

ECE 371
Materials and Devices
HW #5

Due: Thursday 10/24/19 at the beginning of class

*All problems from Neamen 4th Edition Ch. 4.

- 4.3** (a) The maximum intrinsic carrier concentration in a silicon device must be limited to $5 \times 10^{11} \text{ cm}^{-3}$. Assume $E_g = 1.12 \text{ eV}$. Determine the maximum temperature allowed for the device. (b) Repeat part (a) if the maximum intrinsic carrier concentration is limited to $5 \times 10^{12} \text{ cm}^{-3}$.

Hint: since the effective density of states and the exponent depend upon temperature, you cannot solve for T explicitly. You must guess at a temperature, find n_i , and then iterate until both sides of the equation converge.

- 4.5** Two semiconductor materials have exactly the same properties except material A has a bandgap energy of 0.90 eV and material B has a bandgap energy of 1.10 eV. Determine the ratio of n_i of material B to that of material A for (a) $T = 200 \text{ K}$, (b) $T = 300 \text{ K}$, and (c) $T = 400 \text{ K}$.
- 4.6** (a) The magnitude of the product $g_c(E)f_F(E)$ in the conduction band is a function of energy as shown in Figure 4.1. Assume the Boltzmann approximation is valid. Determine the energy with respect to E_c at which the maximum occurs. (b) Repeat part (a) for the magnitude of the product $g_v(E)[1 - f_F(E)]$ in the valence band.
- 4.10** Given the effective masses of electrons and holes in silicon, germanium, and gallium arsenide, calculate the position of the intrinsic Fermi energy level with respect to the center of the bandgap for each semiconductor at $T = 300 \text{ K}$.
- 4.19** The electron concentration in silicon at $T = 300 \text{ K}$ is $n_0 = 2 \times 10^5 \text{ cm}^{-3}$. (a) Determine the position of the Fermi level with respect to the valence band energy level. (b) Determine p_0 . (c) Is this n- or p-type material?
- 4.20** (a) If $E_c - E_F = 0.28 \text{ eV}$ in gallium arsenide at $T = 375 \text{ K}$, calculate the values of n_0 and p_0 . (b) Assuming the value of n_0 in part (a) remains constant, determine $E_c - E_F$ and p_0 at $T = 300 \text{ K}$.
- 4.22** The Fermi energy level in silicon at $T = 300 \text{ K}$ is as close to the top of the valence band as to the midgap energy. (a) Is the material n type or p type? (b) Calculate the values of n_0 and p_0 .
- 4.39** A silicon semiconductor material at $T = 300 \text{ K}$ is doped with arsenic atoms to a concentration of $2 \times 10^{15} \text{ cm}^{-3}$ and with boron atoms to a concentration of $1.2 \times 10^{15} \text{ cm}^{-3}$. (a) Is the material n type or p type? (b) Determine n_0 and p_0 . (c) Additional boron atoms are to be added such that the hole concentration is $4 \times 10^{15} \text{ cm}^{-3}$. What concentration of boron atoms must be added and what is the new value of n_0 ?
- 4.45** A particular semiconductor material is doped at $N_d = 2 \times 10^{14} \text{ cm}^{-3}$ and $N_a = 1.2 \times 10^{14} \text{ cm}^{-3}$. The thermal equilibrium electron concentration is found to be $n_0 = 1.1 \times 10^{14} \text{ cm}^{-3}$. Assuming complete ionization, determine the intrinsic carrier concentration and the thermal equilibrium hole concentration.

- 4.52** Consider GaAs at $T = 300$ K with $N_d = 0$. (a) Plot the position of the Fermi energy level with respect to the intrinsic Fermi energy level as a function of the acceptor impurity concentration over the range of $10^{14} \leq N_a \leq 10^{17} \text{ cm}^{-3}$. (b) Plot the position of the Fermi energy level with respect to the valence-band energy over the same acceptor impurity concentration as given in part (a).
- 4.61** A new semiconductor material is to be “designed.” The semiconductor is to be p type and doped with $5 \times 10^{15} \text{ cm}^{-3}$ acceptor atoms. Assume complete ionization and assume $N_d = 0$. The effective density of states functions are $N_c = 1.2 \times 10^{19} \text{ cm}^{-3}$ and $N_v = 1.8 \times 10^{19} \text{ cm}^{-3}$ at $T = 300$ K and vary as T^2 . A special semiconductor device fabricated with this material requires that the hole concentration be no greater than $5.08 \times 10^{15} \text{ cm}^{-3}$ at $T = 350$ K. What is the minimum bandgap energy required in this new material?

