ECE 595
Network Economics
WNP - Chapter 6
Market Competition Models

Oligopoly

- Three classical strategic form game formulations for competitions among multiple entities (***Oligopoly***):
  - the Cournot model
  - the Bertrand model
  - the Hotelling model
- We use these models to illustrate
  a. the translation of an informal problem statement into a strategic form representation of a game
  b. the analysis of Nash equilibrium when a player can choose his strategy from a continuous set

The Cournot Model

- An economic model used to describe interactions among firms that compete on the amount of output they will produce, which they decide independently of each other simultaneously
- Key features:
  - There are at least two firms producing homogeneous (undifferentiated) products
  - Firms do not cooperate
  - Firms compete by setting production quantities simultaneously. The total output quantity affects the market price
  - The firms are economically rational and act strategically, seeking to maximize profits given their competitors' decisions
- Two firms, $I = \{1, 2\}$
- Each firm i decides its output quantity q_i, under a fixed unit producing cost c_i
- The market-clearing price is a decreasing function of the total quantity $Q = q_1 + q_2$, denoted by $P(Q)$
- ***What is the best quantity choice of each firm?***

The Cournot Game

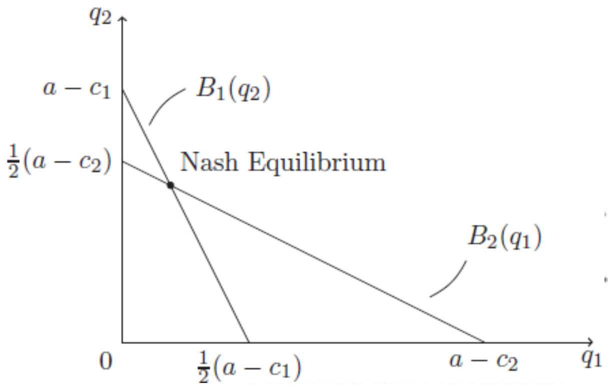
- The set of players is $I = \{1, 2\}$
- The strategy set available to each player $i \in I$ is the set of all nonnegative real numbers, i.e., $q_i \in [0,\infty)$
- The payoff received by each player i is a function of both players' strategies: $\Pi_i(q_i, q_{-i}) = \underbrace{q_i \cdot P(Q)}_{\text{player } i\text{'s revenue from selling } q_i \text{ units of products at a market-clearing price } P(Q)} - \underbrace{c_i \cdot q_i}_{\text{player } i\text{'s production cost}}$
- Given player 2's strategy q_2, player 1's payoff is a function of his quantity q_1

$$\Pi_1(q_1, q_2) = q_1 \cdot P(q_1 + q_2) - c_1 \cdot q_1$$

- The optimal strategy for player 1 needs to satisfy the first-order condition

$$q_1 \cdot P'(q_1 + q_2) + P(q_1 + q_2) - c_1 = 0$$

- Assuming that $P(q_1 + q_2) = a - q_1 - q_2$
- The best response of player 1 is a function of player 2's strategy q_2: $q_1^* = B_1(q_2) = \dfrac{a - q_2 - c_1}{2}$
- Given player 1's strategy q_1, the optimal strategy for player 2: $q_2^* = B_2(q_1) = \dfrac{a - q_1 - c_2}{2}$
- Pure strategy Nash equilibrium $q_1^* = \dfrac{a + c_1 + c_2}{3} - c_1$ $q_2^* = \dfrac{a + c_1 + c_2}{3} - c_2$



The Cournot Game – Example (1/2)

- $P(Q)$=50-2Q
- $C(q)$= 10+2q *(production cost)*
- At the Nash equilibrium we have:

$$\pi_1(q_1,q_2) = q_1\,[50 - 2\,(q_1 + q_2)] - 10 - 2q_1$$

$$\pi_2(q_1,q_2) = q_2\,[50 - 2\,(q_1 + q_2)] - 10 - 2q_2$$

$$\pi_1(q_1^*,q_2^*) \ge \pi_1(q_1,q_2^*) \quad \text{for all } q_1 \ge 0$$

$$\pi_2(q_1^*,q_2^*) \ge \pi_1(q_1^*,q_2) \quad \text{for all } q_2 \ge 0$$

- First-order condition:

$$\frac{\partial \pi_1}{\partial q_1}(q_1^*,q_2^*) = 50 - 4q_1 - 2q_2 - 2 = 0$$

$$\frac{\partial \pi_2}{\partial q_2}(q_1^*,q_2^*) = 50 - 2q_1 - 4q_2 - 2 = 0$$

$$\xrightarrow{\text{Nash equilibrium}} \quad q_1^* = q_2^* = 8$$

The Cournot Game – Example (2/2)

