Time-average Autocorrelation For Energy Signals

$$\underset{\text{time}}{\phi(t)} \triangleq \lim_{T\to\infty} \int_{-T}^{T} x(\lambda)\cdot x(\lambda+\underset{\substack{\uparrow \\ \text{time swift.}}}{\tau})\, d\lambda$$

For Power Signals

$$R(\tau) = \langle x(t)\cdot x(t+\tau) \rangle \leftarrow \text{average}$$

$$\triangleq \lim_{T\to\infty} \frac{1}{2\cdot T}\cdot \int_{-T}^{T} x(t)\cdot x(t+\tau)\, dt$$

For Periodic Signals

autocorrelation function

$$R(\tau) = \frac{1}{T_0} \int_{T_0} x(t)\cdot x(t+\tau)\, dt$$

$$\int_{T_0} \to \int_0^{T_0}$$

Total Power of a Signal.

$$R(0) = \langle x^2(t) \rangle = \int_{-\infty}^{+\infty} S(f)\, df$$

$S(f)$: power spectral density.

Properties

1) $R(0) = \langle x^2(t) \rangle \geq R(\tau)$

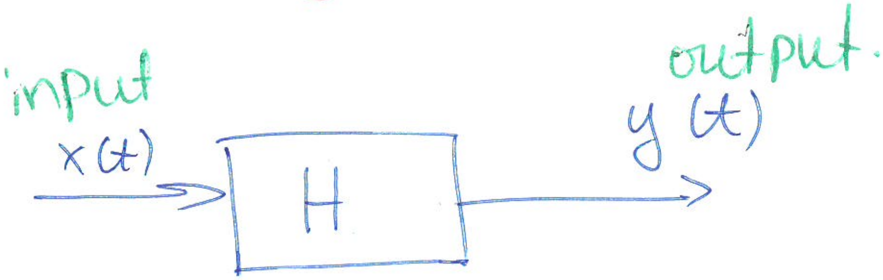
2) $R(-\tau) = R(\tau)$

3) If $x(t)$ does not contain any periodic component, $\lim_{|\tau|\to\infty} R(\tau) = [\langle x(t) \rangle]^2$

4) If $x(t)$ periodic with T_0 $\longrightarrow$ $R(\tau)$ periodic with T_0

5) $R(\tau) \longleftrightarrow S(f) \rightarrow$ non-negative

Signals & Systems



$$y(t) = H[x(t)]$$

Superposition Property.

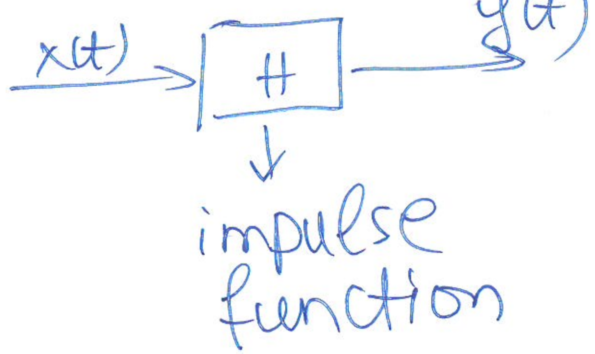
$$y(t) = H[a_1 x_1(t) + a_2 x_2(t)]$$

$$= a_1 H[x_1(t)] + a_2 H[x_2(t)]$$

$$= a_1 y_1(t) + a_2 y_2(t)$$

Time-invariant Property

$$y(t-t_0) = H[x(t-t_0)]$$



$h(t) \triangleq H[\delta(t)]$

impulse response function.

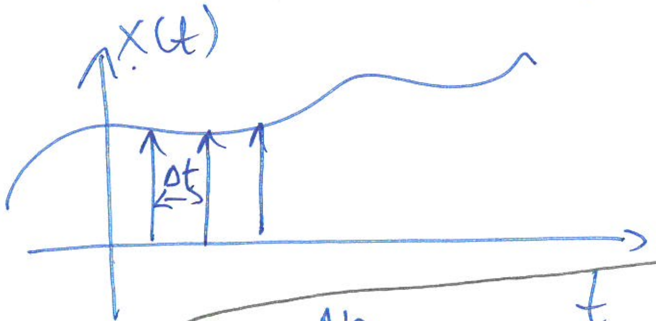
$h(t) \longleftrightarrow H[f]$

input: $x(t) = \sum_{n=1}^{N} a_n \delta(t - t_n)$

output: $y(t) = \sum_{n=}^{N} a_n h(t - t_n)$

Single output → single output

$$y(t) = H[x(t)]$$



$\tilde{x}(t) = \sum_{n=N_1}^{N_2} x(n\Delta t) \cdot \delta(t - n\Delta t)\Delta$

Approximation

$\tilde{y}(t) = \sum_{n=N_1}^{N_2} x(n\Delta t)\, h(t - n\Delta t)\Delta t$

$\Delta t \to 0$

$$y(t) = \int_{-\infty}^{+\infty} x(\lambda) \cdot h(t-\lambda)\, d\lambda$$

1 **Causal System** If the system does not respond before an input is applied.

$$h(t-\lambda) = 0 \quad , \; t < \lambda$$

Bounded input Bounded Output (BIBO) Stable

$$\int_{-\infty}^{+\infty} |h(t)|\, dt < \infty$$

$$h(t) \Leftrightarrow H(f) = |H(f)|\, e^{j \angle H(f)}$$

$$X(f) \longrightarrow \boxed{H(f)} \longrightarrow Y(f)$$

$$Y(f) = H(f) \cdot X(f)$$

Symmetry Properties

$$|H(f)| = |H(-f)| \quad \text{even}$$

$$\angle H(f) = -\angle H(-f) \quad \text{odd}$$

Periodic Signal:

$$x(t) = A \cdot e^{j2\pi f_0 t} \quad \text{input}$$

$$y(t) = \int_{-\infty}^{+\infty} h(\lambda) \cdot A \cdot \underline{e^{j2\pi f_0 (t-\lambda)}} \, d\lambda$$

$$= A e^{j2\pi f_0 t} \underbrace{\int_{-\infty}^{+\infty} h(\lambda) \cdot e^{-j2\pi f_0 \lambda} \, d\lambda}$$

$$= A \cdot e^{j2\pi f_0 t} \cdot H(f_0)$$

Energy Spectral Density

$$G_y(f) = |H(f)|^2 \cdot G_x(f)$$

Power Spectral Density

$$S_y(f) = |H(f)|^2 \, S_x(f)$$