ECE 345 / ME 380: Introduction to Control Systems
Problem Set #2

Dr. Oishi

Due Thursday, September 17, 2020 at 3:30pm

This homework is open note and open book. You are welcome to discuss the problems with other students, but your solutions *must be written independently.* Copying will not be tolerated.

1. (+10 points) Consider the system described by

$$G(s) = \frac{2(s+2)}{s(s+4)(s+10)} \tag{1}$$

   (a) Find the poles and the zeros of $G(s)$.

   (b) Put the transfer function $G(s)$ in proper form, with one polynomial in the numerator and one polynomial in the denominator.

   (c) Find the characteristic equation of $G(s)$.

2. (+10 points) The longitudinal dynamics of a vertical take-off and landing aircraft that is hovering are described by:

$$\begin{aligned} \dot{x} &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -5 & -2 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix} u \\ y &= \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x \end{aligned} \tag{2}$$

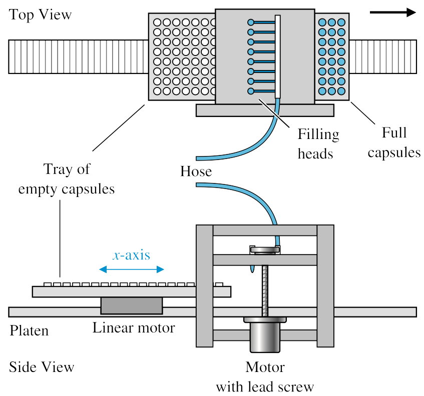
   (a) Find the characteristic equation of this system.

   (b) Where are the poles of the system located?

3. (+10 points) State-space representations are not unique. A single system can be represented in several possible ways. Consider the following two systems:

$$\text{System 1:} \quad \begin{cases} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -3 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 3 \\ -2 \end{bmatrix} u \\ y = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \end{cases}$$

$$\text{System 2:} \quad \begin{cases} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \\ y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \end{cases}$$



(a) Find the transfer function $G_1(s) = C_1(sI - A_1)^{-1}B_1 + D_1$ for System 1.

(b) Find the transfer function $G_2(s) = C_2(sI - A_2)^{-1}B_2 + D_2$ for System 2.

(c) Describe the relationship between G_1 and G_2. What zeros and/or poles do they have in common?

4. (+15 points) Consider the following state-space system, that describes the dynamics of a system for automatically dispensing fluid into capsules. A tray of capsules is guided through the dispenser by linear motor with motor torque $u(t)$ (the input). The tray position is the output, $y(t)$.

$$G(s) = \frac{3}{s^2 + Ks + 3} \tag{3}$$

(a) Using Matlab, follow the steps below for each of $K = 1, 2, 3, 4$. Use the `diary` or `publish` commands to record your code, and hand in the history of Matlab command-line inputs and outputs as well as the *single* plot that you generate. *Note: Please append the Matlab file and plot to your homework, so that you hand in a* ***single*** *.pdf. Multiple files will not be accepted.*

i. First create transfer functions $G_1(s), \cdots, G_4(s)$. For example, for the first system,

```
>> G1 = tf(3, [1 1 3];
```

ii. On a single figure, plot step responses for each of these systems using

```
>> step(G1,G2,G3,G4)
>> legend('G1','G2','G3','G4')
```

(b) Consider the oscillatory nature of the step responses. What happens as K increases? Which value of K produces the most oscillatory response? Which produces the least?