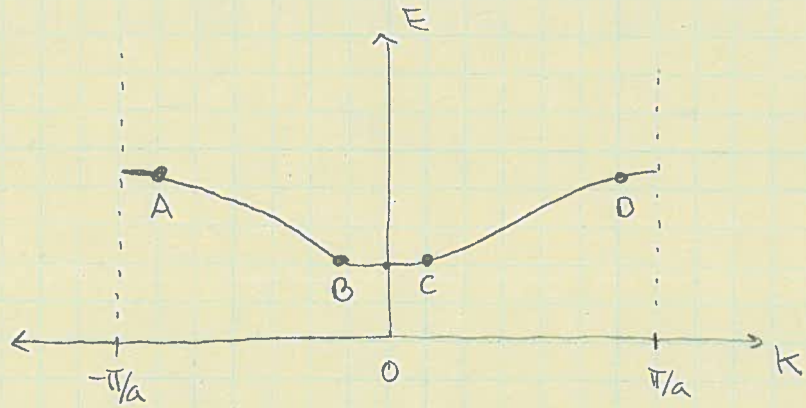
3.15



a) Effective mass $\propto \dfrac{1}{\frac{d^2E}{dk^2}}$

A: $\frac{d^2E}{dk^2}$ is negative $\Rightarrow$ m^* negative

B: $\frac{d^2E}{dk^2}$ is positive $\Rightarrow$ m^* positive

C: $\frac{d^2E}{dk^2}$ is positive $\Rightarrow$ m^* positive

D: $\frac{d^2E}{dk^2}$ is negative $\Rightarrow$ m^* negative

b) Velocity direction $\propto \frac{dE}{dk}$

A: $\frac{dE}{dk} < 0$ $\Rightarrow$ V is in $-x$ direction

B: $\frac{dE}{dk} < 0$ $\Rightarrow$ V is in $-x$ direction

C: $\frac{dE}{dk} > 0$ $\Rightarrow$ V is in $+x$ direction

D: $\frac{dE}{dk} > 0$ $\Rightarrow$ V is in $+x$ direction

$\boxed{3.16}$

The general expression for a parabola is

$$E(k) = \underbrace{C_1}_{\text{curvature}} k^2 + \underbrace{C_2}_{\text{intercept} = 0 \text{ in both curves}}$$

For A: $E(0.08\,\text{Å}^{-1}) = C_1 (0.08\,\text{Å}^{-1})^2 = 0.05\,\text{eV}$

$$\Rightarrow C_1 = 7.8125\ \text{eV}\cdot\text{Å}^2$$

so $E(k) = 7.8125\,k^2 \quad [\text{eV}]$

$$m^* = \frac{\hbar}{\frac{d^2E}{dk^2}} = \frac{\hbar^2}{2\cdot(7.8125)\ \text{eV}\cdot\text{Å}^2}$$

$$m^* = \frac{(1.054\times10^{-34}\ \text{J}\cdot\text{s})^2}{2(7.8125)\text{eV}\cdot\text{Å}^2\cdot(1.6\times10^{-19}\ \text{J/eV})(1\times10^{-10}\ \text{m/Å})^2}$$

$$= 4.44\times10^{-31}\ \text{kg} \Rightarrow \boxed{0.488\,m_0}$$

For B: $E(0.08\,\text{Å}^{-1}) = C_1(0.08\,\text{Å}^{-1})^2 = 0.5\,\text{eV}$

$$\Rightarrow C_1 = 78.125\ \text{eV}\cdot\text{Å}^2$$

so $E(k) = 78.125\ \text{eV}\cdot\text{Å}^2\cdot k^2 \quad [\text{eV}]$, k in Å^{-1}

$$m^* = \frac{(1.054\times10^{-34}\ \text{J}\cdot\text{s})^2}{2(78.125\,\text{eV}\,\text{Å}^2)(1.6\times10^{-19}\ \text{J/eV})(1\times10^{-10}\ \text{m/Å})^2}$$

$$= 4.44\times10^{-32}\ \text{kg} \Rightarrow \boxed{0.0488\,m_0}$$

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$\boxed{3.20}$ $E = E_0 - E_1 \cos(\alpha(k-k_0))$

