ECE 371
Materials and Devices
HW #4
Due: Tuesday 10/15/19 at the beginning of class

*All problems from Neamen 4th Edition Ch. 3. Figures on next page.

3.15 The E versus k diagram for a particular allowed energy band is shown in Figure P3.15. Determine (*a*) the sign of the effective mass and (*b*) the direction of velocity for a particle at each of the four positions shown.

3.16 Figure P3.16 shows the parabolic E versus k relationship in the conduction band for an electron in two particular semiconductor materials. Determine the effective mass (in units of the free electron mass) of the two electrons.

3.20 The energy-band diagram for silicon is shown in Figure 3.25b. The minimum energy in the conduction band is in the [100] direction. The energy in this one-dimensional direction near the minimum value can be approximated by

$$E = E_0 - E_1 \cos \alpha(k - k_0)$$

where k_0 is the value of k at the minimum energy. Determine the effective mass of the particle at $k = k_0$ in terms of the equation parameters.

3.25 Derive the density of states function for a one-dimensional electron gas in GaAs ($m_n^* = 0.067m_0$). Note that the kinetic energy may be written as $E = (\pm\, p)^2/2m_n^*$, which means that there are two momentum states for each energy level.

3.26 (*a*) Determine the total number (#/cm^3) of energy states in silicon between E_c and $E_c + 2kT$ at (*i*) $T = 300$ K and (*ii*) $T = 400$ K. (*b*) Repeat part (*a*) for GaAs.

3.28 (*a*) Plot the density of states in the conduction band of silicon over the range $E_c < E < E_c + 0.4$ eV. (*b*) Repeat part (*a*) for the density of states in the valence band over the range $E_v - 0.4 \text{ eV} < E < E_v$.

3.30 Plot the Fermi–Dirac probability function, given by Equation (3.79), over the range $-0.2 \le (E - E_F) \le 0.2$ eV for (*a*) $T = 200$ K, (*b*) $T = 300$ K, and (*c*) $T = 400$ K.

3.32 Determine the probability that an energy level is occupied by an electron if the state is above the Fermi level by (*a*) kT, (*b*) $5kT$, and (*c*) $10kT$.

3.33 Determine the probability that an energy level is empty of an electron if the state is below the Fermi level by (*a*) kT, (*b*) $5kT$, and (*c*) $10kT$.

3.39 (*a*) Determine for what energy above E_F (in terms of kT) the Fermi–Dirac probability function is within 1 percent of the Boltzmann approximation. (*b*) Give the value of the probability function at this energy.

3.42 Consider the energy levels shown in Figure P3.42. Let $T = 300$ K. (*a*) If $E_1 - E_F = 0.30$ eV, determine the probability that an energy state at $E = E_1$ is occupied by an electron and the probability that an energy state at $E = E_2$ is empty. (*b*) Repeat part (*a*) if $E_F - E_2 = 0.40$ eV.

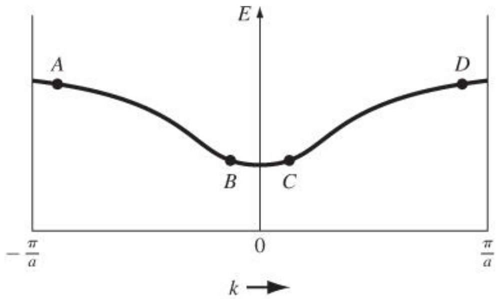


Figure P3.15 | Figure for Problem 3.15.

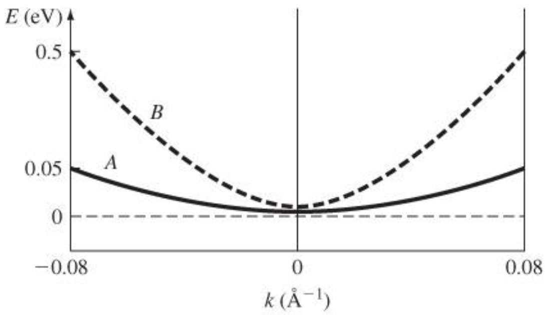


Figure P3.16 | Figure for Problem 3.16.

