$\boxed{11}$

For $G(s) = \dfrac{10(K_P + K_D s)}{s^2}$,

a) Find K_P s.t. parabolic error constant is $100 = K_a$.

b) Find K_D s.t. negative unity feedback system is asymptotically stable.

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a) Type 2 system.

$$K_a = \lim_{s \to 0} s^2 G(s) = 10 K_P$$

For $K_a = 100$, $K_P = 10$. ($\Rightarrow e_{ss} = \frac{1}{10}$ in response to a unit parabola input)

b) $\Delta(s) = D(s) + K N(s)$

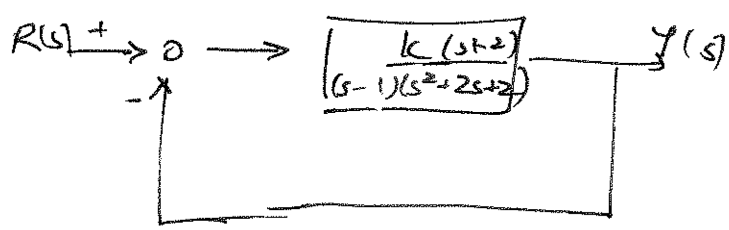
$$= s^2 + 10(K_P + K_D s)$$

$$= s^2 + 10 \cdot K_D s + 100$$

By Hurwitz criterion,

$$10 \cdot K_D > 0 \Rightarrow K_D > 0.$$

$\boxed{2}$



a) Use Nyquist criterion to find K for stabilization.

b) Find # roots of $\frac{Y(s)}{R(s)}$ in RHP, as a function of K.

c) Use a Routh table to determine K that assure stability.

a) $Z = P - N$, and $P = 1$ $\Rightarrow$ Need $N = +1$ (1 counterclockwise encirclement) for stability.

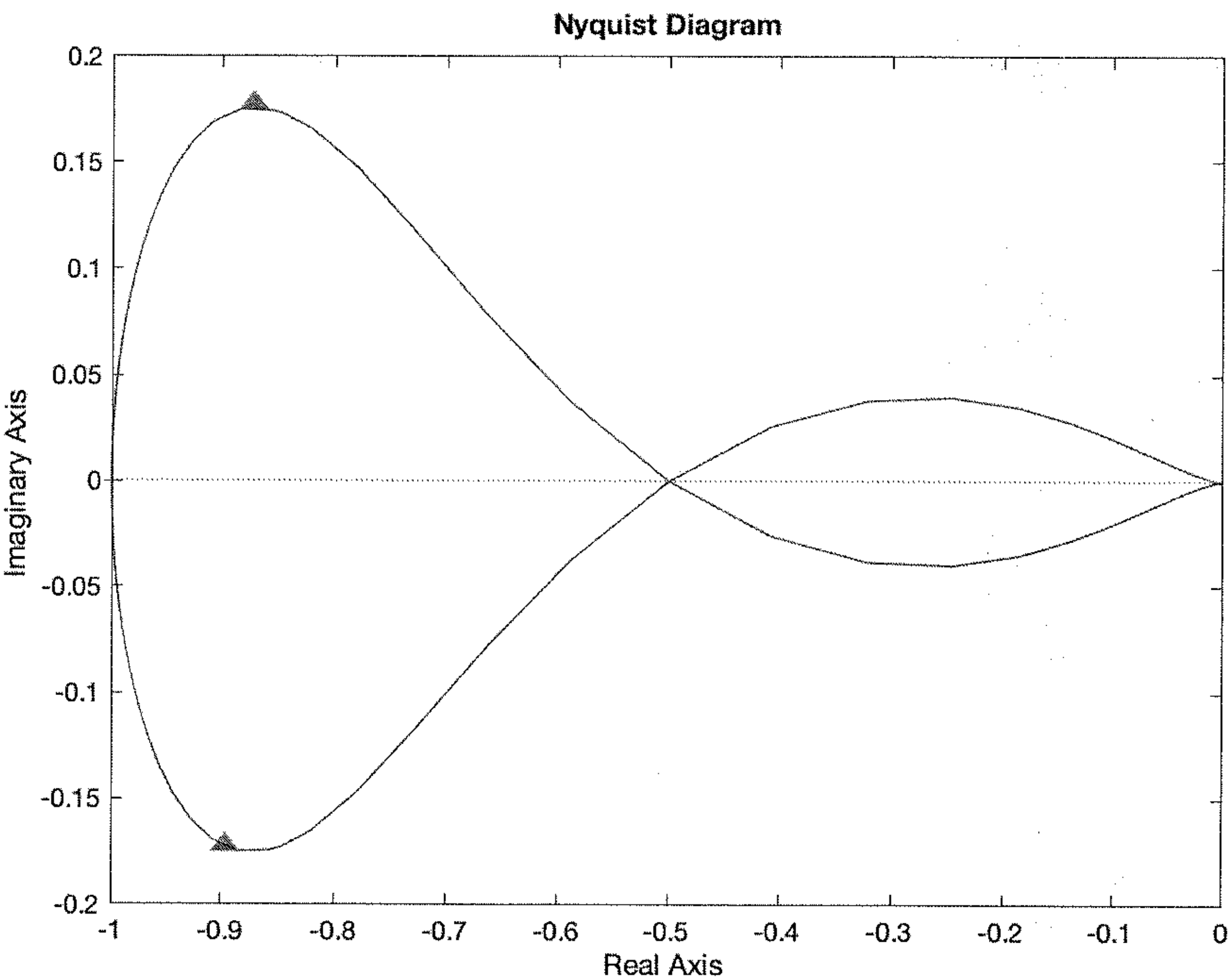
From Nyquist diagram, will have this condition for $K \in (1, 2)$, i.e., $1 \lesssim K \lesssim 2$.

