

**Course ID: ECE 341 Communication Systems- Fall**

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**235D/ Office Hours: Mondays and Wednesdays 11:00am - 12:00pm**

**Lectures: Mondays and Wednesdays 9:30am-10:45 am, Room: EECE 118**

**Department of Electrical and Computer Engineering / University of New Mexico**

### **Homework #2**

**Corresponding to Sections 2.4 – 2.9 of Principles of Communications,  
Rodger E. Zimmer and William H. Tranter, John Wiley, 7th Edition.**

1. Find the Fourier transform of the signals: a)  $x_1(t) = A \exp(-t/\tau) u(t)$ , b)  $x_2(t) = A \exp(t/\tau) u(-t)$ , c)  $x_3(t) = x_1(t) - x_2(t)$ , d)  $x_4(t) = x_1(t) + x_2(t)$ , e)  $x_5(t) = x_1(t-5)$ , and f)  $x_6(t) = x_1(t) - x_1(t-5)$
2. Obtain the time-average autocorrelation function of the signal  $x(t) = 3 + 6\cos(20\pi t) + 3\sin(20\pi t)$  and obtain the power spectral density of the signal. What is its total average power?
3. Determine the autocorrelation function of the signal with power spectral density  $S(f) = 9\delta(f-20) + 9\delta(f+20)$  and give the average power.
4. For the transfer function  $H(f) = j2\pi f / (7 + j2\pi f)$ , determine the unit impulse response of the system.
5. Determine whether or not the filter with impulse response  $h(t) = (1/t) u(t-1)$  is (BIBO) stable.
6. Given a filter with frequency response function  $H(f) = 5 / (4 + j2\pi f)$  and input  $x(t) = \exp(-3t) u(t)$ , obtain the energy spectral densities of the input and output.
7. Prove that  $x(t) = \sin(\omega_0 t)$  and  $\hat{x}(t)$  are orthogonal signals.

To be delivered at instructor's office: 30 September 2019

Good Luck!

1. Find the Fourier transform of the signals: a)  $x_1(t) = A \exp(-t/\tau) u(t)$ , b)  $x_2(t) = A \exp(t/\tau) u(-t)$ , c)  $x_3(t) = x_1(t) - x_2(t)$ , d)  $x_4(t) = x_1(t) + x_2(t)$ , e)  $x_5(t) = x_1(t-5)$ , and f)  $x_6(t) = x_1(t) - x_1(t-5)$

$$\begin{aligned}
 (a) \quad x_1(t) &= A e^{-t/\tau} u(t) \\
 X_1(\omega) &= A \int_0^{\infty} e^{-t/\tau} e^{-j\omega t} dt \\
 &= A \int_0^{\infty} e^{-t(\frac{1}{\tau} + j\omega)} dt \\
 &= A \left( -\frac{1}{\frac{1}{\tau} + j\omega} \right) e^{-t(\frac{1}{\tau} + j\omega)} \Big|_0^{\infty} \\
 &= \frac{-A}{\frac{1}{\tau} + j\omega} (0 - 1) \Big|_{\left(\frac{\tau}{\tau}\right)}
 \end{aligned}$$

$$X_1(\omega) = \frac{A\tau}{1 + j\omega\tau}$$

$$\begin{aligned}
 (b) \quad x_2(t) &= A e^{t/\tau} u(-t) \\
 X_2(\omega) &= A \int_{-\infty}^0 e^{t/\tau} e^{-j\omega t} dt \\
 &= A \int_{-\infty}^0 e^{t(\frac{1}{\tau} - j\omega)} dt
 \end{aligned}$$

$$X_2(\omega) = \frac{A\tau}{1 - j\omega\tau}$$

$$(c) \quad x_3(t) = x_1(t) - x_2(t)$$

$$\begin{aligned}
 X_3(\omega) &= X_1(\omega) - X_2(\omega) \\
 &= \frac{A\tau}{1 + j\omega\tau} - \frac{A\tau}{1 - j\omega\tau} \\
 &= \frac{A\tau(1 - j\omega\tau) - A\tau(1 + j\omega\tau)}{(1 + j\omega\tau)(1 - j\omega\tau)}
 \end{aligned}$$

$$X_3(\omega) = \frac{-2A\tau^2 j\omega}{(\omega\tau)^2 + 1}$$

$$\begin{aligned}
 (d) \quad x_4(t) &= x_1(t) + x_2(t) \\
 X_4(\omega) &= X_1(\omega) + X_2(\omega) \\
 &= \frac{A\tau}{1 + j\omega\tau} + \frac{A\tau}{1 - j\omega\tau} \\
 &= \frac{A\tau(1 - j\omega\tau) + A\tau(1 + j\omega\tau)}{(1 + j\omega\tau)(1 - j\omega\tau)}
 \end{aligned}$$

