Homework 3

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1 Description

Project to calculate the value of given integral $I = \int_{-1}^{1} e^{\cos(kx)} dx$ using fortran program using Trapezoidal and Gauss Quadrature for two different values of k π & π^2.

2 Requirements

- A Fortran compiler tested and working.
- Linux Environment
- Matlab for error plot

3 Compiling the program

3.1 Using Fortran compiler

The following commands should be from the `HW3` directory.

3.1.1 GNU Fortran

```
The following code makes makefile with name Integral and errors
For Integral -
gfortran -c functions.f90 gauss_quadrature.f90 Trapezoidal.f90 HW3.f90
gfortran -o integral functions.o gauss_quadrature.o Trapezoidal.o HW3.o

For relative error-
gfortran -c functions.f90 gauss_quadrature.f90 Trapezoidal.f90 errors.f90
gfortran -o errors functions.o gauss_quadrature.o Trapezoidal.o errors.f90
```

4 List of Source code

Name	Description
functions.f90	This includes the list of required functions
HW3.f90	This program call functions and subroutines to calculate integral values
Trapezoidal.f90	This function compute integral using Trapezoidal method
gauss_quadrature.f90	This function compute integral using gauss quadrature
errors.f90	This program calculate relative error calling functions
data.txt	This is txt file that contains data for plotting in matlab
lglnodes.f90	This subroutine compute nodes used in gauss quadrature
matlaberror.m	This matlab code to plot the error

5 Summary

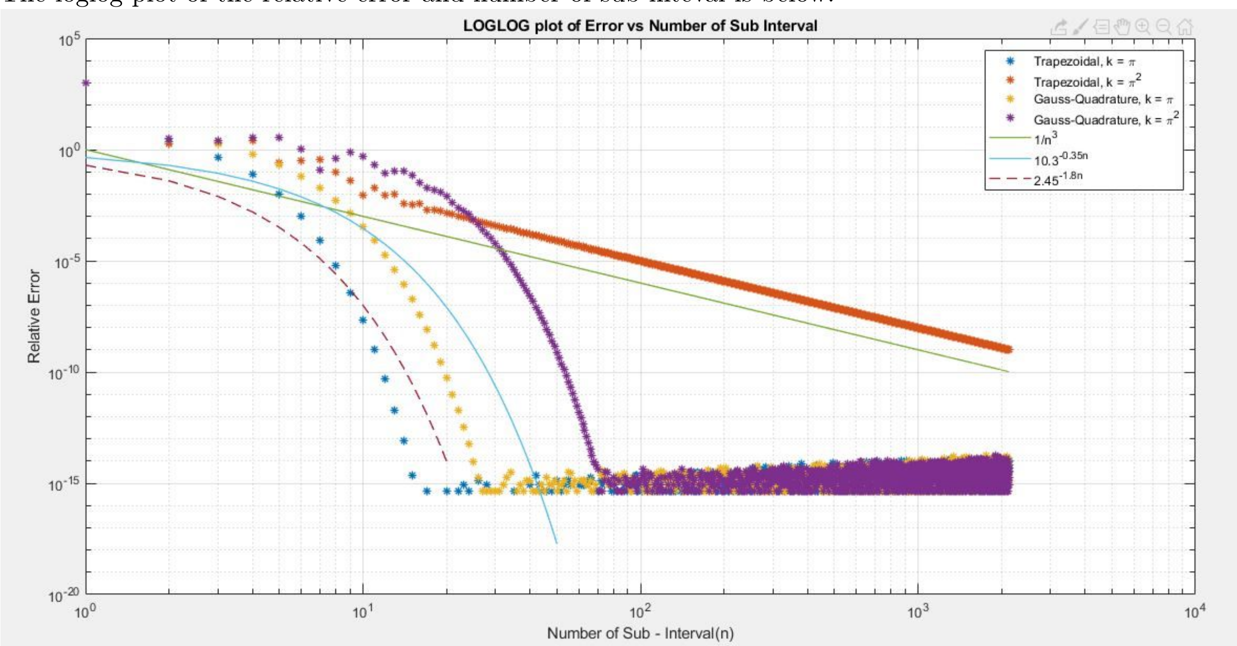
In this project We use the Gauss Quadrature and Trapezoid rule to approximate the given integral

$$\int_{-1}^{1} e^{\cos(kx)} dx$$

for the given two valuse of $k = \pi, \pi^2$ and then plot the error using Matlab. It is difficult to compute exact integral of such function. So, we cannot compute absolute error using exact integral. In this project, we used relative method for error calculation based on the previous integral value and current integral value based on the given tolerance value.

5.1 Error Plot

The loglog plot of the relative error and number of sub inteval is below:-



5.2 Convergence of Trapezoidal rule

$$Sum - Integral = (\sum_{m=1}^{p} \frac{B_m}{m!}(f^{m-1}(a) - f^{m-1}(b))) + R \tag{1}$$

where B_m is the mth Bernoulli number and R is the error term of the order O(n^{-3}). The derivates of given function $f(x) = e^{cos(kx)}$ consist of cosine and sine terms.

$$f'(x) = -k \sin(kx)\ e^{\cos(kx)} \tag{2}$$

$$f''(x) = (-k^2\ \cos(kx) + k^2(\sin(kx))^2\ e^{\cos(kx)} \tag{3}$$

we can see if $k = \pi$, the $(f^{k-1}(a) - f^{k-1}(b))$ term vanishes because of the periodic nature of the cosine (even) and sine (odd) function. so, we expect equation no 1 to converge to R which is $O(n^{-3})$. The error plot for the $k = \pi$ for trapezoidal is similar to $O(n^{-3})$ but it doesn't hold correct for $k = \pi^2$.

5.3 Gauss Quadrature

we used Gauss Quadrature to compute the integral and its error. In Gauss quadrature, the grid-points z_i (nodes) and quadrature weight w_i is calculated using $w(z)$=1 and applying legendre polynomial. The same integral convergence quicker than trapezoidal integral which can clearly visible in Error plot. For the larger sub interval, we can see $\exp\left(\cos(\pi x)\right)$, the error is roughly $2.45^{-1.8n}$ and for $\exp\left(\cos(\pi^2 x)\right)$, the error is roughly $10.3^{-0.35n}$.