# ECE 345 / ME 380: Introduction to Control Systems
# Midterm #1

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October 6, 2016

This midterm is closed-note, closed book, and no electronic devices are allowed. A one-sided, one-page cheat-sheet is allowed and *must be handed in with your midterm.* The cheat-sheet will be returned you with your graded midterm.

For full credit, show all your work.

Student Name                                        Student ID #

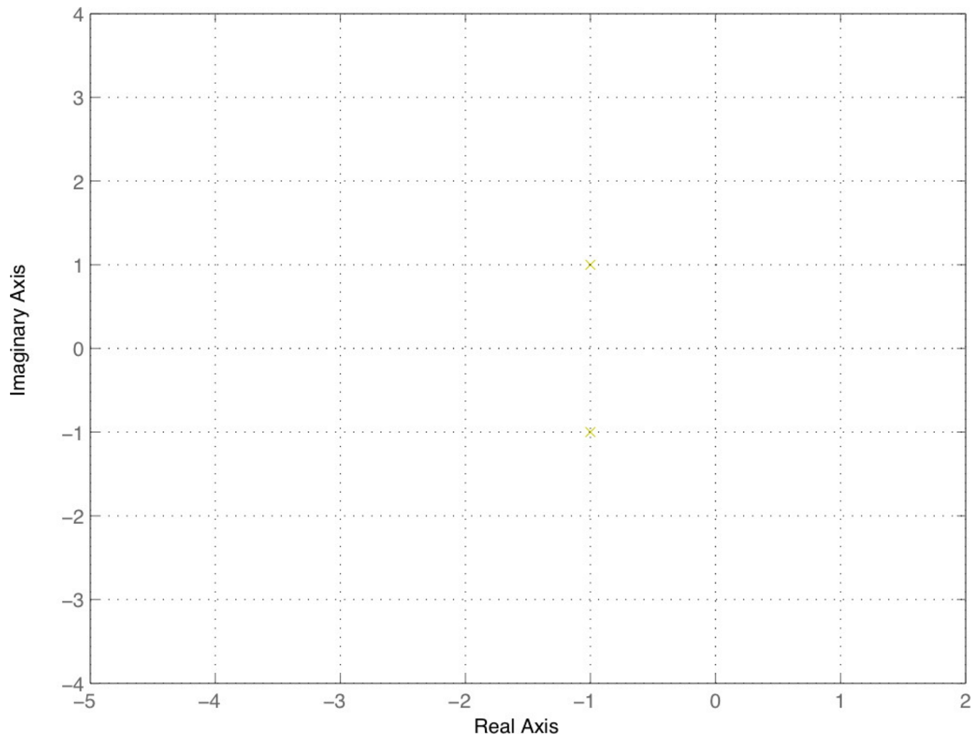
| Problem # | Actual points | Possible points |
|---|---|---|
| 1 | | 20 |
| 2 | | 25 |
| 3 | | 30 |
| 4 | | 15 |
| **Total:** | | 90 |

# 1 Warm Up (20 points)

Consider the system with transfer function

$$G(s) = \frac{2(s+3)}{s^2+1} \tag{1}$$

1. (10 points) Sketch the poles and zeros of the transfer function $G(s)$ on the complex plane plot below. Mark the zeros with '$\circ$' and the poles with '$\times$'.

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Recall that $G(s) = \dfrac{2(s+3)}{s^2+1}$.

2. (10 points) Determine the output response $y(t) = \mathcal{L}^{-1}\{Y(s)\}$ in response to an input $u(t) = e^{-3t} \cdot \mathbf{1}(t)$.

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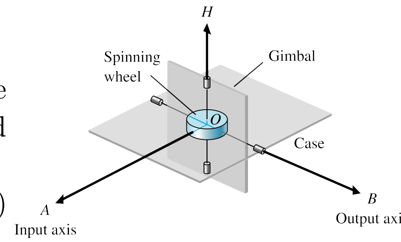
## 2 Gyroscope Dynamics (25 points)




Gyroscopes with a single degree of freedom can be used to sense changes in angular motion. The equations of motion are described by

$$J\ddot{\theta}(t) = -b\dot{\theta}(t) - k\theta(t) + f(\theta) \cdot u(t), \tag{2}$$

with nonlinear term $f(\theta) = H\cos\theta(t)$. Constant values for damping coefficient $b$, the spring constant $k$, and moments of inertia $J$ and $H$ are known, positive values.

1. (10 points) Show that the linearization of $f(\theta)$ about $\theta_0 = 0$ is $f(\theta) \approx H$.

