ECE 345 / ME 380: Introduction to Control Systems
Midterm #2

Dr. Oishi

Due November 13, 2020, by 11:59am

This midterm is open note, open book, and Matlab and electronic resources are allowed. **No communication of any sort regarding the content of the exam is allowed with anyone other than Dr. Oishi.**

For full credit, show all your work.

Please provide your written response on the exam .pdf if possible, adding additional sheets as necessary.

Academic dishonesty is a violation of the UNM Student Code of Conduct. Students suspected of academic dishonesty will be referred for disciplinary action in accordance with University procedures.

By signing below, I affirm that I have completed the midterm independently, under the conditions stated above.

David Kirby — 101652098

Student Name — Student ID #

Problem #	Actual points	Possible points
1	17	20
2	37	40
3	30	30
Total:	84	90

1 BIBO stability (20 points)

Consider the transfer function $G(s) = \dfrac{s}{(s+1)(s^2+3s+a)}$, where a is a real-valued number.

1. (10 points) Use a Routh table to assess asymptotic stability of $G(s)$. Which of the following is most correct?

    (a) The closed-loop system is unstable for $a < -4$ since there are *two* poles in the RHP.

    (b) The closed-loop system is unstable for $a > 4$ since there are *two* poles in the RHP.

    (c) The closed-loop system is unstable for $0 > a > -4$ since there is *one* pole in the RHP.

    (d) The closed-loop system is stable for $a < 0$ since there are *no* poles in the RHP.

-3 X

$$G(s) = \frac{s}{s^3 + 4s^2 + (3+a)s + a}$$

s^3	1	$3+a$
s^2	4	a
s^1	$\frac{4(3+a)-a}{4}$	0
s^0	a	0

s^3	1	$3+a$
s^2	4	a
s^1	$\frac{3a+12}{4}$	0
s^0	a	0

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Recall $G(s) = \dfrac{s}{(s+1)(s^2+3s+a)}$.

2. (10 points) Presume $a = 2$. Which *one* of the following is correct?

   (a) Since $G(s)$ is marginally stable, it is also BIBO stable.

   (b) Since $G(s)$ is BIBO stable, it is also asymptotically stable.

   (c) Since $G(s)$ is asymptotically stable, it is also BIBO stable.

   (d) Since $G(s)$ is asymptotically stable, there may be bounded input trajectories that generate unbounded output trajectories, hence more work is needed to assess BIBO stability.

   (e) Since $G(s)$ is asymptotically unstable, it is also BIBO unstable.

$$G(s) = \frac{s}{s^3 + 4s^2 + 5s + 2}$$

$$\begin{array}{c|cc} s^3 & 1 & 5 \\ s^2 & 4 & 2 \\ s^1 & \frac{9}{2} & 0 \\ s^0 & 2 & 0 \end{array}$$

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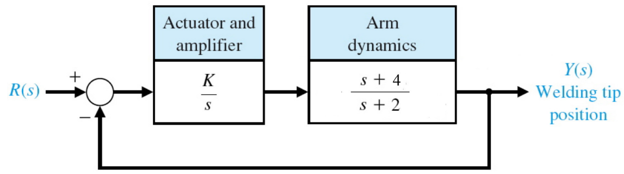
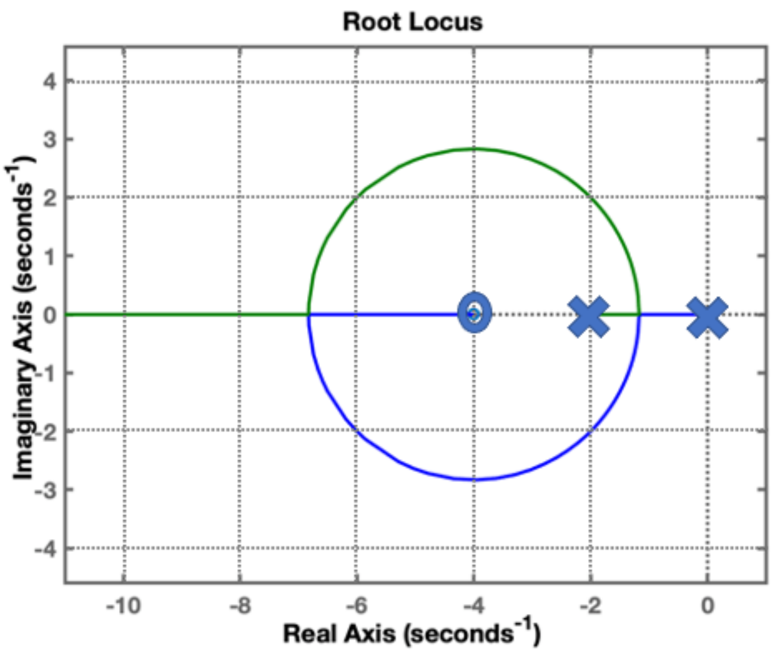
2 Precision welding (40 points)



