Course ID: ECE 341 Communication Systems- Fall
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Homework #3

Corresponding to Chapter 3 of Principles of Communications, Rodger E. Zimmer and William H. Tranter, John Wiley, 7th Edition.

1. Find the transmitted signal as a Fourier series and determine the transmitted power, given a message signal $m(t)=\sum_{k=1}^{5}\frac{10}{k}\sin(2\pi\,\mathrm{k}f_m\,t)$ and the carrier is c(t)=100cos(200πt).

2. The output of an AM modulator is $x_c(t)$=40cos[2π200t]+5cos[2π180t]+ 5cos[2π220t]. Determine the modulation index and the efficiency.

3. The output of an AM modulator is $x_c(t)$=Acos[2π200t]+Bcos[2π180t]+ Bcos[2π220t]. The carrier power is P_0 and the efficiency is E_{ff}. Determine an expression for E_{ff} with respect to P_0, A and B and determine A, B and modulation index (given: P_0=200W and E_{ff}=0.3)

To be delivered at instructor's office: 9 October 2019

Good Luck!

1. Find the transmitted signal as a Fourier series and determine the transmitted power, given a message signal $m(t)=\sum_{k=1}^{5}\frac{10}{k}\sin(2\pi \mathrm{k} f_m t)$ and the carrier is c(t)=100cos(200πt).

$$X_c(t) = m(t)\cdot c(t)$$

$$= \sum_{K=1}^{5} \frac{1000}{K} \sin(2\pi f_m K t)\cos(200\pi t)$$

$$= \sum_{K=1}^{5} \frac{1000}{K} \cos\left(2\pi f_m K t - \frac{\pi}{2}\right)\cos(200\pi t)$$

let $2\pi f_m K t - \frac{\pi}{2} = x$, $200\pi t = y$

$$= \sum_{K=1}^{5} \frac{1000}{K} \cos(x)\cos(y)$$

$$= \sum_{K=1}^{5} \frac{\cancel{1000}}{K}\left[\frac{1}{\cancel{2}}\left(\cos(x-y)+\cos(x+y)\right)\right]$$

let $x-y = A$, $x+y = B$

$$= \sum_{K=1}^{5} \frac{500}{K}\left[\frac{1}{2}\left(e^{jA}+e^{-jA}\right)+\frac{1}{2}\left(e^{jB}+e^{-jB}\right)\right]$$

$$= \sum_{K=1}^{5} \frac{250}{K}\left[e^{jA}+e^{-jA}+e^{jB}+e^{-jB}\right]$$

$$= \sum_{K=1}^{5} \frac{250}{K}\Big[e^{j(2\pi f_m K t - \frac{\pi}{2} - 200\pi t)} + e^{-j(2\pi f_m K t - \frac{\pi}{2} - 200\pi t)} + e^{j(2\pi f_m K t - \frac{\pi}{2} + 200\pi t)} + e^{-j(2\pi f_m K t - \frac{\pi}{2} + 200\pi t)}\Big]$$

$$= \sum_{k=1}^{5} \frac{250}{k}\Big[e^{-j\pi/2} e^{j2\pi t(f_m k - 100)} + e^{j\frac{\pi}{2}} e^{-j2\pi t(f_m k - 100)}$$

$$+ e^{-j\pi/2} e^{j2\pi t(f_m k + 100)} + e^{j\frac{\pi}{2}} e^{-j2\pi t(f_m k + 100)}\Big]$$

$$\frac{250}{k} e^{-j\pi/2} \qquad \rightarrow \left| e^{-j\pi/2} \right| = 1$$

$$\frac{250}{k} e^{j\pi/2}$$

$n = kf_m - 100,$
$kf_m + 100$

must be an integer $\therefore f_m$ must be integer

$$P = \cancel{X_0^2} \xrightarrow{} 0 + \sum_{k=1}^{5} 2|X_n|^2$$

$$= 2\left|\frac{250}{1}\right|^2 + 2\left|\frac{250}{2}\right|^2 + 2\left|\frac{250}{3}\right|^2 + 2\left|\frac{250}{4}\right|^2$$

$$+ 2\left|\frac{250}{5}\right|^2$$

$$= \boxed{182951.39 \text{ W}}$$

Fourier Coeff.

