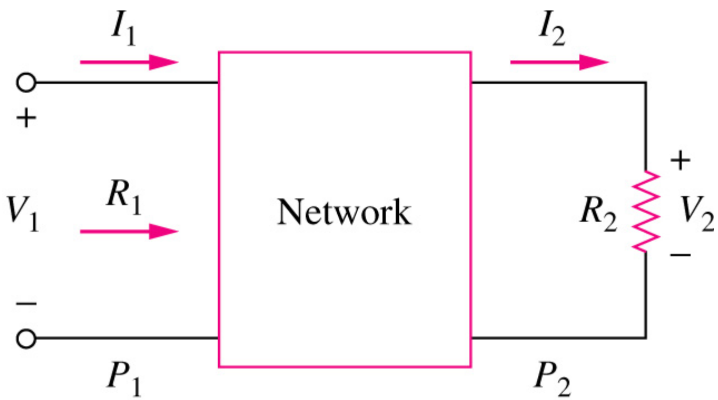
ECE 322L
Electronics 2

Handout

Simple systems transfer functions and Bode plots (Neamen 7.2)

The decibel



The DECIBEL value is a logarithmic measurement of the ratio between two variables

Power gain is expressed in dB by the formula:

$$A_P = 10 \log a_P$$

Voltage gain is expressed by:

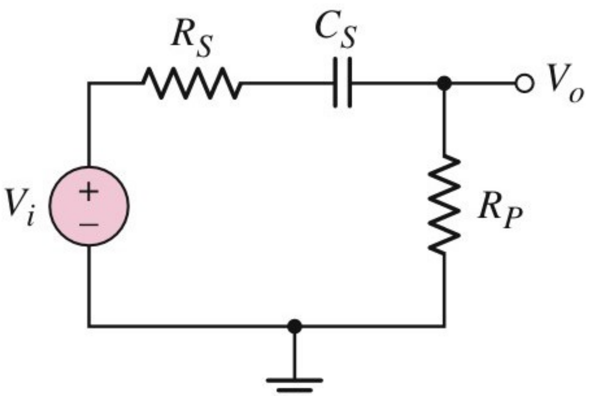
$$A_{V(dB)} = 20 \log a_v$$

$a_p = P_2/P_1$ and $a_v = V_2/V_1$

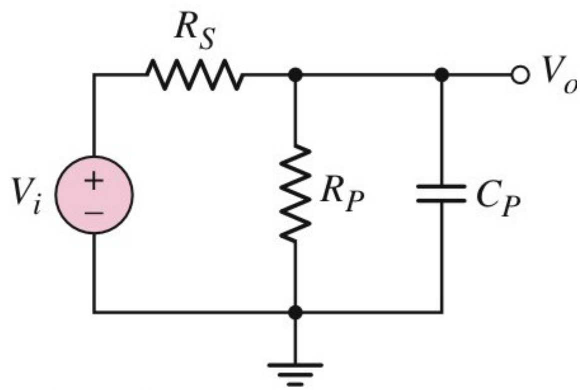
Magnitude	Decibel Value
0.001	-60
0.01	-40
0.1	-20
0.5	-6
$1/\sqrt{2}$	-3
1	0
$\sqrt{2}$	3
2	6
10	20
20	26
100	40

Selected transfer functions

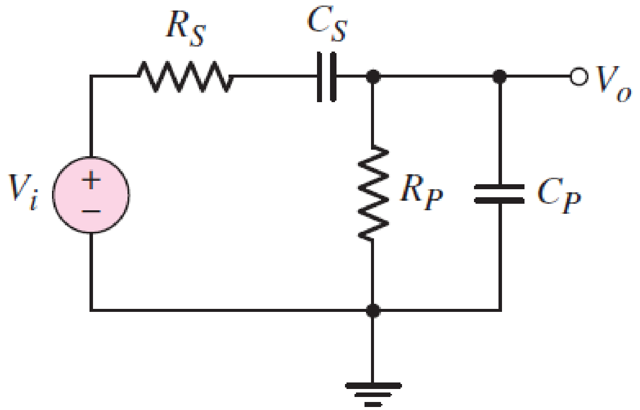
It is useful to determine the frequency response of an amplifier are the transfer functions T(s)= (V_o/V_i) of the circuits below.



Series coupling capacitor circuit



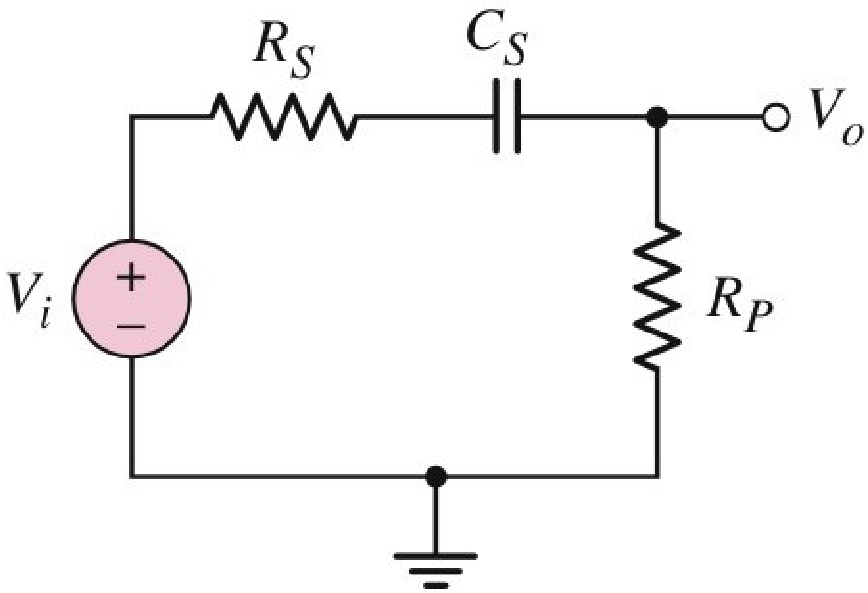
Parallel load capacitor circuit



Series coupling and parallel load capacitor circuit

$$T(s) = K\frac{(s-z_1)(s-z_2)\cdots(s-z_m)}{(s-p_1)(s-p_2)\cdots(s-p_n)}$$

Series coupling capacitor circuit



$$\frac{V_o(s)}{V_i(s)} = \frac{R_P}{R_S + R_P + \dfrac{1}{sC_S}}$$

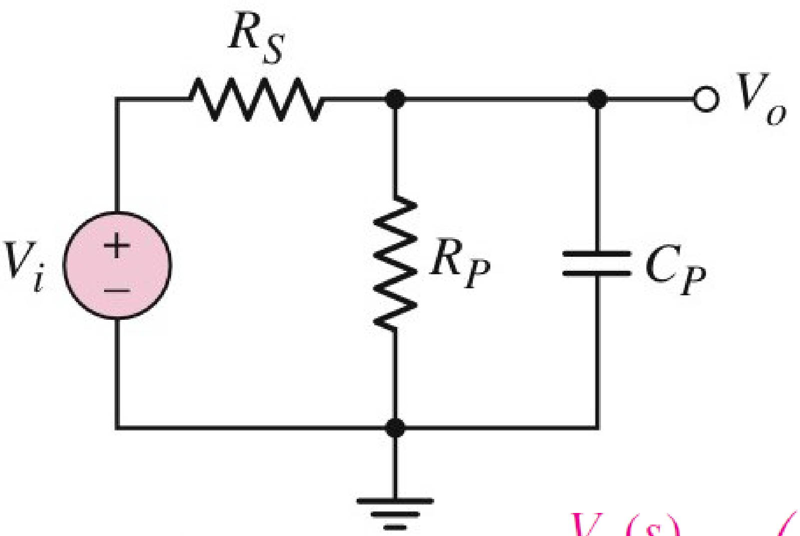
$$\frac{V_o(s)}{V_i(s)} = \frac{sR_PC_S}{1 + s(R_S + R_P)C_S}$$

