ECE 371
Materials and Devices

10/15/19 - Lecture 14

Extrinsic Semiconductors, Donor/Acceptor Statistics, and Charge Neutrality

General Information

- Homework #4 due today
- Homework #5 assigned and is due 10/24
- Reading for next time: 4.3-4.5

Ionization Energy

- Energy required to remove a weakly bound electron from the lattice and put it in the conduction band is called "*ionization energy*"
- Similarly, the energy required to elevate an electron from the lattice into an acceptor level also has an ionization energy (holes)
- Some dopants can function as donors AND acceptors. These are called *amphoteric dopants*. An example is a Si dopant in a GaAs lattice.

Table 4.3 | Impurity ionization energies in silicon and germanium

Impurity	Ionization energy (eV) Si	Ge
Donors		
Phosphorus	0.045	0.012
Arsenic	0.05	0.0127
Acceptors		
Boron	0.045	0.0104
Aluminum	0.06	0.0102

Table 4.4 | Impurity ionization energies in gallium arsenide

Impurity	Ionization energy (eV)
Donors	
Selenium	0.0059
Tellurium	0.0058
Silicon	0.0058
Germanium	0.0061
Acceptors	
Beryllium	0.028
Zinc	0.0307
Cadmium	0.0347
Silicon	0.0345
Germanium	0.0404

For $E_F > E_{Fi} \rightarrow$ n-type

$E_F < E_{Fi} \rightarrow$ p-type

$$n_0 = N_c \exp\left[\frac{-(E_c - E_F)}{kT}\right]$$

$$= n_i \exp\left[\frac{(E_F - E_{Fi})}{kT}\right]$$

$$p_0 = N_v \exp\left[\frac{-(E_F - E_v)}{kT}\right]$$

$$= n_i \exp\left[\frac{-(E_F - E_{Fi})}{kT}\right]$$

Extrinsic Semiconductors (Fermi Level)

- Addition of dopant atoms changes the position of the Fermi energy
  - For $E_F > E_{Fi} \rightarrow n > p \rightarrow$ n-type
  - For $E_F < E_{Fi} \rightarrow p > n \rightarrow$ p-type

shifting the blue curve – the prob. density

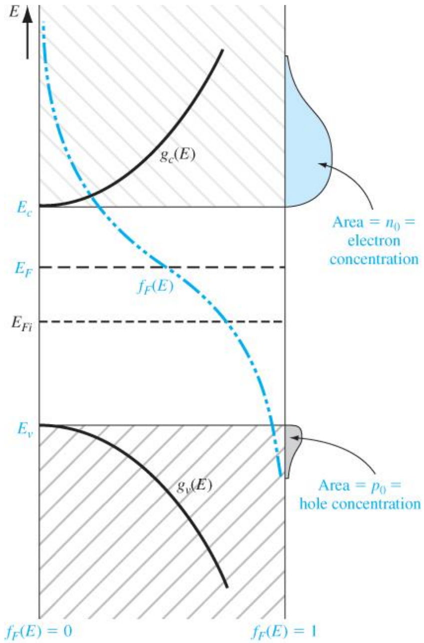

Figure 4.8 | Density of states functions, Fermi–Dirac probability function, and areas representing electron and hole concentrations for the case when E_F is above the intrinsic Fermi energy.

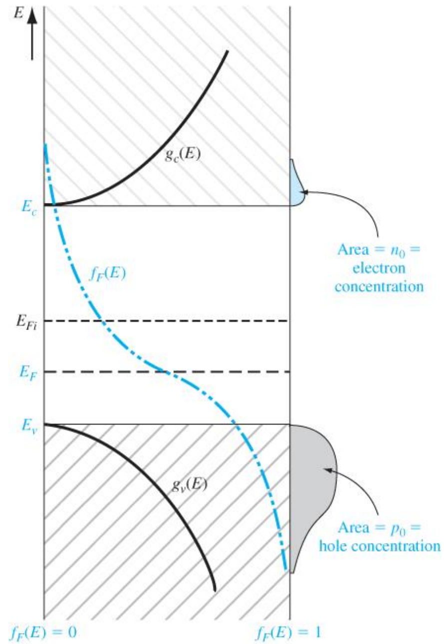

Figure 4.9 | Density of states functions, Fermi–Dirac probability function, and areas representing electron and hole concentrations for the case when E_F is below the intrinsic Fermi energy.