b) From the Nyquist diagram, we see that

$0 < K < 1$: $N = 0 \Rightarrow Z = 1$, 1 pole in RHP

$1 < K < 2$: $N = 1 \Rightarrow Z = 0$, 0 poles in RHP

$2 < K$: $N = -1 \Rightarrow Z = 2$, 2 poles in RHP.



Nyquist Diagram
Imaginary Axis
Real Axis
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-1
-0.9
-0.8
-0.7
-0.6
-0.5
-0.4
-0.3
-0.2
-0.1
0

c) $\Delta(s) = (s-1)(s^2+2s+2) + K(s+2)$

$$= s^3 + s^2 + Ks + \underbrace{(2K-2)}_{=2(K-1)}$$

Routh table:

s^3: 1 $\quad$ K

s^2: 1 $\quad$ $2(k-1)$

s^1: $\dfrac{-\begin{vmatrix} 1 & k \\ 1 & 2(k-1) \end{vmatrix}}{1} = 2-k$ $\quad$ 0

s^0: $\dfrac{-\begin{vmatrix} 1 & 2(k-1) \\ 2-k & 0 \end{vmatrix}}{2-k} = 2(k-1)$ $\quad$ 0

$$\begin{vmatrix} 1 & k \\ 1 & 2(k-1) \end{vmatrix} = 2(k-1) - k = k-2$$

$$\begin{vmatrix} 1 & 2(k-1) \\ 2-k & 0 \end{vmatrix} = -(2-k)\cdot 2(k-1) = 2(k-2)(k-1)$$

$\therefore$ No sign changes if $2-k > 0 \Rightarrow 2 > k$

and $2(k-1) > 0 \Rightarrow K > 1$

13 Put the system described by the diff. eqn

$$y^{(3)} + 3\ddot{y} + 3\dot{y} + y = r$$

into state-space form, with input $r(t)$, output $y(t)$, & state $x(t) = \begin{bmatrix} y(t) \\ \dot{y}(t) \\ \ddot{y}(t) \end{bmatrix}$

$A \in \mathbb{R}^{3\times 3}$, $B \in \mathbb{R}^{3\times 1}$, $C \in \mathbb{R}^{1\times 3}$, $D \in \mathbb{R}^{1\times 1}$.

