David Kirby

ECE 371
Materials and Devices
HW #7
Due: Thursday 11/21/19 at the beginning of class

*All problems from Neamen 4[th] Edition Ch. 7 and Ch. 8.

7.17 Consider the pn junction described in Problem 7.10 for $T = 300$ K. The cross-sectional area of the junction is 2×10^{-4} cm^2 and the applied reverse-biased voltage is $V_R = 2.5$ V. Calculate (*a*) V_{bi}, (*b*) x_n, x_p, W, (*c*) $|E_{max}|$, and (*d*) the junction capacitance.

7.19 A silicon n$^+$p junction is biased at $V_R = 5$ V. (*a*) Determine the change in built-in potential barrier if the doping concentration in the p region increases by a factor of 3. (*b*) Determine the ratio of junction capacitance when the acceptor doping is $3N_a$ compared to that when the acceptor doping is N_a. (*c*) Why does the junction capacitance increase when the doping concentration increases?

7.28 A silicon pn junction at $T = 300$ K has the doping profile shown in Figure P7.28. Calculate (*a*) V_{bi}, (*b*) x_n and x_p at zero bias, and (*c*) the applied bias required so that $x_n = 30\ \mu$m.

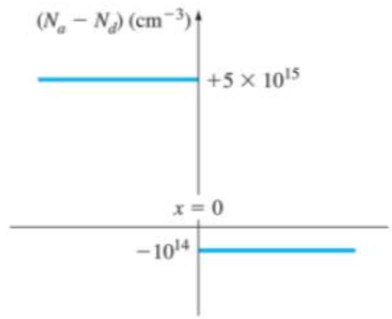


Figure P7.28 | Figure for Problem 7.28.

7.30 A silicon p$^+$n junction has doping concentrations of $N_a = 2 \times 10^{17}$ cm^{-3} and $N_d = 2 \times 10^{15}$ cm^{-3}. The cross-sectional area is 10^{-5} cm^2. Calculate (*a*) V_{bi} and (*b*) the junction capacitance at (*i*) $V_R = 1$ V, (*ii*) $V_R = 3$ V, and (*iii*) $V_R = 5$ V. (*c*) Plot $1/C^2$ versus V_R and show that the slope can be used to find N_d and the intercept at the voltage axis yields V_{bi}.

***7.34** A silicon PIN junction has the doping profile shown in Figure P7.34. The "I" corresponds to an ideal intrinsic region in which there is no impurity doping concentration. A reverse-biased voltage is applied to the PIN junction so that the total depletion width extends from -2 μm to $+2$ μm. (*a*) Using Poisson's equation, calculate the magnitude of the electric field at $x = 0$. (*b*) Sketch the electric field through the PIN junction. (*c*) Calculate the reverse-biased voltage that must be applied.

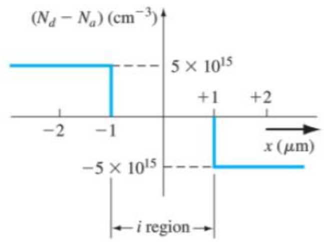


Figure P7.34 | Figure for Problem 7.34.

**Unless otherwise stated in the problem, you may need to use the following values for Si and GaAs materials in the following problems

[**Note:** In the following problems, assume $T = 300$ K and the following parameters unless otherwise stated. For silicon pn junctions: $D_n = 25$ cm^2/s, $D_p = 10$ cm^2/s, $\tau_{n0} = 5 \times 10^{-7}$ s, $\tau_{p0} = 10^{-7}$ s. For GaAs pn junctions: $D_n = 205$ cm^2/s, $D_p = 9.8$ cm^2/s, $\tau_{n0} = 5 \times 10^{-8}$ s, $\tau_{p0} = 10^{-8}$ s.]

