$\boxed{2.5}$

$\Phi_{Au} = 4.90\ eV$ For Au

$\Phi_{Ce} = 1.90\ eV$ For Ce

work function $\Phi = h\nu_0$ where ν_0 is the minimum frequency to remove an electron

$$c = \lambda\nu_0 \longrightarrow \lambda_{max} = \frac{c}{\nu_0}$$

$$\nu_0 = \frac{\Phi}{h} = \frac{4.90\ eV}{4.135\times10^{-15}\ eV\cdot s} = 1.185\times10^{15}\ s^{-1}$$

$$\text{so } \lambda_{max} = \frac{3\times10^{8}\ m/s}{1.185\times10^{15}\ s^{-1}} = 2.53\times10^{-7}\ m$$

$\boxed{= 253\ nm \text{ For Au} \rightarrow \text{ultra violet}}$

similarly for Ce:

$$\nu_0 = \frac{\Phi}{h} = \frac{1.90\ eV}{4.135\times10^{-15}\ eV\cdot s} = 4.595\times10^{14}\ s^{-1}$$

$$\lambda_{max} = \frac{3\times10^{8}\ m/s}{4.595\times10^{14}\ s^{-1}} = 6.52\times10^{-7}\ m$$

$\boxed{= 652\ nm \text{ For Ce} \rightarrow \text{red light}}$

$\boxed{2.10}$

(a) $\lambda = 85\ \text{Å}$

$$p = \frac{h}{\lambda} = \frac{6.625\times10^{-34}\ J\cdot s}{85\times10^{-10}\ m} = \boxed{7.794\times10^{-26}\ \frac{kg\,m}{s}}$$

$$\text{so } V = \frac{p}{m} = \frac{7.794\times10^{-26}\ kg\,m/s}{9.11\times10^{-31}\ kg} = \boxed{8.555\times10^{4}\ m/s}$$

$$E = \frac{1}{2}mv^2 = \frac{1}{2}(9.11\times10^{-31}\ kg)(8.555\times10^{4}\ m/s)^2 = 3.334\times10^{-21}\ J$$

$\boxed{= 0.0208\ eV}$

(b) electron with $v = 8\times10^5$ cm/s

$$p = mv = (9.11\times10^{-31}\ \text{kg})(8\times10^{3}\ \text{m/s}) = \boxed{7.288\times10^{-27}\ \frac{\text{kg m}}{\text{s}}}$$

$$E = \frac{p^2}{2m} = \frac{\left(7.288\times10^{-27}\ \frac{\text{kg}\cdot\text{m}}{\text{s}}\right)^2}{2(9.11\times10^{-31}\ \text{kg})} = 2.915\times10^{-23}\ \text{J}$$

$$\boxed{= 1.822\times10^{-4}\ \text{eV}}$$

$$\lambda = \frac{h}{p} = \frac{6.625\times10^{-34}\ \text{J}\cdot\text{s}}{7.288\times10^{-27}\ \frac{\text{kg m}}{\text{s}}}$$

$$= 9.09\times10^{-8}\ \text{m} = \boxed{90.9\ \text{nm}} = 909\ \text{Å}$$

$\boxed{2.11}$ want $\lambda = 1$ Å from x-ray

(a) what potential voltage must an electron accelerate through to produce $\lambda = 1$ Å?

we need $E_{photon} = E_{electron}$

$$\Longrightarrow \frac{hc}{\lambda} = qV \quad \text{both sides in eV}$$

$$\text{so} \quad V = \frac{hc}{\lambda q} = \frac{(4.135\times10^{-15}\ \text{eV}\cdot\text{s})(3\times10^{8}\ \text{m/s})}{(1\times10^{-10}\ \text{m})(1.602\times10^{-19}\ \text{C})}$$

$$= 7.743\times10^{22}\ \frac{\text{eV}}{\text{C}} \quad \text{but volt} = \text{J/C}$$

$$\text{so} \longrightarrow 7.743\times10^{22}\ \text{eV/C}\cdot\frac{1.6\times10^{-19}\ \text{J}}{\text{eV}}$$

