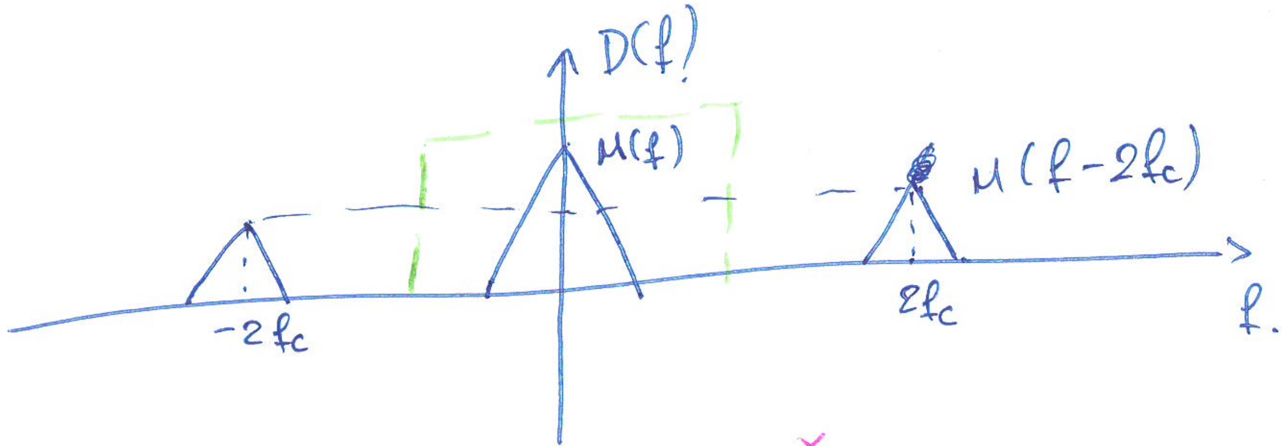


$$d(t) = 2A_c \cdot m(t) \cdot \cos(\overbrace{\omega_c t}^{y}) \cdot \cos(\overbrace{\omega_c t + \theta(t)}^{x})$$

$2\cos x \cos y = \cos(x+y) + \cos(x-y)$

$$= A_c m(t) \left[\cos(2\omega_c t + \theta(t)) + \cos(\theta(t))\right]$$

$$= \cancel{A_c \cdot m(t) \cdot \cos(2\omega_c t + \theta(t))} + A_c m(t) \cdot \cos(\theta(t))$$

Transmitted Signals in SSB

$$x_c(t) = \frac{1}{2} A_c m(t) \cdot \cos(2\pi f_c t) + \frac{1}{2} A_c \cdot \hat{m}(t) \cdot \sin(2\pi f_c t)$$

LSB signal

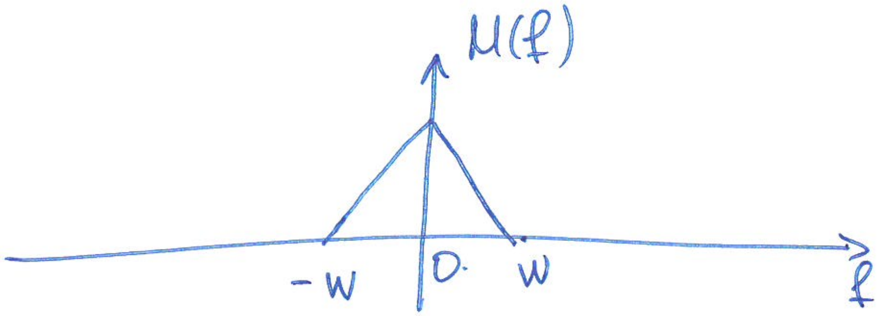
$$x_c(t) = \frac{1}{2} A_c \cdot m(t) \cos(2\pi f_c t) - \frac{1}{2} A_c \cdot \hat{m}(t) \cdot \sin(2\pi f_c t)$$

