ECE 371
Materials and Devices

10/17/19 - Lecture 15

Fermi Level Position, Drift Current, Mobility

General Information

- Homework 4 solutions posted
- Homework 5 assigned and due 10/24
- Midterm #2 on 10/31, covers Ch. 3, 4, 5
- Reading for next time: 5.1-5.2

Charge Neutrality

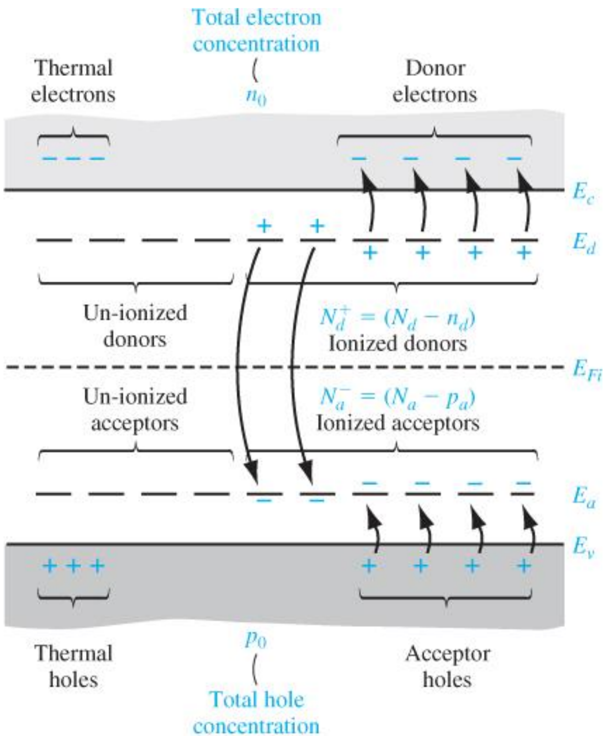

Figure 4.14 | Energy-band diagram of a compensated semiconductor showing ionized and un-ionized donors and acceptors.

- *__Compensated Semiconductor:__* contains both donor and acceptor impurity atoms in the same region

$N_d > N_a$ n-type compensated
$N_a > N_d$ p-type compensated
$N_a = N_d$ completely compensated

- In thermal equilibrium, the net charge density is zero

charge neutrality

$$\boxed{n_0 + N_a^- = p_0 + N_d^+}$$

$$\boxed{n_0 + (N_a - p_a) = p_0 + (N_d - n_d)}$$

Equilibrium Concentrations

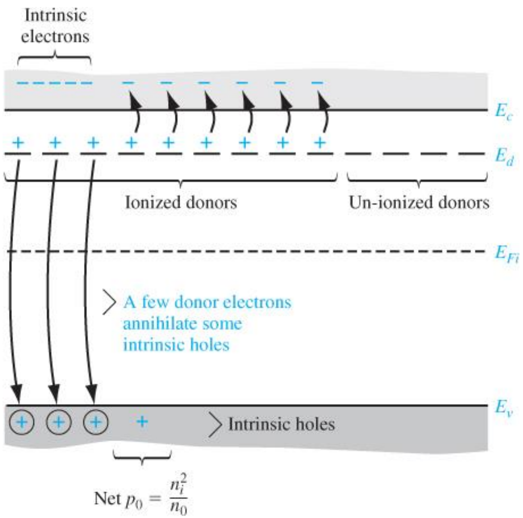

Figure 4.15 | Energy-band diagram showing the redistribution of electrons when donors are added.

Electron concentration in an n-type semiconductor

$$n_0 = \frac{(N_d - N_a)}{2} + \sqrt{\left(\frac{N_d - N_a}{2}\right)^2 + n_i^2}$$

Hole concentration in a p-type semiconductor

$$p_0 = \frac{(N_a - N_d)}{2} + \sqrt{\left(\frac{N_a - N_d}{2}\right)^2 + n_i^2}$$

- Above expressions assume complete ionization
- Calculate majority carrier concentration using the above expression and then calculate minority carrier concentration using $n_i^2 = n_0 p_0$
- Some impurity donors/acceptors annihilate intrinsic holes/electrons
- Net electron concentration is not simply the sum of N_d and n_i
- If $N_a = N_d$, the material behaves as if it is intrinsic

Example 4.9

Objective: Determine the thermal-equilibrium electron and hole concentrations in silicon at $T = 300$ K for given doping concentrations. (*a*) Let $N_d = 10^{16}$ cm^{-3} and $N_a = 0$. (*b*) Let $N_d = 5 \times 10^{15}$ cm^{-3} and $N_a = 2 \times 10^{15}$ cm^{-3}.