$$X_4(\omega) = \frac{2A\tau}{(\omega\tau)^2 + 1}$$

$$\begin{aligned}
 (e) \quad x_5(t) &= x_1(t-5) \\
 X_5(\omega) &= X_1(\omega) e^{-j5\omega}
 \end{aligned}$$

$$X_5(\omega) = \frac{A\tau}{1 + j\omega\tau} e^{-j5\omega}$$

$$\begin{aligned}
 (f) \quad x_6(t) &= x_1(t) - x_1(t-5) \\
 X_6(\omega) &= X_1(\omega) - X_5(\omega) \\
 &= \frac{A\tau}{1 + j\omega\tau} - \frac{A\tau}{1 + j\omega\tau} e^{-j5\omega}
 \end{aligned}$$

$$X_6(\omega) = \frac{A\tau}{1 + j\omega\tau} (1 - e^{-j5\omega})$$

Superposition Theorem

$$a_1 x_1(t) + a_2 x_2(t) \longleftrightarrow a_1 X_1(\omega) + a_2 X_2(\omega)$$

Time-Delay Theorem

$$x(t - t_0) \longleftrightarrow X(\omega) e^{-j2\pi f t_0}$$

2. Obtain the time-average autocorrelation function of the signal  $x(t) = 3 + 6\cos(20\pi t) + 3\sin(20\pi t)$  and obtain the power spectral density of the signal. What is its total average power?

$$x(t) = 3 + 6\cos(20\pi t) + 3\sin(20\pi t) \\ = 3 + 3\sqrt{5} \cos\left[20\pi t + \tan^{-1}\left(\frac{3}{6}\right)\right]$$

$$R(\tau) = \frac{1}{T_0} \int_{T_0} x(t)x(t+\tau) dt$$

$$R(\tau) = \frac{1}{0.1} \int_0^{0.1} \left\{ 3 + 3\sqrt{5} \cos\left[20\pi t + \tan^{-1}\left(\frac{1}{2}\right)\right] \right\} \left\{ 3 + 3\sqrt{5} \cos\left[20\pi(t+\tau) + \tan^{-1}\left(\frac{1}{2}\right)\right] \right\} dt \\ = 10 \int_0^{0.1} \left\{ 9 + 9\sqrt{5} \cos\left[20\pi(t+\tau) + \tan^{-1}\left(\frac{1}{2}\right)\right] + 9\sqrt{5} \cos\left[20\pi t + \tan^{-1}\left(\frac{1}{2}\right)\right] \right. \\ \left. + 45 \cos\left[20\pi(t+\tau) + \tan^{-1}\left(\frac{1}{2}\right)\right] \cos\left[20\pi t + \tan^{-1}\left(\frac{1}{2}\right)\right] \right\} dt \\ = 10 \int_0^{0.1} 9 dt \\ + 90\sqrt{5} \int_0^{0.1} \cos\left[20\pi(t+\tau) + \tan^{-1}\left(\frac{1}{2}\right)\right] dt \quad \rightarrow 0 \\ + 90\sqrt{5} \int_0^{0.1} \cos\left[20\pi t + \tan^{-1}\left(\frac{1}{2}\right)\right] dt \quad \rightarrow 0 \\ + \frac{450}{2} \int_0^{0.1} \cos(20\pi\tau) dt \rightarrow \frac{450}{20} \cos(20\pi\tau) \\ + \frac{450}{2} \int_0^{0.1} \cos\left[40\pi t + 20\pi\tau + 2\tan^{-1}\left(\frac{1}{2}\right)\right] dt \quad \rightarrow 0$$

$$R(\tau) = 9 + \frac{45}{2} \cos(20\pi\tau)$$

$$S(f) = F\left[9 + \frac{45}{2} \cos(20\pi\tau)\right]$$

$$S(f) = \mathfrak{F}[R(\tau)]$$

$$\cos(2\pi f_0 t) \leftrightarrow \frac{1}{2} \delta(f - f_0) + \frac{1}{2} \delta(f + f_0)$$

$$= 9 F[1] + \frac{45}{2} F[\cos(20\pi\tau)]$$

$$S(f) = 9 \delta(f) + \frac{45}{4} \delta(f - 10) + \frac{45}{4} \delta(f + 10)$$

Integrating over all of  $f$  gives:

$$P = 9 + \frac{45}{2} = 31.5 \frac{\text{Watts}}{\text{ohm}}$$

$$R(0) = \langle x^2(t) \rangle = \int_{-\infty}^{\infty} S(f) df$$

$$P = \int_{-\infty}^{\infty} S(f) df = \langle x^2(t) \rangle$$

3. Determine the autocorrelation function of the signal with power spectral density  $S(f) = 9\delta(f-20) + 9\delta(f+20)$  and give the average power.

