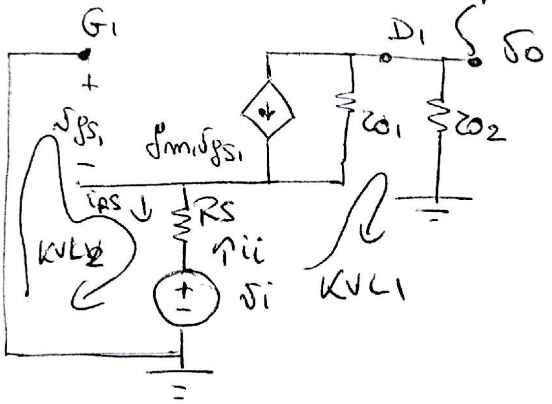
Homework #4 - Solution

(a) Common-gate with an active load. Specifically the load is a current mirror.

(b) Calculate R_i, R_o and G_v

b1) R_i



$R_S = 50\Omega$

$r_{o1} = \dfrac{1}{\lambda_1 I_Q} = 200K\Omega$

$r_{o2} = \dfrac{1}{\lambda_2 I_Q} = 133.33K\Omega$

$g_{m1} = 2\sqrt{K_n I_Q} = 0.52 mA/V$

$R_i = \dfrac{v_i}{i_i}$

KVL 1:

$v_i - R_S i_i + (-i_i - g_{m1} v_{gs1}) r_{o1} - r_{o2} i_i = 0$

$v_i - R_S i_i - r_{o1} i_i - g_{m1} v_{gs1} r_{o1} - r_{o2} i_i = 0$

$v_i - i_i (R_S + r_{o1} + r_{o2}) - g_{m1} v_{gs1} r_{o1} = 0$ (1)

KVL 2:

$v_{gs1} = -v_{s1} = -[R_S i_i + v_i]$ (2)

Combining (1) and (2) yields

$v_i(1 + g_{m1} r_{o1}) - i_i[R_S + r_{o1} + r_{o2} + g_{m1} r_{o1} R_S] = 0$

$$R_i = \frac{v_i}{i_i} = \frac{R_S + r_{o1} + r_{o2} + g_{m1} r_{o1} R_S}{1 + g_{m1} r_{o1}} \simeq \frac{r_{o1} + r_{o2} + g_{m1} r_{o1} R_S}{g_{m1} r_{o1}} =$$

$$= \frac{200K + 133.3K + 0.52m \cdot 200K \cdot 50}{0.52m \cdot 200K} = 1.84K\Omega$$

$R_i = 1.84K\Omega$

$G_v = \dfrac{v_o}{v_i}$

KCL @ the output node

$$\frac{v_o}{r_{o2}} + g_{m1} v_{gs1} + \frac{v_o + v_{gs}}{r_{o1}} = 0 \Rightarrow v_o\left(\frac{1}{r_{o2}} + \frac{1}{r_{o1}}\right) + v_{gs1}\left(g_{m1} + \frac{1}{r_{o1}}\right) = 0 \quad (3)$$

$$v_{gs1} = -v_{s1} = -[R_S i_{RS} + v_i] = -\left[R_S \frac{v_o}{r_{o2}} + v_i\right] \quad (4)$$

Scanned by CamScanner

Combining (3) and (4) yields

$$v_o\left[\frac{1}{r_{o2}}+\frac{1}{r_{o1}}\right]=\left[g_{m1}+\frac{1}{r_{o1}}\right]\cdot\left[v_i-R_S\frac{v_o}{r_{o2}}\right]$$

$$v_o\left[\frac{1}{r_{o2}}+\frac{1}{r_{o1}}\right]+\left[g_{m1}+\frac{1}{r_{o1}}\right]\cdot\frac{R_S}{r_{o2}}v_o=v_i\left[g_{m1}+\frac{1}{r_{o1}}\right]$$

