ECE 345 / ME 380
Fall 2020
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Problem Set #1 Solutions

$\boxed{11}$ a) $s^2 \Theta(s) \cdot J = -k_1 s \Theta(s) - k_0 \Theta(s) + F(s)$

$$\Theta(s)\left[J \cdot s^2 + k_1 s + k_0\right] = F(s)$$

$$\Theta(s) = \frac{1}{Js^2 + k_1 s + k_0} \cdot F(s)$$

$$s H(s) = V \cdot \Theta(s)$$

$$H(s) = \frac{V}{s} \cdot \frac{1}{Js^2 + k_1 s + k_0} \cdot F(s)$$

$$\frac{H(s)}{F(s)} = \frac{V}{Js^3 + k_1 s^2 + k_0 s}$$

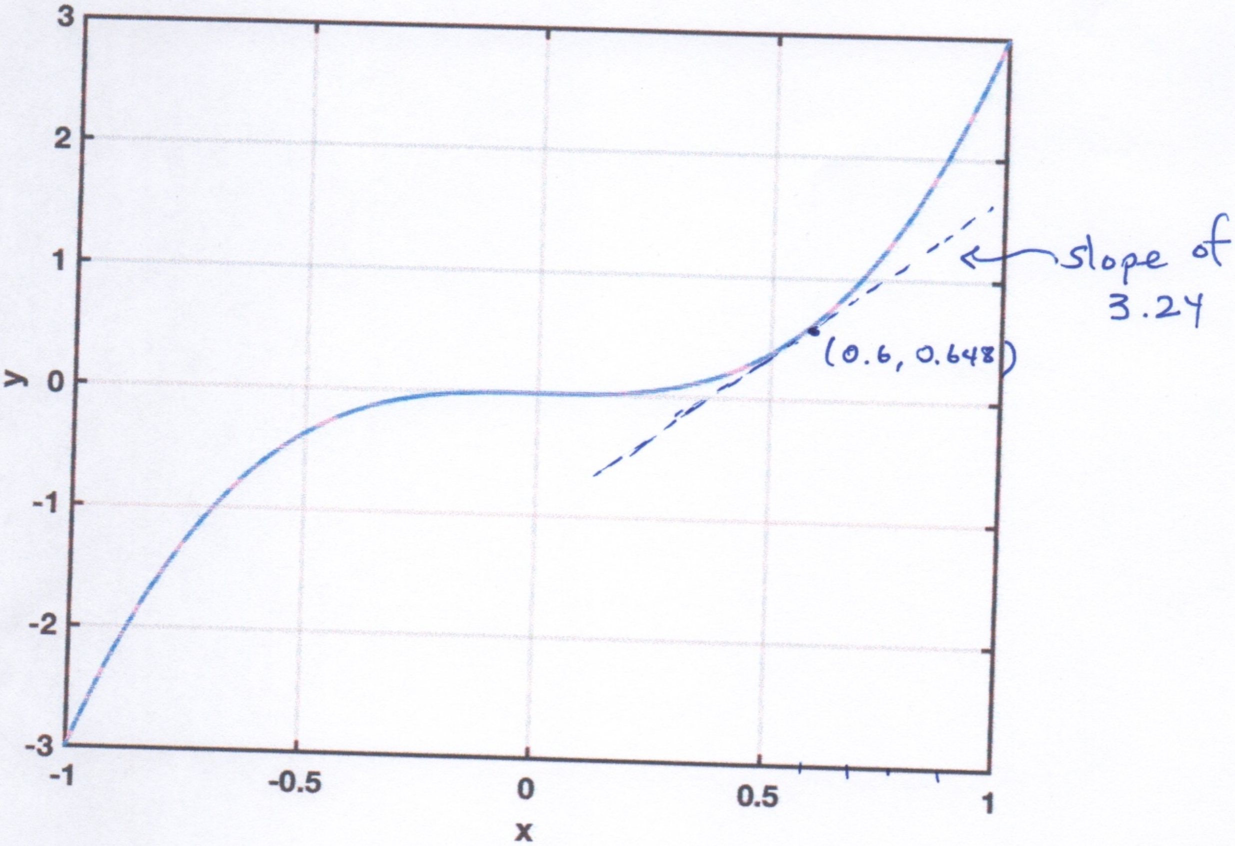
$\boxed{12}$ $f(x) = 3x^3$, $x_0 = 0.6$

$$f(x_0) = 3 \cdot 0.6^3 = 0.648$$

$$f(x) - f(x_0) \simeq \left.\frac{\partial f}{\partial x}\right|_{x = x_0} \cdot (x - x_0)$$

