Chapter 2
Thruster Principles

Electric thrusters propel the spacecraft using the same basic principle as chemical rockets—accelerating mass and ejecting it from the vehicle. The ejected mass from electric thrusters, however, is primarily in the form of energetic charged particles. This changes the performance of the propulsion system compared to other types of thrusters and modifies the conventional way of calculating some of the thruster parameters, such as specific impulse and efficiency. Electric thrusters provide higher exhaust velocities than is available from gas jets or chemical rockets, which either improves the available change in vehicle velocity (called Δv or delta-v) or increases the delivered spacecraft and payload mass for a given Δv. Chemical rockets generally will have exhaust velocities of 3 to 4 km/s, while the exhaust velocity of electric thrusters can approach 10^2 km/s for heavy propellant such as xenon atoms, and 10^3 km/s for light propellants such as helium.

2.1 The Rocket Equation

The mass ejected to provide thrust to the spacecraft is the propellant, which is carried onboard the vehicle and expended during thrusting. From conservation of momentum, the ejected propellant mass times its velocity is equal to the spacecraft mass times its change in velocity. The "rocket equation" describing the relationship between the spacecraft velocity and the mass of the system is derived as follows. The force on a spacecraft, and thus the thrust on the vehicle, is equal to the mass of the spacecraft, *M*, times its change in velocity, *v*:

$$\text{Force} = T = M \frac{dv}{dt} . \qquad (2.1\text{-}1)$$

The thrust on the spacecraft is equal and opposite to the time rate of change of the momentum of the propellant, which is the exhaust velocity of the propellant times the time rate of change of the propellant mass:

$$T = -\frac{d}{dt}\left(m_p v_{\text{ex}}\right) = -v_{\text{ex}} \frac{dm_p}{dt}, \tag{2.1-2}$$

where m_p is the propellant mass on the spacecraft and v_{ex} is the propellant exhaust velocity in the spacecraft frame of reference.

The total mass of the spacecraft at any time is the delivered mass, m_d, plus the propellant mass:

$$M(t) = m_d + m_p. \tag{2.1-3}$$

The mass of the spacecraft changes due to consumption of the propellant, so the time rate of change of the total mass is

$$\frac{dM}{dt} = \frac{dm_p}{dt}. \tag{2.1-4}$$

Substituting Eq. (2.1-4) into Eq. (2.1-2) and equating with Eq. (2.1-1) gives

$$M\frac{dv}{dt} = -v_{\text{ex}} \frac{dM}{dt}, \tag{2.1-5}$$

which can be written as

$$dv = -v_{\text{ex}} \frac{dM}{M}. \tag{2.1-6}$$

For motion in a straight line, this equation is solved by integrating from the spacecraft initial velocity, v_i, to the final velocity, v_f, during which the mass changes from its initial value, $m_d + m_p$, to its final delivered mass, m_d:

$$\int_{v_i}^{v_f} dv = -v_{\text{ex}} \int_{m_d+m_p}^{m_d} \frac{dM}{M}. \tag{2.1-7}$$

The solution to Eq. (2.1-7) is

$$v_i - v_f = \Delta v = v_{\text{ex}} \ln\left(\frac{m_d}{m_d + m_p}\right). \tag{2.1-8}$$

The final mass of a spacecraft delivered after a given amount of propellant has been used to achieve the specified Δv is

$$m_d = (m_d + m_p)e^{-\Delta v / v_{\text{ex}}} . \tag{2.1-9}$$

The specific impulse, Isp, will be shown in Section 2.4 to be equal to the propellant exhaust velocity, v_{ex}, divided by the gravitational acceleration g. The change in velocity of the spacecraft is then

$$\Delta v = \left(\text{Isp}^* g\right) \ln\left(\frac{m_d + m_p}{m_d}\right), \tag{2.1-10}$$

where g is the acceleration by gravity, 9.8067 m/s^2.

Equation (2.1-10) shows that for a given mission with a specified Δv and final delivered mass, m_d, the initial spacecraft wet mass $(m_d + m_p)$ can be reduced by increasing the Isp of the propulsion system, which has implications for the launch vehicle size and cost. High delta-v missions are often enabled by electric propulsion because it offers much higher exhaust velocities and Isp than do conventional chemical propulsion systems.

Equation (2.1-9) can be written in terms of the required propellant mass:

$$m_p = m_d \left[e^{\Delta v / (v_{\text{ex}})} - 1 \right] = m_d \left[e^{\Delta v / (\text{Isp}^* g)} - 1 \right]. \tag{2.1-11}$$

The relationship between the amount of propellant required to perform a given mission and the propellant exhaust velocity (or the propulsion system Isp) shows that the propellant mass increases exponentially with the delta-v required. Thrusters that provide a large propellant exhaust velocity compared to the mission Δv will have a propellant mass that is only a small fraction of the initial spacecraft wet mass.

The exhaust velocity of chemical rockets is limited by the energy contained in the chemical bonds of the propellant used; typical values are up to 4 km/s. Electric thrusters, however, separate the propellant from the energy source (which is now a power supply) and thus are not subject to the same limitations.

