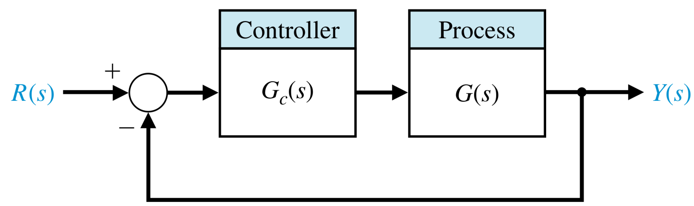
ECE 345 / ME 380: Introduction to Control Systems
Problem Set #4

Dr. Oishi

Due *Tuesday*, November 24, 2020 at 3:30pm

This problem set is open note and open book. You are welcome to discuss the problems with other students, but your solutions and Matlab code *must be written independently*. Copying of written work or Matlab code and results will not be tolerated.

Consider the system in the figure below with $G(s) = \dfrac{8}{s^2 + 4s + 8}$.



This assignment will investigate the use of three different controllers $G_c(s)$ under negative unity feedback: (1) Lead $G_c(s) = K\frac{s+4}{s+10}$, (2) Lag $G_c(s) = K\frac{s+10}{s+4}$, and (3) PID $G_c(s) = K\frac{(s+4)(s+10)}{s}$.

Use `diary` or similar commands to record your Matlab session for the following steps. Hand in your recorded Matlab commands as well as the plots Matlab generates.

1. (+10 points) Consider the lead controller $G_c(s) = K\dfrac{s+4}{s+10}$.

   (a) Plot (via Matlab) or sketch (by hand) the root locus for this system. In Matlab, use `GcG = tf(8*[1 4],conv([1 10],[1 4 8])` to represent the open-loop system $G_c(s)G(s)$, then use `rlocus(GcG)`.

   (b) Using the Hurwitz conditions, find the values of $K > 0$, if any, that will make the closed-loop system asymptotically stable.

   (c) Use the Matlab command `margin(GcG)` to compute the phase margin and gain margin with $K = 1$. Is the system stable with $K = 1$?

2. (+15 points) Consider the lag controller $G_c(s) = K\dfrac{s+10}{s+4}$.

   (a) Plot (via Matlab) or sketch (by hand) the root locus for this system. In Matlab, use `tf` to represent the open-loop system $G_c(s)G(s)$, then use `rlocus(GcG)`.

   (b) Using the Hurwitz conditions, find the values of $K > 0$, if any, that will make the closed-loop system asymptotically stable.

(c) Use the Matlab command `margin(GcG)` to compute the phase margin and gain margin with $K = 1$. Is the system stable with $K = 1$?

(d) What is the gain margin in magnitude (not dB)? Compare this to your answer in Question 2(b).

3. (+10 points) Now compare the lead and lag controllers.

   (a) Compare the order, number of asymptotes, and location of the centroid for the two systems. What is the primary effect of reversing the location of the controller pole and zero?

   (b) Which of the two systems (under lead or lag control) has *more* relative stability? Justify your answer in a single sentence.

4. (+10 points) Lastly, consider the effect of a Proportional-Integral-Derivative (PID) controller $G_c(s) = K\dfrac{(s+10)(s+4)}{s} = 14K + \dfrac{40K}{s} + Ks.$

   (a) Plot (via Matlab) the root locus for this system. Use `tf` to represent the open-loop system $G_c(s)G(s)$, then use `rlocus`.

   (b) Use `rlocfind` to find the value of K that results in a critically damped system.

   (c) Based on your root locus plot, is it possible to destabilize the system by making K sufficiently large? Why or why not?