PROBLEM 1

$v_{out\_ac} = G_V \cdot v_{in\_ac}$

$v_{out\_dc} = v_{out\_ac} + V_C$

↳ COLLECTOR VOLTAGE OR VOLTAGE BETWEEN THE COLLECTOR AND GROUND.

$G_V = ?$

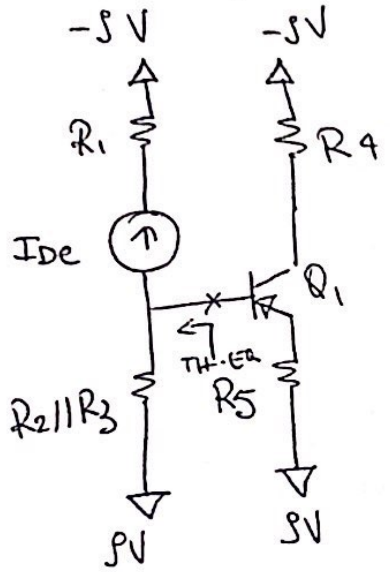
In order to calculate G_V we need to determine the small-signal parameters r_o, r_π, and g_m..

$r_\pi = \dfrac{\beta}{g_m}$ $\quad g_m = \dfrac{I_C}{V_t}$ $\quad V_t = 0.02585\,V\ (300K)$

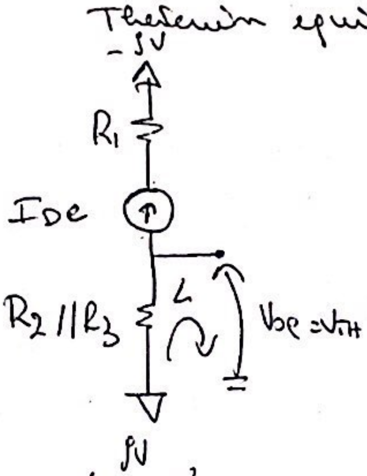
$r_o = \dfrac{V_A + V_{EC}}{I_C}$

Therefore, we need to determine the Q point of the transistor.

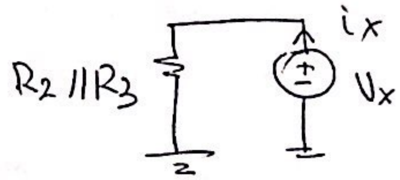
DC analysis



It is convenient to reduce the linear circuit seen from the base of Q1 to its Thevenin equivalent circuit.



$V_{TH} = ?$



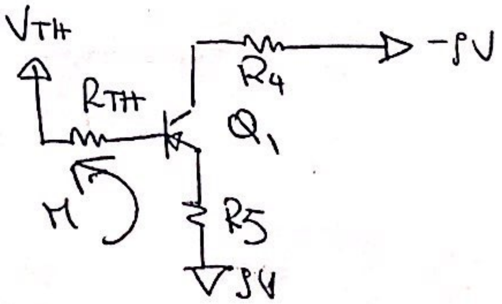
$R_{TH} = \dfrac{V_x}{i_x} = R_2 // R_3$

$R_{TH} = ?$

KVL @ L:

$V_{TH} + (R_2//R_3)\,I_{DC} - 5 = 0$

$V_{TH} = 7\,V$



(1)

(2)

KVL @ H:

$7\,V + I_B \cdot 10K + 0.64 + I_E \cdot 1.5K - 9 = 0$

Assuming operation in the active region, we can write

$I_E = (\beta+1) I_B$; $I_C = \beta I_B$; $I_C = \frac{\beta}{\beta+1} I_E$

Then the KVL @ H can be written in terms of I_B only:

$I_B \cdot [10k + (201)\,1.5K] = 2 - 0.64$

$I_B = 4.36\,\mu A$; $I_C \simeq I_E = 873\,\mu A$.

In order to verify the assumption of active mode operation, we need to calculate the voltages on the base, on the emitter and on the collector. Indeed, a pnp transistor is in active mode if the E-B junction is forward biased (FB) and the B-C junction is reversed biased (RB).

For that to happen

$V_{EB} \geq V_{EB(on)} = 0.64$

&

$V_{CB} < V_{CB(on)} \simeq 0.4 - 0.5\,V$ (Typical values)

$V_C = -9 + 10K\, I_C = -0.27\,V$

$V_B = 7V + I_B \cdot 10K = 7.044V$

$V_E = 9V - I_E \cdot 1.5K = 7.68V$

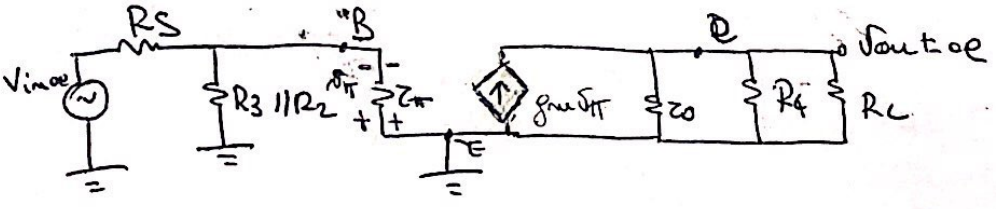
All these equations are obtained using KVL analysis.

