

Course ID: ECE 341 Communication Systems- Fall

Prof. Eirini Eleni Tsiropoulou

eirini@unm.edu / (505) – 277 – 5501

235D/ Office Hours: Mondays and Wednesdays 11:00am - 12:00pm

Lectures: Mondays and Wednesdays 9:30am-10:45 am, Room: EECE 118

Department of Electrical and Computer Engineering / University of New Mexico

Homework #1

**Corresponding to Sections 2.1 – 2.3 of Principles of Communications,
Rodger E. Zimmer and William H. Tranter, John Wiley, 7th Edition.**

1. Sketch the single-sided and double-sided amplitude and phase spectra of the following signal: $x(t)=2\sin(4\pi t+\pi/8)+12\sin(10\pi t)$
2. Given the signal $x(t)=\cos(6\pi t)+2\sin(10\pi t)$, write it as follows: a) the real part of a sum of rotating phasors, b) a sum of rotating phasors plus their complex conjugates, and c) sketch the single-sided and double-sided amplitude and phase spectra of $x(t)$.
3. If a signal is a power signal, find its normalized power and if a signal is an energy signal, find its normalized energy: a) $x(t)=2\cos(4\pi t+2\pi/3)$ and b) $x(t)=e^{-at}u(t)$.
4. Find the exponential Fourier series for the following signals (use the uniqueness property of the Fourier series): a) $x(t)=\cos(2\pi f_0 t)+\sin(4\pi f_0 t)$ and b) $x(t)=\sin(2\pi f_0 t)\cos^2(4\pi f_0 t)$.

To be delivered at instructor's office: 16 September 2019

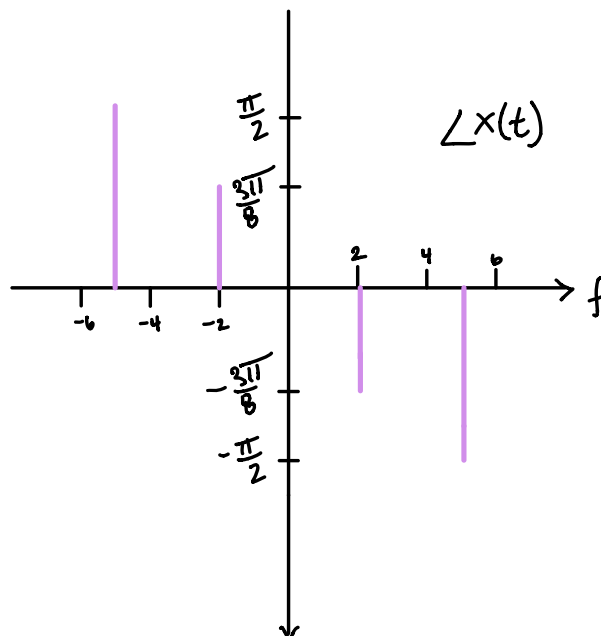
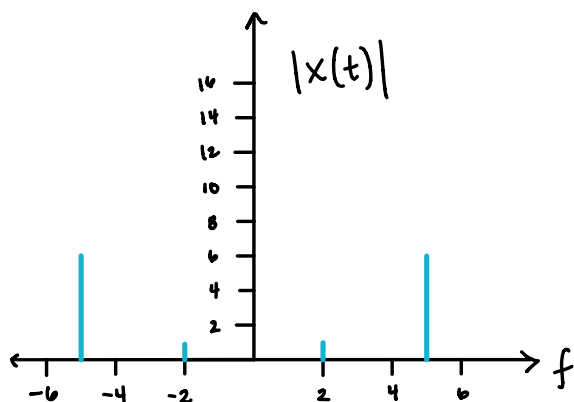
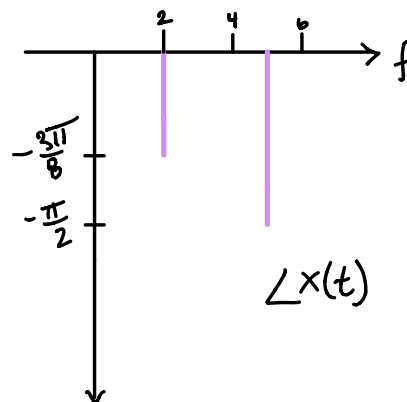
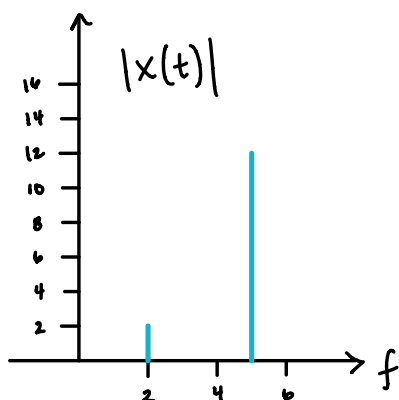
Good Luck!

