

# ECE 371

## Materials and Devices

10/24/19 - Lecture 17

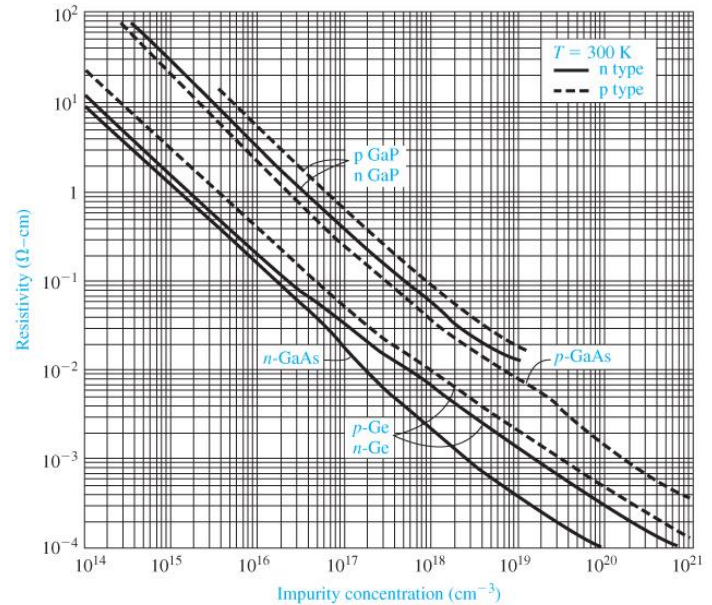
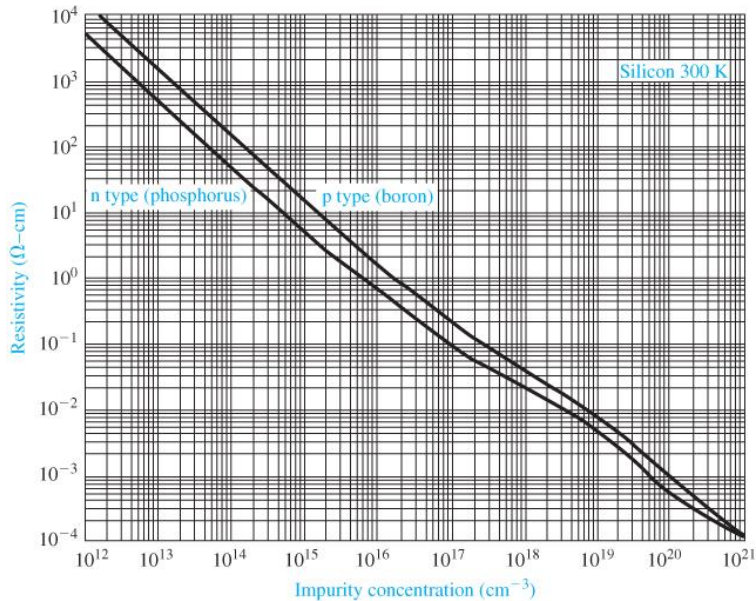
Conductivity/Resistivity, Velocity Saturation,  
Diffusion Current, Graded Impurity Distribution,  
Einstein Relation

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# General Information

- Homework 5 due today before class
- Homework 6 assigned and due 11/07
- Midterm #2 on 10/31, covers Ch. 3, 4, 5. Closed book, one 8.5" x 11" sheet (front and back) allowed.
- Example problems from previous midterms posted
- I will host another review session Monday 10/28 from 3:30-5:30 pm in CHTM 103. We will try to record it again.
- Reading for next time: 5.3 and 7.1 (skip Ch. 6)

# Resistivity ( $\rho$ )



$$\rho = \frac{1}{\sigma} = \frac{1}{e(\mu_n n + \mu_p p)}$$

- $\sigma$  is the conductivity
- Function of mobility and carrier concentration
- Controllable with doping
- p-type resistivity is usually higher than n-type resistivity

