ECE 345 / ME 380: Introduction to Control Systems
Midterm #1

Dr. Oishi

Due October 2, 2020 by 11:59am

This midterm is open note, open book, and Matlab and electronic resources are allowed. **No communication of any sort regarding the content of the exam is allowed with anyone other than Dr. Oishi.**

For full credit, show all your work.

Academic dishonesty is a violation of the UNM Student Code of Conduct. Students suspected of academic dishonesty will be referred for disciplinary action in accordance with University procedures.

By signing below, I affirm that I have completed the midterm independently, under the conditions stated above.

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Problem #	Actual points	Possible points
1	17	20
2	20	20
3	37.5	40
4	10	10
Total:	84.5	90

1 Warm Up (20 points)

Consider the dynamical system described by the transfer function $G(s) = \dfrac{2}{s+4}$, with input $U(s)$ and output $Y(s)$.

1. (10 points) Find the output $y(t)$ of the system in response to an input $u(t) = e^{-2t} \cdot \mathbf{1}(t)$.

$$G(s) = \frac{Y(s)}{U(s)} \qquad Y(s) = G(s)U(s)$$

$$= \left(\frac{2}{s+4}\right)\left(\frac{1}{s+2}\right)\left(\frac{1}{s}\right)$$

$$= \frac{2}{s(s+4)(s+2)}$$

$$= \frac{2}{s^3 + 6s^2 + 8s}$$

$$y(t) = \mathcal{L}^{-1}\{Y(s)\}$$

$$= 2\left[\frac{1}{8} - \frac{1}{4}e^{-2t} + \frac{1}{8}e^{-4t}\right]$$

$$= \frac{1}{4} - \frac{1}{2}e^{-2t} + \frac{1}{4}e^{-4t}$$

Must use partial fractions -3

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Now consider a spacecraft module descending to the surface, that can be described by the equations of motion

$$M\ddot{y}(t) = k \cdot \dot{m}(t) - m(t)g \tag{1}$$

where $m(t)$ is the mass of the module, g is the gravitational constant, and k, M are positive constants.

2. (10 points) Which one of the following correctly describes the system in state-space form, with state vector $x(t) = \begin{bmatrix} y(t) \\ \dot{y}(t) \\ m(t) \end{bmatrix}$, input $u(t) = \dot{m}(t)$, and output $y(t)$?

$C = [0\ 0\ 1]$

(a) $A = \begin{bmatrix} 0 & 0 & 1 \\ -g/M & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 0 \\ k/M \end{bmatrix}$, $\cancel{C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}}$, $D = 0$.

$C = [1\ 0\ 0]$

(b) $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -g/M \\ 0 & 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ k/M \\ 1 \end{bmatrix}$, $\cancel{C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}}$, $D = 0$.

(c) $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & k/M \\ 0 & 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ -g/M \\ 1 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$, $D = 0$.

(d) $A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -k/M \\ 0 & 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, $C = \begin{bmatrix} g/M & 0 & 0 \end{bmatrix}$, $D = 0$.

$$\ddot{y} = -\frac{g}{M} m(t) + \frac{k}{M} u(t)$$

- State equation $\dot{x}(t) = Ax(t) + Bu(t)$
- Output equation $y(t) = Cx(t) + Du(t)$

$$\dot{x} = \begin{bmatrix} \dot{y} \\ \ddot{y} \\ \dot{m} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -\frac{g}{M} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} y \\ \dot{y} \\ m \end{bmatrix} + \begin{bmatrix} \\ \frac{k}{M} \\ \\ \end{bmatrix} u(t)$$

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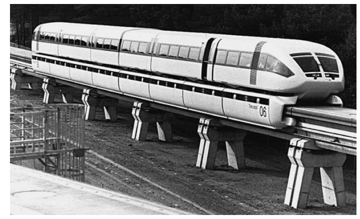
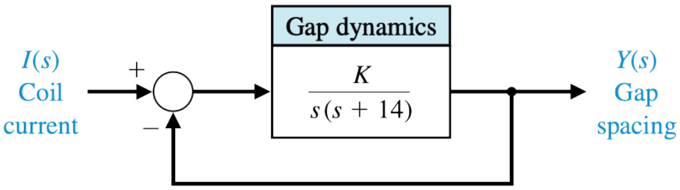
2 MagLev Train (20 points)



