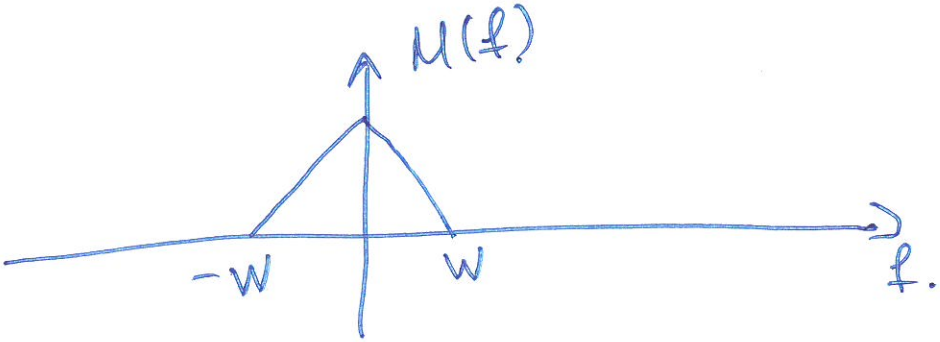
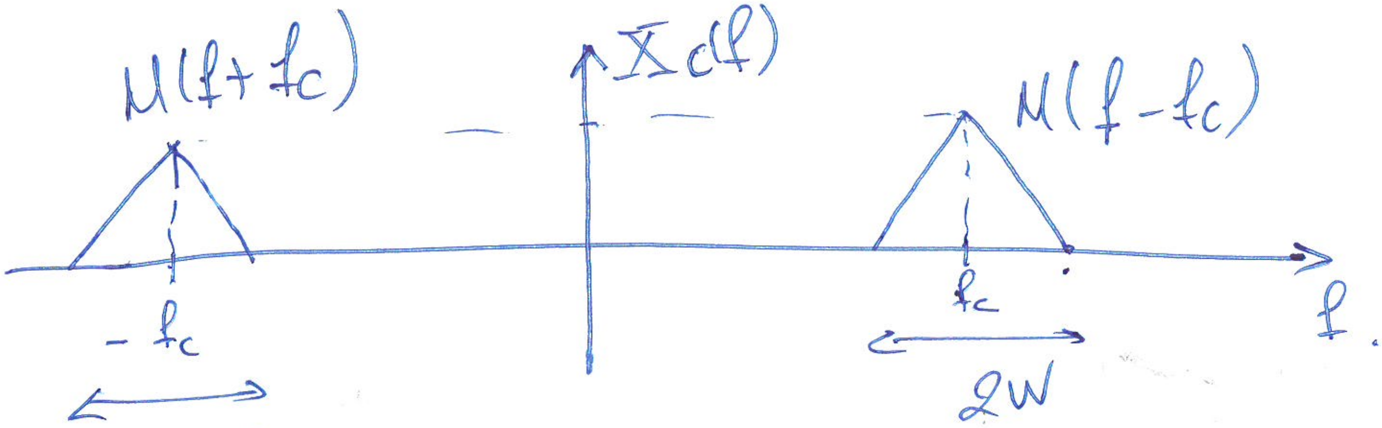
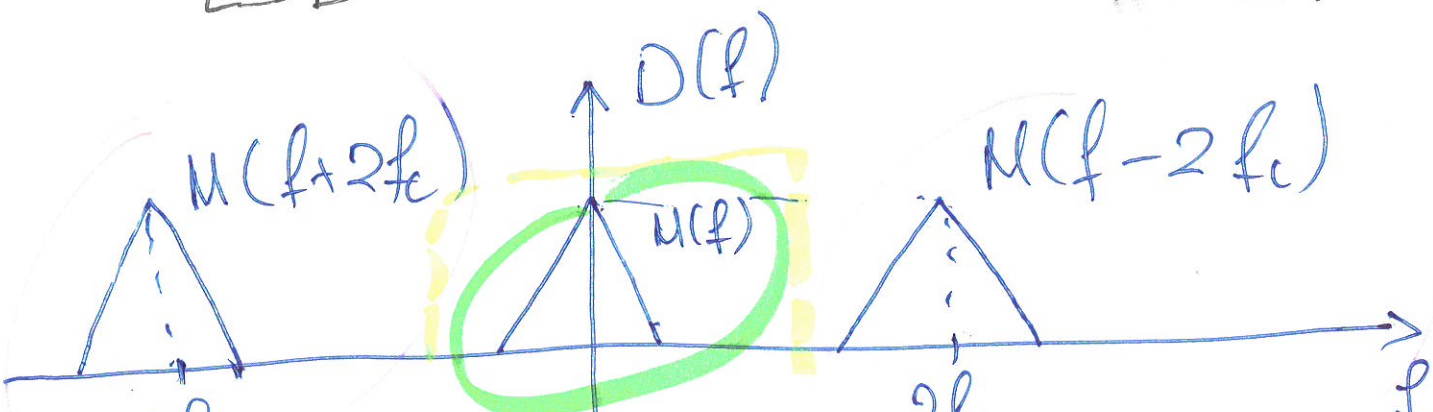
09/25/2019 ①