$C_1 = 10 + 2q_1$

$C_2 = 12 + 8q_2$

- What if **the firms have different costs???**
- Profit functions:

$$\pi_1(q_1,q_2) = q_1\,[50 - 2\,(q_1 + q_2)] - 10 - 2q_1$$

$$\pi_2(q_1,q_2) = q_2\,[50 - 2\,(q_1 + q_2)] - 12 - 8q_2$$

- First-order condition:

$$\frac{\partial \pi_1}{\partial q_1}(q_1^*, q_2^*) = 50 - 4q_1 - 2q_2 - 2 = 0$$

$$\frac{\partial \pi_2}{\partial q_2}(q_1^*, q_2^*) = 50 - 2q_1 - 4q_2 - 8 = 0$$

Nash equilibrium

$$q_1^* = 9,\; q_2^* = 6$$

the low-cost firm (firm 1) produces more and makes higher profits **than the high-cost firm (firm 2)**

Let’s work together!

- $P(Q)=120-Q$
- $C(q)= 30q$ *(production cost)*

[redacted]

1. Write the profit functions
2. Determine the first order conditions
3. Determine the Nash Equilibrium

The Bertrand Model

- An economic model used to describe interactions among firms (sellers) that set prices and their customers (buyers) that choose quantities at that price
- Key features:
  - There are at least two firms producing homogeneous products
  - Firms do not cooperate
  - Firms compete by setting prices simultaneously
  - Consumers buy everything from a firm with a lower price. If all firms charge the same price, consumers randomly select among them
  - The firms are economically rational and act strategically, seeking to maximize profits given their competitors' decisions
- Two firms, $I = \{1, 2\}$
- Each firm i chooses the price p_i, rather than quantity as in the Cournot model
- Consumers buy from the firm with a lower price, and the total consumer demand is a decreasing function of the market price: $D(\min\{p1, p2\})$
- ***What is the best price choice of each firm?***

The Bertrand Game

- The set of players is $I = \{1, 2\}$
- The strategy set available to each player $i \in I$ is the set of all nonnegative real numbers, i.e., $p_i \in [0,\infty)$
- The payoff (profit) received by each player i is a function of both players' strategies, defined by $\Pi_i(p_i, p_{-i}) = (p_i - c_i) \cdot D_i(p_1, p_2)$, where c_i is the unit producing cost and $D_i(p_1, p_2)$ is the consumers' demand to player i
- If player i's price is lower than that of the other player (denoted by $-i$), then he gets the total consumer demand $D(P)$: $D_i(p_1, p_2) = D(p_i)$ if $p_i < p_{-i}$ and $D_i(p_1, p_2) = 0$ if $p_i > p_{-i}$
- If two players' prices are the same, each player gets half of the total consumer demand $D(P)$: $D_i(p_1, p_2) = D(p_i)/2$ if $p_i = p_{-i}$
- Given player 2's strategy p_2, player 1's payoff is a function of his price p_1

$$\Pi_1(p_1, p_2) = \begin{cases} (p_1 - c_i) \cdot D(p_1) & \text{if } p_1 < p_2 \\ 0 & \text{if } p_1 > p_2 \\ (p_1 - c_i) \cdot D(p_1)/2 & \text{if } p_1 = p_2 \end{cases}$$

- Thus, given player 2's strategy p_2, the optimal strategy for player 1 is to select a price p_1 slightly lower than p_2,

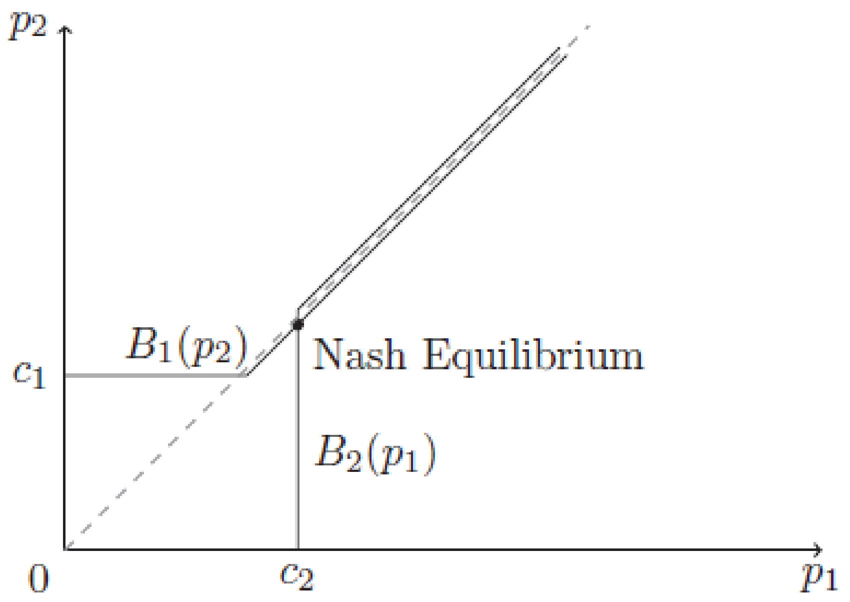
  under the constraint that $p_1 \geq c_1$

- Given player 1's strategy p_1, the optimal strategy for player 2 is to select a price p_2 slightly lower than p_1,

  under the constraint that $p_2 \geq c_2$

$$\begin{array}{ll} p_1^* = [c_2]^-, \quad p_2^* \in [c_2, \infty) & \text{if } c_1 < c_2 \\ p_1^* \in [c_1, \infty), \quad p_2^* = [c_1]^- & \text{if } c_1 > c_2 \\ p_1^* = \ p_2^* = c & \text{if } c_1 = c_2 = c \end{array}$$

The lower producing cost firm will extract all the consumer demand, by setting a price slightly lower than the other firm's producing cost!!!



