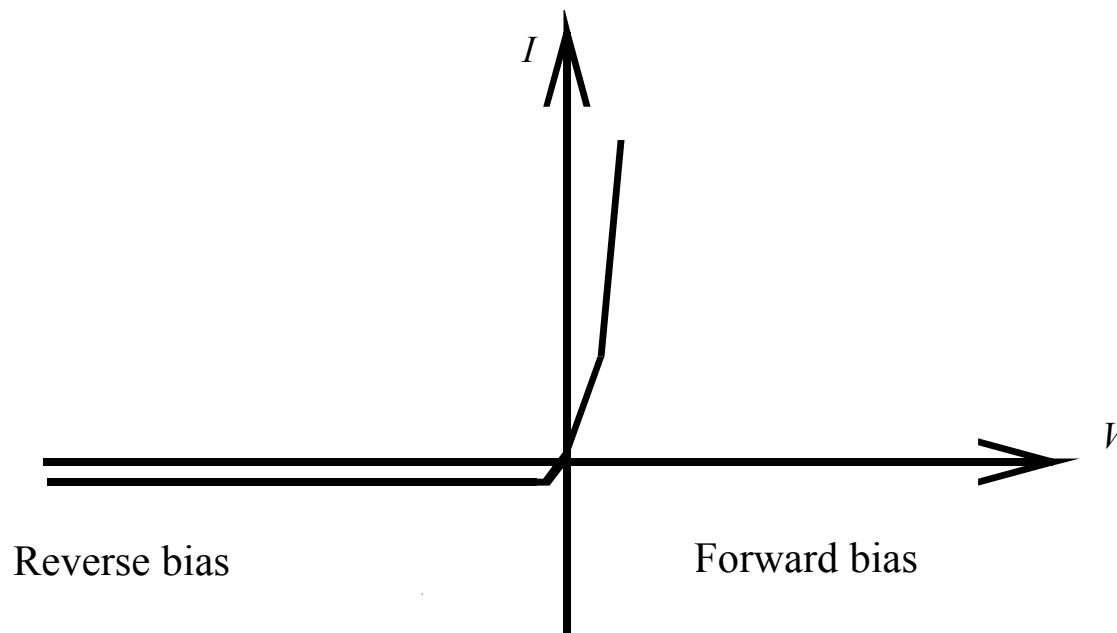


ECE 322L

p-n junctions in equilibrium and in non equilibrium

Operation of a pn junction

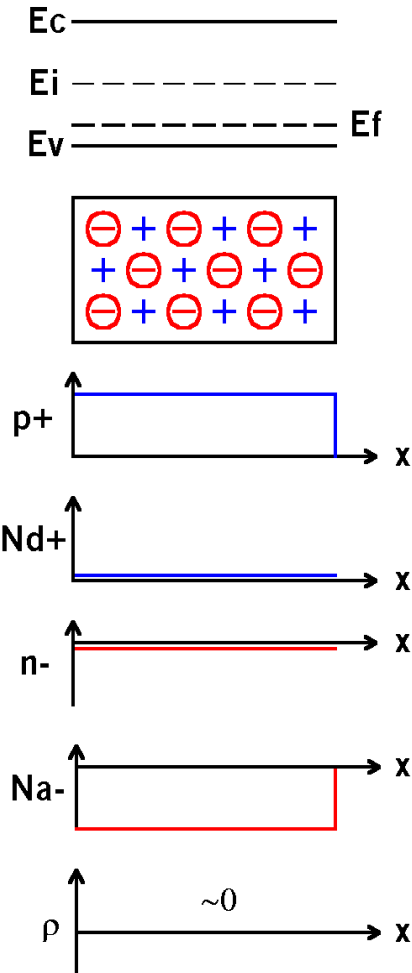
In order to understand the operation of a p-n junction, it is necessary to study its three operation regions:



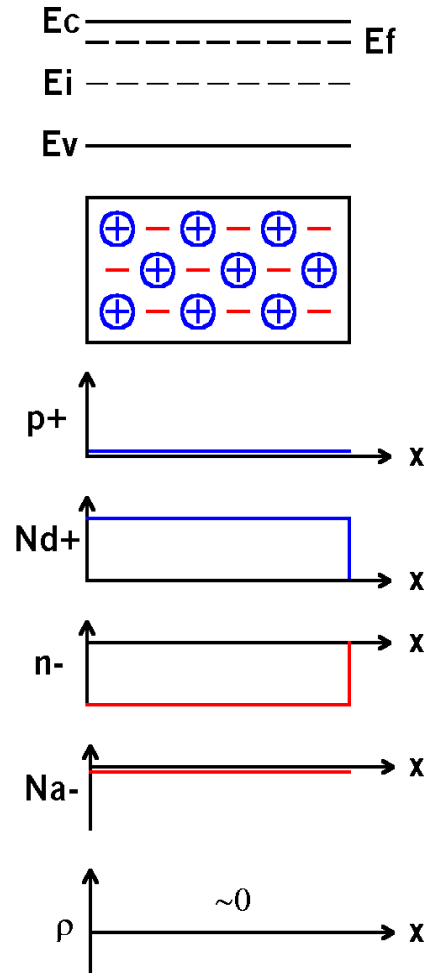
P-N Junction at the equilibrium: charge distribution

Time < 0

P-type piece

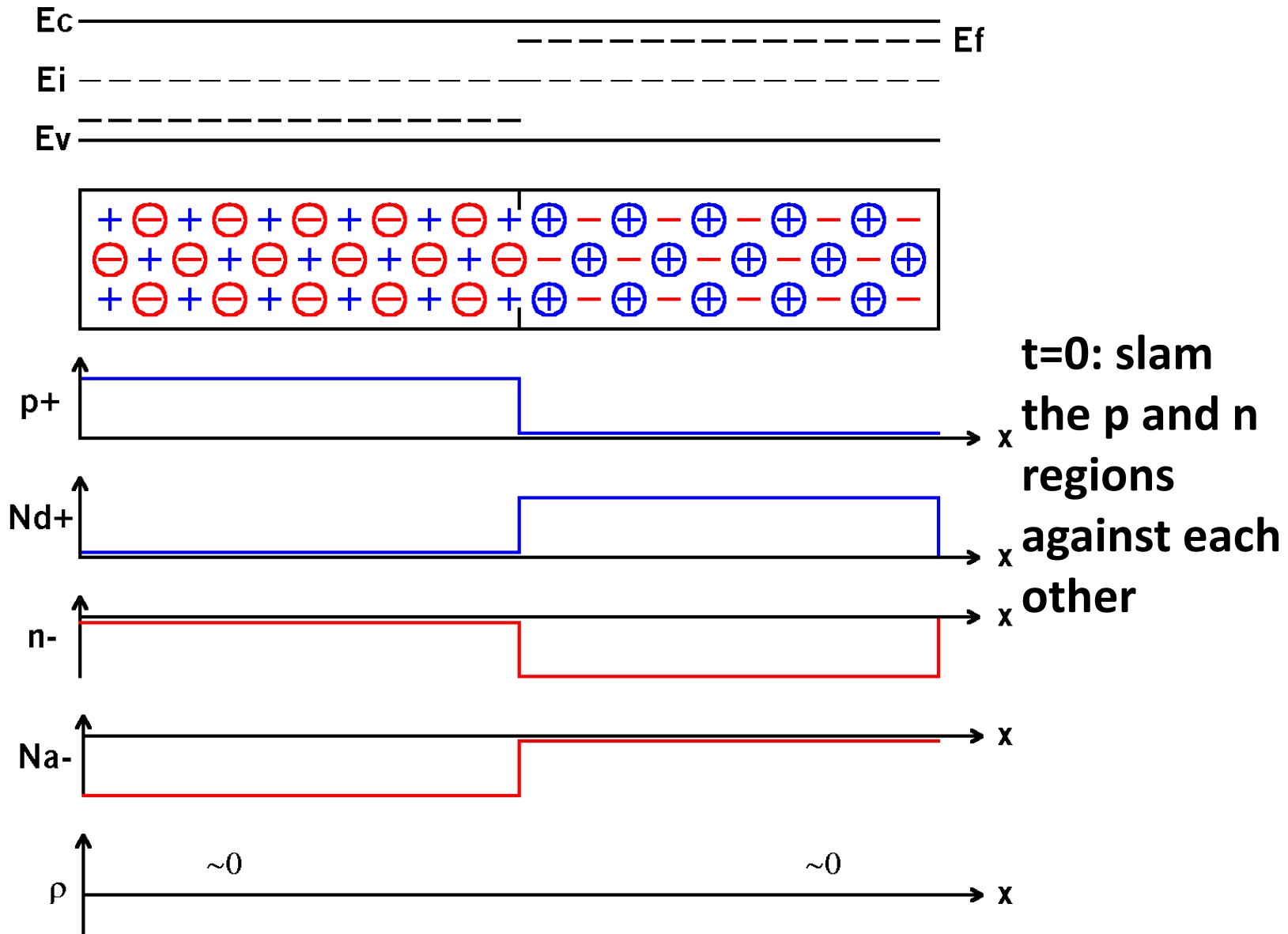


N-type piece



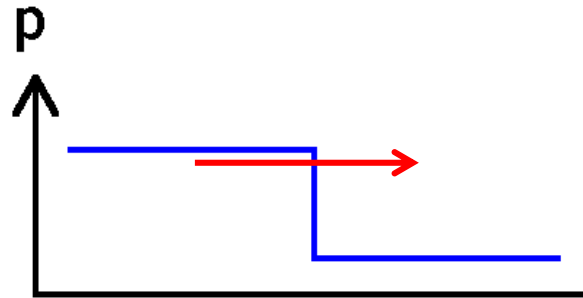
**Time < 0:
p and n
regions
separated**

P-N Junction at the equilibrium: charge distribution



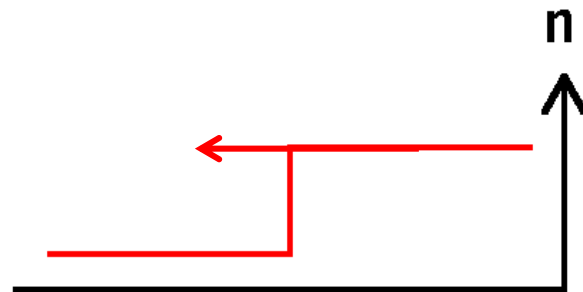
$t > 0$ Gradients drive diffusion

Hole gradient



$$J_{p, \text{diffusion}} = -qD_p \frac{dp}{dx} = \text{current right, holes right}$$

