ECE 345 / ME 380
Introduction to Control Systems
Lecture Notes 5

Dr. Oishi
oishi@unm.edu

September 29, 2020

Learning Objectives

- Reduce a block diagram of multiple subsystems to a single block representing the transfer function from input to output
- Reduce a negative unity feedback system
- Convert block diagrams to signal flow diagrams
- Represent state equations as signal flow graphs

References:

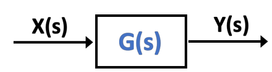
- Nise Chapter 5.1-5.3 (5.4-5.7 optional)

Outline

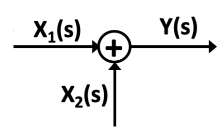
- Block diagram reduction
- **Negative unity feedback**
- Signal flow graphs

Block diagram algebra

- Two basic equations
  - One for every block, $Y(s) = G(s) \cdot X(s)$



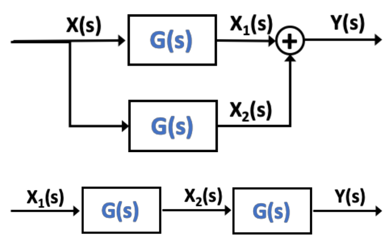
  - One for every junction, $Y(s) = X_1(s) + X_2(s)$



- With these two types of equations, *any* block diagram can be reduced into a simple, input-output relationship.

Block diagram reduction

- Elements of block diagrams
  - Signals: Lines
  - Systems: Blocks
  - Summing junctions
  - Pickoff points

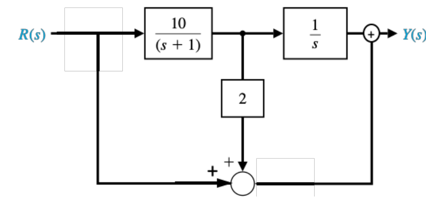


- Graphical representation that facilitates transfer function modeling and analysis
- Rules for manipulating signals and systems to reduce complexity

Block diagram reduction

Clicker question

Which of the following describes the transfer function $\frac{Y(s)}{R(s)}$?



A. $\frac{10}{s(s+1)}$

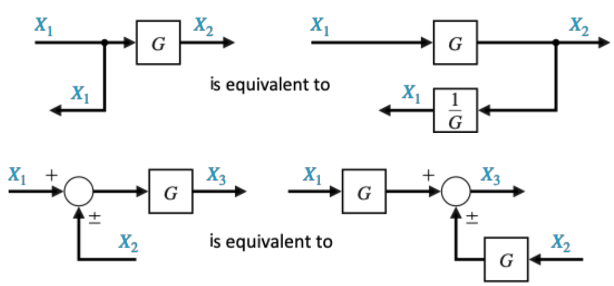
B. $\frac{10}{s(s+1)} + 2$

C. $\frac{s^2+s+30}{s(s+1)}$

D. $\frac{s^2+21s+10}{s(s+1)}$

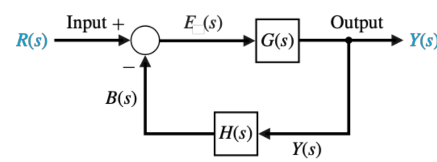
Block diagram reduction

Moving pickoff points or summing junctions can help with breaking tightly integrated systems into manageable chunks that can be reduced.



Block diagram reduction

Feedback form



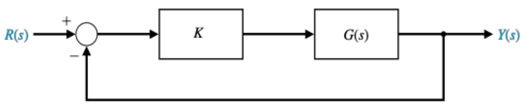
$$
\begin{aligned}
E(s) &= R(s) - B(s) \\
B(s) &= H(s)Y(s) \\
Y(s) &= G(s)E(s)
\end{aligned}
$$

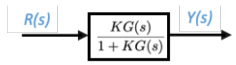
- Using block diagram reduction, $\frac{Y(s)}{R(s)} = \frac{G(s)}{1 + H(s)G(s)}$

Block diagram reduction

Negative unitary feedback

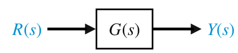
- Simplest controller is often an amplifier (e.g., **P**roportional control with gain K)
- "Unitary" because the feedback loop has $H(s) = 1$





$$Y(s) = KG(s)E(s), \quad E(s) = R(s) - Y(s)$$

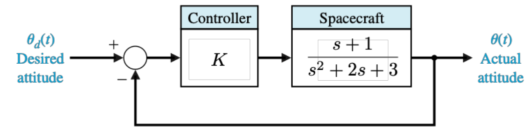
- *Error* drives plant, not the reference signal
- "Open-loop" vs. "Closed-loop" systems



Block diagram reduction

Clicker question

A single-axis spacecraft attitude control system has controller K.



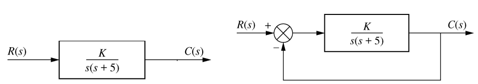
Which of the following describes the transfer function $\frac{\theta(s)}{\theta_d(s)}$?

A. $\frac{s+1}{s^2+2s+3}$
B. $\frac{K(s+1)}{s^2+(2+K)s+(3+K)}$
C. $\frac{K(s+1)}{s^2+2s+3}$
D. $\frac{s+1}{s^2+3s+4}$

Block diagram reduction

Clicker question

Consider the effect of increasing the gain $K > 0$ in open-loop system (left) and in the closed-loop system (right).



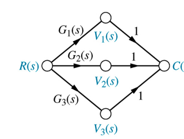
Which of the following statements is *always incorrect*?

A. The poles of the open-loop system are on the real axis, but the poles of the closed-loop system have imaginary components.
B. The zeros of the open-loop system depend on the value of K.
C. The zeros of the closed-loop system and the zeros of the open-loop system are the same.
D. For any $K \neq 0$, the poles of the open-loop system and the poles of the closed-loop system will be different.

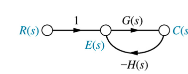
Signal flow graphs

Elements of signal-flow graphs

1. Systems: Branches
2. Signals: Nodes



(d)

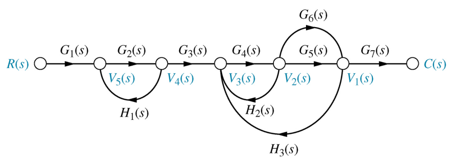


(f)

Signal flow graphs

Mason's rule for signal-flow graph reduction

Loop gain: Product of branch gains traversing a path that starts and ends at the same node
Forward-path gain: Product of branch gains traversing a path from input to output nodes
Nontouching loops: Loops that do not have any nodes in common
Nontouching loop gain: Product of loop gains from non-touching loops



Signal flow graphs

Mason's rule for signal-flow graph reduction

$$G(s) = \frac{\sum_k (T_k(s)\Delta_k(s))}{\Delta(s)} \tag{1}$$

- Let $\Delta(s) = 1-$ loop gains $+$ non-touching loop gains (in pairs) $-$ non-touching loop gains (in threes) $+\cdots$
- For each of the k forward paths in the transfer function,
  - Find the forward path gain $T_k(s)$
  - Calculate $\Delta_k(s) = \Delta(s)-$ loop gains that touch the kth forward path

Signal flow graphs

Lends itself to state-space representations
- Observer canonical form
- Controller canonical form (compare to phase-variable form)

$$G(s) = \frac{C(s)}{R(s)} = \frac{s^2+7s+2}{s^3+9s^2+26s+24}$$