$V_{CB} = -7.314V$ CB junction is reverse biased

$V_{EB} = 0.64\,V$ E-B junction is forward bias

$\Rightarrow$ THE ASSUMPTION OF ACTIVE MODE CONDITION IS VERIFIED.

AC (Small-signal analysis)

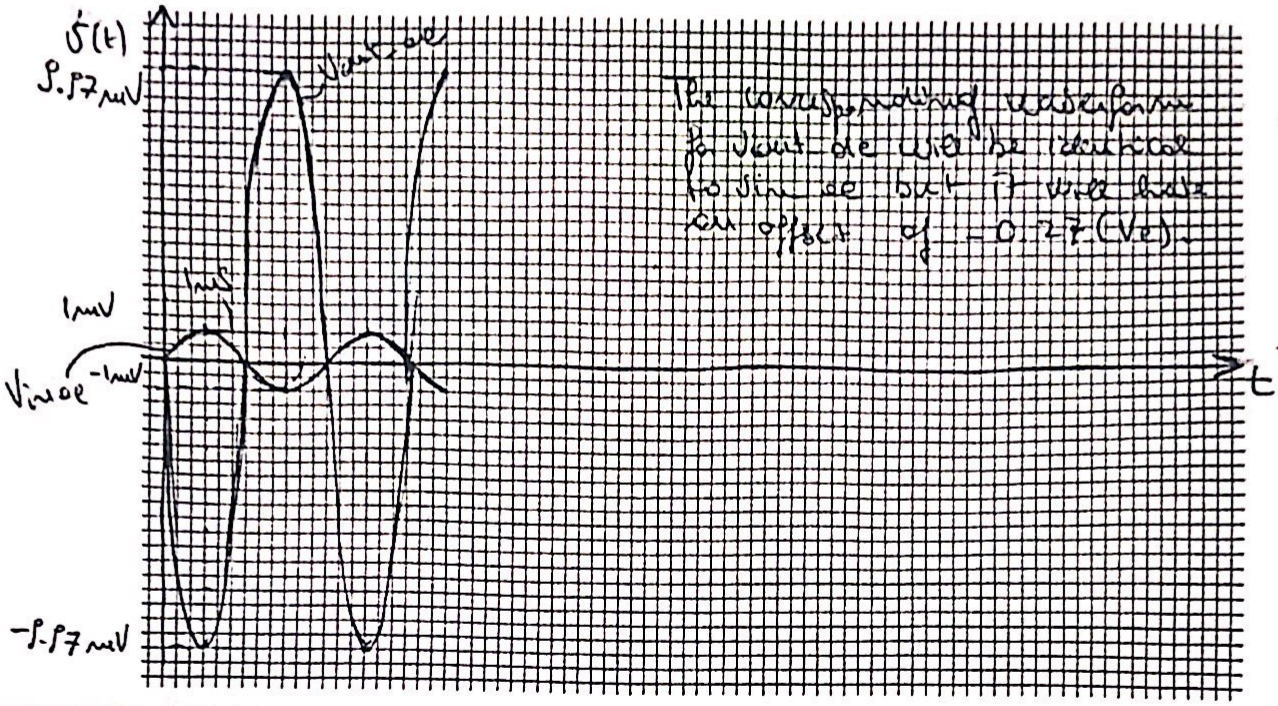


$g_m = \frac{I_C}{V_T} = \frac{873\,\mu A}{0.02585\,V} = 0.03375\,\frac{A}{V}$ $\quad r_\pi = \frac{\beta}{g_m} = \frac{200}{0.00337} = 5.5\,K\Omega$ $\quad r_o = \frac{V_A + V_{EE}}{V_T} = \frac{100 + 7.55}{0.00258} = 123\,K\Omega$

$$G_V = \frac{v_{out\text{-}ac}}{v_{in\text{-}ac}} = \frac{v_{out\text{-}ac}}{v_\pi} \cdot \frac{v_\pi}{v_{in}}$$

$$v_\pi = v_{in}\left(\frac{r_\pi // R_2 // R_3}{R_S + r_\pi // R_2 // R_3}\right) \qquad v_{out\text{-}ac} = -g_m v_\pi \left(r_o // R_4 // R_L\right)$$

$$G_V = -5.57 \text{ V/V}$$



The corresponding waveform for $v_{out\text{-}dc}$ will be identical to $v_{out\text{-}ac}$ but it will have an offset of -0.27 (V_e) with respect to $v(t)=0$ axis.