An automated welding machine must be precise and agile. Consider the welding system on the right.

1. (10 points) Consider the root locus plot of the system, shown below. Is it possible to find a gain K such that the poles of $\frac{Y(s)}{R(s)}$ are located at $-4 \pm 4j$? In a single sentence, describe why or why not. *You do not need to find the value of K, if it is possible to do so.*



It is not possible to find a gain K that would satisfy this constraint; by looking at the plot, we can see that the imaginary part stops below $\pm 3j$.

2. (10 points) Based on the root locus plot above, which one of the following is correct?

   (a) The open-loop system $\frac{K(s+4)}{s(s+2)}$ is asymptotically stable for all $K > 0$ because all of the poles lie in the open left half plane for any $K > 0$.

   (b) The closed-loop system $\frac{Y(s)}{R(s)}$ is asymptotically stable for all $K > 0$ because all of the poles lie in the open left-half plane for any $K > 0$.

   -3

   (c) The closed-loop system $\frac{Y(s)}{R(s)}$ is marginally stable for all $K > 0$ because there is always a pole on the imaginary axis, and all other poles are in the open left half plane.

   (d) The stability of the closed-loop system $\frac{Y(s)}{R(s)}$ cannot be inferred from this plot.

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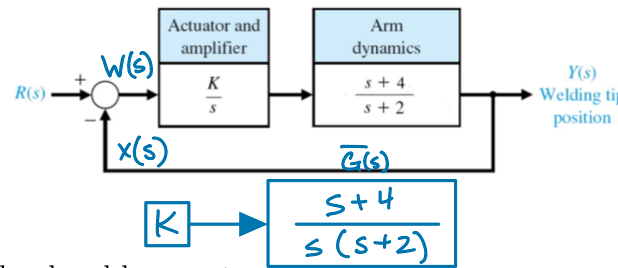
3. (10 points) Which one of the following describes the characteristic equation of the closed-loop system? *Show your work for full credit.*



(a) $0 = s^2 + 2s$

(b) $0 = s^2 + (2 + K)s + 4K$

(c) $0 = (K + 1)s + 4K + 2$

(d) $0 = s^2 + 2s + 6 + K$

4. (10 points) What value of gain K is required to make the closed-loop system have damping ratio $\zeta = 1/\sqrt{2}$ and natural frequency $\omega_n = 2\sqrt{2}$? *Hint: Match coefficients of the characteristic equation and* $0 = s^2 + 2\zeta\omega_n s + \omega_n^2$.

$$W(s) = R(s) - X(s)$$

$$X(s) = Y(s)$$

$$Y(s) = W(s)\ K \cdot \frac{s+4}{s(s+2)}$$

$$Y(s) = \left[R(s) - Y(s)\right] K \cdot \frac{s+4}{s(s+2)}$$

$$Y(s) + Y(s)\ K \cdot \frac{s+4}{s(s+2)} = R(s)\ K \cdot \frac{s+4}{s(s+2)}$$

$$Y(s)\left[1 + K \cdot \frac{s+4}{s(s+2)}\right] s(s+2) = R(s)\ K\ (s+4)$$

$$Y(s)\left[s(s+2) + K\ (s+4)\right] = R(s)\ K\ (s+4)$$

$$\frac{Y(s)}{R(s)} = \frac{K\ (s+4)}{s(s+2) + K\ (s+4)} = \frac{K\ (s+4)}{s^2 + 2s + Ks + 4K}$$

$$\Delta(s) = s^2 + (2+K)s + 4K \longrightarrow s^2 + 2\zeta\omega_n s + \omega_n^2$$

$$2\zeta\omega_n = 2 + K \qquad \omega_n^2 = 4K$$

$$\zeta = \frac{2+K}{4\sqrt{K}} = \frac{\cancel{4}}{\cancel{4}\sqrt{2}} = \frac{1}{\sqrt{2}} \qquad \omega_n = 2\sqrt{K}$$

$$\boxed{K = 2}$$

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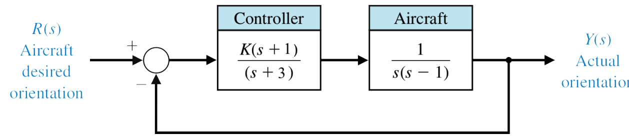
3 Aircraft carrier landing (30 points)

