David Kirby

Course ID: ECE 495/595 Network Economics-Spring
Prof. Eirini Eleni Tsiropoulou, eirini@unm.edu / (505) – 277 – 5501
326B/ Office Hours: Tuesdays and Thursdays 2:00pm-3:00pm
Department of Electrical and Computer Engineering / University of New Mexico

Final Exam
Thursday, May 14, 2020

1. ***(20 points)*** Consider a two-player game, where the players are called A and B. Player A has two strategies, called "Yes" and "No". Player B has two strategies, called "Like" and "Dislike". The game's payoff matrix showing the payoffs to both players for each of the four possible strategy combinations is as follows.

		Player B	
		Like	Dislike
Player A	Yes	(6,18)	(2,16)
	No	(0,0)	(4,2)

A. Is (Yes, Dislike) a likely play? Yes/No? Explain. ***(4 points)***
B. Is (No, Dislike) a likely play? Yes/No? Explain. ***(4 points)***
C. Is (No, Like) a likely play? Yes/No? Explain. ***(4 points)***
D. Is (Yes, Like) a likely play? Yes/No? Explain. ***(4 points)***
E. Consider that the players chose their strategies simultaneously. Determine the Nash equilibrium (or equilibria). ***(4 points)***

2. ***(40 points)*** In the small town of College Park, Maryland, there are only two wireless service providers and they are the only ones offering wireless services to the customers. The local demand for wireless services is given by (P denotes price measured in dollars, Q denotes the total quantity measured in Gbps): $P = 4000 - 4Q$ ($Q<1000$). Both wireless service providers have the same cost function given by (C is total cost measured in dollars and q is output measured in Gbps): $C = 40{,}000 + 400\,q$.
A. Write down the profit function of each wireless service provider. ***(10 points)***
B. Calculate and draw the reaction function of wireless service provider 1 (that is, calculate the profit-maximizing output of wireless service provider 1 for every possible output of wireless service provider 2). Do the same for wireless service provider 2. ***(15 points)***
C. Calculate the Cournot-Nash equilibrium (give the output of each wireless service provider, the total output, the price and the profit of each wireless service provider). ***(15 points)***

3. ***(40 points)*** Consider Hotelling's model (a street of length one, consumers uniformly distributed along the street, each consumer has a transportation cost equal to 2d, where d is the distance traveled). Suppose there are two gas stations, one located at 1/4 and the other located at 1.

A. Determine the location of the consumer who is indifferent between buying from firm 1 and buying from firm 2. ***(20 points)***

B. Calculate the demand functions for the two firms. ***(10 points)***

C. If the two gas stations compete in prices and settle at a Nash equilibrium, will they charge the same price for gasoline? (assume that production costs are zero, that is, firms maximize revenue). ***(10 points)***

Good Luck! Have a great summer 😊 😊 😊

1. ***(20 points)*** Consider a two-player game, where the players are called A and B. Player A has two strategies, called "Yes" and "No". Player B has two strategies, called "Like" and "Dislike". The game's payoff matrix showing the payoffs to both players for each of the four possible strategy combinations is as follows.

		Player B	
		Like	Dislike
Player A	Yes	(6,18)	(2,16)
	No	(0,0)	(4,2)

A. Is (Yes, Dislike) a likely play? Yes/No? Explain. ***(4 points)***
B. Is (No, Dislike) a likely play? Yes/No? Explain. ***(4 points)***
C. Is (No, Like) a likely play? Yes/No? Explain. ***(4 points)***
D. Is (Yes, Like) a likely play? Yes/No? Explain. ***(4 points)***
E. Consider that the players chose their strategies simultaneously. Determine the Nash equilibrium (or equilibria). ***(4 points)***

B.R.S. N.E.?

A) Player B → Dislike : Player A → No (4 > 2) No!
Player A → yes : Player B → Like (18 > 16)

(Yes, Dislike) is not a likely play because it is not a Nash equilibrium (which is for both to maximize profits).

B.R.S N.E.?

B) Player B → Dislike : Player A → No (4 > 2) yes!
Player A → No : Player B → Dislike (2 > 0)
(No, Dislike) is likely to be played because it is a N.E.

C) B.R.S. N.E.?
Player B → Like : Player A → yes (6 > 0) No!
Player A → No : Player B → Dislike (2 > 0)
(No, Like) is not a likely play because it is not a N.E.

B.R.S. N.E.?

D) Player B → Like : Player A → yes (6 > 0) yes!
Player A → yes : Player B → Like (18 > 16)
(yes, Like) is a likely play because it is a N.E.

E) The N.E. will be (yes, Like) as it gives both players the maximum profits.

2. ***(40 points)*** In the small town of College Park, Maryland, there are only two wireless service providers and they are the only ones offering wireless services to the customers. The local demand for wireless services is given by (P denotes price measured in dollars, Q denotes the total quantity measured in Gbps): P = 4000 - 4Q (Q<1000). Both wireless service providers have the same cost function given by (C is total cost measured in dollars and q is output measured in Gbps): C = 40,000 + 400 q.

