ECE 371

Materials and Devices

HW #6

Due: Thursday 11/07/19 at the beginning of class

*All problems from Neamen 4^{th} Edition Ch. 5 and Ch. 7.

5.23 Consider three samples of silicon at $T = 300$ K. The n-type sample is doped with arsenic atoms to a concentration of $N_d = 5 \times 10^{16}$ cm^{-3}. The p-type sample is doped with boron atoms to a concentration of $N_a = 2 \times 10^{16}$ cm^{-3}. The compensated sample is doped with both the donors and acceptors described in the n-type and p-type samples. (*a*) Find the equilibrium electron and hole concentrations in each sample, (*b*) determine the majority carrier mobility in each sample, (*c*) calculate the conductivity of each sample, (*d*) and determine the electric field required in each sample to induce a drift current density of $J = 120$ A/cm^2.

5.29 Consider a sample of silicon at $T = 300$ K. Assume that the electron concentration varies linearly with distance, as shown in Figure P5.29. The diffusion current density is found to be $J_n = 0.19$ A/cm^2. If the electron diffusion coefficient is $D_n = 25$ cm^2/s, determine the electron concentration at $x = 0$.

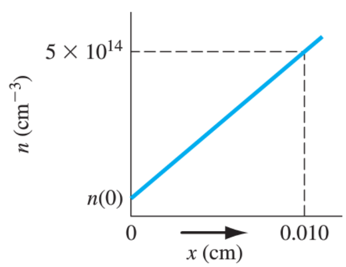


Figure P5.29 | Figure for Problem 5.29.

5.35 The electron concentration in silicon at $T = 300$ K is given by

$$n(x) = 10^{16} \exp\left(\frac{-x}{18}\right) \text{ cm}^{-3}$$

where x is measured in μm and is limited to $0 \le x \le 25$ μm. The electron diffusion coefficient is $D_n = 25$ cm^2/s and the electron mobility is $\mu_n = 960$ cm^2/V-s. The total electron current density through the semiconductor is constant and equal to $J_n = -40$ A/cm^2. The electron current has both diffusion and drift current components. Determine the electric field as a function of x which must exist in the semiconductor.

5.36 The total current in a semiconductor is constant and equal to $J = -10$ A/cm^2. The total current is composed of a hole drift current and electron diffusion current. Assume that the hole concentration is a constant and equal to 10^{16} cm^{-3} and assume that the electron concentration is given by $n(x) = 2 \times 10^{15} e^{-x/L}$ cm^{-3} where $L = 15$ μm. The electron diffusion coefficient is $D_n = 27$ cm^2/s and the hole mobility is $\mu_p = 420$ cm^2/V-s. Calculate (*a*) the electron diffusion current density for $x > 0$, (*b*) the hole drift current density for $x > 0$, and (*c*) the required electric field for $x > 0$.

5.40 Consider an n-type semiconductor at $T = 300$ K in thermal equilibrium (no current). Assume that the donor concentration varies as $N_d(x) = N_{d0} e^{-x/L}$ over the range $0 \le x \le L$ where $N_{d0} = 10^{16}$ cm^{-3} and $L = 10$ μm. (*a*) Determine the electric field as a function of x for $0 \le x \le L$. (*b*) Calculate the potential difference between $x = 0$ and $x = L$ (with the potential at $x = 0$ being positive with respect to that at $x = L$).

5.45' Consider a semiconductor at $T = 300$ K. (*a*) (*i*) Determine the electron diffusion coefficient if the electron mobility is $\mu_n = 1150$ cm^2/V-s. (*ii*) Repeat (*i*) of part (*a*) if the electron mobility is $\mu_n = 6200$ cm^2/V-s. (*b*) (*i*) Determine the hole mobility if the hole diffusion coefficient is $D_p = 8$ cm^2/s. (*ii*) Repeat (*i*) of part (*b*) if the hole diffusion coefficient is $D_p = 35$ cm^2/s.

