Overview of Propability and Random Variables

If there are N possible equally likely and mutually exclusive outcomes (the occurence of a given outcome precludes the occurence of any of the others) to an experiment ("fair"), and if N_A of these outcomes correspond to an event A of interest then the probability of event A

$$P(A) = \frac{N_A}{N}$$

<u>Example 1</u>: We have a deck of cards (52 cards)

1) what is the probability of drawing ace of spades
2)   »   »   »   »   »   » spade ?

<u>Solution</u>:

1) Event A = "Drawing ace of spade"

$$P(A) = \frac{1}{52}$$

2) Event B = "Drawing ~~of~~ a spade"

$$P(B) = \frac{13}{52} = 25\%$$

Example 2

We are tossing two fair coins at the same time

a) What is the probability of having two heads?

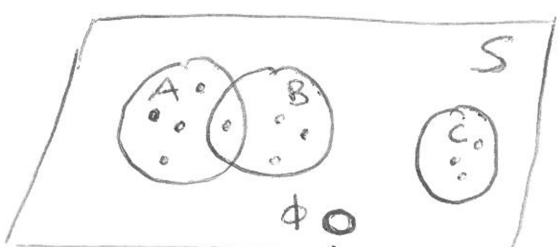
Solution:

$$\text{Outcomes} = \{HH, TT, HT, TH\}$$

$$P(A) = \frac{1}{4} = 25\%$$

Sample Spaces and Axioms of Probability:

Venn Diagrams



$P(\phi) = 0$

$P(S) = 1$.

-> A chance experiment can be viewed geometrically by representing its all possible outcomes as elements of space referred to as sample space S. An event is defined as a colletion of outcomes. An impossible collection of outcomes is referred to as the null event ϕ.

Set Theory :

Events A, B

1) "Event A or B or both" : $C = A \cup B$ (Union) $= A + B$

2) " Both A and B" : $D = A \cap B$ (joint event of A and B).

$= AB$

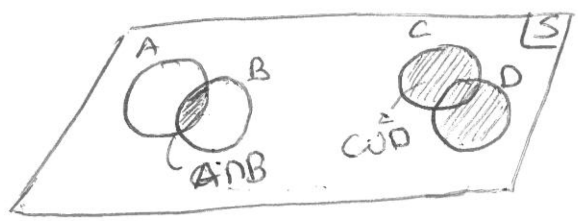
3) "Not A" : $\bar{A}$

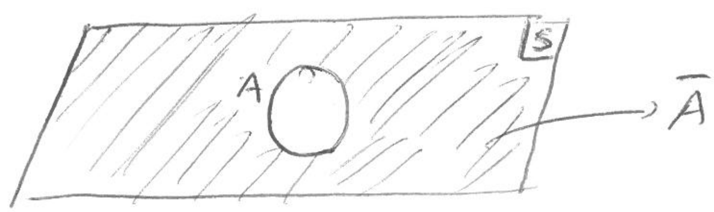
Mutually Exclusive Events :

$P(A \cap B) = 0$

$A \cap B = \phi$

$\underline{\underline{P(A \cup B) = P(A) + P(B)}}$





Relationships :

1) $A \cup \bar{A} = S$

$P(A \cup \bar{A}) = P(A) + P(\bar{A}) = 1 \quad = P$

$P(A) = 1 - P(\bar{A})$.

2) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

3) Conditional Probability of event A given that event B occurred :

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

<u>Baye's Rules</u> :

$$P(B|A) = \frac{P(B) \cdot P(A|B)}{P(A)} \quad , \quad P(A|B) = \frac{P(A)\, P(B|A)}{P(B)}$$

<u>Statistically Independent Events A, B</u> :

$$P(A|B) = P(A) \quad , \quad P(B|A) = P(B)$$

The occurence or non-occurence of event B in no way influences the occurence or non-occurence of A

$$P(A \cap B) = P(A) \cdot P(B)$$

Example 3 : Tossing of two fair coins at the same time:

Event A: "At least one head"

Event B: "A match"

Solution :

$$\text{Outcomes} = \{HH, TT, HT, TH\}$$

$$\text{Event } A = \{HH, HT, TH\}$$

$$P(A) = \frac{3}{4}$$

$$P(A) = P(HH) + P(T\cancel{H}) + P(HT)$$

$$P(HH) = P(H) \cdot P(H) = \frac{1}{2} \cdot \frac{1}{2} = 1/4.$$

$$P(TH) = P(T) \cdot P(H) = \frac{1}{4}$$

$$P(HT) = P(H) \cdot P(T) = 1/4.$$

$$P(A) = 3/4.$$

Event $B = \{HH, TT\}$, $P(B) = \frac{2}{4} = 1/2$.

$$P(B) = P(HH) + P(TT) = P(H) \cdot P(H) + P(T) \cdot P(T) = 1/2.$$

Event C: "At least one head given a match"

$$P(C) = P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{1/4}{1/2} = 1/2$$

$\{HH, TT\}$

Example 4 : Checking if two events are independent.

$\Rightarrow$ A single card is drawn from a deck of 52 cards.

Event A : "the card is a club"

Event B : " " " " " black"

Event C : " " " " " king"

(A,B) , (B,C)

Solution:

(A,B) $\quad P(A) = \frac{13}{52} = \frac{1}{4}.$

$$P(B) = \frac{26}{52} = \frac{1}{2}.$$

$$P(A|B) = \frac{13}{26} = 1/2$$

$$P(A\cap B) = P(A|B) \cdot P(B) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}.$$

$$P(A) \cdot P(B) = \frac{1}{4} \cdot \frac{1}{2} \neq 1/4.$$

A, B not statistical independent

(B, C): $P(B) = 1/2$

$P(C) = \frac{4}{52} = 1/13$

$P(C|B) = \frac{2}{26} = 1/13$

$P(C) \cdot P(B) = \frac{1}{2} \cdot \frac{1}{13}$

$P(C \cap B) = P(C|B) \cdot P(B) = \frac{1}{2} \cdot \frac{1}{13}$

$P(C \cap B) = P(C) \cdot P(B)$

C, B are S.I.

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