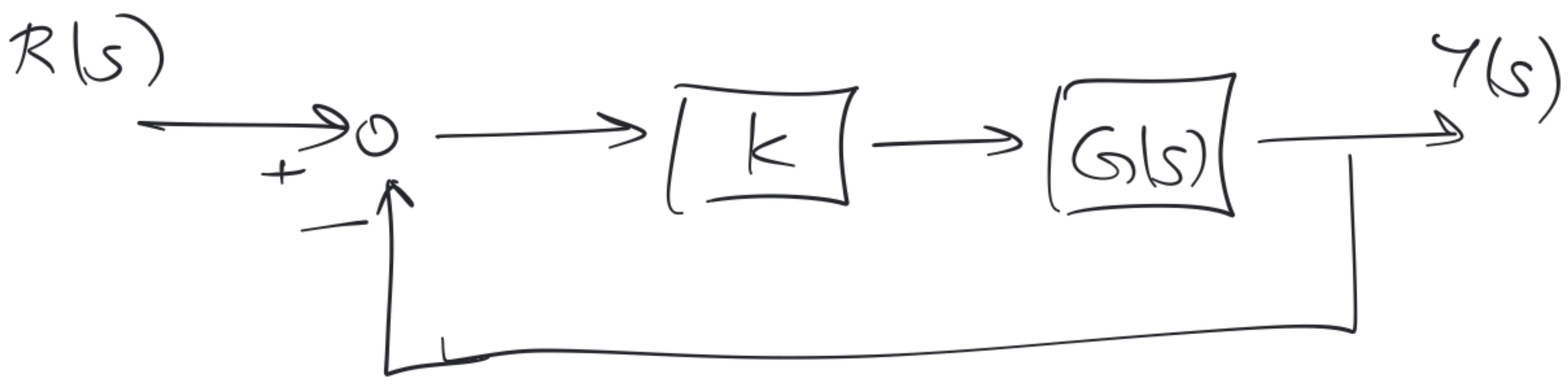
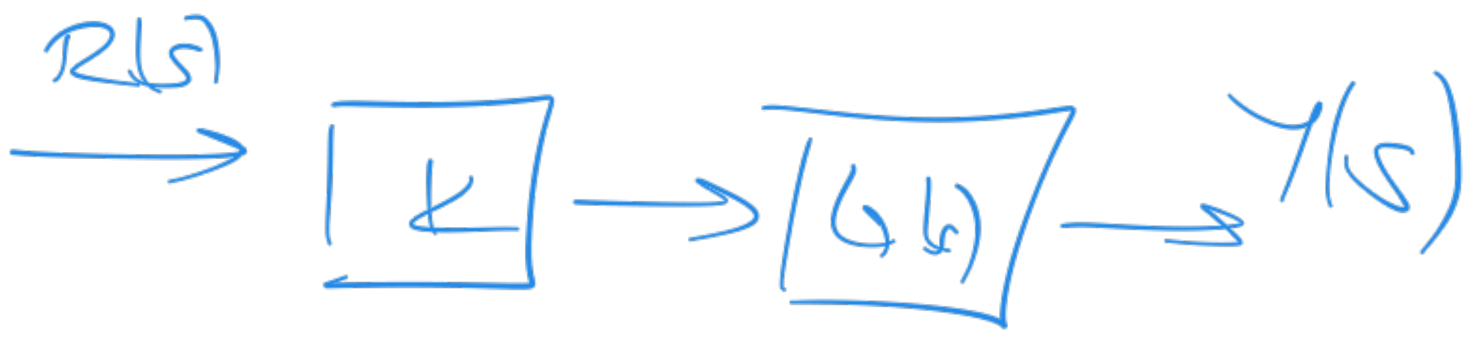