7.4 An abrupt silicon pn junction at zero bias has dopant concentrations of $N_a = 10^{17}$ cm^{-3} and $N_d = 5 \times 10^{15}$ cm^{-3}. $T = 300$ K. (*a*) Calculate the Fermi level on each side of the junction with respect to the intrinsic Fermi level. (*b*) Sketch the equilibrium energy-band diagram for the junction and determine V_{bi} from the diagram and the results of part (*a*). (*c*) Calculate V_{bi} using Equation (7.10), and compare the results to part (*b*). (*d*) Determine x_n, x_p, and the peak electric field for this junction.

7.8 (*a*) Consider a uniformly doped silicon pn junction at $T = 300$ K. At zero bias, 25 percent of the total space charge region is in the n-region. The built-in potential barrier is $V_{bi} = 0.710$ V. Determine (*i*) N_a, (*ii*) N_d, (*iii*) x_n, (*iv*) x_p, and (*v*) $|E_{max}|$. (*b*) Repeat part (*a*) for a GaAs pn junction with $V_{bi} = 1.180$ V.

7.9 Consider the impurity doping profile shown in Figure P7.9 in a silicon pn junction. For zero applied voltage, (*a*) determine V_{bi}, (*b*) calculate x_n and x_p, (*c*) sketch the thermal equilibrium energy-band diagram, and (*d*) plot the electric field versus distance through the junction.

7.12 An "isotype" step junction is one in which the same impurity type doping changes from one concentration value to another value. An n-n isotype doping profile is shown in Figure P7.12. (*a*) Sketch the thermal equilibrium energy-band diagram of the isotype junction. (*b*) Using the energy-band diagram, determine the built-in potential barrier. (*c*) Discuss the charge distribution through the junction.

5.23 Consider three samples of silicon at $T = 300$ K. The n-type sample is doped with arsenic atoms to a concentration of $N_d = 5 \times 10^{16}$ cm^{-3}. The p-type sample is doped with boron atoms to a concentration of $N_a = 2 \times 10^{16}$ cm^{-3}. The compensated sample is doped with both the donors and acceptors described in the n-type and p-type samples. (*a*) Find the equilibrium electron and hole concentrations in each sample, (*b*) determine the majority carrier mobility in each sample, (*c*) calculate the conductivity of each sample, (*d*) and determine the electric field required in each sample to induce a drift current density of $J = 120$ A/cm^2.

<u>arsenic-doped</u> $N_d = 5e^{16}\ cm^{-3}$ n-type $\rightarrow n_o = N_d$

(a) $n_o = \boxed{5e16\ cm^{-3}}$

$p_o = \dfrac{n_i^2}{n_o} = \dfrac{(1.5e^{10})^2}{5e16} = \boxed{4500\ cm^{-3}}$

(b) n-type $\rightarrow$ majority = electrons

$\mu_n \approx \boxed{1100\ cm^2/V\cdot s}$

(c) $\sigma = e\mu_n n_o = \boxed{8.8120\ \mho}$

(d) $J = 120\ \frac{A}{cm^2} = \sigma E \rightarrow E = \boxed{13.6178\ \frac{V}{cm}}$

<u>boron-doped</u> $Na = 2e^{16}\ cm^{-3}$ p-type $\rightarrow p_o = Na$

(a) $\boxed{p_o = 2e16\ cm^{-3}}$

$n_o = \dfrac{n_i^2}{p_o} = \dfrac{(1.5e^{10})^2}{2e16} = \boxed{11250\ cm^{-3}}$

(b) p-type $\rightarrow$ majority = holes

$\mu_p \approx \boxed{400\ cm^2/V\text{-}s}$

(c) $\sigma = e\mu_p p_o = \boxed{1.2817\ \mho}$

<u>compensated</u> $Na = 2e16$ $Nd = 5e16$

(a) $n_o = Nd - Na = \boxed{3e^{16}\ cm^{-3}}$

$$p_o = \frac{n_i^2}{n_o} = \frac{(1.5e10)^2}{3e16} = \boxed{7500\ cm^{-3}}$$

(b) n-type → majority = holes

$\mu_n \approx 1000\ cm^2/V\text{-}s$

(c) $\sigma = e\mu_n n_o = \boxed{4.8065\ \mho}$

5.29 Consider a sample of silicon at $T = 300$ K. Assume that the electron concentration varies linearly with distance, as shown in Figure P5.29. The diffusion current density is found to be $J_n = 0.19$ A/cm². If the electron diffusion coefficient is $D_n = 25$ cm²/s, determine the electron concentration at $x = 0$.