$$\boxed{= 12{,}389\ \text{V}}$$

(b) Just before it hits the target it has maximum energy and it is all kinetic

$$E = qV = (1.6\times10^{-19}\ \text{C})(12{,}389\ \text{J/C}) = 1.982\times10^{-15}\ \text{J}$$

$$E = \frac{p^2}{2m} \Longrightarrow p = \sqrt{2mE}$$

$$p = \sqrt{2(9.11\times10^{-31}\ \text{kg})(1.982\times10^{-15}\ \text{J})} = 6.01\times10^{-23}\ \frac{\text{kg m}}{\text{s}}$$

so $\lambda = \frac{h}{p} = \frac{6.625 \times 10^{-34}\ J\cdot s}{6.01 \times 10^{-23}\ kg\,m/s} = 1.10 \times 10^{-11}\ m$

$$\boxed{= 0.11\ \text{Å}}$$

2.15

(a) $\Delta E\, \Delta t \geq \hbar$

$\Delta E \leq 0.8\ eV$ maximum

so Δt minimum $= \frac{\hbar}{\Delta E} = \frac{\left(\frac{1}{2\pi}\right)(4.135 \times 10^{-15}\ eV\cdot s)}{0.8\ eV}$

$$\boxed{\Delta t = 8.23 \times 10^{-16}\ s}$$

(b) $\Delta p \cdot \Delta x \geq \hbar$

$\Delta x \leq 1.5\ \text{Å}$

so Δp minimum $= \frac{\hbar}{\Delta x} = \frac{\left(\frac{1}{2\pi}\right)(6.625 \times 10^{-34}\ J\cdot s)}{(1.5 \times 10^{-10}\ m)}$

$$\boxed{\Delta p = 7.03 \times 10^{-25}\ \frac{kg\cdot m}{s}}$$

2.17 $\Psi(x,t) = A \cos\left(\frac{\pi x}{2}\right) e^{-j\omega t}$ For $-1 \leq x \leq 3$

Determin A so $\int_{-1}^{3} |\Psi(x,t)|^2 dx = 1$

$$|\Psi(x,t)|^2 = \Psi(x,t) \cdot \Psi^*(x,t) = A \cos\left(\frac{\pi x}{2}\right) e^{-j\omega t} \cdot A^* \cos\left(\frac{\pi x}{2}\right) e^{j\omega t}$$

$$= A \cdot A^* \cos^2\left(\frac{\pi x}{2}\right) = |A|^2 \cos^2\left(\frac{\pi x}{2}\right)$$

so $\int_{-1}^{3} |A|^2 \cos^2\left(\frac{\pi x}{2}\right) dx = 1$

using $\cos^2(x) = \frac{1}{2} + \frac{1}{2}\cos(2x)$

$$|A|^2 \int_{-1}^{3} \left[\frac{1}{2} + \frac{1}{2}\cos \pi x\right] dx = 1$$

so $$|A|^2 = \frac{1}{\frac{1}{2}\left[\int_{-1}^{3} dx + \int_{-1}^{3} \cos \pi x \, dx\right]}$$

performing the integral

$$\frac{1}{2}\left[x\Big|_{-1}^{3} + \cancel{\frac{\sin \pi x}{\pi}\Big|_{-1}^{3}} \to 0\right]$$

$$= \frac{1}{2}\left[3-(-1)\right] = \frac{4}{2} = 2$$

so $|A|^2 = \frac{1}{2}$ so $|A| = \frac{1}{\sqrt{2}} = \boxed{\frac{\sqrt{2}}{2}}$

$\boxed{2.20}$ $\psi(x) = \sqrt{\frac{2}{a}} \cos\left(\frac{\pi x}{a}\right)$ for $-\frac{a}{2} < x < \frac{a}{2}$

