

ECE 345 / ME 380: Introduction to Control Systems

Problem Set #3

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Due Thursday, November 5, 2020 at 3:30pm

1. (+10 points) Consider a negative unity feedback system as in Figure 1 with $G(s) = \frac{(s+1)(s+2)}{s^2(s^2+2s+3)}$.

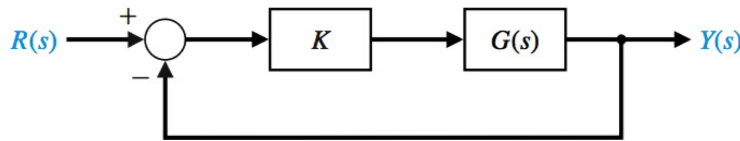


Figure 1: Negative unity feedback system.

- (a) What is the type number of the closed-loop system $\frac{Y(s)}{R(s)}$?

Type 2

- (b) What finite values of K , if any, will yield a steady-state error less than or equal to 0.1, in response to a unit step input?

$$e_{ss} = \frac{1}{1 + K_p} \qquad K_p = \lim_{s \rightarrow 0} KG(s)$$

$$= \lim_{s \rightarrow 0} K \cdot \frac{(s+1)(s+2)}{s^2(s^2+2s+3)}$$

$$= \infty$$

$$e_{ss} = 0$$

e_{ss} is 0, \therefore all values of K will meet this constraint.

- (c) What finite values of K , if any, will yield a steady-state error less than or equal to 0.1, in response to a unit ramp input?

$$e_{ss} = \frac{1}{K_v} \qquad K_v = \lim_{s \rightarrow 0} sKG(s)$$

$$= \lim_{s \rightarrow 0} s \cdot K \cdot \frac{(s+1)(s+2)}{s^2(s^2+2s+3)}$$

$$= \infty$$

$$e_{ss} = 0$$

e_{ss} is 0, \therefore all values of K will meet this constraint.

- (d) What finite values of K , if any, will yield a steady-state error less than or equal to 0.1, in response to a unit parabolic input?

$$e_{ss} = \frac{1}{K_a}$$

$$K_a = \lim_{s \rightarrow 0} s^2 K G(s)$$

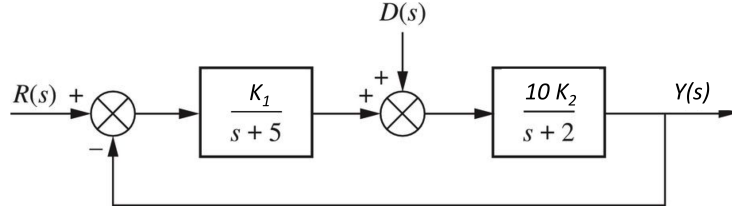
$$= \lim_{s \rightarrow 0} s^2 \cdot K \cdot \frac{(s+1)(s+2)}{s^2(s^2+2s+3)}$$

$$= \frac{2K}{3}$$

$$e_{ss} = \frac{3}{2K} \leq 0.1$$

$$K \geq 15$$

2. (+15 points) Consider the following system, with reference input $r(t)$ and disturbance input $d(t)$.



- (a) Find the transfer function $G_R(s)$ with output $Y(s)$ and input $R(s)$, and the transfer function $G_D(s)$ with output $Y(s)$ and input $D(s)$, such that $Y(s) = G_R(s)R(s) + G_D(s)D(s)$.

$$\text{Let } G_1(s) = \frac{K_1}{s+5} \text{ (our controller) and } G_2(s) = \frac{10K_2}{s+2} \text{ (our plant).}$$

$$G_R(s) = \frac{G_1(s)G_2(s)}{1 + G_1(s)G_2(s)}$$

$$= \frac{10K_1K_2}{(s+5)(s+2) + 10K_1K_2}$$

$$= \frac{10K_1K_2}{s^2 + 7s + 10 + 10K_1K_2}$$

$$G_D(s) = \frac{G_2(s)}{1 + G_1(s)G_2(s)}$$

$$= \frac{10K_2(s+5)}{(s+5)(s+2) + 10K_1K_2}$$

$$= \frac{10K_2(s+5)}{s^2 + 7s + 10 + 10K_1K_2}$$

- (b) Describe the relationship between the characteristic equation of $G_R(s)$ and the characteristic equation of $G_D(s)$.

$$\text{The characteristic equations are the same: } \Delta_R(s) = \Delta_D(s).$$

- (c) Do $K_1 = 250$ and $K_2 = \frac{1}{10}$ meet the following specifications? Why or why not?