PROBLEM 2

(a) Stage 1: CS ; Stage 2: CD

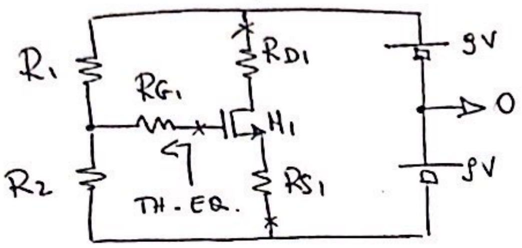
(b) In order to calculate the small-signal voltage gain we'll have to determine the position of the Q point. The reason for this is that the small-signal parameters (g_m, r_o) are related to the coordinates of the Q point (e.g., I_{DQ} and V_{DSQ}).

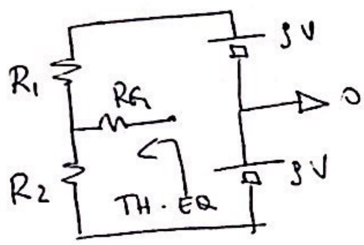
Q POINT – DC analysis

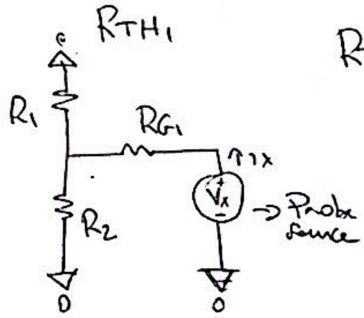
M_1 and M_2 are decoupled in DC due to C_2.

Stage 1 - DC analysis

It is convenient to reduce the circuit seen from the base to its Thevenin equivalent circuit.

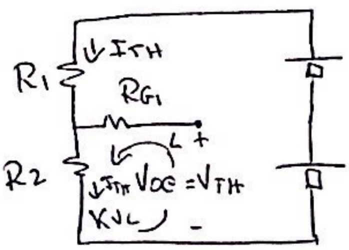






$R_{TH_1} = \frac{V_x}{i_x} = R_{G1} + R_1 // R_2$

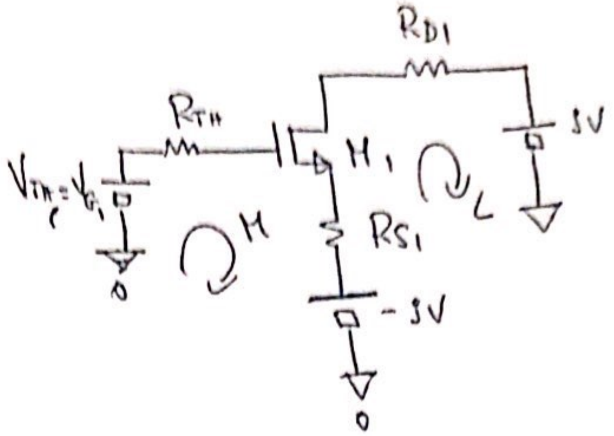
$R_{TH_1} = 128.8\ k\Omega$



KVL @ L: $V_{TH_1} = -9 + R_2 I_{TH_1}$

$I_{TH_1} = \frac{18}{400K + 190K} = 30.51\,\mu A$

$V_{TH_1} = -9 + (30.51\mu) \cdot 190K = -3.2\ V$



KVL @ L:

$-9 + I_{D1} R_{S1} + V_{DS1} + I_{D1} R_{D1} - 9 = 0$

$V_{DS1} = 18 - I_{D1} \cdot 13K$

KVL @ M:

$-9 + I_{D1} R_{S1} + V_{GS1} - V_{TH_1} = 0$

$-9 + I_{D1} 5K + V_{GS1} + 3.2 = 0$

$5.8 = I_{D1} R_{S1} + V_{GS1} \Rightarrow V_{GS1} = 5.8 - I_{D1} 5K$

Assuming that M_1 operates in saturation,

$$I_{D_1} = \frac{1}{2}\left(K_n' \frac{W}{L}\right)(V_{GS_1} - V_T)^2 (1 + \lambda_1 V_{DS_1})$$

$$I_{D1} = \frac{1}{2}(0.0027)(5.8 - I_{D_1} \cdot 5K - 0.5)^2 (1 + 0.1(18 - I_{D1} 13K))$$

$\Downarrow$

ALGEBRA BOX

$\Downarrow$

$45.14M\, I_{D1}^3 - 308.47K I_{D1}^2 + 516.74\, I_{D1} - 0.1308 = 0$

$\Downarrow$

ALGEBRA BOX

$\Downarrow$

$I_{D1} = 930\mu A$

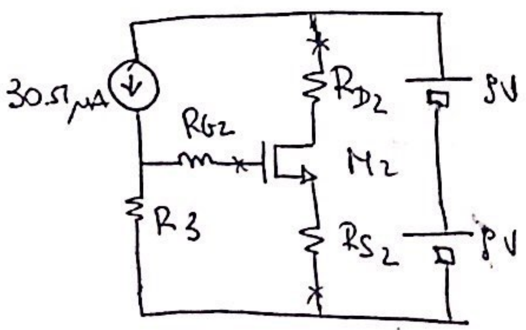
$I_{D1} = 930\mu A \Rightarrow V_{DS1} = 18 - 930\mu \cdot 13K = 5.91 V$

