

ECE 345 / ME 380: Introduction to Control Systems

Problem Set #2

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Due Thursday, September 17, 2020 at 3:30pm

1. (+10 points) Consider the system described by

$$G(s) = \frac{2(s+2)}{s(s+4)(s+10)} \quad (1)$$

- (a) Find the poles and the zeros of $G(s)$.

poles: $s = 0, -4, -10$

zeros: $s = -2$

- (b) Put the transfer function $G(s)$ in proper form, with one polynomial in the numerator and one polynomial in the denominator.

$$G(s) = \frac{2s+4}{s^3+14s^2+40s}$$

- (c) Find the characteristic equation of $G(s)$.

$$\Delta(s) = s^3 + 14s^2 + 40s = 0$$

2. (+10 points) The longitudinal dynamics of a vertical take-off and landing aircraft that is hovering are described by:

$$\begin{aligned} \dot{x} &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -5 & -2 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix} u \\ y &= \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x \end{aligned} \quad (2)$$

- (a) Find the characteristic equation of this system.

$$\begin{aligned} \Delta(s) &= |sI - A| \\ &= \det \begin{bmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 0 & 5 & s+2 \end{bmatrix} \\ &= s^3 + 2s^2 + 5s \end{aligned}$$

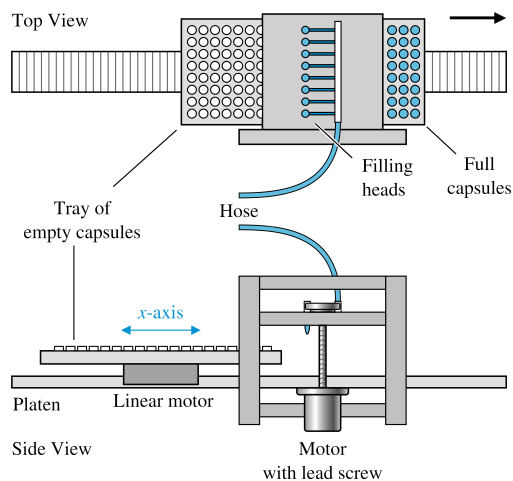
(b) Where are the poles of the system located?

$$\text{Eigenvalues}(A) = 0, -1 \pm 2i$$

3. (+10 points) State-space representations are not unique. A single system can be represented in several possible ways. Consider the following two systems:

$$\text{System 1: } \begin{cases} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -3 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 3 \\ -2 \end{bmatrix} u \\ y = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \end{cases}$$

$$\text{System 2: } \begin{cases} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \\ y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \end{cases}$$



(a) Find the transfer function $G_1(s) = C_1(sI - A_1)^{-1}B_1 + D_1$ for System 1.

$$\begin{aligned} G_1(s) &= -\frac{2}{s+2} + \frac{3}{s+3} \\ &= \frac{s}{s^2 + 5s + 6} \end{aligned}$$

(b) Find the transfer function $G_2(s) = C_2(sI - A_2)^{-1}B_2 + D_2$ for System 2.

$$G_1(s) = \frac{s}{s^2 + 5s + 6}$$

- (c) Describe the relationship between G_1 and G_2 . What zeros and/or poles do they have in common?

Even though the state-space representations are different, G_1 and G_2 have the exact same transfer functions. This is because, as the problem stated, state-space representations are not unique and a single system can be represented in several possible ways.

poles for both: $s = -2, -3$

zeros for both: $s = 0$

4. (+15 points) Consider the following state-space system, that describes the dynamics of a system for automatically dispensing fluid into capsules. A tray of capsules is guided through the dispenser by a linear motor with motor torque $u(t)$ (the input). The tray position is the output, $y(t)$.

$$G(s) = \frac{3}{s^2 + Ks + 3} \quad (3)$$

- (a) Using Matlab, follow the steps below for each of $K = 1, 2, 3, 4$. Use the `diary` or `publish` commands to record your code, and hand in the history of Matlab command-line inputs and outputs as well as the *single* plot that you generate. *Note: Please append the Matlab file and plot to your homework, so that you hand in a **single.pdf**. Multiple files will not be accepted.*

- i. First create transfer functions $G_1(s), \dots, G_4(s)$. For example, for the first system,

```
>> G1 = tf(3, [1 1 3]);
```

- ii. On a single figure, plot step responses for each of these systems using

```
>> step(G1,G2,G3,G4)
```

```
>> legend('G1','G2','G3','G4')
```

Please see the final two pages for Matlab code and figure.

- (b) Consider the oscillatory nature of the step responses. What happens as K increases? Which value of K produces the most oscillatory response? Which produces the least?

K has the effect of being a dampening agent in our step responses. As K increases, the oscillations decrease; thus, $K = 1$ is the least damped and most oscillatory response, while $K = 4$ is the most damped and exhibits the least oscillatory response.

```
G = cell(4,1);
for K = 1:1:4
    G{K}=tf(3, [1 K 3]);G{K}
end
```

ans =

$$\frac{3}{s^2 + s + 3}$$

Continuous-time transfer function.

ans =

$$\frac{3}{s^2 + 2 s + 3}$$

Continuous-time transfer function.

ans =

$$\frac{3}{s^2 + 3 s + 3}$$

Continuous-time transfer function.

ans =

$$\frac{3}{s^2 + 4 s + 3}$$

Continuous-time transfer function.

```
step(G{1},G{2},G{3},G{4});
legend('G1','G2','G3','G4')
```

