Course ID: ECE 341 Communication Systems- Fall
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235D/ Office Hours: Mondays and Wednesdays 11:00am - 12:00pm
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Department of Electrical and Computer Engineering / University of New Mexico

Homework #1

Corresponding to Sections 2.1 – 2.3 of Principles of Communications, Rodger E. Zimmer and William H. Tranter, John Wiley, 7th Edition.

1. Sketch the single-sided and double-sided amplitude and phase spectra of the following signal: $x(t)=2\sin(4\pi t+\pi/8)+12\sin(10\pi t)$

2. Given the signal $x(t)=\cos(6\pi t)+2\sin(10\pi t)$, write it as follows: a) the real part of a sum of rotating phasors, b) a sum of rotating phasors plus their complex conjugates, and c) sketch the single-sided and double-sided amplitude and phase spectra of $x(t)$.

3. If a signal is a power signal, find its normalized power and if a signal is an energy signal, find its normalized energy: a) $x(t)=2\cos(4\pi t+2\pi/3)$ and b) $x(t)=e^{-at}u(t)$.

4. Find the exponential Fourier series for the following signals (use the uniqueness property of the Fourier series): a) $x(t)=\cos(2\pi f_0 t)+\sin(4\pi f_0 t)$ and b) $x(t)=\sin(2\pi f_0 t)\cos^2(4\pi f_0 t)$.

To be delivered at instructor's office: 16 September 2019

Good Luck!

1. Sketch the single-sided and double-sided amplitude and phase spectra of the following signal: x(t)=2sin(4πt+π/8)+12sin(10πt)

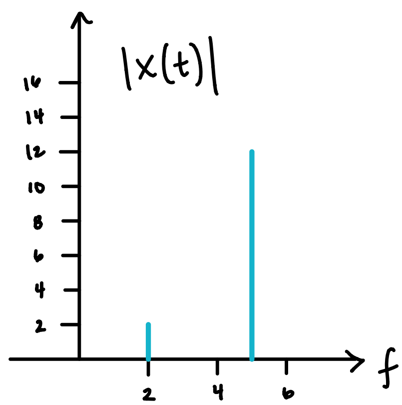
$$x(t) = 2\sin\left(4\pi t + \frac{\pi}{8}\right) + 12\sin(10\pi t)$$

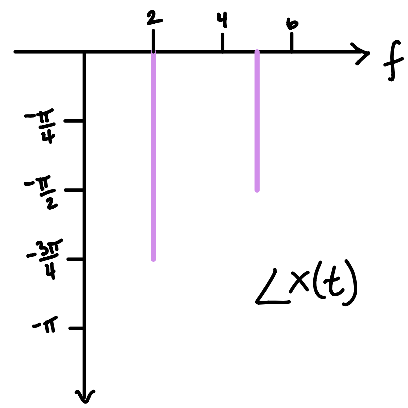
$$= 2\cos\left(4\pi t + \frac{\pi}{8} - \frac{\pi}{2}\right) + 12\cos\left(10\pi t - \frac{\pi}{2}\right)$$

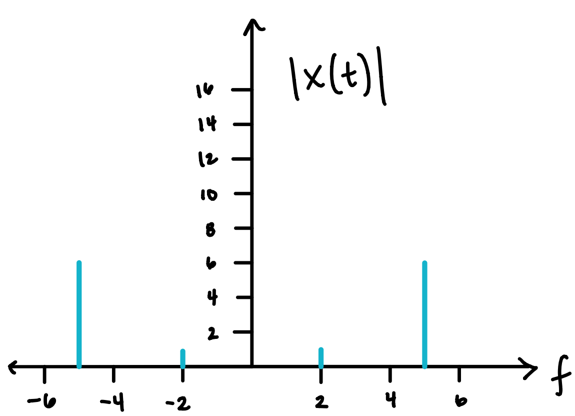
$$= 2\cos\left(4\pi t - \frac{3\pi}{4}\right) + 12\cos\left(10\pi t - \frac{\pi}{2}\right)$$

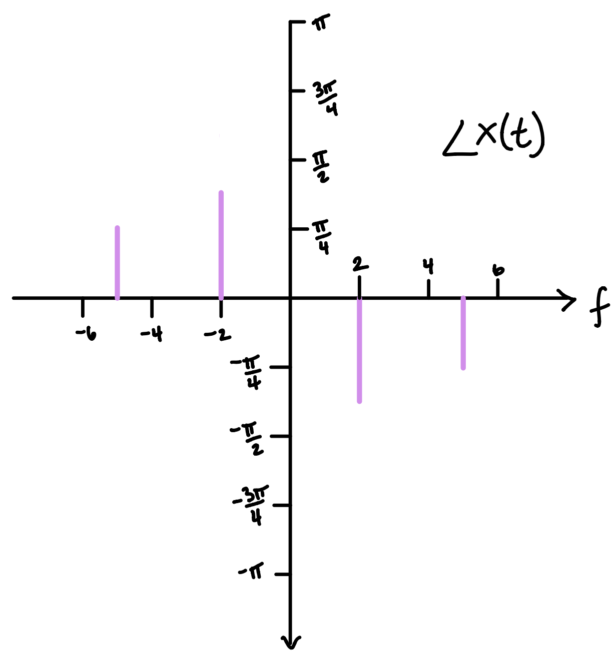
$$\omega_1 = 4\pi \rightarrow f_1 = \frac{\omega_1}{2\pi} = 2\,Hz$$