8.2 A silicon pn junction has impurity doping concentrations of $N_d = 2 \times 10^{15}$ cm^{-3} and $N_a = 8 \times 10^{15}$ cm^{-3}. Determine the minority carrier concentrations at the edges of the space charge region for (*a*) $V_a = 0.45$ V, (*b*) $V_a = 0.55$ V, and (*c*) $V_a = -0.55$ V.

8.5 Consider a GaAs pn junction with doping concentrations $N_a = 5 \times 10^{16}$ cm^{-3} and $N_d = 10^{16}$ cm^{-3}. The junction cross-sectional area is $A = 10^{-3}$ cm^2 and the applied forward-bias voltage is $V_a = 1.10$ V. Calculate the (*a*) minority electron diffusion current at the edge of the space charge region, (*b*) minority hole diffusion current at the edge of the space charge region, and (*c*) total current in the pn junction diode.

8.7 An ideal germanium pn junction diode has the following parameters: $N_a = 4 \times 10^{15}$ cm^{-3}, $N_d = 2 \times 10^{17}$ cm^{-3}, $D_p = 48$ cm^2/s, $D_n = 90$ cm^2/s, $\tau_{p0} = \tau_{n0} = 2 \times 10^{-6}$ s, and $A = 10^{-4}$ cm^2. Determine the diode current for (*a*) a forward-bias voltage of 0.25 V and (*b*) a reverse-biased voltage of 0.25 V.

8.15 A silicon pn junction with a cross-sectional area of 10^{-4} cm^2 has the following properties at $T = 300$ K:

n region	p region
$N_d = 10^{17}$ cm^{-3}	$N_a = 5 \times 10^{15}$ cm^{-3}
$\tau_{p0} = 10^{-7}$ s	$\tau_{n0} = 10^{-6}$ s
$\mu_n = 850$ cm^2/V-s	$\mu_n = 1250$ cm^2/V-s
$\mu_p = 320$ cm^2/V-s	$\mu_p = 420$ cm^2/V-s

(*a*) Sketch the thermal equilibrium energy-band diagram of the pn junction, including the values of the Fermi level with respect to the intrinsic level on each side of the junction. (*b*) Calculate the reverse-saturation current I_s and determine the forward-bias current I at a forward-bias voltage of 0.5 V. (*c*) Determine the ratio of hole current to total current at the space charge edge x_n.

8.17 Consider the ideal long silicon pn junction shown in Figure P8.17. $T = 300$ K. The n region is doped with 10^{16} donor atoms per cm^3 and the p region is doped with 5×10^{16} acceptor atoms per cm^3. The minority carrier lifetimes are $\tau_{n0} = 0.05$ μs and $\tau_{p0} = 0.01$ μs. The minority carrier diffusion coefficients are $D_n = 23$ cm^2/s and $D_p = 8$ cm^2/s. The forward-bias voltage is $V_a = 0.610$ V. Calculate (*a*) the excess hole concentration as a function of x for $x \geq 0$, (*b*) the hole diffusion current density at $x = 3 \times 10^{-4}$ cm, and (*c*) the electron current density at $x = 3 \times 10^{-4}$ cm.

Note: x = 0 is defined at the edge of the depletion region on the n-side (what we normally call x_n)

Hint part (c): The total current (sum of electron and hole) at any point (x) in the structure must be the same. You can find the total current by summing the currents at the depletion region edges, as we did in the derivation of the ideal didoe equation. Since you calculate the hole current at x = 3um in part (c), you can then get the electron current at x = 3um by sustracting the hole current at x = 3um from the total current.

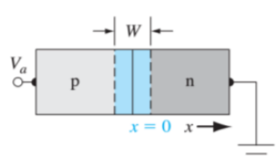


Figure P8.17 | Figure for Problem 8.17.

7.17 Consider the pn junction described in Problem 7.10 for $T = 300$ K. The cross-sectional area of the junction is 2×10^{-4} cm^2 and the applied reverse-biased voltage is $V_R = 2.5$ V. Calculate (*a*) V_{bi}, (*b*) x_n, x_p, W, (*c*) $|E_{max}|$, and (*d*) the junction capacitance.