positive	negative
$\frac{250}{k} e^{-j\pi/2}$	$\frac{250}{k} e^{j\pi/2}$

2. The output of an AM modulator is $x_c(t)=40\cos[2\pi 200t]+5\cos[2\pi 180t]+5\cos[2\pi 220t]$. Determine the modulation index and the efficiency.

$$x_c(t) = 40\cos(2\pi 200t) + 5\cos(2\pi 180t) + 5\cos(2\pi 220t)$$

$$= 40\cos(2\pi 200t) + 5\left[2\cos\left(\frac{2\pi 180t + 2\pi 220t}{2}\right)\cos(40\pi t)\right]$$

$$= 40\underline{\cos(2\pi 200t)} + 10\underline{\cos(2\pi 200t)}\cos(40\pi t)$$

$$= [40 + 10\cos(40\pi t)] \cdot \cos(2\pi 200t)$$

$$= 40\left[1 + \tfrac{1}{4}\cos(40\pi t)\right] \cdot \cos(2\pi 200t)$$

Modulation index $(a) = \boxed{\frac{1}{4}}$

$$x_c(t) = A_c[1 + am_n(t)]\cos(2\pi f_c t) \quad (3.10)$$

Carrier signal = $40\cos(2\pi 200t)$

$$E_{ff} = \frac{\left(\frac{1}{4}\right)^2 \langle \cancel{\cos^2(40\pi t)} \rangle^{1/2}}{1 + \left(\frac{1}{4}\right)^2 \langle \cancel{\cos^2(40\pi t)} \rangle^{\frac{1}{2}}}$$

$$= 0.03030 \longrightarrow \boxed{3.03\%}$$

$$E_{ff} = \frac{a^2\langle m_n^2(t)\rangle}{1 + a^2\langle m_n^2(t)\rangle} \quad (3.15)$$

message power

total power

$P_{info} = P_{message} = \frac{25}{2} + \frac{25}{2} = 25$

$P_{carrier} = \frac{40}{2}$

3. The output of an AM modulator is $x_c(t)=A\cos[2\pi 200t]+B\cos[2\pi 180t]+B\cos[2\pi 220t]$. The carrier power is P_0 and the efficiency is E_{ff}. Determine an expression for E_{ff} with respect to P_0, A and B and determine A, B and modulation index (given: $P_0=200W$ and $E_{ff}=0.3$)

From #2 we found:

$$A\cos(2\pi 200t) + 2B\cos(2\pi 200t)\cos(40\pi t)$$

$$\underbrace{A}_{A_c}\left[1 + \underbrace{\frac{2B}{A}}_{a}\underbrace{\cos(40\pi t)}_{m_n(t)}\right]\cdot\cos(2\pi 200t)$$

$$P_o = \frac{A_c^2}{2} = 200W \longrightarrow \boxed{20 = A_c = A}$$

$$E_{ff} = \frac{\left(\frac{2B}{A}\right)^2\left(\frac{1}{2}\right)}{1+\left(\frac{2B}{A}\right)^2\left(\frac{1}{2}\right)} = \frac{\left(\frac{\cancel{4}B^2}{A^2}\right)\left(\frac{1}{\cancel{2}}\right)}{1+\left(\frac{\cancel{4}B^2}{A^2}\right)\left(\frac{1}{\cancel{2}}\right)} \qquad A = 2P_o$$

$$E_{ff} = \boxed{\frac{B^2}{P_o + B^2}} = 0.3 \rightarrow \boxed{B = 9.258}$$

$$\text{mod index } (a) = \frac{2B}{A} = \boxed{0.9258}$$

$$P_{carrier} = \frac{A^2}{2}$$

$$P_{info} = \frac{B^2}{2} + \frac{B^2}{2} = B^2$$

$$\frac{B^2}{\frac{A^2}{2} + B^2} =$$