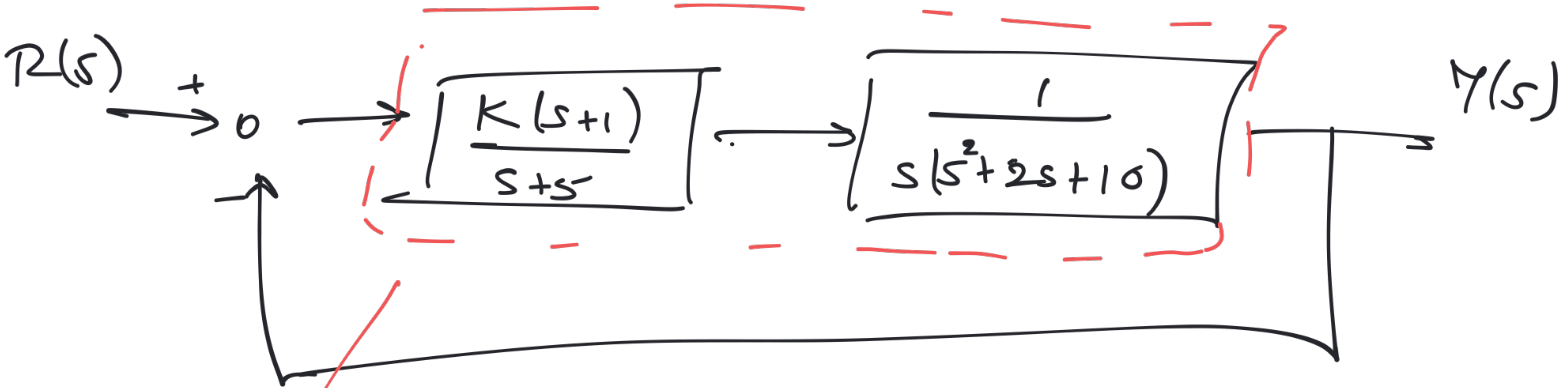
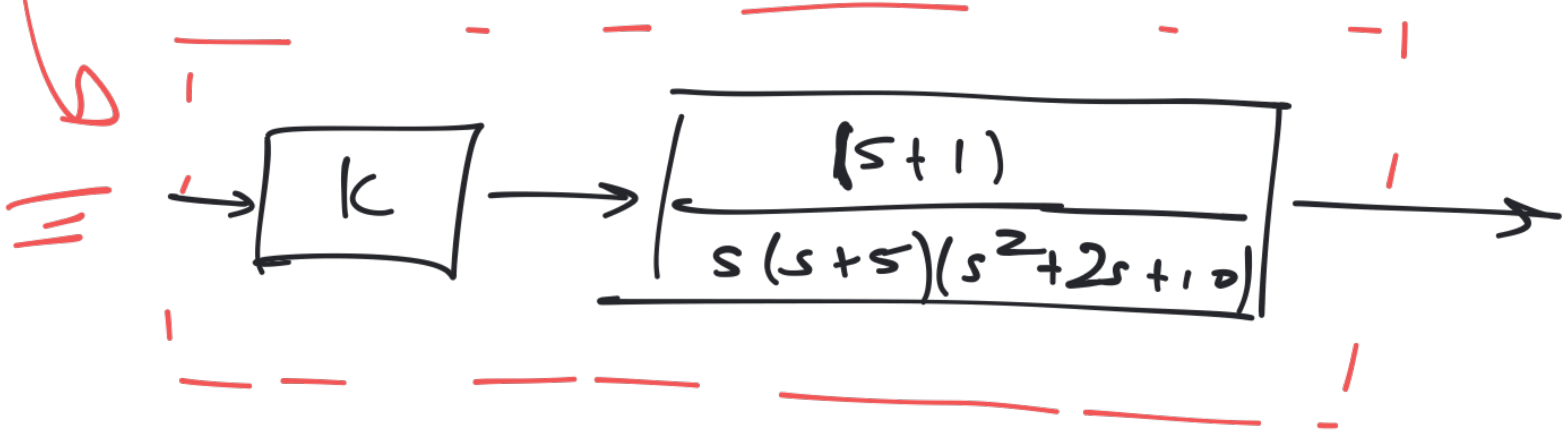
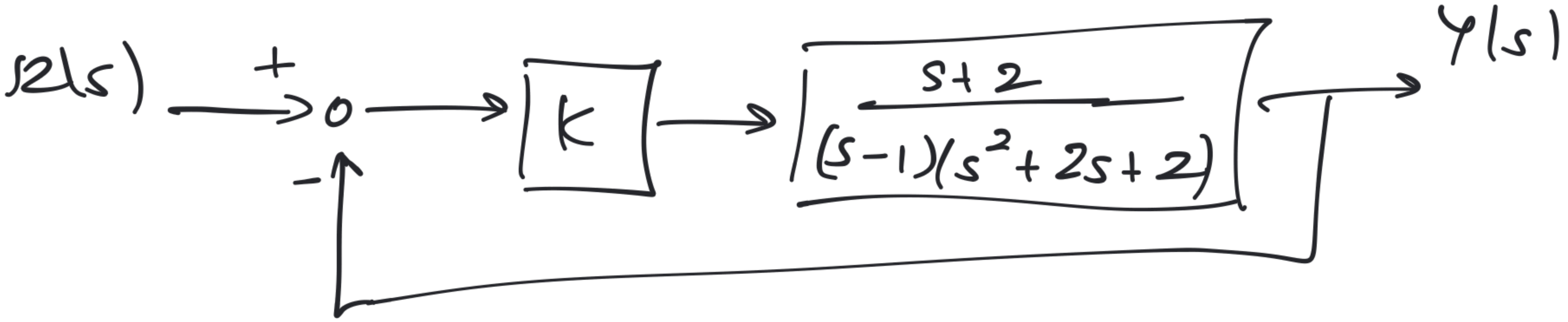