Modern ion and Hall thrusters operating on xenon propellant have exhaust velocities in the range of 20–40 km/s and 10–20 km/s, respectively.

The dramatic benefits of the high exhaust velocities of electric thrusters are clearly seen from Eq. (2.1-11). For example, consider an asteroid rendezvous mission for which it is desired to deliver 500 kg of payload with a mission Δv of 5 km/s. A spacecraft propelled by a chemical engine with a 3-km/s exhaust velocity, corresponding to an Isp of 306 s, would require 2147 kg of propellant to accomplish the mission. In contrast, an ion thruster with a 30-km/s exhaust velocity, corresponding to an Isp of 3060 s, would accomplish the same mission using only 91 kg of propellant. High-Δv missions such as this are often enabled by electric propulsion, allowing either a significant reduction in the amount of required propellant that has to be launched or the ability to increase the spacecraft dry mass for a given wet mass associated with a launch vehicle or mission requirement.

2.2 Force Transfer in Ion and Hall Thrusters

The propellant ionized in ion and Hall thrusters is accelerated by the application of electric fields. However, the mechanism for transferring the thrust from the ion motion to the thruster body, and thereby the spacecraft, is different for ion thrusters and Hall thrusters.

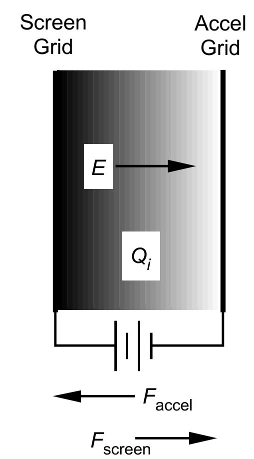

Fig. 2-1. Schematic of ion thruster acceleration region.

In ion thrusters, ions are produced by a plasma source and accelerated electrostaticly by the field applied between two (or more) grids, as illustrated in Fig. 2-1. The voltage applied between the two grids creates a vacuum electric field between the grids of the voltage divided by the gap d. The ions represent additional charge in the gap between the grids that modifies the electric field. Assuming infinitely large grids, the electric field distribution between the grids can be found from the one-dimensional Poisson's Equation:

$$\frac{dE(x)}{dx} = \frac{\rho(x)}{\varepsilon_o} = \frac{qn_i(x)}{\varepsilon_o}, \tag{2.2-1}$$

where ε_o is the permittivity of free space, ρ is the ion charge density in the gap, q is the charge on an ion, and n_i is the ion number density in the gap. Equation (2.2-1) can be integrated from the screen grid to the accel grid to give

$$E(x) = \frac{q}{\varepsilon_o} \int_0^x n_i(x')\,dx' + E_{\text{screen}}\,, \tag{2.2-2}$$

where E_{screen} is the electric field at the screen grid. Assuming that the screen grid is a perfect conductor, its surface charge density, σ, is

$$\sigma = \varepsilon_o\, E_{\text{screen}}\,. \tag{2.2-3}$$

The surface charge is an image charge and is attracted by the ion charge in the gap. Since the field drops to zero inside the conductor, the screen grid feels a force per unit area equal to the charge density times the average field (which is half the field on the outside of the conductor):

$$F_{\text{screen}} = \sigma\,\frac{\left(E_{\text{screen}} + 0\right)}{2} = \frac{1}{2}\varepsilon_o E_{\text{screen}}^2\,, \tag{2.2-4}$$

where F_{screen} is the force on the screen grid. Correspondingly, at the accelerator grid there is an electric field, E_{accel}, and a surface charge density equal to that on the screen grid but of the opposite sign. The accel grid feels a force is in the opposite direction:

$$F_{\text{accel}} = -\sigma\,\frac{\left(E_{\text{accel}} + 0\right)}{2} = -\frac{1}{2}\varepsilon_o E_{\text{accel}}^2\,. \tag{2.2-5}$$

The net thrust on the ion engine is the sum of the forces on the screen and accel grids,

$$T = F_{\text{screen}} + F_{\text{accel}} = \frac{1}{2}\varepsilon_o\left(E_{\text{screen}}^2 - E_{\text{accel}}^2\right), \tag{2.2-6}$$

where T is the force in newtons. The force per unit area on the ions in the gap between the grids can be calculated using the fact that the force on an ion equals its charge times the local electric field, and integrating that force across the gap:

$$F_{\text{ion}} = q\int_0^d n_i(x)E(x)\,dx\,. \tag{2.2-7}$$

Eliminating the ion density $n_i(x)$ using Eq. (2.2-1), the integral can be done directly:

$$F_{\text{ion}} = \varepsilon_o \int_0^d \frac{dE(x)}{dx} E(x)\,dx = \varepsilon_o \int_{E_{\text{screen}}}^{E_{\text{accel}}} E\,dE = \frac{1}{2}\varepsilon_o \left(E_{\text{accel}}^2 - E_{\text{screen}}^2\right). \quad (2.2\text{-}8)$$

The net force on the grids, which is the thrust, is equal and opposite to the electric field forces on the ions between the grids:

$$T = -F_{\text{ion}} = -\frac{1}{2}\varepsilon_o \left(E_{\text{accel}}^2 - E_{\text{screen}}^2\right). \quad (2.2\text{-}9)$$

Therefore, the thrust in an ion engine is transferred by the electrostatic force between the ions and the two grids.