$\psi(x) = 0$ elsewhere

- Find probability of finding the electron between

(a) $0 < x < \frac{a}{4}$

the probability of finding it between $0 < x < \frac{a}{4}$ is $\int_0^{a/4} |\psi(x)|^2 dx$

$$|\psi(x)|^2 = \psi(x) \cdot \psi^*(x) = \frac{2}{a}\cos^2\left(\frac{\pi x}{a}\right)$$

so we have $\frac{2}{a}\int_0^{a/4} \cos^2\left(\frac{\pi x}{a}\right) dx$

using $\cos^2(x) = \frac{1}{2} + \frac{1}{2}\cos(2x)$

$$\frac{2}{a}\int_0^{a/4}\left[\frac{1}{2} - \frac{1}{2}\cos\left(\frac{2\pi x}{a}\right)\right]dx$$

$$\frac{1}{a}\left[x + \frac{a}{2\pi}\sin\left(\frac{2\pi x}{a}\right)\right]_0^{a/4}$$

$$= \frac{1}{\cancel{a}}\left[\frac{\cancel{a}}{4} + \frac{\cancel{a}}{2\pi}\sin\left(\frac{\pi}{2}\right)\right]$$

$$= \frac{1}{4} + \frac{1}{2\pi} = \boxed{0.409}$$

(b) evaluating between $\frac{a}{4} < x < \frac{a}{2}$

$$\frac{1}{a}\left[x + \frac{a}{2\pi}\sin\left(\frac{2\pi x}{a}\right)\right]_{a/4}^{a/2}$$

$$= \frac{1}{a}\left[\frac{a}{4} + \frac{a}{2\pi}\cancel{\sin(\pi)}^{0} - \frac{a}{2\pi}\cancel{\sin\left(\frac{\pi}{2}\right)}^{1}\right]$$

$$= \frac{1}{\cancel{a}}\left[\frac{\cancel{a}}{4} - \frac{\cancel{a}}{2\pi}\right] = \frac{1}{4} - \frac{1}{2\pi} = \boxed{0.091}$$

(c) evaluating between $-\frac{a}{2} < x < \frac{a}{2}$

* it should be $= 1$ since the particle must be somewhere described by the wavefunction

verify

$$\frac{1}{a}\left[x + \frac{a}{2\pi}\sin\left(\frac{2\pi x}{a}\right)\right]_{-a/2}^{a/2}$$

$$= \frac{1}{a}\left[a + \frac{a}{2\pi}\left[\cancel{\sin\pi}^{0} - \cancel{\sin(\pi)}^{0}\right]\right] = \boxed{1}$$

$\boxed{2.22}$ electron in free space: $\psi(x,t) = A e^{j(kx-\omega t)}$

* This problem is ill-defined!

For a free electron: $k = \sqrt{\frac{2mE}{\hbar^2}}$ from Schrodinger Eqn.

$$\text{so} \implies E = \frac{\hbar^2 k^2}{2m} = \hbar\omega$$

$$\implies \omega = \frac{\hbar k^2}{2m}$$

so if $k = 8 \times 10^8\ m^{-1} \longrightarrow \underline{\omega = 3.7 \times 10^{13}\ rad/s}$

- ω cannot be 8×10^{12} rad/s as given in the problem statement!

so I will use $k = 8\times 10^8\ m^{-1}$ and $\omega = 3.7 \times 10^{13}$ rad/s

(a) $V_{phase} = \frac{\omega}{k} = \frac{3.7\times 10^{13}\ rad/s}{8\times 10^8\ m^{-1}} = 46{,}250\ m/s$

$$\lambda = \frac{2\pi}{|k|} = \frac{2\pi}{8\times 10^8\ m^{-1}} = 7.85 \times 10^{-9}\ m \boxed{= 7.85\ nm}$$

For momentum, we need the particle velocity

↳ for a free electron this is the <u>group</u> velocity

$V_g = \frac{d\omega}{dk}$ or you can say $p = mv = \hbar k$

$$\text{so } V = \frac{\hbar k}{m} = \frac{(1.054\times 10^{-34}\ J\cdot s)(8\times 10^8\ m^{-1})}{(9.11\times 10^{-31}\ kg)} = 92{,}558\ m/s$$

$= 2 V_p$

some rounding error here

to see why $V_g = V$ find $\frac{d\omega}{dk} = \frac{d}{dk}\left(\frac{\hbar k^2}{2m}\right) = \frac{\hbar k}{m} = V$ ✓

- so $p = mv = (9.11\times10^{-31} kg)(92,558 m/s) = \boxed{8.43\times10^{-26} \frac{kg\,m}{s}}$