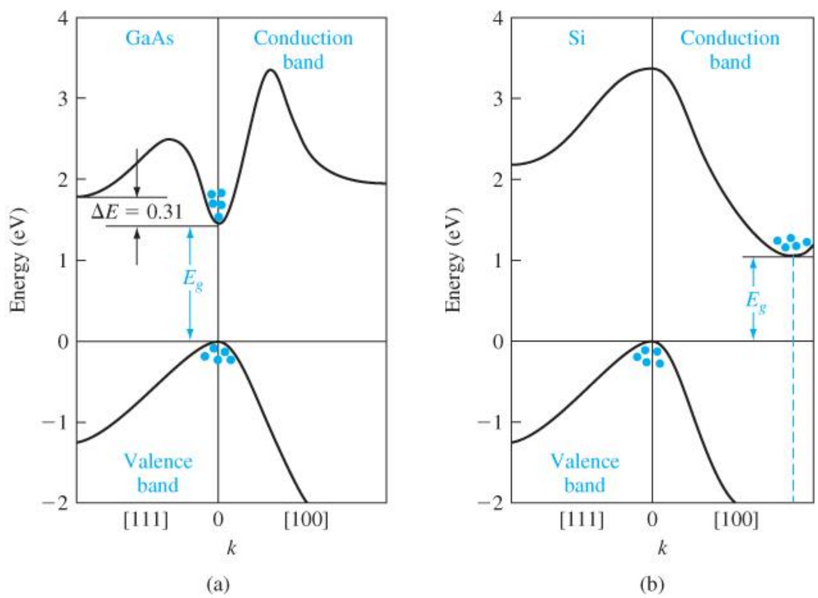


Figure 3.25 | Energy-band structures of (a) GaAs and (b) Si.
(From Sze [12].)

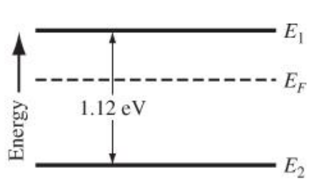


Figure P3.42 | Energy levels
for Problem 3.42.

3.15 The E versus k diagram for a particular allowed energy band is shown in Figure P3.15. Determine (*a*) the sign of the effective mass and (*b*) the direction of velocity for a particle at each of the four positions shown.

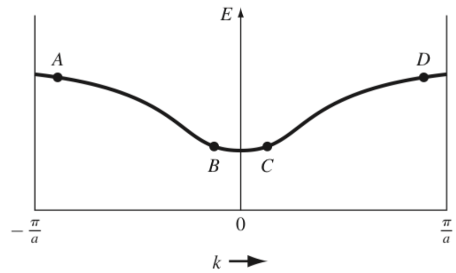


Figure P3.15 | Figure for Problem 3.15.

$\frac{\partial E}{\partial k} \rightarrow$ slope

$\frac{\partial^2 E}{\partial k^2} \rightarrow$ curvature

Particle A:

(a) $\frac{\partial^2 E}{d k^2}$ negative $\therefore$ negative eff. mass

(b) $\frac{\partial E}{d k}$ negative $\therefore$ negative velocity

Particle B:

(a) $\frac{\partial^2 E}{d k^2}$ positive $\therefore$ positive eff. mass

(b) $\frac{\partial E}{d k}$ negative $\therefore$ negative velocity

Particle C:

(a) $\frac{\partial^2 E}{d k^2}$ positive $\therefore$ positive eff. mass

(b) $\frac{\partial E}{d k}$ positive $\therefore$ positive velocity

Particle D:

(a) $\frac{\partial^2 E}{d k^2}$ negative $\therefore$ negative eff. mass

(b) $\frac{\partial E}{d k}$ positive $\therefore$ positive velocity

3.16 Figure P3.16 shows the parabolic E versus k relationship in the conduction band for an electron in two particular semiconductor materials. Determine the effective mass (in units of the free electron mass) of the two electrons.

