ECE 345 / ME 380: Introduction to Control Systems
Collaborative Quiz #4

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Due October 29, 2020, at the end of class (4:45pm)

NASA OSIRIS-REx (`https://tinyurl.com/yyt7x9fb`), a spacecraft launched in 2016, recently completed a touch-and-go maneuver on the surface of an asteroid known as Bennu. Its mission is to bring back samples from the asteroid for future study on earth.

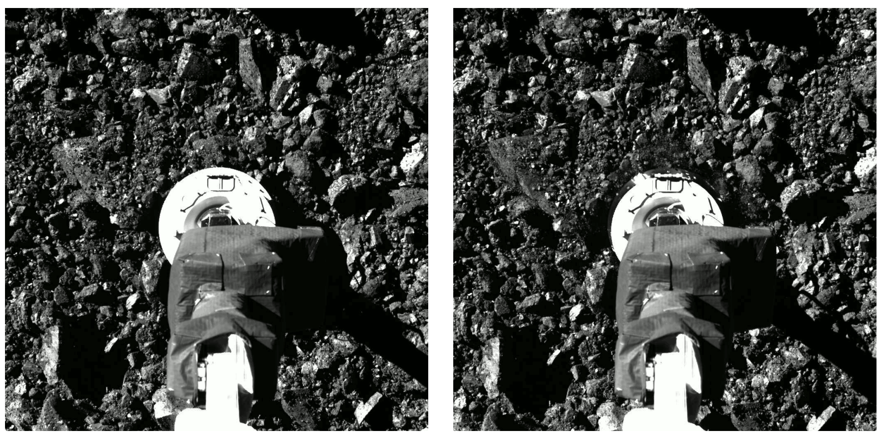


Figure 1: "Captured on Oct. 20 during the OSIRIS-REx mission's Touch-And-Go (TAG) sample collection event, this series of 2 images shows the SamCam imager's field of view at the moment before and after the NASA spacecraft touched down on asteroid Bennu's surface... Preliminary data show the sampling head touched Bennu's surface for approximately 6 seconds, after which the spacecraft performed a back-away burn." From `https://tinyurl.com/yyt7x9fb`, NASA / Goddard / University of Arizona.

We consider the controlled descent of the spacecraft to a desired landing site. We presume that the dynamics of the controlled spacecraft are as shown in Figure 2, with input that is the desired location, and output that it the actual location of the spacecraft. The plant is defined as

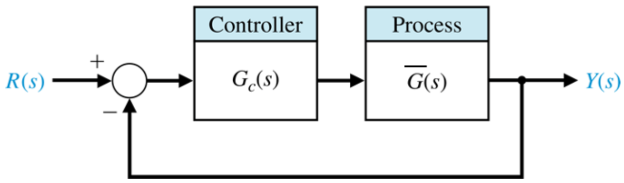
$$\overline{G}(s) = \frac{1}{s} \tag{1}$$

and the controller as

$$G_c(s) = \frac{K}{s+p} \tag{2}$$

for some positive constants K, p.

This objective of this quiz is to design a controller that directs OSIRIS-Rex to a desired target during descent.



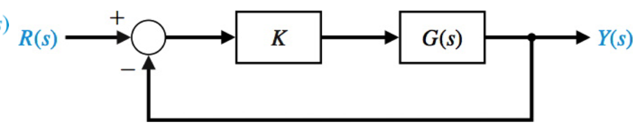


Figure 2: (Left) Closed-loop system with controller $G_c(s)$. (Right) Closed-loop system with proportional feedback for some gain K.

1 Pre-Class Questions

1. Use block diagram manipulation to rewrite the open-loop transfer function $G_c(s)\overline{G}(s)$ as $KG(s)$. Show that $G(s) = \dfrac{1}{s(s+p)}$.

2. What is the type number of the system?

3. Find the characteristic equation $\Delta_{CL}(s)$ of the closed-loop system $\dfrac{Y(s)}{R(s)}$. Describe its the asymptotic stability for $K > 0$, $p > 0$.