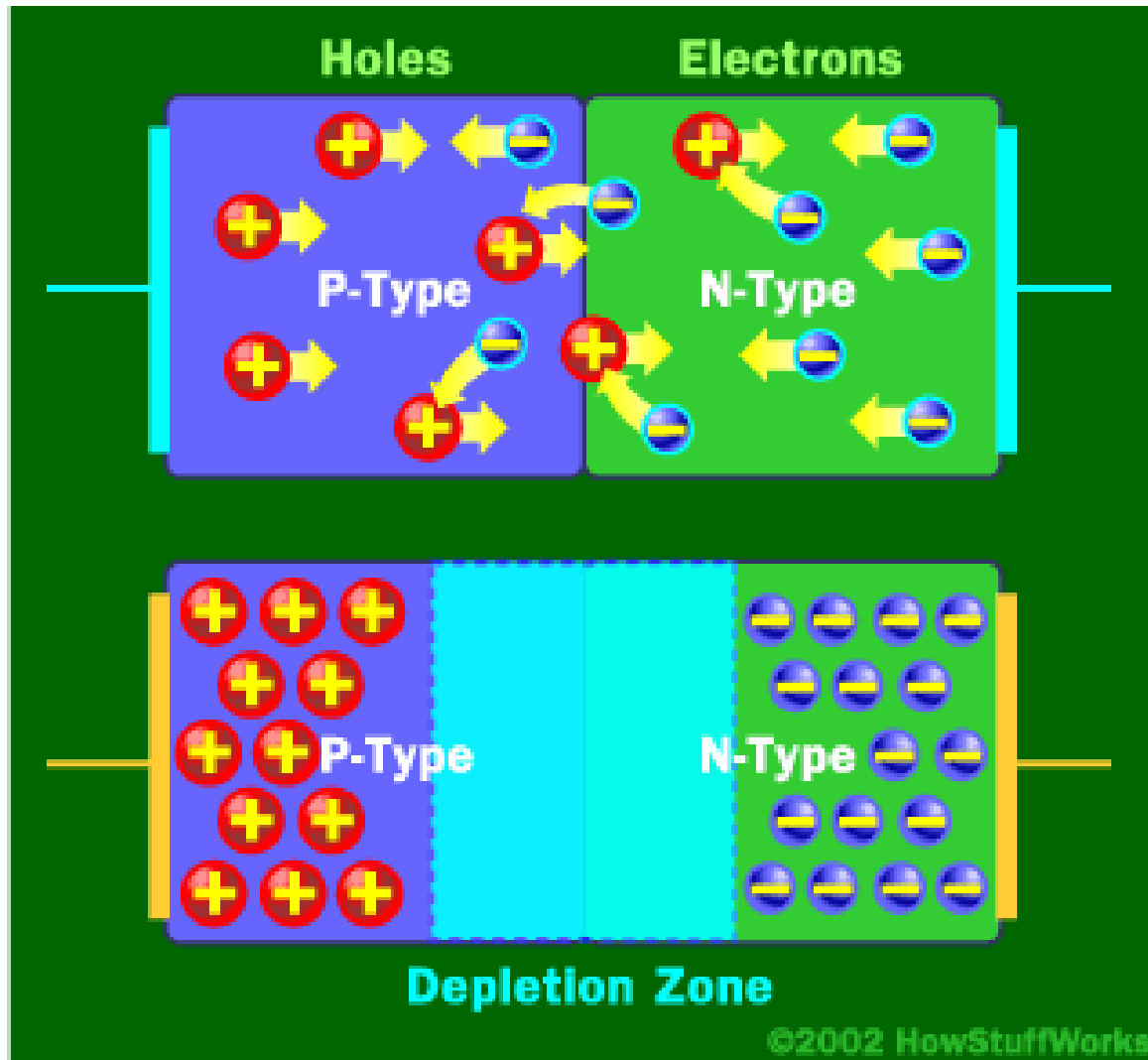
Electron gradient



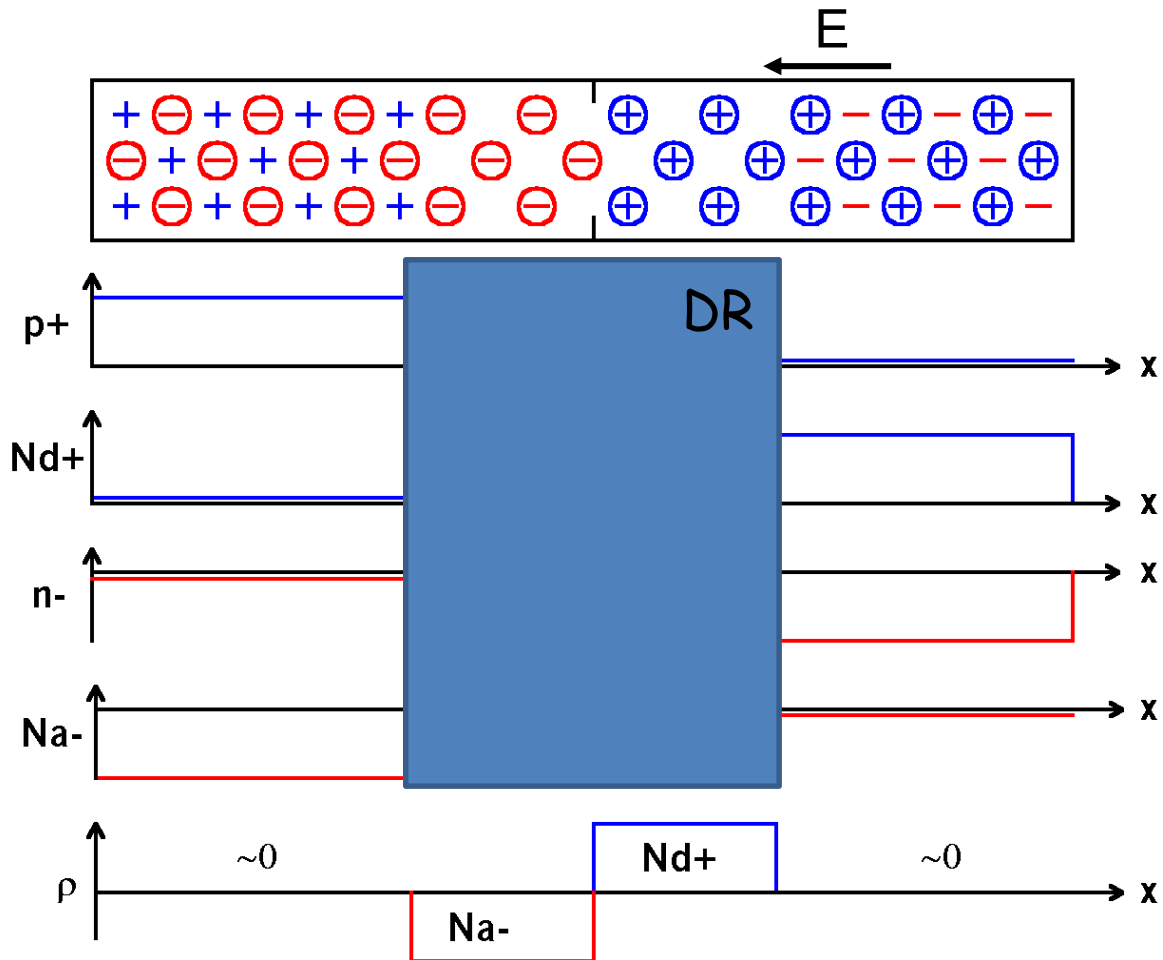
$$J_{n, \text{diffusion}} = -qD_n \frac{dn}{dx} = \text{current right, electrons left}$$

$t > 0$ Gradients drive diffusion

Carriers cross the metallurgical junction and recombine



P-N junction at the equilibrium: charge distribution and transport

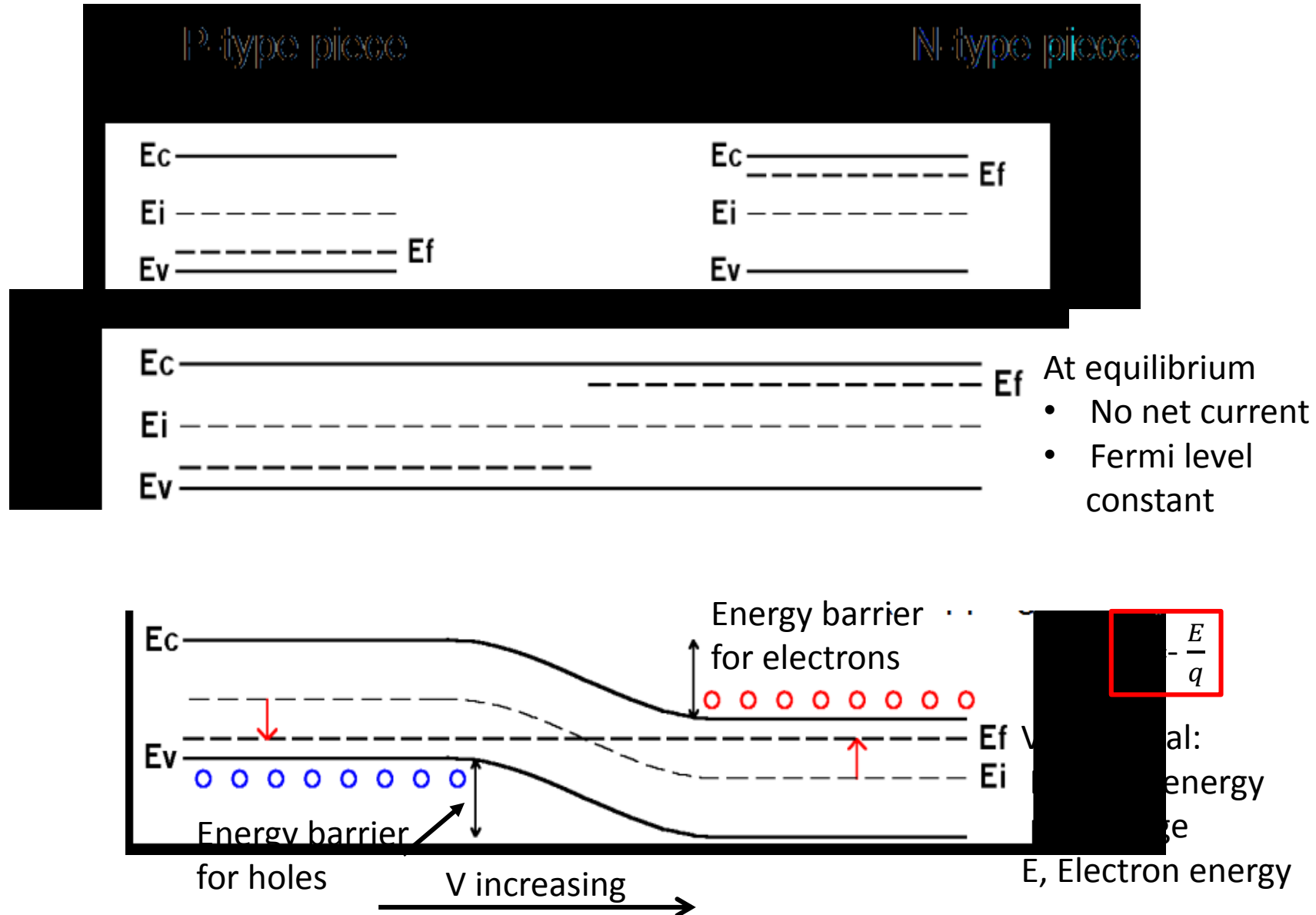


❖ Current Mechanisms

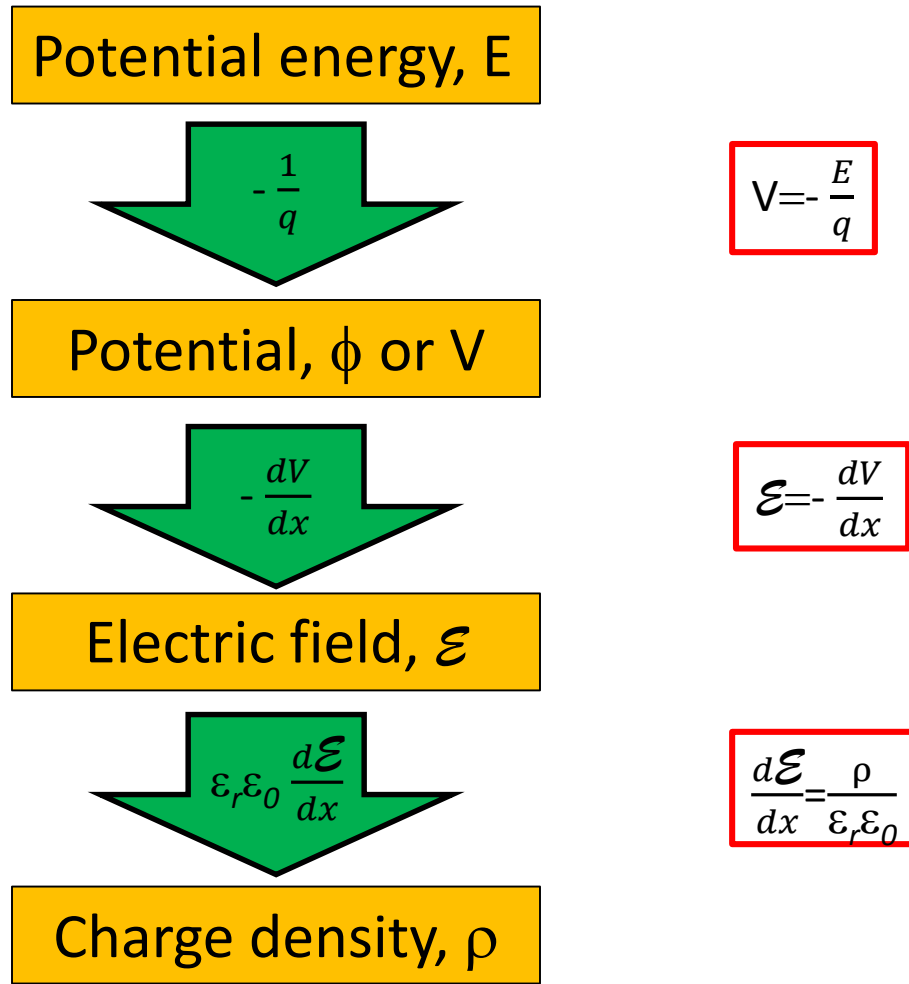
- ❖ Diffusion of the carriers cause an electric field in DR.
- ❖ Drift current is due to the presence of electric field in DR.
- ❖ Diffusion current is due to the majority carriers.
- ❖ Drift current is due to the minority carriers.

DR: Depletion Region

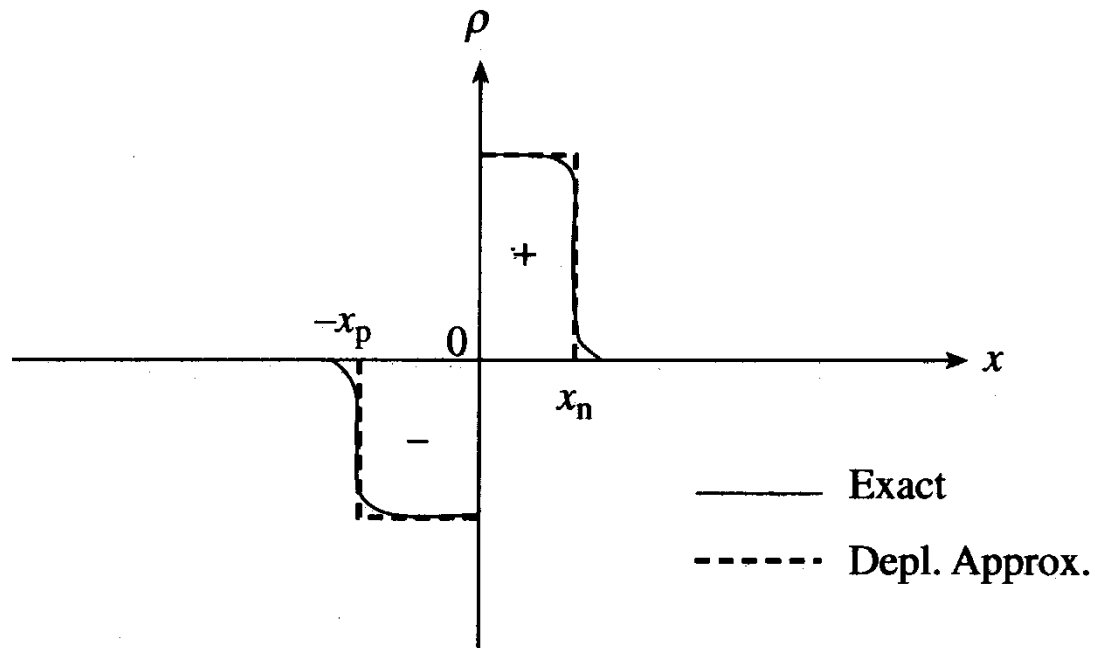
Band diagram of a p-n junction at equilibrium



Energy, potential, electric field and charge density (in 1D)



The depletion approximation



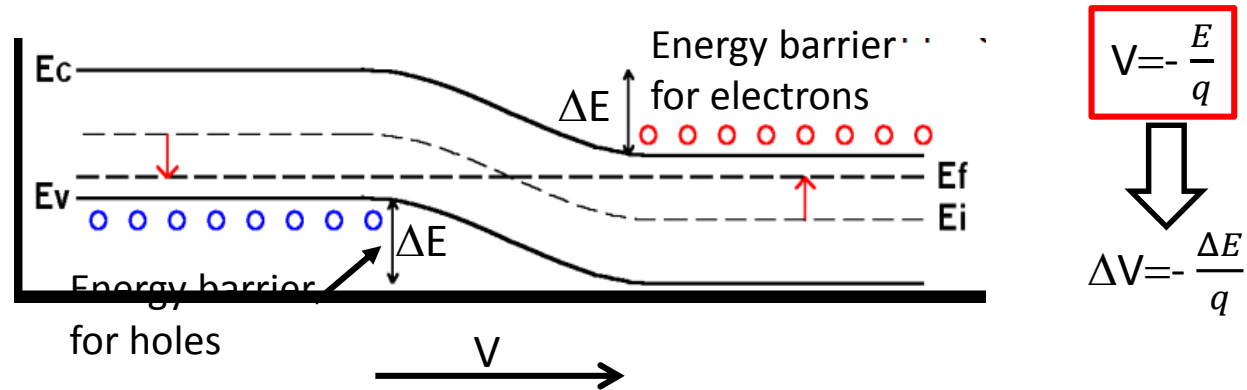
Depletion Region Approximation states that approximately no free carriers exist in the space charge region and no net charge exists outside of the depletion region (known as the quasi-neutral region). Thus,

$$\frac{d\mathcal{E}}{dx} = \frac{\rho}{\epsilon_r \epsilon_o} = \frac{q}{K_S \epsilon_o} (p - n + N_D - N_A) = 0 \quad \text{within the quasi-neutral region}$$

becomes...

$$\frac{dE}{dx} = \frac{q}{\epsilon_r \epsilon_o} (N_D - N_A) \quad \text{within the space charge region}$$