$$m^* = \frac{\hbar^2}{\frac{d^2E}{dk^2}}$$

$$\frac{dE}{dk} = E_1 \sin(\alpha(k-k_0)) \cdot \alpha$$

$$\frac{d^2E}{dk^2} = E_1 \cos(\alpha(k-k_0)) \cdot \alpha^2$$

so $m^*(k) = \dfrac{\hbar^2}{E_1 \alpha^2 \cos(\alpha(k-k_0))}$

m^* is a function of k, so evaluating at k_0

$$\boxed{m^*(k_0) = \frac{\hbar^2}{E_1 \alpha^2}}$$

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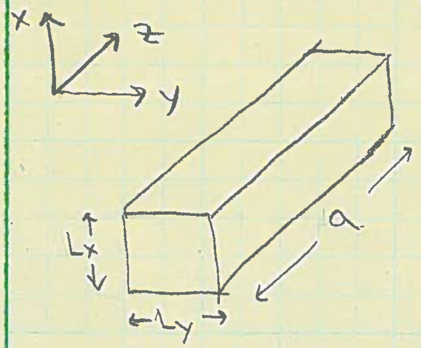
3.25 1-D electron gas in GaAs → two quantized dimensions

$m_n^* = 0.067\, m_0$

$$E = \pm \frac{p^2}{2m_n^*} \Rightarrow \text{two momentum states per energy level}$$



- L_x, L_y are quantum sized
- a is large

- electrons energies are quantized in x and y and continuous is z

- the total energy is

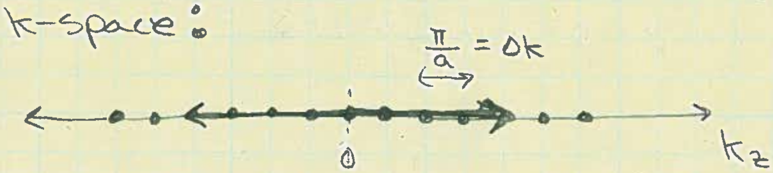
$$E = E_{nx} + E_{ny} + \frac{\hbar^2 k_z^2}{2m_n^*}$$

$$E_{nx} = \frac{\pi^2 \hbar^2 n_x^2}{2m_n^* L_x^2} \qquad E_{ny} = \frac{\pi^2 \hbar^2 n_y^2}{2m_n^* L_y^2}$$

- E_{nx} and E_{ny} are quantized

- Follow a procedure similar to the one on slide 9 of lecture 10

- in k-space:



note: we have to consider + and − k values since we are told there are two momentum states per energy level

$$N_s(k) = \frac{2 \cdot k_z}{(\pi/a)} \cdot \underset{\text{spin}}{2} \cdot \underset{\substack{\text{positive}\\ k_z\text{ only}}}{\frac{1}{2}}$$

$$N_s(k) = \frac{2 k_z a}{\pi}$$

- translate $N_s(k)$ into $N_s(E)$ assuming free electron in z-direction

$$E = E_{nx} + E_{ny} + \frac{\hbar^2 k_z^2}{2m_n^*} \Longrightarrow k_z = \frac{\sqrt{2m_n^*(E - E_{nx} - E_{ny})}}{\hbar}$$

so $$N_s(E) = \frac{2a}{\pi\hbar}\sqrt{2m_n^*(E - E_{nx} - E_{ny})}$$

- divide by unit length $= a$

$$\frac{N_s(E)}{a} = \frac{2}{\pi\hbar}\sqrt{2m_n^*(E - E_{nx} - E_{ny})}$$

- Finally $$\frac{1}{a}\frac{dN_s(E)}{dE} = \frac{1}{2}\frac{2}{\pi\hbar}\sqrt{2m_n^*}\,(E - E_{nx} - E_{ny})^{-\frac{1}{2}}$$

$$\boxed{DOS_{1D} = \frac{1}{\pi\hbar}(2m_n^*)^{1/2}\frac{1}{\sqrt{E - E_{nx} - E_{ny}}}}$$