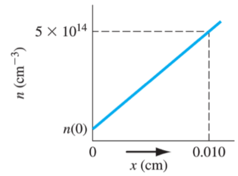


Figure P5.29 | Figure for Problem 5.29.

$$\frac{dn}{dx} = \frac{5e^{14} - n(0)}{0.01 - 0}$$

$$J_n = e D_n \frac{dn}{dx} \rightarrow \frac{J_n}{eD_n} = \frac{5e^{14} - n(0)}{0.01}$$

$$n(0) = (5e^{14}) - (0.01)\left(\frac{J_n}{eD_n}\right) = \boxed{2.5 \times 10^{13}\ cm^{-3}}$$

5.35 The electron concentration in silicon at $T = 300$ K is given by

$$n(x) = 10^{16} \exp\left(\frac{-x}{18}\right) \text{ cm}^{-3}$$

where x is measured in μm and is limited to $0 \leq x \leq 25$ μm. The electron diffusion coefficient is $D_n = 25$ cm^2/s and the electron mobility is $\mu_n = 960$ cm^2/V-s. The total electron current density through the semiconductor is constant and equal to $J_n = -40$ A/cm^2. The electron current has both diffusion and drift current components. Determine the electric field as a function of x which must exist in the semiconductor.

J_n = drift + diffusion

$$= e\mu_n nE + eD_n \frac{dn}{dx}$$

$$\frac{dn}{dx} = \frac{-10^{16}}{18e^{-4}} \exp\left(\frac{-x}{18}\right)$$

$$E = \frac{J_n - eD_n \frac{dn}{dx}}{e\mu_n n}$$

$$\boxed{= \frac{22.2525\left(\frac{-x}{18}\right) - 40}{1.53809\left(\frac{-x}{18}\right)}}$$

5.36 The total current in a semiconductor is constant and equal to $J = -10$ A/cm^2. The total current is composed of a hole drift current and electron diffusion current. Assume that the hole concentration is a constant and equal to 10^{16} cm^{-3} and assume that the electron concentration is given by $n(x) = 2 \times 10^{15} e^{-x/L}$ cm^{-3} where $L = 15$ μm. The electron diffusion coefficient is $D_n = 27$ cm^2/s and the hole mobility is $\mu_p = 420$ cm^2/V-s. Calculate (*a*) the electron diffusion current density for $x > 0$, (*b*) the hole drift current density for $x > 0$, and (*c*) the required electric field for $x > 0$.

$n(x) = 2e15\, e^{-x/L}$ cm^{-3} $\quad L = 15e^{-4}$ cm $\quad J = -10$ A/cm^2

$D_n = 27$ cm^2/s $\quad \mu_p = 420$ cm^2/V-s

$$\frac{dn}{dx} = \frac{-2e15}{L}\exp\left(-\frac{x}{L}\right)$$

(a) $$J_{n_{diff}} = eD_n\frac{dn}{dx} = \frac{(1.6e^{-19})(27)\left(-2e15\exp\left(\frac{-x}{15e^{-4}}\right)\right)}{15e^{-4}}$$

$$= \boxed{-5.76\exp\left(\frac{-x}{15e^{-4}}\right)} \ \frac{A}{cm^2}$$

(b) $$J_{p_{drift}} = J_T - J_{n_{diff}}$$

$$= \boxed{-10 + 5.76\exp\left(\frac{-x}{15e^{-4}}\right)} \ \frac{A}{cm^2}$$

(c) $$J_{p_{drift}} = -e\mu_p p_0 E$$

$$E = \frac{J_{p_{drift}}}{-e\mu_p p_0} = \frac{-10 + 5.76\exp\left(\frac{-x}{L}\right)}{-(1.6e^{-19})(420)(10^{16})}$$

$$= \frac{-10 + 5.76\exp\left(-\frac{x}{L}\right)}{-0.672}$$

$$= \boxed{14.881 - 8.57143\exp\left(\frac{-x}{15e^{-4}}\right)} \ \frac{V}{cm}$$