$$\boxed{n_0 = n_i\, exp\left[\frac{E_F - E_{Fi}}{kT}\right]}$$

$$\boxed{p_0 = n_i\, exp\left[\frac{-(E_F - E_{Fi})}{kT}\right]}$$

The n_0*p_0 Product

- The product of n_0 and p_0 is always a constant for a given material at a given temperature
- The following relationship was derived under the Boltzmann approximation and is valid under thermal equilibrium

$$\boxed{n_0\, p_0 = n_i^2}$$

<u>Terminology:</u>

- Electrons in n-type and holes in p-type → "majority carriers"
- Electrons in p-type and holes in n-type → "minority carriers"

$$n_o p_o = N_c N_v \exp\left[\frac{-(E_F - E_v)}{kT}\right] \exp\left[\frac{-(E_F - E_{Fi})}{kT}\right]$$

$$= N_c N_v \exp\left[\frac{-E_g}{kT}\right] = n_i^2$$

Degenerate Semiconductors

- *Nondegenerate* – individual dopant atoms are far apart and do not interact. Spacing between dopants is large.
- *Degenerate* – concentration of dopant atoms is high and individual dopant atoms interact, splitting the dopant energies into a band.
- E_F can move into the conduction or valence band if $n_0>N_c$ or $p_0>N_v$, respectively. This leads to a large electron or hole concentration.

Boltzmann doesn't apply, need full Fermi-Dirac equations

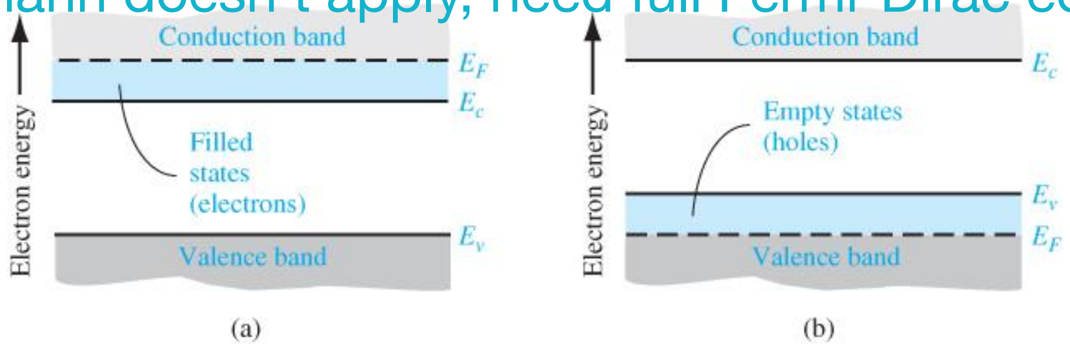

Figure 4.11 | Simplified energy-band diagrams for degenerately doped (a) n-type and (b) p-type semiconductors.

Exercise Problem 4.5

n_0 & p_0

■ **EXERCISE PROBLEM**

Ex 4.5 Determine the thermal-equilibrium concentrations of electrons and holes in silicon at $T = 300$ K if the Fermi energy level is 0.215 eV above the valence-band energy E_v. (Ans. $p_0 = 2.58 \times 10^{15}$ cm^{-3}, $n_0 = 1.87 \times 10^4$ cm^{-3})

$0.215 = E_v$

$1.12 = E_g$

If Fermi level is more than 3kT away from either valence or conductance band, good to go (we will operate in this range from here on out)

$$p_o = N_v \exp\left[\frac{-(E_F - E_V)}{kT}\right]$$

$N_V \rightarrow$ look up

1.04×10^{19}

$$= n_i \exp\left[\frac{-(E_F - E_{Fi})}{kT}\right]$$

$$= (1.04 \times 10^{19}) \exp\left[\frac{-(0.215)}{0.0259}\right]$$

$$= 2.58 \times 10^{15} \ cm^{-3}$$

then

$$n_o = \frac{n_i^2}{p_o} = \frac{(10^{10})^2}{2.58 \times 10^{15}} = 1.87 \times 10^4 \ cm^{-3}$$

will not match back of book exactly, but use the back of book (experimentally derived)