The built-in potential barrier



In a p-n junction in thermal equilibrium a voltage drop is established to keep the net current 0 and the Fermi level constant. This voltage drop occurs across the depletion region and it is indicated as the built-in potential or V_{bi} .

$$qV_{bi} = (E_i - E_F)_{Left} + (E_F - E_i)_{Right}$$

$$p \approx N_a$$

$$N_a = n_i e^{(E_i - E_F)/kT}$$

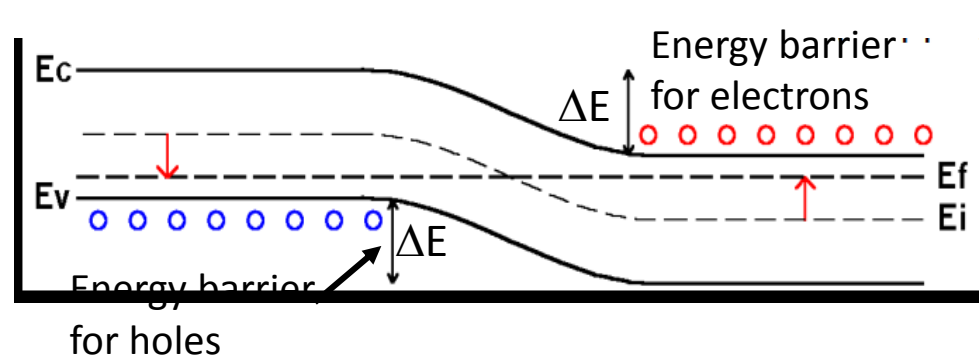
$$(E_i - E_F)_{Left} = kT \ln \left(\frac{N_a}{n_i} \right)$$

$$n \approx N_d$$

$$N_d = n_i e^{(E_F - E_i)/kT}$$

$$(E_F - E_i)_{Right} = kT \ln \left(\frac{N_d}{n_i} \right)$$

The built-in potential barrier



$$V = -\frac{E}{q}$$

$$\Delta V = -\frac{\Delta E}{q}$$

In a p-n junction in thermal equilibrium a voltage drop is established to keep the net current 0 and the Fermi level constant. This voltage drop occurs across the depletion region and it is indicated as the built-in potential or V_{bi} .

$$V_{bi} = \frac{kT}{q} \ln\left(\frac{N_a}{n_i}\right) + \frac{kT}{q} \ln\left(\frac{N_d}{n_i}\right)$$

For $N_A = N_D = 10^{15}/\text{cm}^3$ in silicon at room temperature,
 $V_{bi} \sim 0.6 \text{ V}$

For a non-degenerate semiconductor, $|-qV_{bi}| < |E_g|$

$$\Rightarrow V_{bi} = \frac{kT}{q} \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

Note: N_a acceptor level on the p side;
 N_d donor level on the n side

The built-in potential barrier

- One side very heavily doped so that Fermi level is at band edge.
- e.g. p⁺-n junction (heavy B implant into lightly doped substrate)

$$(E_i - E_F)_{Left} \approx E_i - E_V = E_G / 2$$

$$(E_F - E_i)_{Right} = kT \ln \left(\frac{N_D}{n_i} \right)$$

$$\Rightarrow V_{bi} = \frac{E_G}{2q} + \frac{kT}{q} \ln \left(\frac{N_d}{n_i} \right)$$

The built-in electric field

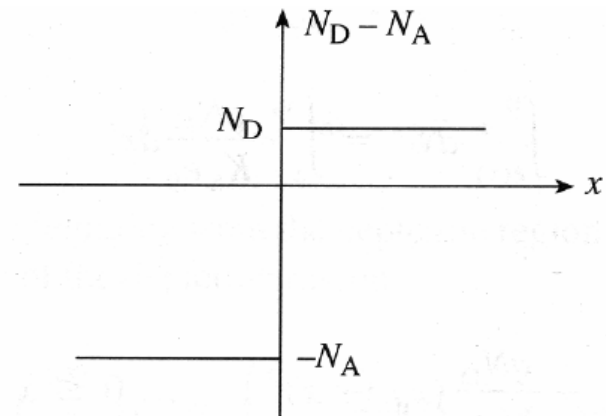
$$\frac{d\mathcal{E}}{dx} = \frac{\rho}{\epsilon_r \epsilon_0}$$

Depletion Region Approximation: Step Junction Solution

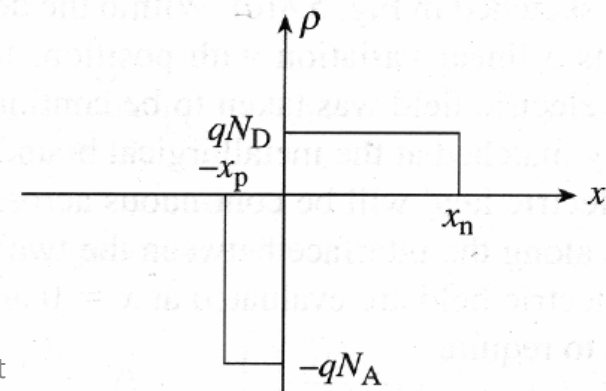
$$\rho = \begin{cases} -qN_A & \text{for } -x_p \leq x \leq 0 \\ qN_D & \text{for } 0 \leq x \leq x_n \\ 0 & \text{for } x \leq -x_p \text{ and } x \geq x_n \end{cases}$$

thus,

$$\frac{dE}{dx} = \begin{cases} \frac{-qN_A}{\epsilon_r \epsilon_0} & \text{for } -x_p \leq x \leq 0 \\ \frac{qN_D}{\epsilon_r \epsilon_0} & \text{for } 0 \leq x \leq x_n \\ 0 & \text{for } x \leq -x_p \text{ and } x \geq x_n \end{cases}$$



(a)



The built-in electric field

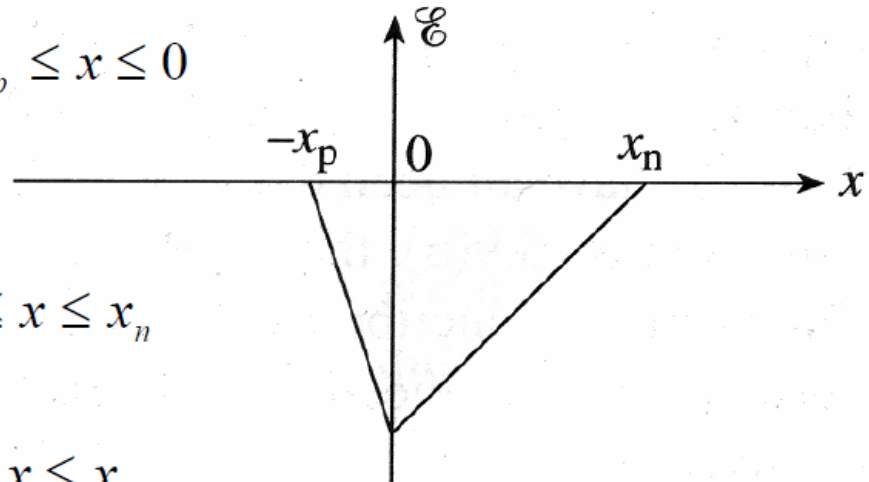
$$\int_0^{\mathcal{E}(x)} d\mathcal{E}' = \int_{-x_p}^x \frac{-qN_A}{\epsilon_r \epsilon_0} dx' \quad \text{for } -x_p \leq x \leq 0$$

$$\mathcal{E}'(x) = -\frac{qN_A}{\epsilon_r \epsilon_0} (x + x_p) \quad \text{for } -x_p \leq x \leq 0$$

and

$$\int_{\mathcal{E}(x)}^0 d\mathcal{E}' = \int_x^{x_n} \frac{qN_D}{\epsilon_r \epsilon_0} dx' \quad \text{for } 0 \leq x \leq x_n$$

$$\mathcal{E}(x) = -\frac{qN_D}{K_S \epsilon_0} (x_n - x) \quad \text{for } 0 \leq x \leq x_n$$



Since $E(x=0^-) = E(x=0^+)$

$$N_A x_p = N_D x_n$$

The built-in potential

$$\mathcal{E} = - \frac{dV}{dx}$$

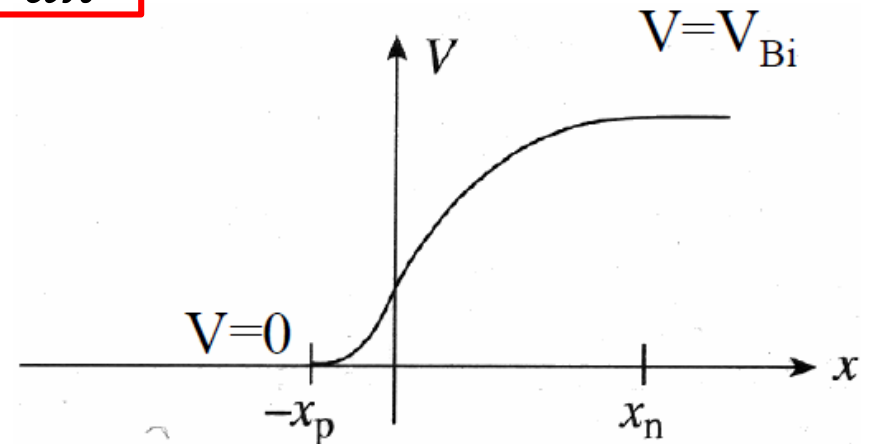
$$\frac{dV}{dx} = \begin{cases} \frac{qN_A}{K_S \epsilon_o} (x_p + x) & \text{for } -x_p \leq x \leq 0 \\ \frac{qN_D}{K_S \epsilon_o} (x_n - x) & \text{for } 0 \leq x \leq x_n \end{cases}$$

or,

$$\int_0^{V(x)} dV' = \int_{-x_p}^x \frac{qN_A}{K_S \epsilon_o} (x_p + x') dx' \quad \text{for } -x_p \leq x \leq 0$$

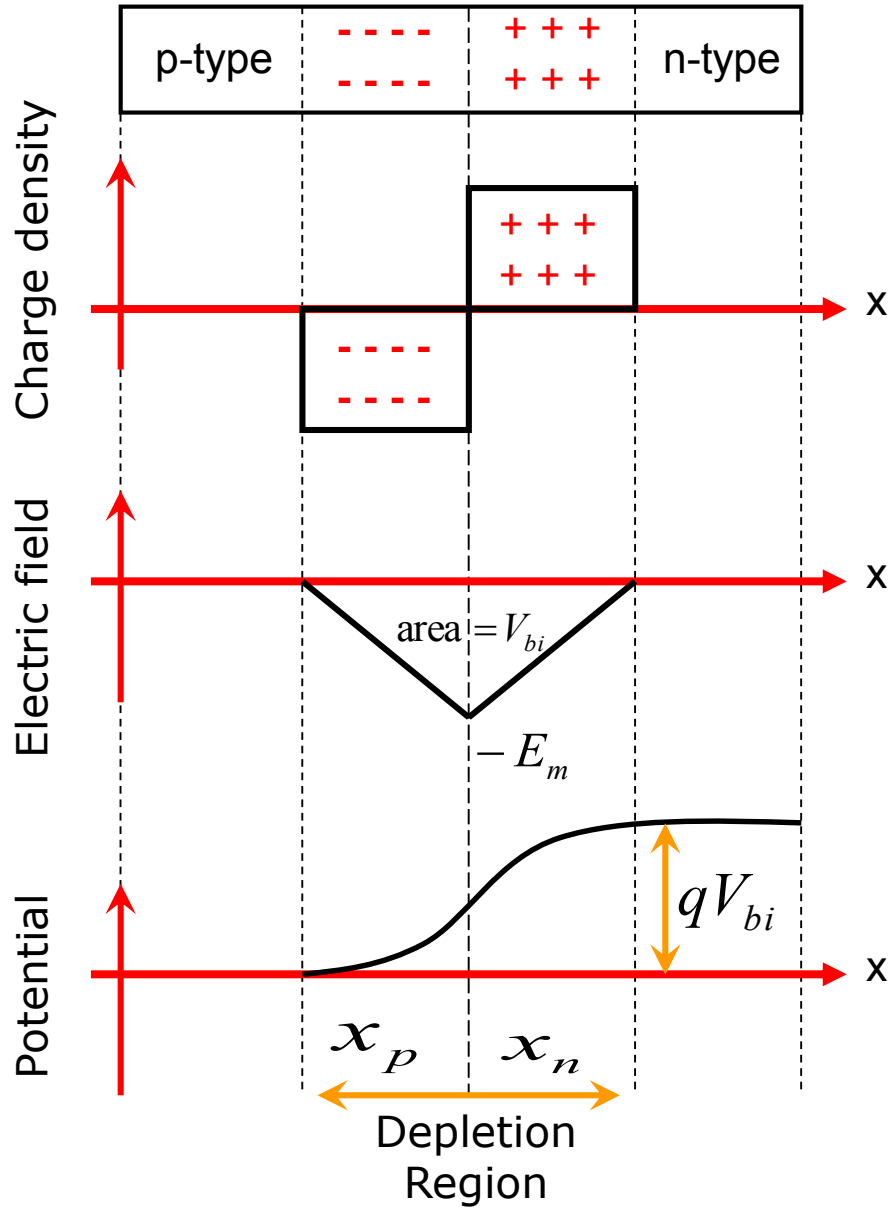
$$\int_{V(x)}^{V_{Bi}} dV' = \int_x^{x_n} \frac{qN_D}{K_S \epsilon_o} (x_n - x') dx' \quad \text{for } 0 \leq x \leq x_n$$

$$V(x) = \begin{cases} \frac{qN_A}{2K_S \epsilon_o} (x_p + x)^2 & \text{for } -x_p \leq x \leq 0 \\ V_{bi} - \frac{qN_D}{2K_S \epsilon_o} (x_n - x)^2 & \text{for } 0 \leq x \leq x_n \end{cases}$$



Note: K_S is the same as ϵ_r in the previous slides

pn junction in equilibrium



Depletion region

Objective: Calculate the width of the depletion region at equilibrium

$$W = x_p + x_n$$

$$V(x) = \begin{cases} \frac{qN_A}{2K_S\epsilon_o}(x_p + x)^2 & \text{for } -x_p \leq x \leq 0 \\ V_{bi} - \frac{qN_D}{2K_S\epsilon_o}(x_n - x)^2 & \text{for } 0 \leq x \leq x_n \end{cases}$$