(a) $V_{bi} = V_T \ln\left(\frac{N_a N_d}{n_i^2}\right) = 0.0259 \ln\left(\frac{(2\times10^{17})(4\times10^{16})}{1.5\times10^{10}}\right) = \boxed{808.1\,\text{mV}}$

(b) $x_n = \sqrt{\frac{2\epsilon_s(V_{bi}+V_r)}{e}\left(\frac{N_a}{N_d}\right)\left(\frac{1}{N_a+N_d}\right)} = \boxed{298.54\ \text{nm}}$

$x_p = \sqrt{\frac{2\epsilon_s(V_{bi}+V_r)}{e}\left(\frac{N_d}{N_a}\right)\left(\frac{1}{N_a+N_d}\right)} = \boxed{59.707\ \text{nm}}$

$W = \sqrt{\frac{2\epsilon_s(V_{bi}+V_r)}{e}\left(\frac{N_a+N_d}{N_a\cdot N_d}\right)} = \boxed{358.24\ \text{nm}}$

(c) $|E_{max}| = \frac{2(V_{bi}+V_r)}{W} = \boxed{18.469\ \frac{\mu V}{m}}$

(d) $C = A\sqrt{\frac{e\epsilon_s N_a\cdot N_d}{2(V_{bi}+V_r)(N_a+N_d)}} = \boxed{5.78\ \text{pF}}$

7.19 A silicon n^+p junction is biased at $V_R = 5$ V. (*a*) Determine the change in built-in potential barrier if the doping concentration in the p region increases by a factor of 3. (*b*) Determine the ratio of junction capacitance when the acceptor doping is $3N_a$ compared to that when the acceptor doping is N_a. (*c*) Why does the junction capacitance increase when the doping concentration increases?

(a) $\Delta V_{bi} = V_T \ln\left(\frac{3 N_a N_d}{n_i^2}\right) - V_T \ln\left(\frac{N_a N_d}{n_i^2}\right) = 0.0259 \ln(3)$

$= \boxed{28.454 \text{ mV}}$

(b) $$\frac{\cancel{A}\sqrt{\dfrac{e\cancel{\epsilon_s} \cdot 3N_a \cdot \cancel{N_d}}{\cancel{2(V_{bi}+V_r)}(\cancel{3N_a+}N_d)}}}{\cancel{A}\sqrt{\dfrac{e\epsilon_s N_a \cdot N_d}{2(V_{bi}+V_r)(N_a+N_d)}}} = \sqrt{3} = \boxed{1.7321}$$

(c) As the doping concentration increases, the width decreases. As we learned in early physics classes, decreasing the width increases the capacitance.

7.28 A silicon pn junction at $T = 300$ K has the doping profile shown in Figure P7.28. Calculate (*a*) V_{bi}, (*b*) x_n and x_p at zero bias, and (*c*) the applied bias required so that $x_n = 30\ \mu$m.

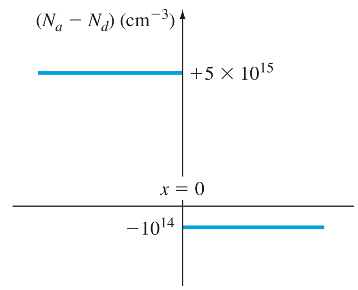


Figure P7.28 | Figure for Problem 7.28.