$V_{GS1} = 5.8 - 930\mu \cdot 5 = 1.15 V$

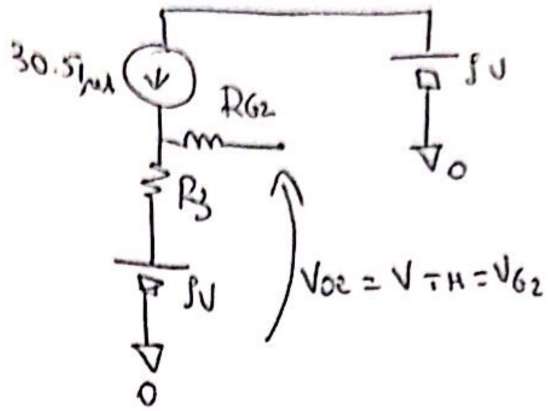
$V_{DS1} \geq V_{GS1} - V_T = 1.15 - 0.5 V$

The saturation assumption is verified for M_1.

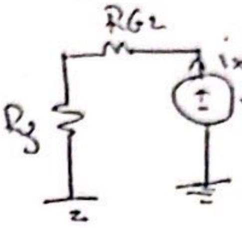
Stage 2 - DC analysis



It is again convenient to use a Thevenin equivalent circuit of the network seen from the gate of M_2.



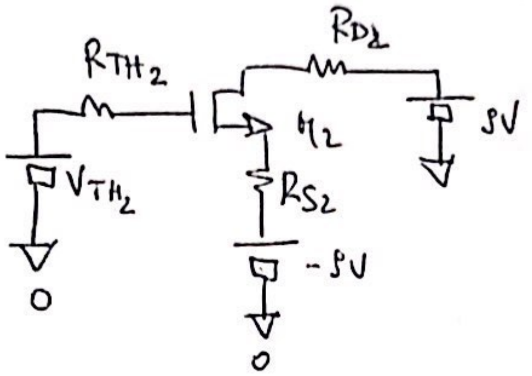
RTH2



VTH

$V_{oc} = V_{TH} = V_{G2} = 30.51\mu(150K) - 5 =$
$= -3.2\ V$

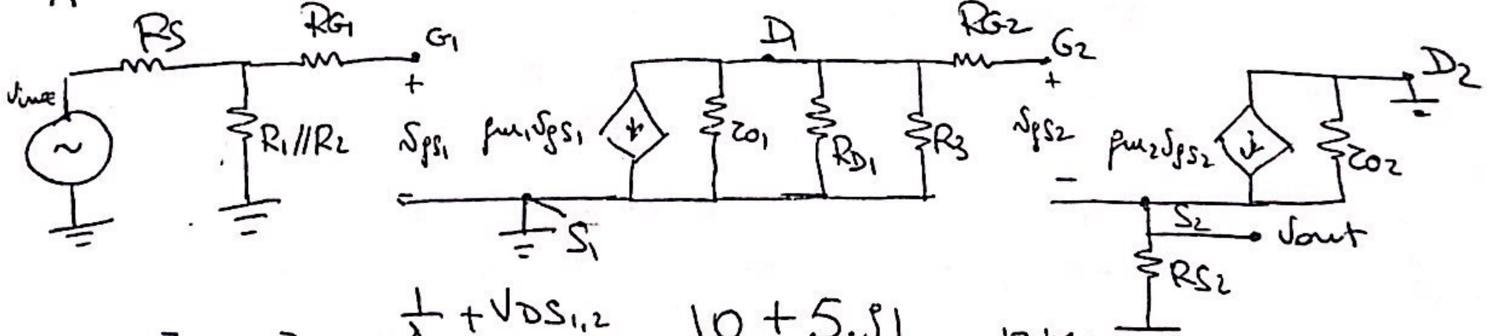
$R_{TH_2} = R_{G2} + R_3$
$R_{TH_2} = 75 + 150K =$
$= 150.075\ K \simeq 150K\Omega$



Now, as $V_{TH2} = V_{TH1}$
$R_{S2} = R_{S1}$, $R_{D2} = R_{D1}$
$R_{TH2} \neq R_{TH1}$ but no current flows in it
The current flowing in R3 is the same value of the current flowing in R1 and R2.