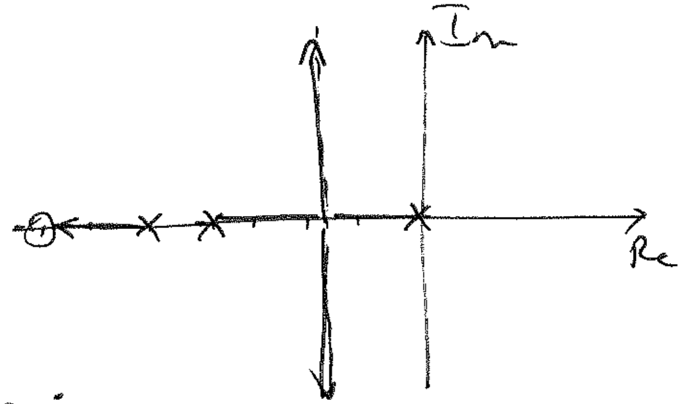
Note that $\dot{x}(t) = \begin{bmatrix} \dot{y}(t) \\ \ddot{y}(t) \\ y^{(3)}(t) \end{bmatrix} = \begin{bmatrix} x_2(t) \\ x_3(t) \\ -x_1(t) - 3x_2(t) - 3x_3(t) + u(t) \end{bmatrix}$

$$= \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -3 & -3 \end{bmatrix}}_{A} x(t) + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}_{B} u(t)$$

and $$y(t) = \underbrace{\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}}_{C} x(t) + \underbrace{0}_{D} \cdot u(t)$$

14 Sketch a root locus for a system whose open-loop plant $G(s)$ has a) poles at $0, -5, -6$ and a zero at -8; or b) poles at $0, -1, -2$ & a zero at $+1$.

a) $G(s) = \dfrac{s+8}{s(s+5)(s+6)}$



$n = 3, \quad m = 1$

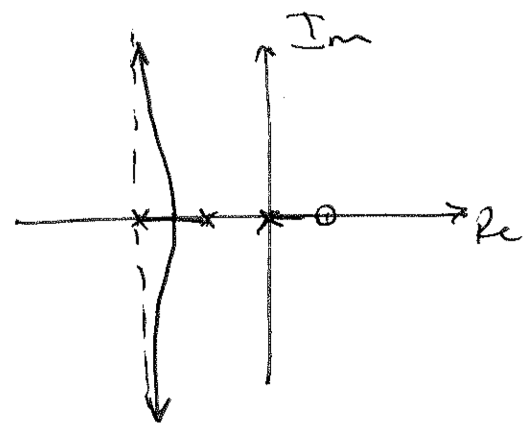
$\Rightarrow n - m = 2$

$\therefore$ 2 asymptotes at $\pm 90^\circ$

centroid at $\dfrac{\sum -p_i + \sum -z_i}{n-m} = \dfrac{(0-5-6)-(-8)}{2}$

$= -3/2.$

b) $G(s) = \dfrac{s-1}{s(s+1)(s+2)}$



$n = 3, \quad m = 1$

$\Rightarrow n - m = 2$

$\therefore$ 2 asymptotes at $\pm 90^\circ$,

centroid at $\dfrac{(0-1-2)-(+1)}{2} = -2.$

$\boxed{5}$

For $G(s) = \dfrac{K}{(s+5)^n}$ for each of $n \in \{2, 3, 4\}$

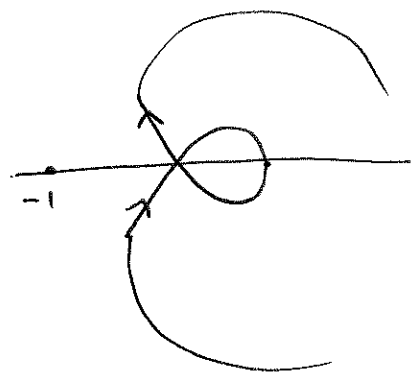
Find the value of K s.t. the negative unity feedback system is asymptotically stable.

- Create Matlab plots of Nyquist diagram, then apply Nyquist criterion. Calculate gain margin.

$Z = P - N$, $P = 0$. Want $N = 0$.

$n = 2$: $K > 0$ will assure stability.