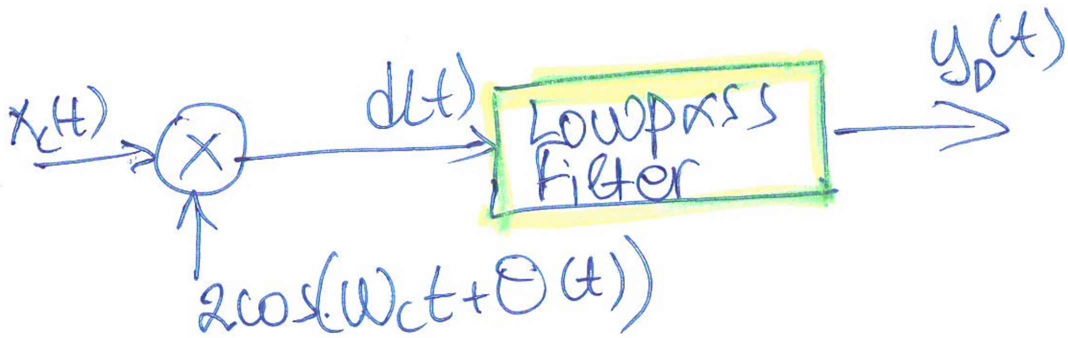




Lower-SideBand
LSB

Upper SideBand.
USB.





$$d(t) = 2A_c m(t)\cos(\omega_c t)\cdot\cos(\omega_c t+\theta(t))$$

$$= A_c m(t)\cos(\theta(t)) + \cancel{A_c m(t)\cdot\cos(4\pi f_c t+\theta(t))}$$

$$\downarrow \boxed{\text{Lowpass filter}}$$

$$y_D(t) = A_c m(t)\cdot\cos(\theta(t))$$

Amplitude Modulation

→ continuous Mod

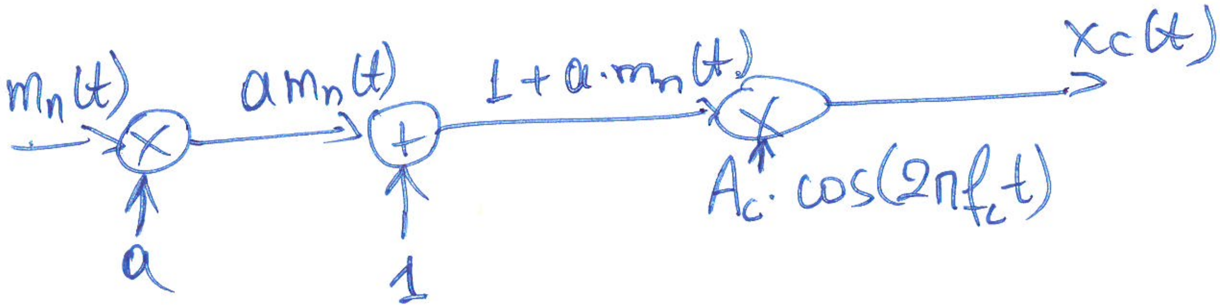
→ linear

$$x_c(t) = A_c[1+a\cdot m_n(t)]\cdot\cos(2\pi f_c t)$$

modulation index

original message signal

$$m_n(t) = \frac{m(t)}{|\min(m(t))|}$$



Total Power of the modulated signal

$$\langle x_c^2(t)\rangle = \langle A_c^2[1+a\cdot m_n(t)]^2\cdot\cos^2(2\pi f_c t)\rangle =$$

$$= \langle\frac{1}{2}A_c^2[1+\cancel{2a m_n(t)}+a^2 m_n^2(t)]\rangle$$

$$= \underbrace{\frac{1}{2}A_c^2}_{\text{carrier power}} + \underbrace{\frac{1}{2}A_c^2 a^2 \langle m_n^2(t)\rangle}_{\text{information power}} = \text{total power}$$

carrier power + information power = total power

Efficiency of the modulation

$$\boxed{Eff = \frac{a^2 \langle m_n^2(t)\rangle}{1 + a^2\langle m_n^2(t)\rangle}}$$

