ELE 5'11 HW#8 Solutions Fall 2013 5; Na=2×10'7 cm-3 Nd = 4×1016 cm-3 T-300 K A = 2 × 10-4 cm2 VR = 2.5 V a) $V_{b} = \frac{kT}{e} \ln \left(\frac{N_a N_d}{n_1^2} \right) = 0.0259 V \ln \left(\frac{(2 \times 10^{17} \text{cm}^3)(4 \times 10^{16} \text{cm}^3)}{(1.5 \times 10^{10})^2} \right)$ = 0.808V b) $Y_n = \left[\frac{265(V_{bi} + V_R)}{e} \left(\frac{N_a}{N_a}\right) \frac{1}{N_a + N_a}\right]^{1/2}$ 1/2 = 2(11.7)(8.85×10" F/cm)(0.808V+2.5V) (2×10"7m3) 2×10"7m3+4×10"6m3)
(1.6×10-19c) (1.6×10-19c) = 2.987 ×105 cm = 0.2987um Xp = Xn Nd = (0.2987um) (11×1016 cm3) = 0.05974 um W= xn + xp = 0.3584 um c) $|E_{\text{max}}| = \frac{2(U_{5.} + V_{R})}{V_{1}} = \frac{2(0.808V + 2.5V)}{0.3584 \times 10^{-4} \text{ cm}}$ = 1.846×105, V/cm d) C'= { = { = EsNaNd } | 12 = { = (Ubi+VR)(Na+Nd) } | $= \frac{(1.6 \times 10^{-19} c)(11.7)(8.85 \times 10^{-14} F/cm)(2 \times 10^{17} cm^{-3})(4 \times 10^{16} cm^{-3})}{2(0.808 V + 2.5 V)(2 \times 10^{17} cm^{-3} + 4 \times 10^{16} cm^{-3})}$ C' = 2.889 × 10-8 F/cm2 ⇒ C=C'.A = |5.778×10 F = 5.78pF

if Na -> 3 Na

50
$$\frac{C'_{\text{New}}}{C'} = \left[\frac{e \, \epsilon_3 \, 3 \, N_a}{2 \, (V_{\text{bi}} + V_R)} \cdot \frac{2 \, (V_{\text{bi}} + V_R)}{e \, \epsilon_5 \, N_a}\right]^{1/2} = \sqrt{3} = 1.732$$

c) Function capacitance increases as Na increases because the space-charge narrows. Since the junction is is kind of like a parallel plate capacitor, we can see that capacitance goes up as the space charge width goes down by

[7.28] Si prijunction T=300K

* note: this plot is Na-Nd, so the p-side is on the left (as usual)

a)
$$V_{bi} = \frac{kT}{e} \ln \left(\frac{|5 \times 10^{15})(1 \times 10^{14})}{(1.5 \times 10^{10})^2} \right) = 0.557 V$$

b)
$$Y_n = \left[\frac{265 \text{ Vb;}}{e} \left(\frac{\text{Na}}{\text{Na}} \right) \frac{1}{\text{Na} + \text{Nd}} \right]^{1/2}$$

50
$$Xp = Xn Nd = 3.56um (1x1014) = 0.071 um (5x105)$$



[7.30]
$$\rho + n$$
 silicon $Na = 2 \times 10^{17} \text{ cm}^{-3}$, $Nd = 2 \times 10^{15} \text{ cm}^{-3}$
 $A = 10^{-5} \text{ cm}^{-3}$

a)
$$V_{bi} = 0.0259V \ln \left(\frac{2 \times 10^{17} \cdot 2 \times 10^{15}}{(1.5 \times 10^{10})^2} \right) = \left[0.731V \right]$$

