ECE 345 / ME 380: Introduction to Control Systems
Exercise: Introduction to Root Locus

Dr. Oishi

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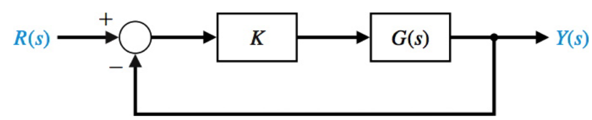
This exercise is designed to provide you with exposure to typical root locus plots, and help you develop some intuition about how the root locus works. The root locus is the set of *all* possible locations of the poles of the *negative unity feedback system*, as K varies from 0 to ∞.

Some Matlab commands that may be useful in this exercise:



- `sys = tf(1, [1 10 0])`
- `pzmap(sys)`
- `rlocus(sys)`
- `sgrid on`
- `rlocfind(sys)`

Questions marked with an asterisk (*) are optional.

1 Single Pole

Consider the following systems, all of which have a single pole, under negative unity feedback.

(1a) $G(s) = \dfrac{1}{s}$

(1b) $G(s) = \dfrac{1}{s+2}$

(1c) $G(s) = \dfrac{s+1}{s+2}$

1. What do the first two root locus diagrams have in common?

2. What is the effect of the zero in the root locus diagram of the third system? Compare the root locus of the system in (1b) with that in (1c).

3. *Construct root locus plots for two more systems: $G(s) = \dfrac{1}{s+200}$ and $G(s) = \dfrac{1}{s-2}$. What do these plots tell you about the effect of the location of the pole in $G(s)$ on the overall shape of the root locus?

4. How many "branches" (i.e., lines originating from poles) are evident in each of these plots?

2 Two Poles

Consider the following transfer functions (all with two poles) under negative unity feedback.

(2a) $G(s) = \dfrac{1}{(s+2)^2}$

(2b) $G(s) = \dfrac{1}{(s+2)^2+1}$

(2c) $G(s) = \dfrac{1}{(s+2)(s-1)}$

1. Describe the most salient features of the root locus diagrams in (2a), (2b), (2c). How many branches are in each of these plots?
2. Compare the root locus of (2c) with that of (1b). What is the effect of adding a second pole?
3. What happens to a root locus of a system with two poles as $K \to \infty$? Does the location of the poles of $G(s)$ change the behavior as $K \to \infty$?
4. Consider the root locus of the system in (2b). Is it possible for the closed-loop system to be overdamped, for any $K > 0$?

3 Two Poles, One Zero

Now consider systems with two poles and a single zero.

(3a) $G(s) = \dfrac{s+3}{s(s+10)}$

(3b) $G(s) = \dfrac{s+7}{s(s+10)}$

(3c) $G(s) = \dfrac{s+11}{s(s+10)}$

1. What do these plots have in common with the transfer functions of the single pole systems in (1a) and (1b)?
2. What do these plots have in common with the transfer function of the of the single pole, single zero system in (1c)?
3. How many branches are in each of these plots? Do all branches go to infinity? If not, where does the terminating branch end?
4. *Replace the pole at $s = 0$ with a pole at $s = 2$. Do any of the plots change substantively?
5. *Now replace the pole at $s = 0$ with a pole at $s = 4$. Do any of the plots change substantively?
6. Consider the following hypothesis: The root locus must be on the real line to the *left* of an odd number of *poles and zeros*. Is this consistent with your observation of the plots in this section? What happens to the locus when it cannot lie on the real line?

4 Root Locus for Controller Design

Consider the negative unity feedback system with $G(s) = \dfrac{1}{(s+2)(s^2+2s+2)}$.

1. What value of gain K will result in a damping ratio of $\zeta = 0.6$ for the two poles closest to the imaginary axis? *Hint: Try `sgrid` and `rlocfind`.*
2. Is it possible to obtain a damping ratio of 0.8? Why or why not?
3. What is the fastest settling time possible for this system? What value of gain K is required in this case?
4. Use `rlocfind` to determine the value of gain K that will destabilize the system. Compare this result to what you obtain through the Hurwitz criterion.