In Hall thrusters, ions are generated in a plasma volume and accelerated by an electric field in the plasma. However, the presence of the transverse magnetic field responsible for the rotational Hall current modifies the force transfer mechanism. Assume for argument that the Hall thruster plasma is locally quasi-neutral ($qn_i \approx qn_e$) in the acceleration region, where n_e is the electron plasma density, and that in the acceleration zone the electric and magnetic fields are uniform. The geometry is shown schematically in Fig. 2-2. The ions are essentially unmagnetized and feel the force of the local electric field, so the force on the ions is

$$\mathbf{F}_{\text{ion}} = 2\pi \iint q\, n_i\, \mathbf{E}\; r dr\, dz \quad (2.2\text{-}10)$$

The electrons in the plasma feel an $\mathbf{E} \times \mathbf{B}$ force and circulate in the system transverse to the electric and magnetic fields with the velocity

$$\mathbf{v}_e = \frac{\mathbf{E} \times \mathbf{B}}{B^2}. \quad (2.2\text{-}11)$$

The electrostatic force on the ions is the negative of the electrostatic force on the electrons due to their sign differences. The electrons are constrained not to move axially by the transverse magnetic field, so the force per unit area on the electrons (to the left) is balanced by the Lorentz force:

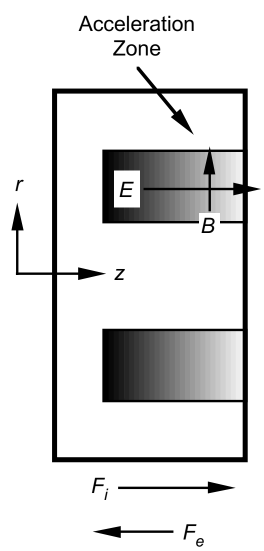

Fig. 2-2. Cross section of a Hall thruster showing electric and magnetic fields.

$$\mathbf{F}_e = -2\pi \iint q\, n_e\, \mathbf{E}\; r dr\, dz \; - 2\pi \iint e n_e \mathbf{v}_e \times \mathbf{B}\; r dr\, dz = 0 \quad (2.2\text{-}12)$$

Using quasi-neutrality and the definition of the Hall current density, $\mathbf{J}_{\text{Hall}} = -en_e\mathbf{v}_e$, the force on the ions is shown to be equal to Lorentz forces on the electrons:

$$\mathbf{F}_i = -2\pi \iint q\, n_i\, \mathbf{E}\; r dr\, dz \; + 2\pi \iint \mathbf{J}_{\text{Hall}} \times \mathbf{B}\; r dr\, dz = 0\,. \tag{2.2-13}$$

Solving Eq. (2.2-13), the force on the ions is then

$$\mathbf{F}_i = \mathbf{J}_{\text{Hall}} \times \mathbf{B} \tag{2.2-14}$$

By Newton's second law, the Hall current force on the magnets is equal and opposite to the Hall current force on the electrons and, therefore, is also equal and opposite to the force on the ions:

$$\mathbf{T} = \mathbf{J}_{\text{Hall}} \times \mathbf{B} = -\mathbf{F}_i\,. \tag{2.2-15}$$

In Hall thrusters the thrust is transferred from the ions to the thruster body through the electromagnetic Lorentz force. These thrusters are sometimes called *electromagnetic thrusters* because the force is transferred through the magnetic field. However, since the ion acceleration mechanism is by the electrostatic field, we will choose to call them *electrostatic thrusters*.

2.3 Thrust

Thrust is the force supplied by the engine to the spacecraft. Since the spacecraft mass changes with time due to the propellant consumption, the thrust is given by the time rate of change of the momentum, which can be written as

$$T = \frac{d}{dt}\left(m_p v_{\text{ex}}\right) = \frac{dm_p}{dt}\, v_{\text{ex}} = \dot{m}_p v_{\text{ex}} \tag{2.3-1}$$

where $\dot{m}_p$ is the propellant mass flow rate in kg/s. The propellant mass flow rate is

$$\dot{m}_p = QM\,, \tag{2.3-2}$$

where Q is the propellant particle flow rate (in particles/s) and M is the particle mass.

The kinetic thrust power of the beam, called the jet power, is defined as

$$P_{\text{jet}} = \frac{1}{2}\dot{m}_p v_{\text{ex}}^2\,. \tag{2.3-3}$$

Using Eq. (2.3-1), the jet power is then

$$P_{\text{jet}} = \frac{T^2}{2\dot{m}_p} . \tag{2.3-4}$$

This expression shows that techniques that increase the thrust without increasing the propellant flow rate will result in an increase in the jet power.

