ECE 345 / ME 380: Introduction to Control Systems
Collaborative Quiz #0

Dr. Oishi

Due Thursday, August 29, 2019 at the end of class

NASA Astronauts recently completed a historic mission, in which they were transported to and from the International Space Station by a private company, SpaceX (`https://www.nytimes.com/interactive/2020/05/26/science/spacex-nasa.html`).

This was the first inhabited flight launched from the US since 2011. While on this mission, the crew completed the docking maneuver, SpaceX has previously demonstrated automated docking capabilities in its Dragon capsules.

> *Once it reached orbit, the tip of the gumdrop-shaped capsule pivoted open, exposing the capsule's docking mechanism underneath. As the Crew Dragon approached the station... it then used a series of lasers, sensors, and software to automatically dock this hardware to an available port on the outside of the ISS.*
>
> Loren Grush, "SpaceX's Crew Dragon capsule successfully docks to the ISS for the first time," The Verge, March 3, 2019.

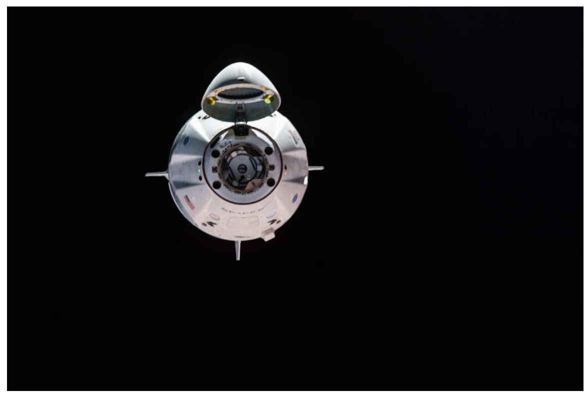


Figure 1: The SpaceX Dragon capsule preparing to dock with the International Space Station. The vehicle launched on May 30, 2020, and returned on August 2, 2020 with two NASA Astronauts. (*Image from NASA.*)

Docking is a critical maneuver, because of the precision needed and because of the potential for damage to both the capsule and the space station. The manual docking performed by NASA astronauts Robert L. Behnken and Douglas G. Hurley, `https://www.youtube.com/watch?v=Jd_aIRkI5ws`, shows the vehicle's thrusters firing in multiple directions to finesse the last stages of docking.

We can model the relative motion of the space capsule with respect to the space station by the Clohessey-Wiltshire-Hill (CWH) dynamics,

$$\ddot{z}(t) = -n^2 \cdot z(t) + n^2 \cdot f(t) \tag{1}$$

in which $z(t)$ is the spacecraft's out-of-plane position with respect to an elliptical orbit, and the thrusters generate the force $f(t)$ in the z-direction. The constant n is the orbital rate, and takes a value of approximately 1×10^{-3} for a low-earth orbit.

The thruster dynamics arise from the the combustion processes needed to create propulsive forces. We model these dynamics as

$$\dot{f}(t) = 0.5(r(t) - f(t)) \tag{2}$$

$sF(s) = 0.5R(s) - 0.5F(s) \rightarrow sF(s) + 0.5F(s) = 0.5R(s)$

$F(s)[s + 0.5] = 0.5R(s)$

① $G_{thr}(s) = \boxed{\frac{F(s)}{R(s)} = \frac{0.5}{s+0.5}}$ $a = 0.5$ $0.5e^{-0.5t} = g_{thr}(t)$

$s^2 Z(s) = -n^2 Z(s) + n^2 F(s) \longrightarrow s^2 Z(s) + n^2 Z(s) = n^2 F(s)$

$Z(s)\left[s^2 + n^2\right] = n^2 F(s)$

$G_{CWH}(s) = \boxed{\frac{Z(s)}{F(s)} = \frac{n^2}{s^2+n^2}}$ $\quad \omega = n \quad \sin(nt) = g_{CWH}(t)$