Example

$a = 0.5$

$\frac{1}{2}A_c^2 = 50W$

$A_c = 10$

$m(t) = 4 \cdot \cos(2\pi f_m t - \frac{\pi}{9}) + 2 \cdot \sin(4\pi f_m t)$

$\min(m(t)) = -4.364$

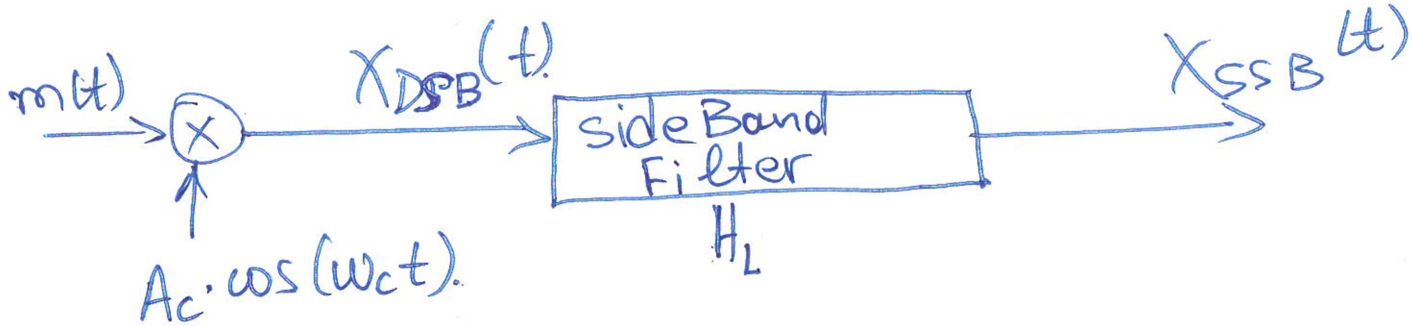
$$m_n(t) = \frac{m(t)}{|\min(m(t))|} = 0.9166 \cdot \cos(2\pi f_m t - \frac{\pi}{9}) + 0.4583 \cdot \sin(4\pi f_m t)$$

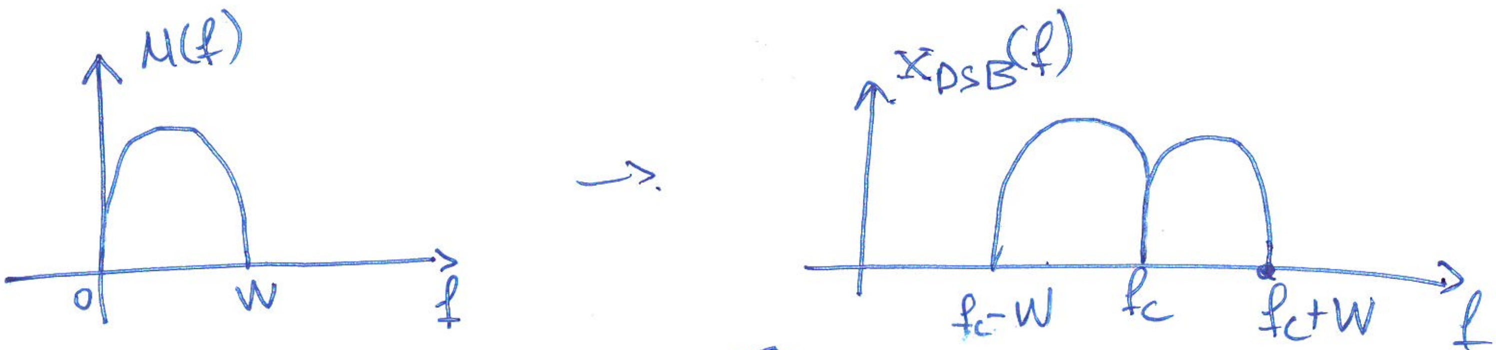
$$x_c(t) = 10\{1 + 0.5[0.9166 \cdot \cos(2\pi f_m t - \frac{\pi}{9}) + 0.4583 \sin(4\pi f_m t)]\} \cos(2\pi f_c t)$$