For ion and Hall thrusters, ions are accelerated to high exhaust velocity using an electrical power source. The velocity of the ions greatly exceeds that of any unionized propellant that may escape from the thruster, so the thrust can be described as

$$T = \frac{dm_p}{dt} v_{\text{ex}} \approx \dot{m}_i v_i , \tag{2.3-5}$$

where $\dot{m}_i$ is the ion mass flow rate and v_i is the ion velocity. By conservation of energy, the ion exhaust velocity is given by

$$v_i = \sqrt{\frac{2qV_b}{M}} , \tag{2.3-6}$$

where V_b is the net voltage through which the ion was accelerated, q is the charge, and M is the ion mass. The mass flow rate of ions is related to the ion beam current, I_b, by

$$\dot{m}_i = \frac{I_b M}{q} . \tag{2.3-7}$$

Substituting Eqs. (2.3-6) and (2.3-7) into Eq. (2.3-5), the thrust for a singly charged propellant $(q = e)$ is

$$T = \sqrt{\frac{2M}{e}} I_b \sqrt{V_b} \text{ [newtons]}. \tag{2.3-8}$$

The thrust is proportional to the beam current times the square root of the acceleration voltage. In the case of Hall thrusters, there is a spread in beam energies produced in the thruster, and V_b represents the effective or average beam voltage. If the propellant is xenon, $\sqrt{2M/e} = 1.65 \times 10^{-3}$, the thrust is given by

$$T = 1.65\, I_b \sqrt{V_b}\ \text{[mN]}, \tag{2.3-9}$$

where I_b is in amperes and V_b is in volts.

Equation (2.3-9) is the basic thrust equation that applies for a unidirectional, singly ionized, monoenergetic beam of ions. The equation must be modified to account for the divergence of the ion beam and the presence of multiply charged ions commonly observed in electric thrusters. The assumption of a monoenergetic ion beam in Eq. (2.3-6) is generally valid for ion thrusters, but is only an approximation for the beam characteristics in Hall thrusters, which will be discussed in Chapter 7.

The correction to the thrust equation for beam divergence is straightforward for a beam that diverges uniformly upon exiting from the thruster. For a thruster with a constant ion current density profile accelerated by uniform electric fields, the correction to the force due to the effective thrust-vector angle is simply

$$F_t = \cos\theta , \tag{2.3-10}$$

where θ is the average half-angle divergence of the beam. If the thrust half angle is 10 deg, then cos $\theta = 0.985$, which represents a 1.5% loss in thrust. If the plasma source is not uniform and/or the accelerator system has curvature, then the thrust correction must be integrated over the beam and grid profiles. For cylindrical thrusters, the correction factor is then

$$F_t = \frac{\int_0^r 2\pi r\, J(r)\, \cos\theta(r)\, dr}{I_b}, \tag{2.3-11}$$

where $J(r)$ is the ion current density which is a function of the radius. The ion current density is usually determined from direct measurement of the current distribution in the plume by plasma probes. For a constant value of $J(r)$, Eq. (2.3-11) reduces to Eq. (2.3-10).

The second correction applied to the thrust equation for electric thrusters accounts for the presence of multiply charged ion species. If the beam contains both singly charged and doubly charged ions such that the total beam current is

$$I_b = I^{+} + I^{++}, \tag{2.3-12}$$

where I^{+} is the singly charged ion current and I^{++} is the doubly charged ion current, the total thrust for the multiple species, T_m, is the sum of the thrust from each species:

$$T_m = I^{+}\sqrt{\frac{2MV_b}{e}} + I^{++}\sqrt{\frac{MV_b}{e}} = I^{+}\sqrt{\frac{2MV_b}{e}}\left(1+\frac{1}{\sqrt{2}}\frac{I^{++}}{I^{+}}\right). \qquad (2.3\text{-}13)$$

The thrust correction factor, α, for thrust in the presence of doubly ionized atoms is defined by the ratio of Eqs. (2.3-13) and (2.3-8), where the beam current in Eq. (2.3-8) is given by Eq. (2.3-12):

$$\alpha = \frac{I^{+} + \frac{1}{\sqrt{2}}I^{++}}{I^{+} + I^{++}} = \frac{1 + 0.707\frac{I^{++}}{I^{+}}}{1 + \frac{I^{++}}{I^{+}}}, \qquad (2.3\text{-}14)$$

where I^{++}/I^{+} is the fraction of double ion current in the beam. A similar correction factor can be easily derived for higher charged ions (see Problem 4), although the number of these species is typically found to be relatively small in most ion and Hall thrusters.

The total thrust correction is the product of the divergence and multiply charged species terms:

$$\gamma = \alpha F_t . \qquad (2.3\text{-}15)$$

The total corrected thrust is then given by

$$T = \gamma \dot{m}_i v_i = \gamma \sqrt{\frac{2M}{e}}\, I_b \sqrt{V_b} . \qquad (2.3\text{-}16)$$

The total thrust for xenon can be simply written as

$$T = 1.65\, \gamma\, I_b \sqrt{V_b} \ \text{[mN]}. \qquad (2.3\text{-}17)$$

For example, assuming an ion thruster with a 10-deg half-angle beam divergence and a 10% doubles-to-singles ratio results in $\gamma = 0.958$. For a thruster producing 2 A of xenon ions at 1500 V, the thrust produced is 122.4 mN.