$n = 3$: $-\dfrac{1}{a} \simeq -1 \times 10^{-3}$



$a \simeq 10^3$

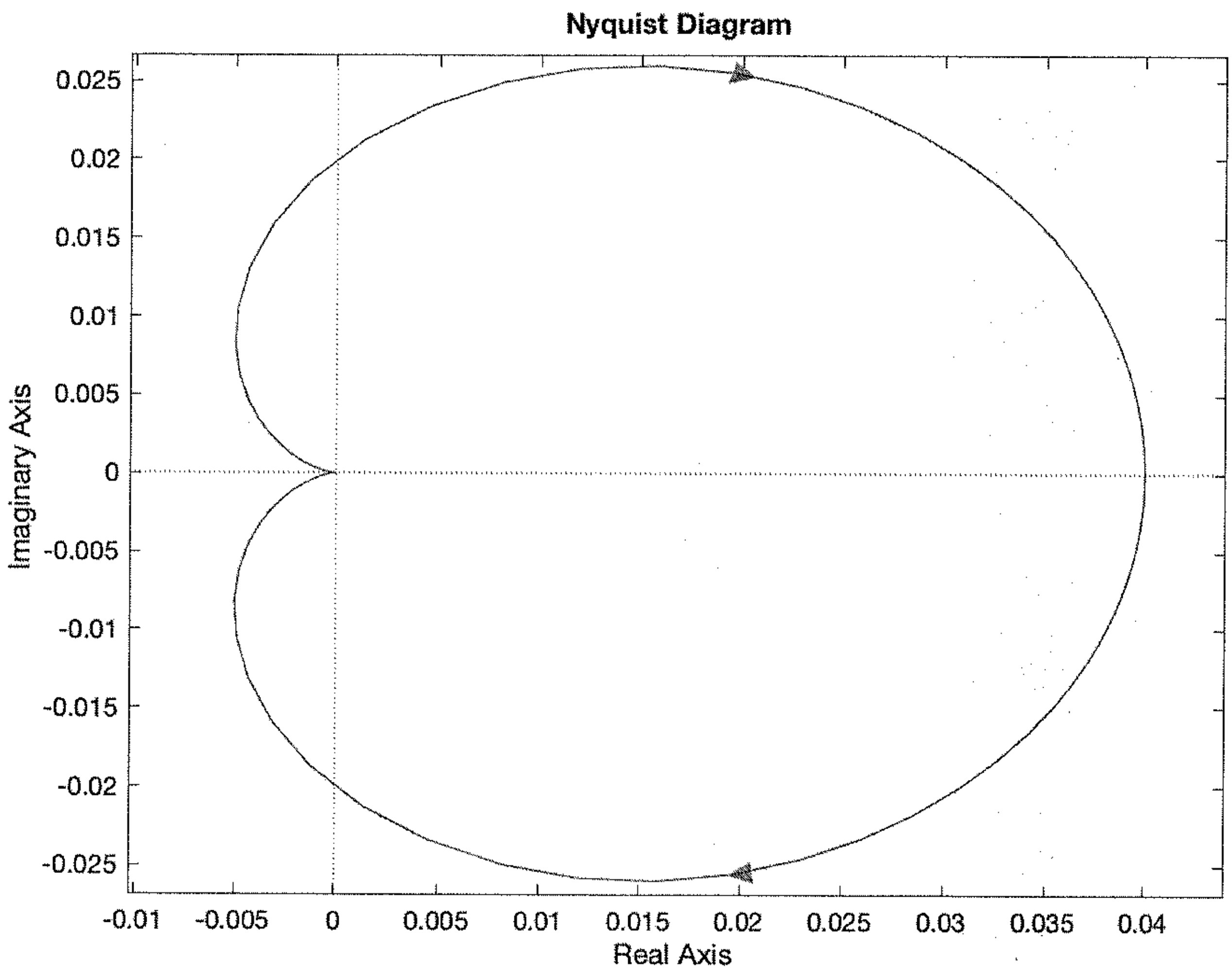
$\therefore K \lesssim 1000$ is ok.