$\frac{\#\ \text{states}}{\text{unit length} \cdot \text{unit E}}$

- we can plug in some values for GaAs

$$DOS_{GaAs_{1D}} = \frac{(2m_n^*)^{1/2}}{\pi\hbar} \cdot \frac{1}{\sqrt{E - E_{nx} - E_{ny}}}$$

$$= \frac{(2(9.11\times 10^{-31}\,kg)(0.067))^{1/2}}{\pi(1.054\times 10^{-34}\,J\cdot s)} \quad \frac{1}{\sqrt{E - E_{nx} - E_{ny}}}$$

$$\boxed{= \frac{1.055\times 10^{18}}{\sqrt{E - E_{nx} - E_{ny}}}\left[J^{-1}\cdot m^{-1}\right]}$$

* constant has units of $m^{-1} \cdot J^{-1/2}$
* E is in J

* Considering all energy levels

$$\boxed{DOS_{1D} = \frac{1}{\pi\hbar}(2m_n^*)^{1/2}\sum_{n_x,n_y}\frac{H(E - E_{nx} - E_{ny})}{\sqrt{E - E_{nx} - E_{ny}}}}$$

$\boxed{3.26}$

a) determine the total number ($\#/cm^3$) of energy states in Si between E_c and $E_c + 2kT$ at (i) $T = 300k$, (ii) $T = 400k$

- assuming this is "bulk" Si
- need conduction band D.O.S.

$$g_c(E) = \frac{4\pi(2m_n^*)^{3/2}}{h^3}\sqrt{E - E_c} \qquad \frac{\#}{\text{unit V} \cdot \text{unit E}}$$

- to get total $\#/cm^3$ we need to integrate with respect to E

$$\frac{\#}{cm^3} = \int_{E_c}^{E_c+2kT} g_c(E)\,dE = \frac{4\pi(2m_n^*)^{3/2}}{h^3}\int_{E_c}^{E_c+2kT} (E-E_c)^{1/2}$$

$$= \frac{2}{3}\cdot\frac{4\pi(2m_n^*)^{3/2}}{h^3}(E-E_c)^{3/2}\Big|_{E_c}^{E_c+2kT}$$

$$= \frac{8\pi(2m_n^*)^{3/2}}{3h^3}\left[(2kT)^{3/2}\right]$$

For Si: $m_n^* = 1.08\,m_0$ (From Table B.4 – D.O.S. effective mass)

(i) $T = 300k$: $\dfrac{8\pi(2(1.68)(9.11\times10^{-31}\,kg))^{3/2}}{3(6.62\times10^{-34}\,J\cdot s)^3}\left[2(1.38\times10^{-23}\,J/k)(300k)\right]^{3/2}$

$$= 6.01\times10^{25}\ m^{-3} = \boxed{6.01\times10^{19}\ cm^{-3}}$$

(ii) $T = 400k$: $\dfrac{8\pi(2(1.08)(9.11\times10^{-31}\,kg))^{3/2}}{3(6.62\times10^{-34}\,J\cdot s)^3}\left[2(1.38\times10^{-23}\,J/k)(400k)\right]^{3/2}$

$$= 9.25\times10^{25}\ m^{-3} = \boxed{9.25\times10^{19}\ cm^{-3}}$$

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b) For GaAs $m_n^* = 0.067 m_0$

(i) Just scale the result for Si at 300K by

$$\left(\frac{m_n^*{}_{GaAs}}{m_n^*{}_{Si}}\right)^{3/2}$$

$$\Longrightarrow \quad 6.01 \times 10^{19}\,cm^{-3}\left(\frac{0.067}{1.08}\right)^{3/2} = \boxed{9.29 \times 10^{17}\,cm^{-3}}$$

(ii) Scale result for Si at 400K by

$$\left(\frac{m_n^*{}_{GaAs}}{m_n^*{}_{Si}}\right)^{3/2}$$

$$\Longrightarrow \quad 9.25 \times 10^{19}\,cm^{-3}\left(\frac{0.067}{1.08}\right)^{3/2} = \boxed{1.43 \times 10^{18}\,cm^{-3}}$$