(a) $V_{bi} = V_T \ln\left(\frac{N_a N_d}{n_i^2}\right) = \boxed{557.41\,mV}$

(b) $x_n = \sqrt{\frac{2\epsilon_s (V_{bi} + V_r)}{e}\left(\frac{N_a}{N_d}\right)\left(\frac{1}{N_a + N_d}\right)}$

$= \boxed{2.6584\,\mu m}$

$x_p = \sqrt{\frac{2\epsilon_s (V_{bi} + V_r)}{e}\left(\frac{N_d}{N_a}\right)\left(\frac{1}{N_a + N_d}\right)}$

$= \boxed{53.167\,nm}$

(c) $x_n = \sqrt{\frac{2\epsilon_s (V_{bi} + V_r)}{e}\left(\frac{N_a}{N_d}\right)\left(\frac{1}{N_a + N_d}\right)}$

$(30 \times 10^{-4}\,cm)^2 = 1.27 \times 10^{-7}(557.41\,mV + V_r)$

$V_r = \boxed{70.309\,V}$

7.30 A silicon p$^+$n junction has doping concentrations of $N_a = 2 \times 10^{17}$ cm^{-3} and $N_d = 2 \times 10^{15}$ cm^{-3}. The cross-sectional area is 10^{-5} cm^2. Calculate (*a*) V_{bi} and (*b*) the junction capacitance at (*i*) $V_R = 1$ V, (*ii*) $V_R = 3$ V, and (*iii*) $V_R = 5$ V. (*c*) Plot $1/C^2$ versus V_R and show that the slope can be used to find N_d and the intercept at the voltage axis yields V_{bi}.

(a) $V_{bi} = V_T \ln\left(\frac{N_a N_d}{n_i^2}\right)$

(i) 730.55 mV

(ii) 730.55 mV

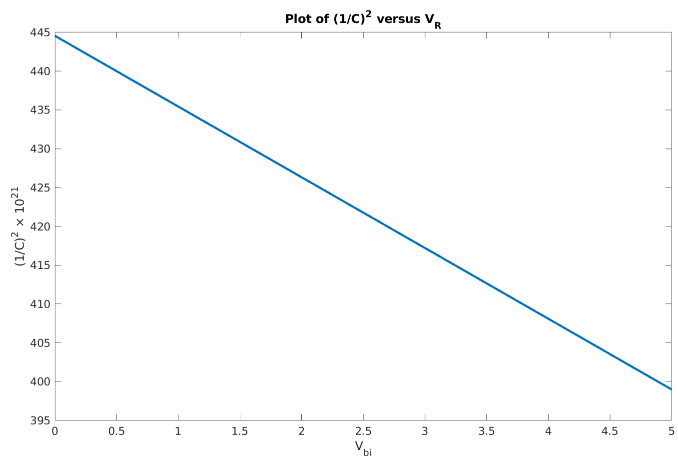
(iii) 730.55 mV

(b) $C = A\sqrt{\frac{e\epsilon_s N_a \cdot N_d}{2(V_{bi} + V_r)(N_a + N_d)}}$

(i) 97.447 fF

(ii) 66.371 fF

(iii) 53.551 fF



***7.34** A silicon PIN junction has the doping profile shown in Figure P7.34. The "I" corresponds to an ideal intrinsic region in which there is no impurity doping concentration. A reverse-biased voltage is applied to the PIN junction so that the total depletion width extends from $-2\ \mu m$ to $+2\ \mu m$. (*a*) Using Poisson's equation, calculate the magnitude of the electric field at $x = 0$. (*b*) Sketch the electric field through the PIN junction. (*c*) Calculate the reverse-biased voltage that must be applied.

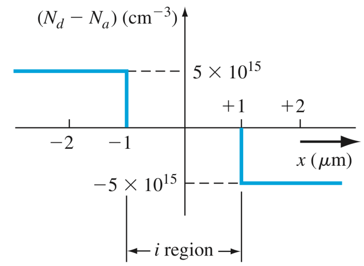


Figure P7.34 | Figure for Problem 7.34.

