ECE 345 / ME 380: Introduction to Control Systems
Final Exam

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December 12, 2013

This final is closed-note, closed-book. A one-page, two-sided cheat-sheet is allowed. No calculators or other electronic devices are allowed. To get full credit, show all of your work.

There is an additional page at the end of the exam if you need more space.

If you finish early, carefully proofread your answers.

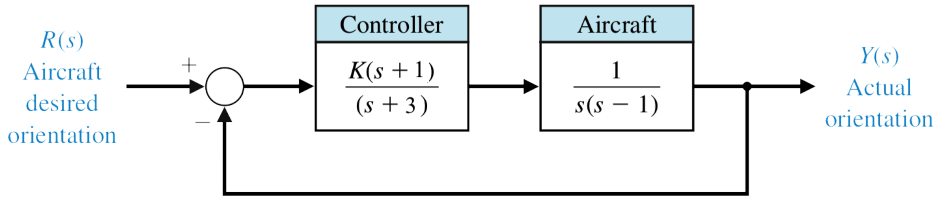
Academic dishonesty is a violation of the UNM Student Code of Conduct. Students suspected of academic dishonesty will be dismissed from the exam and referred for disciplinary action in accordance with University procedures.

Student Name Student ID #

Problem #	Actual points	Possible points
1		15
2		25
3		25
4		30
5		30
6		20
Total:		140

1 Jump-jet aircraft control (15 points)

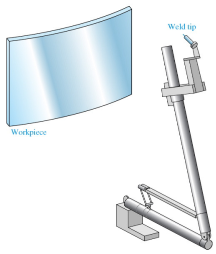
The lateral dynamics of jump-jet aircraft are inherently unstable, and so a controller $G_c(s) = \frac{K(s+1)}{s+3}$ is designed to stabilize the plant dynamics $G(s) = \frac{1}{s(s-1)}$.

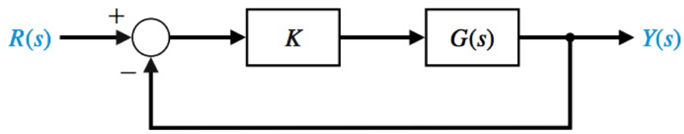


1. (+5 points) Use the Hurwitz criterion to determine all values of $K > 0$ that will assure the *closed-loop system* $\frac{Y(s)}{R(s)}$ is asymptotically stable.

2. (+10 points) Use a Routh table to determine *how many* poles lie in the RHP when $K = 2$.

2 Precision welding (25 points)





Consider a high precision welding arm with a long reach as shown in the figure above. The plant dynamics are $G(s) = \frac{s+4}{s(s+2)}$. The output $y(t)$ is the actual welding tip position, and the input $r(t)$ is the desired welding tip position.

The characteristic equation of the closed-loop system is $\Delta(s) = s^2 + (2+K)s + 4K$.

1. (+10 points) What positive values of K result in a closed-loop system $\frac{Y(s)}{R(s)}$ with settling time of *at most* 2 seconds?

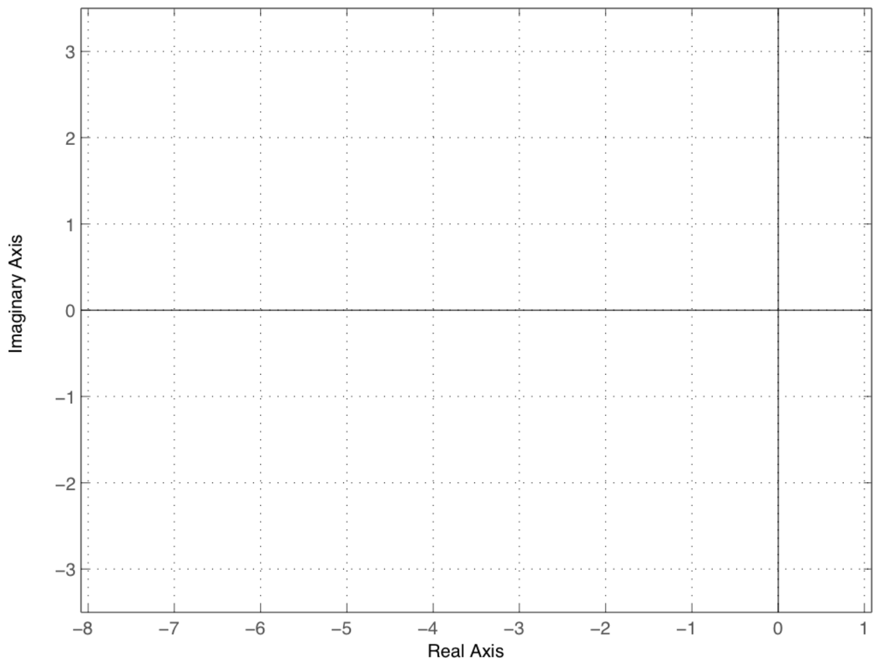
Recall $G(s) = \frac{s+4}{s(s+2)}$.