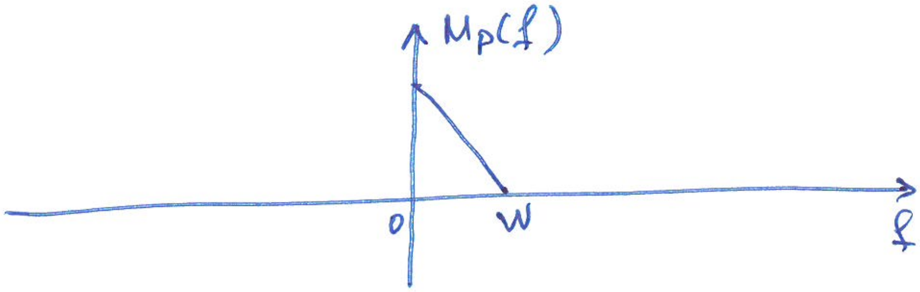
USB signal

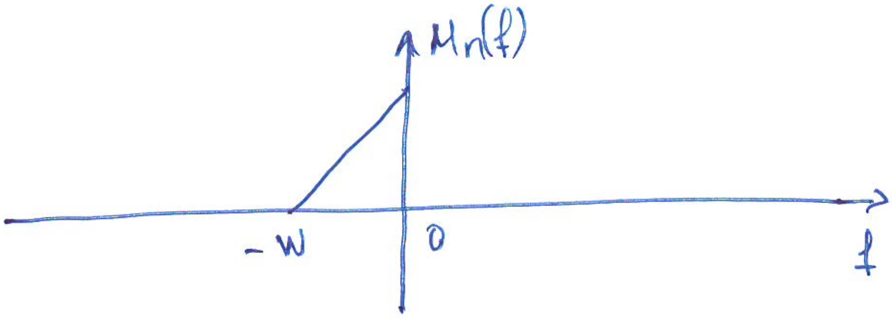
B)

$$M_p(f) = \frac{1}{2} F\{m(t) + j\hat{m}(t)\}$$

$$M_n(f) = \frac{1}{2} F\{m(t) - j\hat{m}(t)\}$$







By definition:

$$X_c(f) = \frac{1}{2} A_c \cdot M_p(f - f_c) + \frac{1}{2} A_c M_n(f + f_c)$$

$$\Updownarrow$$

$$x_c(t) = \frac{1}{4} A_c \left[m(t) + j\hat{m}(t) \right] \cdot e^{j2\pi f_c t} + \frac{1}{4} A_c \left[m(t) - j\hat{m}(t) \right] e^{-j2\pi f_c t}$$

$$= \frac{1}{4} A_c m(t) \left[e^{j2\pi f_c t} + e^{-j2\pi f_c t} \right] + j\frac{1}{4} A_c \hat{m}(t) \cdot \left[e^{j2\pi f_c t} - e^{-j2\pi f_c t} \right]$$

$$= \frac{1}{2} A_c \cdot m(t) \cdot \cos(2\pi f_c t) - \frac{1}{2} A_c \cdot \hat{m}(t) \cdot \sin(2\pi f_c t)$$

USB

Demodulation

A) $$d(t) = \Big[\overbrace{\tfrac{1}{2}A_c m(t)\cos(2\pi f_c t) \pm \tfrac{1}{2}A_c \hat{m}(t)\cdot\sin(2\pi f_c t)}^{x_c(t)\,=\,x_r(t)}\Big]\cdot$$

$$\underbrace{\overset{2\cdot 2}{4}\cos(2\pi f_c t + \theta(t))}_{\text{demodulation carrier}}$$

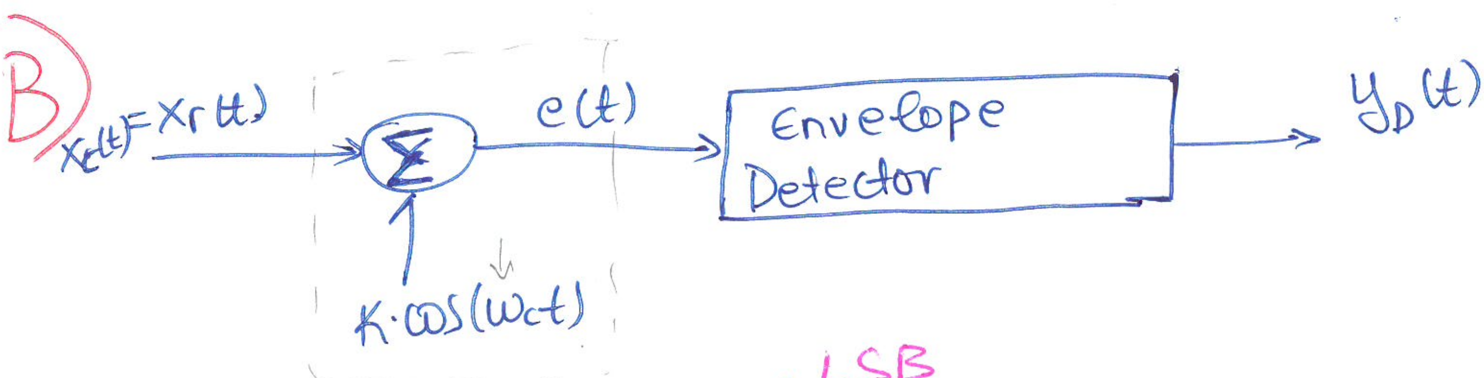
$$= A_c m(t)\cdot\cos(\theta(t)) + \cancel{A_c\cdot m(t)\cdot\cos[4\pi f_c t + \theta(t)]}$$