5.40 Consider an n-type semiconductor at $T = 300$ K in thermal equilibrium (no current). Assume that the donor concentration varies as $N_d(x) = N_{d0}e^{-x/L}$ over the range $0 \le x \le L$ where $N_{d0} = 10^{16}$ cm^{-3} and $L = 10$ μm. (*a*) Determine the electric field as a function of x for $0 \le x \le L$. (*b*) Calculate the potential difference between $x = 0$ and $x = L$ (with the potential at $x = 0$ being positive with respect to that at $x = L$).

$$N_d(x) = N_{do}\exp\left(\frac{-x}{L}\right) \quad ; \quad N_{do} = 10^{16}\ \text{cm}^{-3} \quad L = 10\ \mu\text{m} = 10^{-4}\ \text{cm}$$

(a) $$J = e\mu_n n E + eD_n \frac{dn}{dx}$$

$$n = N_d = N_{do}\exp\left(-\frac{x}{L}\right), \quad J = 0$$

$$0 = \mu_n N_{do}\left[\exp\left(-\frac{x}{L}\right)\right]E + D_n N_{do}\left(-\frac{1}{L}\right)\exp\left(-\frac{x}{L}\right)$$

$$0 = E + \frac{D_n}{\mu_n}\left(-\frac{1}{L}\right) \quad \text{but} \quad \frac{D_n}{\mu_n} = \frac{kT}{e}$$

$$E = \left(\frac{kT}{e}\right)\left(\frac{1}{L}\right) = (0.0259)/10^{-4}\text{cm}$$

$$= \boxed{259\ \text{V/cm}}$$

(b) $$\phi = -\int_0^L E\,dx = -\left(\frac{kT}{e}\right)\left(\frac{1}{L}\right)\int_0^L dx$$

$$= -\frac{kT}{e} = \boxed{-0.0259\ \text{V}}$$

5.45 Consider a semiconductor at $T = 300$ K. (*a*) (*i*) Determine the electron diffusion coefficient if the electron mobility is $\mu_n = 1150$ cm^2/V-s. (*ii*) Repeat (*i*) of part (*a*) if the electron mobility is $\mu_n = 6200$ cm^2/V-s. (*b*) (*i*) Determine the hole mobility if the hole diffusion coefficient is $D_p = 8$ cm^2/s. (*ii*) Repeat (*i*) of part (*b*) if the hole diffusion coefficient is $D_p = 35$ cm^2/s.

(a)

(i) $\dfrac{D_n}{\mu_n} = \dfrac{kT}{e} = 0.0259 \longrightarrow \boxed{D_n = 29.785 \text{ cm}^2/\text{s}}$

(ii) $\boxed{D_n = 160.5 \text{ cm}^2/\text{s}}$

(b)

(i) $\dfrac{D_p}{\mu_p} = \dfrac{kT}{e} = 0.0259 \longrightarrow \boxed{\mu_p = 308.88 \text{ cm}^2/\text{V-s}}$

(ii) $\boxed{\mu_p = 1351.35 \text{ cm}^2/\text{V-s}}$

7.4 An abrupt silicon pn junction at zero bias has dopant concentrations of $N_a = 10^{17}\ \text{cm}^{-3}$ and $N_d = 5 \times 10^{15}\ \text{cm}^{-3}$. $T = 300$ K. (*a*) Calculate the Fermi level on each side of the junction with respect to the intrinsic Fermi level. (*b*) Sketch the equilibrium energy-band diagram for the junction and determine V_{bi} from the diagram and the results of part (*a*). (*c*) Calculate V_{bi} using Equation (7.10), and compare the results to part (*b*). (*d*) Determine x_n, x_p, and the peak electric field for this junction.