(a) $\dfrac{d^2\phi(x)}{dx^2} = \dfrac{-\rho(x)}{\epsilon_s} = \dfrac{-dE(x)}{dx}$

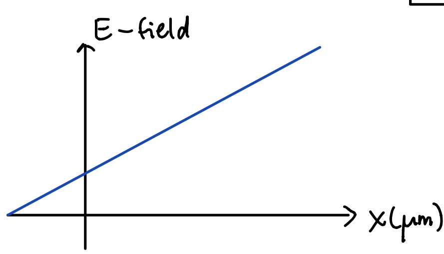
For $-2\mu m < x < -1\ \mu m$

$\rho(x) = +eN_d \longrightarrow E = \dfrac{eN_d x}{\epsilon_s} + c$

$c = E(0) = \dfrac{eN_d x_0}{\epsilon_s} = 0$

$E(0) = \dfrac{eN_d}{\epsilon_s}(x + x_0) = \boxed{77.33\ \dfrac{kV}{cm}}$

(b)



(c) $\phi_1 = \int \dfrac{eN_d}{\epsilon_s}(x + x_0) = 3.8665\ V$

$\phi_2 = E(0)d = 15.466\ V$

$V_r = 2(3.8665) + 15.466$

$= \boxed{23.199\ V}$

8.2 A silicon pn junction has impurity doping concentrations of $N_d = 2 \times 10^{15}$ cm^{-3} and $N_a = 8 \times 10^{15}$ cm^{-3}. Determine the minority carrier concentrations at the edges of the space charge region for (*a*) $V_a = 0.45$ V, (*b*) $V_a = 0.55$ V, and (*c*) $V_a = -0.55$ V.

$N_a > N_d \rightarrow$ p-type $\rightarrow$ minority n

$$p_{no} = \frac{n_i^2}{N_d} = \frac{1.5 \times 10^{10\,2}}{2 \times 10^{15}} = 1.125 \times 10^5 \text{ cm}^{-3}$$

$$n_{po} = \frac{n_i^2}{N_a} = \frac{1.5 \times 10^{10\,2}}{8 \times 10^{15}} = 2.8125 \times 10^4 \text{ cm}^{-3}$$

(a)
$$p_n(x_n) = p_{no} \exp\left(\frac{eV_a}{KT}\right) = 1.125 \times 10^5 \exp\left(\frac{0.45}{0.0259}\right) = \boxed{3.9519 \times 10^{12} \text{ cm}^{-3}}$$

$$n_p(-x_p) = n_{po} \exp\left(\frac{eV_a}{kT}\right) = 2.8125 \times 10^4 \exp\left(\frac{0.45}{0.0259}\right) = \boxed{9.8798 \times 10^{11} \text{ cm}^{-3}}$$

(b)
$$p_n(x_n) = p_{no} \exp\left(\frac{eV_a}{KT}\right) = 1.125 \times 10^5 \exp\left(\frac{0.55}{0.0259}\right) = \boxed{1.8777 \times 10^{14} \text{ cm}^{-3}}$$

$$n_p(-x_p) = n_{po} \exp\left(\frac{eV_a}{kT}\right) = 2.8125 \times 10^4 \exp\left(\frac{0.55}{0.0259}\right) = \boxed{4.6942 \times 10^{13} \text{ cm}^{-3}}$$

(c)
$$p_n(x_n) = p_{no} \exp\left(\frac{eV_a}{KT}\right) = 1.125 \times 10^5 \exp\left(\frac{-0.55}{0.0259}\right) \approx \boxed{0 \text{ cm}^{-3}}$$

$$n_p(-x_p) = n_{po} \exp\left(\frac{eV_a}{kT}\right) = 2.8125 \times 10^4 \exp\left(\frac{-0.55}{0.0259}\right) \approx \boxed{0 \text{ cm}^{-3}}$$

8.5 Consider a GaAs pn junction with doping concentrations $N_a = 5 \times 10^{16}$ cm^{-3} and $N_d = 10^{16}$ cm^{-3}. The junction cross-sectional area is $A = 10^{-3}$ cm^2 and the applied forward-bias voltage is $V_a = 1.10$ V. Calculate the (*a*) minority electron diffusion current at the edge of the space charge region, (*b*) minority hole diffusion current at the edge of the space charge region, and (*c*) total current in the pn junction diode.

