Homework-2

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1 Description

This project was about familiarizing us with A perl script. Here we iterate the perl script on some functions using Newton's Method and modify the template and write the output to a temp file and compile it and test it.

2 Running

2.1 Process to Run modified newton

```
$ cd HW2/CODE
$ perl newtonS.pl
```

2.2 Process to Run newton

```
$ cd HW2/newtonprofsoln
$ perl newtonS.pl
```

3 Files present in Repository

3.1 Files in CODE

File	Description
`a.out`	A raw output file
`newtonS.f90`	The output file of the perl script at each iteration
`newtonS.f90.Template`	A template for running Newton's method on a function
`newtonS.pl`	The perl script that runs Newton's method on a sequence of functions
`tmp.txt`	The output file of the fortran code

3.2 Files present in newtonprofsoln

File	Description
`a.out`	A raw output file
`newtonS.f90`	The output file of the perl script at each iteration
`newtonS.f90.Template`	A template for running Newton's method on a function
`newtonS.pl`	The perl script that runs Newton's method on a sequence of functions
`tmp.txt`	The output file of the fortran code

4 Overview of the project

We have modified The given template `newtonS.f90.Template` to run Newton's method on $f(x) = x$, $f(x) = x^2$, and $f(x) = \sin(x) + \cos\left(x^2\right)$. The modification outputs to `newtonS.f90` . The raw output is printed into `tmp.txt`.We have to consider that the perl script changes the output, it separates the output by comma.The given template has been modified using the do while loop so that it runs (iterates) the newton method until $(E_{\text{abs.}})_n = \text{abs}(x_n - x_{n-1})$ which is less than is less than 10^{-15}. The template initially performed 10 iteration which was more than required for some functions so, we modified given template because for few iteration, it is hard to find out the convergence rate for the function in newton method.Then finally all the data for the project is stored on tmp.txt file.

5 Comma Modification in perl script

The perl code given did the actions listed below to modifies the output to seperate them using comma separate the output.

```
$line =~ s/\s+/ , /g;
$line =  $line . "\n";
$line =~ s/ , $/ /;
$line =~ s/^ , / /;
print $line;
```

6 Convergence of Newton's Method

6.1 $f(x) = x$ Modified Newton's Method

iteration	guess	Δx	$(E_{\text{abs.}})_n = \text{abs}(x_n - x_{n-1})$	$\frac{(E_{\text{abs.}})_n}{(E_{\text{abs.}})_{n-1}}$	$\frac{(E_{\text{abs.}})_n}{\left((E_{\text{abs.}})_{n-1}\right)^2}$
01	0.0000E+00	0.5000E+00	0.5000E+00		
02	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00

After modifying code, newton's method on the function $f(x) - x$ gives an answer regardless of initial guess. Let x_0 be our initial guess and r be the root of $f(x)$ (which happens to be 0).from newton method formula, we have

$$x_1 = x_0 - \frac{f(x)}{f'(x)} = x_0 - \frac{x_0}{1} = 0 \tag{1}$$

It's hard to say what the convergence looking on two data. we can argue that the $f'(x) = 1$ which is non zero. so, convergence is likely quadratic.

6.1.1 $f(x) = x$ Netwon's Method

iteration	guess	Δx
01	0.00000E+00	0.50000E+00
02	0.00000E+00	-0.00000E+00
03	0.00000E+00	-0.00000E+00
04	0.00000E+00	-0.00000E+00
05	0.00000E+00	-0.00000E+00
06	0.00000E+00	-0.00000E+00
07	0.00000E+00	-0.00000E+00
08	0.00000E+00	-0.00000E+00
09	0.00000E+00	-0.00000E+00
10	0.00000E+00	-0.00000E+00

6.2 $f(x) = x^2$ Newton Method

iteration	guess	Δx
01	-0.25000E+00	0.25000E+00
02	-0.12500E+00	0.12500E+00
03	-0.62500E-01	0.62500E-01
04	-0.31250E-01	0.31250E-01
05	-0.15625E-01	0.15625E-01
06	-0.78125E-02	0.78125E-02
07	-0.39062E-02	0.39062E-02
08	-0.19531E-02	0.19531E-02
09	-0.97656E-03	0.97656E-03
10	-0.48828E-03	0.48828E-03

Running Newton's method on $f(x) = x^2$,the guess in the table above is converging linearly. in mathematical term, one can argue that derivative of $f(x)$ at root is zero. So, Newton's method converge linearly.we can derive the convergence as follow.