$$\frac{V_o(s)}{V_i(s)} = \left(\frac{R_P}{R_S + R_P}\right)\left[\frac{s(R_S + R_P)C_S}{1 + s(R_S + R_P)C_S}\right] = K\left(\frac{s\tau_s}{1 + s\tau_s}\right)$$

$$\tau_S = (R_S + R_P)C_S$$

Time constant

Parallel load capacitor circuit



$$\frac{V_o - V_i}{R_S} + \frac{V_o}{R_P} + \frac{V_o}{(1/sC_P)} = 0$$

$$\frac{V_o(s)}{V_i(s)} = \left(\frac{R_P}{R_S + R_P}\right)\left[\frac{1}{1 + s\left(\frac{R_S R_P}{R_S + R_P}\right)C_P}\right]$$

$$\frac{V_o(s)}{V_i(s)} = \left(\frac{R_P}{R_S + R_P}\right)\left[\frac{1}{1 + s(R_S \| R_P)C_P}\right] = K\left(\frac{1}{1 + s\tau_P}\right)$$

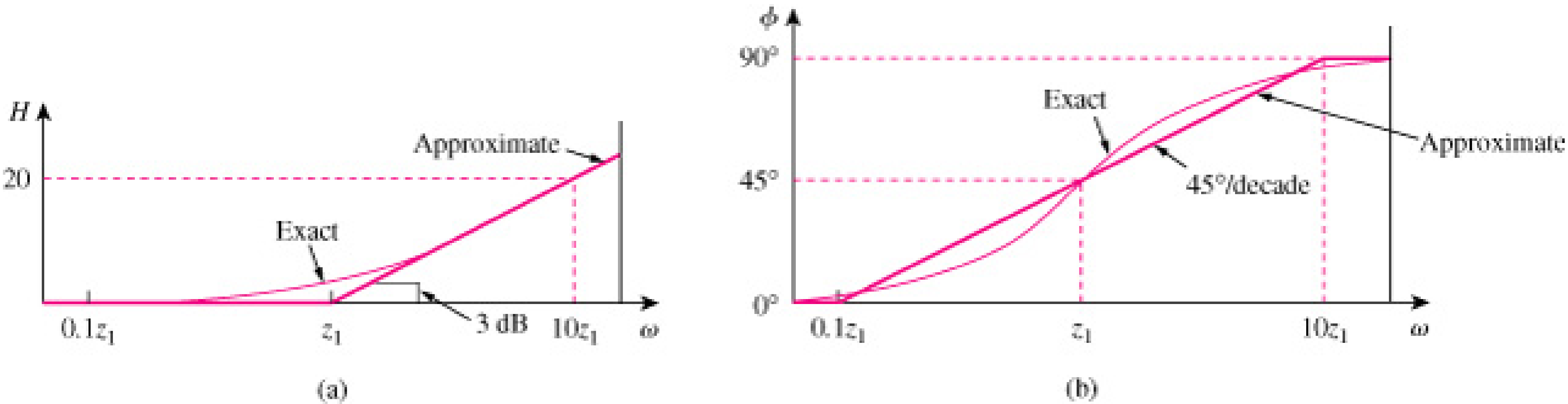
$$\tau_P = (R_S \| R_P)C_P$$

Time constant

Bode plots

Approximate plots of the amplitude and phase of the transfer functions.

Semilog plots of the magnitude (in Decibels) and phase (in degrees) of a transfer function versus frequency.



Bode plots (approximate) and exact plots of the amplitude and phase of a generic transfer function H(ω).

How to sketch the Bode Plots

1. Put the transfer function in STANDARD FORM.

$$T(j\omega) = K\frac{(1-j\omega/z_1)(1-\omega/z_2)\cdots\cdots(1-\omega/z_m)}{(1-\omega/p_1)(1-\omega/p_2)\cdots\cdots(1-\omega/p_n)}$$

2. Write the magnitude and phase equations from the STANDARD FORM.
3. Plot the magnitude (in dB) of each term separately.
4. Add all magnitude terms to obtain the magnitude transfer function.
5. Repeat 2-4 for the phase response.
6. The total magnitude response in Decibel units is the combination of the responses of different terms.
7. The total phase response in degrees is the summation and subtraction of the phase responses of different terms.

How to plot the single factors of the T(s)

Next, we will examine how to plot different terms that may appear in a transfer function, such as K, $(s-z_i)$, and $(s-p_i)$.

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Transfer function in s

$$T(s) = K\frac{(s-z_1)(s-z_2)\cdots(s-z_m)}{(s-p_1)(s-p_2)\cdots(s-p_n)}$$

Transfer function in ω (Obtained from T(s) by setting $s=j\omega$)

$$T(j\omega) = K\frac{(j\omega-z_1)(j\omega-z_2)\cdots\cdots(j\omega-z_m)}{(j\omega-p_1)(j\omega-p_2)\cdots\cdots(j\omega-p_n)}$$

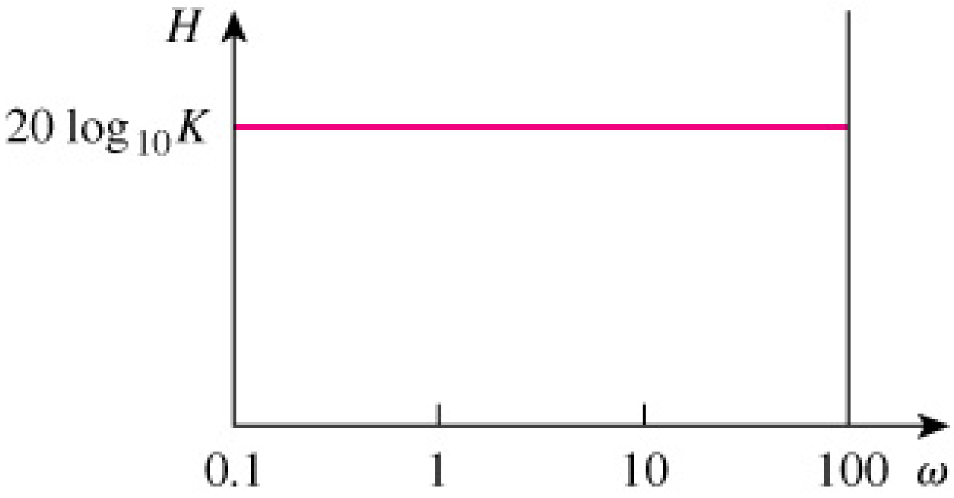
$$T(j\omega) = K\frac{(1-j\omega/z_1)(1-j\omega/z_2)\cdots\cdots(1-j\omega/z_m)}{(1-j\omega/p_1)(1-j\omega/p_2)\cdots\cdots(1-j\omega/p_n)}$$