(a) $J_n = e\sqrt{\frac{D_n}{\tau_{no}}}\,\frac{n_i^2}{N_a}\left(\exp\left(\frac{eV_a}{kT}\right)-1\right) = 1.8519\ A/cm^2$

$I_{diff} = J_n A = \boxed{1.8519\ mA}$

(b) $J_p = e\sqrt{\frac{D_p}{\tau_{po}}}\,\frac{n_i^2}{N_d}\left(\exp\left(\frac{eV_a}{kT}\right)-1\right) = 4.5270\ \frac{A}{cm^2}$

$I_{diff} = J_p A = \boxed{4.5270\ mA}$

(c) $I_{total} = 1.8519\ mA + 4.5270\ mA = \boxed{6.3789\ mA}$

8.7 An ideal germanium pn junction diode has the following parameters: $N_a = 4 \times 10^{15}$ cm^{-3}, $N_d = 2 \times 10^{17}$ cm^{-3}, $D_p = 48$ cm^2/s, $D_n = 90$ cm^2/s, $\tau_{p0} = \tau_{n0} = 2 \times 10^{-6}$ s, and $A = 10^{-4}$ cm^2. Determine the diode current for (*a*) a forward-bias voltage of 0.25 V and (*b*) a reverse-biased voltage of 0.25 V.

(a) $$J_s = e n_i^2 \left(\frac{1}{N_a}\sqrt{\frac{D_n}{\tau_{no}}} + \frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{po}}} \right) = 1.5676 \times 10^{-4}\ \text{A/cm}^2$$

$$I = J_s A \exp\left(\frac{V_a}{V_t}\right) = \boxed{243.93 \times 10\ \mu A}$$

(b) $$I = J_s A\left[\exp\left(\frac{V_a}{V_t}\right) - 1\right] = \boxed{-15.675\ nA}$$

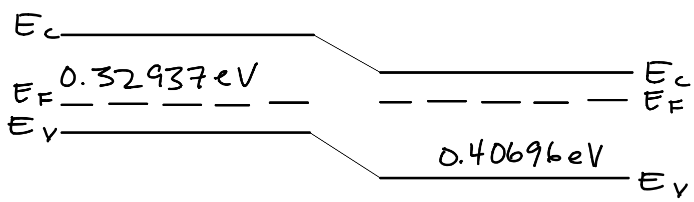
8.15 A silicon pn junction with a cross-sectional area of 10^{-4} cm^2 has the following properties at $T = 300$ K:

n region	p region
$N_d = 10^{17}$ cm^{-3}	$N_a = 5 \times 10^{15}$ cm^{-3}
$\tau_{p0} = 10^{-7}$ s	$\tau_{n0} = 10^{-6}$ s
$\mu_n = 850$ cm^2/V-s	$\mu_n = 1250$ cm^2/V-s
$\mu_p = 320$ cm^2/V-s	$\mu_p = 420$ cm^2/V-s

(*a*) Sketch the thermal equilibrium energy-band diagram of the pn junction, including the values of the Fermi level with respect to the intrinsic level on each side of the junction. (*b*) Calculate the reverse-saturation current I_s and determine the forward-bias current I at a forward-bias voltage of 0.5 V. (*c*) Determine the ratio of hole current to total current at the space charge edge x_n.