(i)
$$V_R = 1 V$$
, $C = 9.80 \times 10^{-14} F$
 $V_R = 3V$, $C = 6.663 \times 10^{-14} F$
 $V_R = 5V$, $C = 5.376 \times 10^{-14} F$

c)
$$C = \frac{1.29 \times 10^{-13}}{\sqrt{V_{bi} + V_R}}$$

$$\frac{1}{C^2} = \frac{V_{bi} + V_R}{1.66 \text{ H} \times 10^{-26}}$$

$$\frac{1.664 \times 10^{-26}}{1.664 \times 10^{-26}} \sqrt{R} + \frac{Vbi}{1.664 \times 10^{-26}}$$

$$50 \quad L = 6.01 \times 10^{26} VR + 4.34 \times 10^{25}$$

$$50 \frac{1}{C^2} = 6.01 \times 10^{26} \text{ Vp} + 4.39 \times 10^{25}$$

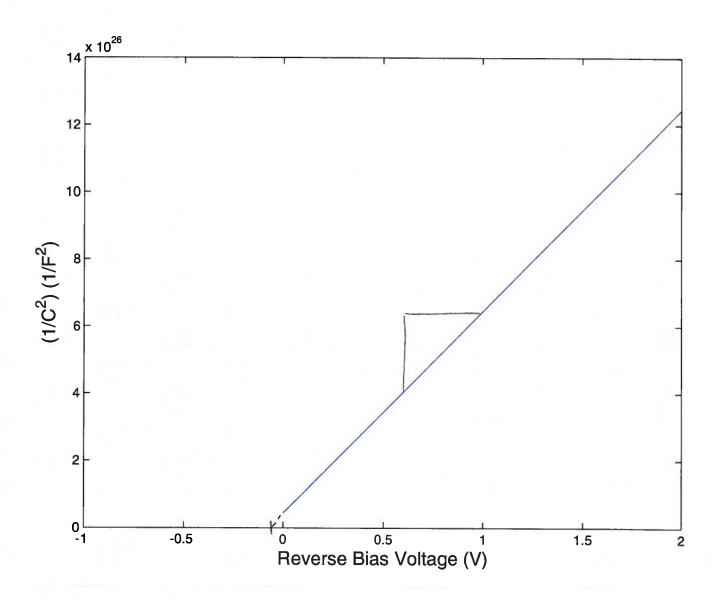
See matlab Plat

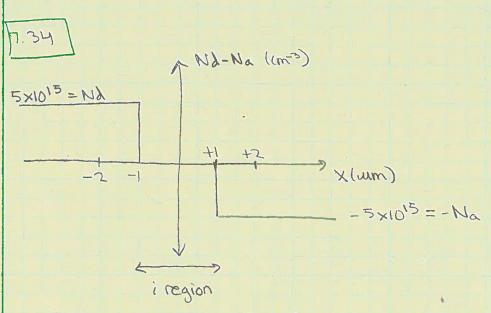


$$C = A \left[\frac{e + s Nd}{2 (Vbi + VR)} \right]^{1/2} \Rightarrow \frac{1}{C^2} = \frac{1}{A^2} \frac{2 (Vbi + VR)}{e + s Nd}$$

$$50 \quad \frac{1}{C^2} = \frac{2}{A^2 + s Nd} \frac{VR}{A^2 + s + s Nd} \frac{2 Vbi}{A^2 + s + s Nd}$$

$$slope \Rightarrow Nd \quad intercept \Rightarrow Vbi$$





- Reverse bias and total depletion width is From - zum to zum

a) Calculate Electric Field Magnitude at x=0 using Poisson

1. Draw charge Density

eNd

$$P(4|am)$$
 $P(x) = \begin{cases} eNd & -2 \le x \le -1 \\ -eNa & 1 \le x \le 2 \end{cases}$
 $V(um)$
 $V(um)$

$$E(x) = \begin{cases} \frac{9(x)}{t^3} dx & \text{call } -2 = -x_n & 1 = x_i \\ -1 = -x_i & 2 = x_0 \end{cases} x_i, x_n, x_p$$

in n-region:
$$E(x) = \begin{cases} eNd & dx = eNd \\ Es & Es \end{cases} + C_1$$

using
$$E(-x_n)=0 \Rightarrow 0=-eNdx_n + C_1 \Rightarrow C_1=eNdx_n$$

in i-region:
$$\rho(x)=0 \Rightarrow dE_i=0 \Rightarrow E_i(x)=C_2$$

Since E-Field must be continuous $E_i(-x_i)=E_i(-x_i)$



-50
$$E_n(-x_i) = \underbrace{eNd}_{E_s}(x_n-x_i) = C_2$$

50 $E_i(x) = \underbrace{eNd}_{E_s}(x_n-x_i) \leftarrow \underbrace{constant}_{E_s}$