- 4.3 (a) The maximum intrinsic carrier concentration in a silicon device must be limited to $5 \times 10^{11} \text{ cm}^{-3}$. Assume $E_g = 1.12 \text{ eV}$. Determine the maximum temperature allowed for the device. (b) Repeat part (a) if the maximum intrinsic carrier concentration is limited to $5 \times 10^{12} \text{ cm}^{-3}$.

$$(a) n_i = 5 \times 10^{11} \text{ cm}^{-3}$$

$$n_i^2 = N_c N_v \exp\left(\frac{-E_g}{kT}\right)$$

$$25 \times 10^{22} = (2.8 \times 10^{19})(1.04 \times 10^{19}) \left(\frac{T}{300}\right)^3 \exp\left[\frac{-1.12}{(0.0259)(T/300)}\right]$$

$$T = 367.408 \text{ K}$$

$$(b) n_i = 5 \times 10^{12} \text{ cm}^{-3}$$

$$T = 417.321 \text{ K}$$

Table 4.1 | Effective density of states function and density of states effective mass values

	$N_c (\text{cm}^{-3})$	$N_v (\text{cm}^{-3})$	m_s^*/m_0	m_p^*/m_0
Silicon	2.8×10^{19}	1.04×10^{19}	1.08	0.56

$$n_i^2 = N_c N_v \exp\left[\frac{-(E_c - E_v)}{kT}\right] = N_c N_v \exp\left[\frac{-E_g}{kT}\right] \quad (4.23)$$

- 4.5 Two semiconductor materials have exactly the same properties except material A has a bandgap energy of 0.90 eV and material B has a bandgap energy of 1.10 eV. Determine the ratio of n_i of material B to that of material A for (a) $T = 200$ K, (b) $T = 300$ K, and (c) $T = 400$ K.

$$\frac{n_i(A)}{n_i(B)} = \frac{\exp\left(\frac{-E_g(A)}{kT}\right)}{\exp\left(\frac{-E_g(B)}{kT}\right)} = \exp\left[\frac{-(E_g(A) - E_g(B))}{kT}\right] \quad \frac{n_i(B)}{n_i(A)} = \exp\left(\frac{-(B-A)}{kT}\right)$$

$$(a) \frac{n_i(A)}{n_i(B)} = \exp\left[\frac{-(0.90 - 1.10)}{2(0.026)(\frac{200}{300})}\right] = 327.506 \quad \frac{n_i(B)}{n_i(A)} = \exp\left(\frac{-(0.90 - 1.10)}{kT}\right)$$

$$(b) \frac{n_i(A)}{n_i(B)} = \exp\left[\frac{-(0.90 - 1.10)}{2(0.026)(\frac{300}{300})}\right] = 47.513$$

$$(c) \frac{n_i(A)}{n_i(B)} = \exp\left[\frac{-(0.90 - 1.10)}{2(0.026)(\frac{400}{300})}\right] = 18.097$$

- 4.6 (a) The magnitude of the product $g_c(E)f_F(E)$ in the conduction band is a function of energy as shown in Figure 4.1. Assume the Boltzmann approximation is valid. Determine the energy with respect to E_c at which the maximum occurs. (b) Repeat part (a) for the magnitude of the product $g_v(E)[1 - f_F(E)]$ in the valence band.

$$(a) g_c f_F \rightarrow \sqrt{E - E_c} \exp \left[-\frac{(E - E_F)}{kT} \right]$$

maximum occurs at

$$E = E_c + \frac{kT}{2}$$

(b) same as part a

$$E = E_v - \frac{kT}{2}$$

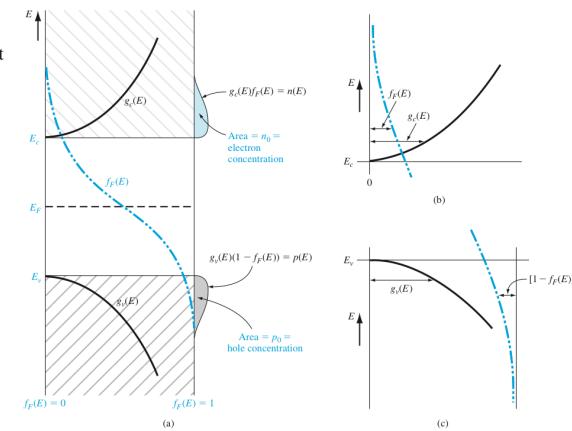


Figure 4.1 | (a) Density of states functions, Fermi-Dirac probability function, and areas representing electron and hole concentrations for the case when E_g is near the midgap energy; (b) expanded view near the conduction-band energy; and (c) expanded view near the valence-band energy.