Standard form of the transfer function

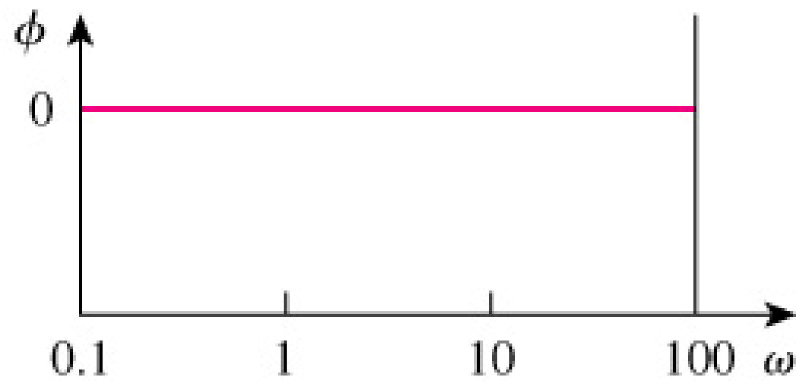
How to plot the single factors of the T(s)

The constant term, K

$$T(j\omega) = K \frac{(1 - j\omega/z_1)(1 - j\omega/z_2) \cdots\cdots (1 - j\omega/z_m)}{(1 - j\omega/p_1)(1 - j\omega/p_2) \cdots\cdots (1 - j\omega/p_n)}$$



(a)



(b)

$$A + jB = Ke^{j\theta}, \text{ where } K = \sqrt{A^2 + B^2} \text{ and } \theta = \tan^{-1}(B/A)$$

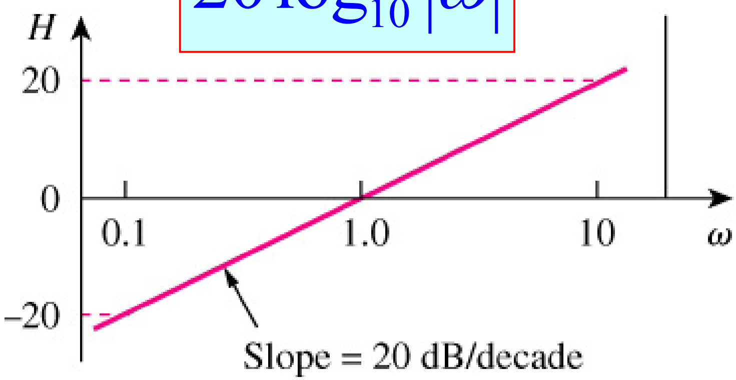
How to plot the single factors of the T(s)

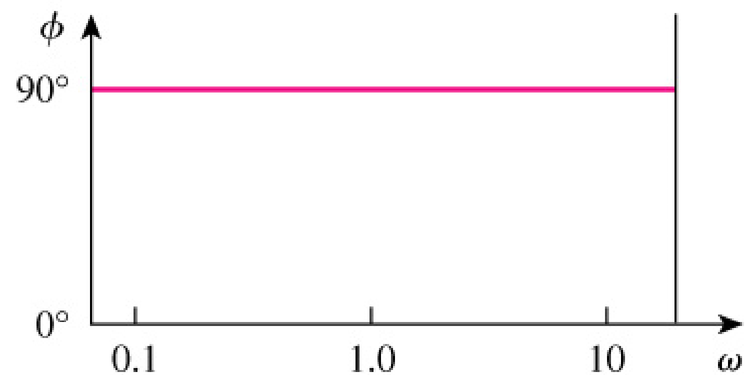
Zeros-Zero at the origin (i.e., z_1=0)

$$T(j\omega) = K\frac{(j\omega)(1 - j\omega/z_2)\cdots\cdots(1 - j\omega/z_m)}{(1 - j\omega/p_1)(1 - j\omega/p_2)\cdots\cdots(1 - j\omega/p_n)}$$

$$|j\omega| = \sqrt{0+\omega^2} = \omega \qquad \phi = \tan^{-1}\frac{\omega}{0} = 90^\circ$$

$$\boxed{20\log_{10}|\omega|}$$





$$A + jB = Ke^{j\theta}, \text{ where } K = \sqrt{A^2+B^2} \text{ and } \theta = \tan^{-1}(B/A)$$

How to plot the single factors of the T(s)

Zeros-Simple zero not at the origin

$$T(j\omega) = K\frac{(1-j\omega/z_1)(1-j\omega/z_2)\cdots\cdots(1-j\omega/z_m)}{(1-j\omega/p_1)(1-j\omega/p_2)\cdots\cdots(1-j\omega/p_n)}$$

$$20log_{10}|1-\omega/z_1| \qquad \phi = \tan^{-1}\frac{\omega}{z_1}$$

Approximate the magnitude response of a simple zero by two linear curves before and after $\omega = z_1$

Approximate the phase response of a simple zero by three linear curves before $\omega = 0.1z_1$ after $\omega = 10z_1$ and between $\omega = 0.1z_1$ and $\omega = 10z_1$

$$H_{dB} = 20\log_{10}|1 - j\omega/z_1| = \begin{cases} 20\log_{10}1 = 0 & \omega \to 0 \\ 20\log_{10}\omega/z_1 & \omega \to \infty \end{cases} \approx \begin{cases} 0\ \text{dB} & \omega \le 0 \\ 20\log_{10}\omega/z_1 & \omega \ge 0 \end{cases}$$

$$\phi = \tan^{-1}\left(\frac{\omega}{z_1}\right) = \begin{cases} 0, & \omega = 0 \\ 45^\circ, & \omega = z_1 \\ 90^\circ, & \omega \to \infty \end{cases}$$

Zeros-Simple zero not at the origin

$$H_{dB} = 20\log_{10}\left|1 - j\,\omega/z_1\right| = \begin{cases} 20\log_{10} 1 = 0 & \omega \to 0 \\ 20\log_{10} \omega/z_1 & \omega \to \infty \end{cases} \approx \begin{cases} 0 \text{ dB} & \omega \leq 0 \\ 20\log_{10} \omega/z_1 & \omega \geq 0 \end{cases}$$

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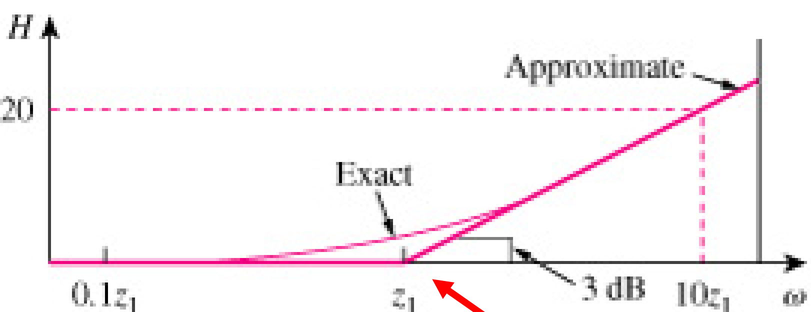
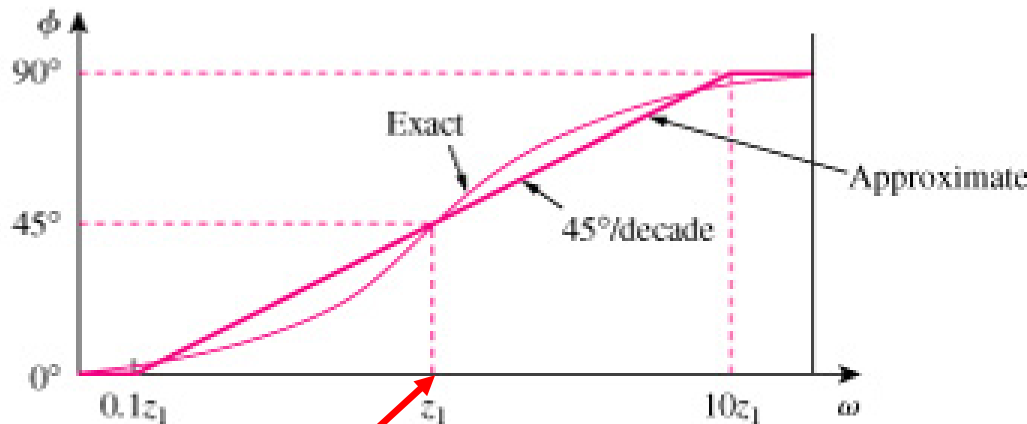


