Take Test: Quiz 6.2. Hilbert spaces

Test Information

Description	
Instructions	
Multiple Attempts	This test allows multiple attempts.
Force Completion	This test can be saved and resumed later.
	Your answers are saved automatically.

QUESTION 1

0.02 points Saved

A vector space has a metric if and only if the equality $<\mathbf{x},\mathbf{x}> = 0$ is satisfied only for $\mathbf{x}=0$.

- ( ) Yes, because if we define a metric in a space whose dot product does not satisfy this property, vector ${\bf x=0}$ may have a nonzero norm.
- ( ) No. A metric can be defined in any vector space, but it cannot be called distance.
- ( ) No. You can always define a metric, but in a non-strict space, it will be biased.
- (•) Yes, because if we define a metric in a space whose dot product does not satisfy this property, two different vectors may have a distance equal to zero.

QUESTION 2

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A matrix $\mathbf{A}$ is positive semi-definite if

- ( ) All its eigenvalues are positive or zero.
- ( ) If it can be expressed as $\mathbf{A} = \mathbf{B}^\mathsf{T}\mathbf{B}$
- ( ) If for any vector a, the product $a^\mathsf{T}\mathbf{A}a$ is nonnegative.
- (•) All the answers are correct.

QUESTION 3

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The Mercer's theorem says that

- [x] If a function $k(\mathbf{x},\mathbf{x}') \in \mathbb{R}$ is a kernel, then it must be a positive semidefinite function.
- [ ] If $k(\mathbf{x},\mathbf{x}') \in \mathbb{R}$ is positive semidefinite, then we can express a mapping function from it.
- [ ] If a positive semidefinite function $k(\mathbf{x},\mathbf{x}') \in \mathbb{R}$ exists, then an explicit transformation into a Hilbert space exists.
- [x] Any positive semidefinite function $k(\mathbf{x},\mathbf{x}') \in \mathbb{R}$ is a dot product in a Hilbert space.

QUESTION 4

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Two different Mercer's kernels

- [ ] Can be used to construct another kernel by subtracting one from another.
- [ ] Cannot be used together if one is a kernel of an infinite-dimensional Hilbert space and the other is finite.
- [x] Can be used to construct another kernel by a linear combination of them with positive coefficients
- [x] Can be used to construct another kernel by multiplying them

QUESTION 5

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The square exponential kernel is a dot product in an infinite dimension Hilbert space because

- (•) The dot product between vectors is less than one, except in the case both vectors are the same in the input space, which proves that any two different vectors of the input space are mapped in the Hilbert space in such a way that they describe a nonzero angle.
- ( ) The dot product between vectors is never zero, which proves that no two vectors in that space are orthogonal.
- ( ) None of the answers are true.
- ( ) The dot product between vectors is less than one, except in the case both vectors are the same in the input space, which proves that no two vectors are equal in the Hilbert space.