2. (+15 points) Sketch the root locus diagram of $G(s)G_c(s)$ on the complex plane below. *You do not need to compute departure / arrival angles.*

   (a) Accurately mark the location of the open-loop poles and zeros.

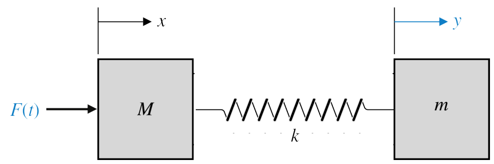
   (b) Compute the number of asymptotes.

   (c) Identify any break-away/break-in points to the real line.

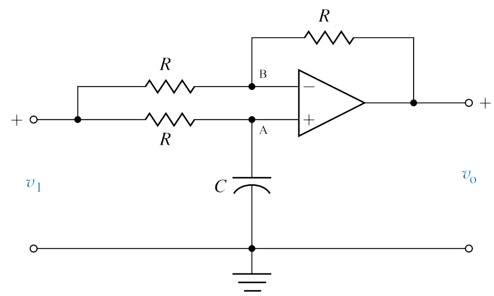


3 Modeling and Transient Behavior (25 points)

Choose *one* of the below systems to answer *either* 1A or 1B. *You do not need to do both problems. You will not receive extra credit for answering both problems; if you do both, only the first one graded will count towards your final score.*



(1A.) Spring mass damper system

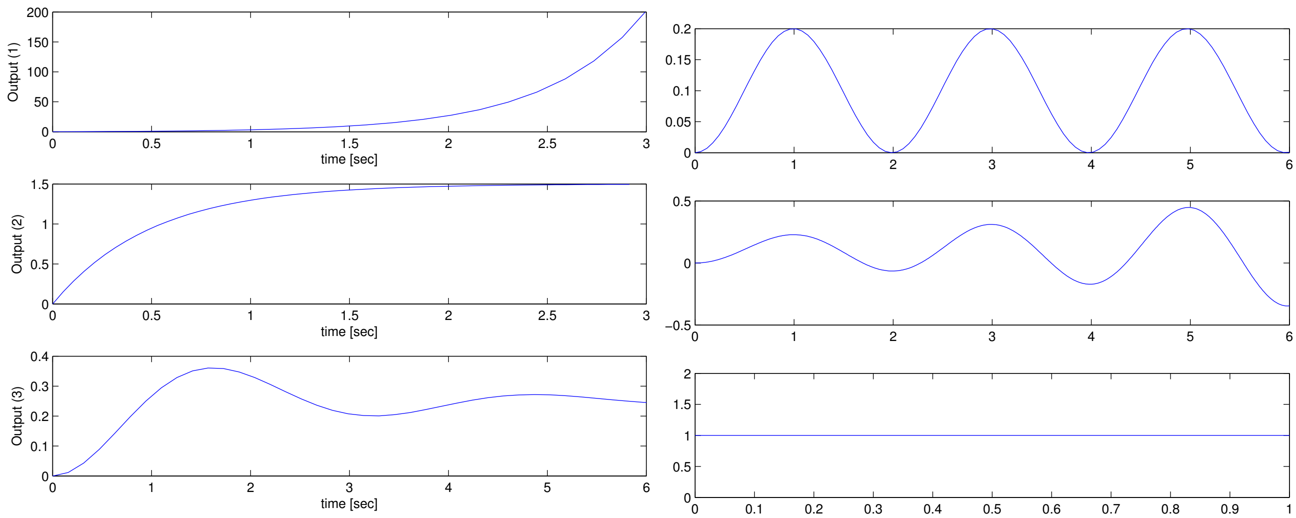


(1B.) Op-amp circuit system

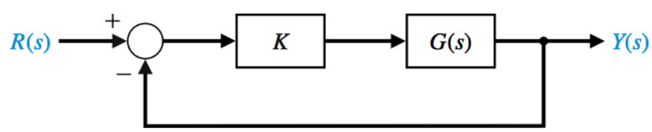
1A. (+10 points) Find the mechanical system transfer function $G(s) = \frac{Y(s)}{F(s)}$, assuming initial conditions are 0. Leave your result in standard form (e.g., one polynomial divided by another polynomial).