As a result the circuits determined for stage ① and ② provide identical values of VDS, VGS, and ID.
$V_{GS2} = V_{GS1}$, $V_{DS2} = V_{DS1}$, $I_{D2} = I_{D1}$

AC (SMALL-SIGNAL) ANALYSIS



$$r_{o1} = r_{o2} = r_o = \frac{\frac{1}{\lambda} + V_{DS1,2}}{I_{D1,2}} = \frac{10 + 5.51}{930\mu} = 17.1K\Omega$$

$$g_{m1} = g_{m2} = g_m = \frac{I_{D1,2}}{\left(\frac{V_{GS1} - V_T}{2}\right)} = \frac{930\mu}{\frac{1.3 - 0.5}{2}} = 2.32\ mA/V$$

$$G_V = \frac{v_{out}}{v_{inac}} = \frac{v_{out}}{v_{gs2}}\cdot\frac{v_{gs2}}{v_{gs1}}\cdot\frac{v_{gs1}}{v_{inac}} = \frac{v_{out}}{v_{gs2}}\cdot\frac{v_{gs2}}{v_{g2}}\cdot\frac{v_{g2}}{v_{gs1}}\cdot\frac{v_{gs1}}{v_{inac}} \quad (7)$$

$$v_{gs1} = v_{inac}\cdot\frac{R_1//R_2}{R_1//R_2+R_S} \Rightarrow \frac{v_{gs1}}{v_{inac}} = \frac{R_1//R_2}{R_1//R_2+R_S} = \frac{128.8k}{128.8k+75} \simeq 1\ V/V$$

$$v_{g2} = v_{gs2} + v_{out}\ ;\ v_{out} = g_{m2}\, v_{gs2}\,(R_{S2}//r_{o2})$$

$$v_{g2} = v_{gs2}\,(1+g_{m2}(R_{S2}//r_{o2})) \Rightarrow \frac{v_{gs2}}{v_{g2}} = \frac{1}{1+g_{m2}(R_{S2}//r_{o2})}$$

$$\frac{v_{gs2}}{v_{g2}} = \frac{1}{1+2.32m\,(5k//17.1k)} = \frac{1}{1+8.97} = 0.1\ V/V$$

$$v_{g2} = -g_{m1}\,(R_{D1}//R_3//r_{o1})\,v_{gs1} \Rightarrow \frac{v_{g2}}{v_{gs1}} = -g_{m1}\,(R_{D1}//R_3//r_{o1})$$

$$\frac{v_{g2}}{v_{gs1}} = -2.32m\,(8k//180k//17.1k) = -2.32m\cdot 5.3k \simeq -12.3\ V/V$$

$$v_{out} = g_{m2}\, v_{gs2}\,(R_{S2}//r_{o2}) \Rightarrow \frac{v_{out}}{v_{gs2}} = g_{m2}\,(R_{S2}//r_{o2})$$

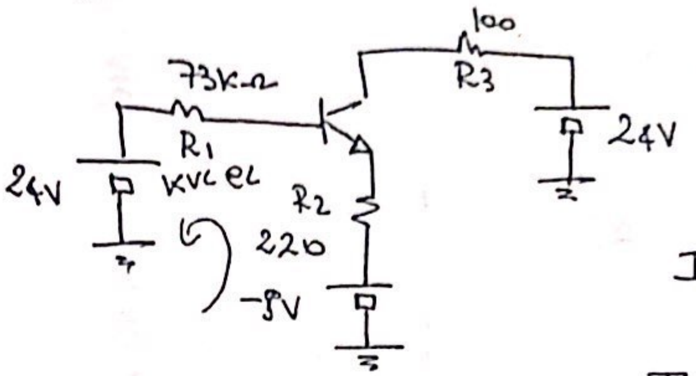
$$\frac{v_{out}}{v_{gs2}} = 2.32m\,(5k//17.1) = 8.97\ V/V$$

$$G_V = 8.97\cdot 0.1\cdot(-12.3)(1) \simeq -11.1\ V/V$$

Problem 13

$V_{out} = 6V\,S_{oc}$ (8)

In order to calculate the gain we need to determine the small signal parameters, which in turn depend on the coordinates of the Q point. Hence, the first thing to do is the DC analysis of the circuit.



KVL @ L:

$-5 + I_E R_2 + 0.7 + I_B R_1 = 24$

$I_E = (\beta+1) I_B$ Assuming that the BJT is in active mode.

$$I_B = \frac{24+5-0.7}{(\beta+1)R_2 + R_1} = 335\,\mu A$$

$I_C = \beta I_B = 100 \cdot 335\mu A = 33.5 mA$

$I_E = (\beta+1) I_B = 101 \cdot 335\mu A = 34.3 mA$

Now we need to verify the assumption of active mode operation for the BJT.