2.4 Specific Impulse

Specific impulse, termed Isp, is a measure of thrust efficiency and is defined as the ratio of the thrust to the rate of propellant consumption. Specific impulse for constant thrust and propellant flow rate is

$$\text{Isp} = \frac{T}{\dot{m}_p g}, \tag{2.4-1}$$

where g is the acceleration of gravity, 9.807 m/s^2. For a xenon thruster, the Isp can be expressed as

$$\text{Isp} = 1.037 \times 10^6 \frac{T[\text{N}]}{Q[\text{sccm}]} = 1.02 \times 10^5 \frac{T[\text{N}]}{Q[\text{mg/s}]}, \tag{2.4-2}$$

where Eq. (2.3-2) and the flow conversions in Appendix B have been used.

Using Eq. (2.3-1) for the thrust in Eq. (2.4-1), the Isp for any thruster is

$$\text{Isp} = \frac{v_{\text{ex}}}{g}, \tag{2.4-3}$$

where v_{ex} is the effective exhaust velocity.

Defining the Isp in terms of the exhaust velocity relative to g is what gives rise to the unusual units of seconds for Isp. In electric thrusters, the thrust is due primarily to the ions. Using Eq. (2.3-5), the Isp is given by

$$\text{Isp} = \frac{v_i}{g} \frac{\dot{m}_i}{\dot{m}_p}. \tag{2.4-4}$$

where v_i is the exhaust velocity for unidirectional, monoenergetic ion exhaust.

The thruster mass utilization efficiency, which accounts for the ionized versus unionized propellant, is defined for singly charged ions as

$$\eta_m = \frac{\dot{m}_i}{\dot{m}_p} = \frac{I_b}{e} \frac{M}{\dot{m}_p}. \tag{2.4-5}$$

In the event that the thruster produces a significant number of multiply charged ions, the expression for the propellant utilization efficiency must be redefined.

For thrusters with both singly and doubly charged ions, the corrected mass utilization efficiency for multiple species is

$$\eta_{m^*} = \alpha_m \frac{I_b}{e} \frac{M}{\dot{m}_p}, \tag{2.4-6}$$

where α_m is a term that accounts for the fact that a doubly charged ion in the beam current carries two charges but only one unit of mass. In a manner similar to the derivation of the thrust correction due to double ions, the mass utilization correction α_m is given by

$$\alpha_m = \frac{1 + \frac{1}{2}\frac{I^{++}}{I^{+}}}{1 + \frac{I^{++}}{I^{+}}}. \tag{2.4-7}$$

For small ratios of double-to-single ion content, α_m is essentially equal to one.

Substituting Eq. (2.3-16) for the thrust and Eq. (2.4-5) for the propellant utilization efficiency into Eq. (2.4-3) yields an expression for the Isp:

$$\text{Isp} = \frac{\gamma \eta_m}{g} \sqrt{\frac{2eV_b}{M}}, \tag{2.4-8}$$

where the propellant utilization efficiency for singly charged ions must be used because Eq. (2.3-16) defines the beam current that way, and again the effective beam voltage must be used for Hall thrusters. Using the values for g and e, the Isp for an arbitrary propellant is

$$\text{Isp} = 1.417 \times 10^3 \gamma \eta_m \frac{\sqrt{V_b}}{\sqrt{M_a}}, \tag{2.4-9}$$

where V_b is the beam voltage in volts and M_a is the ion mass in atomic mass units [1 AMU = 1.6605×10^{-27} kg]. For xenon, the atomic mass $M_a = 131.29$, and the Isp is given by

$$\text{Isp} = 123.6\ \gamma \eta_m \sqrt{V_b}\ . \tag{2.4-10}$$

Using our previous example of a 10-deg half-angle beam divergence and a 10% doubles-to-singles ratio with a 90% propellant utilization of xenon [in Eq. (2.4-5)] at 1500 V, the Isp is 123.6*0.958*0.9* $\sqrt{1500}$ = 4127 s.

Specific impulse is functionally equivalent to gas mileage in a car. Cars with high gas mileage typically don't provide much acceleration, just as thrusters with high Isp don't provide as much thrust for a given input electrical power. Of critical importance is the ratio of the thrust achieved to total power used, which depends on the electrical efficiency of the thruster (to be described in the next section).

2.5 Thruster Efficiency

The mass utilization efficiency, defined in Eq. (2.4-6), describes the fraction of the input propellant mass that is converted into ions and accelerated in the electric thruster. The electrical efficiency of the thruster is defined as the beam power, P_b, out of the thruster divided by the total input power, P_T:

$$\eta_e = \frac{P_b}{P_T} = \frac{I_b V_b}{I_b V_b + P_o}, \tag{2.5-1}$$

where P_o represents the other power input to the thruster required to create the thrust beam. Other power will include the electrical cost of producing the ions, cathode heater or keeper power, grid currents in ion thrusters, etc.