(a)



(b)

CORNER FREQUENCY

BREAK FREQUENCY

3 dB FREQUENCY

How to plot the single factors of the T(s)

Poles-Pole at the origin (i.e., z1=0)

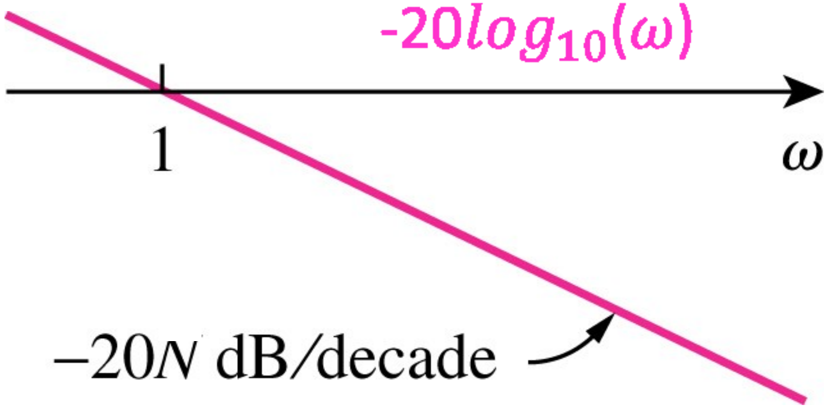
$$T(j\omega) = K\frac{(1-j\omega/z_1)(1-\omega j/z_2)\cdots\cdots(1-j\omega/z_m)}{(j\omega)(1-j\omega/p_2)\cdots\cdots(1-j\omega/p_n)}$$

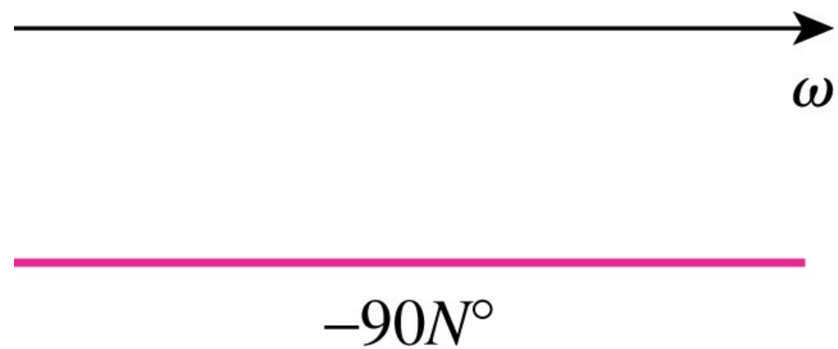
$$20log_{10}|1/(j\omega)| = -20log_{10}|(j\omega)|$$

$$|j\omega| = \sqrt{0+\omega^2} = \omega$$

$$\phi = \tan^{-1} - \frac{1/\omega}{0} = -90°$$

-$20log_{10}(\omega)$





$$A + jB = Ke^{j\theta}, \text{ where } K = \sqrt{A^2+B^2} \text{ and } \theta = \tan^{-1}(B/A)$$

How to plot the single factors of the T(s)

Poles-Simple pole not at the origin

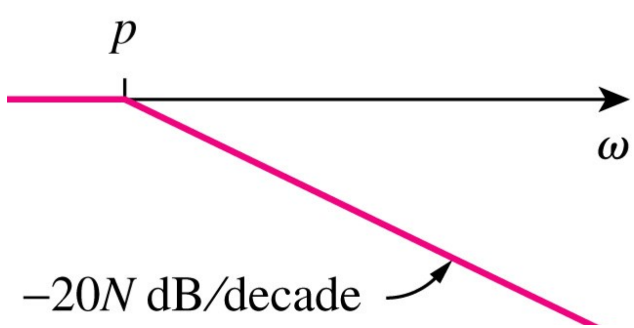
$$T(j\omega) = K\frac{(1-j\omega/z_1)(1-j\omega/z_2)\cdots\cdots(1-j\omega/z_m)}{(1-j\omega/p_1)(1-j\omega/p_2)\cdots\cdots(1-j\omega/p_n)}$$

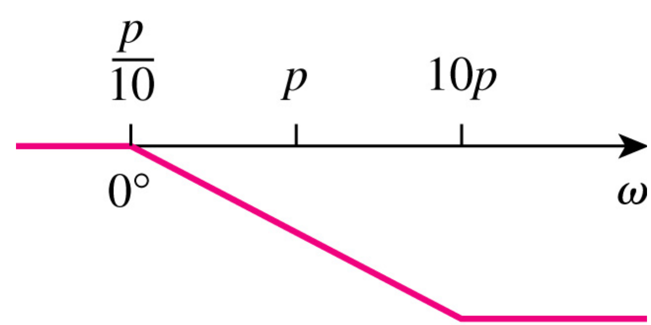
Approximate the magnitude response of a simple zero by two linear curves before and after $\omega = z_1$

Approximate the phase response of a simple zero by three linear curves before $\omega = 0.1z_1$ after $\omega = 10z_1$ and between $\omega = 0.1z_1$ and $\omega = 10z_1$

$$20log_{10}\left|1/(1-\frac{\omega}{p_1})\right| = -20log_{10}\left|1-\frac{\omega}{p_1}\right|$$

$$\phi = -\tan^{-1}\left(\frac{\omega}{p_1}\right) = \begin{cases} 0, & \omega=0 \\ -45°, & \omega=p_1 \\ -90°, & \omega\to\infty \end{cases}$$





$$A + jB = Ke^{j\theta}, \text{ where } K = \sqrt{A^2+B^2} \text{ and } \theta = \tan^{-1}(B/A)$$

How to sketch the Bode Plots

1) Draw the line of each individual term of the transfer function on the graph

2) Follow the combined gain-pole-zero at the origin line back to the left side of the graph.

3) Add the constant offset, 20 log10(K), to the value where the pole/zero at the origin line intersects the left side of the graph.