- now $E = \frac{1}{2}mv^2 = (0.5)(9.11\times10^{-31} kg)(92,558 m/s)^2 = 3.9\times10^{-21} J$

$= 0.0244 \text{ eV}$

or $E = \hbar\omega = (1.054\times10^{-34} J\cdot s)(3.7\times10^{13} rad/s)$

$= 3.9\times10^{-21} J = 0.0244 \text{ eV}$ ✓

(b) Again, the problem is ill-defined

if $k = 1.5\times10^{9} m^{-1} \Longrightarrow \omega$ should be 1.3×10^{14} rad/s

so $v_{phase} = \frac{\omega}{k} = \frac{1.3\times10^{14} rad/s}{-1.5\times10^{9} m^{-1}} = \boxed{-86,667 m/s}$

so $v_{group} = 2\cdot v_p = 173,333 m/s = v$

$\lambda = \frac{2\pi}{|k|} = \frac{2\pi}{1.5\times10^{9} m^{-1}} \boxed{= 4.19 \text{ nm}}$

$p = m\cdot v = (9.11\times10^{-31} kg)(173,333 m/s) = \boxed{-1.58\times10^{-25} \frac{kg\,m}{s}}$

$E = \frac{1}{2}mv^2 = (0.5)(9.11\times10^{-31} kg)(173,333 m/s)^2 = 1.369\times10^{-20} J$

$\boxed{= 0.0855 \text{ eV}}$

* note that − sign means particle moves in −x direction

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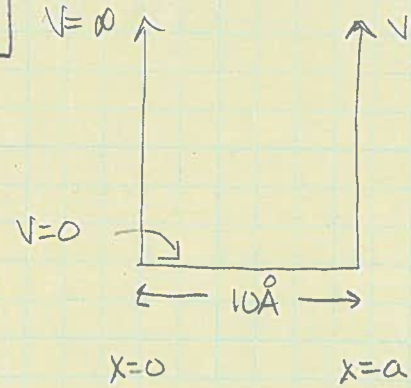
$\boxed{2.26}$



(a) The 1st three energy levels are calculated using (2.38)

$$E_n = \frac{\hbar^2 n^2 \pi^2}{2ma^2} \quad \text{For } n = 1, 2, 3$$

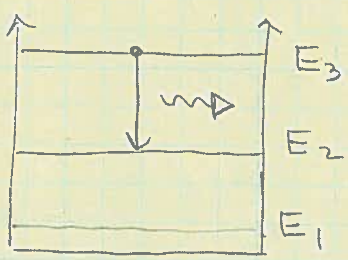
$$E_1 = \frac{(1.054\times10^{-34}\ J\cdot s)^2 (1)^2 \pi^2}{2(9.11\times10^{-31}\ kg)(10\times10^{-10}\ m)^2} = 6.02\times10^{-20}\ J$$

$$\boxed{= 0.376\ eV}$$

$$E_2 = E_1 \cdot 4 = \boxed{1.504\ eV}$$

$$E_3 = E_1 \cdot 9 = \boxed{3.384\ eV}$$

(b)



$$\Delta E = E_3 - E_2 = 3.384\ eV - 1.504\ eV$$

$$\Delta E = 1.88\ eV$$

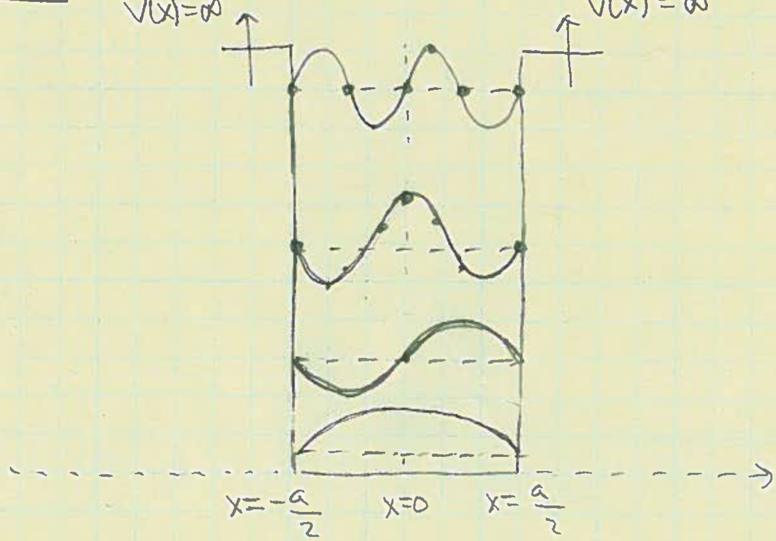
must equal photon energy to conserve energy during the transition