The cost of producing ions is described by an ion production efficiency term, sometimes called the discharge loss:

$$\eta_d = \frac{\text{Power to produce the ions}}{\text{Current of ions produced}} = \frac{P_d}{I_b}, \tag{2.5-2}$$

where η_d has units of watts per ampere (W/A) or equivalently electron-volts per ion (eV/ion). Contrary to most efficiency terms, it is desirable to have η_d as small as possible since this represents a power loss. For example, if an ion thruster requires a 20-A, 25-V discharge to produce 2 A of ions in the beam, the discharge loss is then 20*25/2 = 250 eV/ion.

The performance of a plasma generator is usually characterized by plotting the discharge loss versus the propellant utilization efficiency. An example of this is shown in Fig. 2-3. At low propellant efficiencies, the neutral pressure in the thruster is high and the performance curves are relatively flat. As the propellant efficiency is increased, the neutral pressure in the thruster decreases, the

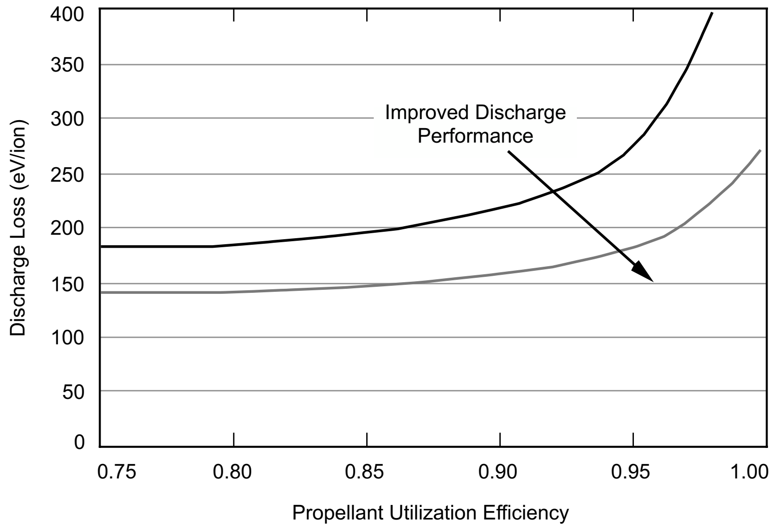

Fig. 2-3. Ion thruster performance curves consisting of discharge loss versus propellant utilization efficiency.

electron temperature increases, and the loss mechanisms in the thruster become larger. Thrusters are normally operated near the knee of this curve such that high mass utilization efficiency is achieved without excessive discharge loss. Optimized thruster designs result in lower discharge losses and low loss at high propellant efficiency.

The total efficiency of an electrically powered thruster is defined as the jet power divided by the total electrical power into the thruster:

$$\eta_T = \frac{P_{\text{jet}}}{P_{\text{in}}} . \tag{2.5-3}$$

Using Eq. (2.3-4) for the jet power, the efficiency of any electric propulsion thruster is

$$\eta_T = \frac{T^2}{2\dot{m}_p P_{\text{in}}} . \tag{2.5-4}$$

Measurements made of the thruster's input electrical power, input mass flow rate, and thrust output (measured in the vacuum system by a thrust stand) during testing can be used to calculate the total efficiency of the thruster using Eq. (2.5-4). This is the preferred technique for determining the efficiency of Hall thrusters because the beam parameters (current and velocity) are not known outright from measurements of electrical or gas flow parameters external to the vacuum system.

In ion thrusters, the beam is nearly monoenergetic, the exhaust velocity can be found from the net acceleration voltage applied to the thruster [using Eq. (2.3-6)], and the beam current is measured by the high voltage power supply. This allows the total efficiency to be accurately calculated from the electrical and gas flow inputs to the thruster. Using Eq. (2.3-16) for the thrust, Eq. (2.3-6) for the exhaust velocity, and Eq. (2.4-5) for the propellant flow rate, the total efficiency in Eq. (2.5-4) can be written as

$$\eta_T = \frac{\gamma \eta_m T v_i}{2\dot{m}_i P_{\text{in}}} = \gamma^2 \eta_m \frac{I_b V_b}{P_{\text{in}}} . \tag{2.5-5}$$

The input power into the thruster, from Eq. (2.5-1), is

$$P_{\text{in}} = \frac{P_b}{\eta_e} = \frac{I_b V_b}{\eta_e} , \tag{2.5-6}$$

Substituting Eq. (2.5-6) into Eq. (2.5-5) gives

$$\eta_T = \gamma^2 \eta_e \eta_m . \tag{2.5-7}$$

Measurements of the input propellant flow rate and electrical parameters (currents and voltages), and knowledge of the thrust correction factors from thruster plume measurements or code predictions, permit the total efficiency of ion thrusters to be calculated with high accuracy using Eq. (2.5-7).