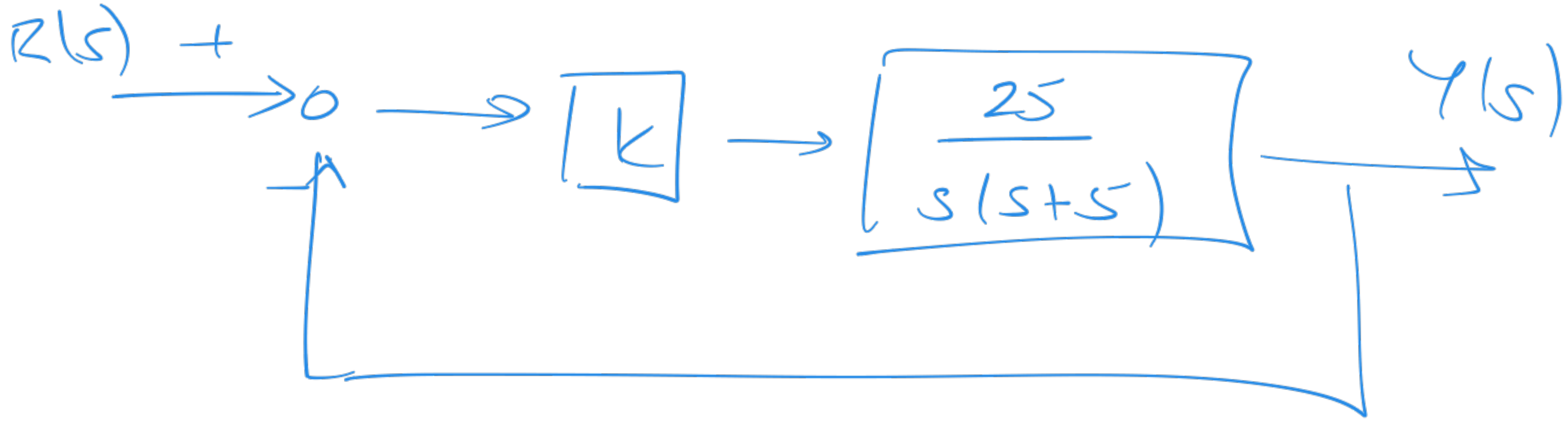


$$\frac{Y(s)}{R(s)} = \frac{K G(s)}{1 + K G(s)}$$

$$G(s) = \frac{N(s)}{D(s)}$$

$$= \frac{K N(s)}{D(s) + K N(s)}$$

$$\frac{Y(s)}{R(s)} = K G(s)$$



$N(s) = 25$

$D(s) = s(s+5)$

1) What is the closed loop t.f? $\rightarrow$ $\dfrac{Y(s)}{R(s)} = \dfrac{K \cdot 25}{s(s+5) + K \cdot 25}$

2) What are its transient response characteristics? – settling time, peak time, overshoot?

