ECE 345 / ME 380: Introduction to Control Systems
Exercise: Introduction to Bode and Nyquist Diagrams

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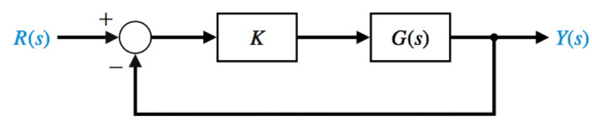
This exercise is designed to provide you with exposure to Bode diagrams and Nyquist plots, and help you develop some intuition about how frequency response methods work to assess relative stability. The Bode diagram is a plot of the steady-state response to sinusoidal inputs at various frequencies. The Nyquist diagram is essentially a polar plot of the Bode diagram.



Some Matlab commands that may be useful in this exercise:

- `sys = tf(1, [1 10 0])`
- `bode(sys)`
- `margin(sys)`
- `nyquist(sys)`
- `rlocus(sys)`

1 The effect of gain

Consider the following systems under negative unity feedback, which all have the same characteristic equation.

$$G_1(s) = \frac{0.1}{(s+1)^3} \tag{1}$$

$$G_2(s) = \frac{1}{(s+1)^3} \tag{2}$$

$$G_3(s) = \frac{10}{(s+1)^3} \tag{3}$$

1. Create transfer functions in Matlab for each of the three systems, for example, `sys1 = tf(0.1, conv([1 1],[1 2 1]))` for $G_1(s)$.
2. Create three Bode plots on the same plot, using `bode(sys1,sys2,sys3)`. Then use the command `legend('G1(s)','G2(s)','G3(s)')` to create a legend which distinguishes the systems by color.
3. Are the phase plots of three Bode plots at all different? The magnitude plots? If so, how?
4. Each plot presumes a gain of $K = 1$. Note that $G_2(s) = 10 \cdot G_1(s)$, and $G_3(s) = 10 \cdot G_2(s)$. By what amount are the gain plots of $G_2(s)$ and $G_3(s)$ shifted, as compared to $G_1(s)$? *Hint: Transform the low frequency gain values from dB to magnitude.*
5. What would the Bode plot of $G_1(s)$ look like with $K = 10$? With $K = 100$? How would these plots compare to plots of $G_2(s)$ and $G_3(s)$?

2 Relative stability via Bode diagrams

1. Use `margin(sys1)` to calculate phase margin and gain margin of $G_1(s)$. Then use `grid on` to plot grid lines on a logarithmic scale. Repeat on *separate* figures for the other two systems.

2. Use phase margin and gain margin to determine which of the three systems are stable with $K = 1$.

3. Consider the gain margin for $G_1(s)$. By what multiplicative factor can the gain be increased and still ensure stability of the closed-loop system?

4. Consider the gain margin for $G_3(s)$. By what multiplicative factor should the gain be decreased to ensure stability of the closed-loop system?

3 Stability via Nyquist

1. On a new figure, use `nyquist(sys1, sys2, sys3)` to generate Nyquist plots of all 3 systems on the same figure. Use `legend('G1(s)','G2(s)','G3(s)')` to create a legend which distinguishes the systems by color.

2. Describe the relationship between the three Nyquist diagrams, in terms of their overall shape.

3. Count the number of encirclements for each of the three systems. What is N in each of the three cases?

4. Consider the Nyquist criterion, $Z = P - N$. For all three systems, we know that $P = 0$ because there are *no* open-loop poles in the RHP. Find Z, the number of poles of the closed-loop system in the RHP, for all three systems. Which of the three systems are stable, according to the Nyquist criterion?

5. Is this assessment of stability consistent with stability via phase margin and gain margin in Question 2.2? Why or why not?

4 Stability via Root Locus

1. Use `rlocus(sys1)` to create root locus plots for $G_1(s)$, and repeat for systems $G_2(s)$ and $G_3(s)$. What do the plots have in common?

2. Use `axis([-4 1 -4 4])` to change the plotting boundaries, then use `rlocfind(sys1)` to find the gain required to destabilize the first system. That is, place the crosshairs at an intersection of the locus with the imaginary axis. Repeat for the other two systems. How do the values of K differ for each of the three systems?

3. For each system, compare your computed value of K with the gain margin (in magnitude, not dB) found in Question 2.1.

4. Use `roots` to find the roots of the characteristic equation of the closed-loop system with $G_3(s)$ when $K = 1$. Compare your answer to the value of Z for the Nyquist criterion in Question 3.4.