# Relation of Resistivity to Resistance

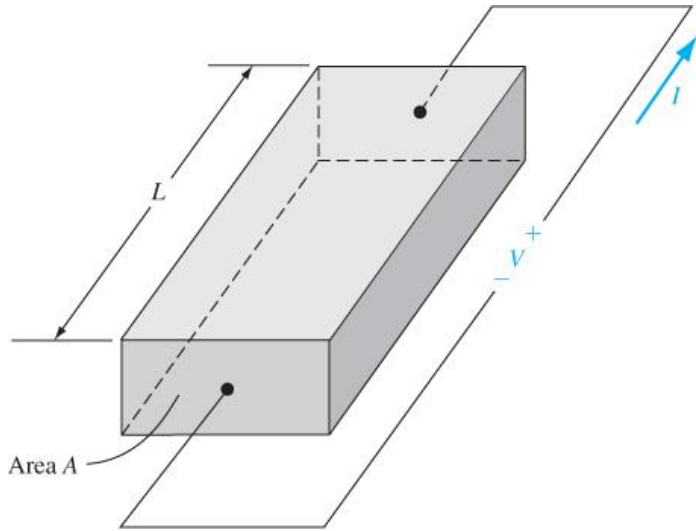


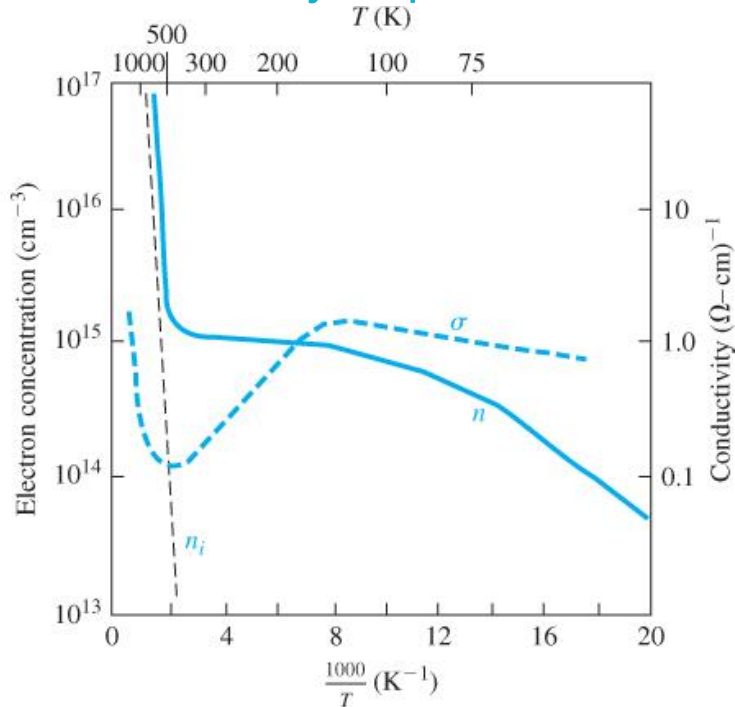
Figure 5.5 | Bar of semiconductor material as a resistor.

$$R = \frac{\rho L}{A}$$

- Current density:  $J = I/A$
- $A$  is the cross-sectional area
- $L$  is the length
- Resistivity is a material property
- Resistance is dependent upon geometry

# $n_0$ and $\sigma$ vs. Temperature

conductivity depends on mobility & carrier concentration



**Figure 5.6** | Electron concentration and conductivity versus inverse temperature for silicon.  
(After Sze [14].)

- At high  $T$ ,  $n_i$  increases and dominates  $n_0$  and  $\sigma$
- Around room temperature,  $n_0$  is almost constant (complete ionization) but  $\sigma$  decreases with increasing  $T$  since  $\mu_n$  decreases
- At low  $T$ , freeze out begins and  $n_0$  and  $\sigma$  decrease

### Ex 5.3

p-type @ 300K,  $N_A = 2.8 \times 10^{17} \text{ cm}^{-3}$   
 $N_D = 8 \times 10^{16} \text{ cm}^{-3}$

Find

(a)  $\mu_p$

(b)  $\sigma$

(c)  $\rho$

① Find  $p_o = \frac{N_A - N_D}{2} + \sqrt{\frac{N_A - N_D}{2}^2 + n_i^2}$

can reduce to  $N_A - N_D$  since  $N_A - N_D \gg n_i$

$$p_o \approx 2 \times 10^{17} \text{ cm}^{-3} \approx N_A$$

② total impurity concentration

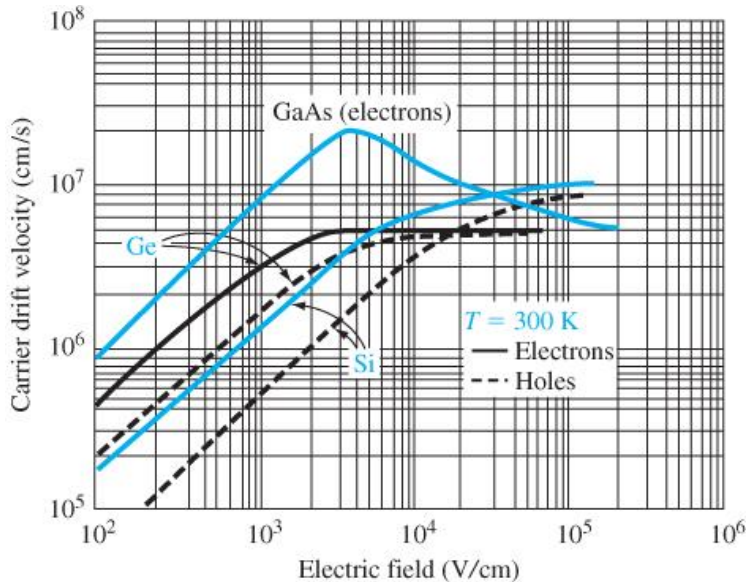
$$N_i = N_A + N_D = 3.6 \times 10^{17} \text{ cm}^{-3}$$

