ECE 345: Introduction to Control Systems
Midterm #2

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This exam is closed-note, closed book, and no electronic devices are allowed. A two-sided, two-page cheat-sheet is allowed and *must be handed in with your midterm.* The cheat-sheet will be returned you with your graded exam.

For full credit, show all your work.

Student Name Student ID #

Problem #	Actual points	Possible points
1		45
2		20
3		30
Total:		95

1 Helicopter flight management system (45 points)

Consider a flight management system for helicopter motion in the lateral plane (e.g., looking down on the helicopter from above, as in Figure 1), with input that is a reference trajectory in the forward direction, and output that is the actual forward distance of the helicopter. A flight management system should be designed so the helicopter can track a ramp input with *finite, non-zero* steady-state error, and meet certain transient performance criteria.

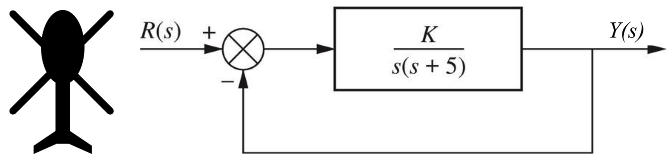

Figure 1: (Left) Top-down view of helicopter. (Right) Helicopter dynamics with desired position $R(s)$ and actual position $Y(s)$.

1. (15 points)

   (a) What is the type number of the closed-loop system $\frac{Y(s)}{R(s)}$?

   (b) Calculate the steady-state error of the closed-loop system to a unit ramp input in $R(s)$.

   (c) What value(s) of K, if any, will assure that steady-state error in response to a unit ramp will be *at most* 0.05?

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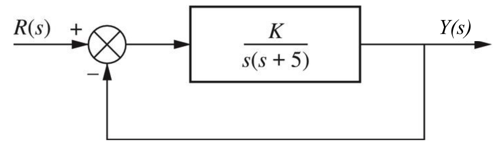
Recall that the closed-loop system is described by the transfer function $\frac{Y(s)}{R(s)}$.



The helicopter should additionally meet transient performance specifications: peak time should be less than or equal to $\pi/2$ seconds.

2. (10 points) What value(s) of K will meet the transient performance specifications?

3. (5 points) Can both steady-state and transient performance specifications be met (e.g., those in Question 1(d) and Question 2)? If so, with what value(s) of K?

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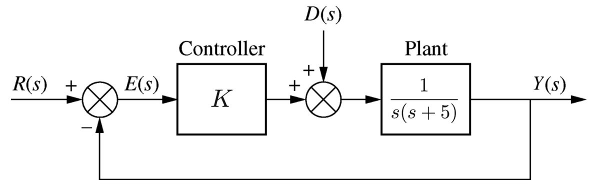
Now consider the case in which a disturbance enters the system as shown, so that



$$Y(s) = G_R(s)R(s) + G_D(s)D(s) \tag{1}$$

We wish to design the system so that the output is unaffected by disturbance inputs that are of the form of a unit step.

4. (15 points)

   (a) Find $G_D(s)$.

   (b) Compute the steady-state response to a disturbance input that is a unit step, for $K = 10$.

   (c) Is disturbance rejection adequate with this gain? Why or why not?

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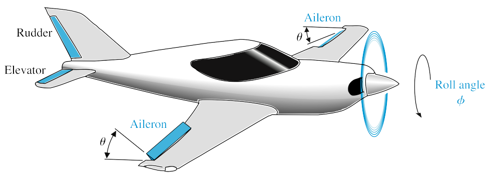
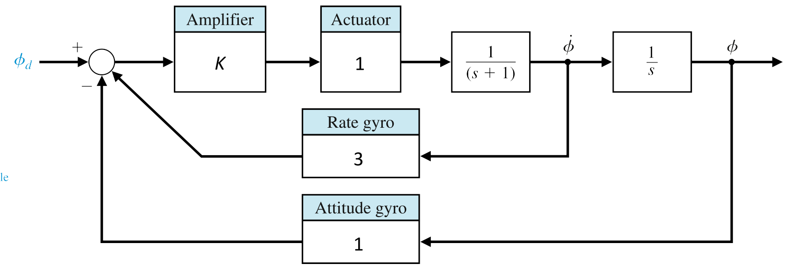
2 Lateral dynamics of a general aviation aircraft (20 points)





An autopilot controls the roll angle $\phi(t)$ to track a desired roll angle $\phi_d(t)$ by adjusting aileron surfaces. Deflections of the ailerons generate torque due to air pressure on these surfaces, which generates a rolling motion. The ailerons are controlled by a hydraulic actuator.

1. (10 points) Show that the closed-loop system has the transfer function $\dfrac{\phi(s)}{\phi_d(s)} = \dfrac{K}{s^2 + (3K+1)s + K}$.