Donor/Acceptor Statistics

- Pauli exclusion principle applies to donors and acceptors
- n_d and p_a are the electron and hole concentrations in donor and acceptor states, respectively
- E_d and E_a are the donor and acceptor energy levels, respectively

Twice as many valence bands

- Factors of ½ and ¼ are $1/g$, where g is the degeneracy factor

how many remain in the donor level

Total donors are in there

$$n_d = \frac{N_d}{1 + \frac{1}{2} exp\left[\frac{E_d - E_F}{kT}\right]} = N_d - N_d^+$$

haven't been ionized up to the conduction band

Density of Electrons in donor states

$$p_a = \frac{N_a}{1 + \frac{1}{4} exp\left[\frac{E_F - E_a}{kT}\right]} = N_a - N_a^-$$

Density of Holes in acceptor states

Complete Ionization $\rightarrow \frac{n_d}{n_d+n_0} = 0$

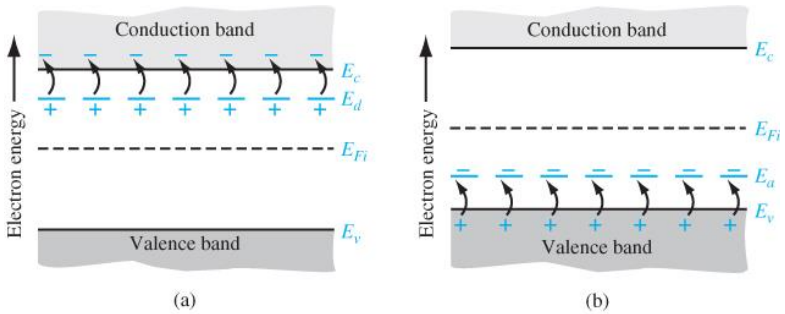

Figure 4.12 | Energy-band diagrams showing complete ionization of (a) donor states and (b) acceptor states.

Donors left

$$\frac{n_d}{n_d + n_0} = \frac{1}{1 + \frac{N_c}{2N_d} exp\left[\frac{-(E_c - E_d)}{kT}\right]}$$

total concentration

$$\frac{p_a}{p_a + p_0} = \frac{1}{1 + \frac{N_v}{4N_a} exp\left[\frac{-(E_a - E_v)}{kT}\right]}$$

+ = ionized atoms (e.g. Phos has lost its valence electron)

- N_d and N_a are the donor and acceptor concentrations
- $E_c - E_d$ and $E_a - E_v$ are the ionization energies for electrons and holes
- At room temperature (300 K) in silicon and GaAs, nearly all donor and acceptor atoms are ionized ("complete ionization") for shallow donors and acceptors
- Donor states become empty and acceptor states become full of electrons when ionized
- Some materials (e.g. – GaN) have deep donors or acceptors and only a fraction of the dopants are ionized

*see in-class notes for details

Example 4.7

Objective: Determine the fraction of total electrons still in the donor states at $T = 300$ K.

EXAMPLE 4.7

Consider phosphorus doping in silicon, for $T = 300$ K, at a concentration of $N_d = 10^{16}$ cm^{-3}.

$E_c - E_d \rightarrow$ lookup table 4.3 P in Si $= 0.045$ eV

■ **Solution**

Using Equation (4.55), we find

$$\frac{n_d}{n_0 + n_d} = \frac{1}{1 + \dfrac{2.8 \times 10^{19}}{2(10^{16})} \exp\left(\dfrac{-0.045}{0.0259}\right)} = 0.0041 = 0.41\%$$

$N_c \rightarrow$ lookup table 4. $= 2.8 \times 10^{19}$

■ **Comment**

This example shows that there are very few electrons in the donor state compared with the conduction band. Essentially all of the electrons from the donor states are in the conduction band and, since only about 0.4 percent of the donor states contain electrons, the donor states are said to be completely ionized.

