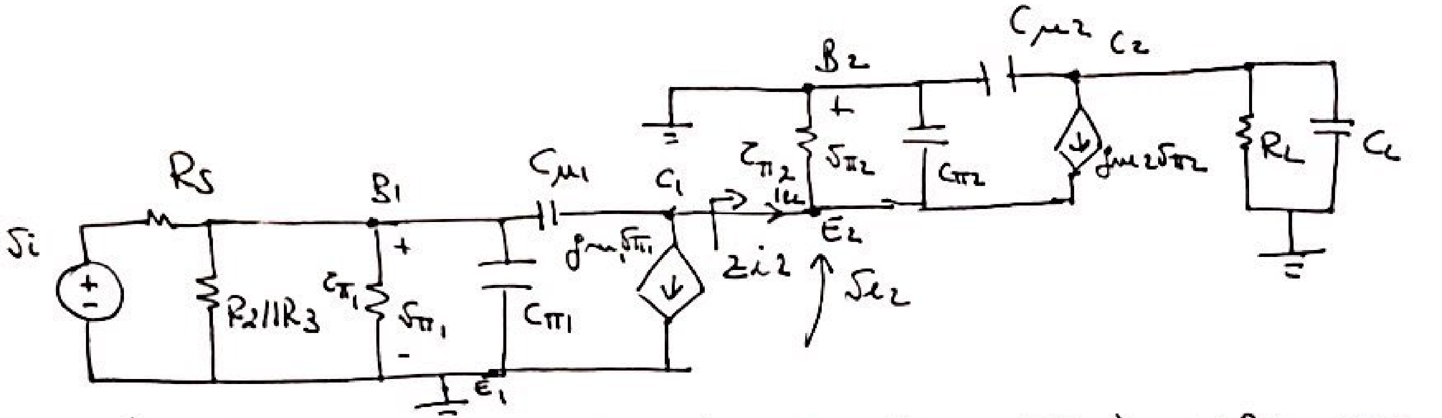
Homework 10 - Solution

Assumption: $V_A = \infty$

Bandwidth = $f_H - f_L$

Determining f_H

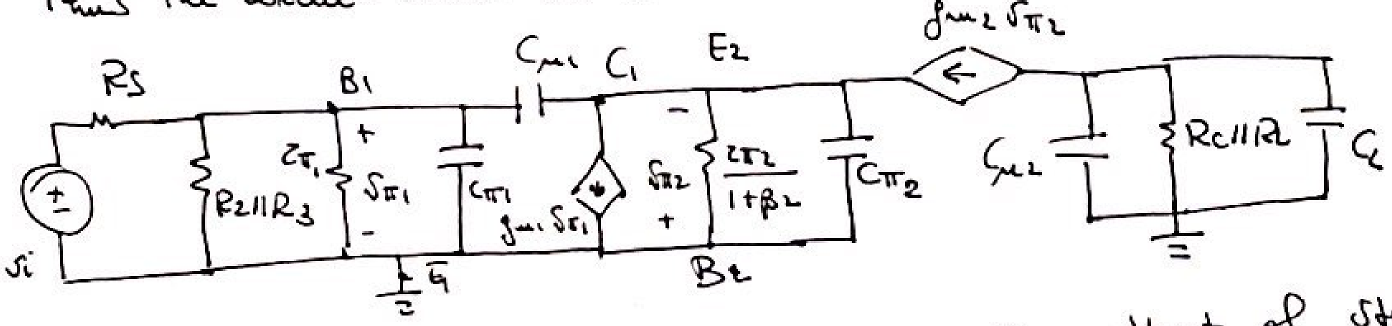
High frequency small-signal equivalent circuit



$$Z_{i2}^{-1} = \frac{i_{e2}}{v_{e2}} = \frac{1}{r_{\pi 2}} + g_{m2} + \underset{\substack{\downarrow \\ j\omega}}{s} C_{\pi 2} = \left(\frac{1 + g_{m2} r_{\pi 2}}{r_{\pi 2}} + sC_{\pi 2} \right) = \frac{1+\beta_2}{r_{\pi 2}} + sC_{\pi 2}$$

$$Z_{i2} = \frac{r_{\pi 2}}{\beta + 1} \parallel \frac{1}{sC_{\pi 2}}$$

Thus the circuit above can be reduced to



$C_{\mu 1}$ is a capacitor between the input and the output of stage 1.