$n = 4$: $-\dfrac{1}{a} \simeq -\dfrac{1}{2}$

$a \simeq 2$

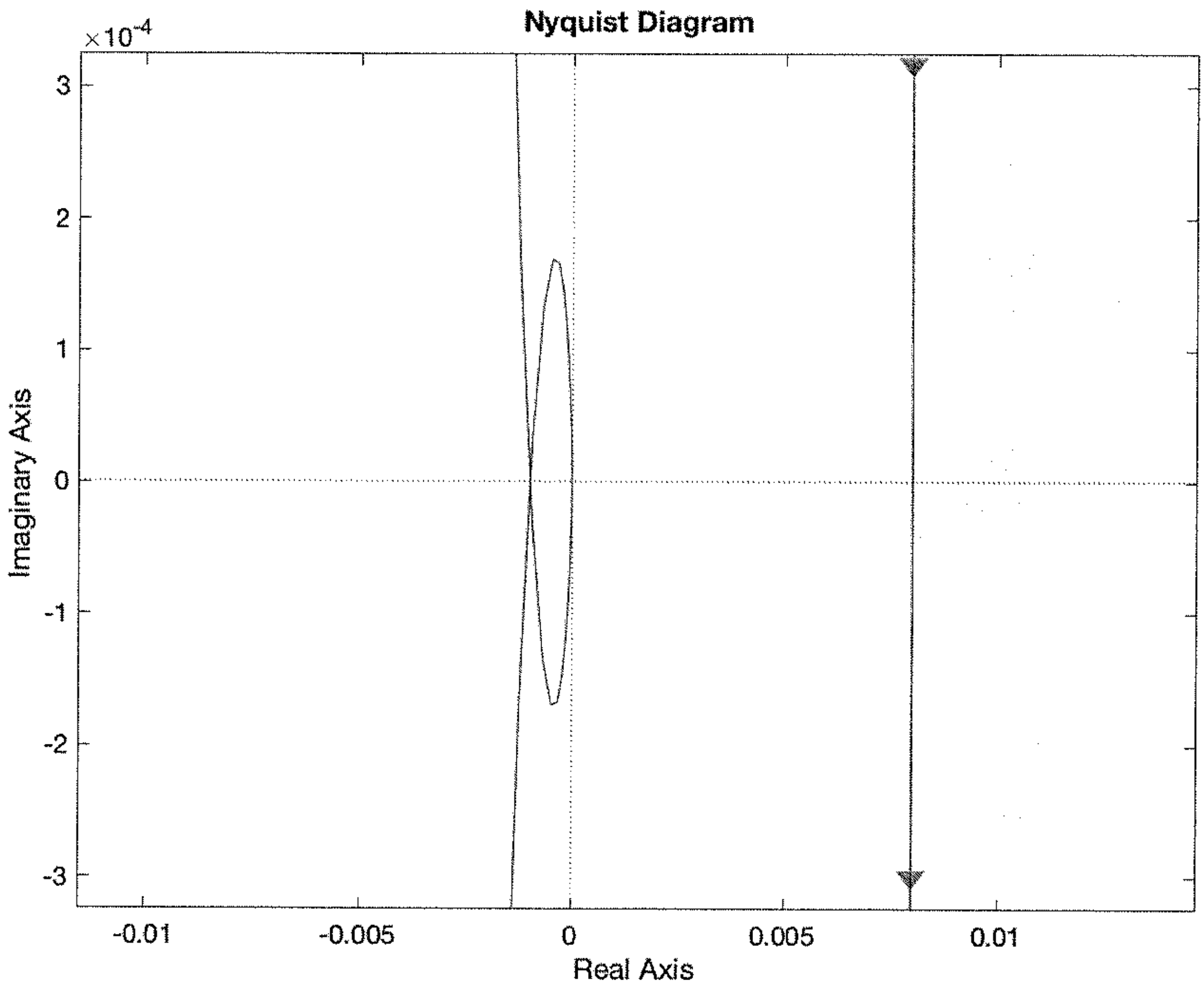
$\therefore K < 2$ is ok.