$\boxed{3.28}$ a) Plot DOS in conduction band for Si over $E_c < E < E_c + 0.4eV$

b) Repeat a) in valence band for $E_v - 0.4eV < E < E_v$

a) $$g_c(E) = \frac{4\pi(2m_n^*)^{3/2}}{h^3}\sqrt{E - E_c}$$

SEE attached Matlab code and plot

b) $$g_v(E) = \frac{4\pi(2m_p^*)^{3/2}}{h^3}\sqrt{E_v - E}$$

SEE attached Matlab code and plot

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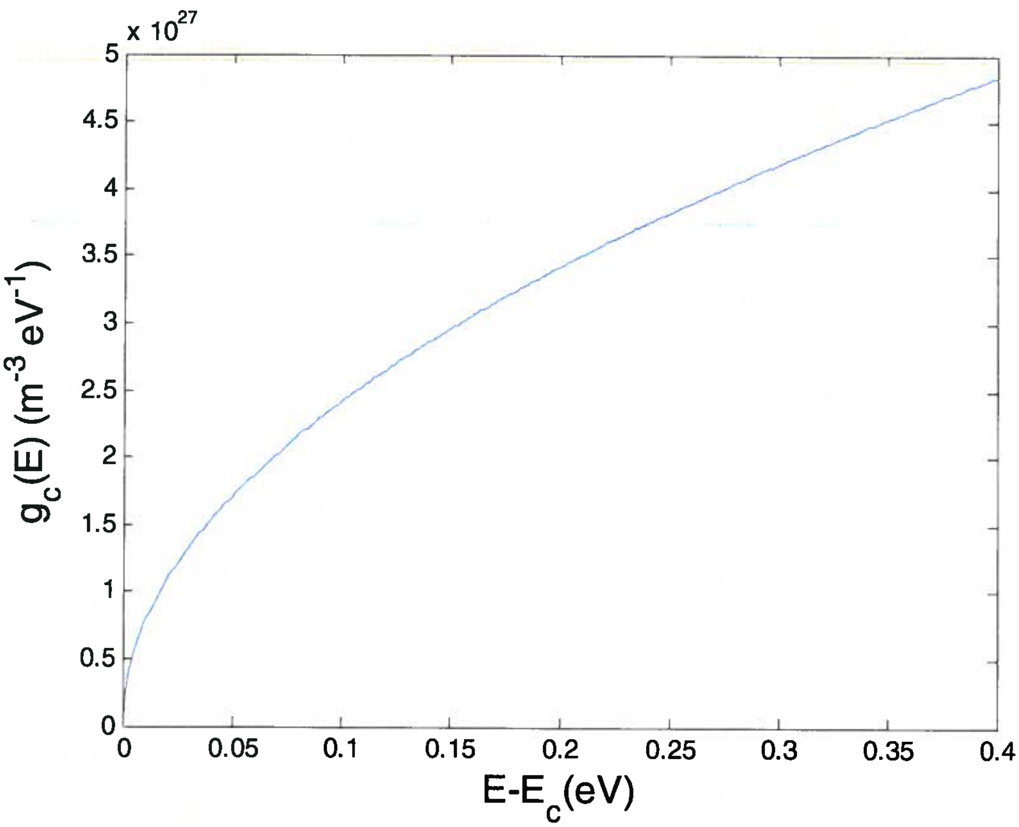
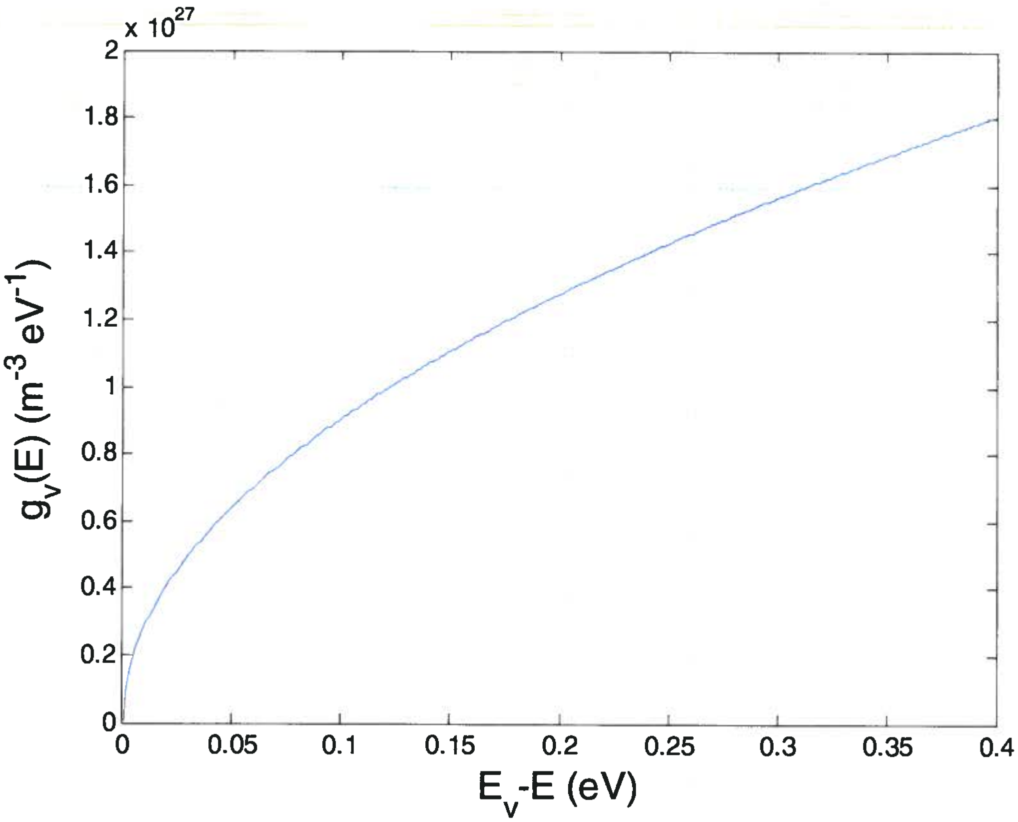
3.28

```
clear all;

m0 = 9.11e-31; %kg
mn = 1.08.*m0;
h = 6.622e-34; %J s

E_Ec = 0:.001:0.4;
gc = (1.6e-19).^(3/2).*((4.*pi.*(2.*mn).^(3/2))./(h.^3))*(E_Ec).^0.5

plot(E_Ec,gc)

xlabel('E-E_{c}(eV)', 'FontSize', 14)
ylabel('g_{c}(E) (m^{-3} eV^{-1})', 'FontSize', 14)
```



```
clear all;

m0 = 9.11e-31; %kg
mp = 0.56.*m0;
h = 6.622e-34; %J s

Ev_E = 0:.001:0.4;
gv = (1.6e-19).^(3/2).*((4.*pi.*(2.*mp).^(3/2))./(h.^3))*(Ev_E).^0.5

plot(Ev_E,gv)

xlabel('E_{v}-E (eV)', 'FontSize', 14)
ylabel('g_{v}(E) (m^{-3} eV^{-1})', 'FontSize', 14)
```



