# ECE 345 / ME 380: Introduction to Control Systems
# Collaborative Quiz #3

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**Due Thursday, Oct. 15, 2020 at the end of class**

Solar collectors are often outfitted with solar trackers, to maximize efficiency by increasing the time the solar panels are exposed to direct sunlight (as opposed to diffuse sunlight). The amount of energy that can be gathered is dependent upon the light intercepted by the panel, as well as the angle of incidence of the sunlight on the panel. (See the first two minutes of the video at `https://youtu.be/KfKrNlxBLZI` for additional information on solar tracking.) Solar trackers orient the solar collector to track the sun from east to west as the day proceeds.

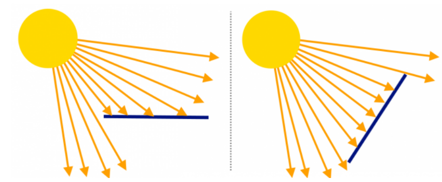


Figure 1: From `http://www.lauritzen.biz/blog/?p=49`.

We consider a single-axis system, which rotates the solar collector about a single axis of rotation. A torque is applied to rotate the sun collector, to make the actual angle $y(t)$ track a desired angle $r(t)$ that represents the angle of the sun.

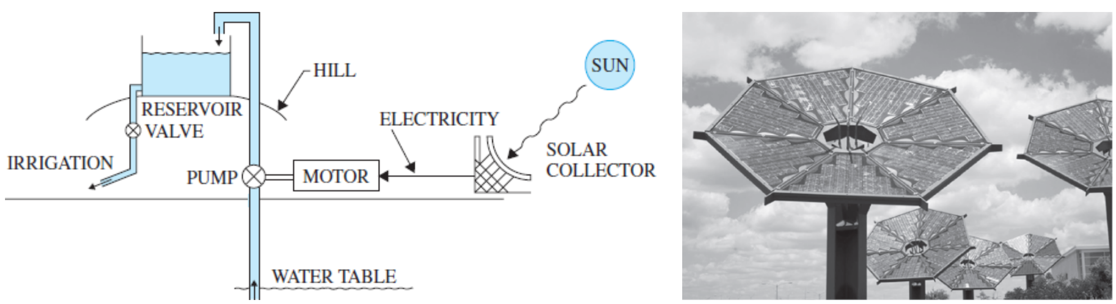



Figure 2: Solar collector used to extract water. Images from Golnaraghi & Kuo, 2017.

The transfer function of the *open-loop system* (Figure 3(a), next page), with input $r(t)$ that is the desired angle of the solar tracker, and output $y(t)$ that is actual angle of the solar tracker, is

$$G(s) = \frac{1}{s^3 + 7s^2 + 12s} \tag{1}$$

Note that we can rewrite this as $G(s) = \frac{N(s)}{D(s)}$, with $N(s) = 1$ and $D(s) = s(s+3)(s+4)$.

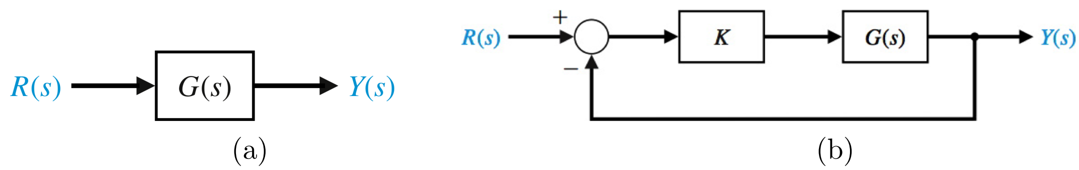



Figure 3: (a) Open-loop system. (b) Closed-loop system under negative unitary feedback.