$$G(s) = \frac{1}{(s+5)^2}$$



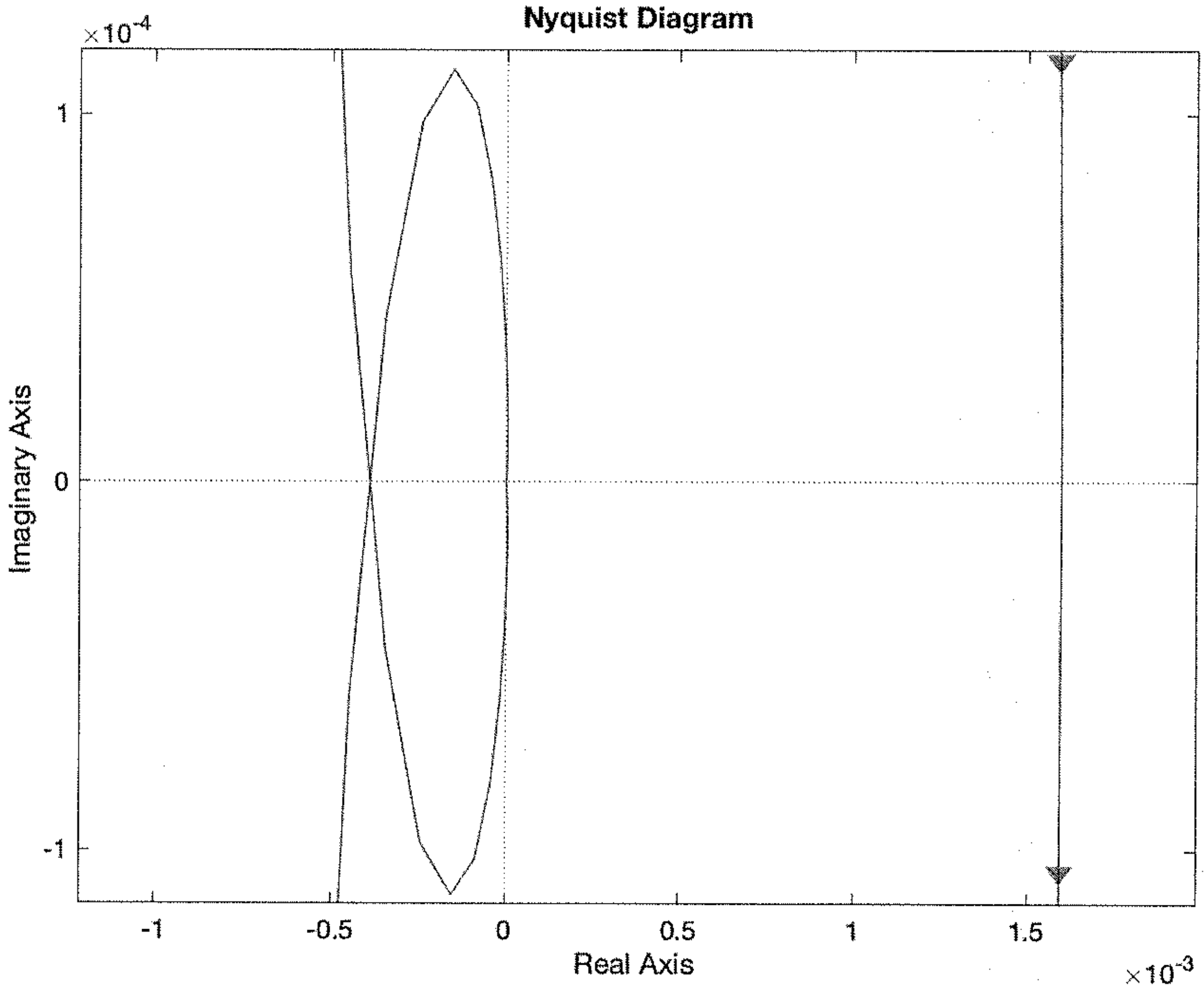
Nyquist Diagram
Imaginary Axis
Real Axis
0.025
0.02
0.015
0.01
0.005
0
-0.005
-0.01
-0.015
-0.02
-0.025
-0.01
-0.005
0
0.005
0.01
0.015
0.02
0.025
0.03
0.035
0.04

$$G(s) = \frac{1}{(s+5)^3}$$



Nyquist Diagram
×10^-4
3
2
1
0
-1
-2
-3
-0.01
-0.005
0
0.005
0.01
Imaginary Axis
Real Axis

$$G(s) = \frac{1}{(s+5)^4}$$


Nyquist Diagram
×10^-4
1
0
-1
Imaginary Axis
-1
-0.5
0
0.5
1
1.5
Real Axis
×10^-3

