ECE 345 / ME 380: Introduction to Control Systems
Problem Set #3

Dr. Oishi

Due Thursday, November 5, 2020 at 3:30pm

This homework is open note and open book. You are welcome to discuss the problems with other students, but your solutions and Matlab code *must be written independently.* Copying will not be tolerated.

1. (+10 points) Consider a negative unity feedback system as in Figure 1 with $G(s) = \dfrac{(s+1)(s+2)}{s^2(s^2+2s+3)}$.

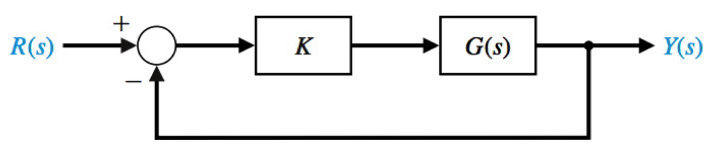
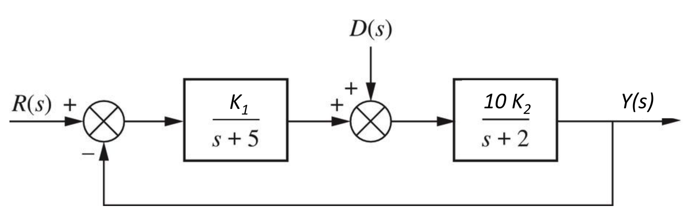


Figure 1: Negative unity feedback system.

   (a) What is the type number of the closed-loop system $\frac{Y(s)}{R(s)}$?
   (b) What finite values of K, if any, will yield a steady-state error less than or equal to 0.1, in response to a unit step input?
   (c) What finite values of K, if any, will yield a steady-state error less than or equal to 0.1, in response to a unit ramp input?
   (d) What finite values of K, if any, will yield a steady-state error less than or equal to 0.1, in response to a unit parabolic input?

2. (+15 points) Consider the following system, with reference input $r(t)$ and disturbance input $d(t)$.

   (a) Find the transfer function $G_R(s)$ with output $Y(s)$ and input $R(s)$, and the transfer function $G_D(s)$ with output $Y(s)$ and input $D(s)$, such that $Y(s) = G_R(s)R(s)+G_D(s)D(s)$.



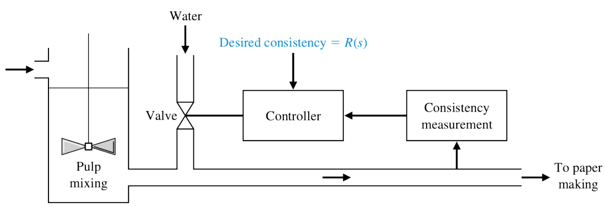
(b) Describe the relationship between the characteristic equation of $G_R(s)$ and the characteristic equation of $G_D(s)$.

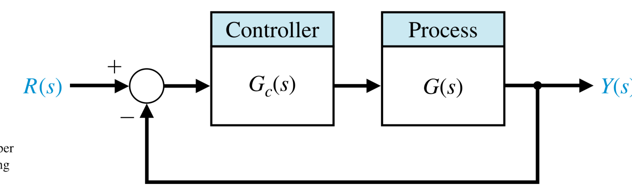
(c) Do $K_1 = 250$ and $K_2 = \frac{1}{10}$ meet the following specifications? Why or why not?

   i. The steady-state *output response* due to a unit step disturbance input is 0.02.

   ii. The steady-state *error* due to a unit step reference input is 0.05.

3. (+15 points) Pulp dilution is an important part of the paper-making process. We model the dynamics of pulp diluation with plant $G(s) = \dfrac{s+2}{s^2+2s+3}$.





(a) Consider the controller $G_c(s) = \dfrac{K}{s+1}$. What is the type number of the closed-loop system $\frac{Y(s)}{R(s)}$?

(b) What value of $K > 0$ will ensure that the steady-state error in response to a unit step input is at most 0.01?

(c) Now consider the controller $G_c(s) = \dfrac{K}{s(s+1)}$. What is the type number of the closed-loop system $\frac{Y(s)}{R(s)}$?

(d) What value of $K > 0$ will ensure that the steady-state error in response to a unit step input is at most 0.01?

(e) Consider the gain $K = 150$. Which controller has the best steady-state performance? Why?