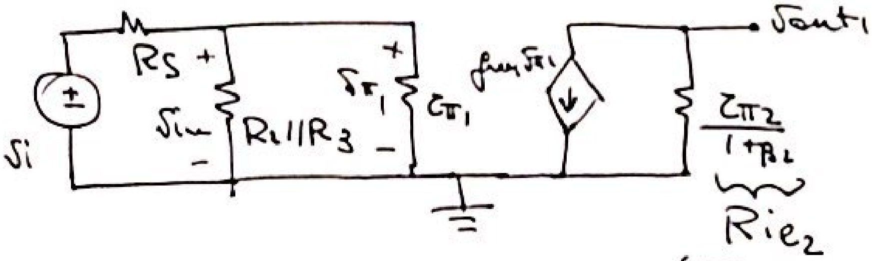
Using the Miller effect

$$C_{Mi} = C_{\mu 1} [1 - A_{midband}] \quad \text{and} \quad C_{Mo} = C_{\mu 1} \left[1 - \frac{1}{A_{midband}} \right]$$

As the gain of a CE stage is typically large we can take

$C_{Mo} \simeq C_{\mu 1}$.

To determine C_{Mi} we need to calculate the amplifier voltage gain at midband.



(Input resistance of stage 2).

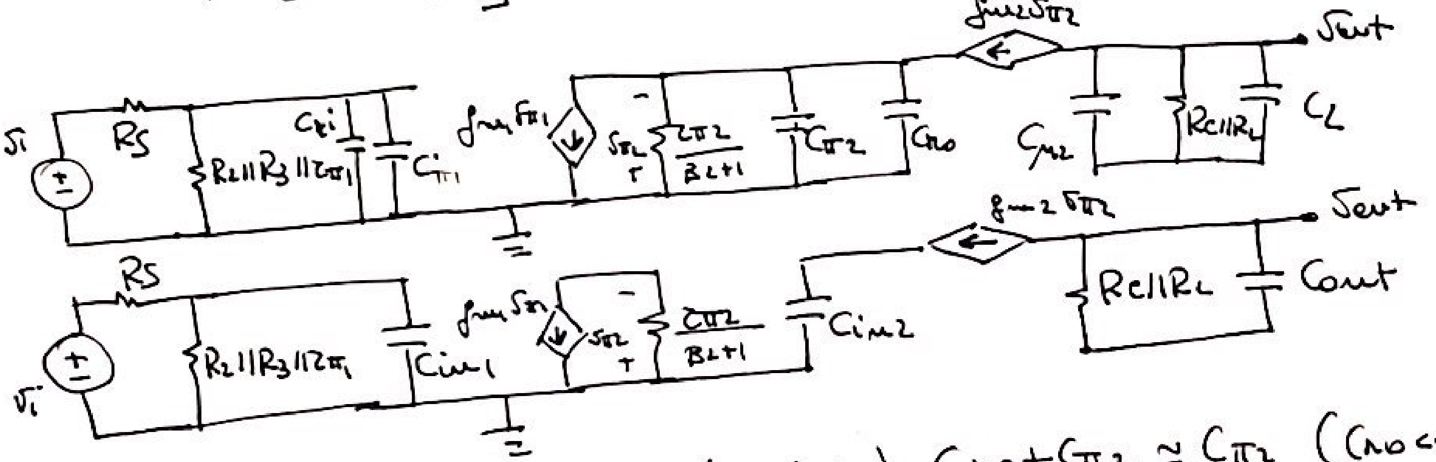
$$A_{v\,midband} = \frac{v_{out_1}}{v_{in_1}} = -g_{m_1}\frac{r_{\pi 2}}{(1+\beta_2)}$$

Thus

$$C_{Mi} = C_{\mu 1}\left[1+\frac{g_{m1}\, r_{\pi 2}}{1+\beta_2}\right]$$

. Assuming $r_{\pi 1} \simeq r_{\pi 2}$ (Reasonable assumption because Q_1 and Q_2 are biased at the same I_{CQ}).

$$C_{Mi} \simeq C_{\mu 1}\left[1+\frac{\beta_2}{1+\beta_2}\right] \simeq C_{\mu 1}\cdot 2 = 2C_{\mu}$$



$C_{in_1} = C_{\pi 1} + C_{Mi}$ $\quad$ C_{in2} (or C_{out_1}) $= C_{Mo} + C_{\pi 2} \simeq C_{\pi 2}$ $\quad$ $((C_{Mo} \ll C_{\pi 2})$

C_{out} (or C_{out_2}) $= C_L + C_{\mu 2}$

$$\tau_{in_1} = C_{in_1} R_{eq\,in_1} = C_{in_1}\cdot(R_S \| R_2 \| R_3 \| r_{\pi 1}) = (C_{\pi 1} + C_{Mi})(R_S \| R_2 \| R_3 \| r_{\pi 1})$$