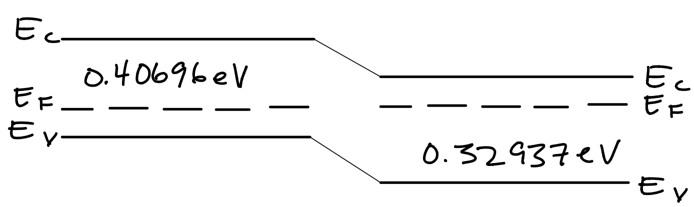
(a) p-side

$$E_{Fi} - E_F = kT \ln\left(\frac{N_a}{n_i}\right) = \boxed{0.40696\,\text{eV}}$$

n-side

$$E_F - E_{Fi} = kT \ln\left(\frac{N_d}{n_i}\right) = \boxed{0.32937\,\text{eV}}$$

(b) $V_{bi} = 0.40696 + 0.32937 = \boxed{0.73632\,\text{eV}}$



(c) $$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right) = 0.0259 \ln\left(\frac{10^{17} \cdot 5e^{15}}{(1e10)^2}\right) = \boxed{0.73632\,\text{eV}}$$

(d) $$x_n = \left[\frac{2\epsilon_s V_{bi}}{e}\left(\frac{N_a}{N_d}\right)\frac{1}{N_a + N_d}\right]^{1/2}$$

$$= \left[\frac{(2)(11.7)(8.85e^{-14})(0.73632)}{(1.6e^{-19})}\left(\frac{10^{17}}{5e^{15}}\right)\frac{1}{10^{17} + 5e^{15}}\right]^{1/2}$$

$$\boxed{= 0.42577\,\mu\text{m}}$$

$$x_p = \left[\frac{2\epsilon_s V_{bi}}{e}\left(\frac{N_d}{N_a}\right)\frac{1}{N_a + N_d}\right]^{1/2}$$

$$= \left[\frac{(2)(11.7)(8.85e^{-14})(0.73632)}{(1.6e^{-19})}\left(\frac{5e^{15}}{10^{17}}\right)\frac{1}{10^{17} + 5e^{15}}\right]^{1/2}$$

$$\boxed{= 21.2887 \text{ nm}}$$

$$|E_{max}| = \frac{e N_d x_n}{\epsilon_s} = \boxed{32.9 \text{ kV/cm}}$$

7.8 (*a*) Consider a uniformly doped silicon pn junction at $T = 300$ K. At zero bias, 25 percent of the total space charge region is in the n-region. The built-in potential barrier is $V_{bi} = 0.710$ V. Determine (*i*) N_a, (*ii*) N_d, (*iii*) x_n, (*iv*) x_p, and (*v*) $|E_{max}|$. (*b*) Repeat part (*a*) for a GaAs pn junction with $V_{bi} = 1.180$ V.

(a)

(i) $x_n = 0.25W = 0.25(x_n + x_p)$

$$0.75x_n = 0.25x_p$$

$$3x_n = x_p$$

$$3 = \frac{x_p}{x_n}$$

$$N_d x_n = N_a x_p \rightarrow \frac{x_p}{x_n} = \frac{N_d}{N_a} = 3$$

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

$$0.710 = 0.0259 \ln\left(\frac{N_a \cdot 3N_a}{(1.5\times10^{10})^2}\right)$$

$$\frac{3N_a^2}{(1.5\times10^{10})^2} = \exp\left(\frac{0.710}{0.0259}\right) \rightarrow \boxed{N_a = 7.7663\times10^{15}\ cm^{-3}}$$