$\boxed{6}$ For $G(s) = \dfrac{K}{(s+a)(s+30)}$ under negative unity feedback, find

a) K, a (if any) s.t. $T_r \simeq 1$ second, $\zeta \simeq 0.5$

b) steady-state error in response to a unit step to a unit ramp input.

$$\Delta_{CL}(s) = (s+a)(s+30) + K$$
$$= s^2 + (30+a)s + (30a + K)$$

Set equal to $s^2 + 2\zeta\omega_n s + \omega_n^2$ to find ζ, ω_n in terms of K, a.

$$\Rightarrow 2\zeta\omega_n = 30 + a \qquad \omega_n^2 = 30a + K$$
$$\Rightarrow \omega_n = \sqrt{30a + K}$$

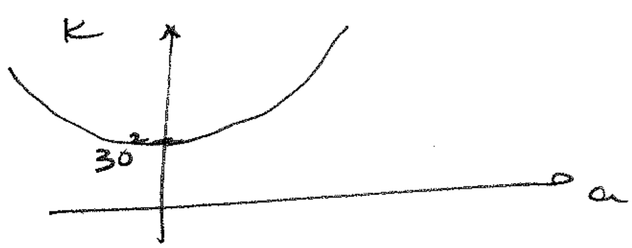
$$\zeta = \frac{30+a}{2\omega_n}$$

$$\zeta = \frac{30+a}{2\sqrt{30a+K}} = 0.5 \quad , \qquad T_r = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 1$$

$$30 + a = \sqrt{30a+K} \qquad \pi = \omega_n\sqrt{1-\zeta^2}$$

$$30^2 + 60a + a^2 = 30a + K \qquad \frac{\pi^2}{\omega_n^2} = 1 - \zeta^2$$

$$a^2 + 30a + 30^2 = K$$



$$\frac{\pi^2}{30a+K} = 1 - \frac{(30+a)^2}{4(30a+K)}$$

$$4\pi^2 = 4(30a+K) - (30+a)^2$$

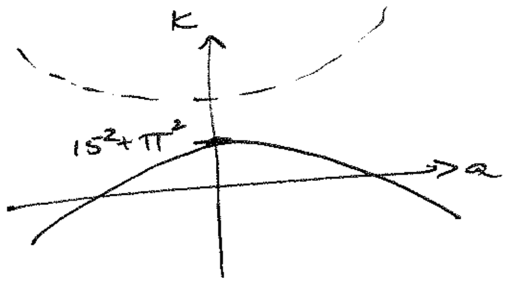
$$4\pi^2 = -(a^2 + 60a + 30^2) + 120a + 4K$$

$$4\pi^2 = -a^2 - 60a - 30^2 + 120a + 4k$$

$$4\pi^2 = -a^2 + 60a + (4k - 30^2)$$

$$a^2 - 60a + 4\pi^2 + 30^2 = 4k$$

$$\frac{a^2}{4} - 15a + \pi^2 + 15^2 = k$$



$\therefore$ curves will not intersect.

No values of k, a that satisfy both requirements.