$$\begin{aligned}
&\text{Let } f(r) = 0\\
&\text{Assume } f'(r) = 0\\
&\text{Let } e_n = (x_n - r) \quad \forall x_n \in \mathbb{R}
\end{aligned}$$

$$\begin{aligned}
&f(x_n) = f(r) + (x_n - r)f'(r) + \frac{(x_n - r)^2}{2} f''(r) + \ldots\\
\implies &f(x_n) = f(r) + (x_n - r)f'(r) + \frac{(x_n - r)^2}{2} f''(\xi_1) \qquad \text{for some } \xi_1 \in [x_n, r]\\
\implies &f(x_n) = \frac{(x_n - r)^2}{2} f''(\xi_1)
\end{aligned}$$

$$\begin{aligned}
&f'(x_n) = f'(r) + (x_n - r)f''(\xi_2) \qquad \text{for some } \xi_2 \in [x_n, r]\\
\implies &f'(x_n) = (x_n - r)f''(\xi_2)
\end{aligned}$$

$$\begin{aligned}
&x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}\\
\implies &(x_{n+1} - r) = (x_n - r) - \frac{f(x_n)}{f'(x_n)}\\
\implies &(x_{n+1} - r) = (x_n - r) - \frac{\frac{(x_n - r)^2}{2} f''(\xi_1)}{(x_n - r)f''(\xi_2)}\\
\implies &e_{n+1} = e_n - \frac{{e_n}^2 \frac{f''(\xi_1)}{2}}{e_n f''(\xi_2)}\\
\implies &e_{n+1} = e_n \left(1 - \frac{\frac{f''(\xi_1)}{2}}{f''(\xi_2)}\right)\\
\implies &e_{n+1} = C \cdot e_n \qquad \text{for some } C \in \mathbb{R}
\end{aligned}$$

We use modified Newton's Method to convergence the function quadratically.

$$x_{n+1} = x_n - m\frac{f(x_n)}{f'(x_n)}$$

where m is the multiplicity of the root.

For the case of $f(x) = x^2$, this will again cause Newton's method to converge quickly as we notice on the table.

$$x_1 = x_0 - 2\frac{f(x)}{f'(x)} = x_0 - 2\frac{{x_0}^2}{2x_0} = x_0 - x_0 = 0$$

is again the root of $f(x) = x^2$.

6.2.1 $f(x) = x^2$ with Modified Netwon's Method

iteration	guess	Δx	$(E_{\text{abs.}})_n = \text{abs}(x_n - x_{n-1})$	$\frac{(E_{\text{abs.}})_n}{(E_{\text{abs.}})_{n-1}}$	$\frac{(E_{\text{abs.}})_n}{\left((E_{\text{abs.}})_{n-1}\right)^2}$
01	0.0000E+00	0.25000E+00	0.5000E+00		
02	0.0000E+00	NaN	0.0000E+00	0.0000E+00	0.0000E+00

6.3 $f(x) = \sin(x) + \cos\left(x^2\right)$

we can analyze the convergence of $f(x) = \sin(x) + \cos\left(x^2\right)$ by looking on the table above but it is tough to find only looking some iteration. $\frac{(E_{\text{abs.}})_n}{(E_{\text{abs.}})_{n-1}}$ nor $\frac{(E_{\text{abs.}})_n}{\left((E_{\text{abs.}})_{n-1}\right)^2}$ were constant; however, $\frac{(E_{\text{abs.}})_n}{\left((E_{\text{abs.}})_{n-1}\right)^2}$ varied much less than $\frac{(E_{\text{abs.}})_n}{(E_{\text{abs.}})_{n-1}}$ (the former expression is around $7-8$). so the convergence is most likely quadratic.

iteration	guess	Δx	$(E_{\text{abs.}})_n = \text{abs}(x_n - x_{n-1})$	$\frac{(E_{\text{abs.}})_n}{(E_{\text{abs.}})_{n-1}}$	$\frac{(E_{\text{abs.}})_n}{\left((E_{\text{abs.}})_{n-1}\right)^2}$
01	-0.93510E+00	-0.43510E+00	0.43510E+00		
02	-0.85464E+00	0.80463E-01	0.80463E-01	0.18492E+00	0.42501E+00
03	-0.84939E+00	0.52514E-02	0.52514E-02	0.65265E-01	0.81112E+00
04	-0.84936E+00	0.21273E-04	0.21273E-04	0.40508E-02	0.77138E+00
05	-0.84936E+00	0.34744E-09	0.34744E-09	0.16332E-04	0.76775E+00
06	-0.84936E+00	-0.62288E-16	0.11102E-15	0.31954E-06	0.91970E+03