EXAMPLE 4.9

Recall that $n_i = 1.5 \times 10^{10}$ cm^{-3} in silicon at $T = 300$ K.

■ **Solution**

(*a*) From Equation (4.60), the majority carrier electron concentration is

$$n_0 = \frac{10^{16}}{2} + \sqrt{\left(\frac{10^{16}}{2}\right)^2 + (1.5 \times 10^{10})^2} \cong 10^{16}\ \text{cm}^{-3}$$

The minority carrier hole concentration is found to be

$$p_0 = \frac{n_i^2}{n_0} = \frac{(1.5 \times 10^{10})^2}{10^{16}} = 2.25 \times 10^4\ \text{cm}^{-3}$$

(*b*) Again, from Equation (4.60), the majority carrier electron concentration is

$$n_0 = \frac{5 \times 10^{15} - 2 \times 10^{15}}{2} + \sqrt{\left(\frac{5 \times 10^{15} - 2 \times 10^{15}}{2}\right)^2 + (1.5 \times 10^{10})^2} \cong 3 \times 10^{15}\ \text{cm}^{-3}$$

The minority carrier hole concentration is

$$p_0 = \frac{n_i^2}{n_0} = \frac{(1.5 \times 10^{10})^2}{3 \times 10^{15}} = 7.5 \times 10^4\ \text{cm}^{-3}$$

Concentration vs. Temperature

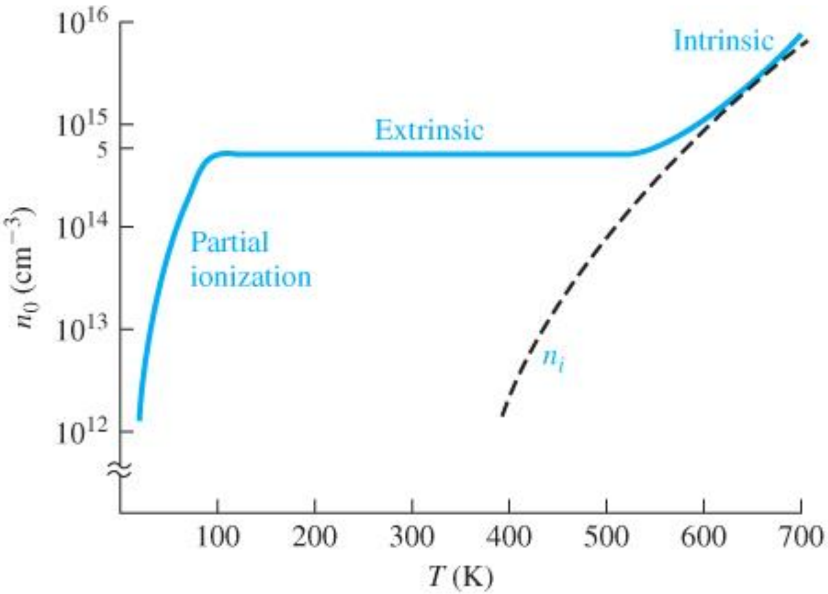

Figure 4.16 | Electron concentration versus temperature showing the three regions: partial ionization, extrinsic, and intrinsic.

Fermi Level Position

- Fermi level as a function of doping concentration and temperature
- Assuming Boltzmann approximation and 100% ionization

n-type

$$\boxed{E_c - E_F = kT\ ln\left(\frac{N_c}{N_d}\right)}$$

p-type

$$\boxed{E_F - E_v = kT\ ln\left(\frac{N_v}{N_a}\right)}$$

- If there is any compensation, replace $N_d \rightarrow N_d - N_a$ for n-type and $N_a \rightarrow N_a - N_d$ for p-type
- We can also find a form relating the Fermi level and the carrier density

n-type

$$\boxed{E_F - E_{Fi} = kT\ ln\left(\frac{n_0}{n_i}\right)}$$

p-type

$$\boxed{E_{Fi} - E_F = kT\ ln\left(\frac{p_0}{n_i}\right)}$$

Example 4.12

DESIGN EXAMPLE 4.12

Objective: Determine the required donor impurity concentration to obtain a specified Fermi energy.

Silicon at $T = 300$ K contains an acceptor impurity concentration of $N_a = 10^{16}$ cm^{-3}. Determine the concentration of donor impurity atoms that must be added so that the silicon is n type and the Fermi energy is 0.20 eV below the conduction-band edge.