so $\Delta E = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{\Delta E}$

$$\lambda = \frac{(4.135\times10^{-15}\ eV\cdot s)(3\times10^{8}\ m/s)}{1.88\ eV}$$

$$= 6.60\times10^{-7}\ m = \boxed{660\ nm}$$

$\boxed{2.29}$ Fig P2.29



- Derive and sketch wave functions for four lowest energy levels ($n=1,2,3,4$)

- We must consider the new boundary conditions

$$\psi(x=-\tfrac{a}{2}) = \psi(x=\tfrac{a}{2}) = 0$$

- The general solution to the time-independent Schrodinger equation for an infinite potential well is

$$\psi(x) = A\cos kx + B\sin kx$$

$$\text{where } k = \sqrt{\frac{2mE}{\hbar^2}}$$

- Applying the boundary condition at $x = \frac{a}{2}$ gives

$$\psi(a/2) = A\cos\left(\frac{ka}{2}\right) + B\sin\left(\frac{ka}{2}\right) = 0$$

$$= A\cos\left(\frac{ka}{2}\right) + B\sin\left(\frac{ka}{2}\right) = 0$$

→ this can be zero if : even multiple of $\frac{\pi}{2}$ (i.e, integer multiple of π)

$$A = 0 \text{ and } \frac{ka}{2} = n\pi \Rightarrow k = \frac{2n\pi}{a} \quad n=1,2,3\ldots$$

$$\text{or } B = 0 \text{ and } \frac{ka}{2} = \frac{(2n-1)\pi}{2} \Rightarrow k = \frac{(2n-1)\pi}{2a} \quad n=1,2,3\ldots$$

odd multiple of $\frac{\pi}{2}$

- so we have even parity and odd parity solutions

$$\psi(x) = A\cos(kx) \quad \text{(even)}$$

$$\psi(x) = B\sin(kx) \quad \text{(odd)}$$

- and we can say the cos(kx) solutions are for

$$\frac{ka}{2} = n\frac{\pi}{2} \quad \text{with } n = 1, 3, 5 \ldots$$

and the sin(kx) solutions are for

$$\frac{ka}{2} = n\frac{\pi}{2} \quad \text{with } n = 2, 4, 6 \ldots$$

so

$$\boxed{\begin{aligned} \psi_1(x) &= A_1\cos\left(\frac{\pi}{a}x\right) \\ \psi_2(x) &= B_2\sin\left(\frac{2\pi}{a}x\right) \\ \psi_3(x) &= A_3\cos\left(\frac{3\pi}{a}x\right) \\ \psi_4(x) &= B_4\sin\left(\frac{4\pi}{a}x\right) \end{aligned}}
\qquad
\boxed{\begin{aligned} E_1 &= \frac{\hbar^2\pi^2}{2ma^2} \\ E_2 &= \frac{4\hbar^2\pi^2}{2ma^2} \\ E_3 &= \frac{9\hbar^2\pi^2}{2ma^2} \\ E_4 &= \frac{16\hbar^2\pi^2}{2ma^2} \end{aligned}}$$

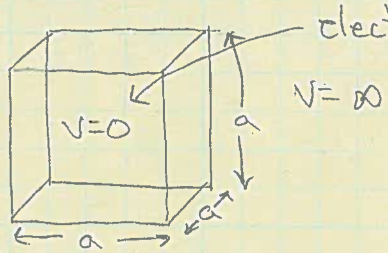
- see beginning for sketches.