(ii) $N_d = 3N_a = \boxed{23.2990\times10^{15}\ cm^{-3}}$

(iii)
$$x_n = \left[\frac{2\epsilon_s V_{bi}}{e}\left(\frac{N_a}{N_d}\right)\frac{1}{N_a+N_d}\right]^{1/2}$$

$$= \left[\frac{(2)(11.7)(8.85\times10^{-14})(0.710)}{1.6\times10^{-19}}\left(\frac{1}{3}\right)\frac{1}{31.065\times10^{15}}\right]^{1/2}$$

$$= \boxed{99.2328\ nm}$$

(iv) $x_p = 3x_n = \boxed{297.698\ nm}$

(v)
$$|E_{max}| = \frac{eN_d x_n}{\epsilon_s} = \frac{(1.6\times10^{-19})(23.2990\times10^{15}\ cm^{-3})(99.2328\ nm)}{(11.7)(8.85\times10^{-14})}$$

$$= \boxed{3.57577\times10^{4}\ V/cm}$$

(b)

(i) $x_n = 0.25W = 0.25(x_n + x_p)$

$$0.75x_n = 0.25x_p$$

$$3x_n = x_p$$

$$3 = \frac{x_p}{x_n}$$

$$N_d x_n = N_a x_p \rightarrow \frac{x_p}{x_n} = \frac{N_d}{N_a} = 3$$

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

$$1.18 = 0.0259 \ln\left(\frac{N_a \cdot 3N_a}{(1.8\times10^{10})^2}\right)$$

$$\frac{3N_a^2}{(1.8\times10^{6})^2} = \exp\left(\frac{1.18}{0.0259}\right) \rightarrow \boxed{N_a = 8.12656\times10^{15}\ cm^{-3}}$$

(ii) $N_d = 3N_a = \boxed{2.43797\times10^{16}\ cm^{-3}}$

(iii)
$$x_n = \left[\frac{2\epsilon_s V_{bi}}{e}\left(\frac{N_a}{N_d}\right)\frac{1}{N_a+N_d}\right]^{1/2}$$

$$= \left[\frac{(2)(13.1)(8.85\times10^{-14})(1.18)}{1.6\times10^{-19}}\left(\frac{1}{3}\right)\frac{1}{3.2506\times10^{16}}\right]^{1/2}$$

$$= \boxed{13.2363\ \mu m}$$

(iv) $x_p = 3x_n = \boxed{39.7089\ \mu m}$

(v)
$$|E_{max}| = \frac{eN_d x_n}{\epsilon_s} = \frac{(1.6\times10^{-19})(2.43797\times10^{16}\ cm^{-3})(13.236\ \mu m)}{(13.1)(8.85\times10^{-14})}$$

$$= \boxed{4.4574\times10^{4}\ V/cm}$$

7.9 Consider the impurity doping profile shown in Figure P7.9 in a silicon pn junction. For zero applied voltage, (*a*) determine V_{bi}, (*b*) calculate x_n and x_p, (*c*) sketch the thermal equilibrium energy-band diagram, and (*d*) plot the electric field versus distance through the junction.

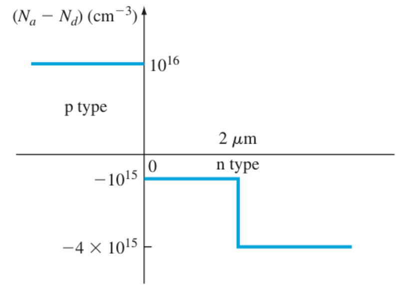


Figure P7.9 | Figure for Problem 7.9.

(a) $N_a = 10^{16} \qquad N_d = 10^{15}$

$$V_{bi} = \frac{kT}{e} \ln\left(\frac{N_d N_a}{n_i^2}\right) = \boxed{0.635 \text{ V}}$$

(b)
$$x_n = \left[\frac{2\epsilon_s V_{bi}}{e}\left(\frac{N_a}{N_d}\right)\frac{1}{N_a + N_d}\right]^{1/2}$$

$$= \left[\frac{(2)(11.7)(8.85\times10^{-14})(0.635)}{1.6\times10^{-19}}\left(\frac{10^{16}}{10^{15}}\right)\frac{1}{10^{16}+10^{15}}\right]^{1/2}$$