- 4.10 Given the effective masses of electrons and holes in silicon, germanium, and gallium arsenide, calculate the position of the intrinsic Fermi energy level with respect to the center of the bandgap for each semiconductor at $T = 300$ K.

$$E_{Fi} - E_{\text{midgap}} = \frac{3}{4} kT \ln \left(\frac{m_p^*}{m_n^*} \right) \quad (4.26b)$$

Table 4.1 | Effective density of states function and density of states effective mass values

	N_c (cm^{-3})	N_v (cm^{-3})	m_n^*/m_0	m_p^*/m_0
Silicon	2.8×10^{19}	1.04×10^{19}	1.08	0.56
Gallium arsenide	4.7×10^{17}	7.0×10^{18}	0.067	0.48
Germanium	1.04×10^{19}	6.0×10^{18}	0.55	0.37

Silicon

$$\frac{3}{4} kT \ln \left(\frac{m_p^*}{m_n^*} \right) = -1.2758 \times 10^{-2} \text{ eV}$$

Germanium

$$\frac{3}{4} kT \ln \left(\frac{m_p^*}{m_n^*} \right) = -7.7004 \times 10^{-3} \text{ eV}$$

Gallium arsenide

$$\frac{3}{4} kT \ln \left(\frac{m_p^*}{m_n^*} \right) = +3.8250 \times 10^{-2} \text{ eV}$$

- 4.19 The electron concentration in silicon at $T = 300$ K is $n_0 = 2 \times 10^5 \text{ cm}^{-3}$. (a) Determine the position of the Fermi level with respect to the valence band energy level. (b) Determine p_0 . (c) Is this n- or p-type material?

$$n_0 = N_c \exp \left[\frac{-(E_c - E_F)}{kT} \right]$$

$$(a) n_0 = N_c \exp \left[-\frac{(E_c - E_F)}{kT} \right] \rightarrow kT \ln \left(\frac{n_0}{N_c} \right) = -(E_c - E_F)$$

$$E_c - E_F = kT \ln \left(\frac{N_c}{n_0} \right) = 0.0259 \ln \left(\frac{2.8 \times 10^{19}}{2.0 \times 10^5} \right)$$

$$= 843.6320 \times 10^{-3} \text{ eV}$$

$$E_F - E_v = 1.12 - 843.6320 \times 10^{-3} \text{ eV}$$

$$= 276.3680 \times 10^{-3} \text{ eV}$$

$$(b) p_0 = \frac{n_i^2}{n_0} = \frac{(1.5 \times 10^{10})^2}{2 \times 10^5} = 1.125 \times 10^{15} \text{ cm}^{-3}$$

Table 4.2 | Commonly accepted values of n_i at $T = 300$ K

Silicon	$n_i = 1.5 \times 10^{10} \text{ cm}^{-3}$
Gallium arsenide	$n_i = 1.8 \times 10^{16} \text{ cm}^{-3}$
Germanium	$n_i = 2.4 \times 10^{15} \text{ cm}^{-3}$

(c) P-type

$$p_0 = n_v \exp \left[\frac{-(E_F - E_v)}{kT} \right]$$

$$= 1.04 \times 10^{-19} \left[\frac{-(276.36 \times 10^{-3})}{0.0259} \right]$$

- 4.20 (a) If $E_c - E_F = 0.28$ eV in gallium arsenide at $T = 375$ K, calculate the values of n_0 and p_0 . (b) Assuming the value of n_0 in part (a) remains constant, determine $E_c - E_F$ and p_0 at $T = 300$ K.