1. For what values of K is this system stable?
2. What gain K is required to ensure a phase margin of 30°?





a) Use the Nyquist criterion to find K for stabilization.

b) Find the number of roots of $\frac{Y(s)}{R(s)}$ in the RHP, as a function of K.

c) Use a Routh table to determine K that assures stability.

---

a) $Z = P - N$, $P = 1$ $\therefore$ Need $N = +1$ for stability

$\Rightarrow$ 1 counterclockwise encirclement of -1.

By examination of the Nyquist diagram,

$0 < K < 1$: $\quad N = 0 \Rightarrow Z = 1$, 1 pole in RHP.

$1 < K < 2$: $\quad N = +1 \Rightarrow Z = 0$, 0 poles in RHP.

$2 < K$: $\quad N = -1 \Rightarrow Z = 2$, 2 poles in RHP.

stable.

c) Routh table

$$\Delta(s) = D(s) + KN(s)$$

$$= (s-1)(s^2+2s+2) + K(s+2)$$

$$= s^3 + s^2 + Ks + (2K-2)$$

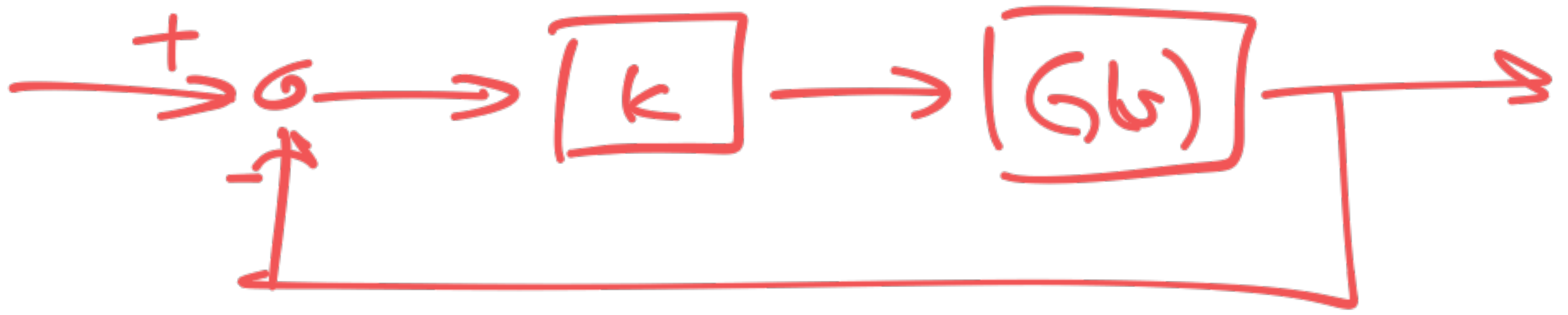
$$\begin{array}{llll}
s^3: & 1 & K \\
s^2: & 1 & 2K-2 \\
s^1: & \dfrac{-\begin{vmatrix} 1 & K \\ 1 & 2K-2 \end{vmatrix}}{1} = 2-K & 0 \\
s^0: & \dfrac{-\begin{vmatrix} 1 & 2K-2 \\ 2-K & 0 \end{vmatrix}}{2-K} = 2K-2 & 0
\end{array}$$