■ **Solution**

From Equation (4.64), we have

$$E_c - E_F = kT \ln\left(\frac{N_c}{N_d - N_a}\right)$$

which can be rewritten as

$$N_d - N_a = N_c \exp\left[\frac{-(E_c - E_F)}{kT}\right]$$

Then

$$N_d - N_a = 2.8 \times 10^{19} \exp\left[\frac{-0.20}{0.0259}\right] = 1.24 \times 10^{16} \text{ cm}^{-3}$$

or

$$N_d = 1.24 \times 10^{16} + N_a = 2.24 \times 10^{16} \text{ cm}^{-3}$$

Fermi Level vs. Temperature

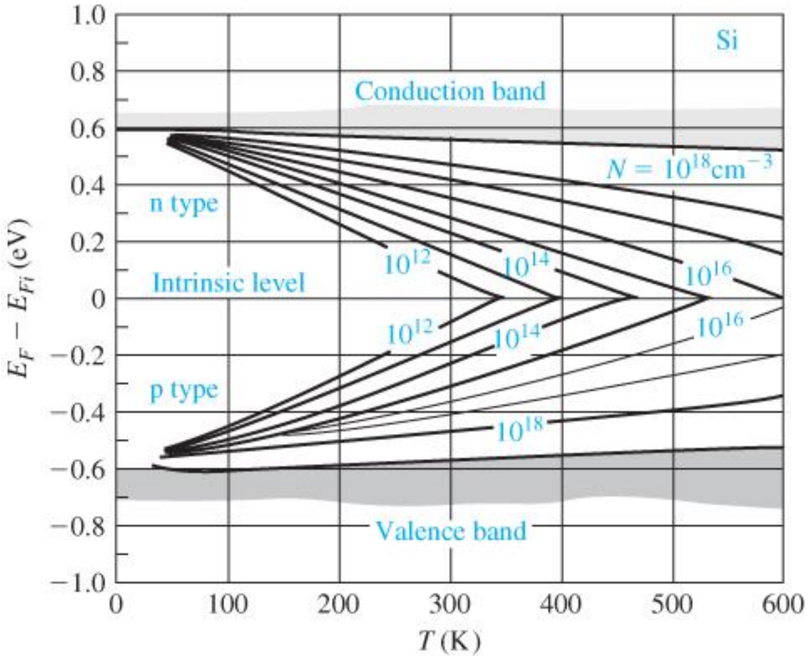

Figure 4.19 | Position of Fermi level as a function of temperature for various doping concentrations. *(From Sze [14].)*

Recall:

$$n_i^2 = N_c N_v \, exp\left[-\frac{E_g}{kT}\right]$$

- At high temperatures, η_i goes up and E_F moves closer to E_{Fi}
- At low temperatures, freeze out occurs and Boltzmann approximation is not valid
  - $E_F > E_d$ (n-type)
  - $E_F < E_a$ (p-type)
- Fermi level is a function of temperature

Fermi Levels of Two Materials in Contact

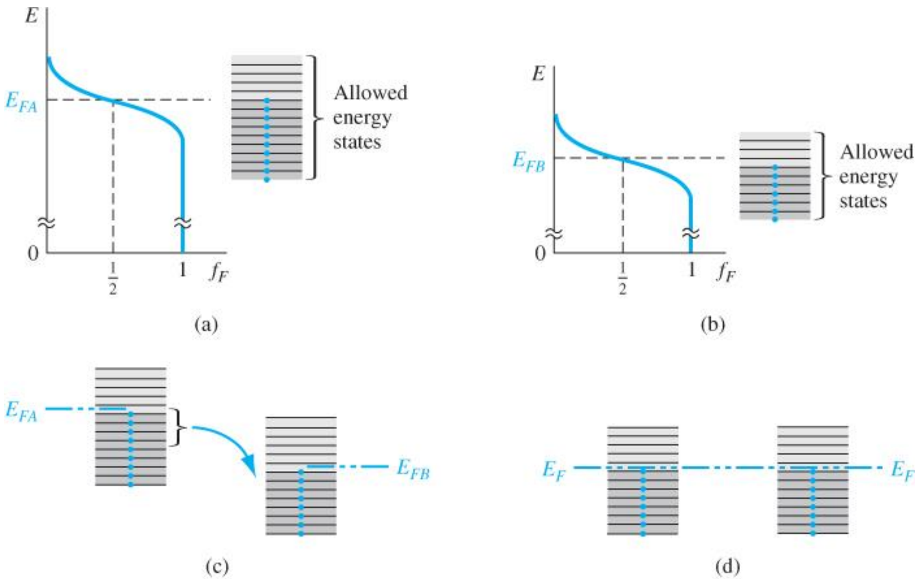

Figure 4.20 | The Fermi energy of (a) material A in thermal equilibrium, (b) material B in thermal equilibrium, (c) materials A and B at the instant they are placed in contact, and (d) materials A and B in contact at thermal equilibrium.