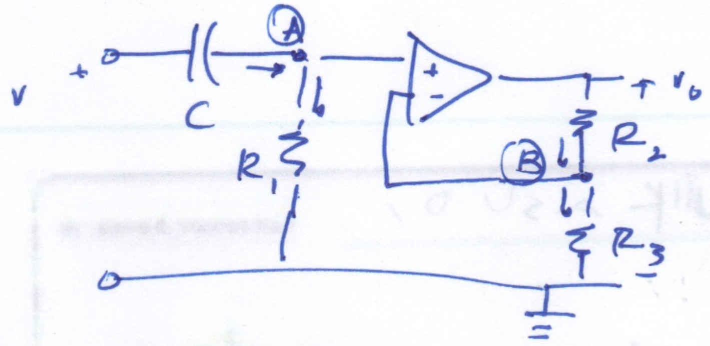
$$\underbrace{y - 0.648}_{\Delta y} \simeq 9x^2\Big|_{x=0.6} \underbrace{(x - 0.6)}_{\Delta x}$$

Linearization: $\Delta y = 3.24 \cdot \Delta x$



3
2
1
0
-1
-2
-3
y
-1
-0.5
0
0.5
1
x
(0.6, 0.648)
slope of
3.24

$\boxed{13}$



a)

KVL at A: $C\left(\frac{dv}{dt} - \frac{dv_A}{dt}\right) = \frac{v_A}{R_1}$

KVL at B: $\frac{v_o - v_B}{R_2} = \frac{v_B}{R_3}$

Op-Amp properties: $v_A = v_B$

$\Rightarrow$ A. $Cs\left[V(s) - V_A(s)\right] = \frac{V_A(s)}{R_1}$

$$R_1 C s V(s) = V_A(s)\left[1 + R_1 C s\right]$$

$$V(s) = V_A(s)\,\frac{(1 + R_1 C s)}{R_1 C s}$$

$\Rightarrow$ B. $\frac{V_o(s) - V_A(s)}{R_2} = \frac{V_A(s)}{R_3}$

$$R_3 V_o(s) - R_3 V_A(s) = V_A(s) \cdot R_2$$

$$R_3 V_o(s) = V_A(s)(R_2 + R_3)$$

$$\frac{R_3}{R_2 + R_3} \cdot V_o(s) = V_A(s)$$

$$\therefore \quad V(s) = \frac{1+R_1Cs}{R_1Cs} \cdot \frac{R_3}{R_2+R_3} \cdot V_o(s)$$

$$\frac{V_o(s)}{V(s)} = \frac{R_1Cs}{1+R_1Cs} \cdot \frac{R_2+R_3}{R_3}$$

$$= \frac{\left(\frac{R_2+R_3}{R_3}\right) \cdot R_1 C \cdot s}{1+R_1Cs}$$

$$= \frac{\left(\frac{1}{R_1C}\right)\left(\frac{R_2+R_3}{R_3}\right) \cdot R_1C \cdot s}{s + 1/R_1C} \longleftarrow \text{proper form.}$$

b) $V(s) = \frac{1}{s}$, $\frac{V_o(s)}{V(s)} = \frac{1 \cdot 3 \cdot 1 \cdot s}{s+1} = \frac{3s}{s+1}$

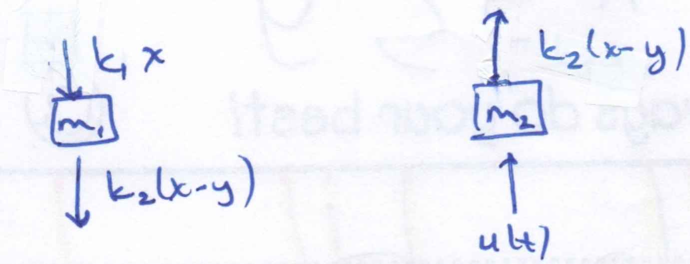
$$\therefore \quad V_o(s) = \frac{V_o(s)}{V(s)} \cdot V(s)$$

$$= \frac{3\cancel{s}}{s+1} \cdot \frac{1}{\cancel{s}}$$

$$= \frac{3}{(s+1)} \implies v_o(t) = \begin{cases} 3e^{-t} & \text{for } t \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

$\boxed{14}$ Presume "+" is $\uparrow$, + to assess FBD, so that $x>0$, $y>0$, $x-y>0$, i.e., the top spring is in compression + the bottom in tension.