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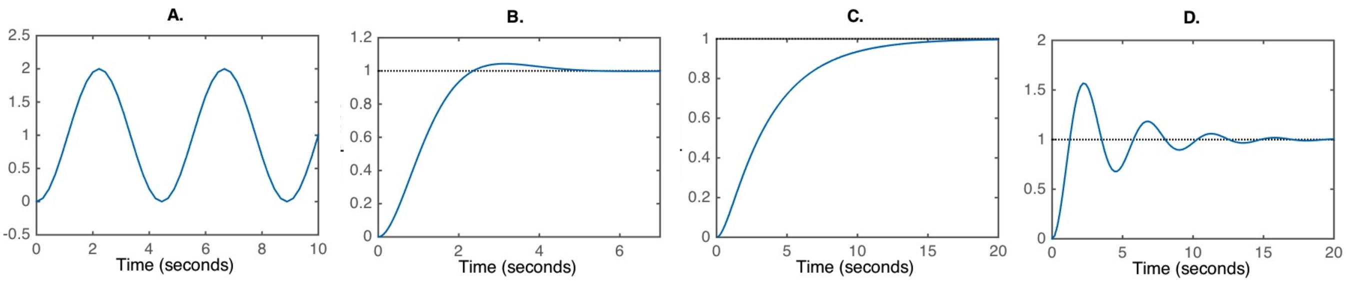
The linearized system has transfer function $\dfrac{\theta(s)}{U(s)} = \dfrac{H/J}{s^2 + \frac{b}{J}s + \frac{k}{J}}$. Presume that $\frac{H}{J} = 2$, $\frac{b}{J} = 2$, and $\frac{k}{J} = 2$ (dimensionless units).

2. (10 points) Calculate the damping ratio and natural frequency.

3. (5 points) Which one of the following plots correctly depicts the step response of the transfer function? For full credit, describe your reasoning briefly, in a single sentence.

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## 3 Insulin delivery system (30 points)

Consider an insulin delivery system, with input that is the desired dosage of insulin, and output that is blood sugar level. We represent the dynamics of the system via the state-space system

$$\begin{aligned} \dot{x} &= \begin{bmatrix} 0 & -4K \\ 1 & -4 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} r(t) \\ y &= \begin{bmatrix} 0 & 4K \end{bmatrix} x \end{aligned} \tag{3}$$

with $K > 0$ an unknown, positive constant that represents throughput of the pump.

1. (a) (5 points) Compute the state transition matrix, $\Phi(s)$.
   (b) (5 points) Use your answer from Part 1(a) to show that the system has the transfer function $G(s) = \dfrac{4K}{s^2 + 4s + 4K}$. *You will not receive full credit for using methods that do not make use of* $\Phi(s)$.

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Recall that $G(s) = \dfrac{4K}{s^2 + 4s + 4K}$.

2. (10 points) What values of $K$, if any, will satisfy the following two constraints: 1) peak time is less than or equal to $\pi/2$ seconds, and 2) settling time is less than or equal to 4 seconds? For full credit, describe *all* values of $K$ that satisfy these constraints.

3. (10 points) Which of the following is correct? *More than one statement may be true.*

   (a) The system is critically damped for $K \leq 1$.

   (b) The system is underdamped for low values of $K$ and overdamped for high values of $K$.

   (c) The system is overdamped for low values of $K$ and underdamped for high values of $K$.

   (d) As $K \to \infty$, the system becomes undamped.

   (e) Altering $K$ changes the natural frequency, but not the damping ratio.

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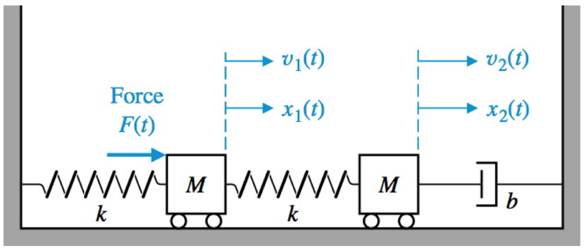
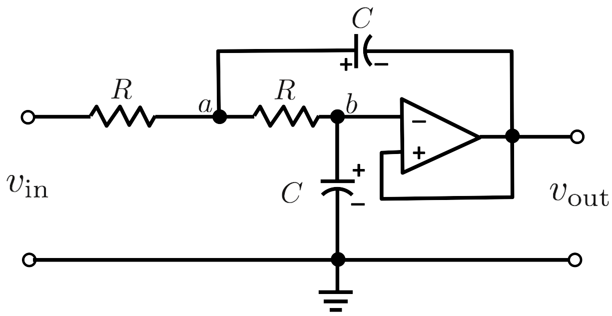
# 4 RLC Op-Amp Circuit / Spring-Mass-Damper System (15 points)

Choose *one* of the below systems to answer *either* 1A or 1B. *You do not need to do both problems and will not receive extra credit for doing more than one problem.*



(1A.) Spring mass damper system



(1B.) Op-amp circuit system

1A. (15 points) Find the mechanical system transfer function $G(s) = \frac{X_2(s)}{F(s)}$ with $M = k = b = 1$ (dimensionless units), assuming initial conditions are 0. The two masses are on a frictionless surface.

1B. (15 points) Find the circuit transfer function $G(s) = \frac{V_{\text{out}}(s)}{V_{\text{in}}(s)}$ with $R = C = 1$ (dimensionless units), assuming initial conditions are 0. Nodes $a$ and $b$ are labelled for convenience.

*End of exam.*