# 1 Questions to be completed before class

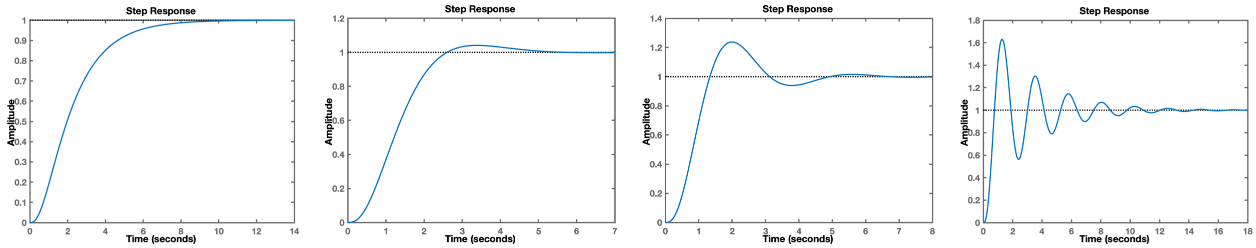
1. Use the Matlab command `roots` to find the location of the poles and zeros of $G(s)$.
2. Describe the asymptotic stability of the open-loop system, based on the location of the poles.
3. Use Matlab to construct the step response of the open-loop system.
4. Find the transfer function $G_{CL}(s)$ for the *closed-loop system* for some positive gain $K > 0$.

# 2 Questions to be completed in class

1. Considering your responses to Questions 1.1 and 1.4, which of the following describes the effect of feedback on $\frac{Y(s)}{R(s)}$? *More than answer may be correct.*
   (a) Both the zeros and the poles of the open-loop system are different from the zeros and the poles of the closed-loop system.
   (b) The open-loop and closed-loop transfer functions are different only by a factor of $K$.
   (c) The zeros of the open-loop system and the closed-loop system are the same, but the poles are different.
   (d) The zeros of the closed-loop system are $K$ times the zeros of the open-loop system.

2. Using your response to Questions 1.2 and 1.3, which of the following best describes the BIBO stability of the open-loop system $G(s)$? *(More than one response may be correct.)*
   (a) The open-loop system is BIBO unstable because *all* bounded inputs produce an unbounded output.
   (b) The open-loop system is BIBO unstable because *at least one* bounded input produces an unbounded output.
   (c) The open-loop system is BIBO stable because *at least one* bounded input produces a bounded output.
   (d) The open-loop system is BIBO stable because it is asymptotically stable.
   (e) The open-loop system is BIBO stable because it is marginally stable.

To assess transient performance, we presume that the closed-loop characteristic equation can be described by two poles near the origin, and a third pole on the real line that is sufficiently far away that the transient response is dominated by the pair of poles closest to the origin.




$K = 5$ $K = 10$ $K = 20$ $K = 50$

Figure 4: Step response of the closed-loop system with various values of gain $K$.

3. Based solely on the observed characteristics of the step responses of $G_{CL}(s)$ in Figure 4, assess which of the following hypotheses are be true. *More than one response may be correct.*

   (a) As $K$ increases, damping ratio decreases.

   (b) As $K$ increases beyond the value associated with critical damping, overshoot decreases.

   (c) For low values of $K$, system is overdamped, and for high values of $K$, the system is underdamped.

   (d) For low values of $K$, system is underdamped, and for high values of $K$, the system is overdamped.

4. Consider your answer to Question 1.4. Using the Hurwitz criterion, determine what values of $K > 0$ will assure that the closed-loop system is asymptotically stable.

5. Compute the step response of the closed-loop system with $K = 100$ *over 0 to 20 seconds only*, using the 'tfinal' option in Matlab:

   ```
   [Y,...]  = step(SYS,TFINAL)
   ```

   and include a plot with your submission to Learn. Describe the physical behavior of the system in this case: What happens to the sun tracker angle in response to a step input? Does the step response differ from the step responses in Figure 4, and if so, in what way?

## If you finish early

- Consider the case in which the pair of poles nearest to the origin are exactly on the imaginary axis, and the third pole is somewhere on the negative real line. Will the sun tracker oscillate in a step response? Why or why not?
- Why must a third-order polynomial always have one root on the real line?
- Is it possible for the closed-loop system with gain $K$ to have only one pole in the RHP?
- For a third-order characteristic equation $\Delta(s) = s^3 + a_2 s^2 + a_1 s + a_0 K$ which can be written as $0 = \Delta(s) = (s + \alpha)(s^2 + 2\zeta\omega_n s + \omega_n^2)$, with $a_i$ known constants, to make the dominant poles have a damping ratio of $\zeta = 0.5$, what value should $K$ have? (You should also be able to find $\omega_n$ and $\alpha$ in terms of $a_i$.)