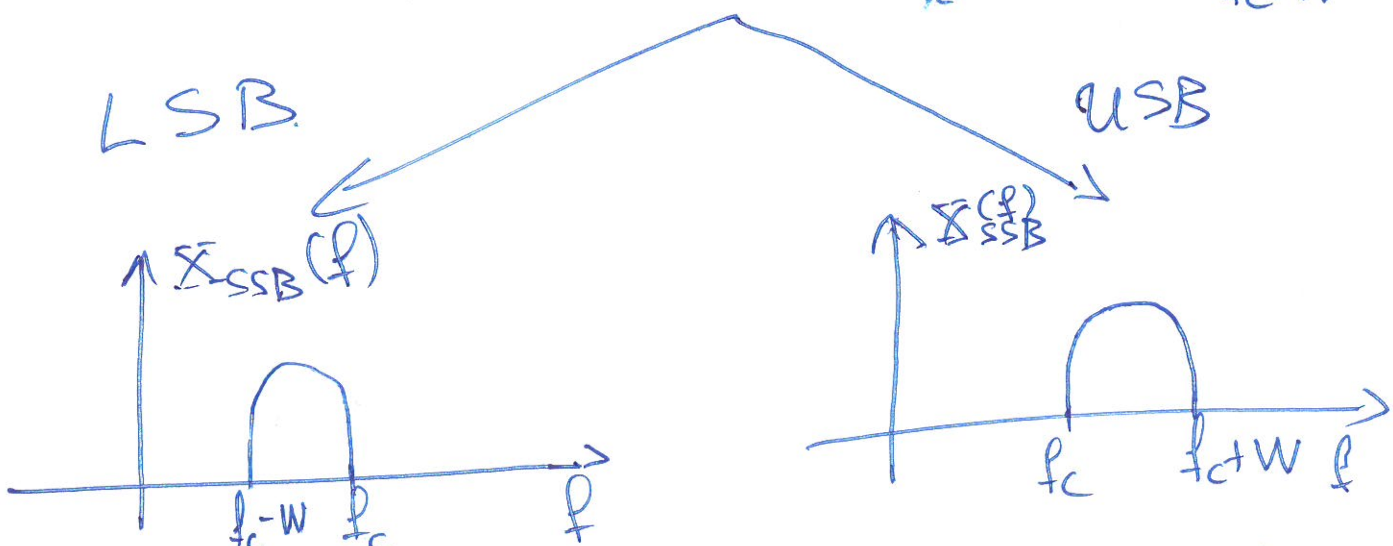
$$\langle m_n^2(t)\rangle = \frac{1}{2}0.9166^2 + \frac{1}{2}0.4583^2 = 0.5251$$