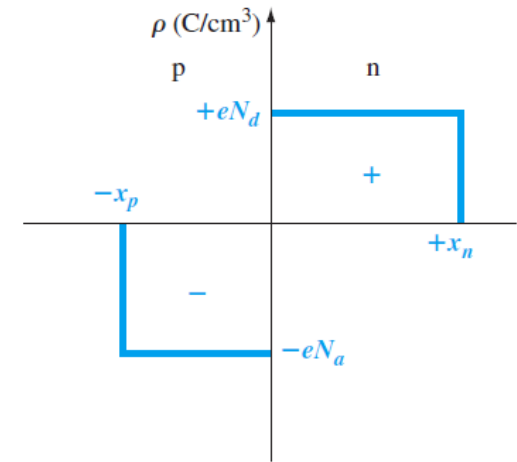
At $x=0$,

$$\frac{qN_A}{2K_S\epsilon_o}(x_p)^2 = V_{bi} - \frac{qN_D}{2K_S\epsilon_o}(x_n)^2$$

$$\text{Using, } x_p = \frac{(x_n N_D)}{N_A}$$

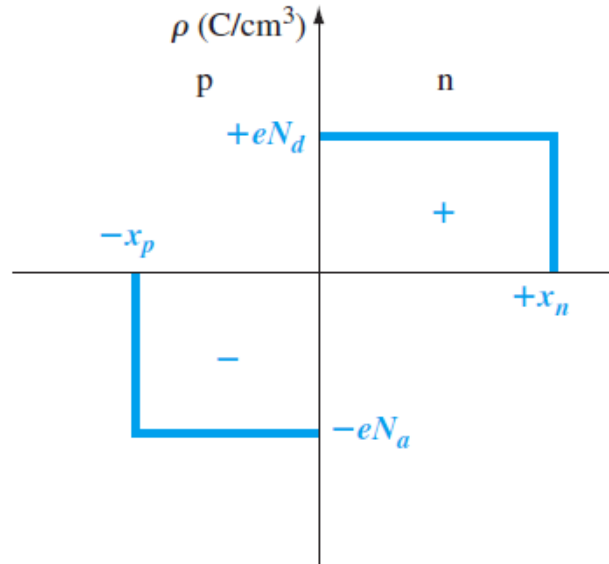
$$x_n = \sqrt{\frac{2K_S\epsilon_o}{q} \frac{N_A}{N_D(N_A + N_D)} V_{bi}} \quad \text{and} \quad x_p = \sqrt{\frac{2K_S\epsilon_o}{q} \frac{N_D}{N_A(N_A + N_D)} V_{bi}}$$

$$W = x_p + x_n = \sqrt{\frac{2K_S\epsilon_o}{q} \frac{(N_A + N_D)}{N_A N_D} V_{bi}}$$



Note: K_S is the same as ϵ_r in the previous slides

pn junction in equilibrium: width of the depletion region



$$x_p = \left\{ \frac{2\epsilon_s V_{bi}}{e} \left[\frac{N_d}{N_a} \right] \left[\frac{1}{N_a + N_d} \right] \right\}^{1/2}$$

$$x_n = \left\{ \frac{2\epsilon_s V_{bi}}{e} \left[\frac{N_a}{N_d} \right] \left[\frac{1}{N_a + N_d} \right] \right\}^{1/2}$$

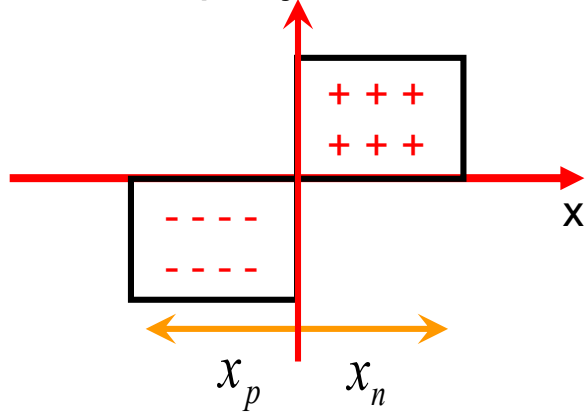
$$W = \left\{ \frac{2\epsilon_s V_{bi}}{e} \left[\frac{N_a + N_d}{N_a N_d} \right] \right\}^{1/2}$$

ECE 322L-Handout

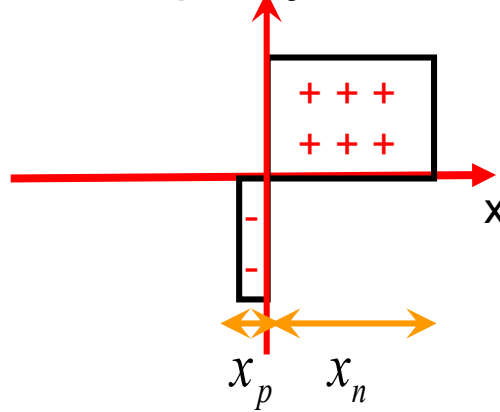
Note: $\epsilon_s = \epsilon_0 \epsilon_r$

One-sided junctions

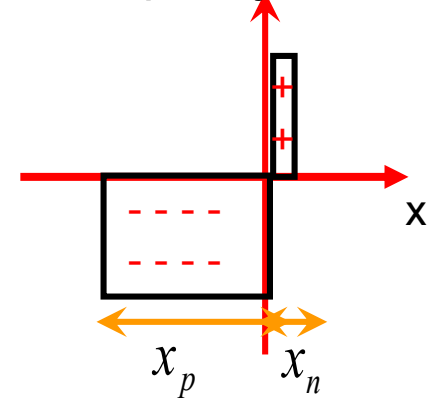
p-n junction



p⁺-n junction



p-n⁺ junction



$$N_a x_p = N_d x_n \quad W = x_p + x_n = \sqrt{\frac{2\epsilon_s V_{bi}}{q} \left(\frac{1}{N_a} + \frac{1}{N_d} \right)}$$

If $N_a \gg N_d$, as in a P⁺N junction,

$$|x_p| = |x_n| N_d / N_a \cong 0$$

$$W = \sqrt{\frac{2\epsilon_s V_{bi}}{q N_d}} \approx |x_n|$$

If $N_d \gg N_a$, as in a P⁺N junction,

$$|x_n| = |x_p| N_a / N_d \cong 0$$

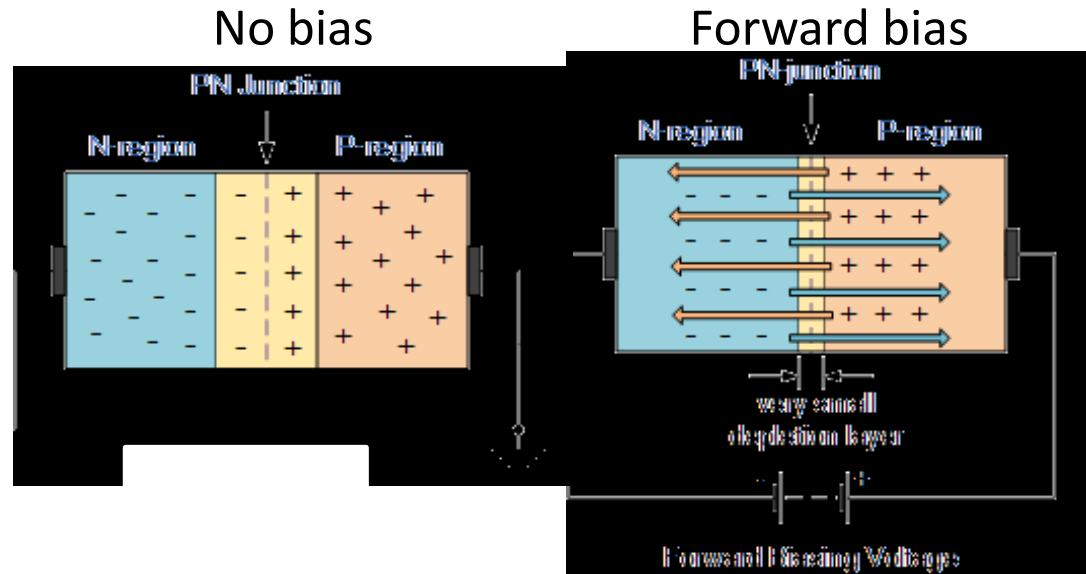
$$W = \sqrt{\frac{2\epsilon_s V_{bi}}{q N_a}} \approx |x_p|$$

P-n junction in equilibrium: summary

- Net current across the junction is zero as drift and diffusion current balance each other out
- Fermi level is constant with respect to x
- Bands bend across the junction
- A built-in electric field and a built-in potential are established
- The built-in potential is a barrier for majority carriers
- The width of the depletion region, the built-in potential, and the built-in electric field are determined by the doping level on the p- and n side of the junction
- The width of the depletion region is also related to the built-in potential

pn junction in non equilibrium

pn junction under forward bias



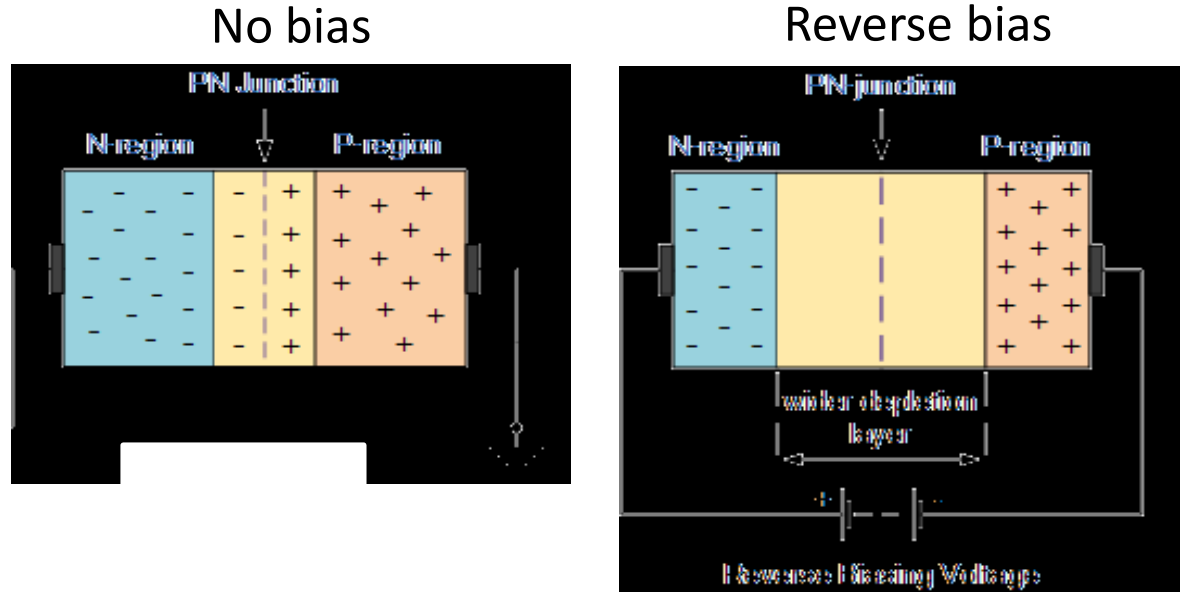
Negative voltage to n side positive to p side

- More electrons supplied to n-side, more holes supplied to the p-side
- Depletion region gets smaller

$$W = \left[\frac{2K_S \epsilon_0}{q} \frac{(N_A + N_D)}{N_D N_A} (V_{bi} - V_{fwd}) \right]^{1/2}$$

pn junction in non equilibrium

pn junction under reverse bias



Positive voltage to n side negative to p side

- Holes are attracted from the p-side towards the contact
- Electrons are attracted from the n side towards the contact
- The depletion region will be larger

$$W = \left[\frac{2K_S \epsilon_0}{q} \frac{(N_A + N_D)}{N_D N_A} (V_{bi} + V_{rev}) \right]^{1/2}$$

pn junction in non equilibrium

General equation for the depletion width

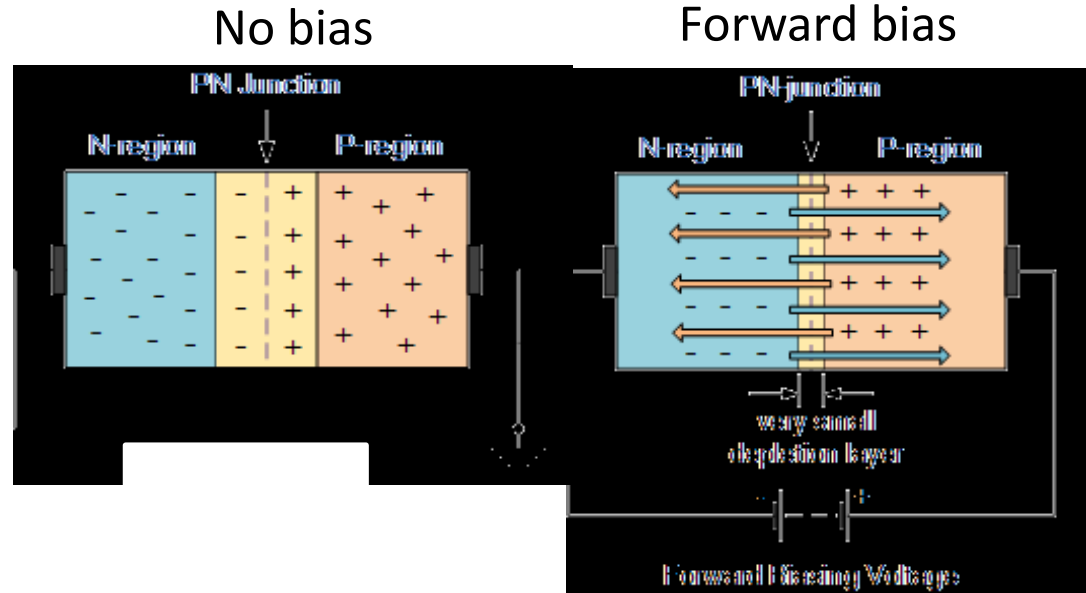
Convention: $V_{appl} = +$ for forward bias
 $V_{appl} = -$ for reverse bias

$$W = \left[\frac{2K_S \epsilon_0}{q} \frac{(N_A + N_D)}{N_D N_A} (V_{bi} - V_{appl}) \right]^{1/2}$$

$$x_n = W(V_{appl}) \left(\frac{N_A}{N_A + N_D} \right) \quad x_p = W(V_{appl}) \left(\frac{N_D}{N_A + N_D} \right)$$

