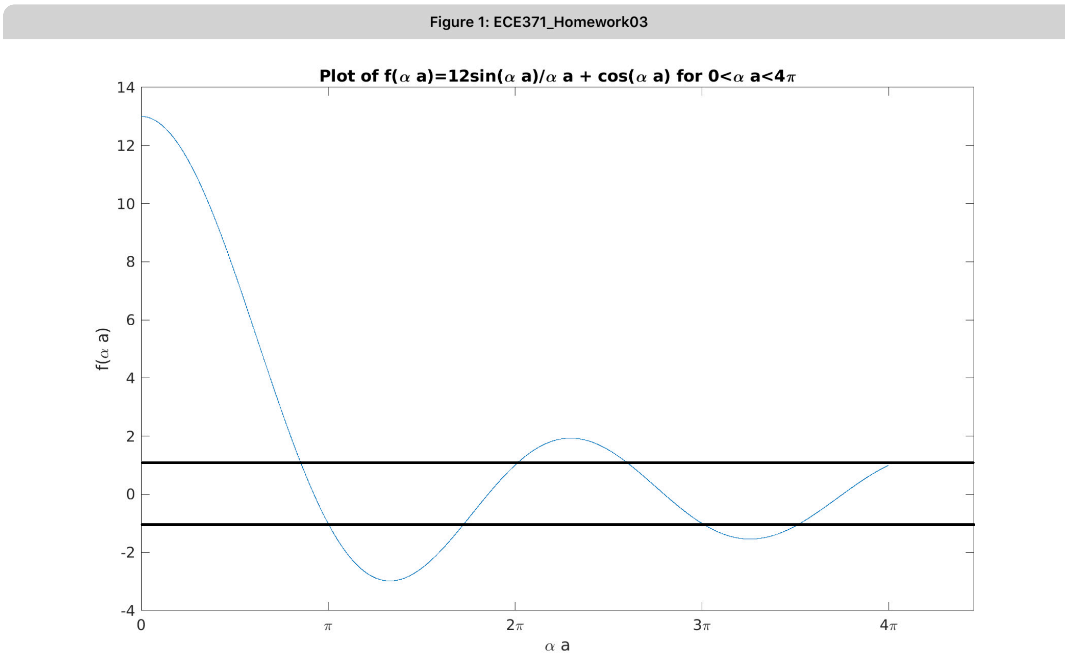
3.1 Consider Figure 3.4b, which shows the energy-band splitting of silicon. If the equilibrium lattice spacing were to change by a small amount, discuss how you would expect the electrical properties of silicon to change. Determine at what point the material would behave like an insulator or like a metal.



(b)

The equilibrium lattice spacing (a_0) affects the bandgap energy and therefore the properties of the material. A smaller bandgap (larger a_0) would act more like a semiconductor, while a larger bandgap (smaller a_0) would act more like an insulator.

3.5 (*a*) Plot the function $f(\alpha a) = 12(\sin \alpha a)/\alpha a + \cos \alpha a$ for $0 \le \alpha a \le 4\pi$. Also, given the function $f(\alpha a) = \cos ka$, indicate the allowed values of αa that will satisfy this equation. (*b*) Determine the values of αa at (*i*) $ka = \pi$ and (*ii*) $ka = 2\pi$.



(a) If $f(\alpha a) = \cos(ka)$, then $f(\alpha a)$ can only exist between -1 and 1.

(b)

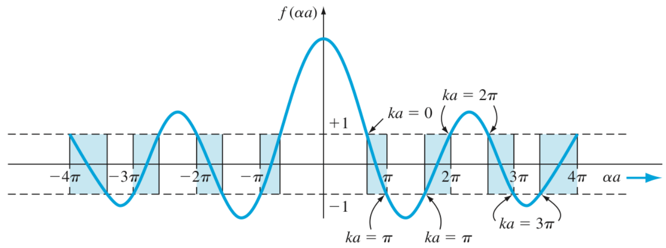
(i) The values for αa for $ka = \pi$ are odd π integers $(\pi, 3\pi, 5\pi \ldots)$

$\alpha a = \pi$, $\alpha a = 1.729\pi$

(ii) The values for αa for $ka = 2\pi$ are even π integers $(2\pi, 4\pi, 6\pi)$

$\alpha a = 2\pi$, $\alpha a = 2.617\,\pi$

3.8 Using the parameters of Problem 3.5 for a free electron and letting $a = 4.2$ Å, determine the width (in eV) of the forbidden energy bands that exist at (*a*) $ka = \pi$ and (*b*) $ka = 2\pi$. (Refer to Figure 3.8c).



(c)

(a) $f(\alpha a) = 12\dfrac{\sin(\alpha a)}{\alpha a} + \cos(\alpha a) = \cos(ka)$

$ka = \pi \longrightarrow \left(\sqrt{\dfrac{2mE}{\hbar^2}}\right)(4.2\text{Å}) = \pi \longrightarrow E_1 = \dfrac{\pi^2\hbar^2}{(4.2)^2 2m} = 3.415\times10^{-19}\ J$

$E_1 = 2.1317\ eV$

$ka = 1.729\pi \longrightarrow E_2 = \dfrac{(1.729\pi)^2\hbar^2}{(4.2\text{Å})^2 2m} = 1.021\times10^{-18}\ J$

$\uparrow$ from 3.5

$E_2 = 6.3726\ eV$

$$\boxed{\Delta E = 4.2409\ eV}$$

(b) $ka = 2\pi \longrightarrow E_1 = \dfrac{4\pi^2\hbar^2}{(4.2\text{Å})2m} = 1.3661\times10^{-18}\ J$

$E_1 = 8.5268\ eV$

$ka = 2.617\pi \rightarrow E_2 = \dfrac{(2.617\pi)^2\hbar^2}{(4.2\text{Å})\,2m} = 2.3391\times10^{-18}\ J$

$E_2 = 14.5993\ eV$

$$\boxed{\Delta E = 6.0725\ eV}$$

3.9 Using the parameters in Problem 3.5 for a free electron and letting $a = 4.2$ Å, determine the width (in eV) of the allowed energy bands that exist for (*a*) $0 < ka < \pi$ and (*b*) $\pi < ka < 2\pi$.

(a) $0 < ka < \pi$

$$E_1 = \frac{\pi^2 \hbar^2}{(4.2\text{Å})2m} = 3.415 \times 10^{-19} \text{ J} = 2.1317 \text{ eV}$$

$$E_2 = \frac{(0.859\pi)^2 \hbar^2}{(4.2\text{Å})2m} = 2.520 \times 10^{-19} \text{ J} = 1.5729 \text{ eV}$$

$$\boxed{\Delta E = 0.5588}$$

(b) $\pi < ka < 2\pi$

$$E_1 = \frac{(2\pi)^2 \hbar^2}{(4.2\text{Å})2m} = 1.3661 \times 10^{-18} \text{ J} = 8.5268 \text{ eV}$$

$$E_2 = \frac{(1.729\pi)^2 \hbar^2}{(4.2\text{Å})2m} = 1.021 \times 10^{-18} \text{ J} = 6.3726 \text{ eV}$$

$$\boxed{\Delta E = 2.1542 \text{ eV}}$$