1B. (+10 points) Find the circuit transfer function $G(s) = \frac{V_0(s)}{V_1(s)}$, assuming initial conditions are 0. Leave your result in standard form (e.g., one polynomial divided by another polynomial).

2. (+5 points) Which *one* of the following six trajectories below most closely characterizes the step response of a stable, underdamped system? Mark the selected plot with a '$\star$'.



3. (+10 points) Consider the system below with $G(s) = \dfrac{6}{(s+3)(s+1)}$ and gain $K > 0$. What is the steady-state error e_{ss} of the closed-loop system $\frac{Y(s)}{R(s)}$ in response to a unit step input?



4 Stability (30 points)

Consider the system in state-space form below.

$$\begin{aligned} \dot{x} &= \begin{bmatrix} -3 & 1 \\ -2 & 0 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u \\ y &= \begin{bmatrix} 2 & 1 \end{bmatrix} x + 0 \cdot u \end{aligned} \tag{1}$$

1. (+10 points) Which of the following most closely describes the BIBO stability of the above system?

   (a) BIBO stable because the system is asymptotically stable

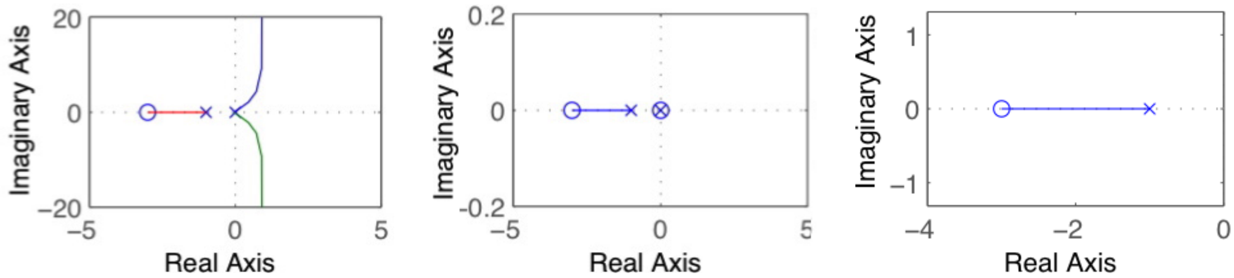
   (b) BIBO unstable because the system is asymptotically unstable, and there is no pole-zero cancellation

   (c) BIBO stability cannot be determined

   (d) BIBO stable because for $u(t) = \mathbf{1}(t)$, $y(t)$ is also bounded

   (e) Both (a) and (d)

Consider the following root locus diagrams of three systems (a), (b), and (c). You may assume that there are no co-located pole pairs and that there are no co-located zero pairs.



(a) (b) (c)

2. (+10 points) Which (if any) of the above three closed-loop systems are *asymptotically stable* for all $K > 0$?

   (a) System (a)

   (b) System (b)

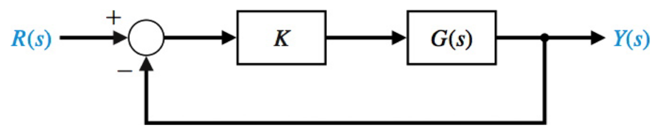
   (c) System (c)

   (d) System (b) and system (c)

