

# ECE 371

## Materials and Devices

10/29/19 - Lecture 18

Einstein Relation, PN Junctions, Built-In  
Potential, Built-In Electric Field

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# General Information

- Homework 6 assigned and due 11/07
- Homework 5 solutions posted
- Midterm #2 on 10/31, covers Ch. 3, 4, 5. Closed book, one 8.5" x 11" sheet (front and back) allowed.
- Example problems from previous midterm posted
- Video of midterm 2 review session posted
- Reading for next time: 7.2 and 7.3

# The Einstein Relation

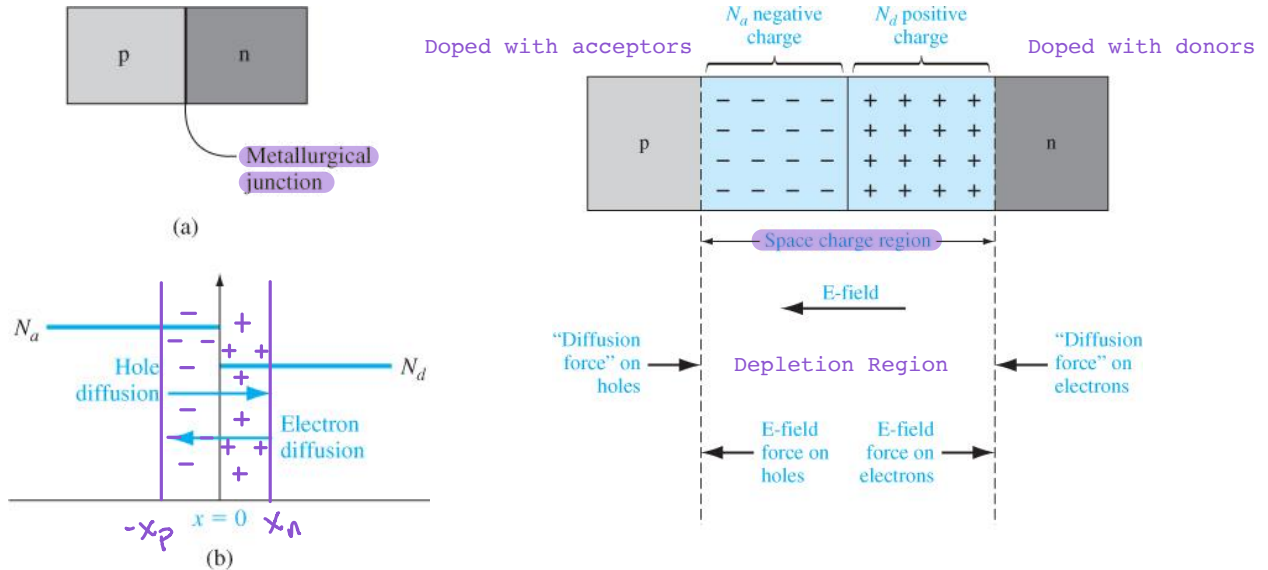
$$\frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = \frac{kT}{e}$$

- Einstein relations relate the diffusion constants to the mobilities
- Temperature dependence of the diffusion constants is the result of the temperature dependence of the mobilities (e.g., lattice and ionized impurity scattering)
- The diffusion constants are ~40X smaller than the mobilities at room temperature

**Table 5.2** | Typical mobility and diffusion coefficient values at  $T = 300 \text{ K}$  ( $\mu = \text{cm}^2/\text{V}\cdot\text{s}$  and  $D = \text{cm}^2/\text{s}$ )