$$\omega_2 = 10\pi \rightarrow f_2 = \frac{\omega_2}{2\pi} = 5\,Hz$$









2. Given the signal x(t)=cos(6πt)+2sin(10πt), write it as follows: a) the real part of a sum of rotating phasors, b) a sum of rotating phasors plus their complex conjugates, and c) sketch the single-sided and double-sided amplitude and phase spectra of x(t).

(a) $x(t) = \cos(6\pi t) + 2\cos\left(10\pi t - \frac{\pi}{2}\right)$

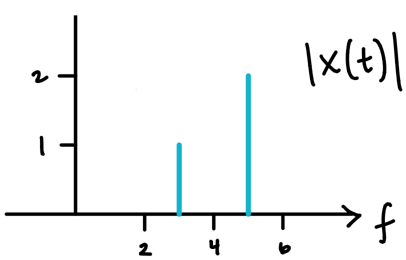
$$\mathrm{Re}\left[e^{j6\pi t} + 2e^{j(10\pi t - \pi/2)}\right]$$

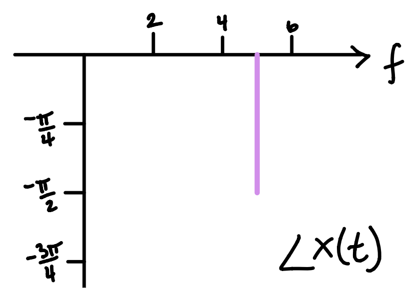
(b) $$\frac{e^{j6\pi t} + e^{-j6\pi t}}{2} + \cancel{2}\left(\frac{e^{j(10\pi t - \pi/2)} - e^{-j(10\pi t - \pi/2)}}{\cancel{2}}\right)$$

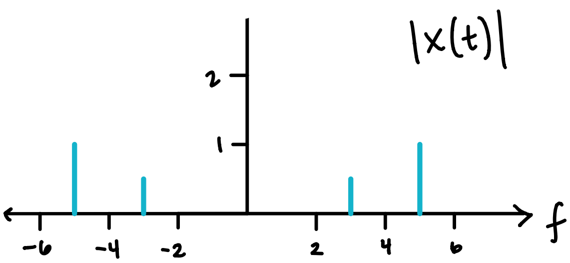
$$x(t) = \frac{e^{j6\pi t} + e^{-j6\pi t}}{2} + e^{j(10\pi t - \pi/2)} - e^{-j(10\pi t - \pi/2)}$$

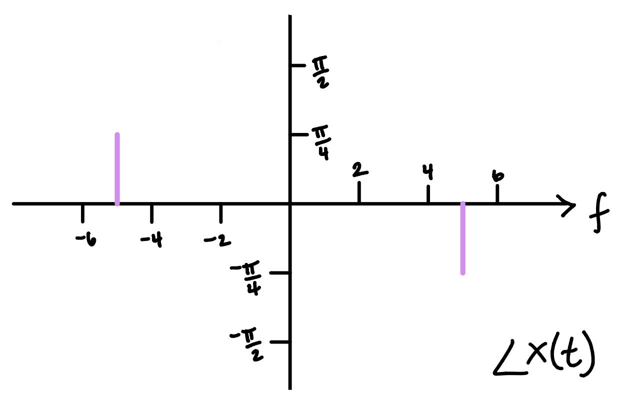
(c) $\omega_1 = 6\pi \rightarrow f = \frac{6\cancel{\pi}}{2\cancel{\pi}} = 3Hz$ (1)

$\omega_2 = 10\pi \rightarrow f = \frac{10\cancel{\pi}}{2\cancel{\pi}} = 5Hz$ (2)









3. If a signal is a power signal, find its normalized power and if a signal is an energy signal, find its normalized energy: a) $x(t)=2\cos(4\pi t+2\pi/3)$ and b) $x(t)=e^{-at}u(t)$.