$$\text{using Fig 5.3} \rightarrow \mu_p = 2.0 \times 10^2 \frac{\text{cm}^2}{\text{V}\cdot\text{s}}$$

$$\begin{aligned} \text{③ } \sigma &\approx e \mu_p p = (1.6 \times 10^{-19}) (200 \frac{\text{cm}^2}{\text{V}\cdot\text{s}}) (2 \times 10^{17}) \\ &= 6.4 \frac{1}{\Omega\cdot\text{cm}} \end{aligned}$$

$$\text{④ } \rho = \frac{1}{\sigma} = 0.156 \Omega\cdot\text{cm}$$

# \*Velocity Saturation\*



Materials	$v_s$ (cm/s)
Si	1.0e7
GaAs	7.2e6
SiC	2.2e7
GaN	2.5e7

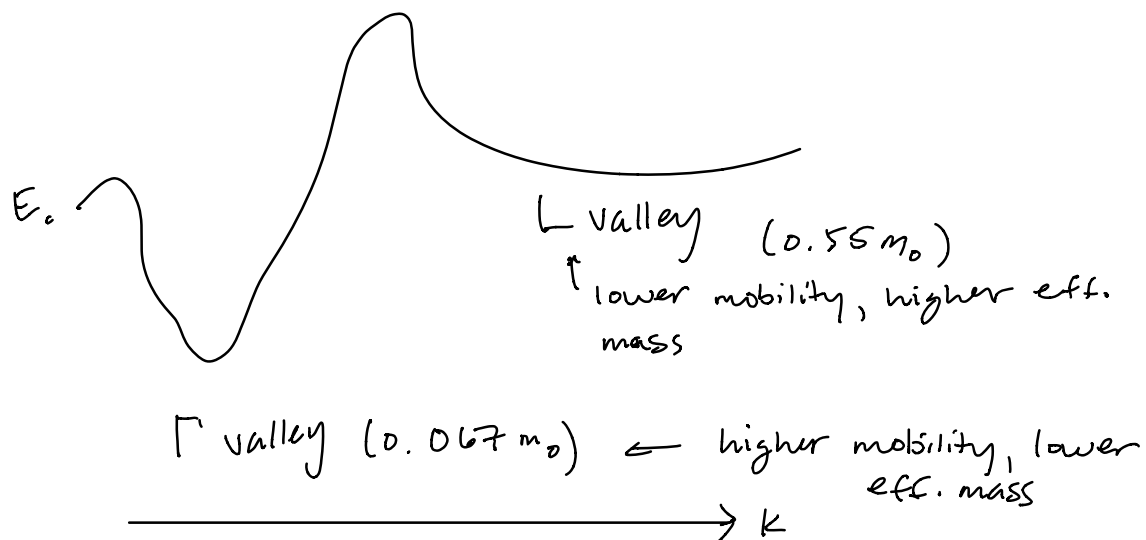
- Saturation velocity is the maximum velocity a carrier can attain in the semiconductor
- Determines the ultimate “speed limit” or frequency limit of transistors
- Mobility is a function of electric field at high field strengths
- When  $v_d$  saturates, so does  $J_{drf}$
- Saturation caused by interaction with phonons

for low fields:  $V_d = \mu E$  (no phonon modes)  
but as E-field goes up, saturation of velocity &  
drift current

mobility is function of E-field

phonon scattering (vibrating of atoms on lattice)

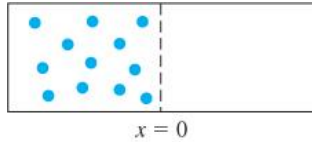
GaAs different — look at dispersion curve



negative differential voltage (neg. resistance)  
can be used in RLE oscillator devices to cancel out  $R$



# Diffusion Current Density



**Figure 5.9** | Container divided by a membrane with gas molecules on one side.

- Diffusion: process by which particles flow from a region of high concentration to a region of low concentration