(a) $$E_F - E_{Fi} = 0.0259 \ln\left(\frac{10^{17}}{1.5\times10^{10}}\right) = 0.40696\,\text{eV}$$

$$E_{Fi} - E_F = 0.0259 \ln\left(\frac{5\times10^{15}}{1.5\times10^{10}}\right) = 0.32937\,\text{eV}$$



(b) $$I_s = A e n_i^2 \left(\frac{1}{N_a}\sqrt{\frac{D_n}{\tau_{no}}} + \frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{po}}}\right) = \boxed{4.6574\,\text{fA}}$$

$$I = I_s \exp\left(\frac{V_a}{V_t}\right) = \boxed{1.1277\,\mu\text{A}}$$

(c) $$I_p = \frac{e n_i^2 A}{N_d}\sqrt{\frac{D_p}{\tau_{po}}}\left[\exp\left(\frac{V_a}{V_t}\right) - 1\right] = 40.0873\,\text{fA}$$

$$I_{total} = A e n_i^2 \left(\frac{1}{N_a}\sqrt{\frac{D_n}{\tau_{no}}} + \frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{po}}}\right)\left[\exp\left(\frac{V_a}{V_t}\right) - 1\right] = 1.128\,\mu\text{A}$$

$$\therefore \text{ratio} = \frac{40.0873\,\text{fA}}{1.128\,\mu\text{A}} = \boxed{3.5538\times10^{-10}}$$

8.17 Consider the ideal long silicon pn junction shown in Figure P8.17. $T = 300$ K. The n region is doped with 10^{16} donor atoms per cm^3 and the p region is doped with 5×10^{16} acceptor atoms per cm^3. The minority carrier lifetimes are $\tau_{n0} = 0.05\ \mu s$ and $\tau_{p0} = 0.01\ \mu s$. The minority carrier diffusion coefficients are $D_n = 23\ cm^2/s$ and $D_p = 8\ cm^2/s$. The forward-bias voltage is $V_a = 0.610$ V. Calculate (*a*) the excess hole concentration as a function of x for $x \geq 0$, (*b*) the hole diffusion current density at $x = 3 \times 10^{-4}$ cm, and (*c*) the electron current density at $x = 3 \times 10^{-4}$ cm.

Note: x = 0 is defined at the edge of the depletion region on the n-side (what we normally call x_n)

Hint part (c): The total current (sum of electron and hole) at any point (x) in the structure must be the same. You can find the total current by summing the currents at the depletion region edges, as we did in the derivation of the ideal didoe equation. Since you calculate the hole current at x = 3um in part (c), you can then get the electron current at x = 3um by sustracting the hole current at x = 3um from the total current.

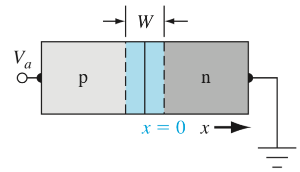


Figure P8.17 | Figure for Problem 8.17.

(a) $p_{no} = \dfrac{n_i^2}{N_d} = 22k/cm^{-3}$

$L_p = \sqrt{D_p \tau_{po}} = 2.8284\ \mu m$

$$\delta p_n = p_{no}\left[\exp\left(\frac{V_a}{V_t}\right) - 1\right]\exp\left(\frac{x_n - x}{L_p}\right) = \boxed{3.8084\times10^{14}\exp\left(\frac{-x}{2.8284\times10^{-6}}\right)}$$

(b) $J_p(x) = -eD_p\left(\dfrac{d(\delta p_n)}{dx}\right)$, $\dfrac{d(\delta p_n)}{dx} = -\dfrac{3.8084\times10^{14}}{2.8284\times10^{-6}}\exp\left(\dfrac{-x}{2.8284\times10^{-6}}\right)$

$= \boxed{0.5966\ A/cm^2}$

(c) $J_n = J_{po} + J_{no} - J_p$

$J_{po} = -eD_p\left(\dfrac{d(\delta p_n)}{dx}\right) = 1.715\ A/cm^2$, $J_{no} = -\dfrac{eD_n n_{po}}{\sqrt{D_n \tau_{no}}}\dfrac{n_i^2}{N_a}\left[\exp\left(\dfrac{V_a}{V_t}\right) - 1\right]$

$= 0.261\ A/cm^2$

$J_n = \boxed{1.384\ A/cm^2}$