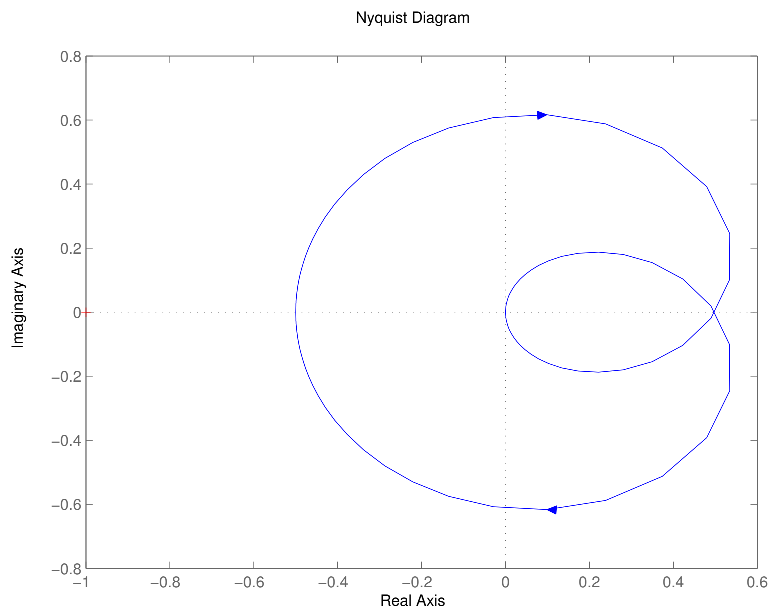
   (e) None of the systems are asymptotically stable for all $K > 0$

Now consider a system under negative unitary feedback, with

$$G(s) = \frac{s-1}{s^2+2s+2}$$



The Nyquist diagram of $KG(s)$ is plotted below, for $K = 1$.



3. (+10 points) Use the Nyquist criterion to determine whether the closed-loop system under negative unity feedback with $K = 1$ is asymptotically stable, marginally stable, or unstable.

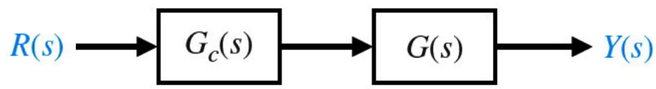
4. BONUS (+5 points)

   (a) Estimate the gain margin G_M from the Nyquist diagram above.

   (b) Describe the stability of the closed-loop system when $K = 2G_M$.

5 Feedback with a Disturbance (30 points)

First consider the open-loop system with transfer function $G_c(s)G(s)$. The controller is $G_c(s) = \left(s + \frac{1}{2}\right)$, and the plant is $G(s) = \frac{1}{s^2 + 3s + 2}$.



1. (+10 points) Compute the output $y(t)$ due to a reference input $r(t) = \frac{1}{2}e^{-\frac{1}{2}t} \cdot \mathbf{1}(t)$. Your answer *must* be in the time-domain.

Recall $G_c(s)G(s) = \frac{s+\frac{1}{2}}{s^2+3s+2}$.

2. (+10 points) Which *one* of the following systems accurately represents $G_c(s)G(s)$ in phase variable form?

   (a) $A = \begin{bmatrix} 0 & -1 \\ 2 & -3 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \end{bmatrix}, C = \begin{bmatrix} \frac{1}{2} \\ 1 \end{bmatrix}, D = 0$

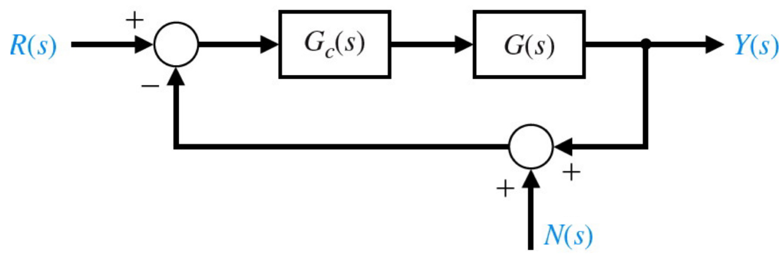
   (b) $A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = \begin{bmatrix} \frac{1}{2} & 1 \end{bmatrix}, D = 0$

   (c) $A = \begin{bmatrix} 0 & 1 \\ -3 & -2 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, C = \begin{bmatrix} 0 & \frac{1}{2} \end{bmatrix}^T, D = 0$

   (d) $A = \begin{bmatrix} 0 & -2 \\ 1 & -3 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = \begin{bmatrix} \frac{1}{2} & 1 \end{bmatrix}, D = 0$

Recall $G_c(s)G(s) = \frac{s+\frac{1}{2}}{s^2+3s+2}$.

Now consider the closed-loop system $\frac{Y(s)}{R(s)}$ with reference input $r(t)$, and noise input $n(t)$.