a)



$$\Sigma F = m_1 a$$

$$m_1 \ddot{x}(t) = -k_1 x(t) - k_2 (x(t) - y(t))$$

$$\ddot{x}(t) = -2x(t) + y(t)$$

$$\Sigma F = m_2 a$$

$$m_2 \ddot{y}(t) = k_2 (x(t) - y(t)) + u(t)$$

$$\ddot{y}(t) = x(t) - y(t) + u(t)$$

b) $s^2 X(s) = -2X(s) + Y(s)$ $\qquad$ $s^2 Y(s) = X(s) - Y(s) + U(s)$

$X(s)[s^2+2] = Y(s)$ $\qquad$ $Y(s)[s^2+1] = X(s) + U(s)$

$$= \frac{1}{s^2+1}\left(X(s) + U(s) \right)$$

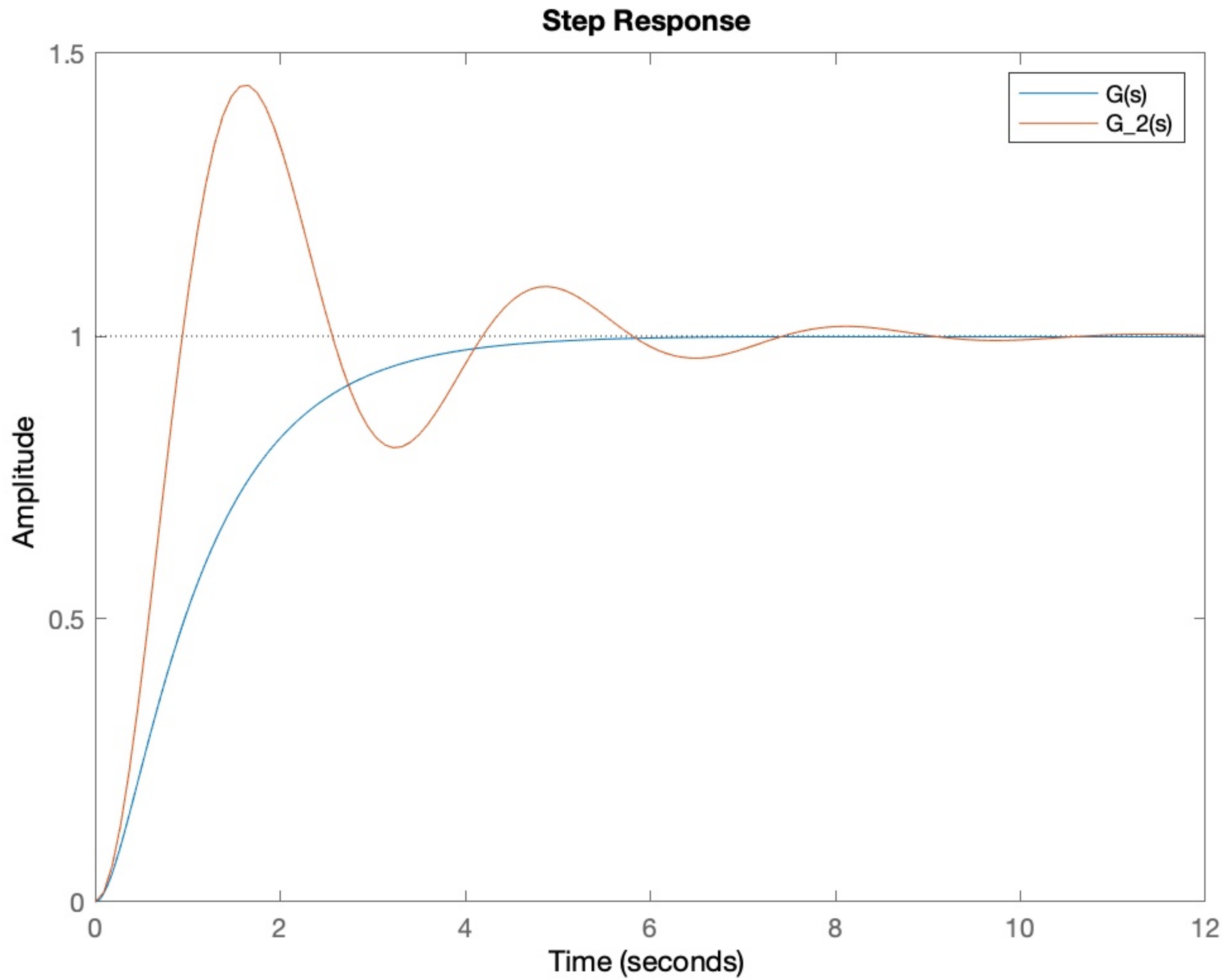
$$X(s)\left((s^2+2)(s^2+1) - 1 \right) = U(s)$$

$$\frac{X(s)}{U(s)} = \frac{1}{s^4 + 3s^2 + 1}$$

5.

diary.txt

```
>> conv([1 1],[1 4])

ans =

     1     5     4

>> roots([1 5 4])

ans =

    -4
    -1

>> G = tf(4, [1 5 4])

G =

          4
  -------------
  s^2 + 5 s + 4

Continuous-time transfer function.

>> step(G)
```


Step Response
G(s)
G_2(s)
Amplitude
Time (seconds)
0
0.5
1
1.5
0
2
4
6
8
10
12