$V_{BE} > V_{BE}(on) = 0.7\,V$ (B-E Junction FB)

$V_{CB} < V_{CB}(on) = 0.7\,V$ (B-C Junction RB)

$V_C = 24 - R_3 I_C = 20.6\,V$

$V_B = 24 - R_1 I_B = -0.76\,V$

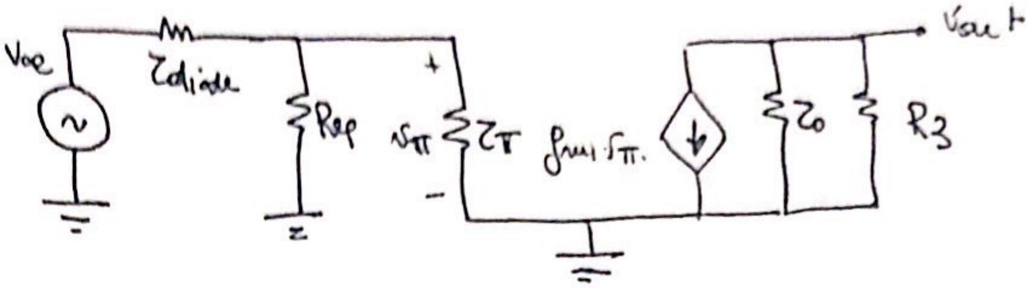
$V_E = -5V + I_E R_2 = -1.46\,V$

$\left.\begin{matrix} V_C > V_B \\ V_B > V_E \end{matrix}\right\} \Rightarrow$ Active mode assumption is verified.

$$g_m = \frac{I_C}{V_T} = \frac{33.5m}{0.0255} = 1.3057\,\frac{A}{V} \qquad r_o = \frac{V_A + V_{CE}}{I_C} = \frac{15+22.06}{33.5m} = 1.052k\Omega$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{1.3057} = 76.35\,\Omega$$

Small-signal equivalent circuit.



$$R_{eq} = R_1 // (R_4 + R_5)$$

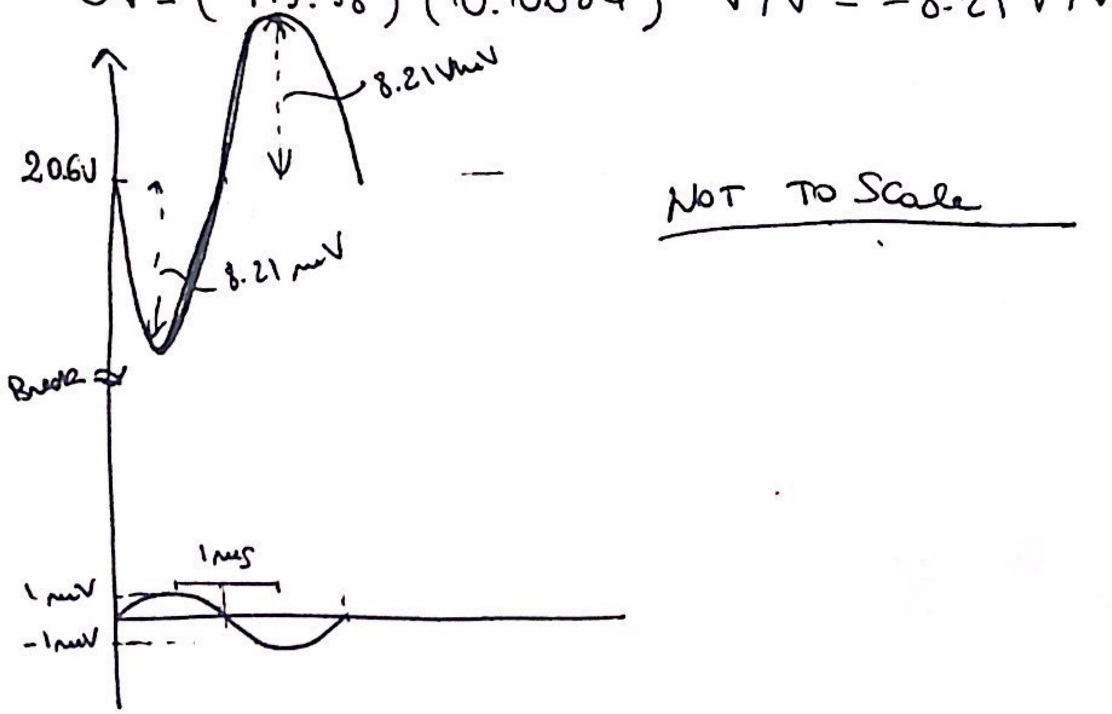
$$G_V = \frac{v_{out}}{v_{ac}} = \frac{v_{out}}{v_\pi} \cdot \frac{v_\pi}{v_{ac}}$$

$$v_{out} = -g_m v_\pi (r_o // R_3)$$

$$v_\pi = v_{ac} \frac{R_{eq} // r_\pi}{R_{eq} // r_\pi + r_{diode}}$$

$$G_V = -g_m (r_o // R_3) \left[\frac{R_{eq} // r_\pi}{R_{eq} // r_\pi + r_{diode}} \right]$$