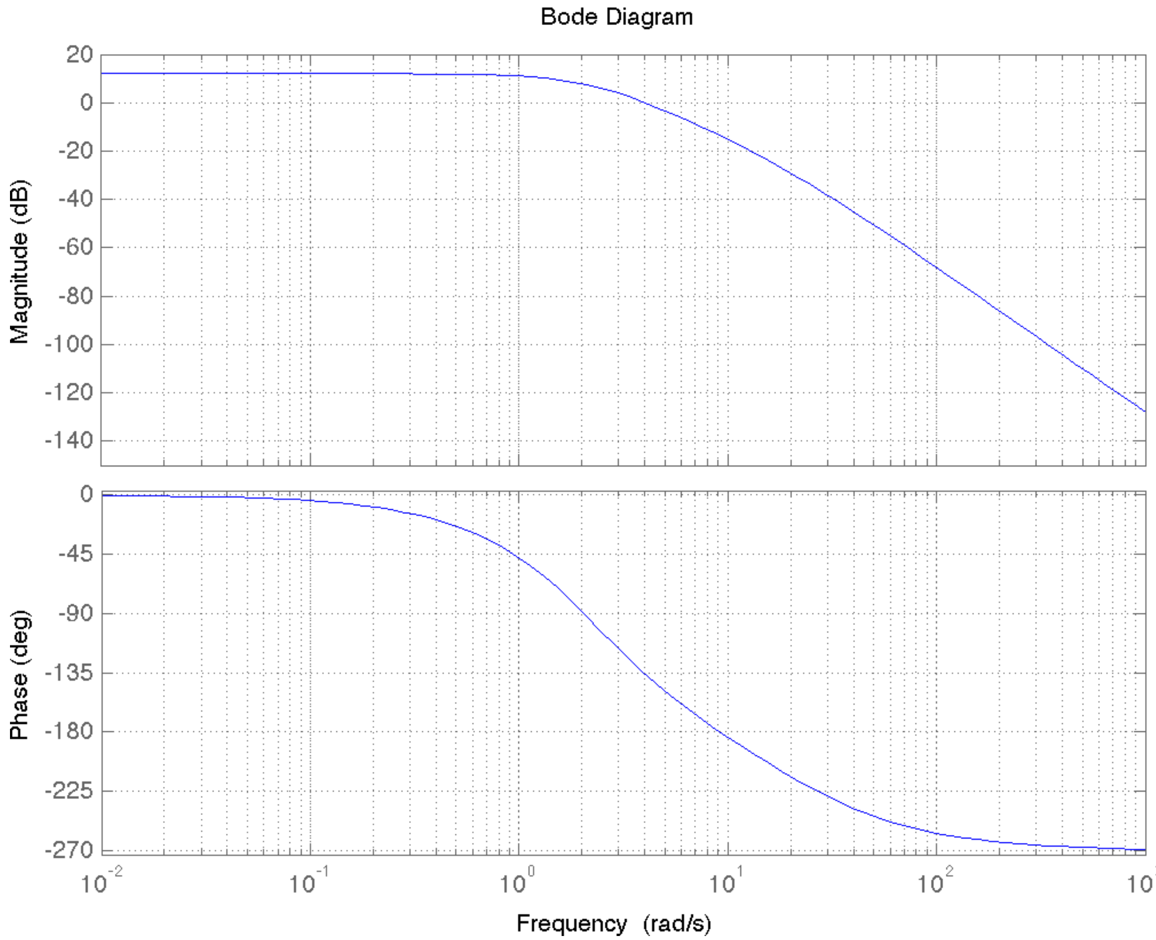
3. (+10 points) Use block diagram manipulation to determine the effect of the reference input $R(s)$ and the disturbance input $N(s)$ on the output $Y(s)$. That is, find $G_R(s)$ and $G_N(s)$ such that

$$Y(s) = G_R(s)R(s) + G_N(s)N(s) \tag{2}$$

Substitute in the numerical values for $G_c(s)G(s)$. Put your answer in standard form, that is, one polynomial divided by another polynomial.

6 Anti-Skid Braking System (20 points)

Consider the Bode diagram of a simplified model $KG(s)$, $K = 1$, of an anti-skid braking system that regulates wheel slip to maximize the car's traction.



1. (a) (+5 points) Find the phase margin Φ_M, and mark the frequency ω_{Φ_M} at which phase margin is measured on the Bode diagram above.

   (b) (+5 points) Find the gain margin G_M in dB, and mark the frequency ω_{G_M} at which gain margin is measured on the Bode diagram above.

2. (a) (+5 points) Is the negative unity feedback system stable with $K = 1$? Why or why not?

   (b) (+5 points) Will the negative unity feedback system be unstable for $K = 10$? Briefly justify your answer in a single sentence.