$\boxed{2.30}$ 3D infinite potential well

$V(x) = 0$ for $0 < x < a$
$0 < y < a$
$0 < z < a$

$V(x) = \infty$ elsewhere

* Note: this potential profile is similar to that of the experimentally achievable "quantum dot" or "quantum box"



show $E_{n_x n_y n_z} = \frac{\hbar^2 \pi^2}{2ma^2}\left(n_x^2 + n_y^2 + n_z^2\right)$

$n_x = 1, 2, 3 \ldots$
$n_y = 1, 2, 3 \ldots$
$n_z = 1, 2, 3 \ldots$

- In 3D, Schrodinger's equation is

$$-\frac{\hbar^2}{2m}\nabla^2 \Psi(\vec{r}, t) + V(\vec{r})\Psi(\vec{r}, t) = j\hbar \frac{\partial \Psi(\vec{r}, t)}{\partial t}$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ is the Laplacian operator

and $\vec{r} = x\hat{e}_x + y\hat{e}_y + z\hat{e}_z$

unit vector in z direction

so $\Psi(\vec{r}, t) = \Psi(x, y, z, t)$

so we have:

$$-\frac{\hbar^2}{2m}\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right]\Psi(x,y,z,t) + V(x,y,z)\Psi(x,y,z,t)$$
$$= j\hbar \frac{\partial \Psi(x,y,z,t)}{\partial t}$$

- we can guess a solution to SE of

$$\Psi(x,y,z,t) = X(x)\,Y(y)\,Z(z)\,\Phi(t)$$

only depends upon t

only depends upon x only depends upon y only depends upon z

- plugging the solution into SE in 3D we get

$$-\frac{\hbar^2}{2m}\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right]X(x)Y(y)Z(z)\Phi(t) + V(x,y,z)\,X(x)Y(y)Z(z)\Phi(t)$$

$$= j\hbar\frac{\partial}{\partial t}X(x)Y(y)Z(z)\Phi(t)$$

- inside the box $V(x,y,z) = 0$

$$-\frac{\hbar^2}{2m}\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right]X(x)Y(y)Z(z)\Phi(t) = j\hbar\frac{\partial}{\partial t}X(x)Y(y)Z(z)\Phi(t)$$

$$-\frac{\hbar^2}{2m}\left[Y(z)Z(z)\Phi(t)\frac{\partial^2 X(x)}{\partial x^2} + X(x)Z(z)\Phi(t)\frac{\partial^2 Y(y)}{\partial y^2} + X(x)Y(y)\Phi(t)\frac{\partial^2 Z(z)}{\partial z^2}\right] = j\hbar\,X(x)Y(y)Z(z)\frac{\partial\Phi(t)}{\partial t}$$

now divide both sides by $X(x)Y(y)Z(z)\Phi(t)$

$$-\frac{\hbar^2}{2m}\left[\frac{1}{X(x)}\frac{\partial^2 X(x)}{\partial x^2}+\frac{1}{Y(y)}\frac{\partial^2 Y(y)}{\partial y^2}+\frac{1}{Z(z)}\frac{\partial^2 Z(z)}{\partial z^2}\right] = j\hbar\frac{1}{\Phi(t)}\frac{\partial\Phi(t)}{\partial t}$$

- each side must equal a constant, call it "E"
- for the quantization and energy levels we care about the time-independent part

$$-\frac{\hbar^2}{2m}\left[\frac{1}{X(x)}\frac{\partial^2 X(x)}{\partial x^2}+\frac{1}{Y(y)}\frac{\partial^2 Y(y)}{\partial y^2}+\frac{1}{Z(z)}\frac{\partial^2 Z(z)}{\partial z^2}\right] = E$$

if we let $k = \sqrt{\frac{2mE}{\hbar^2}}$ as is done in the 1D case described in the book we have

$$\frac{1}{X(x)}\frac{\partial^2 X(x)}{\partial x^2}+\frac{1}{Y(y)}\frac{\partial^2 Y(x)}{\partial y^2}+\frac{1}{Z(z)}\frac{\partial^2 Z(z)}{\partial z^2} = -k^2$$

