ECE 322L-Lecture 5-In class problem-Solution

$v_i = 25\sin(\omega t)$ mV $\quad V_{DD} = 3.3\ V \quad R_D = 8\ k\Omega$

$V_{TN} = 0.4\ V \quad K_N = 0.5\ mA/V^2 \quad \lambda = 0.02\ V^{-1} \quad I_{DQ} = 0.15\ mA$

$i_D = ? \quad v_{DS} = ?$

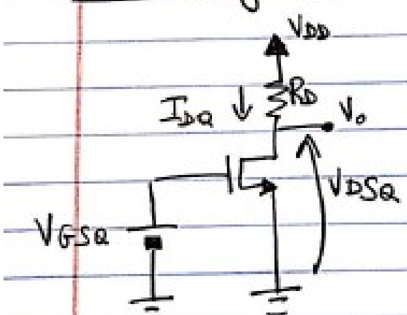
In the small-signal approximation $\left(v_{gs} \ll 2(V_{GSQ} - V_{TN})\right)$

$i_D = I_{DQ} + i_d$ and $v_{DS} = V_{DSQ} + v_{ds}$

We can assume that the small-signal condition is verified and calculate the DC and the AC portion of the i_D and v_{DS} using a DC circuit and an AC circuit, respectively. Once we'll have performed all our calculations we'll have to verify that the assumption of small signal is verified.

Let's start with the DC analysis.

DC analysis



KVL @ the output loop:

$V_{DSQ} = V_{DD} - R_D I_{DQ}$

$V_{DSQ} = 3.3 - 8K \cdot 0.15m = 2.1V$

$V_{DSQ} = 2.1$

Assuming that the transistor operates in saturation:

$I_{DQ} = K_N (V_{GSQ} - V_{TN})^2 (1 + \lambda V_{DSQ})$

$(V_{GSQ} - V_{TN})^2 = \dfrac{I_{DQ}}{K_N(1 + \lambda V_{DSQ})} \Rightarrow$

$V_{GSQ} = V_{TN} + \sqrt{\dfrac{0.15m}{0.5m(1 + 0.02 \cdot 2.1)}}$

Notice that I selected the solution with the positive sign as the one with the negative sign will yield $V_{GSQ} < V_{TN}$

$V_{GSQ} = 0.948$ V

Q point: $I_{DQ} = 0.15\,mA$; $V_{DSQ} = 2.1$ V ; $V_{GSQ} = 0.948$

$$V_{DS,SAT} = V_{GSQ} - V_{TN} = 0.948 - 0.4 = 0.548\text{ V}$$

✓ $V_{DS} > V_{DS,SAT} \Rightarrow$ The assumption of the transistor operating in saturation is verified.

Now that we have calculated V_{GSQ}, we can also verify that the small-signal condition is satisfied.

$$v_{gs_{max}} \ll 2(V_{GSQ} - V_{TN})$$

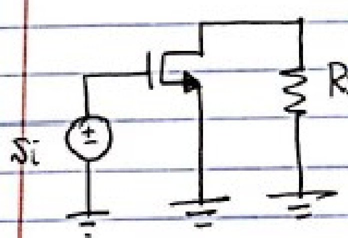
$$v_{gs_{max}} = 25\,mV \qquad 2(V_{GSQ} - V_{TN}) = 2(0.948 - 0.4) = 1.09$$

Hence

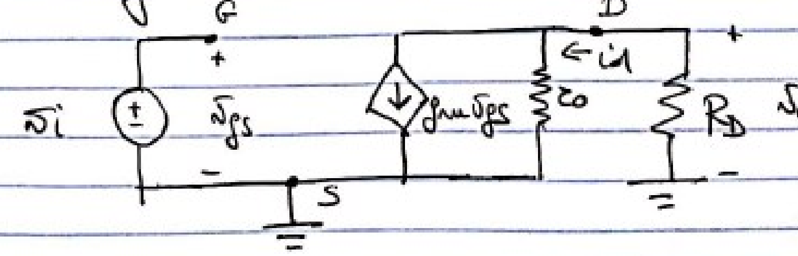
$$v_{gs_{max}} \ll 2(V_{GSQ} - V_{TN}) \quad ✓$$

Now we can proceed with the AC analysis.

AC circuit ($V_{GSQ} = 0$, $V_{DD} = 0$)



Small-signal circuit



$v_{ds} = -g_m v_{gs} (r_o // R_D)$

$$r_o = \frac{1}{\lambda I_{DQ}} = \frac{1}{0.02 \cdot 0.15m} = 333 k\Omega$$

$$g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{0.5m \cdot 0.15m} = 0.548 mA/V$$

$v_{ds} \simeq -g_m v_{gs} R_D$ ($r_o >> R_D$)

$v_{gs} = v_i$

$v_{ds} = -g_m v_i R_D = 0.548m \cdot 25 \sin(\omega t) \cdot 8K = -0.11 \sin(\omega t) V$

$i_d \simeq g_m v_i = 0.548m \cdot 25 \sin(\omega t)$
(Considering the current flowing in r_o negligible, which is reasonable given that $r_o = 333 K\Omega$)

Finally $v_{DS} = 2.1 - 0.11 \sin(\omega t) V$
$i_D = 0.15 + 0.0137 \sin(\omega t) mA$

Note that the maximum amplitude of the input signal (i.e. v_i) has increased from 25 mV to ~110 mV while going through the given circuit. Hence the circuit amplified the input signal with a small signal voltage gain of

$$A_v = \frac{v_o}{v_i} \approx 4.4 V/V$$