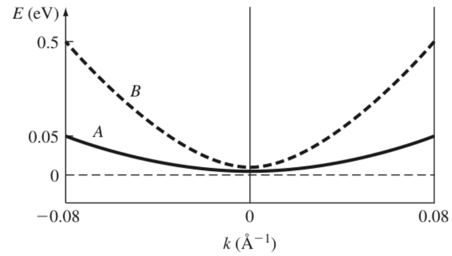


Figure P3.16 | Figure for Problem 3.16.

$$k = 0.08\,\text{Å}^{-1} \longrightarrow \frac{1}{k} = 12.5\,\text{Å} = 12.5\times10^{-10}\,m$$

$$k = 8\times10^{8}\,m^{-1}$$

Material A:

$$E - E_c = \frac{k^2\hbar^2}{2m} \longrightarrow m = \frac{k^2\hbar^2}{2(E-E_c)q} = \frac{(8\times10^8)^2\hbar^2}{2(0.05)q} = 4.442\times10^{-31}\,kg$$

$$\frac{m}{m_o} = \boxed{0.4877}$$

Material B:

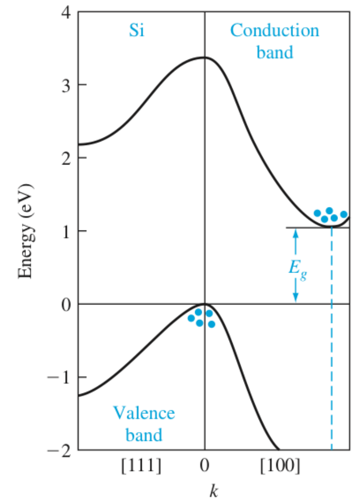
$$E - E_c = \frac{k^2\hbar^2}{2m} \longrightarrow m = \frac{k^2\hbar^2}{2(E-E_c)q} = \frac{(8\times10^8)^2\hbar^2}{2(0.5)q} = 4.442\times10^{-32}\,kg$$

$$\frac{m}{m_o} = \boxed{0.04877}$$

3.20 The energy-band diagram for silicon is shown in Figure 3.25b. The minimum energy in the conduction band is in the [100] direction. The energy in this one-dimensional direction near the minimum value can be approximated by

$$E = E_0 - E_1 \cos \alpha(k - k_0)$$

where k_0 is the value of k at the minimum energy. Determine the effective mass of the particle at $k = k_0$ in terms of the equation parameters.



Indirect bandgap

$$E = E_0 - E_1 \cos(\alpha(k - k_0))$$

$$\frac{dE}{dk} = E_1 \alpha \sin(\alpha(k - k_0))$$

$$\frac{d^2E}{dk} = E_1 \alpha^2 \cos(\alpha(k - k_0))$$

$$\text{eff mass } (k = k_0) = E_1 \alpha^2 \cancel{\cos(\alpha(k - k_0))}^{\,1} \Big|_{k = k_0}$$

$$= E_1 \alpha^2$$

$$\frac{1}{\hbar^2}\frac{d^2E}{dk^2} = \frac{-2C_2}{\hbar^2} = \frac{1}{m^*} \qquad (3.56)$$

$$\boxed{m^* = \frac{\hbar^2}{E_1 \alpha^2}}$$

3.25 Derive the density of states function for a one-dimensional electron gas in GaAs ($m_n^* = 0.067m_0$). Note that the kinetic energy may be written as $E = (\pm\, p)^2/2m_n^*$, which means that there are two momentum states for each energy level.

$$V_s = \frac{\pi}{a} \qquad g(E) = \frac{1}{V} \cdot \frac{dN}{dE} \qquad V_{line} = k = \sqrt{\frac{2m_n^* E}{\hbar^2}}$$

$$N = \frac{V_{line}}{V_s} = \frac{a\sqrt{\frac{2m_n^* E}{\hbar^2}}}{\pi} \rightarrow \frac{dN}{dE} = \frac{m_n^*}{\pi\hbar\sqrt{2m_n^* E}}$$

$$g(E) = \frac{1}{\pi\hbar}\,\frac{\sqrt{2m_n^*}}{\sqrt{E}} = \boxed{\frac{1.055 \times 10^{18}}{\sqrt{E}}}$$

3.26 (*a*) Determine the total number (#/cm³) of energy states in silicon between E_c and $E_c + 2kT$ at (*i*) $T = 300$ K and (*ii*) $T = 400$ K. (*b*) Repeat part (*a*) for GaAs.

from Appendix table B.4:

$m_n^*(Si) = 1.08$

$m_n^*(GaAs) = 0.067$

(a)