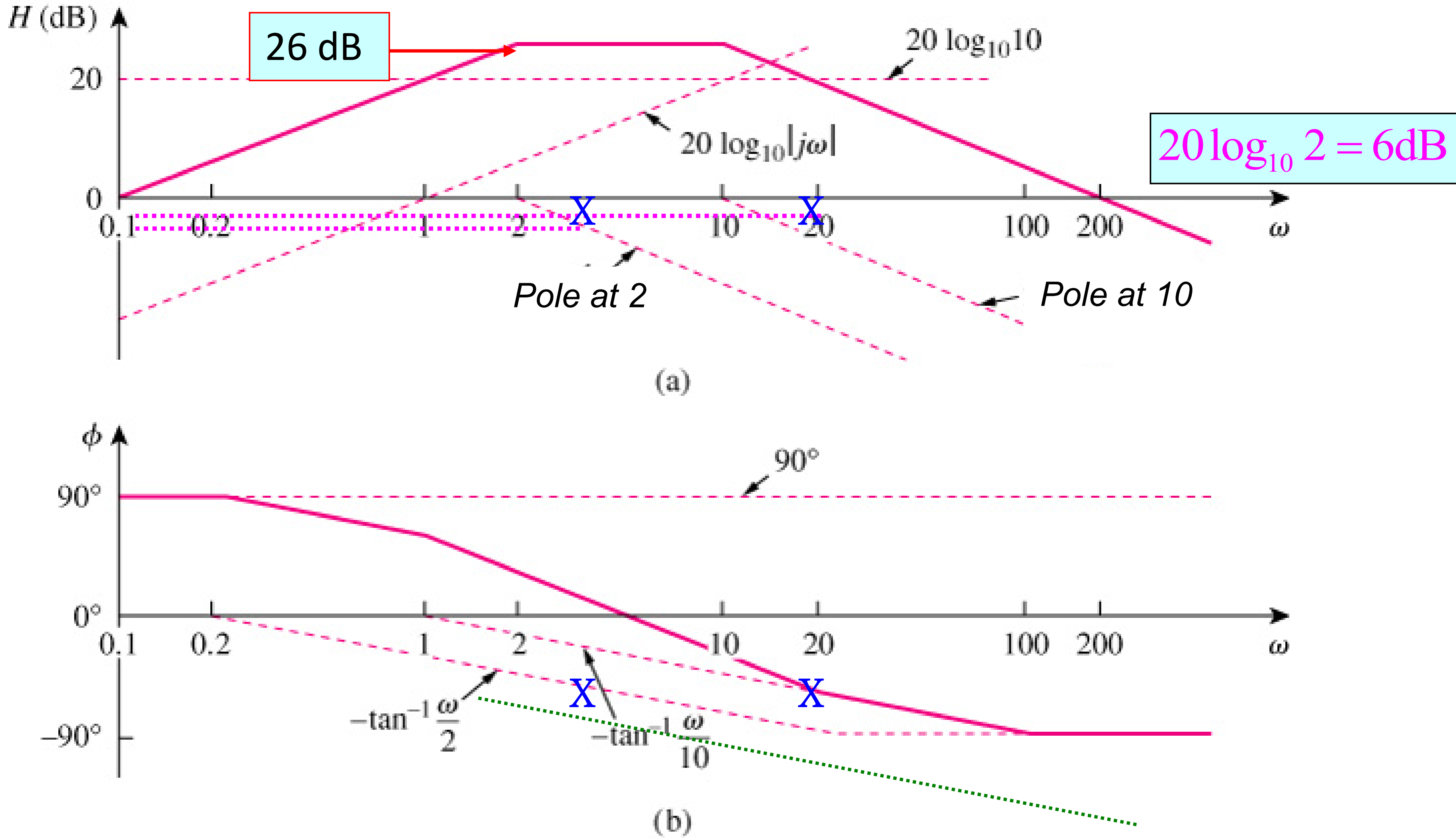
4) Apply the effect of the poles/zeros not at the origin working from left (low values) to right (higher values) of the poles/zeros.

Problem 1

Sketch the Bode Plots for the transfer function below

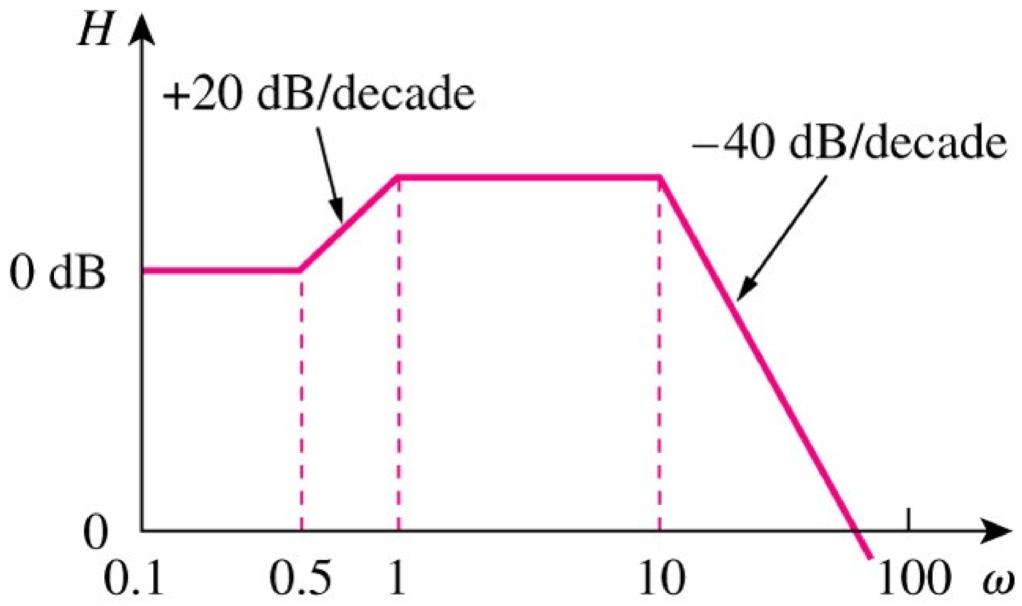
$$H(\omega) = \frac{200\, j\omega}{(j\omega - 2)(j\omega - 10)}$$

Problem 1, solution



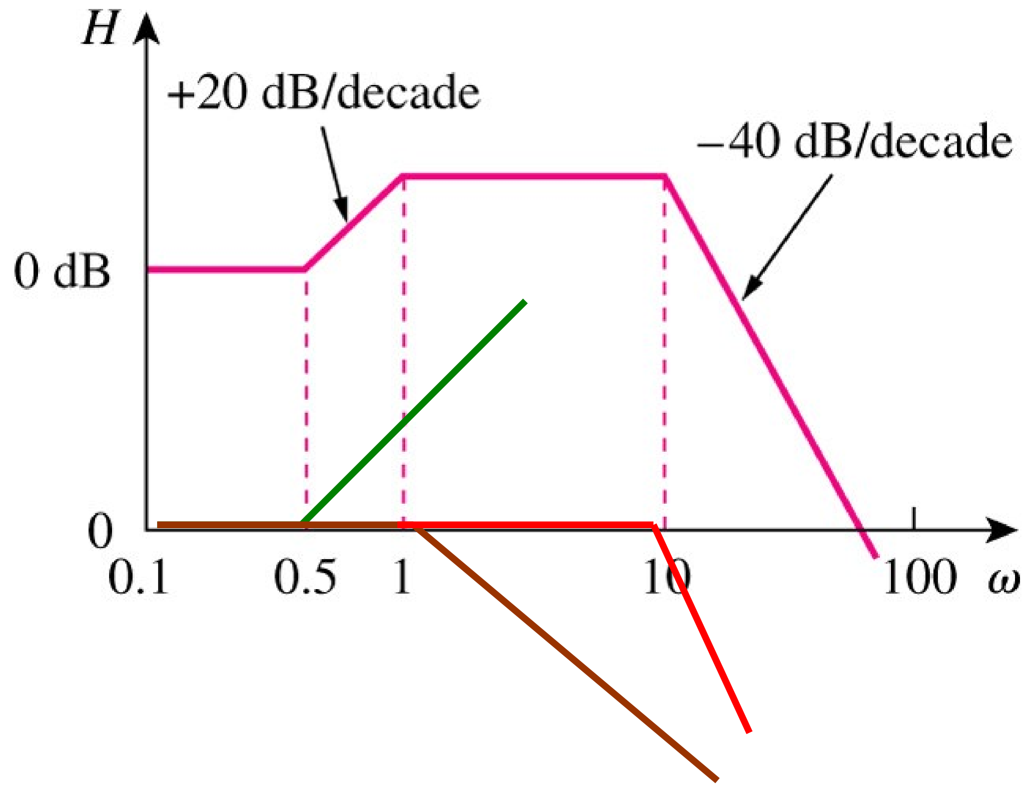
Problem 2

Obtain the transfer function for the Bode plot given.