1. Sketch the single-sided and double-sided amplitude and phase spectra of the following signal: $x(t) = 2\sin(4\pi t + \pi/8) + 12\sin(10\pi t)$

$$\begin{aligned}
 x(t) &= 2\sin\left(4\pi t + \frac{\pi}{8}\right) + 12\sin(10\pi t) \\
 &= 2\cos\left(4\pi t + \frac{\pi}{8} - \frac{\pi}{2}\right) + 12\cos\left(10\pi t - \frac{\pi}{2}\right) \\
 &= 2\cos\left(4\pi t - \frac{3\pi}{8}\right) + 12\cos\left(10\pi t - \frac{\pi}{2}\right)
 \end{aligned}$$

$$\omega_1 = 4\pi \rightarrow f_1 = \frac{\omega_1}{2\pi} = 2 \text{ Hz}$$

$$\omega_2 = 10\pi \rightarrow f_2 = \frac{\omega_2}{2\pi} = 5 \text{ Hz}$$



2. Given the signal $x(t) = \cos(6\pi t) + 2\sin(10\pi t)$, write it as follows: a) the real part of a sum of rotating phasors, b) a sum of rotating phasors plus their complex conjugates, and c) sketch the single-sided and double-sided amplitude and phase spectra of $x(t)$.

$$(a) x(t) = \cos(6\pi t) + 2\cos(10\pi t - \frac{\pi}{2})$$

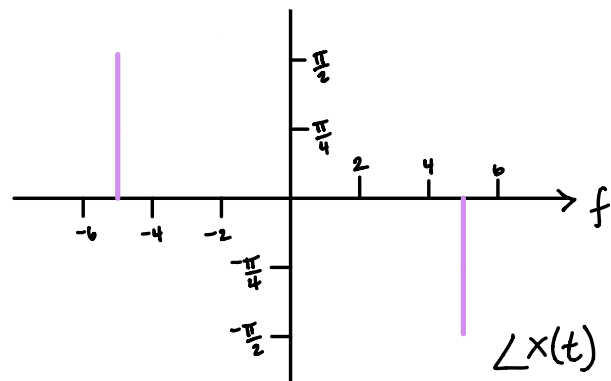
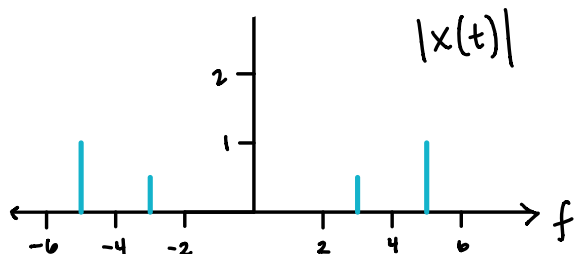
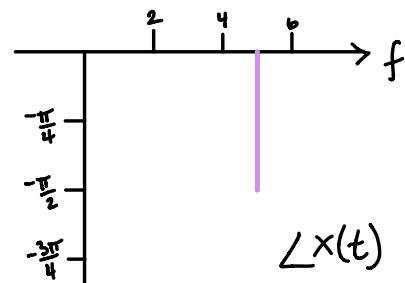
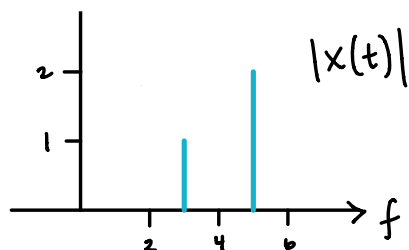
$$\text{Re} \left[e^{j6\pi t} + 2e^{j(10\pi t - \frac{\pi}{2})} \right]$$

$$(b) \frac{e^{j6\pi t} + e^{-j6\pi t}}{2} + \cancel{\left(\frac{e^{j(10\pi t - \frac{\pi}{2})} - e^{-j(10\pi t - \frac{\pi}{2})}}{2} \right)}$$

$$x(t) = \frac{e^{j6\pi t} + e^{-j6\pi t}}{2} + e^{j(10\pi t - \frac{\pi}{2})} - e^{-j(10\pi t - \frac{\pi}{2})}$$

$$(c) \omega_1 = 6\pi \rightarrow f = \frac{6\pi}{2\pi} = 3\text{Hz} \quad (1)$$

$$\omega_2 = 10\pi \rightarrow f = \frac{10\pi}{2\pi} = 5\text{Hz} \quad (2)$$



3. If a signal is a power signal, find its normalized power and if a signal is an energy signal, find its normalized energy: a) $x(t) = 2\cos(4\pi t + 2\pi/3)$ and b) $x(t) = e^{-at}u(t)$.