Maglev trains use magnetic levitation to keep the train car suspended above the guideway, through magnets on the bottom of the guideway which attract magnets on the wraparound, which pull the vehicle up toward the guideway. A controller is required to stabilize the system, resulting in the block diagram:



1. (10 points) Show that the block diagram above can be simplified to the transfer function

$$\frac{Y(s)}{I(s)} = \frac{K}{s^2 + 14s + K}. \tag{2}$$

$$\frac{Y(s)}{I(s)} = \frac{G(s)}{1 + G(s)}$$

$$= \frac{\frac{K}{s(s+14)}}{1 + \frac{K}{s(s+14)}}$$

$$= \frac{K}{s(s+14)\left(1 + \frac{K}{s+14}\right)}$$

$$= \frac{K}{s^2 + 14s + K}$$

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Recall that $\frac{Y(s)}{I(s)} = \frac{K}{s^2 + 14s + K}$.

2. (a) (5 points) Use the Final Value Theorem to compute the steady-state output response ~~$y(t)$~~ y_{ss} to a unit step input in $i(t)$.

   (b) (5 points) For what values of $K > 0$ will the steady-state value be equal to 1?

(a) $$Y(s) = G(s) \cdot I(s) = \frac{K}{s^2 + 14s + K} \cdot \frac{1}{s}$$

$$= \frac{K}{s(s^2 + 14s + K)}$$

$$= \frac{K}{s^3 + 14s^2 + Ks}$$

F.V.T.

$$y_{ss} = \lim_{t \to \infty} y(t) \rightarrow \lim_{s \to 0} sY(s)$$

$$= \lim_{s \to 0} \cancel{s} \frac{K}{\cancel{s}(s^2 + 14s + K)}$$

$$= \frac{K}{K} = 1$$

(b) The steady-state will be equal to 1 for all values of $K > 0$

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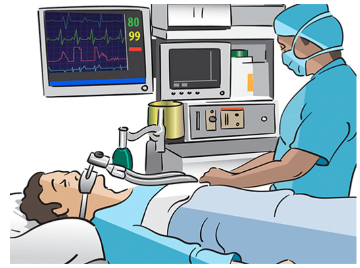
3 Mean arterial pressure control (40 points)



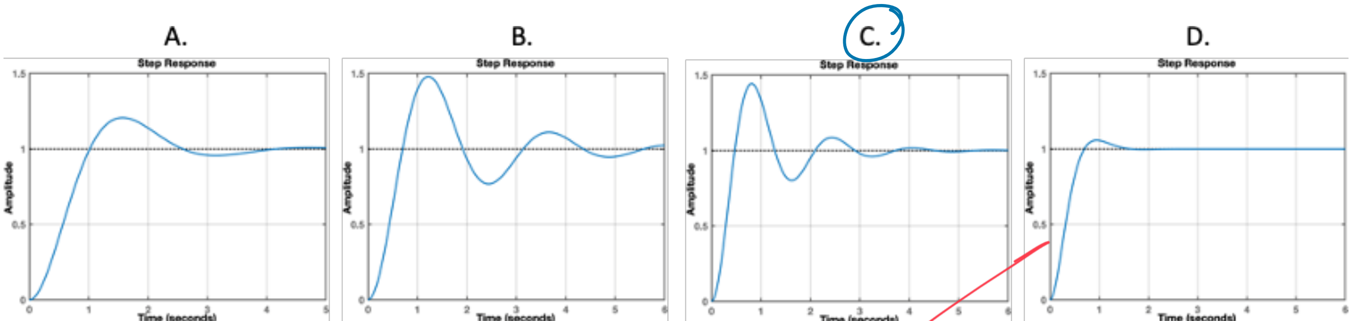
A system that controls mean arterial pressure can help anesthetists stabilize patients during surgery. We consider the design of such a system, in which the patient's hemodynamics are represented by

$$G(s) = \frac{4K}{s^2 + 2\sqrt{K}s + 4K} \tag{3}$$

with $K > 0$ an unknown, positive constant. The system should satisfy the transient response characteristics: 1) settling time less or equal to than 4 seconds, and 2) peak time less than or equal to $\frac{\pi}{3}$ seconds.