Problem 2, solution

Obtain the transfer function for the given Bode plot.
Hint: The transfer function has no zeros or poles in the origin.



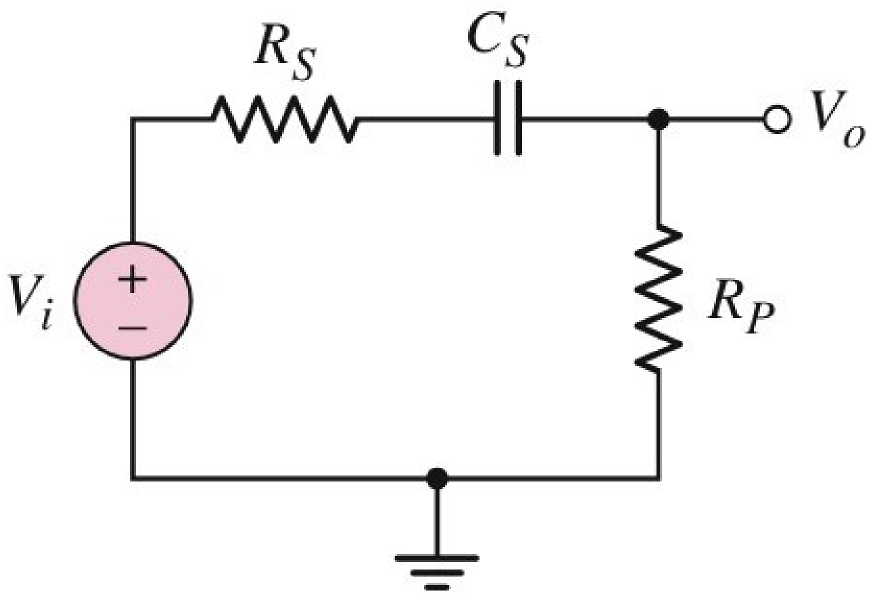
A zero at $\omega = 0.5,\quad 1 + j\omega/0.5$

A pole at $\omega = 1,\quad \dfrac{1}{1 + j\omega/1}$

Two poles at $\omega = 10,\quad \dfrac{1}{(1 + j\omega/10)^2}$

$$H(\omega) = \frac{1 + j\omega/0.5}{(1 + j\omega/1)(1 + j\omega/10)^2} = \frac{(1/0.5)(0.5 + j\omega)}{(1/100)(1 + j\omega)(10 + j\omega)^2} = \frac{200\,(s + 0.5)}{(s+1)(s+10)^2}$$

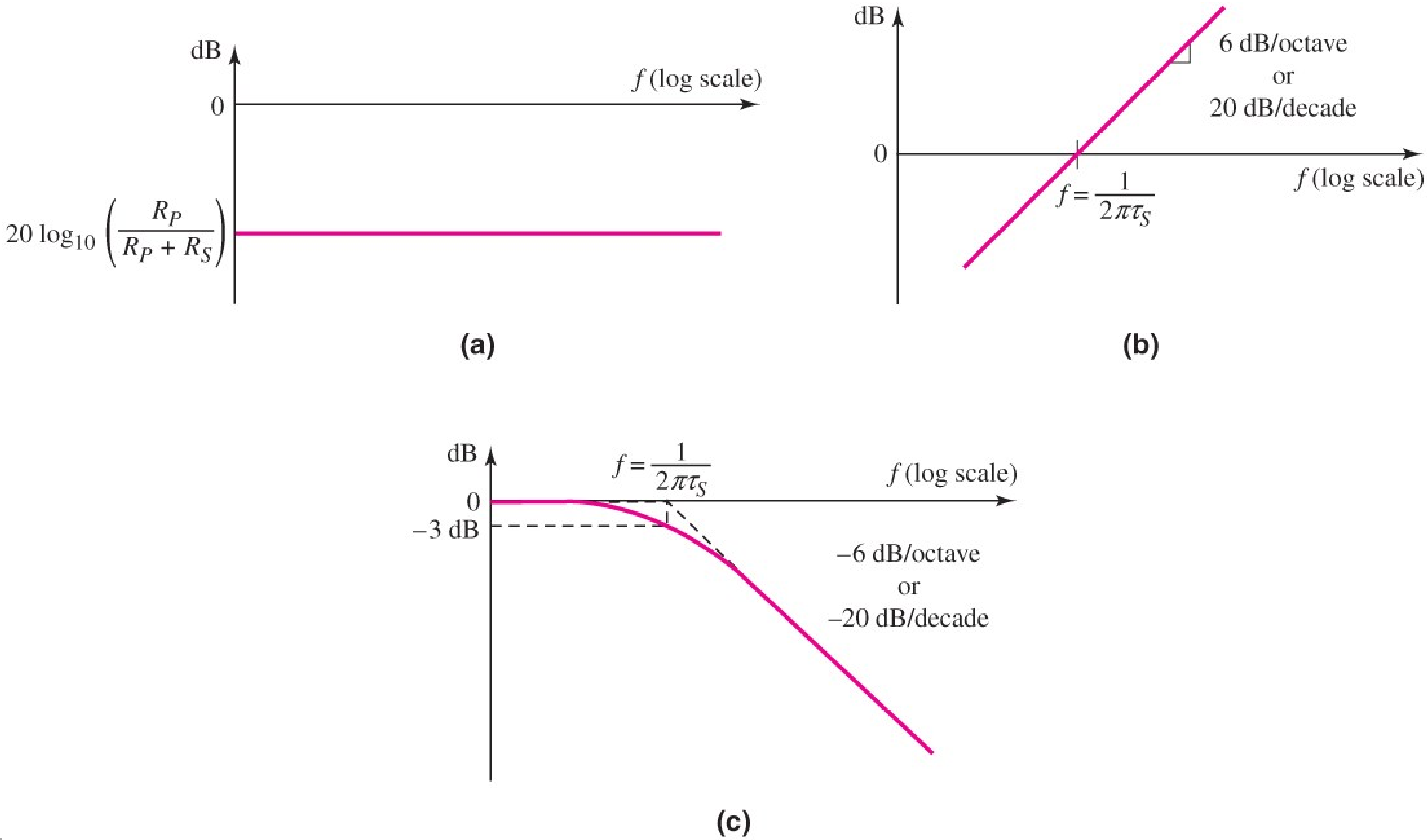
Series coupling capacitor circuit



$$\frac{V_o(s)}{V_i(s)} = \left(\frac{R_P}{R_S + R_P}\right)\left[\frac{s(R_S + R_P)C_S}{1 + s(R_S + R_P)C_S}\right] = K\left(\frac{s\tau_s}{1 + s\tau_s}\right)$$