$$G_V = (-119.98)(0.0684) \ V/V = -8.21 \ V/V$$



NOT TO Scale

Problem 4

(a) Stage 1: CS - Stage 2: CD

(b) $G_V = \frac{v_{out}}{v_{in}}$

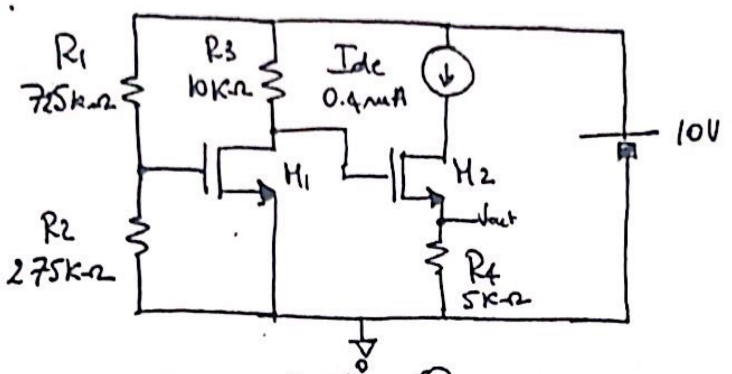
G_V will depend on the parameters of the small-signal circuit

$g_m = 2\sqrt{K_n I_{DQ}}$ $\qquad r_o \simeq \frac{1}{\lambda I_{DQ}}$

One needs to perform the DC analysis of the circuit to be able to calculate g_m and r_o.

The 2 stages are coupled in DC.

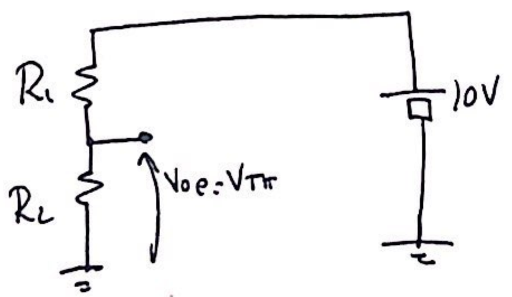
DC CIRCUIT



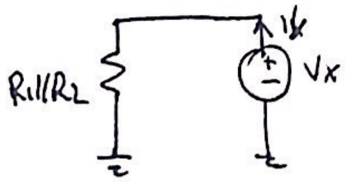
DC analysis for stage ①

Using the Thevenin equivalent of the circuit seen from the gate of M_1, I obtain

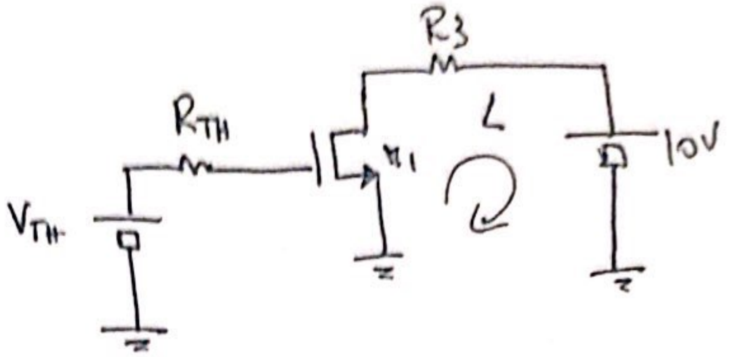
$$V_{TH} = \frac{10 \cdot 275}{275+725} = 2.75V$$



$$R_{TH} = 275k // 725k = 199.4k\Omega$$



$$R_{TH} = \frac{V_x}{i_x}$$



Since the source is grounded $V_{GS1} = V_{G1} = V_{TH} = 2.75$

$V_{DS1} = 10 - I_{D1} R_3$ (From a KVL @ L)

Assuming that M1 is in saturation

$I_{D1} = K_n (V_{GS1} - V_{TN})^2 (1 + \lambda V_{DS1})$

$I_{D1} = 125\,\mu\,(2.75 - 1)^2 (1 + 0.1\,[10 - I_{D1} \cdot 10K])$

$I_{D1} = (0.0003828)\,[2 - I_{D1} \cdot 1K] \Rightarrow I_{D1} = 0.0007656 - 0.3828\, I_{D1}$

$$I_{D1} = \frac{0.0007656}{1 + 0.3828} = 554\ \mu A.$$

Now let's verify that the assumption of saturation for M1 is correct.

$V_{DS1} > V_{GS1} - V_{TN}$ (Condition for saturation)

$V_{DS1} = 10 - [(554\,\mu) \cdot 10K] = 4.46\,V$

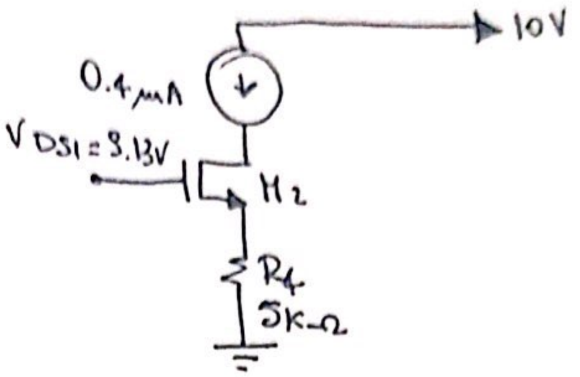
$V_{GS1} = 2.75V$ $\Rightarrow$ $V_{GS1} - V_{TN} = 1.75V$

$V_{TN} = 1\,V$

$V_{DS1} = 4.46V > V_{GS} - V_{TN}$ The saturation assumption is verified for M1

DC analysis for stage 2

Since the DC current source forces a constant current through the drain and source, M2 should be in saturation.