$$R(\tau) = \mathcal{F}^{-1}[9\delta(f-20) + 9\delta(f+20)]$$

$$R(\tau) = 18\cos(40\pi\tau)$$

$$P = R(0) = 18\cos(0) = 18$$

$$P = 18 \frac{\text{Watts}}{\text{ohm}}$$

$$R(\tau) = \mathcal{F}^{-1}[S(f)]$$

$$\cos(2\pi f_0 t) \leftrightarrow \frac{1}{2}\delta(f - f_0) + \frac{1}{2}\delta(f + f_0)$$

$$P = \int_{-\infty}^{\infty} S(f) df = \langle x^2(t) \rangle$$

$$R(0) = \langle x^2(t) \rangle = \int_{-\infty}^{\infty} S(f) df$$

4. For the transfer function  $H(f) = j2\pi f / (7 + j2\pi f)$ , determine the unit impulse response of the system.

$$H(f) = \frac{j2\pi f}{7 + j2\pi f} \rightarrow H(\omega) = \frac{j\omega}{7 + j\omega}$$

$$\frac{7 - 7 + j\omega}{7 + j\omega} = 1 - \frac{7}{7 + j\omega}$$

$$h(t) = \delta(t) - 7e^{-7t}u(t)$$

$$(-jt)^n f(t) \left| \frac{d^n F(\omega)}{d\omega^n} \right.$$

$$u(t)e^{-\alpha t} \left| \frac{1}{\alpha + j\omega} \right.$$

5. Determine whether or not the filter with impulse response  $h(t) = (1/t) u(t-1)$  is BIBO stable.

$$\int_{-\infty}^{\infty} \frac{1}{t} u(t-1) dt$$

$$\int_1^{\infty} \frac{1}{t} dt = \ln(t) \Big|_1^{\infty} \rightarrow \infty$$

The filter is unstable

$$\int_{-\infty}^{\infty} |h(t)| dt < \infty$$

6. Given a filter with frequency response function  $H(f) = 5/(4 + j2\pi f)$  and input  $x(t) = \exp(-3t) u(t)$ , obtain the energy spectral densities of the input and output.

input

$$G_x(\omega) = |X(\omega)|^2 \quad X(\omega) = \frac{1}{3 + j\omega}$$

$$= \left| \frac{1}{3 + j\omega} \right|^2$$

$$G(f) = |X(f)|^2$$

$$\boxed{G_x(\omega) = \frac{1}{9 + \omega^2}}$$

output

$$G_y(\omega) = |H(\omega)|^2 G_x(\omega)$$

$$= \left| \frac{5}{4 + j\omega} \right|^2 \frac{1}{9 + \omega^2}$$

$$= \left[ \frac{25}{16 + \omega^2} \right] \left[ \frac{1}{9 + \omega^2} \right]$$

$$G_y(f) = |H(f)|^2 G_x(f)$$

7. Prove that  $x(t) = \sin(\omega_0 t)$  and  $\hat{x}(t)$  are orthogonal signals.

$$\sin(2\pi f_0 t) \leftrightarrow \frac{1}{2j} \delta(f - f_0) - \frac{1}{2j} \delta(f + f_0)$$

$$x(t) = \sin(\omega_0 t) = \sin(2\pi f_0 t)$$

$$X(f) = \frac{1}{2j} \delta(f - f_0) - \frac{1}{2j} \delta(f + f_0)$$

$$\hat{X}(f) = \frac{1}{2j} \delta(f - f_0) e^{-j\frac{\pi}{2}} - \frac{1}{2j} \delta(f + f_0) e^{j\frac{\pi}{2}}$$

$$\begin{aligned} \hat{x}(t) &= \sin\left(2\pi f_0 t - \frac{\pi}{2}\right) \\ &= -\cos(2\pi f_0 t) \end{aligned}$$

$$\lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T x(t) \hat{x}(t) dt$$

$$\lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T x(t) \hat{x}(t) dt = 0 \text{ (power signals)}$$

$$= \lim_{T \rightarrow \infty} -\frac{1}{2T} \int_{-T}^T \sin(\omega_0 t) \cos(\omega_0 t) dt$$

$$= \lim_{T \rightarrow \infty} -\frac{1}{2T} \int_{-2\pi}^{2\pi} \frac{1}{2} \sin(2\omega_0 t) dt$$

$$= \lim_{T \rightarrow \infty} -\frac{1}{4T} \int_{-2\pi}^{2\pi} \sin(2\omega_0 t) dt$$

$$= \lim_{T \rightarrow \infty} +\frac{1}{4T} \left( \frac{1}{2\omega_0} \cos(2\omega_0 t) \right) \Big|_{-T}^T = 0$$

since the limit goes to 0, they are orthogonal