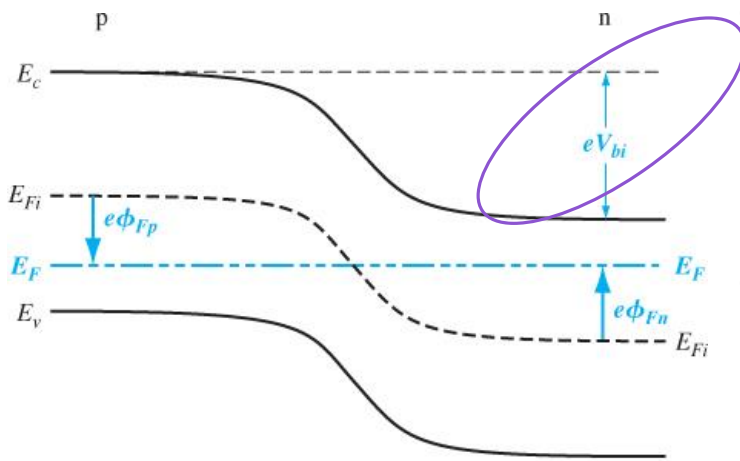
	$\mu_n$	$D_n$	$\mu_p$	$D_p$
Silicon	1350	35	480	12.4
Gallium arsenide	8500	220	400	10.4
Germanium	3900	101	1900	49.2

# pn Junction - Basics



- Majority carriers diffuse to the opposite side and become minority carriers where they **recombine**
- Diffusion current is balanced by drift current Can't have net current, no external forces
- Depletion approximation is assumed (step-like junction)
- Doping on the n and p sides is assumed to be uniform
- In the space charge (depletion) region electrons and holes are swept out by the electric field

# pn Junction – Built in Potential



**Figure 7.3** | Energy-band diagram of a pn junction in thermal equilibrium.

- A potential barrier forms across the junction for holes moving left to right and electrons moving right to left
- $N_a$  and  $N_d$  now refer to the total net impurity concentration on either side of the junction
- The built-in potential varies with the doping concentrations, but only slightly
- $V_{bi}$  for silicon is  $\sim 0.7$  V
- For very high doping,  $V_{bi}$  is approximately  $E_g/e$

$$V_{bi} = \frac{kT}{e} \ln \left( \frac{N_d N_a}{n_i^2} \right)$$

Doping dependence so ex GaAs  $\rightarrow$  higher  $V_{bi}$

Align the p and n-type

$E_c$  \_\_\_\_\_

$E_F$  - - - - -

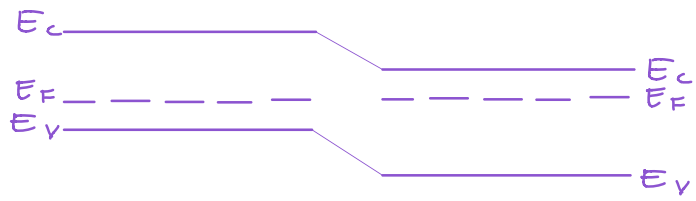
$E_v$  \_\_\_\_\_

P-type

\_\_\_\_\_  $E_c$   
- - - - -  $E_F$

\_\_\_\_\_  $E_v$

N-type



# Example 7.1

**Objective:** Calculate the built-in potential barrier in a pn junction.

**EXAMPLE 7.1**

Consider a silicon pn junction at  $T = 300$  K with doping concentrations of  $N_a = 2 \times 10^{17} \text{ cm}^{-3}$  and  $N_d = 10^{15} \text{ cm}^{-3}$ .

## ■ Solution

The built-in potential barrier is determined from Equation (7.10) as

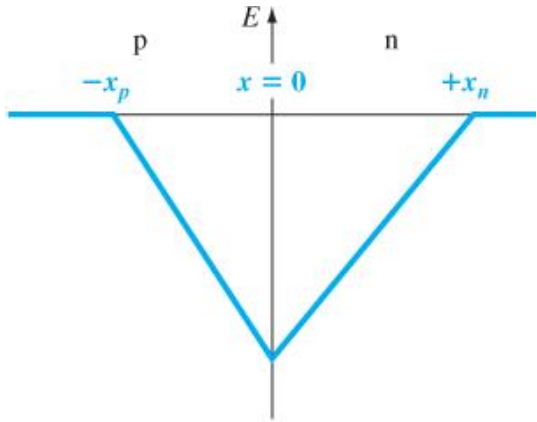
$$V_{bi} = V_t \ln \left( \frac{N_a N_d}{n_i^2} \right) = (0.0259) \ln \left[ \frac{(2 \times 10^{17})(10^{15})}{(1.5 \times 10^{10})^2} \right] = 0.713 \text{ V}$$

If we change the doping concentration in the p region of the pn junction such that the doping concentrations become  $N_a = 10^{16} \text{ cm}^{-3}$  and  $N_d = 10^{15} \text{ cm}^{-3}$ , then the built-in potential barrier becomes  $V_{bi} = 0.635 \text{ V}$ .