Freeze Out

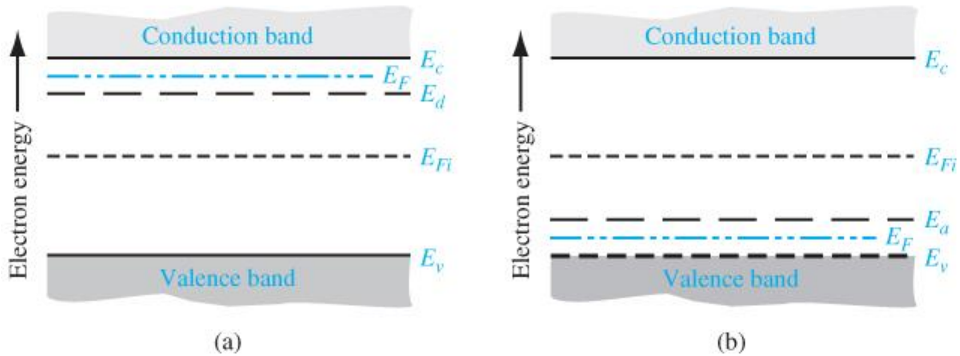

Figure 4.13 | Energy-band diagram at $T = 0$ K for (a) n-type and (b) p-type semiconductors.

- At T = 0 K, all electrons occupy the lowest possible energy states
- $n_d = N_d$ and $N_d^+ = 0$ at T = 0 K (states completely full)
- $p_a = N_a$ and $N_a^- = 0$ at T = 0 K (states completely empty)
- For n-type, we must have $E_F > E_d$
- For p-type, we must have $E_F < E_a$

*see in-class notes for details

Charge Neutrality

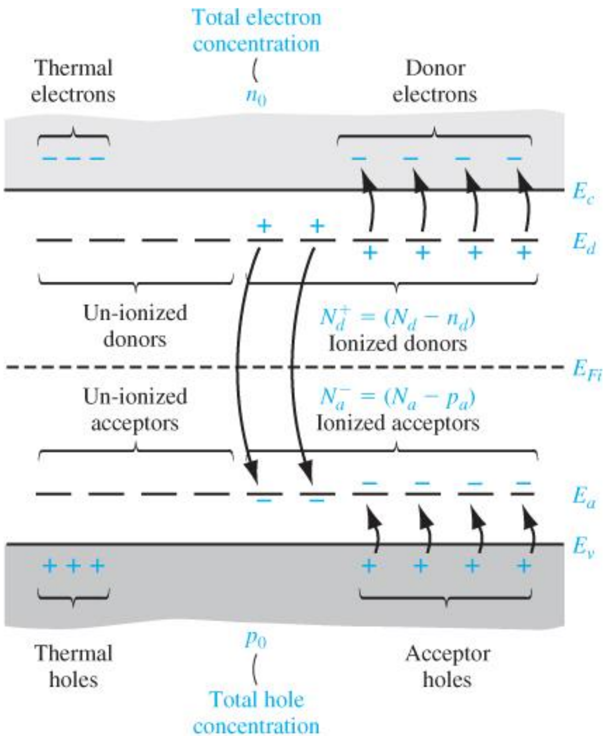

Figure 4.14 | Energy-band diagram of a compensated semiconductor showing ionized and un-ionized donors and acceptors.

- *Compensated Semiconductor:* contains both donor and acceptor impurity atoms in the same region

$N_d > N_a$ n-type compensated
$N_a > N_d$ p-type compensated
$N_a = N_d$ completely compensated

- In thermal equilibrium, the net charge density is zero

charge neutrality

$$\boxed{n_0 + N_a^- = p_0 + N_d^+}$$

$$\boxed{n_0 + (N_a - p_a) = p_0 + (N_d - n_d)}$$

Equilibrium Concentrations

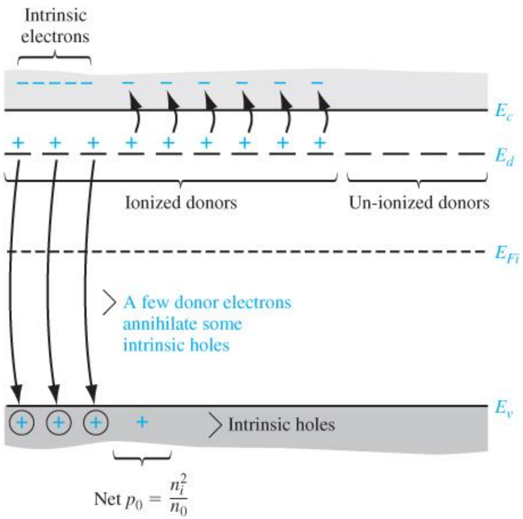

Figure 4.15 | Energy-band diagram showing the redistribution of electrons when donors are added.