- In thermal equilibrium, the Fermi levels of materials in contact with each other are the same
- Relevant for pn-junction theory (Ch. 7)

Carrier Transport

- We now know the carrier densities n_0 and p_0 (Ch. 4)
- Next: Determine processes by which electrons and holes move in the semiconductor

- ***Drift:*** movement of carriers due to electric fields
- ***Diffusion:*** movement of carriers due to concentration gradients

Drift Current Density

- Net movement of charges due to an electric field gives rise to a drift current: $J_{drf} = \rho v_d$, where ρ is the volume charge density and v_d is the average drift velocity
- For holes, $\rho = ep$, where e is the magnitude of the electronic charge and p is the hole concentration
- Holes will accelerate, experience a collision with an ionized impurity or thermally vibrating lattice atom, and then begin accelerating again
- Introduce concept of ***mobility*** – describes how well a particle will move in a crystal under an applied electric field
- The average drift velocity for holes is proportional to the mobility and the electric field, $v_{dp} = \mu_p E$, so the drift current for holes is $J_{p|drf} = e\mu_p p E$
- The total drift current including electrons is given by $J_{drf} = e(\mu_n n + \mu_p p)E$
- Both drift currents are in the same direction as the electric field

*see in-class notes

Mobility Effects

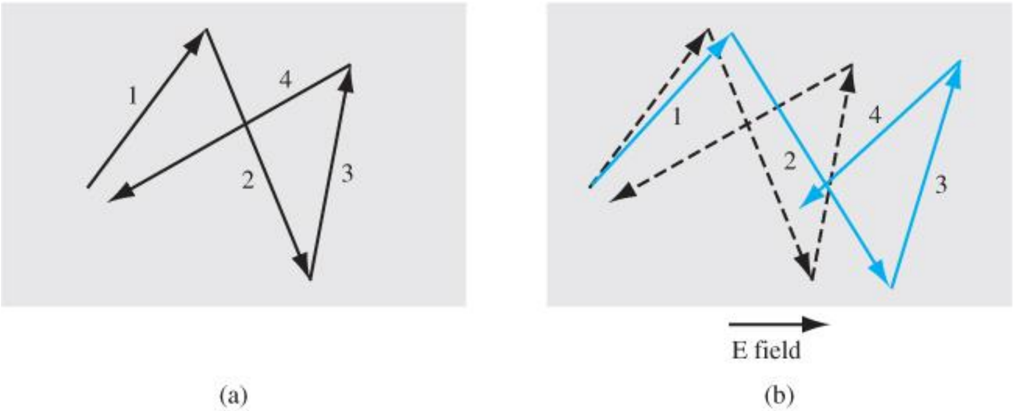

Figure 5.1 | Typical random behavior of a hole in a semiconductor (a) without an electric field and (b) with an electric field.

τ_c = mean time between collisions
m_c^* is the conductivity effective mass (see App. F)

carrier mobility

$$\mu_n = \frac{e\tau_{cn}}{m_{cn}^*}$$

$$\mu_p = \frac{e\tau_{cp}}{m_{cp}^*}$$

$$\frac{1}{\mu} = \frac{1}{\mu_I} + \frac{1}{\mu_L}$$

Two scattering mechanisms are dominant in semiconductors:

1. Phonon (lattice) scattering – due to thermal vibrations, $\mu_L \propto T^{-3/2}$
2. Ionized Impurity scattering – due to Coulomb interaction with ionized impurities, $\mu_I \propto T^{3/2}/N_I$