## ■ Comment

The built-in potential barrier changes only slightly as the doping concentrations change by orders of magnitude because of the logarithmic dependence.

# Built-In Electric Field



**Figure 7.5** | Electric field in the space charge region of a uniformly doped pn junction.

- Built-in electric field is present even without an applied bias
- E-field is negative since it points in the  $-x$  direction
- Maximum magnitude of E-field occurs at the junction

Higher doped side has larger depletion region

$$E(x) = \frac{-eN_a}{\epsilon_s}(x + x_p) \text{ for } -x_p \leq x \leq 0$$

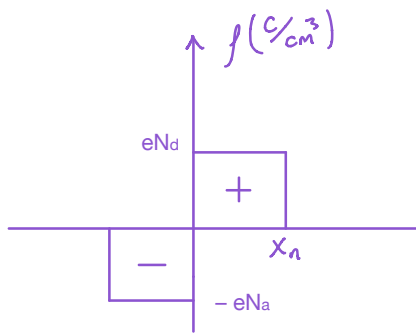
$$E(x) = \frac{-eN_d}{\epsilon_s}(x_n - x) \text{ for } 0 \leq x \leq x_n$$

$$N_a x_p = N_d x_n$$

# of charges per area on the n and p sides are equal



## Electric Field



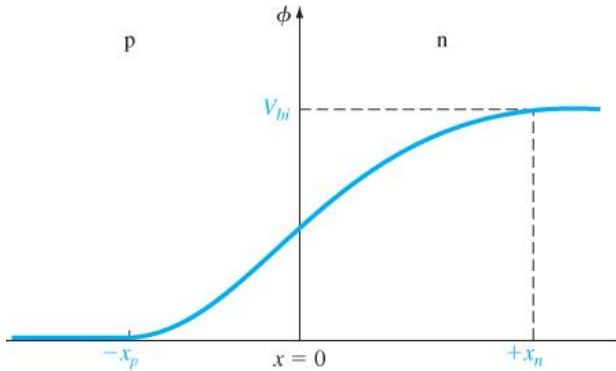
$$\rho_p = -eN_a$$

$$\rho_n = eN_d$$

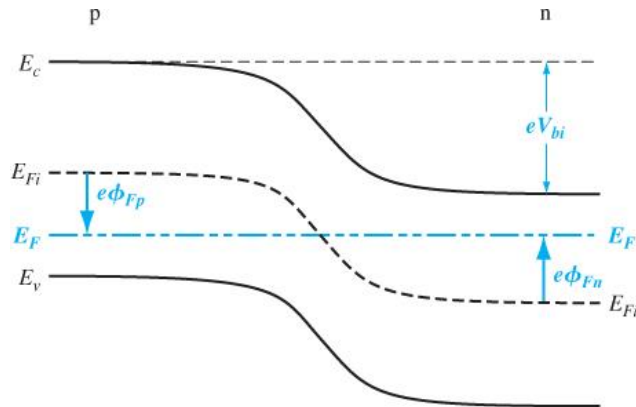
$\rho$  is volume charge density

$\epsilon_s$  is permittivity of semiconductor

# Built-In Potential



**Figure 7.6** | Electric potential through the space charge region of a uniformly doped pn junction.



- Quadratic dependence of potential on distance
- Plot applicable to electrons and holes
- Holes:
  - Lower potential on the p-side
  - Lower potential energy on the p-side
- Electrons:
  - Lower potential on the p-side
  - Lower potential energy on the n-side
- Built-in potential causes “diode” behavior – blocks current at zero bias

$$V_{bi} = \frac{e}{2\epsilon_s} (N_d x_n^2 + N_a x_p^2)$$