4. Show that the damping ratio ζ and natural frequency ω_n of the closed-loop system can be written in terms of positive constants K, p as

$$\zeta = \frac{p}{2\sqrt{K}}, \quad \omega_n = \sqrt{K} \tag{3}$$

2 In-Class Questions

1. Consider your response to Question 1.2. As K increases from 0^+ to ∞, the steady-state error of the closed-loop system in response to a *step input* $R(s) = \frac{1}{s}$ will

   (a) Decrease

   (b) Remain unchanged

   (c) Increase but remain bounded

   (d) Increase and become unbounded

2. Consider a command to guide the spacecraft along a path of interest, so that the reference trajectory is described by the unit ramp input $r(t) = t \cdot \mathbf{1}(t)$. Presume that constants K, p are known, finite values. Which *one* of the following best describes the spacecraft's ability to track the desired trajectory?

   (a) The spacecraft will be able to track the desired path with no steady-state error.

   (b) After the transient phenomena has died out, there will be a constant error of magnitude p/K in location.

   (c) The error in location will eventually settle to the value K/p.

   (d) The spacecraft will converge to a location that is $\frac{K}{K+p}$ away from the desired location.

   (e) The location error will grow over time, becoming unbounded as $t \to \infty$.

We now focus on the problem of controller design for multiple objectives. Specifically, we wish to select p and K such that both of the following criteria are met:

- *Transient performance:* the settling time of the output to a step command in $r(t)$ is less than or equal to 8 seconds, and
- *Steady-state performance:* the steady-state error to a ramp command in $r(t)$ is less than or equal to 0.1 m.

3. Consider your response to Question 1.4. Will $K = 16$ and $p = 2$ satisfy both transient and steady-state performance criteria? Why or why not? *Justification is required for full credit.*

4. Implement the closed-loop transfer function in MATLAB as the system object `sys`, with $p = 2$ and $K = 25$. Using the following code, construct the output response of the closed-loop system to the input $r(t) = t \cdot \mathbf{1}(t)$ using `lsim`, then plot the error $e(t) = r(t) - y(t)$:

```
t = 0:.01:10;
r = t;
[y,tout] = lsim(sys,r,t);
plot(tout, r'-y);
xlabel('Time [sec]'); ylabel('Error [m]');
```

   Consider your response to Question 1.3, and the fact that the trajectory does not converge to 0. Is this plot evidence of marginal stability? Why or why not?

   Note: Please include your plot with your submission.

Disturbances may affect the spacecraft descent, such as model inaccuracy, variations in the gravity field, and nonlinearities in the spacecraft thrusters. Consider the following model which shows how a disturbance impacts the system.

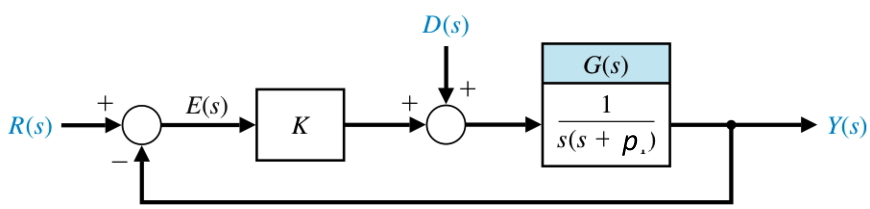


Figure 3: Closed-loop spacecraft landing system with disturbance input $d(t)$.

5. Consider your response to Question 1.3. Find the steady-state response y_{ss} due to a step input in $D(s)$ (while $R(s) = 0$). Which *one* of the following describes the relationship between y_{ss} and K?

   (a) As K increases, y_{ss} decreases.

   (b) As K increases, y_{ss} increases.

   (c) As K increases, y_{ss} is unchanged, and has a non-zero value.

   (d) As K increases, y_{ss} remains 0.

3 If your group finishes early

Instead of a proportional controller K, now consider a proportional-derivative controller $K(s) = K(s+z)$ for some constant $z > 0$.

- Repeat the Pre-Class Questions #1 and #3 with unknown parameters K, p, and z.
- Repeat In-Class questions #1, #5 for your PD-controlled system.
- Compare your PD controller results to those you obtained earlier via proportional control. Which controller works "better"? Why?

Consider the effect of the a disturbance input as shown in Figure 3.

- How would your choice of K, p change if in addition to the transient and steady-state criteria given earlier, the steady-state response to a step input in the disturbance $d(t)$ must be less than or equal to 0.01? Do there exist values of K, p that satisfy all three criteria?