Using our previous example of an ion thruster with 10-deg half-angle divergence, 10% double ion current, 90% mass utilization efficiency, and 250 eV/ion to produce a 2-A beam at 1500 V, the electrical efficiency is

$$\eta_e = \frac{2 * 1500}{2 * 1500 + 250 * 2} = 0.857 ,$$

and the total efficiency is

$$\eta_T = (0.958)^2 \ (0.857) \ (0.9) = 0.708 ,$$

which says that the thruster converts 70.8% of the supplied electrical energy into useful kinetic energy imparted to the spacecraft.

Thrusters with high exhaust velocities, and thus high Isp's, are desirable to maximize a mission payload mass. It was shown in Eq. (2.4-9) that to achieve high Isp, it is necessary to operate at a high ion acceleration voltage and high mass utilization efficiency. Reductions in ion mass also increase the Isp, but at the cost of thrust at the same power level. This is seen by examining the thrust-to-total input power ratio. The total power is just the beam power divided by the electrical efficiency, so the thrust-to-power ratio using Eq. (2.5.1) is

$$\frac{T}{P_T} = \frac{T\eta_e}{P_b}. \tag{2.5-8}$$

The beam power is the beam current times the beam voltage. Using Eq. (2.3-16) for the thrust and Eq. (2.4-8) to put this in terms of Isp, the thrust per unit input power is

$$\frac{T}{P_T} = \frac{2\gamma^2 \eta_m \eta_e}{g \ \mathrm{Isp}} = \frac{2}{g} \frac{\eta_T}{\mathrm{Isp}}. \tag{2.5-9}$$

Equation (2.5-9) shows that for a given input power and total thruster efficiency, increasing the Isp reduces the thrust available from the electric engine. This trade of thrust for Isp at a constant input power can only be improved if higher efficiency ion thrusters are employed.

2.6 Power Dissipation

The power into a thruster that does not result in thrust must be dissipated primarily by radiating the unused power into space. If the thruster electrical efficiency is accurately known, the dissipated power is

$$P_{\text{dissipated}} = P_{\text{in}}\left(1 - \eta_e\right). \tag{2.6-1}$$

If the electrical efficiency is not well known, alternative techniques can be used to determine the dissipated power. For example, in an ion thruster, the power in the beam is well known, and a simple difference between the total input power and the beam power represents the dissipated power. The various input powers can be measured externally to the thruster on the power supplies. For example, assuming the heaters have been turned off and the hollow cathodes are self-heating, the power into the ion thruster is given by

$$\begin{aligned} P_{\text{in}} &= I_b V_b + I_d V_d + I_{\text{ck}} V_{\text{ck}} + I_{\text{nk}} V_{\text{nk}} \\ &\quad + I_{A1}\left(V_b + V_a\right) + I_{A2} V_a + I_{\text{DE}1} V_b + I_{\text{DE}2} V_G, \end{aligned} \tag{2.6-2}$$

where the subscript "b" represents the beam current and voltage, "d" is the discharge current and voltage, "ck" is the cathode keeper current and voltage, "nk" is the neutralizer keeper current and voltage, "$A1$" represents beam ions incident on the accel grid, "$A2$" represents charge exchange ions at the accel grid potential V_a, "$I_{\text{DE}1}$" represents the decel grid (if present) current from beam ions, and "$I_{\text{DE}2}$" represents the decel grid current from backstreaming ions from the beam plume. In reality, the accel and decel grid power are very small compared to the other power levels in the thruster.

The power that must be dissipated by the thruster is Eq. (2.6.2) minus the beam power:

$$\begin{aligned} P_{\text{in}} &= I_d V_d + I_{\text{ck}} V_{\text{ck}} + I_{\text{nk}} V_{\text{nk}} + I_{A1}\left(V_b + V_a\right) \\ &+ I_{A2} V_a + I_{\text{DE}1} V_b + I_{\text{DE}2} V_G . \end{aligned} \tag{2.6-3}$$

Using the same ion thruster example used previously in this chapter, producing a 2-A beam at 1500 V as an example, Table 2-1 shows some example electrical parameters for a generic ion thruster. Assuming 10% of the grid currents are due to direct interception, using the table parameters in Eq. (2.5-12) gives a dissipated power of 528.3 W. Since the discharge power in this example is

Table 2-1. Example of ion thruster parameters used for power dissipation calculation.

Parameter	Term	Nominal
Discharge voltage	V_d	25
Discharge current (A)	I_d	20
Beam voltage	V_B	1500
Beam current (A)	I_B	2
Discharge keeper voltage	V_{ck}	10
Discharge keeper current (A)	I_{ck}	1
Neutralizer keeper voltage	V_{nk}	10
Neutralizer keeper current (A)	I_{nk}	1
Accel current (mA)	I_A	20
Accel voltage	V_A	250
Decel current (mA)	I_{DE}	2
Coupling voltage	V_G	20

500 W, the other power levels are relatively insignificant. However, the thruster will have to be of sufficient size to radiate all this power to space at a reasonable temperature that the materials and construction are designed to handle.