pn junction in non equilibrium

pn junction under forward bias

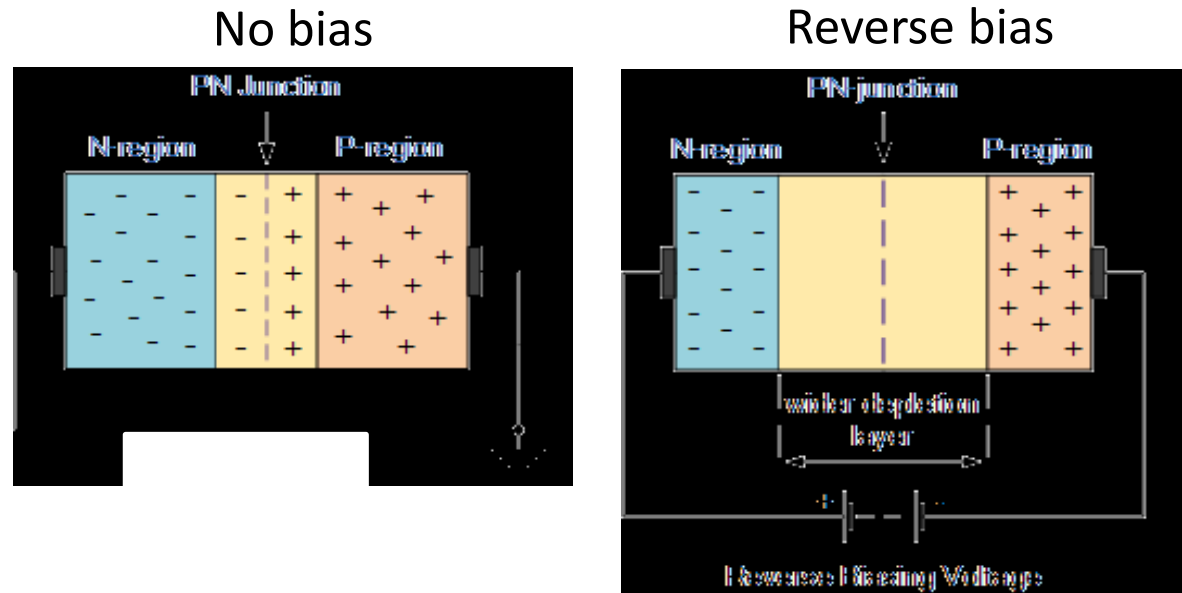


Negative voltage to n side positive to p side

- Band-bending decreases due to the applied voltage
- A current flows across the junction resulting in the Fermi level being not constant

pn junction in non equilibrium

pn junction under reverse bias

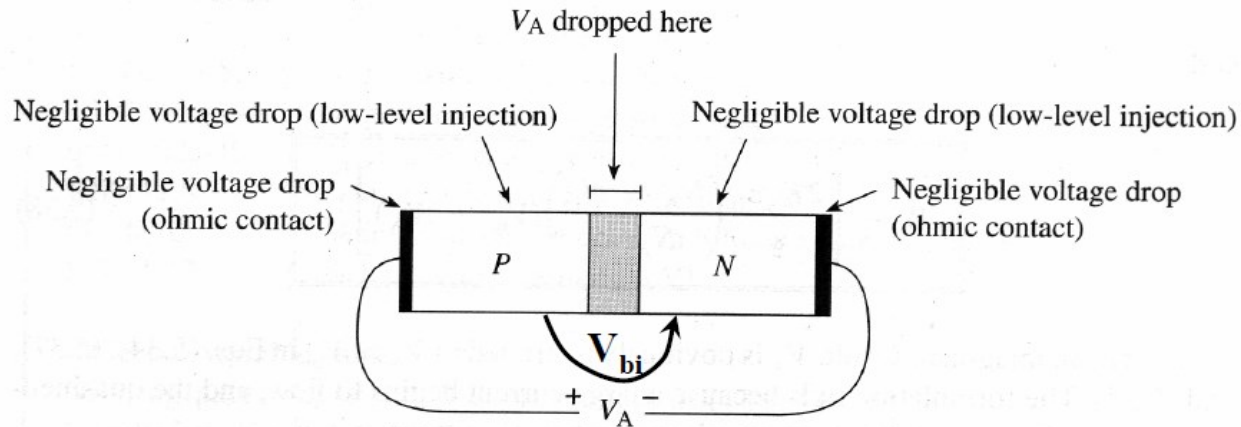


Positive voltage to n side negative to p side

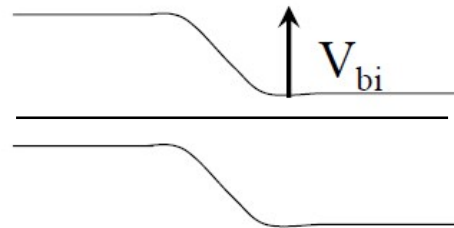
- Band-bending increases due to the applied voltage
- A current flows across the junction resulting in the Fermi level being not constant

pn junction in non equilibrium

Band bending

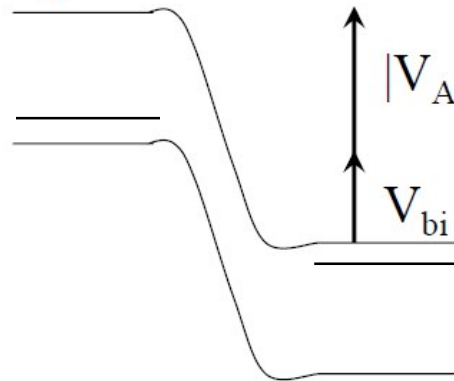


$V_A = 0$: No Bias



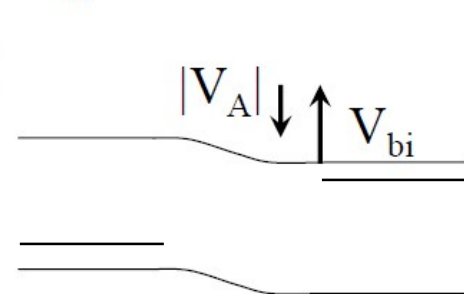
Fermi level constant

$V_A < 0$: Reverse Bias



Two QFLs

$V_A > 0$: Forward Bias

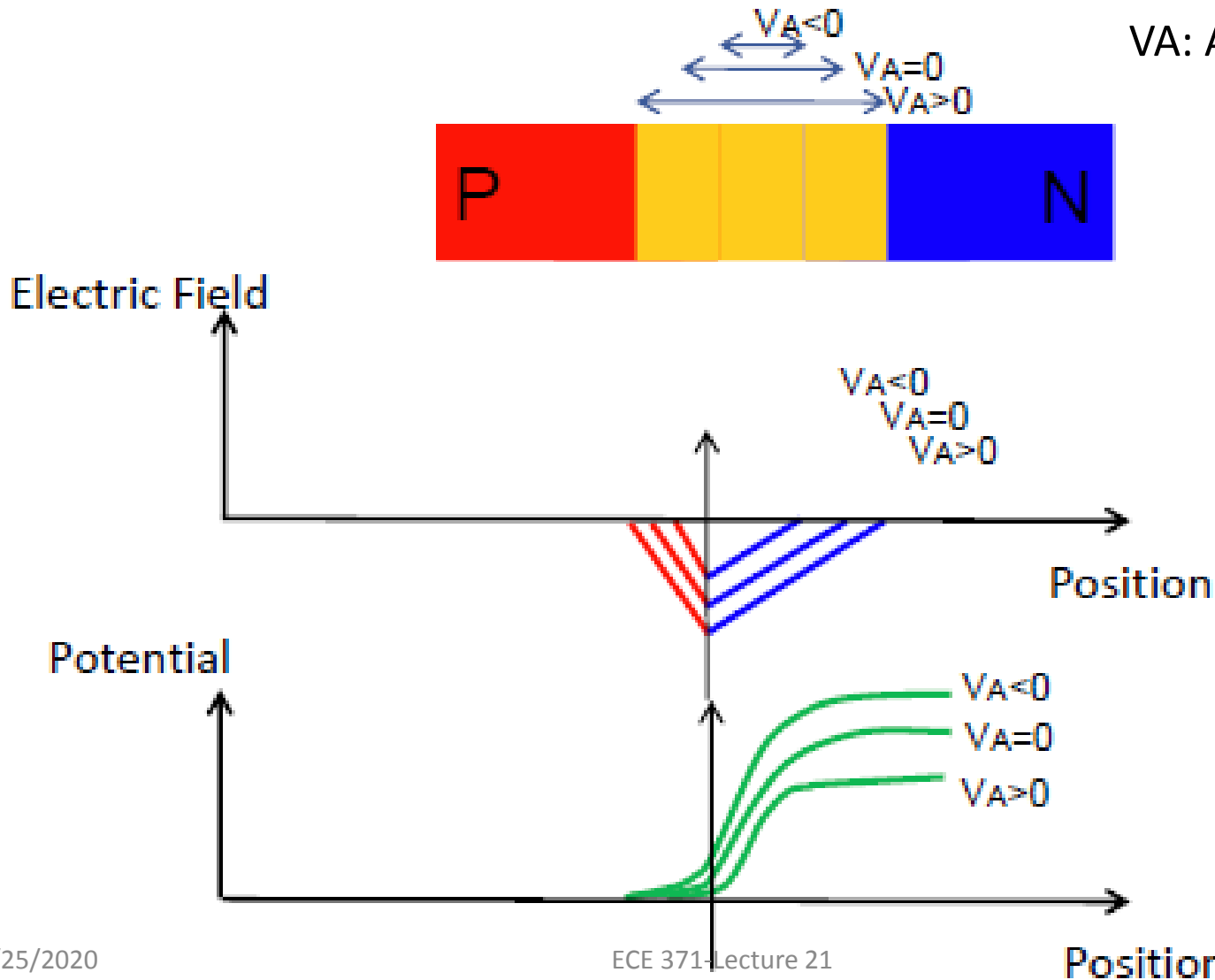


Two QFLs

pn junction in non equilibrium

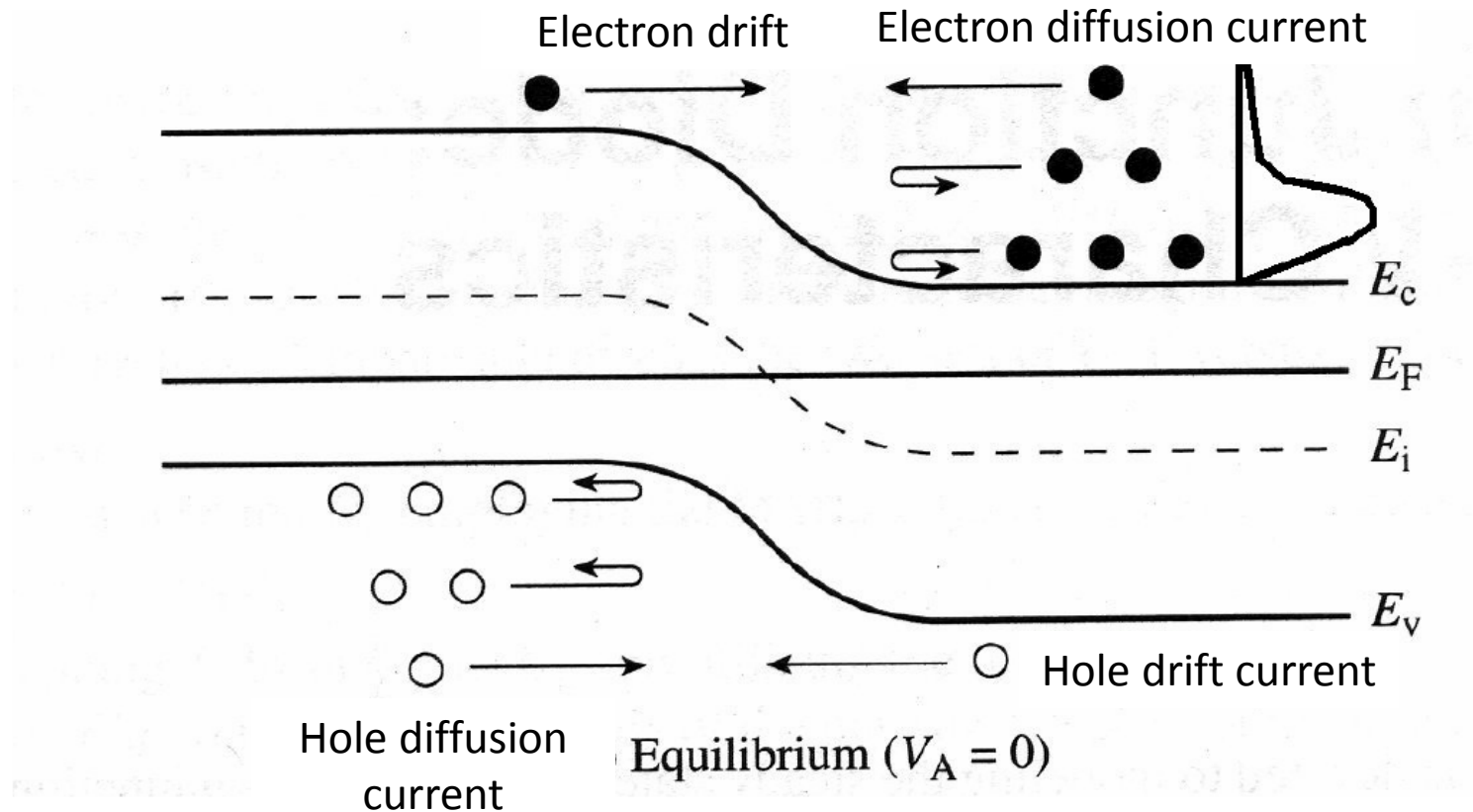
Electric field and potential

V_A : Applied voltage

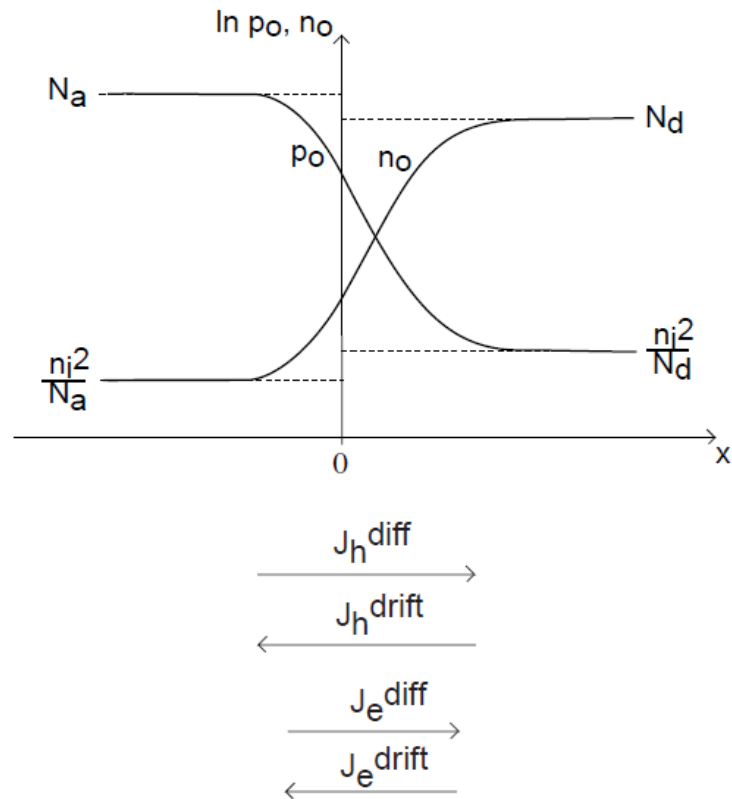


Current through a pn junction

In Equilibrium, the Total current balances due to the sum of the individual components