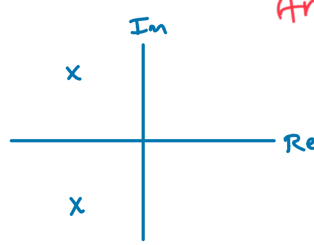
$T_s \leq 4$ $\qquad T_p \leq \frac{\pi}{3}$

1. (10 points) Which of the following step responses depict *both* desired transient response characteristics? *More than one may be correct.*



negative ∴ complex roots

$$\frac{-2\sqrt{K} \pm \sqrt{4K - 16K}}{2}$$



And D

< 2.5

Either B or C, but B settles too slowly.

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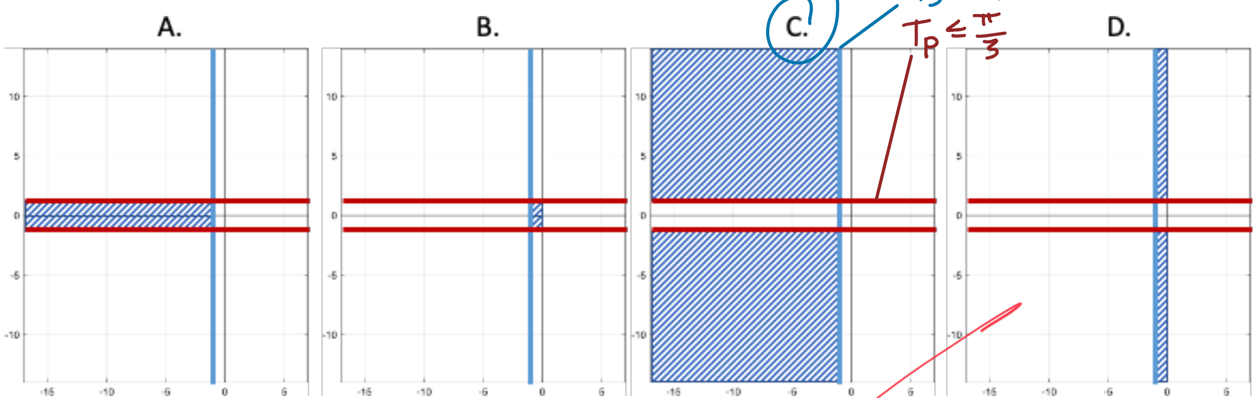
Recall that $G(s) = \dfrac{4K}{s^2 + 2\sqrt{K}s + 4K}$.

2. (10 points) Which one of the four plots below most accurately depicts all pole locations that would generate step responses that satisfy *both* transient response characteristics? For full credit, show your work to justify your response.

$T_s \le 4$

$T_p \le \frac{\pi}{3}$



3. (10 points) Compute the damping ratio ζ and the natural frequency ω_n in terms of K. Which one of the following is true?

   (a) For high values of $K > 0$, the system is overrdamped.

   (b) For low values of $K > 0$, the system is underdamped.

   (c) For low values of $K > 0$, the system is undamped.

   (d) The system is overdamped for all values of $K > 0$.

   (e) The system is underdamped for all values of $K > 0$.

$2\zeta\omega_n = 2\sqrt{K}$ $\qquad \omega_n^2 = 4K$

$\zeta = \frac{1}{2}$ $\qquad \omega_n = 2\sqrt{K}$

2.