$$n_0 = N_c \exp \left[\frac{-(E_c - E_F)}{kT} \right]$$

$$(a) \quad kT_{T=375} = 0.03231$$

$$N_c_{T=375} = 6.56845 \times 10^{17}$$

$$n_0 = N_c \exp \left[-\frac{(E_c - E_F)}{kT} \right]$$

$$= 6.56845 \times 10^{17} \exp \left[\frac{-0.28 \text{ eV}}{0.03231} \right]$$

$$= 1.1335 \times 10^{14} \text{ cm}^{-3}$$

$$N_v_{T=375} = 9.78280 \times 10^{18}$$

$$n_i^2 = N_c N_v \exp \left[-\frac{E_g}{kT} \right]$$

$$= (6.56845 \times 10^{17})(9.78280 \times 10^{18}) \exp \left[\frac{-1.42}{0.03231} \right]$$

$$= 5.29614 \times 10^{17}$$

$$p_0 = \frac{n_i^2}{n_0} = 4.67232 \times 10^3 \text{ cm}^{-3}$$

$$(b) \quad n_0 = N_c \exp \left[-\frac{(E_c - E_F)}{kT} \right] \rightarrow E_c - E_F = kT \ln \left(\frac{N_c}{n_0} \right)$$

$$E_c - E_F = 0.0259 \ln \left(\frac{4.67232 \times 10^3}{1.1335 \times 10^{14}} \right) = 0.2157 \text{ eV}$$

$$E_F - E_v = 1.42 - 0.2157 \text{ eV} = 1.2043 \text{ eV}$$

$$p_0 = 7 \times 10^{18} \exp \left[\frac{-1.2043}{0.0259} \right] = 4.479 \times 10^{-2} \text{ cm}^{-3}$$

Table 4.1 | Effective density of states function and density of states effective mass values

	$N_c (\text{cm}^{-3})$	$N_v (\text{cm}^{-3})$	m_n^*/m_0	m_p^*/m_0
Silicon	2.8×10^{19}	1.04×10^{19}	1.08	0.56
Gallium arsenide	4.7×10^{17}	7.0×10^{18}	0.067	0.48
Germanium	1.04×10^{19}	6.0×10^{18}	0.55	0.37

$$N_c = 2 \left(\frac{2\pi m_n^* kT}{h^2} \right)^{\frac{3}{2}}$$

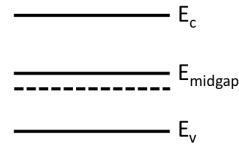
$$N_v = 2 \left(\frac{2\pi m_p^* kT}{h^2} \right)^{\frac{3}{2}}$$

$$n_i^2 = N_c N_v \exp \left[-\frac{E_g}{kT} \right]$$

$T = 300 \text{ K}$	$E_g (\text{eV})$	$n_i (\text{cm}^{-3})$
Si	1.12	1.5×10^{10}
GaAs	1.42	1.8×10^6
Ge	0.66	2.4×10^{13}

- 4.22 The Fermi energy level in silicon at $T = 300$ K is as close to the top of the valence band as to the midgap energy. (a) Is the material n type or p type? (b) Calculate the values of n_0 and p_0 .

(a) P-type



$$(b) E_F - E_v = \frac{1.12}{4} = 0.28$$

$$p_0 = N_v \exp \left[\frac{-E_g}{kT} \right] = 1.04 \times 10^{19} \exp \left[\frac{-0.28}{0.0259} \right]$$

$$= 2.0570 \times 10^{14} \text{ cm}^{-3}$$

$$E_c - E_F = 1.12 - 0.28 = 0.84$$

$$n_0 = N_c \exp \left[\frac{-E_g}{kT} \right] = 2.8 \times 10^{19} \exp \left[\frac{-0.84}{0.0259} \right]$$

$$= 2.16658 \times 10^5 \text{ cm}^{-3} \quad \text{further proving it is p-type}$$

- 4.39 A silicon semiconductor material at $T = 300$ K is doped with arsenic atoms to a concentration of $2 \times 10^{15} \text{ cm}^{-3}$ and with boron atoms to a concentration of $1.2 \times 10^{15} \text{ cm}^{-3}$. (a) Is the material n type or p type? (b) Determine n_0 and p_0 . (c) Additional boron atoms are to be added such that the hole concentration is $4 \times 10^{15} \text{ cm}^{-3}$. What concentration of boron atoms must be added and what is the new value of n_0 ?