The Hotelling Model

- An economic model used to study the effect of locations on the competition among two or more firms
- Key features:
  - There are two firms selling the same good. The firms have different locations, which are represented by two points in the interval [0,1]
  - The customers are uniformly distributed along the interval. Customers incur a transportation cost as well as a purchasing cost
  - The firms are economically rational and act strategically, seeking to maximize profits given their competitors' decisions
- Consider a one mile long beach on a hot summer day
- Two identical ice-cream shops on both ends of the beach: store1 at $x = 0$ and store 2 at $x = 1$
- The customers are uniformly distributed with density 1 along this beach
- Customers incur a transportation cost w per unit of length (e.g., the value of time spent in travel)
  - A customer at location $x \in [0, 1]$ will incur a transportation cost of $w \cdot x$ when going to store 1 and $w \cdot (1 - x)$ when going to store 2
- Each customer buys one icecream and obtains a satisfaction level of $\bar{s}$
- Each store $i \in \{1, 2\}$ chooses a unit price p_i
- A customer will choose a store that has the less generalized cost, i.e., price plus transportation cost
- Each store wants to choose the price to maximize its own profit, by considering the unit cost

The Hotelling Game

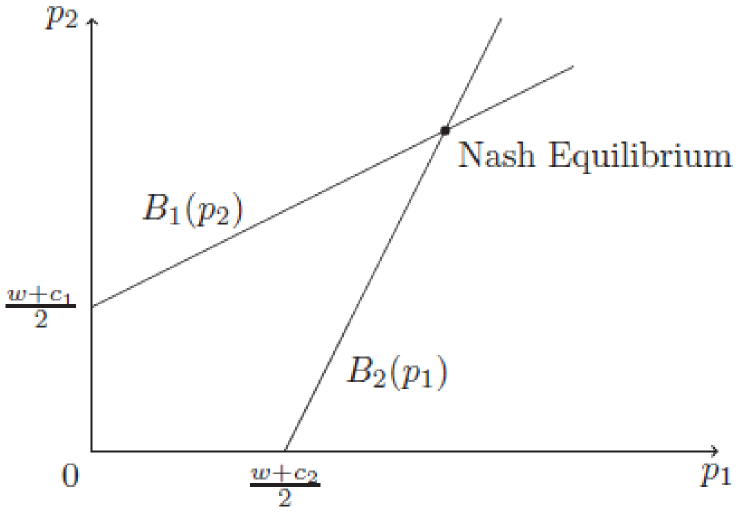
- The set of players is $I = \{1, 2\}$
- The strategy set available to each player $i \in I$ is the set of all nonnegative real numbers, i.e., $p_i \in [0,\infty)$
- The payoff received by each player i is a function of both players' strategies: $\Pi_i(p_i, p_{-i}) = (p_i - c_i) \cdot D_i(p_1, p_2)$, where c_i is the unit producing cost and $D_i(p_1, p_2)$ is the ratio of consumers coming to player i
- Location of the customer who is indifferent of choosing either store, $x = l(p_1, p_2)$, where x is given by $p_1 + w \cdot x = p_2 + w \cdot (1 - x)$
- Players' respective demand ratios $D_1(p_1, p_2) = l(p_1, p_2) = \dfrac{p_2 - p_1 + w}{2w}$ $D_2(p_1, p_2) = 1 - l(p_1, p_2) = \dfrac{p_1 - p_2 + w}{2w}$
- Given player 2's price $p2$, the profit of player 1 is

$$\Pi_1(p_1, p_2) = (p_1 - c_1) \cdot \frac{p_2 - p_1 + w}{2w}$$

First-order condition

$$\underbrace{p_1^* = B_1(p_2) = \frac{p_2 + w + c_1}{2} \quad \text{similarly...} \quad p_2^* = B_2(p_1) = \frac{p_1 + w + c_2}{2}}$$

$$p_1^* = \frac{3w + c_1 + c_2}{3} + \frac{c_1}{3}, \quad p_2^* = \frac{3w + c_1 + c_2}{3} + \frac{c_2}{3}$$



- The classic Hotelling model assumes firms compete purely on price with fixed locations
- In a more general case, firms can choose different locations so as to attract more consumers