$T_s = \frac{4}{\zeta\omega_n} \le 4 \quad \Rightarrow$ either A or C

$1 \le \zeta\omega_n$

$T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} \le \frac{\pi}{3} \quad \Rightarrow C$

$3 \le \omega_n\sqrt{1-\zeta^2}$

Recall that $G(s) = \dfrac{4K}{s^2 + 2\sqrt{K}s + 4K}$.

4. (10 points) Which *one* of the following most correctly describes the restrictions on K that assures that both transient response requirements are met? Show your work for full credit.

   (a) $1 \leq K$

   (b) $1 \leq K \leq 3$

   (c) $K \leq 3$

   (d) $3 \leq K$

   (e) No value of $K > 0$ will satisfy both requirements.

$$T_s = \frac{4}{\zeta \omega_n} \leq 4$$

$$\frac{4}{\sqrt{K}} \leq 4$$

$$K \geq 1$$

$$T_p = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}} \leq \frac{\pi}{3}$$

$$\frac{\pi}{2\sqrt{K}\sqrt{1-\frac{1}{4}}} \leq \frac{\pi}{3}$$

$$K \geq 3$$

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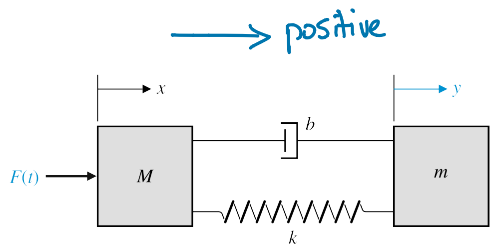
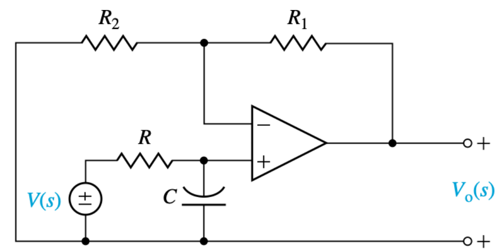
4 RLC Op-Amp Circuit / Spring-Mass-Damper System (10 points)

Choose *one* of the below systems to answer *either* 4A or 4B. *You will not receive extra credit for doing both. If you do more than one problem, your highest score will be used.*



(4A.) Spring mass damper system

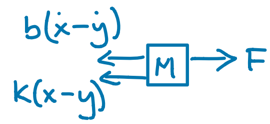


(4B.) Op-amp circuit system

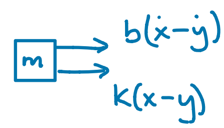
4A. (10 points) Find the mechanical system transfer function $G(s) = \frac{Y(s)}{F(s)}$ with $M = m = k = b = 1$ (dimensionless units), assuming initial conditions are 0.

4B. (10 points) Find the circuit transfer function $G(s) = \frac{V_0(s)}{V(s)}$ with $R_1 = R_2 = R = C = 1$ (dimensionless units), assuming initial conditions are 0.

4A Eqns of motion



$$\cancel{M}\ddot{x} = F - \cancel{b}(\dot{x}-\dot{y}) - \cancel{k}(x-y)$$



$$\cancel{m}\ddot{y} = \cancel{b}(\dot{x}-\dot{y}) + \cancel{k}(x-y)$$

$$F = \ddot{x} + \dot{x} + x - \dot{y} - y$$

$$\ddot{y} + \dot{y} + y - \dot{x} - x = 0$$

$$F(s) = s^2X(s) + sX(s) + X(s) - sY(s) - Y(s)$$

$$0 = s^2Y(s) + sY(s) + Y(s) - sX(s) - X(s)$$

$$\begin{bmatrix} F(s) \\ 0 \end{bmatrix} = \begin{bmatrix} X(s) \\ Y(s) \end{bmatrix} \cdot \begin{bmatrix} s^2+s+1 & -(s+1) \\ -(s+1) & s^2+s+1 \end{bmatrix}$$

$$\frac{Y(s)}{F(s)} = \frac{s+1}{(s^2+s+1)^2 - (s+1)^2}$$

$$= \frac{s+1}{s^4+2s^3+2s^2}$$

End of exam.