(i)

$$N = \int_{E_c}^{E_c+2kT} \frac{4\pi(2m_n^*)^{3/2}}{h^3}\sqrt{E-E_c}\,dE$$

$$= \frac{4\pi(2m_n^*)^{3/2}}{h^3}\cdot\frac{2}{3}\cdot(E-E_c)^{3/2}\Big|_{E_c}^{E_c+2kT}$$

$$= \frac{4\pi(2m_n^*)^{3/2}}{h^3}\cdot\frac{2}{3}\cdot(2kT)^{3/2}$$

$$= \frac{4\pi\left[(2(1.08)(9.11\times10^{-31}))\right]^{3/2}}{(6.625\times10^{-34})^3}\cdot\frac{2}{3}\cdot\left[2\,(1.38\times10^{-23})(300)\right]^{3/2}$$

$$= 5.993\times10^{25}\ m^{-3} = \boxed{5.993\times10^{19}\ cm^{-3}}$$

(ii)

$$N = \frac{4\pi\left[(2(1.08)(9.11\times10^{-31}))\right]^{3/2}}{(6.625\times10^{-34})^3}\cdot\frac{2}{3}\cdot\left[2(1.38\times10^{-23})(400)\right]^{3/2}$$

$$= 9.226\times10^{25}\ m^{-3} = \boxed{9.226\times10^{19}\ cm^{-3}}$$

(b)

(i)

$$N = \frac{4\pi\left[\left(2(0.067)(9.11\times10^{-31})\right)\right]^{3/2}}{(6.625\times10^{-34})^3}\cdot\frac{2}{3}\cdot\left[2(1.38\times10^{-23})(300)\right]^{3/2}$$

$$= 9.2596\times10^{23}\ m^{-3} = \boxed{9.2596\times10^{17}\ cm^{-3}}$$

(ii)

$$N = \frac{4\pi\left[\left(2(0.067)(9.11\times10^{-31})\right)\right]^{3/2}}{(6.625\times10^{-34})^3}\cdot\frac{2}{3}\cdot\left[2(1.38\times10^{-23})(400)\right]^{3/2}$$

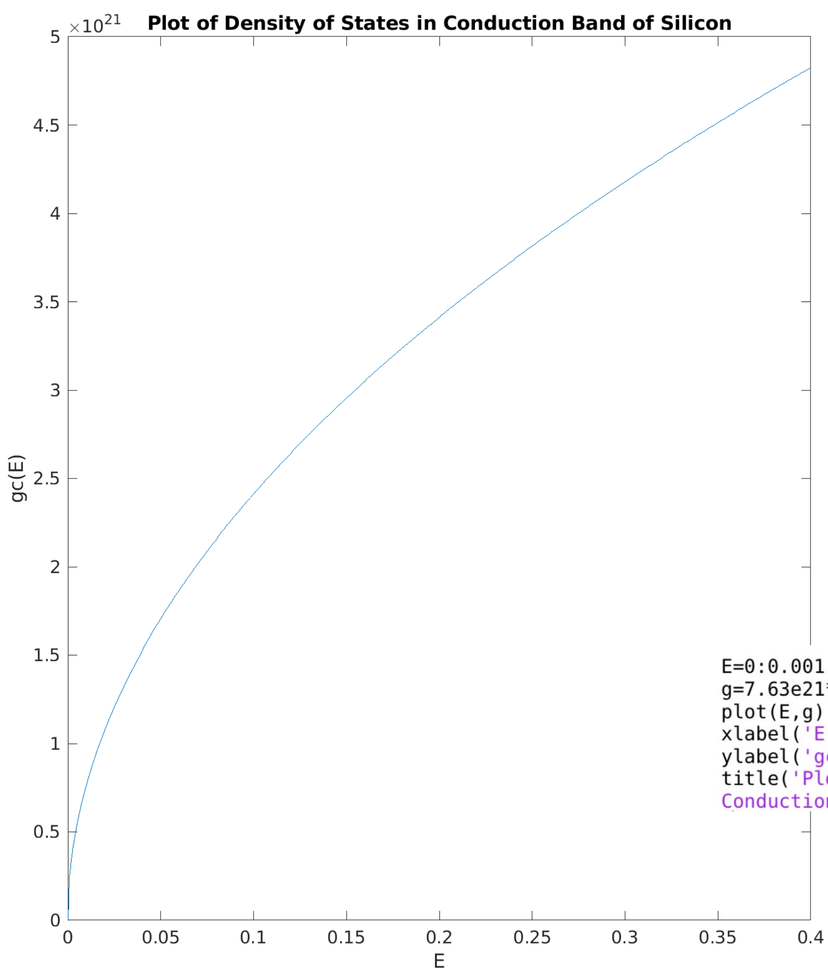
$$= 1.4256\times10^{24}\ m^{-3} = \boxed{1.4256\times10^{18}\ cm^{-3}}$$

3.28 (*a*) Plot the density of states in the conduction band of silicon over the range $E_c < E < E_c + 0.4$ eV. (*b*) Repeat part (*a*) for the density of states in the valence band over the range $E_v - 0.4 \text{ eV} < E < E_v$.