A. Write down the profit function of each wireless service provider. ***(10 points)***

B. Calculate and draw the reaction function of wireless service provider 1 (that is, calculate the profit-maximizing output of wireless service provider 1 for every possible output of wireless service provider 2). Do the same for wireless service provider 2. ***(15 points)***

C. Calculate the Cournot-Nash equilibrium (give the output of each wireless service provider, the total output, the price and the profit of each wireless service provider). ***(15 points)***

$P = 4000 - 4Q$

$C = 40{,}000 + 400q$

WISP 1 $= q_1$

WISP 2 $= q_2$

A) profit functions

$$\pi_1(q_1 q_2) = q_1[P] - C$$

$$= q_1[4000 - 4(q_1 + q_2)] - 40\,000 - 400q_1$$

$$= -4q_1^2 - 4q_1q_2 + 3600q_1 - 40\,000$$

$$\pi_2(q_1 q_2) = q_2[P] - C$$

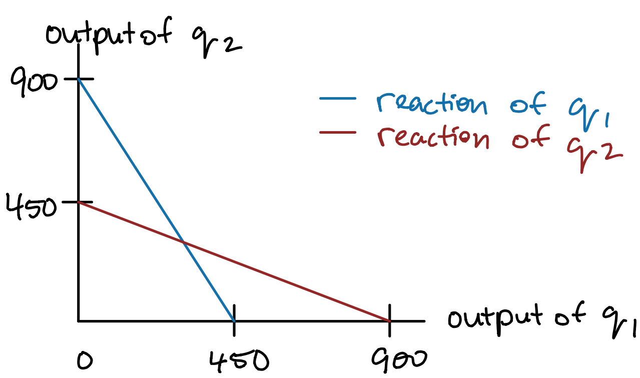
$$= q_2[4000 - 4(q_1 + q_2)] - 40\,000 - 400q_2$$

$$= -4q_2^2 - 4q_1q_2 + 3600q_2 - 40{,}000$$

B) reaction functions

$$\frac{\partial \pi_1}{\partial q_1} = 0 \Rightarrow q_1 = -\frac{1}{2}(q_2 - 900) = \boxed{450 - \frac{q_2}{2}} = q_1$$

$$\frac{\partial \pi_2}{\partial q_2} = 0 \Rightarrow q_2 = -\frac{1}{2}(q_1 - 900) = \boxed{450 - \frac{q_1}{2}} = q_2$$



c) <u>Cournot - Nash equilibrium</u> (q optimal, best strategy for both companies)

$$0.5q_1 + q_2 = 450$$
$$q_1 + 0.5q_2 = 450 \Rightarrow q_1 = q_2 = \boxed{300}$$

$Q = q_1 + q_2 = \boxed{600}$ total output

$P = 4000 - 4Q = \boxed{1600}$ price

$\pi_1 = \pi_2 = -4q_2^2 - 4q_1q_2 + 3600q_2 - 40{,}000 = \boxed{320\,000}$

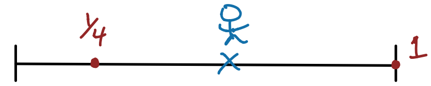
profit of each WISP

3. ***(40 points)*** Consider Hotelling's model (a street of length one, consumers uniformly distributed along the street, each consumer has a transportation cost equal to 2d, where d is the distance traveled). Suppose there are two gas stations, one located at 1/4 and the other located at 1.

A. Determine the location of the consumer who is indifferent between buying from firm 1 and buying from firm 2. ***(20 points)***

B. Calculate the demand functions for the two firms. ***(10 points)***

C. If the two gas stations compete in prices and settle at a Nash equilibrium, will they charge the same price for gasoline? (assume that production costs are zero, that is, firms maximize revenue). ***(10 points)***



uniformly distributed

transportation cost : 2d

A)

$$p_1 + 2\left(x - \frac{1}{4}\right) = p_2 + 2(1-x)$$

$$p_1 + 2x - \frac{1}{2} = p_2 + 2 - 2x$$

$$x = \frac{p_2 - p_1}{4} + \frac{5}{8}$$

B)

__Demand functions__

$$D_1(p_1, p_2) = x = \frac{p_2 - p_1}{4} + \frac{5}{8}$$

$$D_2(p_1, p_2) = 1 - x = \frac{p_1 - p_2}{4} + \frac{3}{8}$$

C)

__Nash Equilibrium__ (maximize profits)

$$\pi_1(p_1, p_2) = p_1 D_1 - \cancel{c}^{\,0} = p_1 \cdot \frac{p_2 - p_1}{4} + \frac{5}{8} = \frac{p_1 p_2}{4} - \frac{p_1^2}{4} + \frac{5}{8} p_1$$

$$\pi_2(p_1, p_2) = p_2 D_2 - \cancel{c}^{\,0} = p_2 \cdot \frac{p_1 - p_2}{4} + \frac{3}{8} = \frac{p_1 p_2}{4} - \frac{p_2^2}{4} + \frac{3}{8} p_2$$

$$\frac{\partial \pi_1}{\partial p_1} = 2p_2 - 4p_1 + 5 = 0$$

$$\Rightarrow \quad p_1 = \frac{13}{6} \; ; \; p_2 = \frac{11}{6}$$

$$\frac{\partial \pi_2}{\partial p_2} = 2p_1 - 4p_2 + 3 = 0$$

So no, they would __not__ charge the same amount.