Collaborative Quiz #1

Solutions

ECE 345/ME 380

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1. $I \cdot s^2 \Theta(s) = -mgl\,\Theta(s) - \tau(s)$

$$\Theta(s)\left[Is^2 + mgl\right] = -\tau(s)$$

$$\frac{\Theta(s)}{\tau(s)} = \frac{-1}{Is^2 + mgl}$$

$$= \frac{-1/I}{s^2 + mgl/I}$$

2. $s^2 + \frac{mgl}{I^2}$

3. $x_1 = \theta \implies \dot{x}_1 = \dot{\theta} = x_2$

$x_2 = \dot{\theta}$ $\quad \dot{x}_2 = \ddot{\theta} = -\frac{mgl}{I}\cdot\theta - \frac{1}{I}\cdot u$

So $A = \begin{bmatrix} 0 & 1 \\ -\frac{mgl}{I} & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ -1/I \end{bmatrix}$

then $y = \theta = x_1$

$\therefore C = [1 \;\; 0]$, $D = 0$.

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1. $G(s) = \dfrac{b_1 s + b_0}{s^2 + a_1 s + a_0} = \dfrac{-1/I}{s^2 + mgl/I}$

$\Rightarrow b_1 = 0, \quad b_0 = -1/I$

$a_1 = 0, \quad a_0 = \dfrac{mgl}{I}$

$\therefore A_p = \begin{bmatrix} 0 & 1 \\ -\frac{mgl}{I} & 0 \end{bmatrix}, \quad B_p = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$C_p = \begin{bmatrix} -1/I & 0 \end{bmatrix}, \quad D_p = 0$

$\Rightarrow$ (d) is correct.

2. $H(s) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} s & -1 \\ +\frac{mgl}{I} & s \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ -1/I \end{bmatrix} + 0$

$= \dfrac{1}{s^2 + \frac{mgl}{I}} \cdot \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} s & +1 \\ -\frac{mgl}{I} & s \end{bmatrix} \begin{bmatrix} 0 \\ -1/I \end{bmatrix}$

$= \dfrac{-1/I}{s^2 + mgl/I}$

$\Rightarrow$ (a) is correct.

3. (b) is correct.

4. Because the only non-zero entry of $\tilde{B}$ is in the third row, we know the input has a direct effect on $z(t)$. The $\tilde{A}$ matrix shows that $\theta(t)$ is affected by $\dot{\theta}(t)$ (via the (1,2) element being non-zero), which is in turn affected by $\theta(t) + z(t)$ (via the (2,1) + (2,3) elements being non-zero). Since we know $z(t)$ is impacted by the input, $\theta(t)$ is indirectly affected by the input. ∴ (a) is correct.

5. $$\frac{J\, z(t_{trap})}{I} \geq \sqrt{g/\ell}$$

$$z(t_{trap}) \geq \frac{I}{J}\sqrt{g/\ell}$$

$J \downarrow$ means $z(t_{trap}) \uparrow$ in order to meet the tripping threshold.

$J \uparrow$ means $z(t_{trap}) \downarrow$.

We see from Fig. 2 that higher values of $z(t_{trap})$ take longer to achieve, because $t_{trap} \uparrow$.

Similarly, $z(t_{trap}) \downarrow$ means $t_{trap} \downarrow$.

∴ We want increased flywheel inertia J.