(a) $$g_c(E) = \frac{4\pi (2m_n^*)^{3/2}}{h^3}\sqrt{E-E_c}$$

$$= \frac{4\pi\left[2(1.08)(9.11\times10^{-31})\right]^{3/2}}{(6.625\times10^{-34})^3}\sqrt{E-E_c} \rightarrow 0 < E < 0.4$$

$$= 7.63\times10^{21}\sqrt{E-E_c}$$

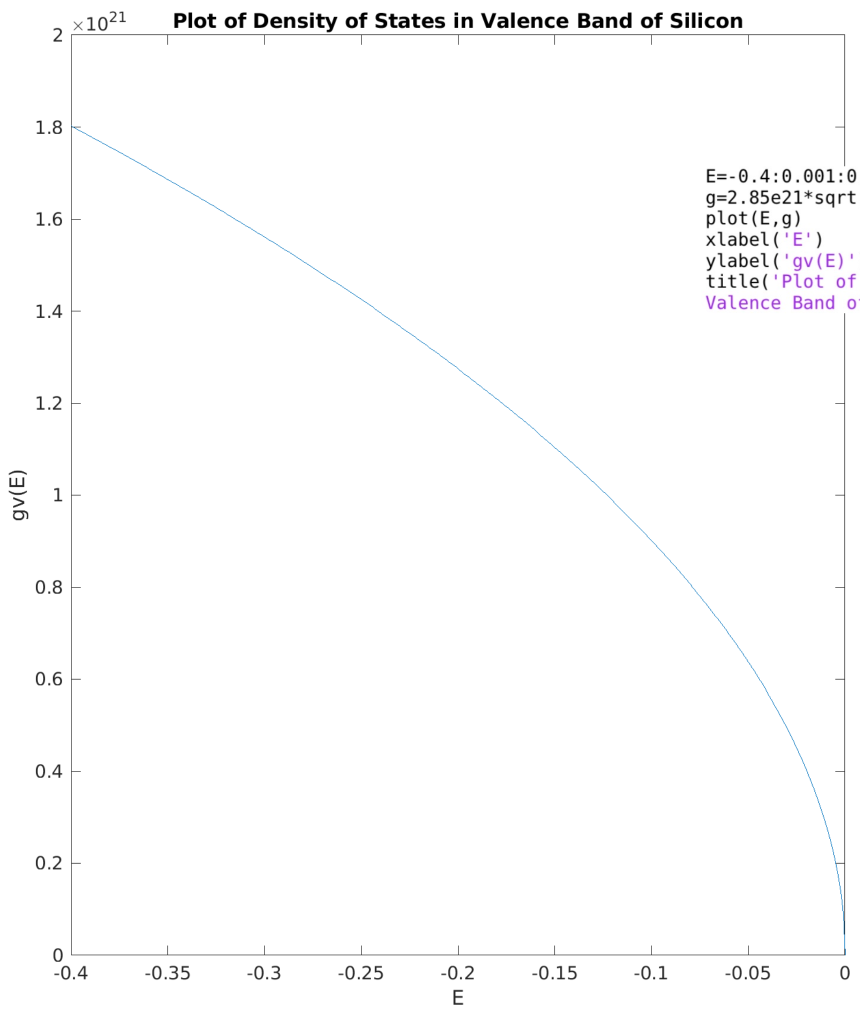



```
E=0:0.001:0.4;
g=7.63e21*sqrt(E)
plot(E,g)
xlabel('E')
ylabel('gc(E)')
title('Plot of Density of States in Conduction Band of Silicon')
```

(b) $g_v(E) = \dfrac{4\pi(2m_p^*)^{3/2}}{h^3}\sqrt{E_v - E}$

$= \dfrac{4\pi\left[2(0.56)(9.11\times10^{-31})\right]^{3/2}}{(6.625\times10^{-34})^3}\sqrt{E_v - E}$ $\rightarrow -0.4 < E < 0$