in p-region:
$$E_{\rho}(x) = -\left(\frac{eN_{\alpha}}{t_{3}}dx = -\frac{eN_{\alpha}x}{t_{3}} + C_{3}\right)$$

$$E_{\rho}(x_{\rho}) = 0 \implies -\frac{eN_{\alpha}x_{\rho}}{t_{3}} + C_{3} = 0$$

$$\Rightarrow C_3 = \underbrace{eNaxp}_{E_5}$$

$$\Rightarrow 0 | E_p(x) = \underbrace{eNa}_{E_5}(xp-x) | \leftarrow \text{negative slope}$$

$$50 E(0) = E_{1}(x) = \frac{eNd}{e_{3}}(x_{n}-x_{1})$$

$$E(0) = \frac{(1.6 \times 10^{-19} c)(5 \times 10^{15} cm^{3})}{(11.7)(9.85 \times 10^{-14} F/cm)} (2um - 1um)(\frac{1 \times 10^{-4} cm}{1um})$$

$$E(x)$$

$$7.726 \times 10^{4} \text{ V/cm} = E_{1}(x)$$

$$E_{0}(x)$$

$$E_{0}(x)$$

$$X(um)$$



c) The easiest way is to use the area under E(x) curve to get VTOTAL = VA + VB; since

approach 1

p(x) = - (E(x) dx => VTOTAL

then VR = VTOTAL - Voi

From part (b), the n and p sections give an area of 2. 1/2. 1 um. (1×104 cm). 7.726×104 v/cm

base height

= 7.726 V

The intrinsic section gives: 2mm (1x104 cm). 7.726x104 V/m

= 15.452 V

50 V TOTAL = 23.178 V

and $V_{b_1} = \frac{kT \ln \left(\frac{NaNd}{n_1^2} \right) = 0.0259V \ln \left(\frac{(5 \times 10^{15})^2}{(1.5 \times 10^{10})^2} \right) = 0.6587V$

SO VA = VTOTAL - VO; = 23.178V-0.6587V

VR = 22.519 V

* You can also get this by explicitly integrating the expressions for E(x) you found in part (b) see approach 2

* Note: If you try to use equation (7.34) you will not get the right answer since this only applies For a basic pn-junction

- approach?

n-region:
$$\phi_n(x) = -\left(\frac{e N\lambda}{6s}(x+x_n) dx = -\frac{eN\lambda}{6s}(x^2+x_nx) + C_1^2\right)$$

now decide on boundary condition: is $\phi(x)$ maximum or minimum at $x = -x_n$?

Ly because $E(x)$ is positive and $\phi(x) = -\frac{1}{1}E(x) dx$
 $\Rightarrow \phi(x)$ gets smaller as use go From $-x_n$ to x_n

Ly another way to see it is to look at the energy for electrons is lowest here

Potential energy

Potential for electrons is highest here

Potential for holes is also highest here

 $\phi(x) = -\frac{eN\lambda}{2}(x^2+x_nx) + C_1^2$
 $\Rightarrow \phi(-x_n) = 0$

so to get (1 : p(-xn)=0.

 $\Rightarrow \frac{-eNd}{\epsilon_3} \left(\frac{\chi_n^2 - \chi_n^2}{2} \right) + \zeta_1 = 0$

p(Xp) = - VTOTAL

* let the potential of xn=0, so the potential at xp is negative (-Viotal)

$$-\frac{eNd}{63}\frac{xn^{2}}{2}=C_{1}' + \frac{60}{50}\frac{d_{n}(x)}{d_{n}(x)}=-\frac{eNd}{245}\left(x^{2}+2x_{n}x+x_{n}^{2}\right)$$

$$\Phi(x) = -\frac{2E_3}{2E_3}(X+X_n)^2$$

i-region:
$$E_i(x) = \underbrace{eNd}_{E_S}(x_n - x_i)$$

$$\phi_i(x) = - \left(\underbrace{eNd}_{E_S}(x_n - x_i) dx = - \underbrace{eNd}_{E_S}(x_n - x_i) x + C_2 \right)$$