$\boxed{3.30}$ $$f_F(E) = \frac{1}{e^{(E-E_F)/kT} + 1}$$

$-0.2 \le (E-E_F) \le 0.2$ eV

a) $T = 200\,k$

b) $T = 300\,k$

c) $T = 400\,k$

* SEE Attached Matlab code and plot

$\boxed{3.32.}$ Determine the probability that an energy level is occupied by an electron if the state is above the Fermi level by

(a) kT (b) $5kT$ (c) $10\,kT$

$$f_F(E) = \frac{1}{e^{(E-E_F)/kT} + 1}$$

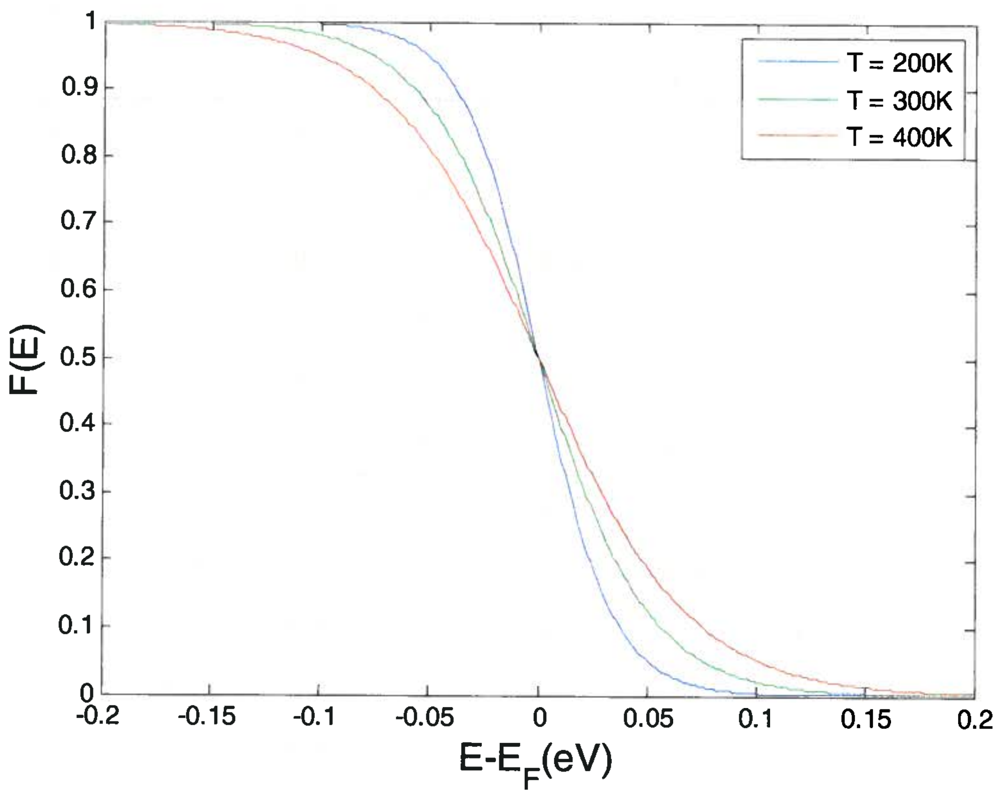
(a) $E - E_F = kT \implies f_F(E) = \dfrac{1}{e^1 + 1} = \boxed{0.269}$

(b) $E - E_F = 5kT \implies f_F(E) = \dfrac{1}{e^5 + 1} = \boxed{6.69 \times 10^{-3}}$

(c) $E - E_F = 10kT \implies f_F(E) = \dfrac{1}{e^{10} + 1} = \boxed{4.54 \times 10^{-5}}$

3.30

```
clear all;

%T = 200; %K
%T = 300; %K
T = 400; %K

k = 8.62e-5; %eV/K

E_Ef = -0.2:.001:0.2; %eV

F = 1./(exp(E_Ef./(k.*T)) + 1);

plot(E_Ef,F)

xlabel('E-E_{F}(eV)', 'FontSize', 14)
ylabel('F(E)', 'FontSize', 14)
legend('T = 200K', 'T = 300K', 'T = 400K', 'Location', 'NorthEast')
hold all;
```



$\boxed{3.33}$ Determine the probability that an energy level is empty of an electron if the state is below the fermi level by