$$\mp A_c \hat{m}(t)\cdot\sin(\theta(t)) \pm \cancel{A_c \hat{m}(t)\cdot\sin[4\pi f_c t + \theta(t)]}$$

$$y_D(t) = \underbrace{A_c m(t)\cdot\cos(\theta(t))}_{\text{message signal}} \mp \underbrace{A_c \hat{m}(t)\cdot\sin(\theta(t))}_{\text{distortion}}$$

Output signal

B)

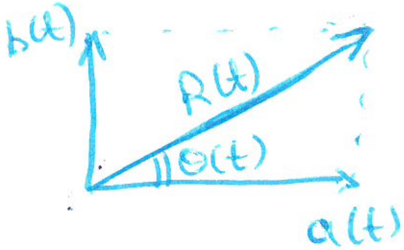


LSB

$$x_r(t) = \frac{1}{2}A_c m(t)\cos(2\pi f_c t) \pm \frac{1}{2}A_c\cdot\hat{m}(t)\cdot\sin(2\pi f_c t)$$

USB

$$e(t) = \underbrace{\left(\frac{1}{2}A_c m(t) + K\right)}_{a(t)}\cos(2\pi f_c t) \pm \underbrace{\frac{1}{2}A_c}_{b(t)}\hat{m}(t)\cdot\sin(2\pi f_c t)$$



$$a(t) = R(t)\cdot\cos(\theta(t))$$

$$b(t) = R(t)\sin(\theta(t))$$

$$e(t) = a(t)\cdot\cos(2\pi f_c t) \pm b(t)\sin(2\pi f_c t)$$

$$e(t) = R(t)\left[\cos(\overset{x}{\theta(t)})\cdot\cos(\overset{y}{2\pi f_c t}) \pm \sin(\theta(t))\sin(2\pi f_c t)\right]$$

$$e(t) = R(t)\cdot\cos\left[2\pi f_c t \mp \theta(t)\right]$$

$$R(t) = \sqrt{a^2(t) + b^2(t)}$$

$$y_D(t) = \sqrt{\left(\tfrac{1}{2}A_c m(t) + K\right)^2 + \left(\tfrac{1}{2}A_c \hat{m}(t)\right)^2}$$

If $\quad \frac{1}{2}A_c m(t) + K >>> \frac{1}{2}A_c \hat{m}(t)$.

$$\Rightarrow y_D(t) \cong \frac{1}{2}A_c m(t) + K$$

Example:

$$m(t) = \cos(2\pi f_1 t) - 0.4\cdot\cos(4\pi f_1 t) + 0.9\cos(6\pi f_1 t)$$

$$\hat{m}(t) = \sin(2\pi f_1 t) - 0.4\cdot\sin(4\pi f_1 t) + 0.9\sin(6\pi f_1 t)$$

A) $$x_c(t) = \frac{1}{2}A_c m(t)\cos(2\pi f_c t) \overset{\text{LSB}}{\underset{\text{USB}}{\pm}} \frac{1}{2}A_c\cdot\hat{m}(t)\sin(2\pi f_c t)$$

$x_c(t) = R(t) \cdot \cos(2\pi f_c t + \theta(t))$ ⑤

$\left(k \ll \frac{1}{2} A_c m(t)\right)$

B)

$$R(t) = \frac{A_c}{2}\sqrt{m^2(t) + \hat{m}(t)}$$

$$\tan\theta(t) = \frac{b(t)}{a(t)}$$

$$\boxed{\theta(t) = \tan^{-1}\left(\frac{\cancel{\frac{1}{2}A_c}\,\hat{m}(t)}{\cancel{\frac{1}{2}A_c}\,m(t)}\right)}$$

Phase: $2\pi f_c t + \theta(t)$

$$\frac{d}{dt}\left[2\pi f_c t + \theta(t)\right] = 2\pi f_c \pm \frac{d}{dt}\left[\tan^{-1}\left(\frac{\hat{m}(t)}{m(t)}\right)\right]$$