$$\Delta(s) = s^2 + 2\zeta\omega_n s + \omega_n^2 = s^2 + 5s + 25K$$

$$\Rightarrow 2\zeta\omega_n = 5 \qquad \omega_n^2 = 25K$$

$$\zeta = \frac{5}{2\omega_n} \qquad \omega_n = 5\sqrt{K}$$

$$= \frac{5}{2 \cdot 5 \cdot \sqrt{K}} = \frac{1}{2\sqrt{K}}$$

$$T_s = \frac{4}{\zeta\omega_n} = \frac{4}{5/2} = \frac{8}{5} = 1.6$$

$$T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} =$$

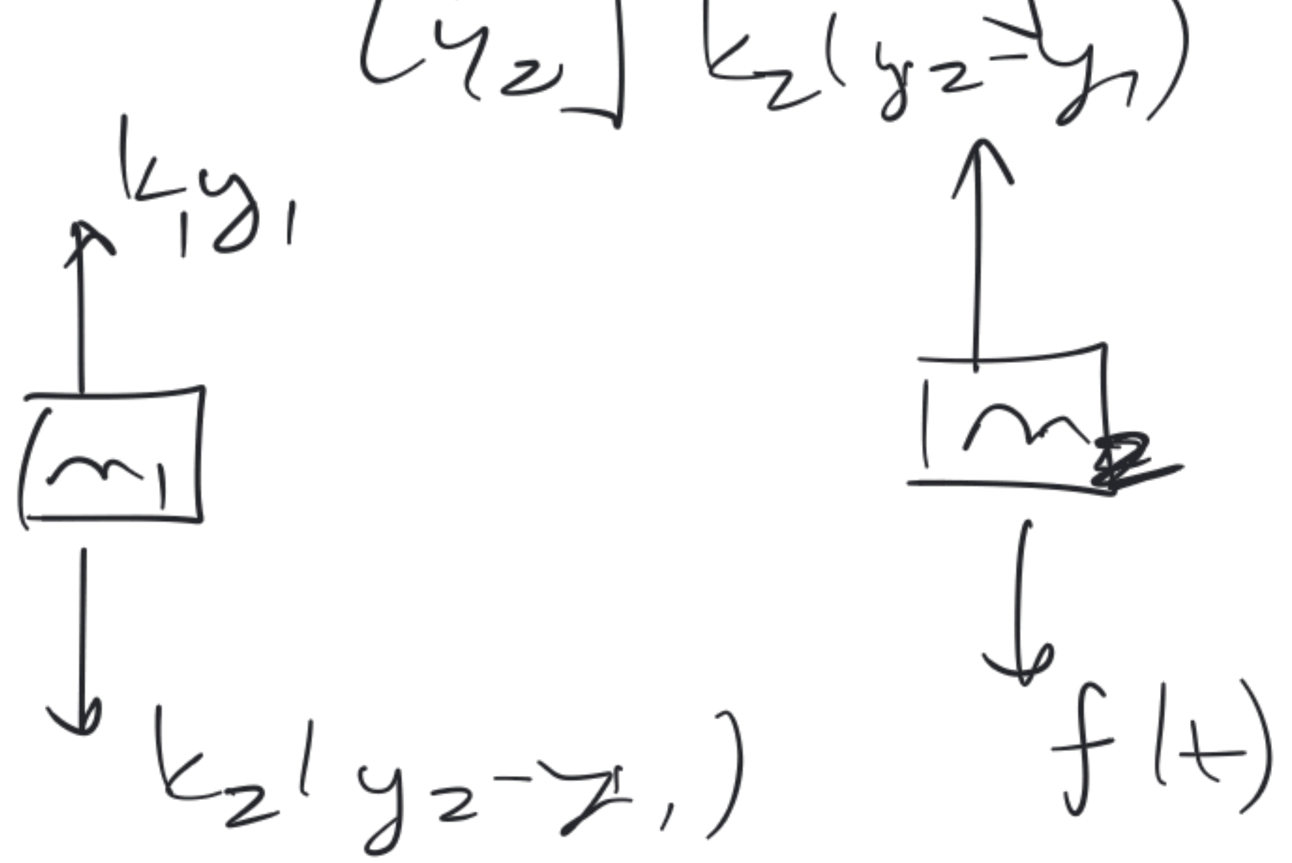
$$M_p = 100 \cdot e^{-\zeta\pi/\sqrt{1-\zeta^2}}$$

1. Find eqns of motion

2. Put in state-space form

w/ $x = \begin{bmatrix} y_1 \\ \dot{y}_1 \\ y_2 \\ \dot{y}_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$