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b) Type 0 plant

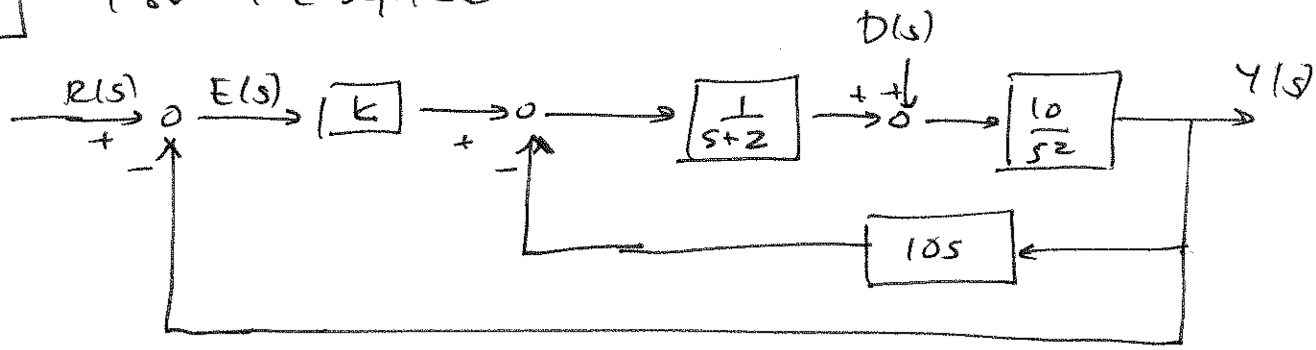
$\Rightarrow e_{ss} = \dfrac{1}{1+k_p}$, $k_p = \lim_{s\to 0} G(s)$ for unit step.

$$k_p = \frac{k}{a \cdot 30}$$

$$\therefore e_{ss} = \frac{1}{1 + \frac{k}{a \cdot 30}} = \frac{30a}{30a + k}$$

$\Rightarrow e_{ss} = \infty$ for unit ramp.

7 For the system



a) Find e_{ss} in response to a ramp input (when $D(s)=0$)

b) Find y_{ss} in response to a step disturbance input ($R(s)=0$)

c) Values of k to ensure stability (when $D(s)=0$)

$$Y(s) = \frac{10}{s^2}\left(D(s) + \frac{1}{s+2}\left(k\left(R(s)-Y(s)\right)\right) - 10s \cdot Y(s) \right)$$

$$= \frac{10}{s^2}\cdot D(s) + \frac{10}{s^2(s+2)}\cdot k \cdot R(s) - Y(s)\left[\frac{10}{s^2}\cdot\frac{1}{s+2}\cdot k + \frac{10}{s^2}\cdot\frac{1}{s+2}\cdot 10s\right]$$

$$Y(s)\left[1 + \frac{10}{s^2}\cdot\frac{1}{s+2}\left(k+10s\right)\right] = \frac{10}{s^2}\cdot D(s) + \frac{10k}{s^2(s+2)}R(s)$$

$$Y(s)\left[s^2(s+2) + 10k + 100s\right] = 10(s+2)\cdot D(s) + 10k\cdot R(s)$$

$$Y(s) = \underbrace{\frac{10k}{s^3+2s^2+100s+10k}}_{G_R(s)}\cdot R(s) + \underbrace{\frac{10(s+2)}{s^3+2s^2+100s+10k}}_{G_D(s)}\cdot D(s)$$

a) $E(s) = -Y(s) + R(s)$

$$= \left(\frac{-10k}{s^3 + 2s^2 + 100s + 10k} + 1 \right) \cdot R(s)$$

$$= \frac{s^3 + 2s^2 + 100s}{s^3 + 2s^2 + 100s + 10k} \cdot R(s)$$

$$\lim_{s \to 0} s E(s) = \lim_{s \to 0} \left(\quad \right) \cdot \frac{1}{s^2} \cdot s$$

$$= \frac{100}{10k} = \frac{10}{k}.$$

b) $Y(s) = G_0(s) \cdot R(s), \quad R(s) = \frac{1}{s}$

$$y_{ss} = \lim_{s \to 0} sY(s) = \lim_{s \to 0} G_0(s) \cdot \frac{1}{s} \cdot s$$

$$= \frac{10 \cdot 2}{10k} = \frac{2}{k}.$$

c) $\Delta_{cl}(s) = s^3 + 2s^2 + 100s + 10k.$

Hurwitz criterion: $1 > 0$, $2 > 0$, $100 > 0$, $10k > 0$

and $2 \cdot 100 - 10k \cdot 1 > 0$

$200 > 10k$

$\underline{20 > k}$