$$\Rightarrow \frac{eNd}{ts}(x_n-x_i)x_i+C_2'=-\frac{eNd}{2ts}(x_n-x_i)^2$$

$$= -\frac{eNd}{2ts} \left(\frac{\chi_{n-1}}{\chi_{n-1}} \right)^{2} + \frac{eNd}{2(\chi_{n-1})\chi_{i}}$$

$$= -\frac{eNd}{2ts} \left[\frac{\chi_{n-1}}{\chi_{i}} \right]^{2} + \frac{2(\chi_{n-1}\chi_{i})\chi_{i}}{\chi_{i}}$$

$$C_2' = -\frac{eNd}{2E_s} \left[(x_n - x_i)(x_n + x_i) \right]$$

so
$$\phi$$
; $(x) = -\frac{end}{Es} \left[(x_n - x_i)x + (x_n - x_i)(x_n + x_i) \right]$

$$\phi_i(x) = \frac{-e N \lambda (x_n - x_i)}{2 \epsilon s} \left[2x + (x_n + x_i) \right]$$

p-region:
$$\phi_p(x) = -\left(\frac{eNa}{ts}(x_p-x)dx = -\frac{eNa}{ts}(x_p-x)dx = -\frac{eNa}{ts}(x_p-x)dx\right)$$

boundary condition \$ (x;) = \$\p(x;)\$



50 - VTOTAL =
$$\frac{-(1.6 \times 10^{-19} c)(5 \times 10^{15} cm^{3})}{2(11.7)(9.85 \times 10^{-14} F/cm)} (2 mm)^{2} - 2(2 mm)(lum) + (lum)^{2}$$

+ (2um-lum)(3um+2um)(1x10-4cm)



on
$$p$$
-side $P_{p_0} = N_0 \Rightarrow n_1^2 = N_0 \cdot n_{p_0}$

$$\Rightarrow n_{p_0} = \frac{n_1^2}{N_0}$$

on n-side
$$\rho_{no} = \frac{n_i^2}{N \lambda}$$

so
$$Np = \frac{N_i^2}{Na} exp\left(\frac{eVa}{kT}\right)$$
 $P_n = \frac{N_i^2}{NA} exp\left(\frac{eVa}{kT}\right)$

$$N_{p} = \frac{(1.5 \times 10^{10})^{2}}{(9 \times 10^{15})} \exp\left(\frac{V_{a}}{0.0259}\right) = 28125 \exp\left(\frac{V_{a}}{0.0259}\right)$$

$$\rho_n = \frac{(1.5 \times 10^{10})^2}{2 \times 10^{15}} \exp\left(\frac{V_a}{0.0259}\right) = 112500 \exp\left(\frac{V_a}{0.0259}\right)$$

a)
$$V_a = 0.45V$$
 $\longrightarrow N_p = 9.88 \times 10^{12} \text{ cm}^3$
 $P_n = 3.95 \times 10^{12} \text{ cm}^3$

b)
$$V_{\alpha} = 0.55 \text{V} - 9 \prod_{p=1.88 \times 0^{14} \text{cm}^{-3}}$$

$$P_{n} = 4.69 \times 10^{13} \text{cm}^{-3}$$

c)
$$V_{\alpha} = -0.55V$$
 \longrightarrow $P_{\alpha} \approx 0$

$$F_n(-xp) = e O_n np_o \left[exp\left(\frac{e V_a}{kT}\right) - 1 \right]$$

$$N\rho_0 = \frac{N_1^2}{N\alpha} = \frac{(1.8 \times 10^6 \text{ cm}^{-3})^2}{5 \times 10^{16} \text{ cm}^{-3}} = 6.48 \times 10^{-5}$$

we are not given the, so assume it is 1075 From Exs. ?