$$Eff = \frac{0.5251 \cdot 0.25}{1 + 0.25 \cdot 0.5251} = 0.116$$

Single-SideBand Modulation



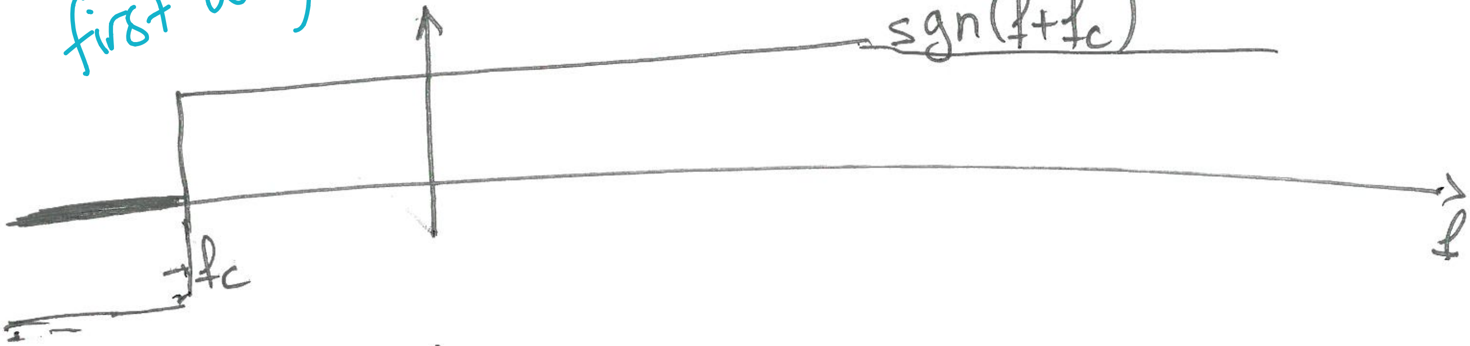


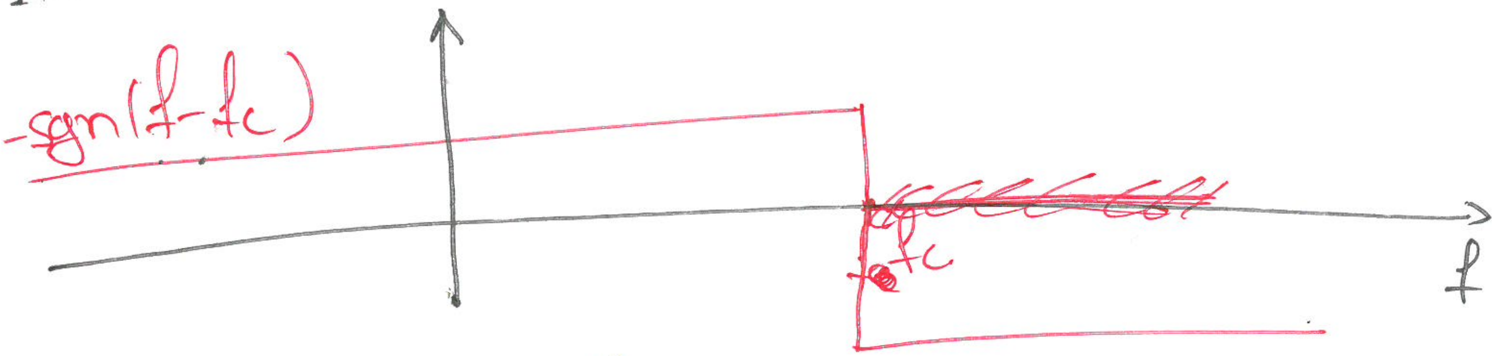


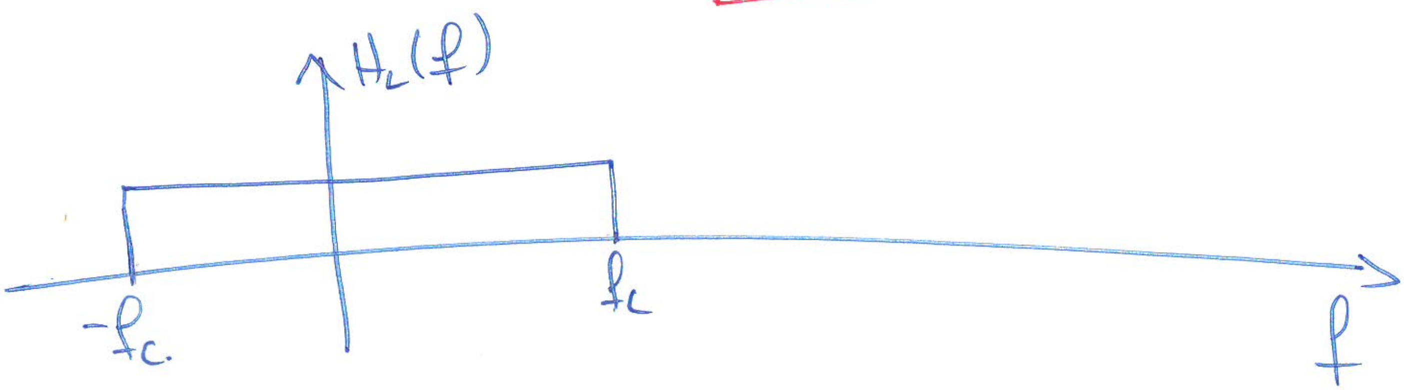
A) Transfer function of the filter (DSB → SSB)