Series coupling capacitor circuit-Bode plot

$$T(s) = K_2\left(\frac{s\tau}{1+s\tau}\right)$$



Series coupling capacitor circuit-Bode plot

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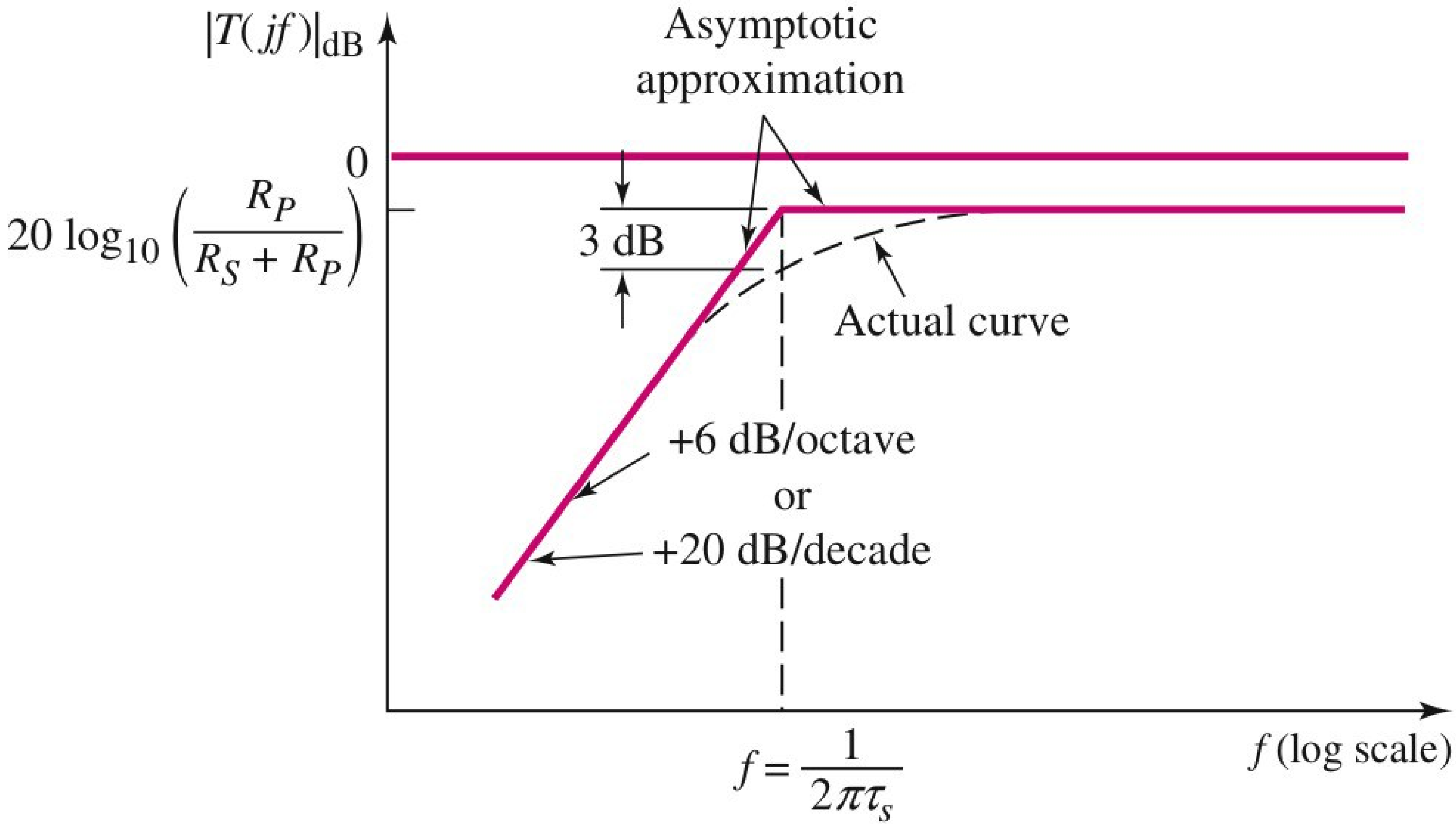
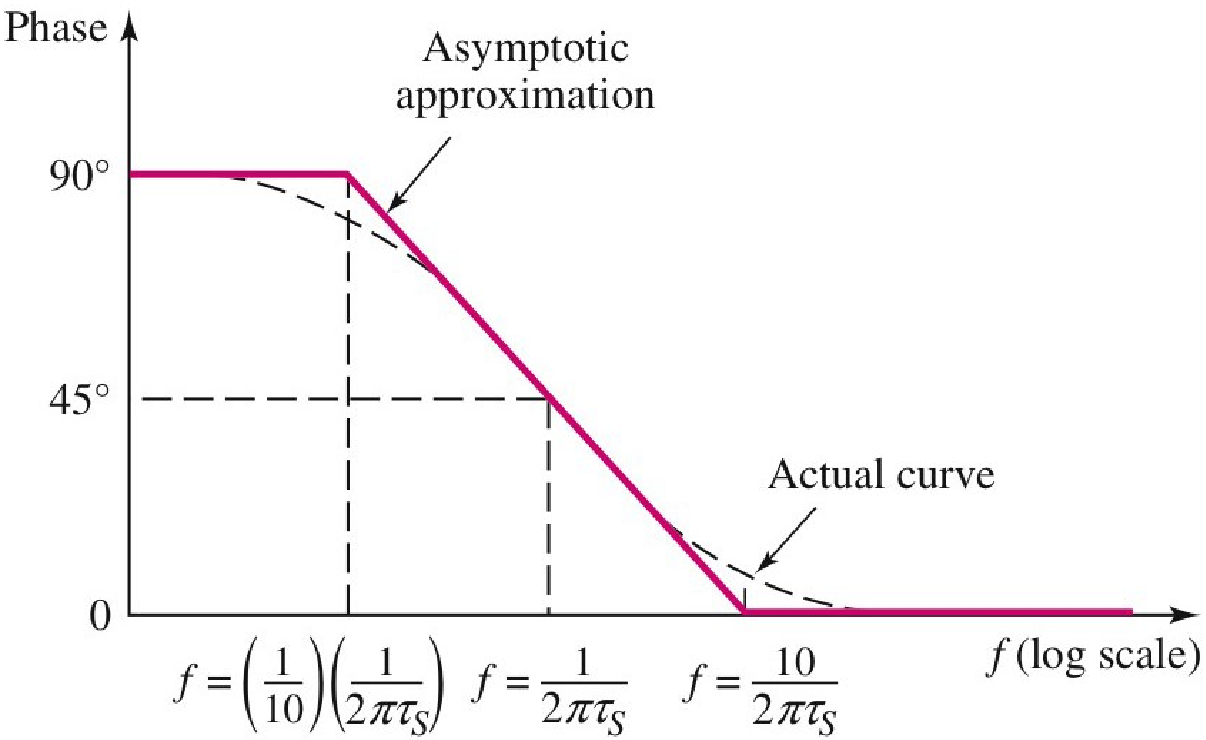
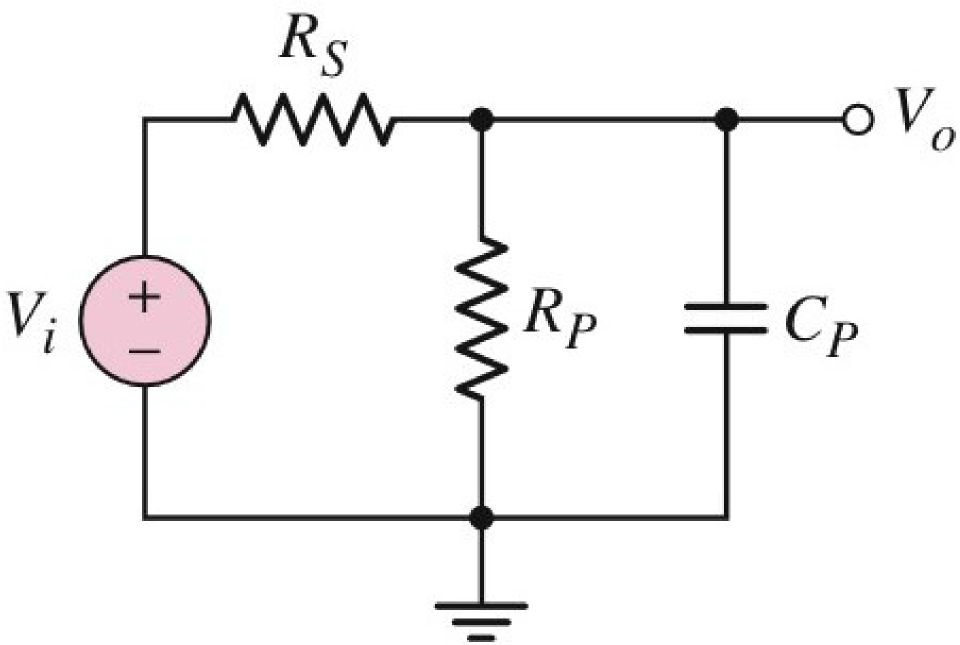