50
$$\mp n(-xp) = (1.6 \times 10^{-19} c)(220 cm2/s)(6.48 \times 10^{-5} cm3) \left[exp \left(\frac{1.1}{6.0259} \right) - 1 \right]$$

$$P_{no} = \frac{M_1^2}{Nd} = \frac{(1.8 \times 10^6 \text{ cm}^3)^2}{1 \times 10^{16} \text{ cm}^3} = 3.24 \times 10^{-4} \text{ cm}^{-3}$$

$$\overline{f_{p}(x_{n})} = \frac{(1.6 \times 10^{-19} c)(10.4 cm^{2}/s)(3.24 \times 10^{-4} cm^{-3})}{\sqrt{(10.4 cm^{2}/s)(5 \times 10^{-8} s)}} \left[exp(\frac{1.1}{0.0259}) - 1 \right]$$

$$= 2.083 Alim^{2}$$





$$J_{s} = en_{i}^{2} \left[\frac{1}{N_{a}} \sqrt{\frac{D_{n}}{\tau_{no}}} + \frac{1}{N_{d}} \sqrt{\frac{D_{p}}{\tau_{p0}}} \right]$$

$$= (1.6 \times 10^{-19})(2.4 \times 10^{13})^{2}$$

$$\times \left[\frac{1}{4 \times 10^{15}} \sqrt{\frac{90}{2 \times 10^{-6}}} + \frac{1}{2 \times 10^{17}} \sqrt{\frac{48}{2 \times 10^{-6}}} \right]$$

$$J_{s} = 1.568 \times 10^{-4} \text{ A/cm}^{2}$$

(a)
$$I = AJ_s \exp\left(\frac{V_a}{V_t}\right)$$

 $= (10^{-4})(1.568 \times 10^{-4}) \exp\left(\frac{0.25}{0.0259}\right)$
 $= 2.44 \times 10^{-4} \text{ A}$
or $I = 0.244 \text{ mA}$

(b)
$$I = -I_s = -AJ_s = -(10^{-4})(1.568 \times 10^{-4})$$

= -1.568×10⁻⁸ A

8.15

(a) p-side;

$$E_{Fi} - E_F = kT \ln \left(\frac{N_a}{n_i} \right)$$
$$= (0.0259) \ln \left(\frac{5 \times 10^{15}}{1.5 \times 10^{10}} \right)$$

or

$$E_{Fi} - E_F = 0.329 \,\mathrm{eV}$$

Also on the n-side;

$$E_F - E_{Fi} = kT \ln \left(\frac{N_d}{n_i} \right)$$
$$= (0.0259) \ln \left(\frac{10^{17}}{1.5 \times 10^{10}} \right)$$

or

$$E_F - E_{Fi} = 0.407 \,\text{eV}$$

(b) We can find

$$D_n = (1250)(0.0259) = 32.4 \text{ cm}^2/\text{s}$$

$$D_p = (320)(0.0259) = 8.29 \text{ cm}^2/\text{s}$$

Now

$$J_{S} = en_{i}^{2} \left[\frac{1}{N_{a}} \sqrt{\frac{D_{n}}{\tau_{nO}}} + \frac{1}{N_{d}} \sqrt{\frac{D_{p}}{\tau_{pO}}} \right]$$
$$= (1.6 \times 10^{-19})(1.5 \times 10^{10})^{2}$$

$$\times \left[\frac{1}{5 \times 10^{15}} \sqrt{\frac{32.4}{10^{-6}}} + \frac{1}{10^{17}} \sqrt{\frac{8.29}{10^{-7}}} \right]$$

OI

$$J_s = 4.426 \times 10^{-11} \,\text{A/cm}^2$$

Then

$$I_s = AJ_s = (10^{-4})(4.426 \times 10^{-11})$$

01

$$I_S = 4.426 \times 10^{-15} \,\mathrm{A}$$

We find

$$I = I_S \exp\left(\frac{V_D}{V_t}\right)$$

= $\left(4.426 \times 10^{-15}\right) \exp\left(\frac{0.5}{0.0259}\right)$

or

$$I = 1.07 \times 10^{-6} \text{ A} = 1.07 \,\mu\text{ A}$$

(c) The hole current is

$$I_p = e n_i^2 A \cdot \frac{1}{N_d} \sqrt{\frac{D_p}{\tau_{po}}} \exp\left(\frac{V_D}{V_t}\right)$$