Landing on an aircraft carrier is difficult partly because the landing surface constantly moves. Consider an automated landing system, which aids the aircraft in tracking an orientation dictated by the landing surface.



We model the actual orientation of the aircraft as $y(t)$, the desired orientation as $r(t)$, and describe the open-loop aircraft dynamics as $KG(s) = K\frac{s+1}{s(s+3)(s-1)}$, for some $K > 0$. The characteristic equation of the closed-loop system is $\Delta(s) = s^3 + 2s^2 + (K-3)s + K$.

We wish for the closed-loop system $\frac{Y(s)}{R(s)}$ to have steady-state error with magnitude less than or equal to 0.15 in response to a unit ramp, i.e., $|e_{ss}| \leq 0.15$.

1. (10 points) Use type number to find all values of K that ensure that the steady-state error criterion is met. *Solutions that do not use type number will not receive full credit.*

Type 1

$$|e_{ss}| = \left|\frac{1}{K_v}\right|$$

$$K_v = \lim_{s\to 0} sKG(s) = \lim_{s\to 0} s\cdot K\cdot \frac{s+1}{s(s+3)(s-1)} = -\frac{K}{3}$$

$$|e_{ss}| = \left|\frac{-K}{3}\right| \leq 0.15$$

$$K \geq 20$$

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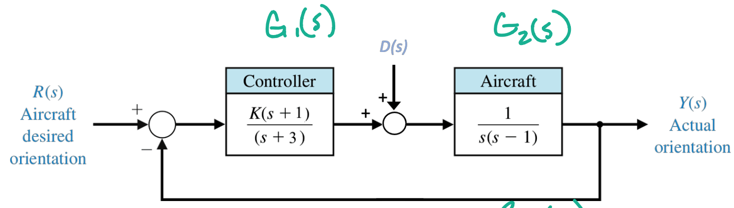
Now consider the case in which an external disturbance affects the system. The output response of the closed-loop system can be written as



$$Y(s) = G_D(s) \cdot D(s) + G_R(s) \cdot R(s) \qquad (1)$$

$$G_D(s) = \frac{G_2(s)}{1 + G_1(s)\,G_2(s)}$$

where $G_D(s) = \frac{1}{s^3+2s^2+(K-3)s+K}$ and $G_R(s) = \frac{K(s+1)}{s^3+2s^2+(K-3)s+K}$.

$$G_D(s) = \frac{s+3}{s^3+2s^2+(K-3)s+K}$$

2. (10 points) What values of K assure that the steady-state response due to a unit step disturbance, presuming $R(s) = 0$ (i.e., no reference input), is less than or equal to 0.05?

3. (10 points) Which one of the following is true?

   (a) As K increases, the steady state response due to a unit step disturbance *increases*.

   (b) As K increases, the steady state response due to a unit step disturbance *decreases*.

   (c) The steady-state response to a unit step disturbance is 0.05, for any $K > 0$.

   (d) The steady-state response to a unit step disturbance is 0, for any $K > 0$.

2) $y_{ss} \leq 0.05$ due to $D(s) = \frac{1}{s}$, $R(s) = 0$

$$Y(s) = G_D(s) \cdot D(s)$$

$$= \frac{1}{s^3+2s^2+(K-3)s+K} \cdot \frac{1}{s}$$

$$= \frac{1}{s\left(s^3+2s^2+(K-3)s+K\right)}$$

F.V.T.

$$y_{ss} = \lim_{s\to 0} sY(s)$$

$$= \lim_{s\to 0} \cancel{s}\left(\frac{1}{\cancel{s}\left(s^3+2s^2+(K-3)s+K\right)}\right)$$

$$= \frac{1}{K} \leq 0.05$$

$$K \geq 20$$

End of exam.