Series coupling capacitor circuit-Bode plots

$$T(s) = K_2\left(\frac{s\tau}{1+s\tau}\right)$$



Parallel load capacitor circuit

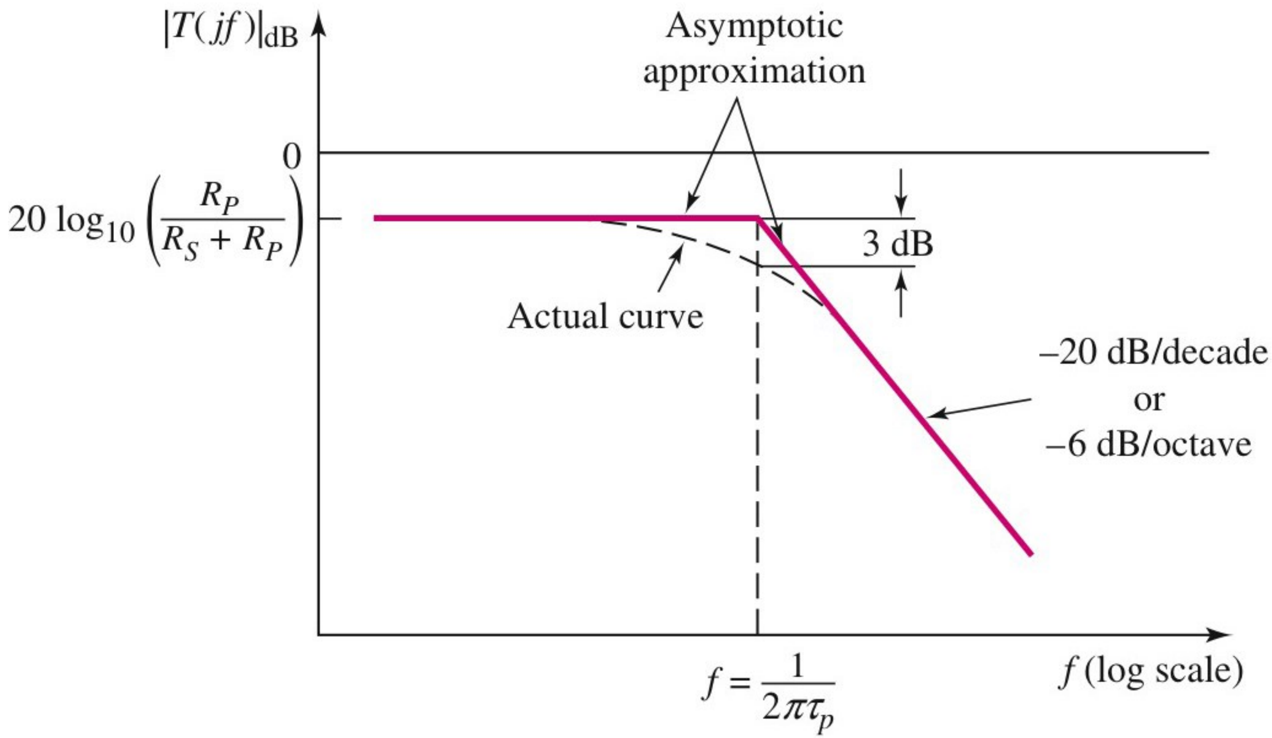


$$\frac{V_o(s)}{V_i(s)} = \left(\frac{R_P}{R_S + R_P}\right)\left[\frac{1}{1 + s(R_S \| R_P)C_P}\right] = K\left(\frac{1}{1 + s\tau_P}\right)$$

$$\tau_P = (R_S \| R_P)C_P$$

Time constant

Parallel load capacitor circuit-Bode plots



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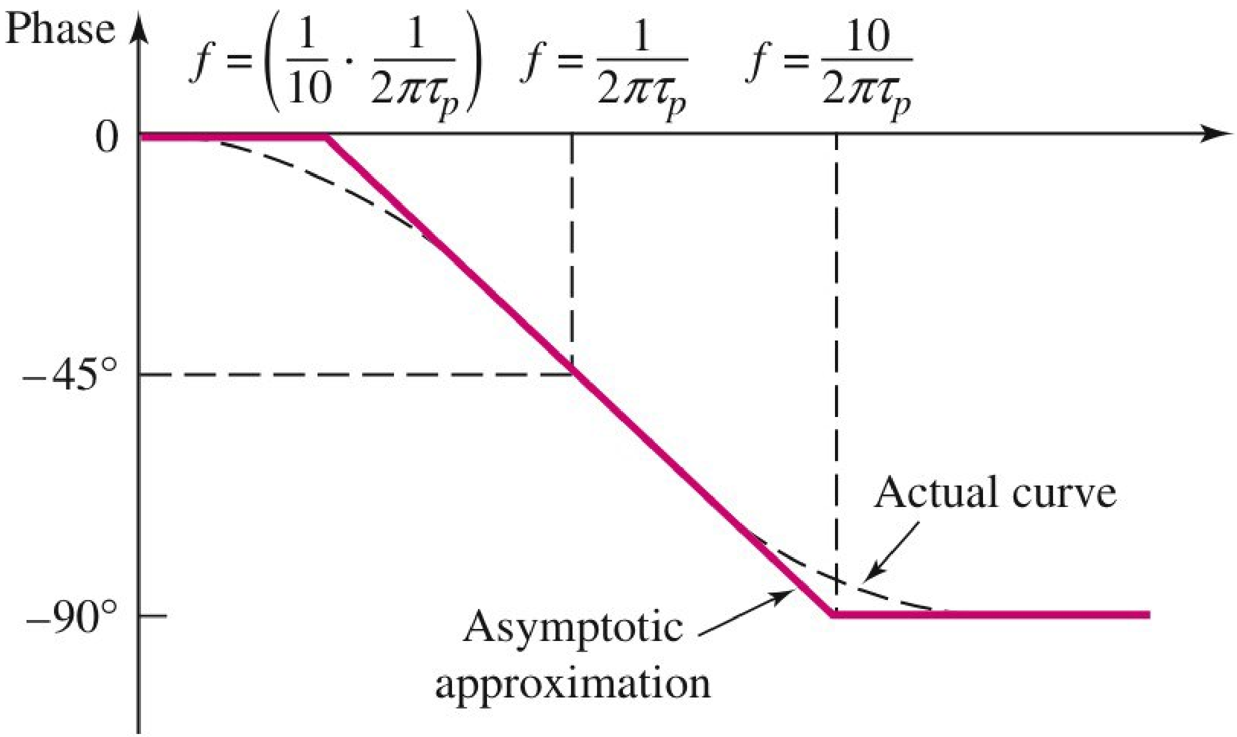
Time constant

Parallel load capacitor circuit-Bode plots



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Time constant