(a) $2\cos(4\pi t + \frac{2\pi}{3})$ periodic → power

$\omega = 4\pi \rightarrow f = \frac{\cancel{4\pi}}{\cancel{2\pi}} = 2\text{Hz} \rightarrow T = \frac{1}{f} = \frac{1}{2}$

$$P = \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2 dt$$

$u = 4\pi t + \frac{2\pi}{3}$

$du = 4\pi\, dt$

$\frac{du}{\pi} = 4dt$

t	u
$-1/4$	$-\pi/3$
$1/4$	$5\pi/3$

$$= 2\int_{-1/4}^{1/4} 4\cos^2(4\pi t + \frac{2\pi}{3})dt$$

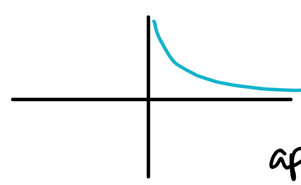
$$= \frac{2}{\pi}\int_{-\pi/3}^{5\pi/3} \cos^2(u)\, du$$

$$= \frac{2}{\pi}\int_{-\pi/3}^{5\pi/3} \frac{1}{2}(1+\cos(2u))\, du$$

$$= \frac{1}{\pi}\left(u + \frac{1}{2}\sin(2u)\right)\Big|_{-\pi/3}^{5\pi/3}$$

$$P = \boxed{2\text{ W}}$$

(b) $x(t) = e^{-\alpha t}u(t)$



aperiodic → energy

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$= \int_{-\infty}^{\infty} e^{-2\alpha t}u^2(t)\, dt = -\frac{1}{2\alpha}e^{-2\alpha t}\Big|_{-\infty}^{\infty} = 0 - (-\frac{1}{2\alpha})$$

$$\boxed{E = \frac{1}{2\alpha}\text{ J}}$$

4. Find the exponential Fourier series for the following signals (use the uniqueness property of the Fourier series): a) $x(t)=\cos(2\pi f_0 t)+\sin(4\pi f_0 t)$ and b) $x(t)=\sin(2\pi f_0 t)\cos^2(4\pi f_0 t)$.

(a) $x(t) = \cos(2\pi f_0 t) + \sin(4\pi f_0 t)$

$$= \cos(2\pi f_0 t) + \sin(4\pi f_0 t)$$

$$= \frac{e^{j2\pi f_0 t} + e^{-j2\pi f_0 t}}{2} + \frac{e^{j4\pi f_0 t} - e^{-j4\pi f_0 t}}{2j}$$

$\omega_0 = 2\pi f_0$

$$x(t) = \frac{1}{2}e^{j\omega_0 t} + \frac{1}{2}e^{-j\omega_0 t} + \frac{1}{2j}e^{j2\omega_0 t} - \frac{1}{2j}e^{-j2\omega_0 t}$$

$$x(t) = \begin{cases} \frac{1}{2} & , t = \pm 1 \\ \frac{1}{2j} & , t = 2 \\ -\frac{1}{2j} & , t = -2 \end{cases}$$

(b) $x(t) = \sin(2\pi f_0 t)\cos^2(4\pi f_0 t)$ $\qquad \omega_0 = 2\pi f_0$

$$= \sin(2\pi f_0 t)\left(\frac{1}{2} + \frac{1}{2}\cos(8\pi f_0 t)\right)$$

$$= \frac{1}{2}\sin(\omega_0 t) + \frac{1}{2}\sin(\omega_0 t)\cos(4\omega_0 t)$$

$x = \omega_0 t$

$y = 4\omega_0 t$

$$= \frac{1}{2}\sin(x)\cos(y) + \frac{1}{2}\sin(x)$$

$$= \frac{1}{2}\left[\frac{1}{2}\left(\sin(x+y) + \sin(x-y)\right) + \sin(x)\right]$$

$$= \frac{1}{4}\left[\sin(x+y) + \sin(x-y) + 2\sin(x)\right]$$

$$x(t) = \frac{1}{4}\left[\sin(5\omega_0 t) + \sin(-3\omega_0 t) + 2\sin(\omega_0 t)\right]$$

$$= \frac{1}{4}\left[\frac{e^{j5\omega_0 t} - e^{-j5\omega_0 t} + e^{-j3\omega_0 t} - e^{j3\omega_0 t} + 2e^{j\omega_0 t}}{2j} - 2e^{-j\omega_0 t}\right]$$

$$= \frac{1}{8j}e^{j5\omega_0 t} - \frac{1}{8j}e^{-j5\omega_0 t} + \frac{1}{8j}e^{-j3\omega_0 t} - \frac{1}{8j}e^{j3\omega_0 t} + \frac{1}{4j}e^{j\omega_0 t} - \frac{1}{4}je^{-j\omega_0 t}$$

$$x(t) = \begin{cases} \frac{1}{8j}, & t = 5, -3 \\ -\frac{1}{8j}, & t = -5, 3 \\ \frac{1}{4j}, & t = 1 \\ -\frac{1}{4j}, & t = -1 \end{cases}$$

Product Identities

$$\sin(x)\cos(y) = \tfrac{1}{2}\left[\sin(x+y) + \sin(x-y)\right]$$

$$\cos(x)\sin(y) = \tfrac{1}{2}\left[\sin(x+y) - \sin(x-y)\right]$$

$$\cos(x)\cos(y) = \tfrac{1}{2}\left[\cos(x-y) + \cos(x+y)\right]$$

$$\sin(x)\sin(y) = \tfrac{1}{2}\left[\cos(x-y) - \cos(x+y)\right]$$