Mobility vs. Temperature

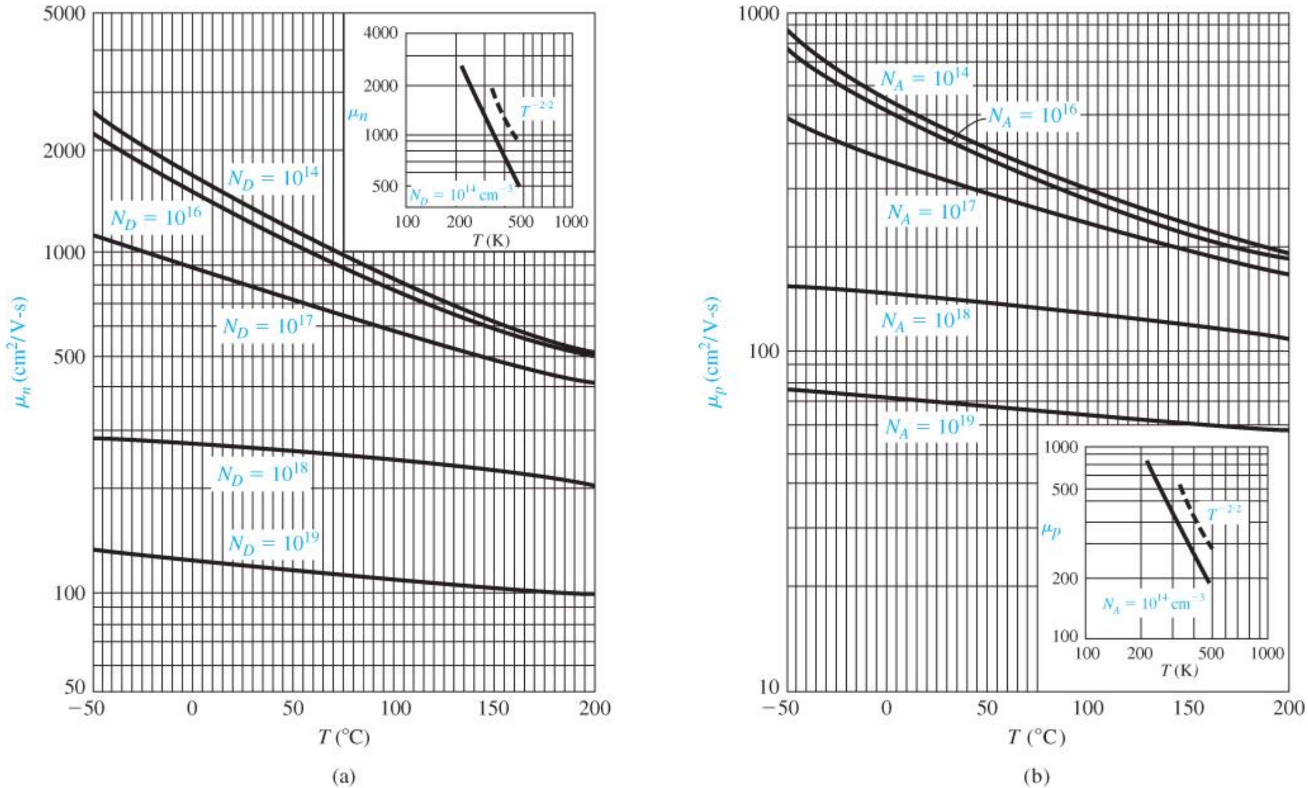

Figure 5.2 | (a) Electron and (b) hole mobilities in silicon versus temperature for various doping concentrations. Inserts show temperature dependence for "almost" intrinsic silicon.
(From Pierret [8].)

Mobility vs. Impurity Concentration

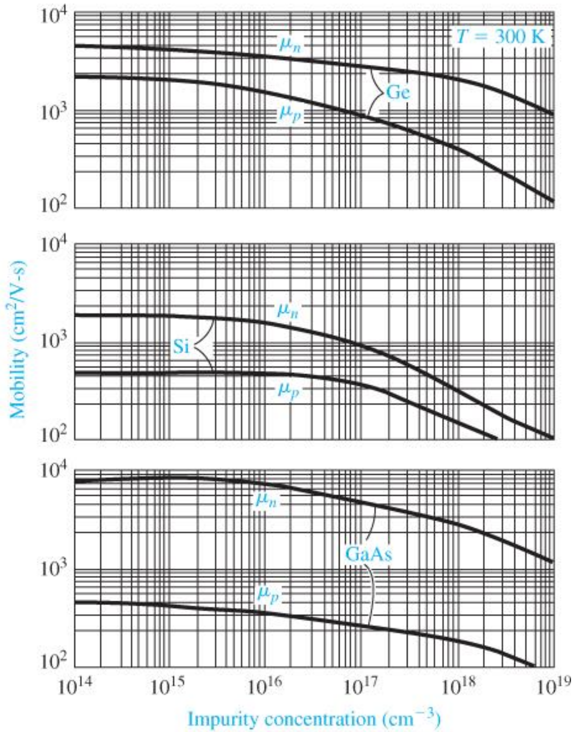

Figure 5.3 | Electron and hole mobilities versus impurity concentrations for germanium, silicon, and gallium arsenide at T = 300 K.
(From Sze [14].)

Table 5.1 | Typical mobility values at T = 300 K and low doping concentrations

	μ_n (cm²/V-s)	μ_p (cm²/V-s)
Silicon	1350	480
Gallium arsenide	8500	400
Germanium	3900	1900

- Higher impurity concentration implies higher probability of collision
- At higher temperatures, impurity scattering goes down
- Undoped silicon dominated by phonon (lattice) scattering