2. (10 points) Is the closed-loop system BIBO stable for $K = 2$? Why or why not?

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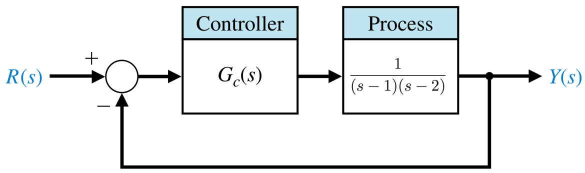
3 Lead control vs. lag control (30 points)



Consider a negative unity feedback system as shown, with controller $G_c(s)$. We consider two controllers:

$$\begin{aligned} \text{Lag controller:} \quad G_{\text{lag}}(s) &= K\frac{s+5}{s} \\ \text{Lead controller:} \quad G_{\text{lead}}(s) &= K\frac{s}{s+5} \end{aligned}$$

Note that the *open-loop* transfer function $G_c(s) \cdot \frac{1}{(s-1)(s-2)}$ is unstable under either lead or lag control.

1. Lag control: $G_c(s) = G_{\text{lag}}(s)$

    (a) (5 points) Find the characteristic equation of the closed-loop system $\frac{Y(s)}{R(s)}$ under lag control.

    (b) (10 points) Use the Hurwitz criterion to show that the closed-loop system $\frac{Y(s)}{R(s)}$ under lag control is unstable for all values of gain $K > 0$.

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Recall the plant is described by $\frac{1}{(s-1)(s-2)}$, and that $G_{\text{lead}}(s) = K\frac{s}{s+5}$.

2. Lead control: $G_c(s) = G_{\text{lead}}(s)$

   (a) (5 points) Find the characteristic equation of the closed-loop system $\frac{Y(s)}{R(s)}$ under lead control.

   (b) (10 points) Use the Routh table to identify the number of poles responsible for the instability of the system under lead control for low values of K, and all value(s) of gain K for which the closed-loop system is stable.

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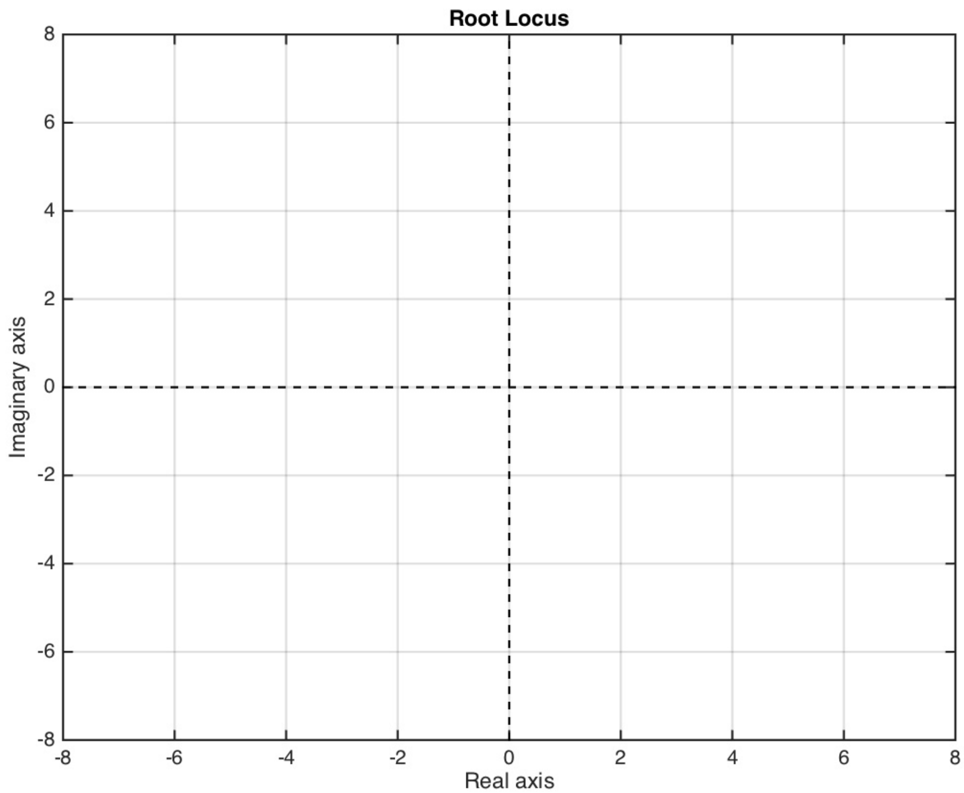
BONUS: (10 points) Sketch the root locus on the diagram below for the system under lead control. Identify a) the number of branches, b) branch segments on the real line, c) the number of asymptotes, and d) the centroid and angles of the asymptotes.



End of exam.