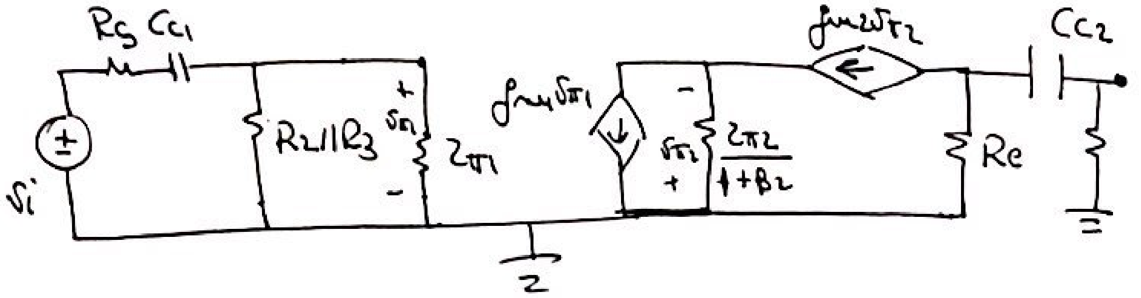
$$\tau_{in2} = C_{in2} R_{eq\,in2} \simeq C_{\pi 2}\cdot\left(\frac{r_{\pi 2}}{1+\beta_2}\right)$$

$$\tau_{out} = C_{out} R_{eq\,out} \simeq C_L (R_C \| R_L)$$

For typical values of capacitors and resistors in the given circuit, the dominant time constant is τ_{out} (the highest of the 3).

Thus $f_H \simeq \frac{1}{2\pi \tau_{out}} \simeq \frac{1}{2\pi (C_L + C_{\mu 2})(R_e \| R_L)}$

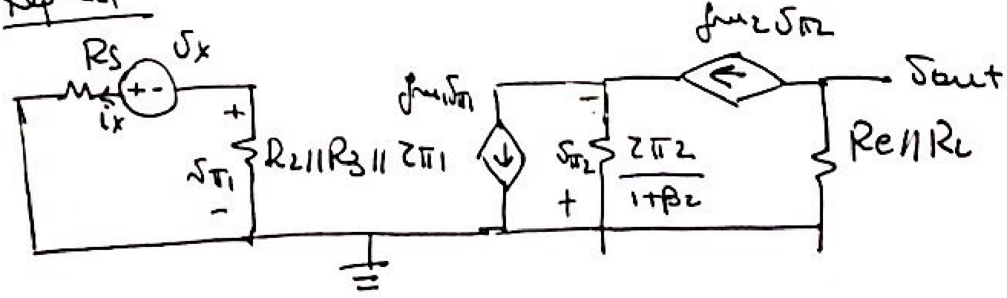
Assuming C_B and $C_E >> C_{C1}$ and C_{C2}, we can determine f_L from the circuit below



$f_L \simeq \sum_i^N \frac{1}{R_{eq_i} C_i}$ $\quad C_i$: C_{C2}, C_{C1} $\quad N = 2$

$f_L \simeq \frac{1}{R_{eq C_{C2}} C_{C2}} + \frac{1}{R_{eq C_{C1}} C_{C1}}$

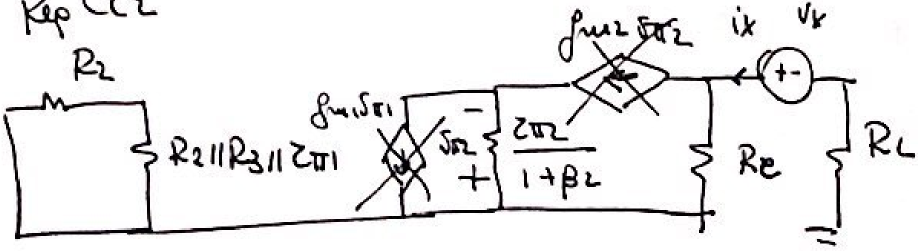
Rep Cc1



$R_{eq C_{C1}} = \frac{v_x}{i_x}$

$R_{eq C_{C1}} = R_S + R_2 \| R_3 \| r_{\pi 1}$

Rep CC2



$R_{eq\, CC1} = \frac{v_x}{i_x} = R_e \| R_L$

$v_{\pi 1} = 0 \Rightarrow g_m v_{\pi 1} = 0$

$v_{\pi 2} = 0 \Rightarrow g_m v_{\pi 2} = 0$

$$f_L \simeq \frac{1}{2\pi\left(R_S + R_2 \| R_3 \| r_{\pi 1}\right) C_{c1}} + \frac{1}{2\pi C_{c2}\left(R_C \| R_L\right)} \simeq$$

$$\simeq \frac{1}{2\pi\left(R_S + r_{\pi}\right) C_{c1}} + \frac{1}{2\pi C_{c2}\left(R_C \| R_L\right)}$$

Assuming $C_{c1} \simeq C_{c2}$ $\quad f_L \simeq \dfrac{1}{2\pi\left(R_S + r_{\pi 1}\right) C_{c1}}$ $\quad$ because $(R_S + r_{\pi 1}) \ll R_C \| R_L$ typically

Thus

$$BW = f_H - f_L \simeq \frac{1}{2\pi\left(C_L + C_{\mu 2}\right)\left(R_C \| R_L\right)} - \frac{1}{2\pi\left(R_S + r_{\pi 1}\right) C_{c1}}$$

AS $f_H \gg f_L$ the $BW \simeq f_H$ (Note: Don't panic if you had made this assumption to begin with and just calculated f_H. You will receive full credit for your work).