$$n_i = 1.5 \times 10^{10}$$

$$(a) N_A = 1.2 \times 10^{15} < N_D = 2 \times 10^{15} \rightarrow \boxed{\text{n-type}}$$

$$(b) p_0 = \frac{N_A - N_D}{2} + \sqrt{\left(\frac{N_A - N_D}{2}\right)^2 + n_i^2}$$

$$= \boxed{2.81248 \times 10^5 \text{ cm}^{-3}}$$

$$n_0 = \frac{n_i^2}{p_0} = \boxed{3.00006 \times 10^{14} \text{ cm}^{-3}}$$

$$p_0 = \frac{N_a - N_d}{2} + \sqrt{\left(\frac{N_a - N_d}{2}\right)^2 + n_i^2} \quad (4.62)$$

$$(c) p'_0 = 4 \times 10^{15}$$

$$4 \times 10^{15} = (N'_A + N_A) - N_D$$

$$N'_A = \boxed{4.8 \times 10^{15}}$$

$$n'_0 = \frac{n_i^2}{p'_0} = \boxed{5.625 \times 10^4 \text{ cm}^{-3}}$$

- 4.45 A particular semiconductor material is doped at $N_d = 2 \times 10^{14} \text{ cm}^{-3}$ and $N_a = 1.2 \times 10^{14} \text{ cm}^{-3}$. The thermal equilibrium electron concentration is found to be $n_0 = 1.1 \times 10^{14} \text{ cm}^{-3}$. Assuming complete ionization, determine the intrinsic carrier concentration and the thermal equilibrium hole concentration.

$$n_0 = \frac{(N_d - N_a)}{2} + \sqrt{\left(\frac{N_d - N_a}{2}\right)^2 + n_i^2}$$

n-type → majority carrier = electrons

$$n_0 = \frac{(N_d - N_a)}{2} + \sqrt{\left(\frac{N_d - N_a}{2}\right)^2 + n_i^2}$$

$$1.1 \times 10^{14} = 4 \times 10^{13} + \sqrt{1.6 \times 10^{27} + n_i^2}$$

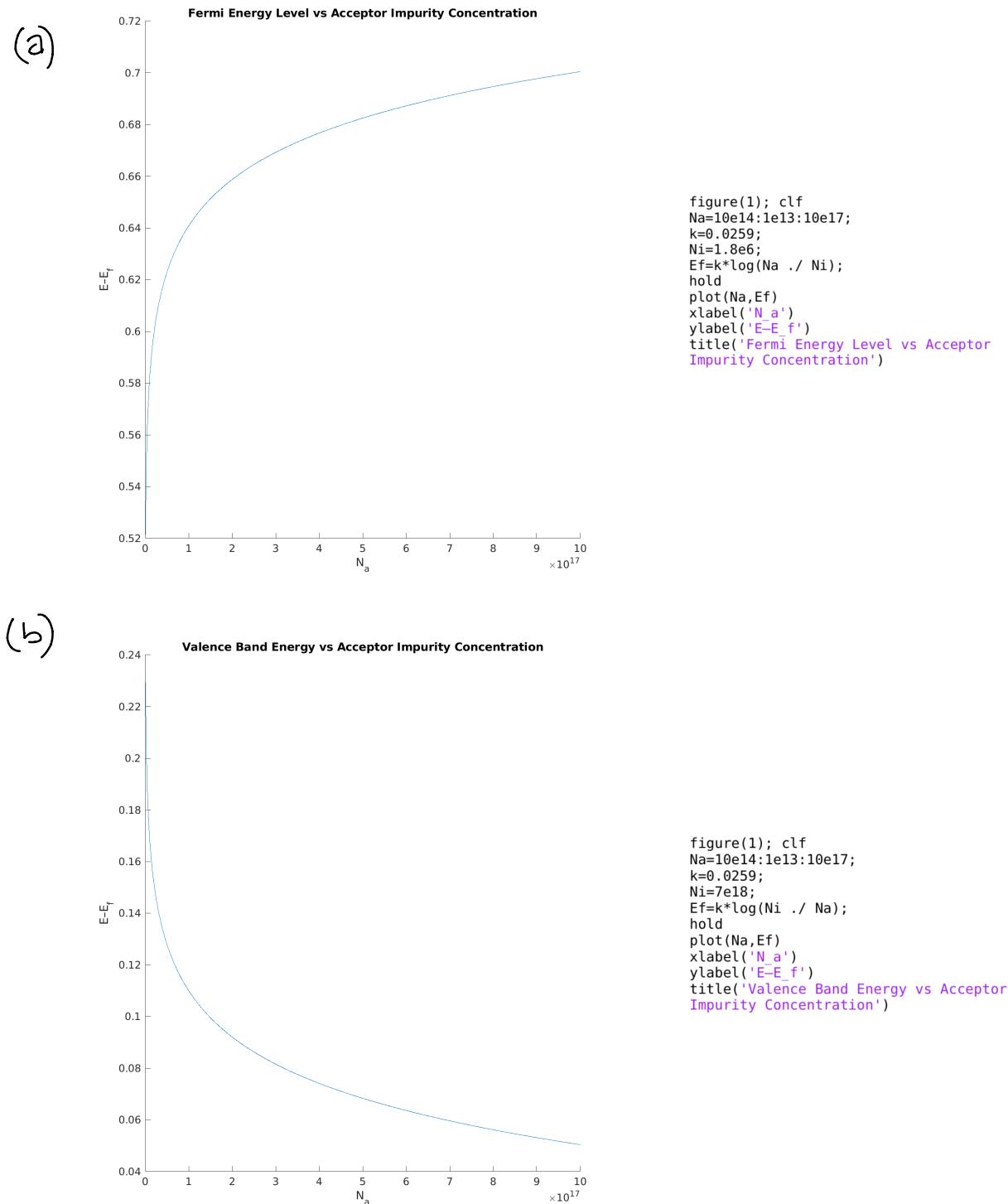
$$7.0 \times 10^{13} = \sqrt{1.6 \times 10^{27} + n_i^2}$$