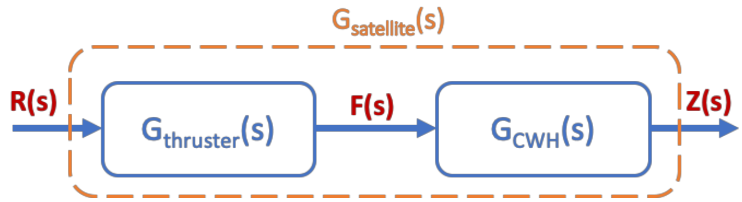


Figure 2: Block diagram that describes the relationship between desired thrust $r(t)$, actual thrust $f(t)$, and satellite position $z(t)$.

in which $r(t)$ is the desired propulsive force.

In the following, we consider the block diagram in Figure 2, that describes the effect of the thrusters on satellite position in the out-of-orbit direction.

We presume that all initial conditions are zero (i.e., $z(0) = 0,\ \dot{z}(0) = 0,\ f(0) = 0$).

1 Questions to be completed before class

1. Construct the transfer function $G_{\text{thruster}}(s) = \dfrac{F(s)}{R(s)}$, that is, with input $R(s)$ and output $F(s)$.
   see above

2. Compute the output $F(s)$ of $G_{\text{thruster}}(s)$ when the input is $r(t) = \cancel{\mathbb{1}(t)}$ $\frac{1}{s}$, the unit step. *Hint: First take the Laplace transform of the input.*
   $F(s) = G_{Th}(s) \cdot R(s) = \frac{0.5}{s+0.5} \cdot \frac{1}{s} = \frac{0.5}{s^2+0.5s}$

3. Construct the transfer function $G_{\text{CWH}}(s) = \dfrac{Z(s)}{F(s)}$, that is, with input $F(s)$ and output $Z(s)$.
   see above

4. Use your answers to Pre-Class questions #1 and #3 to show that the transfer function of the satellite, with input $r(t)$ and output $z(t)$, is

$$G_{\text{satellite}(s)} = \frac{0.5 \cdot n^2}{(s+0.5)(s^2+n^2)} \tag{3}$$

$\frac{\text{out}}{\text{in}} = \frac{Z(s)}{R(s)} = \underbrace{\frac{Z(s)}{F(s)}}_{G_{CWH}} \cdot \underbrace{\frac{F(s)}{R(s)}}_{G_{Thr.}(s)}$

$\left[\frac{n^2}{s^2+n^2}\right] \cdot \left[\frac{0.5}{s+0.5}\right] = \frac{0.5 \cdot n^2}{(s+0.5)(s^2+n^2)}$ ✓

2 Questions to be completed in class, in groups

1. Consider your response to Pre-Class Question #4. Which of the following also correctly describes the transfer function $\dfrac{Z(s)}{R(s)}$?

   (a) $G_{\text{satellite}}(s) = G_{\text{thruster}}(s) \cdot (s^2 + n^2)$

   (b) $G_{\text{satellite}}(s) = G_{\text{thruster}}(s) + \dfrac{n^2}{s^2+n^2}$

   (c) $G_{\text{satellite}}(s) = G_{\text{thruster}}(s) \cdot \dfrac{n^2}{s^2+n^2}$

   (d) $G_{\text{satellite}}(s) = \dfrac{G_{\text{thruster}}(s)}{1 + G_{\text{thruster}}(s)}$

2. Using the complex plane on the answer sheet, please sketch the poles and zeros of $G_{\text{satellite}}(s)$.

Consider the just the first subsystem in Figure 2, $G_{\text{thruster}}(s)$, that describes the effect of the desired thrust $r(t)$ on the actual thrust $f(t)$. Presume that we apply a unit step input, that is, $r(t) = \mathbf{1}(t)$. This would occur if we wanted a sudden, immediate change in thrust.

3. What is the resulting thrust in steady-state, $f_{ss} = \lim_{t\to\infty} f(t)$? *Hint: Use the Final Value Theorem, in combination with your response to Question #2 in the Pre-Class work.*

   (a) $f_{ss} = 1$

   (b) $f_{ss} = \frac{1}{0.5}$

   (c) $f_{ss} = \sin(0.5 \cdot t)$

   (d) $f_{ss} \to \infty$

$F(s) = \frac{0.5}{s^2+0.5s}$

F.V.T.