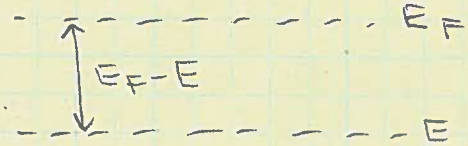
(a) kT (b) $5kT$ (c) $10kT$

For an empty state we look at

$$1 - F(E) = 1 - \frac{1}{e^{(E-E_F)/kT} + 1}$$

$$= \frac{e^{(E-E_F)/kT} \cancel{+1} \cancel{-1}}{e^{(E-E_F)/kT} + 1}$$

$$= \frac{1}{e^{(E_F-E)/kT} + 1}$$



(a) $1 - F(E) = \dfrac{1}{e^{1} + 1} = \boxed{0.269}$

(b) $1 - F(E) = \dfrac{1}{e^{5} + 1} = \boxed{6.69 \times 10^{-3}}$

(c) $1 - F(E) = \dfrac{1}{e^{10} + 1} = \boxed{4.54 \times 10^{-5}}$

$\boxed{3.39}$

$$F_F(E) = \frac{1}{e^{(E-E_F)/kT} + 1}$$

$$F_B(E) = e^{-(E-E_F)/kT} \qquad \text{Boltzmann}$$

(a) Find $(E-E_F)$ where $F_F(E)$ is within 1% of $F_B(E)$

↳ since $F_B(E)$ is always larger we write (see Ex 3.8)

$$\frac{\frac{1}{e^{(E-E_F)/kT}} - \frac{1}{e^{(E-E_F)/kT}+1}}{\frac{1}{e^{(E-E_F)/kT}+1}} = 0.01$$

$$\Rightarrow \quad \frac{e^{(E-E_F)/kT}+1}{e^{(E-E_F)/kT}} - 1 = 0.01$$

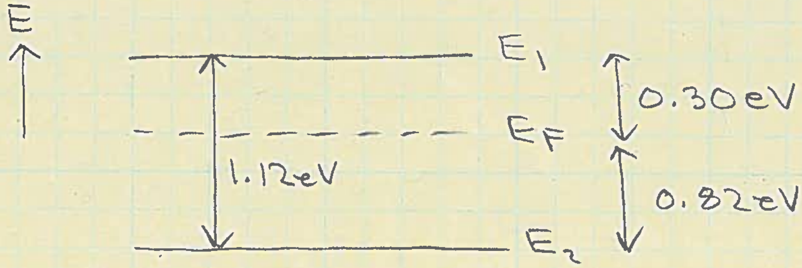
$$\frac{\cancel{e^{(E-E_F)/kT}} + 1 - \cancel{e^{(E-E_F)/kT}}}{e^{(E-E_F)/kT}} = 0.01$$

$$\rightarrow \quad e^{-(E-E_F)/kT} = 0.01$$

$$(E-E_F) = -\ln(0.01)\cdot kT \; \boxed{= 4.6\,kT}$$

(b) $$F_F(E) = \frac{1}{e^{4.6}+1} = \boxed{0.00995 \simeq 0.01}$$

3.42



T=300K

a) $E_1 - E_F = 0.30\,eV$ → what is probality of occupation at $E = E_1$?

→ what is probability of non-occupation at $E = E_2$?

$$f_F(E_1) = \frac{1}{e^{(E_1-E_F)/kT}+1}$$

$$= \frac{1}{e^{(0.3/0.0259)}+1} = \boxed{9.32\times10^{-6}}$$

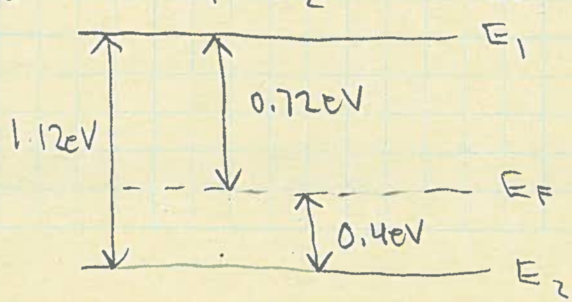
$$1-f_F(E_2) = \frac{1}{e^{(E_F-E_2)/kT}+1} \quad , \; E_F - E_2 = 1.12-0.3 = 0.82\,eV$$

$$= \frac{1}{e^{(0.82/0.0259)}+1} = \boxed{1.78\times10^{-14}}$$

⟹ so since $E_1 \gg E_F$ the chances of occupation are low

⟹ since $E_2 \ll E_F$, the chances of E_2 being occupied (empty) are high (low)

(b) If $E_F - E_2 = 0.4\,eV$



$$f_F(E_1) = \frac{1}{e^{(0.72/0.0259)}+1} \boxed{= 8.45\times10^{-13}}$$

$$1-f_F(E_2) = \frac{1}{e^{(0.4/0.0259)}+1} \boxed{= 1.96\times10^{-7}}$$