$V_{S2} = 0.4m \cdot 5k = 2V$

$V_{GS2} = V_{G2} - V_{S2} = V_{DS1} - V_{S2} =$

$= 4.46\,V - 2V = 2.46\,V$

$I_{D2} = K_n (V_{GS2} - V_{TN})^2 (1 + \lambda V_{DS2})$

$I_{D2} = 0.4\,mA$

$0.4m = .125\mu\,(2.46 - 1)^2 (1 + 0.1\,V_{DS2}) \Rightarrow 0.4m = 0.00026(1 + 0.1\,V_{DS2})$

$\dfrac{0.4m}{0.00026} = 1 + 0.1\,V_{DS2} \Rightarrow 0.501 = 0.1\,V_{DS2} \Rightarrow V_{DS2} = 5.01\,V$

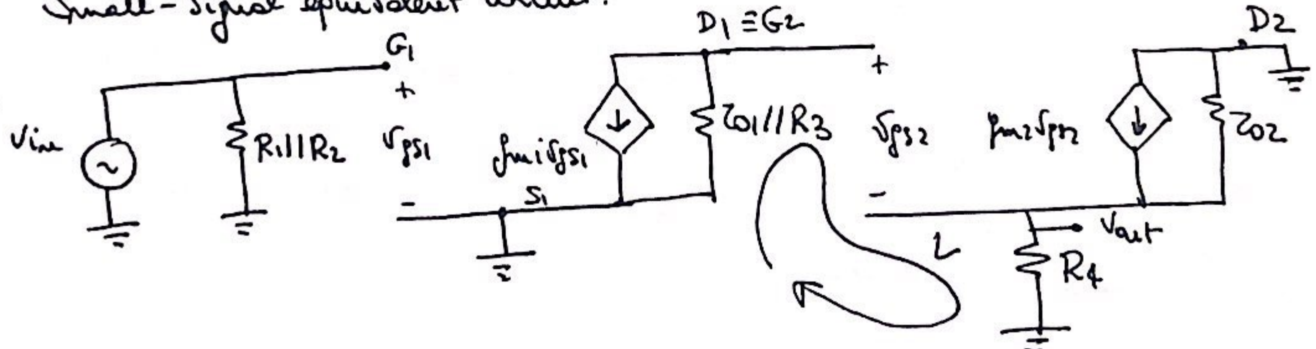
$V_{DS2} = 5.01 > V_{GS2} - V_{TN} \quad (V_{GS2} - V_{TN} = 1.46V)$

$g_{m1} = \dfrac{I_{D1}}{\frac{V_{GS1} - V_T}{2}} = 633\mu A/V \qquad g_{m2} = \dfrac{I_{D2}}{\frac{V_{GS2} - V_T}{2}} = 548\mu A/V$

$r_{o1} = \dfrac{1}{\lambda K_n (V_{GS1} - V_T)^2} = 26.1k\Omega \qquad r_{o2} = 37.53\,k\Omega$

We can now calculate the small-signal voltage gain Gv.

Small-signal equivalent circuit.



$$G_V = \frac{v_{out}}{v_{in}} = \frac{v_{out}}{v_{gs2}} \cdot \frac{v_{gs2}}{v_{gs1}} \cdot \frac{v_{gs1}}{v_{in}}$$

$$v_{out} = g_{m2} v_{gs2} (R_4 // r_{o2})$$

$$KVL\ @L : (-g_{m1} v_{gs1})(r_{o1} // R_3) = g_{m2} v_{gs2} (R_4 // r_{o2}) + v_{gs2}$$

$$v_{in} = v_{gs1}$$

$$\frac{v_{gs2}}{v_{gs1}} = \frac{-g_{m1} (r_{o1} // R_3)}{1 + g_{m2} (R_4 // r_{o2})}$$

$$\frac{v_{out}}{v_{gs2}} = g_{m2} (R_4 // r_{o2})$$

$$G_V = g_m (R_4 // r_{o2}) \left[\frac{-g_{m1} (R_3 // r_{o1})}{1 + g_{m2} (R_4 // r_{o2})} \right] (1) =$$

$$= \underbrace{-g_{m1} (R_3 // r_{o1})}_{\text{CS gain}} \underbrace{\left[\frac{g_{m2} (R_4 // r_{o2})}{1 + g_{m2} (R_4 // r_{o2})} \right]}_{\text{CD gain}}$$

$$G_V = -(0.000633)(7230) \left[\frac{0.000548 \cdot 4412}{1 + 0.000548 \cdot 4412} \right] =$$

$$= -4.577 \cdot 0.707 = -3.24\ V/V$$

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