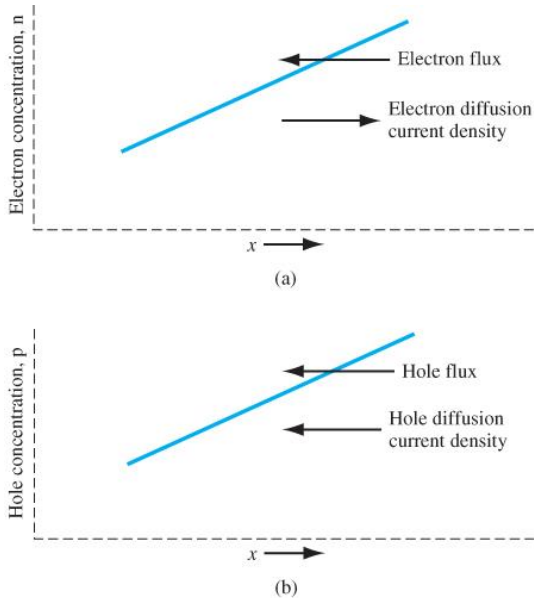
Electron and hole diffusion current densities

$$J_{nx|diff} = eD_n \frac{dn}{dx}$$

\*units of A/cm<sup>2</sup>

$$J_{px|diff} = -eD_p \frac{dp}{dx}$$

- $D_n$  and  $D_p$  are the diffusion constants (units of cm<sup>2</sup>/s) and quantify how well carriers move as a result of a concentration gradient
- Electron diffusion current is in the opposite direction to electron particle flux
- Hole diffusion current is in the same direction as hole particle flux



**Figure 5.11** | (a) Diffusion of electrons due to a density gradient. (b) Diffusion of holes due to a density gradient.

# Total Current

- The total current in a semiconductor is the sum of the drift and diffusion currents for both electrons and holes

$D_n$  = diffusion coefficient  $\left[\frac{\text{cm}^2}{\text{s}}\right]$

$$J = en\mu_n E_x + ep\mu_p E_x + eD_n \frac{dn}{dx} - eD_p \frac{dp}{dx} \left[\frac{\text{A}}{\text{cm}^2}\right]$$

Drift current terms that depend upon electric field

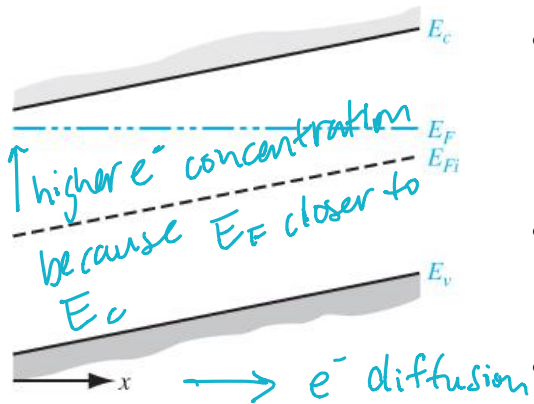
slope of  
electron concentration  
Diffusion current terms  
that depend upon  
concentration gradients

\*For a given situation, we can often neglect some of the terms in the equation for total current

# Test Your Understanding 5.5

**TYU 5.5** The hole concentration in silicon varies linearly from  $x = 0$  to  $x = 0.01$  cm. The hole diffusion coefficient is  $D_p = 10$  cm<sup>2</sup>/s, the hole diffusion current density is 20 A/cm<sup>2</sup>, and the hole concentration at  $x = 0$  is  $p = 4 \times 10^{17}$  cm<sup>-3</sup>. What is the value of the hole concentration at  $x = 0.01$  cm? (Ans.  $2.75 \times 10^{17}$  cm<sup>-3</sup>)

# Graded Impurity Doping



**Figure 5.12** | Energy-band diagram for a semiconductor in thermal equilibrium with a nonuniform donor impurity concentration.

- Electrons on the side with higher doping concentration diffuse to the side with lower doping concentration
- After electrons diffuse, ionize acceptors are left behind
- An electric field is induced by the charge separation to oppose the diffusion process
- Quasi-neutrality is assumed so that  $n_0 \approx N_d$

Fermi level flat because equilibrium

$$E_x(x) = \frac{1}{e} \frac{dE_{Fi}}{dx}$$

\*If the intrinsic Fermi level is not constant, there is an electric field present in the semiconductor

$$E_x(x) = - \left( \frac{kT}{e} \right) \frac{1}{N_d(x)} \frac{dN_d(x)}{dx}$$

\*Induced electric field as a function of impurity concentration as a function of x

If  $E_{Fi}$  is sloped  $\rightarrow$  E-field exists

# The Einstein Relation

$$\frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = \frac{kT}{e}$$

- Einstein relations relate the diffusion constants to the mobilities
- Temperature dependence of the diffusion constants is the result of the temperature dependence of the mobilities (e.g., lattice and ionized impurity scattering)
- The diffusion constants are ~40X smaller than the mobilities at room temperature

**Table 5.2** | Typical mobility and diffusion coefficient values at  $T = 300$  K ( $\mu = \text{cm}^2/\text{V}\cdot\text{s}$  and  $D = \text{cm}^2/\text{s}$ )

	$\mu_n$	$D_n$	$\mu_p$	$D_p$
Silicon	1350	35	480	12.4
Gallium arsenide	8500	220	400	10.4
Germanium	3900	101	1900	49.2