Electron concentration in an n-type semiconductor

$$n_0 = \frac{(N_d - N_a)}{2} + \sqrt{\left(\frac{N_d - N_a}{2}\right)^2 + n_i^2}$$

Hole concentration in a p-type semiconductor

$$p_0 = \frac{(N_a - N_d)}{2} + \sqrt{\left(\frac{N_a - N_d}{2}\right)^2 + n_i^2}$$

- Above expressions assume complete ionization
- Calculate majority carrier concentration using the above expression and then calculate minority carrier concentration using $n_i^2 = n_0 p_0$
- Some impurity donors/acceptors annihilate intrinsic holes/electrons
- Net electron concentration is not simply the sum of N_d and n_i
- If $N_a = N_d$, the material behaves as if it is intrinsic

Example 4.9

Objective: Determine the thermal-equilibrium electron and hole concentrations in silicon at $T = 300$ K for given doping concentrations. (*a*) Let $N_d = 10^{16}$ cm^{-3} and $N_a = 0$. (*b*) Let $N_d = 5 \times 10^{15}$ cm^{-3} and $N_a = 2 \times 10^{15}$ cm^{-3}.

EXAMPLE 4.9

Recall that $n_i = 1.5 \times 10^{10}$ cm^{-3} in silicon at $T = 300$ K.

■ **Solution**

(*a*) From Equation (4.60), the majority carrier electron concentration is

$$n_0 = \frac{10^{16}}{2} + \sqrt{\left(\frac{10^{16}}{2}\right)^2 + (1.5 \times 10^{10})^2} \cong 10^{16} \text{ cm}^{-3}$$

The minority carrier hole concentration is found to be

$$p_0 = \frac{n_i^2}{n_0} = \frac{(1.5 \times 10^{10})^2}{10^{16}} = 2.25 \times 10^4 \text{ cm}^{-3}$$

(*b*) Again, from Equation (4.60), the majority carrier electron concentration is

$$n_0 = \frac{5 \times 10^{15} - 2 \times 10^{15}}{2} + \sqrt{\left(\frac{5 \times 10^{15} - 2 \times 10^{15}}{2}\right)^2 + (1.5 \times 10^{10})^2} \cong 3 \times 10^{15} \text{ cm}^{-3}$$

The minority carrier hole concentration is

$$p_0 = \frac{n_i^2}{n_0} = \frac{(1.5 \times 10^{10})^2}{3 \times 10^{15}} = 7.5 \times 10^4 \text{ cm}^{-3}$$

Concentration vs. Temperature

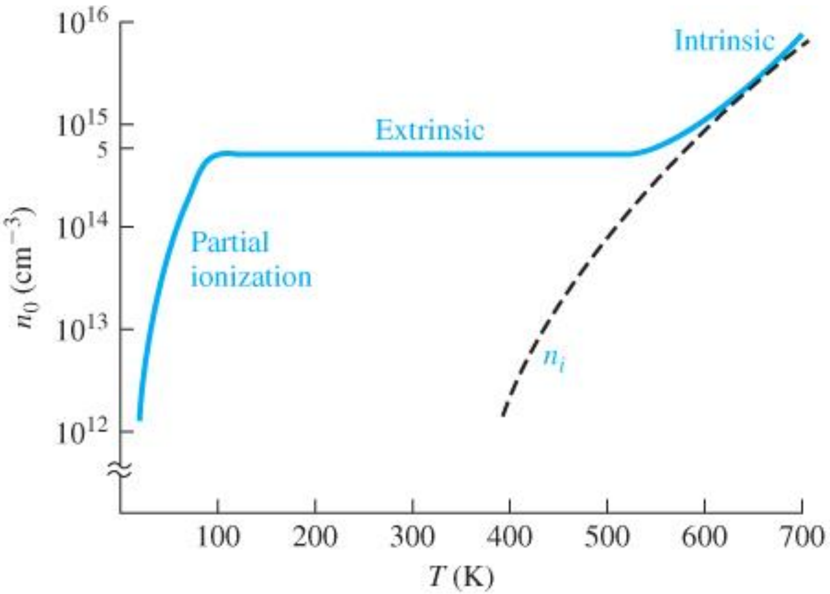

Figure 4.16 | Electron concentration versus temperature showing the three regions: partial ionization, extrinsic, and intrinsic.