No sign changes in 1st column if:

$2-K > 0 \implies 2 > K$

and $2K-2 > 0 \implies K > 1$.

$$G(s) = \frac{1}{(s+a)(s+30)}$$



For $G(s)$ under negative unity feedback with gain K, find:

a) K, a (if any) such that rise time is approx. 1 second, and damping ratio is 0.5, approximately

b) steady-state error in response to a unit ramp input.

$$\Delta_{CL}(s) = D(s) + KN(s)$$
$$= s^2 + (30+a)s + (30a + K)$$

$$0 = s^2 + 2\zeta\omega_n s + \omega_n^2$$

$$2\zeta\omega_n = 30 + a$$

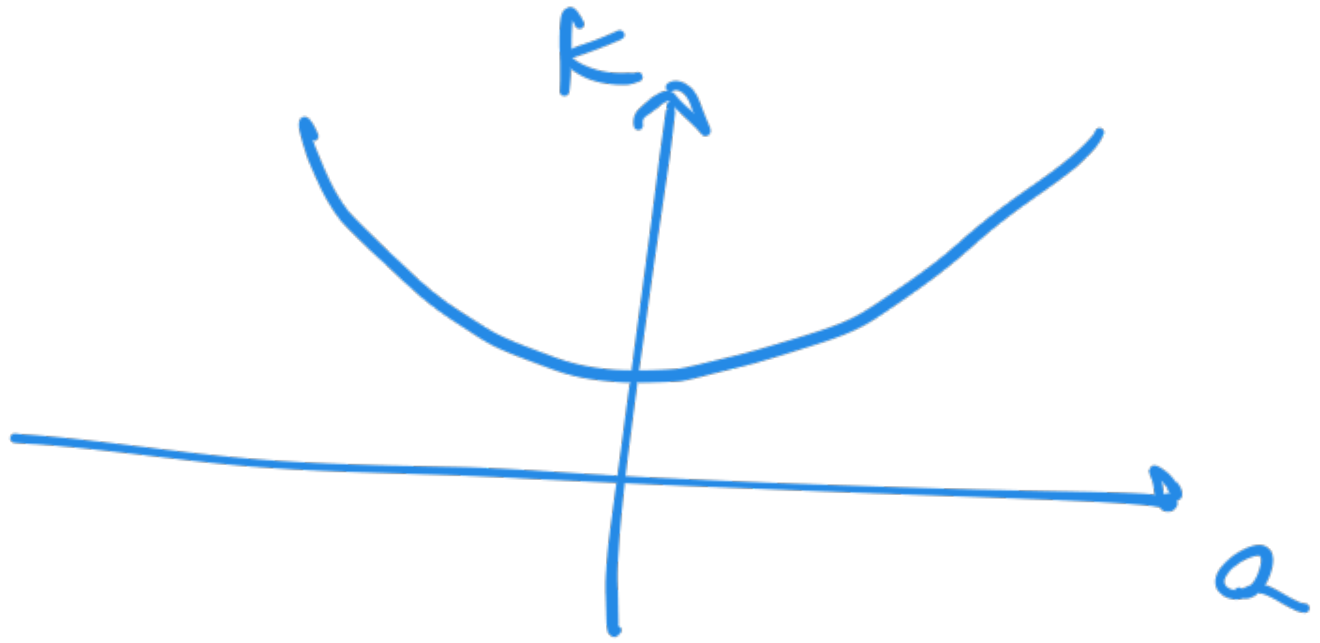
$$\omega_n^2 = 30a + K \Rightarrow \omega_n = \sqrt{30a+K}$$

$$\zeta = \frac{30+a}{2\sqrt{30a+K}} = 0.5$$

$$T_r = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 1$$

algebra...

$$4\pi^2 = -(a^2 + 60a + 30^2) + 120a + 4K$$



---

b) Type 0, presuming $a \neq 0$.

$\therefore$ e_{ss} is infinite.

Unit step input?

$$e_{ss} = \frac{1}{1+K_p}, \quad K_p = \lim_{s\to 0} KG(s)$$

$$= \frac{K}{a+30}$$

$$\Rightarrow e_{ss} = \frac{30a}{30a+K}$$