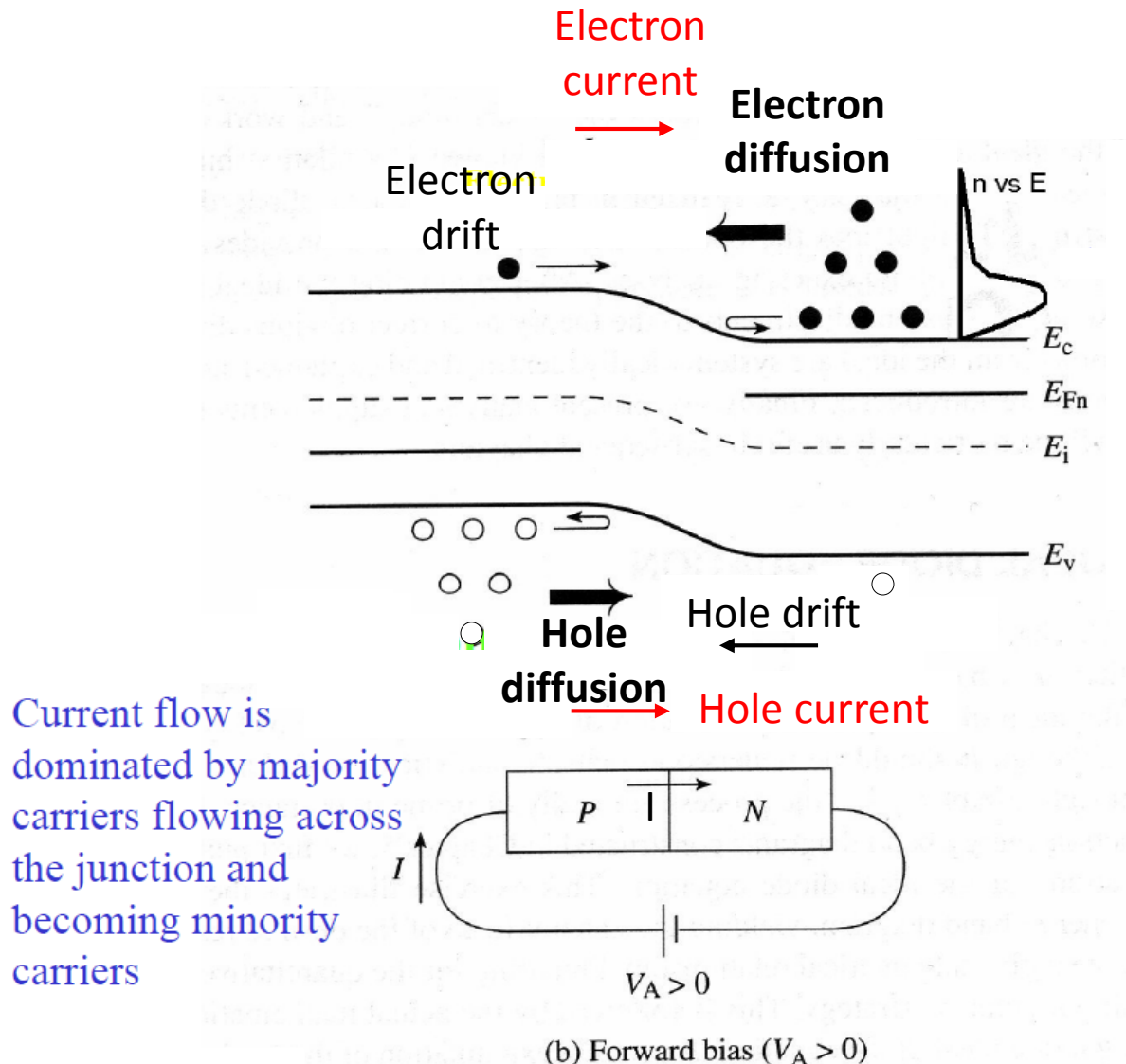
Carrier profiles at the equilibrium



In equilibrium: dynamic balance between drift and diffusion for electrons and holes inside SCR.

$$|J_{\text{drift}}| = |J_{\text{diff}}|$$

Current through a pn junction



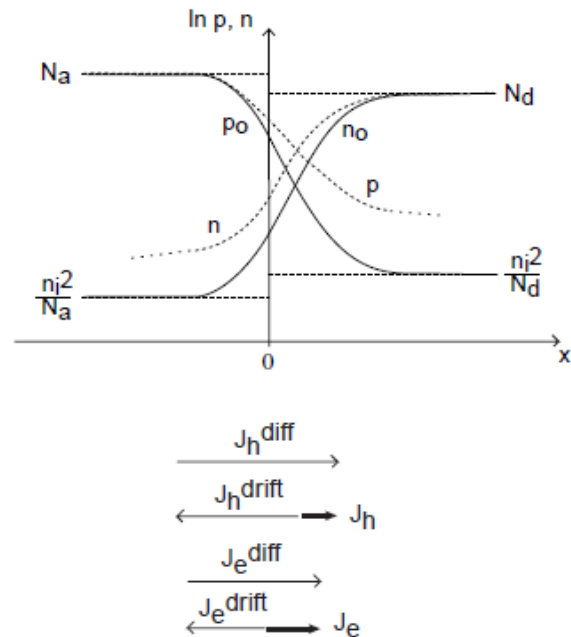
Current flow is proportional to $e^{(V_A/V_{ref})}$ due to the exponential decay of carriers into the majority carrier bands

Current through a pn junction

Consider holes flowing into the n-region. They will flow through the depletion region with small losses due to recombination, as the electron concentration is small compared with the bulk. When holes reach the n-side boundary of the depletion region the concentration of electrons available for recombination increases and the concentration of holes will decrease with distance, depending on the cross-section for recombination. Ultimately, all holes will have recombined with electrons. The required electrons are furnished through the external contact from the power supply. On the p-side, electrons undergo a similar process. The holes required to sustain recombination are formed at the external contact to the p-region by electron flow toward the power supply, equal to the electron flow toward the n-contact.

Carrier profiles under forward bias

For $V > 0$, $\phi_B - V \downarrow \Rightarrow |E_{SCR}| \downarrow \Rightarrow |J_{drift}| \downarrow$

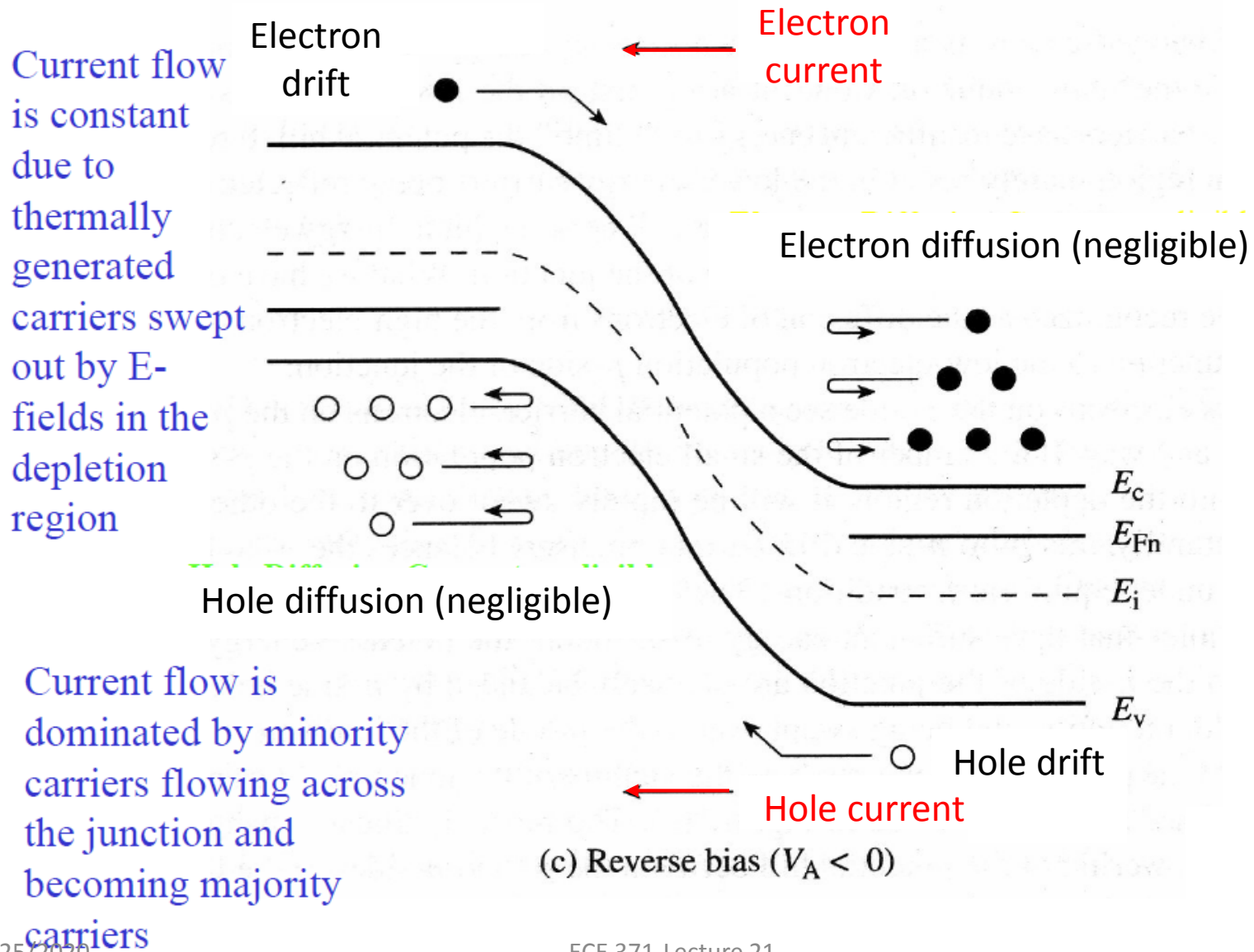


Current balance in SCR broken:

$$|J_{drift}| < |J_{diff}|$$

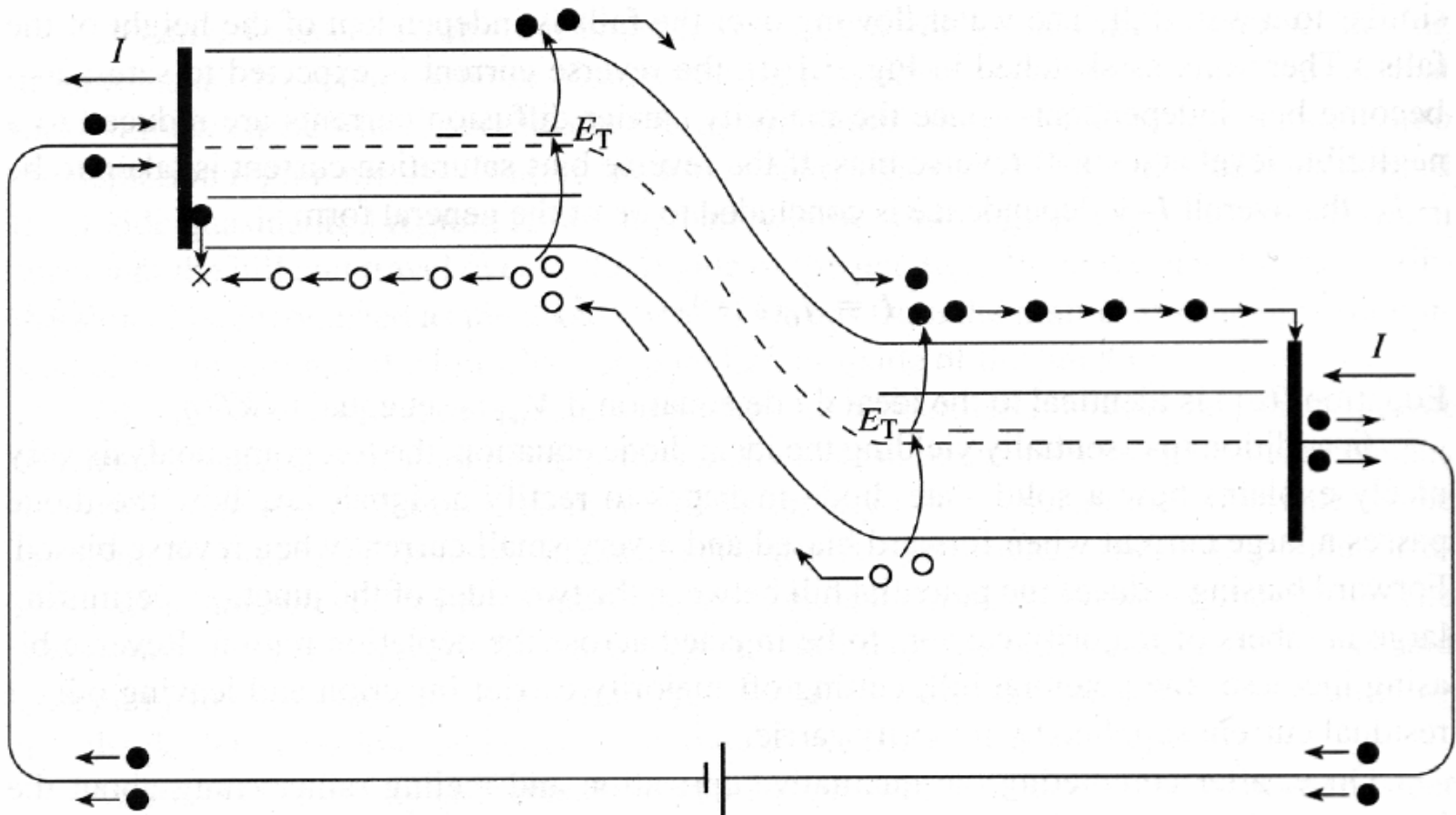
Net diffusion current in SCR \Rightarrow minority carrier **injection** into QNRs.

Current through a pn junction



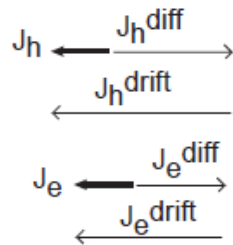
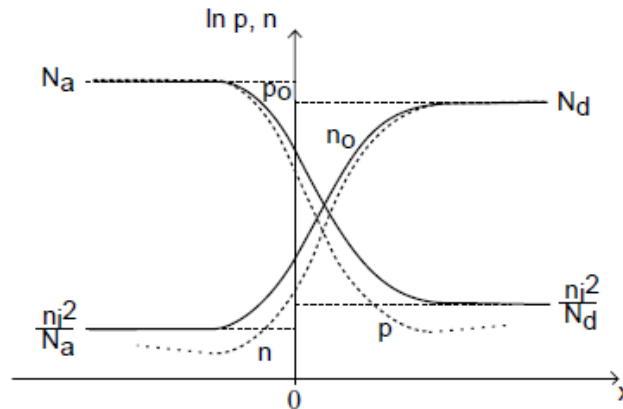
Current through a pn junction

Where does the reverse bias current come from? Generation near the depletion region edges “replenishes” the current source.