1. FBD



$$m_1 \ddot{y}_1 = +k_2(y_2-y_1) - k_1 y_1$$

$$m_2 \ddot{y}_2 = -k_2(y_2-y_1) + f(t)$$

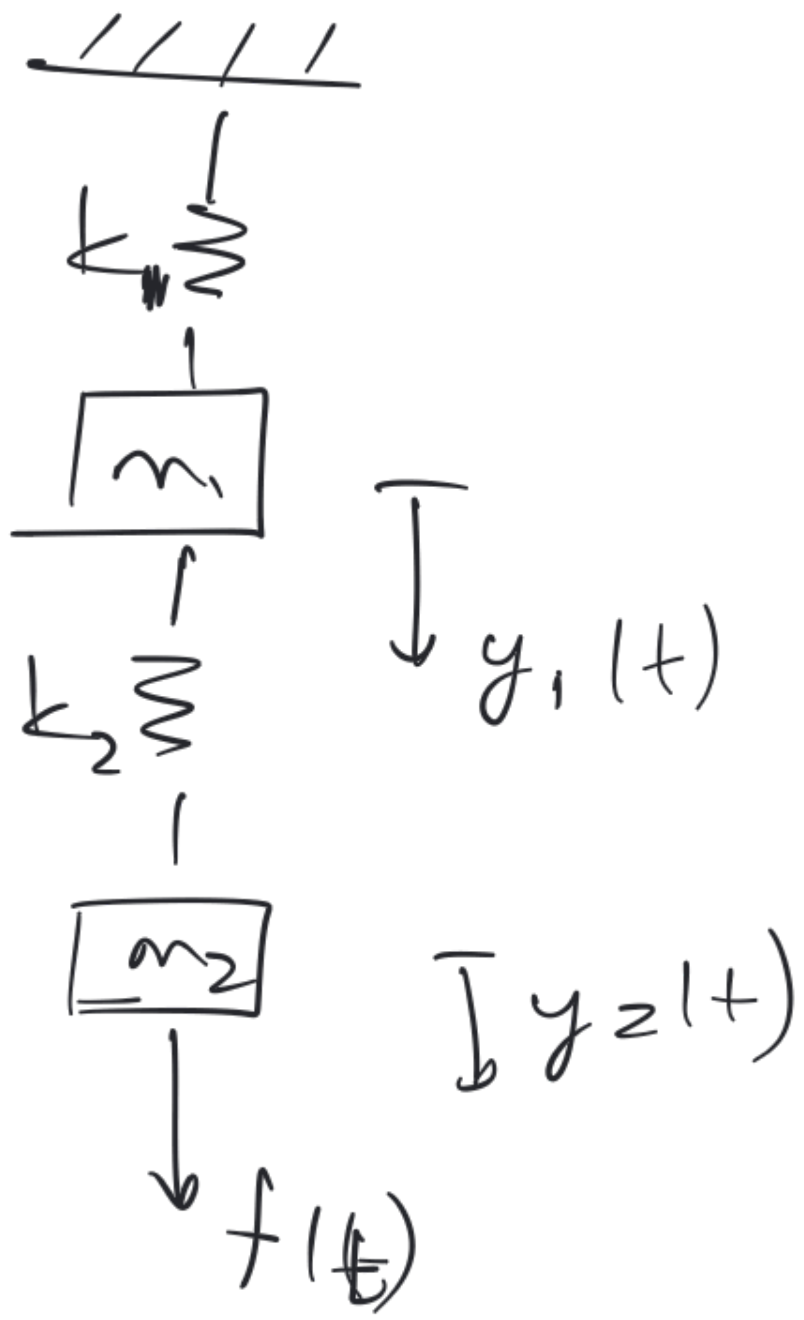
input is $f(t)$

output is $y_1(t)$

For FBD, presume

$y_1 > 0$, $y_2 > 0$

$y_2 - y_1 > 0$



$$m_1 \ddot{y}_1 = -k_1 y_1 + k_2 (y_2 - y_1)$$

$$m_2 \ddot{y}_2 = -k_2 (y_2 - y_1) + f(t)$$

$$\dot{x} = \begin{bmatrix} \dot{y}_1 \\ \ddot{y}_1 \\ \dot{y}_2 \\ \ddot{y}_2 \end{bmatrix} = \begin{bmatrix} -\frac{k_1+k_2}{m_1} & 0 & \frac{k_2}{m_1} & 0 \\ k_2/m_2 & 0 & \frac{-k_2}{m_2} & 0 \end{bmatrix} \begin{bmatrix} y_1 \\ \dot{y}_1 \\ y_2 \\ \dot{y}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/m_2 \end{bmatrix} u(t)$$

$$\ddot{y}_1 = -\frac{k_1}{m_1} y_1 + \frac{k_2}{m_1} (y_2 - y_1)$$

$$= \frac{-(k_1+k_2)}{m_1} y_1 + \underline{0}\, \dot{y}_1 + \frac{k_2}{m_1} y_2 + \underline{0}\, \dot{y}_2 + \underline{0}\, u(t)$$

$$\ddot{y}_2 = \frac{-k_2}{m_2} (y_2 - y_1) + \frac{1}{m_2} f(t)$$

$$= \underline{+k_2/m_2}\, y_1 + \underline{0}\, \dot{y}_1 + \underline{-k_2/m_2}\, y_2 + \underline{0}\, \dot{y}_2 + \underline{1/m_2}\, u(t)$$

$$m_1 \ddot{y}_1 = -k_1 y_1 + k_2 (y_2 - y_1)$$

$$m_2 \ddot{y}_2 = -k_2 (y_2 - y_1) + f(t)$$

$$\dot{x} = \begin{bmatrix} \dot{y}_1 \\ \ddot{y}_1 \\ \dot{y}_2 \\ \ddot{y}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_1+k_2}{m_1} & 0 & \frac{k_2}{m_1} & 0 \\ 0 & 0 & 0 & 1 \\ k_2/m_2 & 0 & -\frac{k_2}{m_2} & 0 \end{bmatrix}}_{A} \begin{bmatrix} y_1 \\ \dot{y}_1 \\ y_2 \\ \dot{y}_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1/m_2 \end{bmatrix}}_{B} u(t)$$