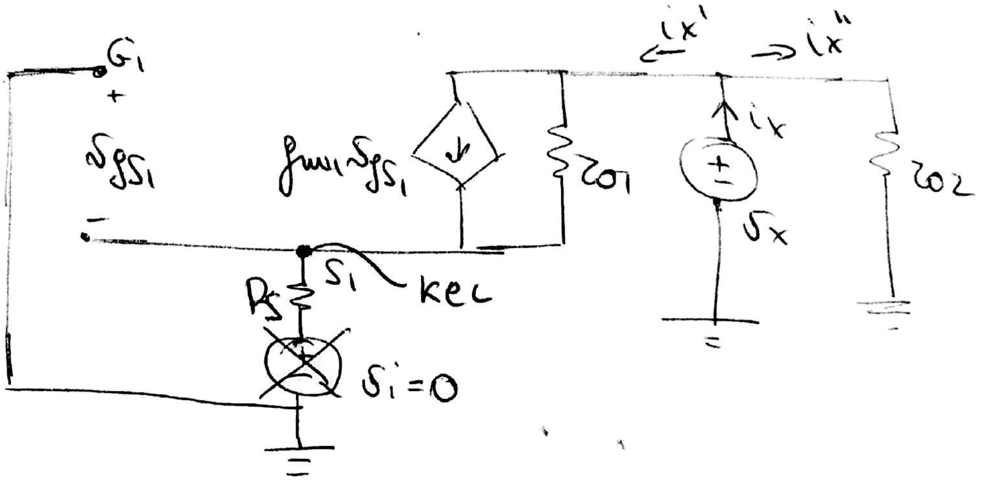
$$v_o\left[\frac{1}{r_{o2}}+\frac{1}{r_{o1}}+\frac{g_{m1}R_S}{r_{o2}}+\frac{R_S}{r_{o1}r_{o2}}\right]=v_i\left[g_{m1}+\frac{1}{r_{o1}}\right]$$

$$G_V=\frac{v_o}{v_i}=\frac{g_{m1}+\frac{1}{r_{o1}}}{\frac{1}{r_{o1}}+\frac{1}{r_{o2}}+\frac{g_{m1}R_S}{r_{o2}}+\frac{R_S}{r_{o1}r_{o2}}}\approx\frac{0.92m+5\cdot10^{-6}}{5\cdot10^{-6}+7.5\cdot10^{-6}+0.23\cdot10^{-6}+0.00}$$

$$=\frac{0.92m}{12.72\mu}=72.72$$

$$G_V=72.72$$

$$R_o=\frac{V_x}{i_x}$$



$$i_x=i_x'+i_x''$$

$$\frac{1}{R_o}=\frac{i_x}{v_x}=\frac{i_x'+i_x''}{v_x}=\frac{i_x'}{v_x}+\frac{i_x''}{v_x}=\frac{i_x'}{v_o}+(r_{o2})^{-1}$$

$$\frac{1}{R_o}=\frac{i_x'}{v_o}+\frac{1}{r_{o2}}$$

$$\underline{\text{KCL}}:\ \frac{v_{S1}}{R_S}-g_{m1}v_{gs1}+\frac{v_{S1}-v_x}{r_{o1}}=0\qquad v_{gs1}=-v_{S1}$$

$$v_{S1}\left(\frac{1}{R_S}+g_{m1}+\frac{1}{r_{o1}}\right)=\frac{v_x}{r_{o1}}\qquad v_{S1}=i_x'R_S$$

$$i_x'R_S\left(\frac{1}{R_S}+g_{m1}+\frac{1}{r_{o1}}\right)=\frac{v_x}{r_{o1}}\Rightarrow\frac{i_x'}{v_x}=\frac{1}{r_{o1}+g_{m1}R_Sr_{o1}+R_S}=$$

$$=\frac{1}{200k+0.92m\cdot200k\cdot50+50}=4.7\mu\Omega^{-1}\qquad R_o\approx82\,k\Omega$$