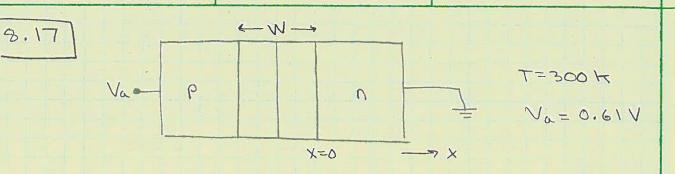
$$= (1.6 \times 10^{-19})(1.5 \times 10^{10})^{2} (10^{-4}) \left(\frac{1}{10^{17}}\right)$$
$$\times \sqrt{\frac{8.29}{10^{-7}}} \exp\left(\frac{V_{D}}{V_{A}}\right)$$

or

$$I_p = 3.278 \times 10^{-16} \exp\left(\frac{V_D}{V_t}\right)$$
 (A)

Then

$$\frac{I_p}{I} = \frac{J_p}{J_s} = \frac{3.278 \times 10^{-16}}{4.426 \times 10^{-15}} = 0.0741$$



$$N_d = 10^{16} \text{ cm}^{-3}$$
 $N_a = 5 \times 10^{16} \text{ cm}^{-3}$
 $T_{no} = 0.05 \text{ us}$ $T_{po} = 0.01 \text{ us}$
 $D_n = 23 \text{ cm}^2/\text{s}$ $D_p = 8 \text{ cm}^2/\text{s}$

a) Find excess hole conc as a Function of x on the n-side

$$\delta P_n = P_{no} \left[\exp \left(\frac{e V_a}{kt} \right) - i \right] \exp \left(\frac{\chi_n - \chi}{L_p} \right)$$

but since we defined x=0 at the typical xn point

$$P_{NO} = \frac{n_1^2}{NA} = \frac{(1.5 \times 10^{10})^2}{1 \times 10^{16}} = 22500 \text{ cm}^{-3}$$

50
$$\delta P_n(x) = (22500 \text{ cm}^3) \left[\exp \left(\frac{0.61}{0.0254} \right) - 1 \right] \exp \left(\frac{-x}{2.828 \times 10^{41} \text{ cm}} \right)$$

$$\delta P_n(x) = 3.808 \times 10^{14} \exp\left(\frac{-x}{2.828 (um)}\right)$$
 [cm⁻³]



$$\delta \rho_n (3 \times 10^{-4} \, \text{cm}) = 3.868 \times 10^{14} \, \text{exp} \left(\frac{-3}{2.828} \right) = 1.318 \times 10^{14} \, \text{cm}^3$$

so
$$\overline{Jp(3um)} = -eD_P \frac{d(\delta P_n(x))}{dx}$$
 $x = 3um$

$$\frac{d \, \delta P_n(x)}{dx} = -3.808 \times 10^{14} \, \exp\left(\frac{-x}{2.828 \times 10^{-4} \, \text{cm}}\right) \cdot \frac{1}{2.828 \times 10^{-4} \, \text{cm}}$$

$$\frac{d \delta \rho_n(x)}{dx} \Big|_{x=3 \times 10^{-4} \text{cm}} = -4.66 \times 10^{17} \text{cm}^{-4}$$

$$N\rho_0 = \frac{N_1^2}{N\rho_0} = 4.5 \times 10^3 \text{ cm}^{-3}$$

50 Fro =
$$(1.6 \times 10^{-19} c)(23 cm^2/5)(4.5 \times 10^3 cm^3)$$
 $\left[\exp\left(\frac{6.61}{60259}\right) - 1\right]$

also
$$\overline{4p_0} = \frac{(1.6 \times 10^{-19} c)(8 cm^2/8)(22500 cm^3)}{2.828 \times 10^{-4} cm} \left[\exp \left(\frac{0.61}{0.0254} \right) - 1 \right]$$

GO Itor = Ino + IPo = 0.2615 A/cm² + 1.724 A/cm²

| Fror = 1.985 A/cm²

* From is the same everywhere

SO In (3 mm) + Fp (3 mm) = + TOT

=> In (3 mm) = Frot - Fp (3 mm)

= 1.985 - 0.5965 = 1.389 A/cm²