$f_{ss} = \lim_{t\to\infty} f(t) \to \lim_{s\to 0} sF(s) = \lim_{s\to 0} \frac{0.5\cancel{s}}{s^{\cancel{2}}+0.5\cancel{s}} \to 1$

Now consider the entire system, $G_{\text{satellite}}(s)$, i.e., both subsystems combined, as well as your response to Pre-Class Question #4.

4. Which of the following describes how the output $z(t)$ could be computed for an input signal $r(t)$?

   (a) $z(t) = \mathcal{L}^{-1}\{G_{\text{CWH}}(s)\} + \mathcal{L}^{-1}\{G_{\text{thruster}}(s) \cdot R(s)\}$

   (b) $z(t) = \mathcal{L}^{-1}\{G_{\text{CWH}}(s) + sR(s)G_{\text{thruster}}(s)\}$

   (c) $z(t) = \mathcal{L}^{-1}\{G_{\text{satellite}}(s) \cdot R(s)\}$

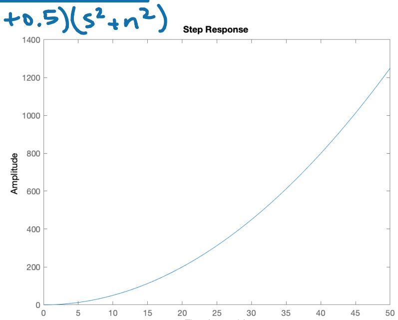
   (d) $z(t) = \mathcal{L}^{-1}\{G_{\text{satellite}}(s)\} \cdot \mathcal{L}^{-1}\{R(s)\}$

$z(t) = \mathcal{L}^{-1}[Z(s)]$

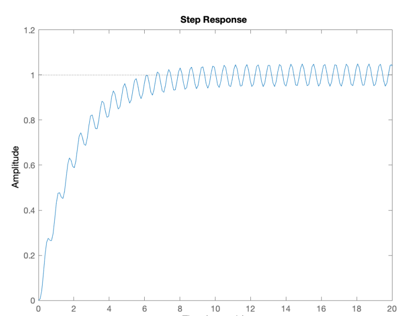
$G_{sat}(s) = \frac{Z(s)}{R(s)} \to Z(s) = G_{sat}(s) \cdot R(s)$

5. By looking solely at the terms in $G_{\text{satellite}}(s)$ (i.e., not doing detailed calculations), which of the following plots is most likely to represent $z(t)$ when the desired thrust $r(t)$ is a unit step? *Hint: If you were to do partial fraction expansion on $Y(s)$, what terms would appear?* Justify your response in 1-2 sentences.

$G_{sat}(s) = \frac{0.5 \cdot n^2}{(s+0.5)(s^2+n^2)}$



(a)



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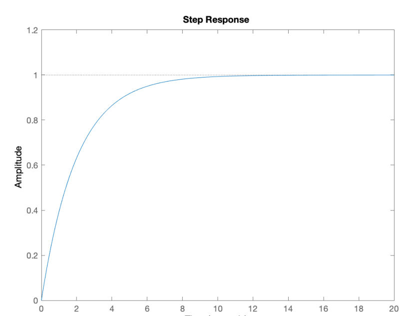

(c)

Please check your answers carefully before uploading your group's answer sheet to Learn.

3 If your group finishes early

Other points to consider (not necessary to hand in):

1. Calculate $Z(s)$ and $z(t)$ in response to a step input in $r(t)$.
2. What is the order of the numerator and denominator in $G_{\text{satellite}}(s)$?
3. Look at the Laplace transform table from Lecture Notes 2. For some arbitrary Laplace transform $X(s)$, what is the order of the polynominal in the denominator that is associated with a sinusoidal signal $x(t)$?