- i. The steady-state *output response* due to a unit step disturbance input is 0.02.

$$y_{ss} \leq 0.02 \text{ due to } D(s) = \frac{1}{s} \text{ and pressure } R(s) = 0.$$

$$Y(s) = \frac{10K_2(s+5)}{s^2 + 7s + 10 + 10K_1K_2} \cdot D(s)$$

$$= \frac{s+5}{s^2 + 7s + 260} \cdot \frac{1}{s}$$

F.V.T.

$$\begin{aligned}
 y_{ss} &= \lim_{s \rightarrow 0} sY(s) \\
 &= \lim_{s \rightarrow 0} s \left(\frac{s+5}{s^2+7s+260} \cdot \frac{1}{s} \right) \\
 &= \frac{1}{52} = 0.01923 \leq 0.02 \quad \checkmark
 \end{aligned}$$

ii. The steady-state error due to a unit step reference input is 0.05.

$$e_{ss} \leq 0.05 \text{ for } R(s) = \frac{1}{s} \text{ and pressure } D(s) = 0.$$

$$\frac{Y(s)}{R(s)} = \frac{10K_1K_2}{(s+5)(s+2) + 10K_1K_2} \quad \text{Type 0 system}$$

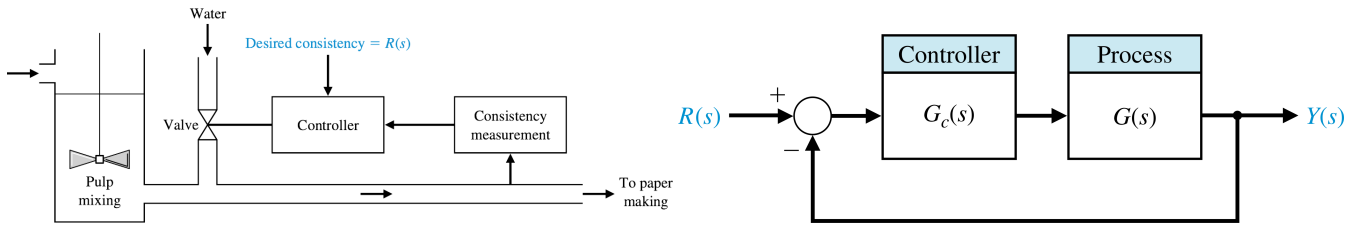
$$e_{ss} = \frac{1}{1 + K_p}$$

$$\begin{aligned}
 K_p &= \lim_{s \rightarrow 0} KG(s) \\
 &= \lim_{s \rightarrow 0} G_1(s)G_2(s) \\
 &= \lim_{s \rightarrow 0} \left(\frac{K_1}{s+5} \right) \left(\frac{10K_2}{s+2} \right) \\
 &= K_1K_2 \\
 &= 25
 \end{aligned}$$

$$e_{ss} = \frac{1}{26} = 0.03846 \leq 0.05 \quad \checkmark$$

Yes, these values of K_1 & K_2 meet the specifications.

3. (+15 points) Pulp dilution is an important part of the paper-making process. We model the dynamics of pulp dilution with plant $G(s) = \frac{s+2}{s^2+2s+3}$.



(a) Consider the controller $G_c(s) = \frac{K}{s+1}$. What is the type number of the closed-loop system $\frac{Y(s)}{R(s)}$?

Type 0

(b) What value of $K > 0$ will ensure that the steady-state error in response to a unit step input is

at most 0.01?

$$e_{ss} = \frac{1}{1 + K_p}$$

$$\begin{aligned} K_p &= \lim_{s \rightarrow 0} KG(s) \\ &= \lim_{s \rightarrow 0} G_1(s)G_2(s) \\ &= \lim_{s \rightarrow 0} \left(\frac{K_1}{s+5} \right) \left(\frac{10K_2}{s+2} \right) \\ &= K_1K_2 \\ &= 25 \end{aligned}$$

$$e_{ss} = \frac{1}{26} = 0.03846 \leq 0.05 \quad \checkmark$$

- (c) Now consider the controller $G_c(s) = \frac{K}{s(s+1)}$. What is the type number of the closed-loop system $\frac{Y(s)}{R(s)}$?
- (d) What value of $K > 0$ will ensure that the steady-state error in response to a unit step input is at most 0.01?
- (e) Consider the gain $K = 150$. Which controller has the best steady-state performance? Why?