- now $k^2 = k_x^2 + k_y^2 + k_z^2$

- we get three equations

(1) $\frac{\partial^2 X(x)}{\partial x^2} + k_x^2 X(x) = 0$

(2) $\frac{\partial^2 Y(y)}{\partial y^2} + k_y^2 Y(y) = 0$

(3) $\frac{\partial^2 Z(z)}{\partial z^2} + k_z^2 Z(z) = 0$

general solution to one of these is

$X(x) = A_1 \sin(k_x x) + A_2 \cos(k_x x)$

since $X(0) = 0$ from boundary conditions $\Rightarrow A_2 = 0$

so $\boxed{X(x) = A_1 \sin(k_x x)}$

also since $X(a)$ must be zero we get the condition

$X(a) = A_1 \sin(k_x a) = 0 \Longrightarrow \boxed{k_x = \frac{n_x \pi}{a}}$ $n_x = 1, 2, 3, \ldots$

- similarly for $Y(y)$ and $Z(z)$

$\boxed{\begin{array}{l} Y(y) = A_3 \sin(k_y y) \\ Z(z) = A_5 \sin(k_z z) \end{array}}$ $\Rightarrow \boxed{k_y = \frac{n_y \pi}{a}}$ $n_y = 1, 2, 3 \ldots$

$\Rightarrow \boxed{k_z = \frac{n_z \pi}{a}}$ $n_z = 1, 2, 3 \ldots$

so $\boxed{\psi(x,y,z) = A \sin(k_x x) \sin(k_y y) \sin(k_z z)}$

and $E = \frac{\hbar^2 k^2}{2m}$; $k^2 = k_x^2 + k_y^2 + k_z^2$

$\Longrightarrow E = \frac{\hbar^2 (k_x^2 + k_y^2 + k_z^2)}{2m} = \frac{\hbar^2}{2m} \frac{\pi^2}{a^2} (n_x^2 + n_y^2 + n_z^2)$

$\boxed{E = \frac{\hbar^2 \pi^2}{2ma^2} (n_x^2 + n_y^2 + n_z^2)}$ $n_x = 1,2,3..$ $n_y = 1,2,3..$ $n_z = 1,2,3..$

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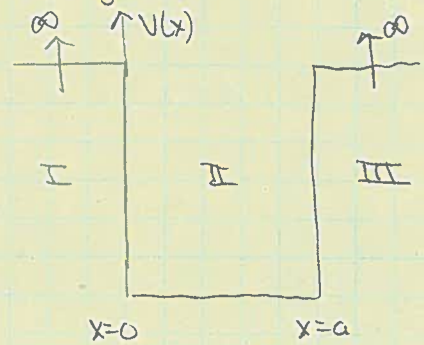
$\boxed{2.32}$ Consider a proton in a 1D infinite potential well in Fig. 2.6



(a) The fact that we are considering a proton instead of an electron only changes the mass, so the derivation is the same as the one given in section 2.3.2 in the book

$$\boxed{E_n = \frac{\hbar^2 n^2 \pi^2}{2ma^2} \qquad n = 1, 2, 3, \ldots}$$

* m = mass of proton

(b) - lowest possible energy is when $n=1$
- next highest is for $n=2$

so we have $\Delta E = \dfrac{4\hbar^2\pi^2}{2ma^2} - \dfrac{\hbar^2\pi^2}{2ma^2} = \dfrac{3\hbar^2\pi^2}{2ma^2}$

For (i) $a = 4$ Å we have

$$\Delta E = \frac{3(1.054\times10^{-34}\,J\cdot s)^2\pi^2}{2(1.67\times10^{-27}\,kg)(4\times10^{-10}\,m)^2} = 6.155\times10^{-22}\,J$$
$$\boxed{= 0.00385\ eV}$$

For (ii) $a = 0.5$ cm

$$\Delta E = \frac{3(1.054\times10^{-34}\,J\cdot s)^2\pi^2}{2(1.67\times10^{-27}\,kg)(0.5\times10^{-2}\,m)^2} = 3.939\times10^{-36}\,J$$
$$\boxed{= 2.46\times10^{-17}\ eV}$$

* so as the well gets wider, the energy difference goes down, that is there is almost a continuum of energies for a significaltly wide well!

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