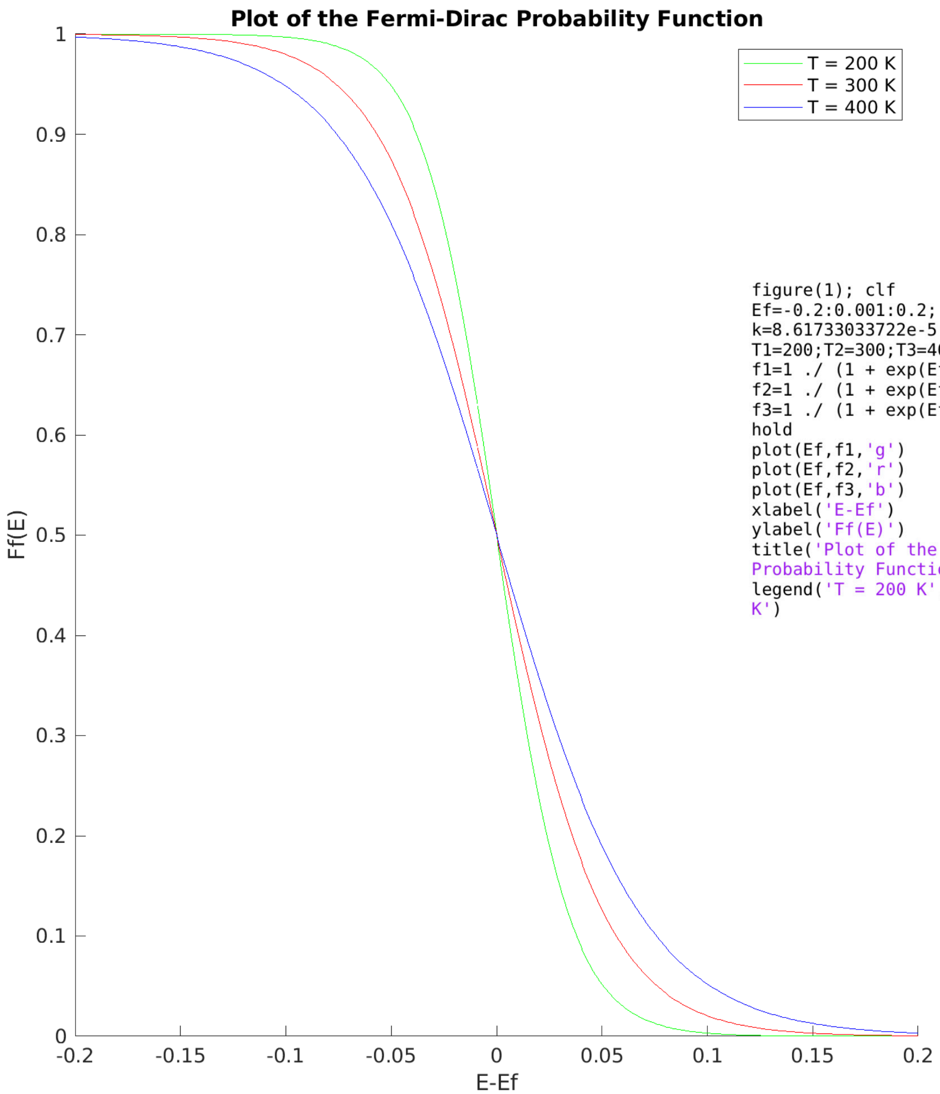
$= 2.85\times10^{21}\sqrt{E_v - E}$




```
E=-0.4:0.001:0;
g=2.85e21*sqrt(-E)
plot(E,g)
xlabel('E')
ylabel('gv(E)')
title('Plot of Density of States in
Valence Band of Silicon')
```

3.30 Plot the Fermi–Dirac probability function, given by Equation (3.79), over the range $-0.2 \le (E - E_F) \le 0.2$ eV for (*a*) $T = 200$ K, (*b*) $T = 300$ K, and (*c*) $T = 400$ K.