Two kinds of Modulation

Continuous — Pulse

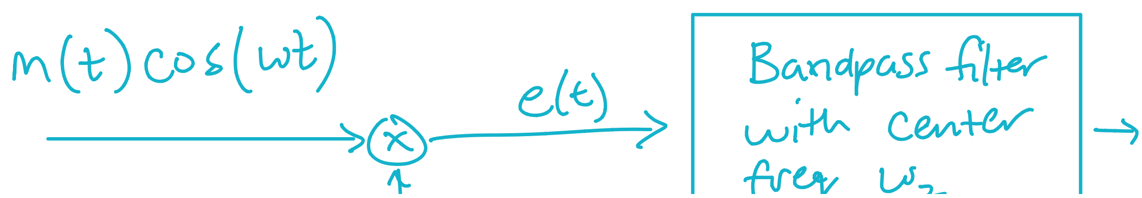
Linear | Angle

AM | FM, ΦM

TDMA - Time-delay Modulation

Translation of frequency

OFDMA & FDMA use this method *
(will learn in Digital systems)



local oscillator

$2\cos[(\omega_1 + \omega_2)t]$

$x_c(t) =$
$m(t)\cos(\omega_2 t)$

amplitude modulation: [...]

pulse position: keep initial position, send info in different positions at same pulse

which time position

pulse width: info captured in width of signal

Pulse Modulation

Pulse Ampl. Mod. (P.A.M.) — Width (P.W.M) — Position (P.P.M)

↓

$$h(t) = \Pi\left(\frac{t - \frac{1}{2}\tau}{\tau}\right)$$

↓

$$m_c(t) = m_s(t) * h(t)$$

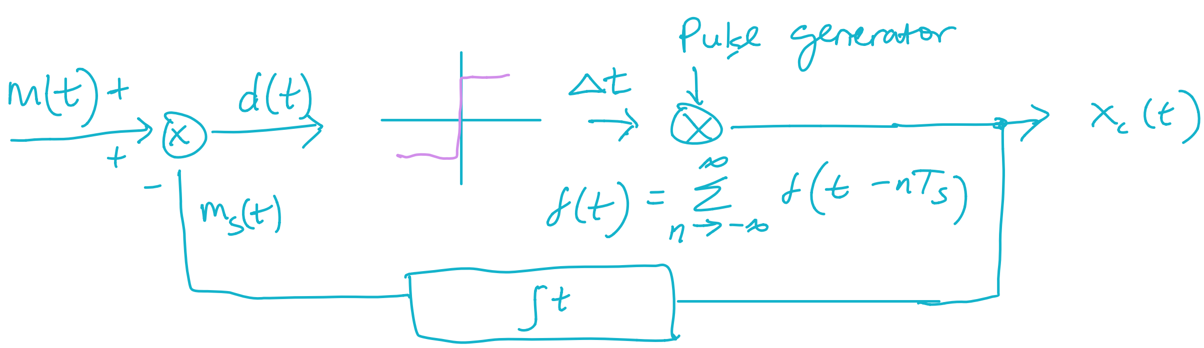
$$= m(nT_s) \cdot \Pi\left(\frac{(t - nT_s) + \frac{1}{2}\tau}{\tau}\right)$$

Digital Pulse Modulation

- Delta Mod
- Pulse Code Mod

Compare delta | samples that we took and see how the deviate

limiter sets the boundaries of signal



$$d(t) = m(t) - m_s(t)$$

$$x_c(t) = \Delta(nT_s) \cdot \delta(t - nT_s)$$

$$m_s(t) = \Delta(nT_s) \cdot \int^{t} \delta(x - nT_s)\,dx$$

Pulse Code Modulation

(Encoding = mapping amplitude to a specific code)

$m(t) \rightarrow$ [sampler] $\rightarrow$ [quantization] $\rightarrow$ [encoding]

Time-Division Multiplexing

1 message signal

(1 user transmitting)

timeslot

Cont. Wave Mod

Linear

Angular

Pulse Mod

Angle Mod. Techniques

PM   FM

general signal:

$$x_c(t) = A_c \cos\left[2\pi f_c t + \phi t\right]$$

Instant. phase | $\theta(t) = 2\pi f_c t + \phi t$ — phase deviation

Instant freq | $f(t) \triangleq \frac{1}{2\pi} \frac{d\theta(t)}{dt}$

$$= f_c + \frac{1}{2\pi} \frac{d\phi(t)}{dt}$$ — freq. deviation

Phase mod | $\phi(t) = k_p \cdot m(t)$