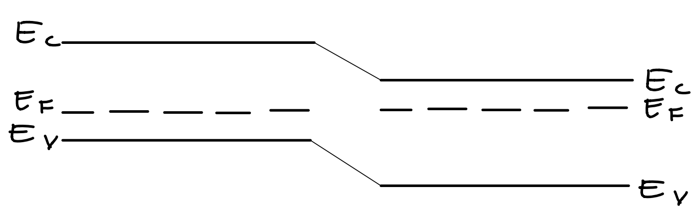
$$x_n = \boxed{0.863806 \ \mu m}$$

$$x_p = \left[\frac{2\epsilon_s V_{bi}}{e}\left(\frac{N_d}{N_a}\right)\frac{1}{N_a + N_d}\right]^{1/2}$$

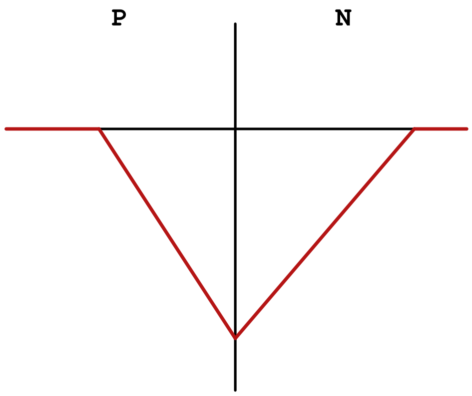
$$= \left[\frac{(2)(11.7)(8.85\times10^{-14})(0.635)}{1.6\times10^{-19}}\left(\frac{10^{15}}{10^{16}}\right)\frac{1}{10^{16}+10^{15}}\right]^{1/2}$$

$$x_p = \boxed{0.0864 \ \mu m}$$

(c)



Electric Field vs. Distance



$$|E_{max}| = \frac{e N_d x_n}{\epsilon_s}$$

$$= \frac{(1.6\times10^{-19})(10^{15})(0.864\,\mu m)}{(11.7)(8.85\times10^{-14})}$$

$$= 1.33596 \ V/cm$$

7.12 An "isotype" step junction is one in which the same impurity type doping changes from one concentration value to another value. An n-n isotype doping profile is shown in Figure P7.12. (*a*) Sketch the thermal equilibrium energy-band diagram of the isotype junction. (*b*) Using the energy-band diagram, determine the built-in potential barrier. (*c*) Discuss the charge distribution through the junction.

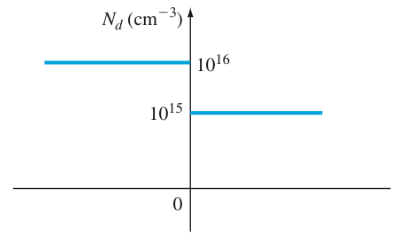
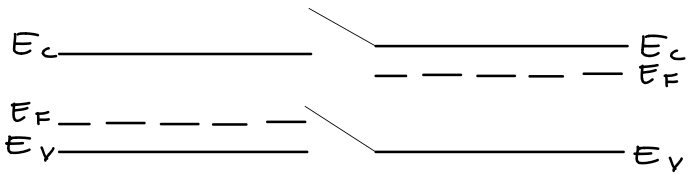


Figure P7.12 | Figure for Problem 7.12.

(a)



(b) $E_F - E_{Fi} = kT \ln\left(\frac{N_d}{n_i}\right) = 0.0259 \ln\left(\frac{10^{16}}{1.5\times10^{10}}\right)$

$= 0.34732 \text{ eV}$

$E_F - E_{Fi} = kT \ln\left(\frac{N_d}{n_i}\right) = 0.0259 \ln\left(\frac{10^{15}}{1.5\times10^{10}}\right)$

$= 0.28768 \text{ eV}$

$V_{bi} = 0.34732 - 0.28768 = 0.05964 \text{ V}$

(c) Unsure, haven't covered this in class yet.