$$\boxed{\frac{N(E)}{g(E)} = f_F(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)}} \tag{3.79}$$




```
figure(1); clf
Ef=-0.2:0.001:0.2;
k=8.61733033722e-5;
T1=200;T2=300;T3=400;
f1=1 ./ (1 + exp(Ef ./ (k * T1)));
f2=1 ./ (1 + exp(Ef ./ (k * T2)));
f3=1 ./ (1 + exp(Ef ./ (k * T3)));
hold
plot(Ef,f1,'g')
plot(Ef,f2,'r')
plot(Ef,f3,'b')
xlabel('E-Ef')
ylabel('Ff(E)')
title('Plot of the Fermi-Dirac
Probability Function')
legend('T = 200 K','T = 300 K','T = 400
K')
```

3.32 Determine the probability that an energy level is occupied by an electron if the state is above the Fermi level by (*a*) kT, (*b*) $5kT$, and (*c*) $10kT$.

$$f(E) = \frac{1}{1+\exp\left(\frac{E-E_F}{kT}\right)}$$

(a) $$f(E) = \frac{1}{1+\exp\left(\frac{kT}{kT}\right)} = \boxed{0.2689}$$

(b) $$f(E) = \frac{1}{1+\exp\left(\frac{5kT}{kT}\right)} = \boxed{6.6929\times10^{-3}}$$

(c) $$f(E) = \frac{1}{1+\exp\left(\frac{10kT}{kT}\right)} = \boxed{4.5398\times10^{-5}}$$

3.33 Determine the probability that an energy level is empty of an electron if the state is below the Fermi level by (*a*) kT, (*b*) $5kT$, and (*c*) $10kT$.

$$1 - f(E) = 1 - \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)} = \frac{1}{1 + \exp\left(\frac{E_F - E}{kT}\right)}$$

(a) $1 - f(E) = \boxed{0.2689}$

(b) $1 - f(E) = \boxed{6.6929 \times 10^{-3}}$

(c) $1 - f(E) = \boxed{4.5398 \times 10^{-5}}$

3.39 (*a*) Determine for what energy above E_F (in terms of kT) the Fermi–Dirac probability function is within 1 percent of the Boltzmann approximation. (*b*) Give the value of the probability function at this energy.

(a) $$\frac{1+\exp\left(\frac{E-E_F}{kT}\right)}{\exp\left(\frac{E-E_F}{kT}\right)} - 1 = 0.01$$

$$E = E_F + kT\ln(100) = \boxed{E_F + 4.6052\,kT}$$

(b) $$f(E) = \frac{1}{1+\exp\left(\frac{E-E_F}{kT}\right)} = \frac{1}{1+\exp\left(\frac{4.6052\cancel{kT}}{\cancel{kT}}\right)}$$

$$= \boxed{9.9001\times10^{-3}}$$

3.42 Consider the energy levels shown in Figure P3.42. Let $T = 300$ K. (*a*) If $E_1 - E_F = 0.30$ eV, determine the probability that an energy state at $E = E_1$ is occupied by an electron and the probability that an energy state at $E = E_2$ is empty. (*b*) Repeat part (*a*) if $E_F - E_2 = 0.40$ eV.

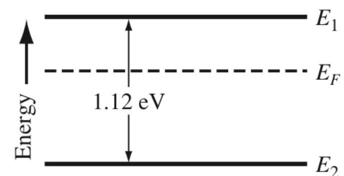


Figure P3.42 | Energy levels for Problem 3.42.

(a) $$f(E) = \frac{1}{1+\exp\left(\frac{E_1 - E_F}{kT}\right)}$$

$$= \frac{1}{1+\exp(0.3/0.0259)} = \boxed{9.323 \times 10^{-6}}$$

prob that occupied

$$1 - f(E) = 1 - \frac{1}{1+\exp\left(\frac{E_2 - E_F}{kT}\right)} = \frac{1}{1+\exp\left(\frac{E_F - E_2}{kT}\right)}$$

$$= \frac{1}{1+\exp(-0.82/0.0259)} = \boxed{1.7788 \times 10^{-14}}$$

prob. that empty

(b) $E_F - E_2 = 0.4$

$$f(E) = \frac{1}{1+\exp\left(\frac{E_1 - E_F}{kT}\right)} = \frac{1}{1+\exp(-0.72/0.0259)}$$

$$= \boxed{8.4518 \times 10^{-13}}$$

$$1 - f(E) = 1 - \frac{1}{1+\exp(-0.4/0.0259)} = \boxed{1.9622 \times 10^{-7}}$$