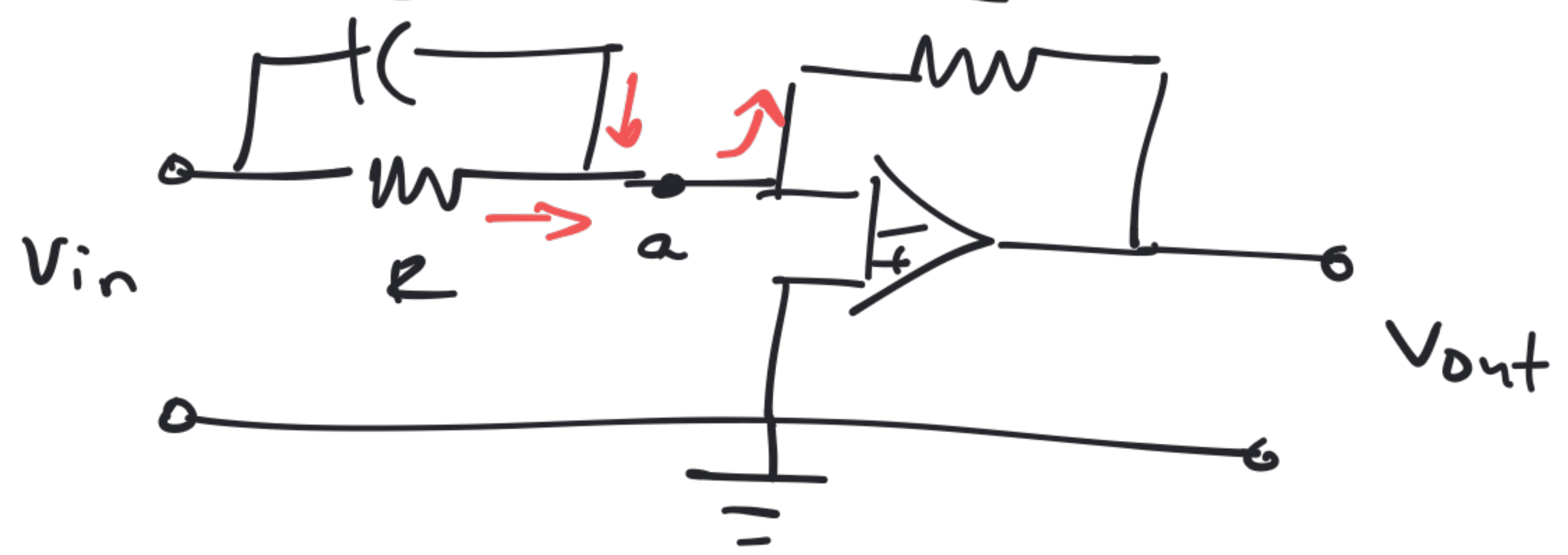
$$\dot{y}_1 = \underline{\ 0\ }\, y_1 + \underline{\ 1\ }\, \dot{y}_1 + \underline{\ 0\ }\, y_2 + \underline{\ 0\ }\, \dot{y}_2 + \underline{\ 0\ }\, u(t)$$

$$\dot{y}_2 = \underline{\ 0\ }\, y_1 + \underline{\ 0\ }\, \dot{y}_1 + \underline{\ 0\ }\, y_2 + \underline{\ 1\ }\, \dot{y}_2 + \underline{\ 0\ }\, u(t)$$

$$y = Cx + Du, \qquad y = y_1 \qquad \Longrightarrow C = [1 \quad 0 \quad 0 \quad 0]$$

$$= \underline{\ 1\ }\, y_1 + \underline{\ 0\ }\, \dot{y}_1 + \underline{\ 0\ }\, y_2 + \underline{\ 0\ }\, \dot{y}_2 + \underline{\ 0\ }\, u(t) \qquad D = 0$$

1. Find eqn of motion
2. Find transfer function.



1. KCL at node a

$$C\frac{d}{dt}(v_{in} - v_a) + \frac{v_{in} - v_a}{R} = \frac{v_a - v_{out}}{R}$$

ideal op-amp properties: $v_a = 0$

$$\Rightarrow C\dot{v}_{in} + \frac{v_{in}}{R} = \frac{-v_{out}}{R}$$

2. Laplace transform

$$sC\,V_{in}(s) + \frac{1}{R}V_{in}(s) = \frac{-1}{R}V_{out}(s)$$

$$V_{in}(s)\left(sC + \frac{1}{R}\right) = -\tfrac{1}{R}V_{out}(s)$$

$$\frac{V_{out}(s)}{V_{in}(s)} = -(RCs+1)$$

Given the state-space system

$$\dot{x} = \underbrace{\begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}}_{A} x + \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{B} u$$

$$y = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{C} x \qquad D = 0$$

find its equivalent transfer function.

$$\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$$

$$G(s) = C(sI - A)^{-1}B + D$$

$$= \begin{bmatrix} 1 & 0 \end{bmatrix} \underbrace{\begin{bmatrix} s & -1 \\ +2 & s+3 \end{bmatrix}^{-1}}_{\Phi(s)} \begin{bmatrix} 0 \\ 1 \end{bmatrix} + 0$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$= \frac{1}{s^2+3s+2} \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} s+3 & +1 \\ -2 & s \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= \frac{1}{s^2+3s+2}$$