Carrier profiles under reverse bias

For $V < 0$, $\phi_B - V \uparrow \Rightarrow |E_{SCR}| \uparrow \Rightarrow |J_{drift}| \uparrow$

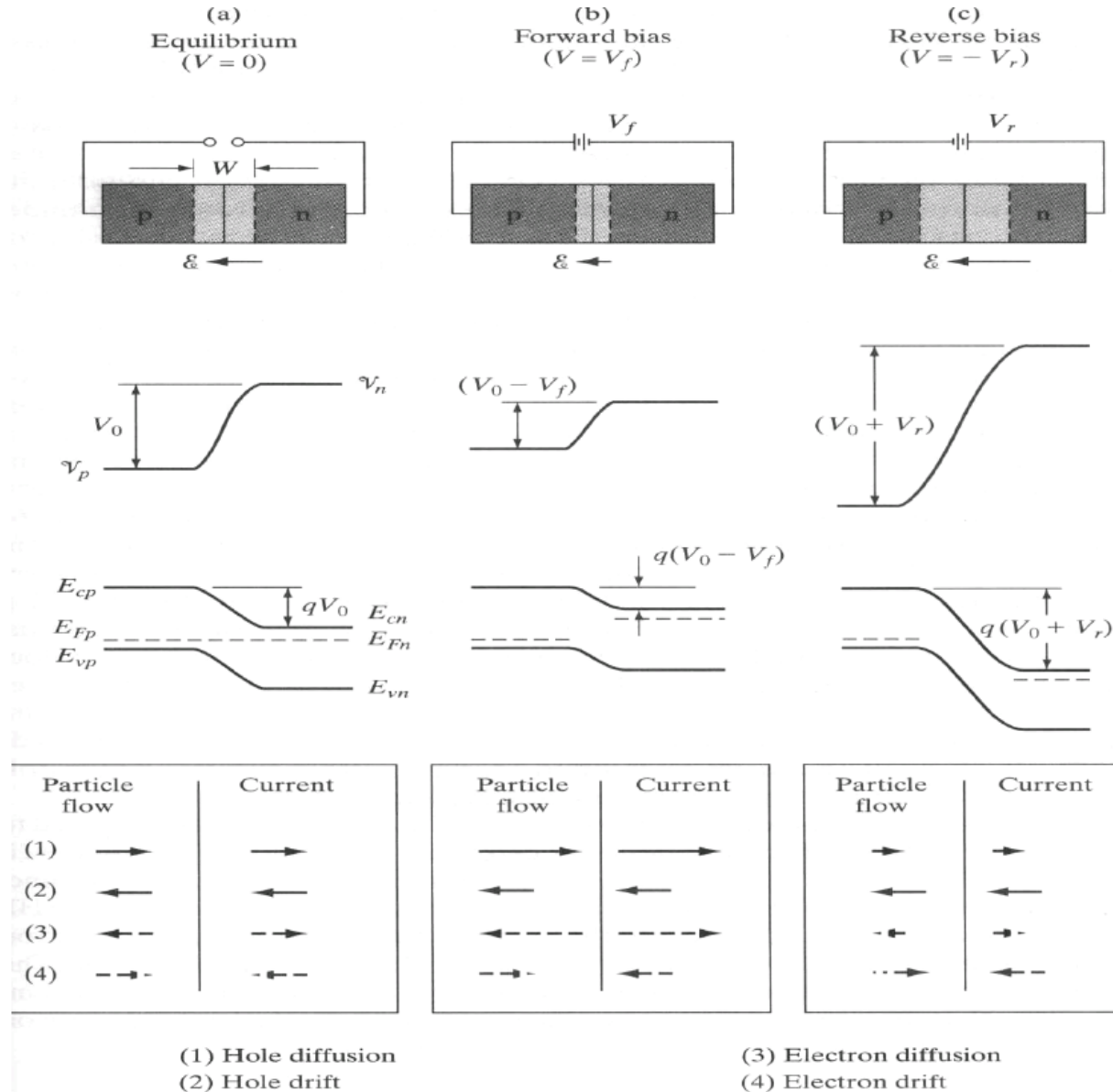


Current balance in SCR broken:

$$|J_{drift}| > |J_{diff}|$$

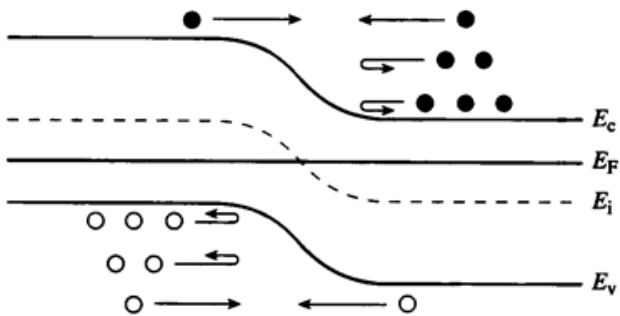
Net drift current in SCR \Rightarrow minority carrier **extraction** from QNRs.

Carrier flow and currents: summary

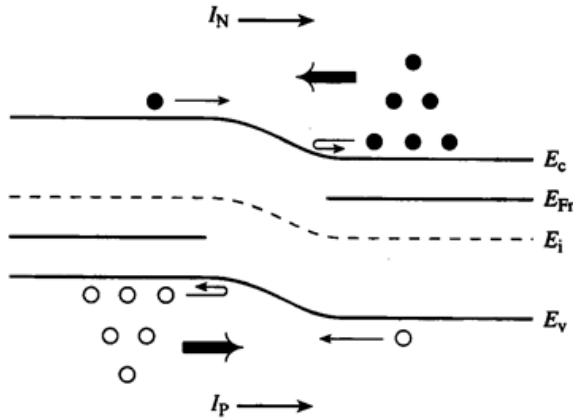


Current Flow (Qualitative View)

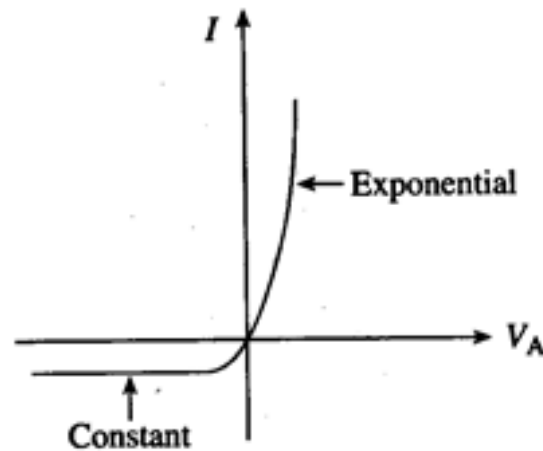
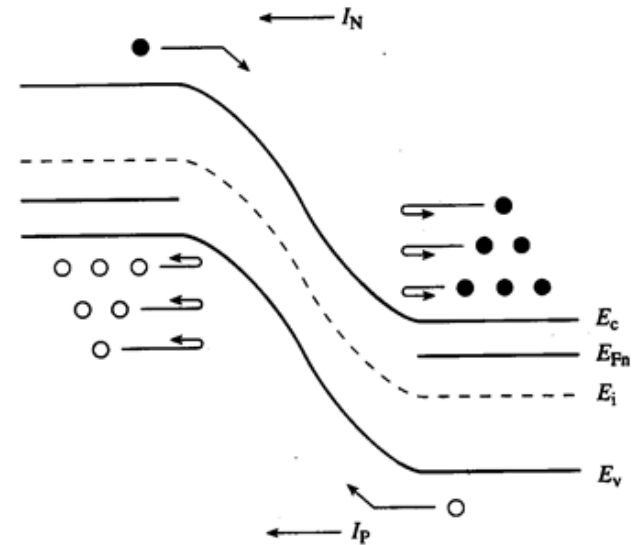
Equilibrium ($V_A = 0$)



Forward Bias ($V_A > 0$)



Reverse Bias ($V_A < 0$)



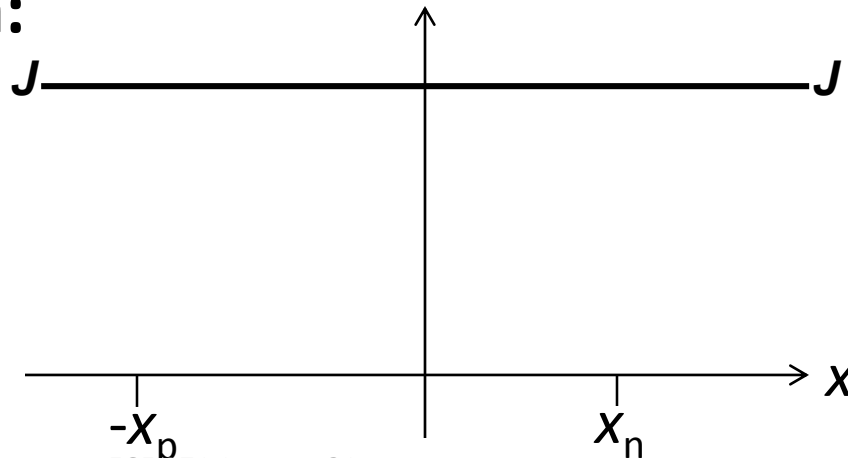
Components of Current Flow

- Current density $J = J_n(x) + J_p(x)$

$$J_n(x) = q\mu_n n \mathcal{E} + qD_n \frac{dn}{dx} = q\mu_n n \mathcal{E} + qD_n \frac{d(\Delta n)}{dx}$$

$$J_p(x) = q\mu_p p \mathcal{E} - qD_p \frac{dp}{dx} = q\mu_p p \mathcal{E} - qD_p \frac{d(\Delta p)}{dx}$$

- J is constant throughout the diode, but $J_n(x)$ and $J_p(x)$ vary with position:



Ideal Diode Analysis: Assumptions

- Non-degenerately doped step junction
- Steady-state conditions
- Low-level injection conditions in quasi-neutral regions
- Recombination-generation negligible in depletion region

$$\Rightarrow \frac{dJ_n}{dx} = 0, \quad \frac{dJ_p}{dx} = 0$$

i.e. J_n & J_p are constant inside the depletion region

“Game Plan” for Obtaining Diode I - V

1. Solve minority-carrier diffusion equations in quasi-neutral regions to obtain excess carrier distributions $\Delta n_p(x, V_A), \Delta p_n(x, V_A)$
 - boundary conditions:
 - p side: $\Delta n_p(-x_p), \Delta n_p(-\infty)$
 - n side: $\Delta p_n(x_n), \Delta p_n(\infty)$
2. Find minority-carrier current densities in quasi-neutral regions

$$J_n(x, V_A) = qD_n \frac{d(\Delta n_p)}{dx} \qquad J_p(x, V_A) = -qD_p \frac{d(\Delta p_n)}{dx}$$

3. Evaluate J_n at $x=-x_p$ & J_p at $x=x_n$ to obtain total current density J :

$$J(V_A) = J_n(-x_p, V_A) + J_p(x_n, V_A)$$

Minority carriers diffusion equations

(Special case of the ambipolar transport equation)

$$\frac{d^2 \Delta p_n}{dx^2} = \frac{\Delta p_n}{D_p \tau_p} = \frac{\Delta p_n}{L_p^2} \quad \text{n-side}$$

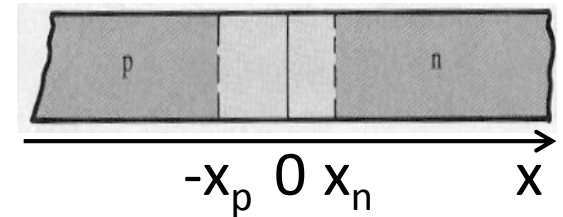
$$\frac{d^2 \Delta n_p}{dx^2} = \frac{\Delta n_p}{D_n \tau_n} = \frac{\Delta n_p}{L_n^2} \quad \text{p-side}$$

- We need to find concentration of excess minority carriers
- In order to do that we will solve the minority carriers diffusion equation

- We need boundary conditions

$$\Delta p_n(x \rightarrow \infty) = 0, \Delta n_p(x \rightarrow -\infty) = 0$$

$$\Delta p_n(x = x_n) = ?, \Delta n_p(x = -x_p) = ?$$



Carrier Concentrations at $-x_p$, x_n

Consider the **equilibrium** ($V_A = 0$) carrier concentrations:

p side

$$p_{p0}(-x_p) = N_A$$

$$n_{p0}(-x_p) = \frac{n_i^2}{N_A}$$

n side

$$n_{n0}(x_n) = N_D$$

$$p_{n0}(x_n) = \frac{n_i^2}{N_D}$$

If low-level injection conditions hold in the quasi-neutral regions when the applied voltage $V_A \neq 0$, then

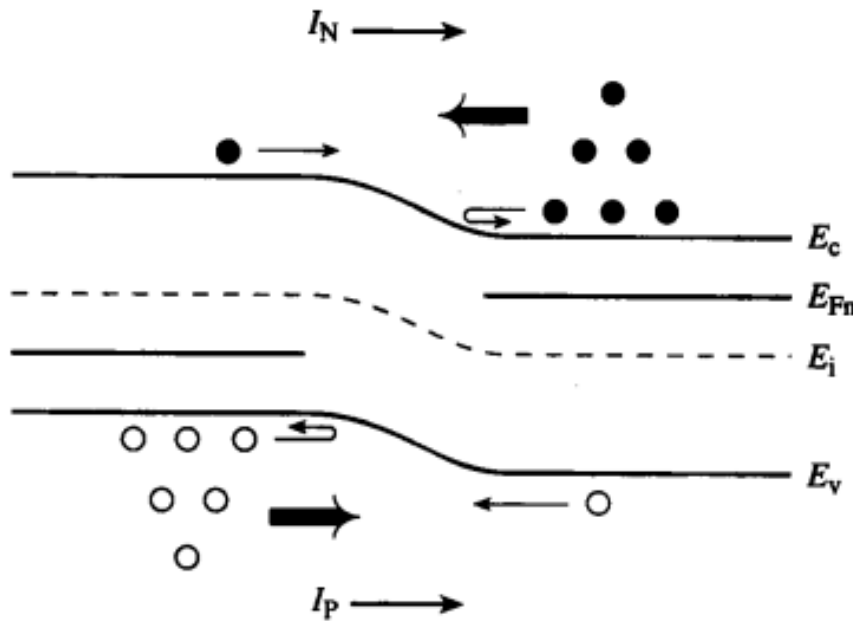
$$p_p(-x_p) = N_A$$

$$n_n(x_n) = N_D$$

“Law of the Junction”

The voltage applied to a pn junction falls mostly across the depletion region (assuming low-level injection in the quasi-neutral regions).