(a) $2\cos(4\pi t + \frac{2\pi}{3})$

periodic \rightarrow power

$$\omega = 4\pi \rightarrow f = \frac{4\pi}{2\pi} = 2 \text{ Hz} \rightarrow T = \frac{1}{f} = \frac{1}{2}$$

$$P = \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt$$

$$= 2 \int_{-1/4}^{1/4} 4\cos^2(4\pi t + \frac{2\pi}{3}) dt$$

$$= \frac{2}{\pi} \int_{-\pi/3}^{5\pi/3} \cos^2(u) du$$

$$= \frac{2}{\pi} \int_{-\pi/3}^{5\pi/3} \frac{1}{2} (1 + \cos(2u)) du$$

$$= \frac{1}{\pi} \left(u + \frac{1}{2} \sin(2u) \right) \Big|_{-\pi/3}^{5\pi/3}$$

$$u = 4\pi t + \frac{2\pi}{3}$$

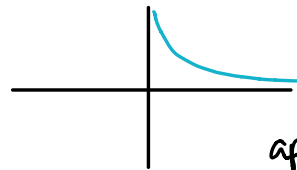
$$du = 4\pi dt$$

$$\frac{du}{\pi} = 4dt$$

t	u
$-\frac{1}{4}$	$-\frac{\pi}{3}$
$\frac{1}{4}$	$\frac{5\pi}{3}$

$$P = \boxed{2 \text{ W}}$$

(b) $x(t) = e^{-at}u(t)$



aperiodic \rightarrow energy

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$= \int_{-\infty}^{\infty} e^{-2at} u^2(t) dt = -\frac{1}{2a} e^{-2at} \Big|_{-\infty}^{\infty} = 0 - \left(-\frac{1}{2a}\right)$$

$$\boxed{E = \frac{1}{2a} \text{ J}}$$

4. Find the exponential Fourier series for the following signals (use the uniqueness property of the Fourier series): a) $x(t) = \cos(2\pi f_0 t) + \sin(4\pi f_0 t)$ and b) $x(t) = \sin(2\pi f_0 t) \cos^2(4\pi f_0 t)$.

$$\begin{aligned}
 (a) \quad x(t) &= \cos(2\pi f_0 t) + \sin(4\pi f_0 t) \\
 &= \cos(2\pi f_0 t) + \sin(4\pi f_0 t) \\
 &= \frac{e^{j2\pi f_0 t} + e^{-j2\pi f_0 t}}{2} + \frac{e^{j4\pi f_0 t} - e^{-j4\pi f_0 t}}{2j}
 \end{aligned}$$

$$\omega_0 = 2\pi f_0$$

$$x(t) = \frac{1}{2} e^{j\omega_0 t} + \frac{1}{2} e^{-j\omega_0 t} + \frac{1}{2j} e^{j2\omega_0 t} - \frac{1}{2j} e^{-j2\omega_0 t}$$

$$x(t) = \begin{cases} \frac{1}{2} & , t = \pm 1 \\ \frac{1}{2j} & , t = 2 \\ -\frac{1}{2j} & , t = -2 \end{cases}$$

$$\begin{aligned}
 (b) \quad x(t) &= \sin(2\pi f_0 t) \cos^2(4\pi f_0 t) & \omega_0 = 2\pi f_0 \\
 &= \sin(2\pi f_0 t) \left(\frac{1}{2} + \frac{1}{2} \cos(8\pi f_0 t) \right) \\
 &= \frac{1}{2} \sin(\omega_0 t) + \frac{1}{2} \sin(\omega_0 t) \cos(4\omega_0 t) \\
 &= \frac{1}{2} \sin(x) \cos(y) + \frac{1}{2} \sin(x) & \begin{matrix} x = \omega_0 t \\ y = 4\omega_0 t \end{matrix} \\
 &= \frac{1}{2} \left[\frac{1}{2} (\sin(x+y) + \sin(x-y)) + \sin(x) \right] \\
 &= \frac{1}{4} [\sin(x+y) + \sin(x-y) + 2\sin(x)]
 \end{aligned}$$

$$x(t) = \frac{1}{4} \left[\sin(5\omega_0 t) + \sin(-3\omega_0 t) + 2\sin(\omega_0 t) \right]$$

$$= \frac{1}{4} \left[\frac{e^{j5\omega_0 t} - e^{-j5\omega_0 t} + e^{-j3\omega_0 t} - e^{j3\omega_0 t} + 2e^{j\omega_0 t} - 2e^{-j\omega_0 t}}{2j} \right]$$

$$= \frac{1}{8j} e^{j5\omega_0 t} - \frac{1}{8j} e^{-j5\omega_0 t} + \frac{1}{8j} e^{-j3\omega_0 t} - \frac{1}{8j} e^{j3\omega_0 t} + \frac{1}{4j} e^{j\omega_0 t} - \frac{1}{4j} e^{-j\omega_0 t}$$

$$x(t) = \begin{cases} \frac{1}{8j}, & t = 5, -3 \\ -\frac{1}{8j}, & t = -5, 3 \\ \frac{1}{4j}, & t = 1 \\ -\frac{1}{4j}, & t = -1 \end{cases}$$

Product Identities

$$\sin(x) \cos(y) = \frac{1}{2} [\sin(x+y) + \sin(x-y)]$$

$$\cos(x) \sin(y) = \frac{1}{2} [\sin(x+y) - \sin(x-y)]$$

$$\cos(x) \cos(y) = \frac{1}{2} [\cos(x-y) + \cos(x+y)]$$

$$\sin(x) \sin(y) = \frac{1}{2} [\cos(x-y) - \cos(x+y)]$$