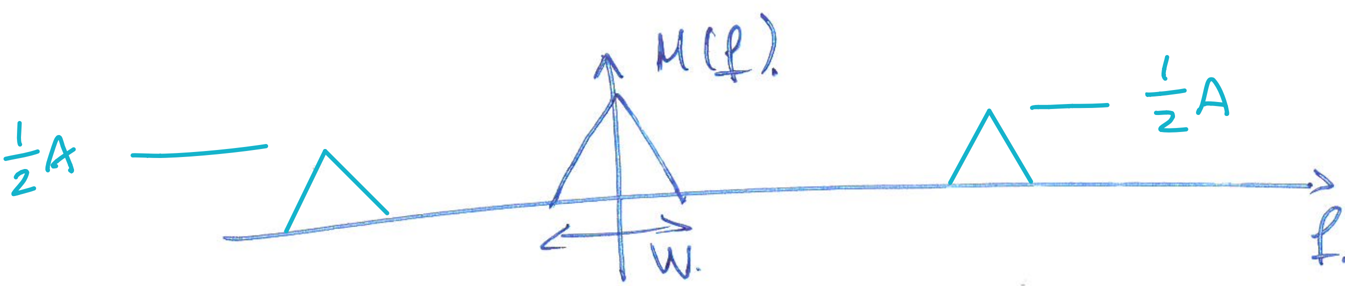
$$H_L(f) = \frac{1}{2}\left[\mathrm{sgn}(f + f_c) - \mathrm{sgn}(f - f_c)\right]$$

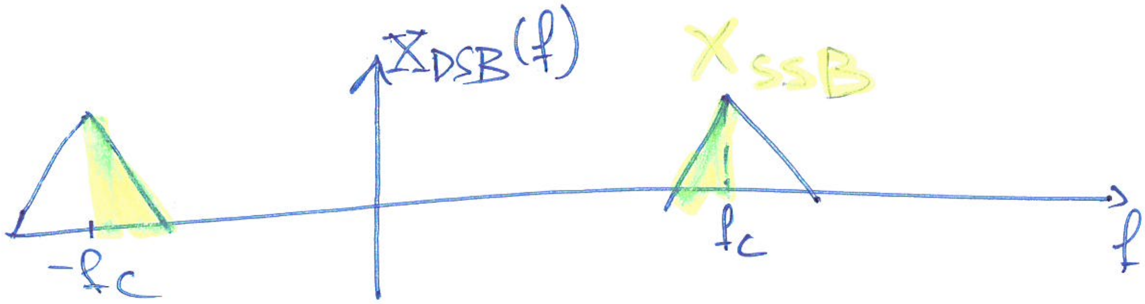
first way to demod sign











$$X_{SSB}(f) = X_{DSB}(f)\, H_L(f)$$

$$= \frac{1}{4} A_c \left[M(f+f_c) \cdot \mathrm{sgn}(f+f_c) + M(f-f_c) \cdot \mathrm{sgn}(f-f_c) \right]$$

$$- \frac{1}{4} A_c \left[M(f+f_c)\, \mathrm{sgn}(f-f_c) + M(f-f_c) \cdot \mathrm{sgn}(f-f_c) \right]$$

demodulated signal $\longrightarrow$ $A_c\, m(t) \cos(\theta t)$

Transmitted Signals in S.S.B.

$$x_c(t) = \frac{1}{2} A_c\, m(t) \cdot \cos(2\pi f_c t) + \frac{1}{2} A_c\, \hat{m}(t) \sin(2\pi f_c t)$$

↑ Lower sideband

$$x_c(t) = \frac{1}{2} A_c\, m(t) \cdot \cos(2\pi f_c t) - \frac{1}{2} A_c\, \hat{m}(t) \sin(2\pi f_c t)$$

↑ upper sideband

Another way to write the signal

- exponential
- trigonometric

– analytic $\longrightarrow$ $m(t) + j\hat{m}(t)$ positive signals

hilbert

$m(t) - j\hat{m}(t)$ negative signals

$$d(t) = \underbrace{A_c\, m(t)\cos(\theta t)}_{\text{message signal}} \mp \underbrace{A_c\, \hat{m}(t)\sin(\theta t)}_{\text{carrier distortion}}$$

(useful to see pwr)

carrier = received

$x_c(t) = x_r(t)$

Ex

$$m(t) = \cos(2\pi f_1 t) - 0.4\cos(2\pi f_1 t) + 0.9\cos(6\pi f_1 t)$$

① given signal

② find analytic

$$\hat{m}(t) = \sin(2\pi f_1 t) - 0.4\sin(4\pi f_1 t) + 0.9\sin(6\pi f_1 t)$$

First way — U.S.B or L.S.B.
(−) (+)

$$x_c(t) = \frac{1}{2}A_c\, m(t)\cos(2\pi f_c t) \pm \frac{1}{2}A_c\,\hat{m}(t)\sin(2\pi f_c t)$$

simplify by representing with R(t)

$$x_c(t) = R(t)\cos(2\pi f_c t + \theta t)$$