We can draw 2 quasi-Fermi levels in the depletion region:



$$p = n_i e^{(E_i - F_P)/kT}$$

$$n = n_i e^{(F_N - E_i)/kT}$$

$$pn = n_i^2 e^{(F_N - F_P)/kT}$$

$$pn = n_i^2 e^{qV_A/kT}$$

Excess Carrier Concentrations at $-x_p$, x_n

$$pn = n_i^2 e^{qV_A / kT}$$

p side

$$p_p(-x_p) = p_{po}(-x_p) = N_A$$

$$n_p(-x_p) = \frac{n_i^2 e^{qV_A / kT}}{N_A}$$
$$= n_{po} e^{qV_A / kT}$$

$$n_p(-x_p) = n_{po}(-x_p) + \Delta n_p(-x_p) =$$
$$= n_{po}(-x_p) + \Delta n_p(-x_p) \Rightarrow$$

$$\Rightarrow \Delta n_p(-x_p) = n_p(-x_p) - n_{po}(-x_p)$$

$$\Delta n_p(-x_p) = \frac{n_i^2}{N_A} (e^{qV_A / kT} - 1)$$

n side

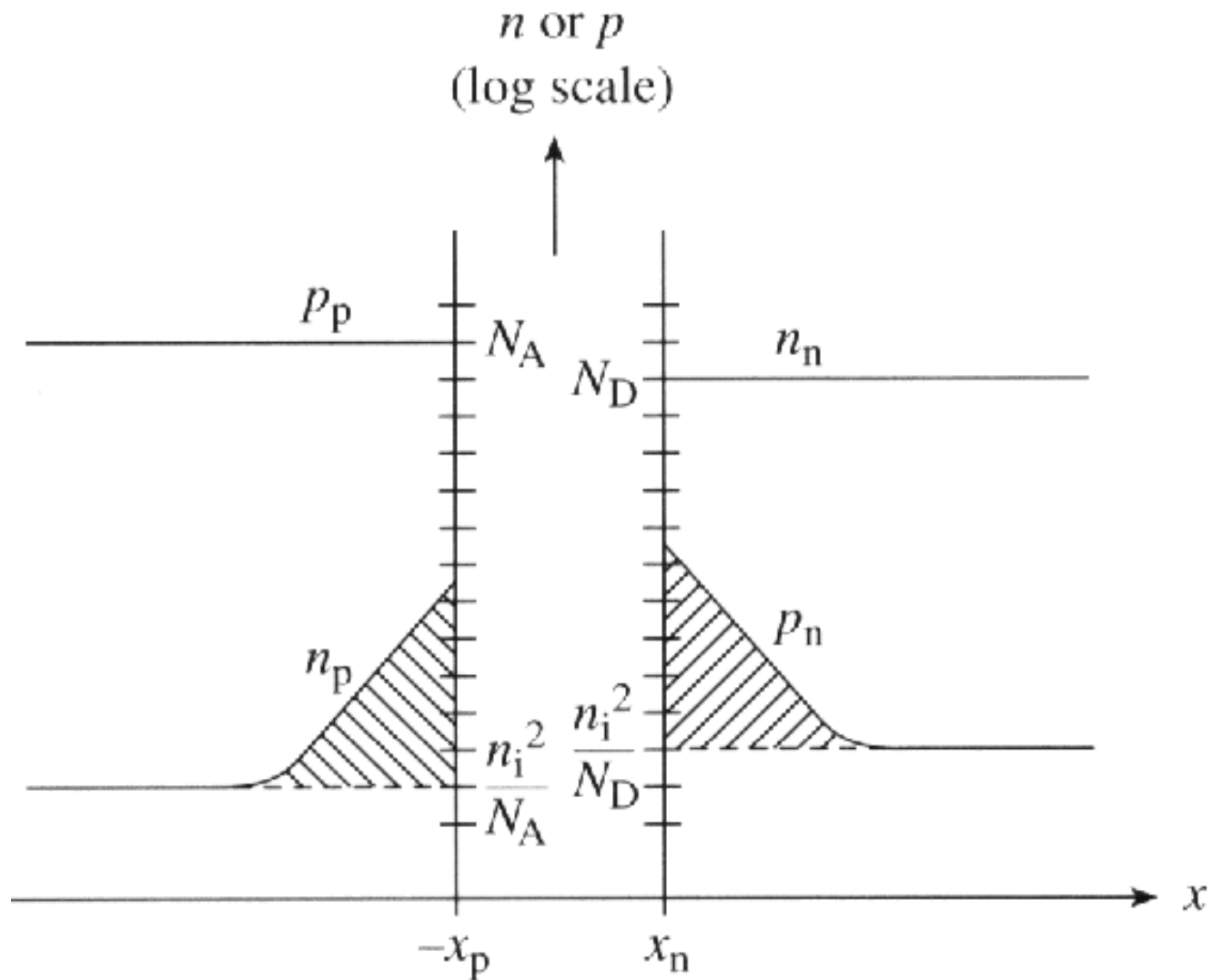
$$n_n(x_n) = N_D$$

$$p_n(x_n) = \frac{n_i^2 e^{qV_A / kT}}{N_D}$$

$$= p_{n0} e^{qV_A / kT}$$

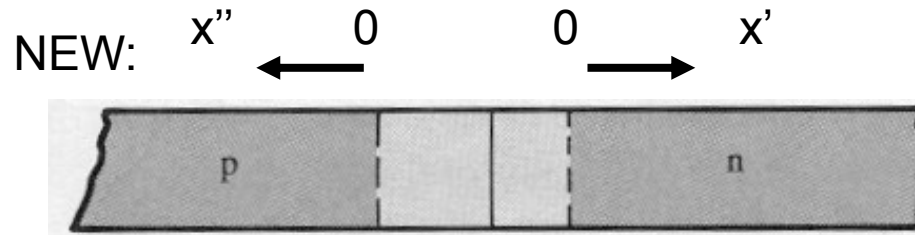
$$\Delta p_n(x_n) = \frac{n_i^2}{N_D} (e^{qV_A / kT} - 1)$$

Carrier Concentration Profiles under Forward Bias



Excess Carrier Distribution (n side)

- From the minority carrier diffusion equation: $\frac{d^2 \Delta p_n}{dx^2} = \frac{\Delta p_n}{D_p \tau_p} = \frac{\Delta p_n}{L_p^2}$
- We have the following boundary conditions:
 $\Delta p_n(x_n) = p_{no}(e^{qV_A/kT} - 1)$ $\Delta p_n(\infty) \rightarrow 0$
- For simplicity, use a new coordinate system:



- Then, the solution is of the form: $\Delta p_n(x') = A_1 e^{x'/L_p} + A_2 e^{-x'/L_p}$

Excess Carrier Distribution (n and p side)

$$\Delta p_n(x') = A_1 e^{x'/L_p} + A_2 e^{-x'/L_p}$$

From the $x = \infty$ boundary condition:

From the $x = x_n$ boundary condition:

$$\text{Therefore } \Delta p_n(x') = p_{no} (e^{qV_A/kT} - 1) e^{-x'/L_p}, \quad x' > 0$$

Similarly, for the p-side we can derive

$$\Delta n_p(x'') = n_{po} (e^{qV_A/kT} - 1) e^{-x''/L_n}, \quad x'' > 0$$

Total Current Density

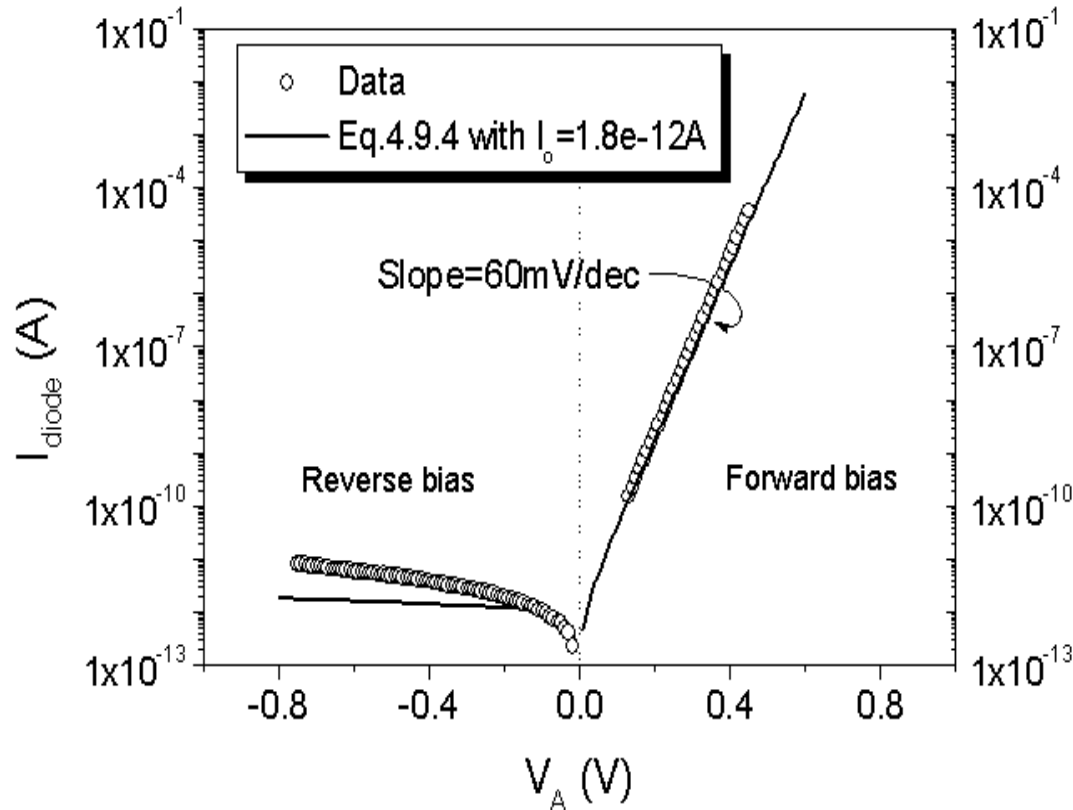
p side: $J_n = -qD_n \frac{d\Delta n_p(x'')}{dx''} = q \frac{D_n}{L_n} n_{p0} (e^{qV_A/kT} - 1) e^{-x''/L_n}$

n side: $J_p = -qD_p \frac{d\Delta p_n(x')}{dx'} = q \frac{D_p}{L_p} p_{n0} (e^{qV_A/kT} - 1) e^{-x'/L_p}$

$$J = J_n \Big|_{x=-x_p} + J_p \Big|_{x=x_n} = J_n \Big|_{x''=0} + J_p \Big|_{x'=0}$$

$$J = qn_i^2 \left[\frac{D_n}{L_n N_A} + \frac{D_p}{L_p N_D} \right] (e^{qV_A/kT} - 1)$$

Ideal Diode Equation



$$I = I_0(e^{qV_A/kT} - 1)$$

$$I_0 = Aqn_i^2 \left(\frac{D_p}{L_p N_D} + \frac{D_n}{L_n N_A} \right)$$

Diode Saturation Current I_0

- I_0 can vary by orders of magnitude, depending on the semiconductor material and dopant concentrations:

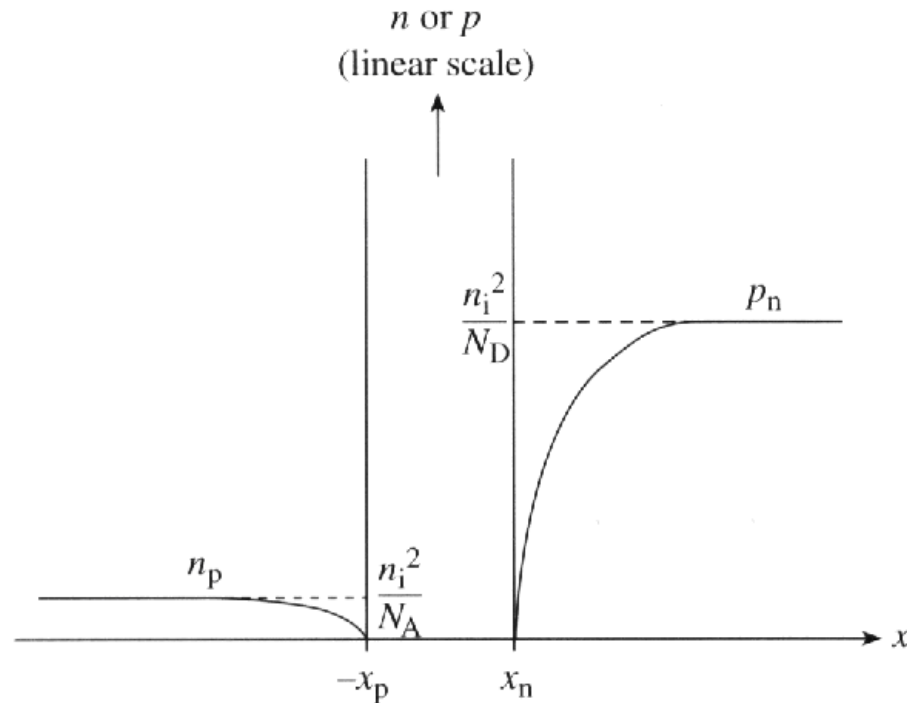
$$I_0 = Aqn_i^2 \left(\frac{D_p}{L_p N_D} + \frac{D_n}{L_n N_A} \right)$$

- In an asymmetrically doped (one-sided) pn junction, the term associated with the more heavily doped side is negligible:

- If the p side is much more heavily doped, $I_0 \cong Aqn_i^2 \left(\frac{D_p}{L_p N_D} \right)$

- If the n side is much more heavily doped, $I_0 \cong Aqn_i^2 \left(\frac{D_n}{L_n N_A} \right)$

Carrier Concentration Profiles under Reverse Bias



- Depletion of minority carriers at edges of depletion region
- The only current which flows is due to **drift** of minority carriers across the junction. This current is fed by diffusion of minority carriers toward junction (supplied by thermal generation).

Current through a p-n junction: summary

- Under forward bias ($V_A > 0$), the potential barrier to carrier diffusion is reduced \rightarrow minority carriers are “injected” into the quasi-neutral regions.
 - The minority-carrier concentrations at the edges of the depletion region change with the applied bias V_A , by the factor $e^{qV_A/kT}$
 - The excess carrier concentrations in the quasi-neutral regions decay to zero away from the depletion region, due to recombination.

pn junction diode current
$$I = qAn_i^2 \left[\frac{D_n}{L_n N_A} + \frac{D_p}{L_p N_D} \right] (e^{qV_A/kT} - 1)$$

- I_0 can be viewed as the drift current due to minority carriers generated within a diffusion length of the depletion region