$$4.9 \times 10^{27} = 1.6 \times 10^{27} + n_i^2$$

$$3.3 \times 10^{27} = n_i^2 \rightarrow n_i = \boxed{5.74456 \times 10^{13} \text{ cm}^{-3}}$$

$$p_0 = \frac{n_i^2}{n_0} = \boxed{3.00 \times 10^{13} \text{ cm}^{-3}}$$

- 4.52** Consider GaAs at $T = 300$ K with $N_d = 0$. (a) Plot the position of the Fermi energy level with respect to the intrinsic Fermi energy level as a function of the acceptor impurity concentration over the range of $10^{14} \leq N_a \leq 10^{17} \text{ cm}^{-3}$. (b) Plot the position of the Fermi energy level with respect to the valence-band energy over the same acceptor impurity concentration as given in part (a).



- 4.61 A new semiconductor material is to be "designed." The semiconductor is to be p type and doped with $5 \times 10^{15} \text{ cm}^{-3}$ acceptor atoms. Assume complete ionization and assume $N_d = 0$. The effective density of states functions are $N_c = 1.2 \times 10^{19} \text{ cm}^{-3}$ and $N_v = 1.8 \times 10^{19} \text{ cm}^{-3}$ at $T = 300 \text{ K}$ and vary as T^2 . A special semiconductor device fabricated with this material requires that the hole concentration be no greater than $5.08 \times 10^{15} \text{ cm}^{-3}$ at $T = 350 \text{ K}$. What is the minimum bandgap energy required in this new material?

$$N_A = 5 \times 10^{15} \text{ cm}^{-3} \quad N_D = 0$$

$$n_i^2 = (1.2 \times 10^{19})(1.8 \times 10^{19}) \exp\left[-\frac{E_g}{kT}\right]$$

$$p_0 \leq 5.08 \times 10^{15} \text{ cm}^{-3} \text{ at } 350 \text{ K}$$

$$2.58 \times 10^{15} = \sqrt{6.25 \times 10^{30} + n_i^2}$$

$$p_0 = \frac{N_a - N_d}{2} + \sqrt{\left(\frac{N_a - N_d}{2}\right)^2 + n_i^2} \quad (4.62)$$

$$n_i^2 = 4.064 \times 10^{29} \rightarrow n_i = 6.37495 \times 10^{14} \text{ cm}^{-3}$$

$$4.064 \times 10^{29} = 2.16 \times 10^{38} \left(\frac{350}{300}\right)^4 \exp\left[\frac{-E_g}{0.0259\left(\frac{350}{300}\right)}\right]$$

$$-E_g \geq 3.02167 \times 10^{-2} \ln(1.01558 \times 10^{-9}) =$$

$$E_g \geq \boxed{0.62572 \text{ eV}}$$

$$n_i^2 = N_c N_v \exp\left[-\frac{E_g}{kT}\right]$$

T = 300 K	E _g (eV)	n _i (cm ⁻³)
Si	1.12	1.5e10
GaAs	1.42	1.8e6
Ge	0.66	2.4e13