Unlike ion thrusters, power dissipation in Hall thrusters is not easily obtained from the external power supply readings. However, two techniques can be used to estimate the dissipated power. First, the dissipated power can be inferred from measurements of the thruster efficiency and the beam power (ion current and energy), which involves calculating the difference between the total beam power and the input electrical power. Another technique is to assume that the dissipated power is primarily the loss due to the electron current flowing from the exterior cathode through the thruster to the high voltage anode. If the fraction of the discharge current that becomes beam ions can be determined from the external diagnostics, then the difference between the discharge current and beam current times the discharge voltage is approximately the dissipated power. This technique neglects the ionization power and energy carried by the electrons in the beam, and so produces only a rough estimate. Hall thruster efficiency and performance useful in determining the power dissipation are described in detail in Chapter 7.

2.7 Neutral Densities and Ingestion in Electric Thrusters

In electric propulsion thrusters, the propellant is injected as a neutral gas into a chamber or region where ionization takes place. Accurately knowing the flow rate of the propellant gas is important in determining the performance and efficiency of the thruster and allows the operator to find the impact of finite pumping speed of test chambers on the thruster operation. The gas flow into the thruster, which is sometimes called the throughput, is often quoted in a number of different units. The most common units are standard cubic centimeters per minute (sccm) for ion thrusters and mg/s for Hall thrusters. Additional flow rate units include atoms per second, equivalent amperes, and torr-liter per second (torr-l/s). Conversion factors between these systems of flow units are derived in Appendix B.

The neutral pressure in the thruster discharge chamber or in the vacuum system follows standard gas law [1,2]:

$$PV = N\mathrm{k}T \ , \tag{2.7-1}$$

where P is the pressure in pascals, V is the volume, N is the number of particles, k is Boltzman's constant (1.38×10^{-23} W/s/K), and T is the temperature in kelvins. Since there are 133.32 pascals per torr, the number density of the neutral gas is

$$n = \frac{P_T \text{ [torr]} * 133.32 \text{ [pascal/ torr]}}{1.38 \times 10^{-23} \text{ [J / K]} * T \text{[K]}} \\ = 9.66 \times 10^{24} * \frac{P_T}{T} \left[\frac{\text{particles}}{m^3} \right], \tag{2.7-2}$$

where P_T is the pressure in the vacuum system in torrs and T is the gas temperature in kelvins. It should be noted that the pressure must be corrected for the gas type in whatever measurement system is used to obtain the actual pressure data. As an example, for a pressure of 10^{-6} torr and a temperature of 290 K, the density of gas atoms is 3.3×10^{16} per cubic meter.

The pressure in a vacuum system [3] in which a thruster is being tested is determined by the gas flow rate and the pumping speed

$$P = \frac{Q}{S}, \tag{2.7-3}$$

where Q is the total propellant throughput and S is the pumping speed. The most common units for pumping speed are liters per second, so utilizing a throughput in torr-l/s directly provides the pressure in the vacuum system in torr. The conversions of different flow units to torr-l/s can be obtained from Appendix B.

The finite pressure in the test vacuum system causes a backflow of neutral gas into the thruster that may artificially improve the performance. This ingestion of facility gas by the thruster can be calculated if the pressure in the chamber is known by evaluating the flux of neutral gas from the chamber into the thruster ionization region. The equivalent flow into the thruster is then the injected flow Q plus the equivalent ingested flow. The ingested flow (in particles per second) is given by

$$Q_{\text{ingested}} = \frac{n\bar{c}}{4} A * \eta_c, \tag{2.7-4}$$

where n is the neutral density in the chamber, $\bar{c}$ is the gas thermal velocity, A is the total open area fraction of the thruster to the vacuum system, and η_c is a correction factor related to the conductance into the thruster from the vacuum system. The neutral gas density is given by Eq. (2.7-2), and the gas thermal velocity is given by

$$\bar{c} = \sqrt{\frac{8kT}{\pi M}}, \tag{2.7-5}$$

where M is the atom mass in kg. The conductance correction factor is sometimes called the Clausing factor [4] and describes the conductance reduction due to the finite axial length of the effective entrance aperture(s) to the thruster. This factor is generally negligible for Hall thrusters but appreciable for the apertured grids of ion thrusters. Due to the large diameter-to-length ratio of the accelerator grid apertures in ion thrusters, the Clausing factor is usually calculated by Monte-Carlo gas flow codes. An example of a simple spreadsheet Monte-Carlo code for calculating the Clausing factor for ion thruster grids is given in Appendix G.

The ingested flow of gas from the finite pressure in the vacuum system is then

$$Q_{\text{ingested}} = \frac{133.2\,P}{4\,\mathrm{k}T}\sqrt{\frac{8\,\mathrm{k}T}{\pi M}}\;\frac{A*\eta_c}{4.479\times 10^{17}}\ [\text{sccm}]\,. \tag{2.7-6}$$

This expression for the ingested flow can be rewritten as

$$Q_{\text{ingested}} = 7.82\times 10^{8}\,\frac{P*A*\eta_c}{\sqrt{T\,M_a}}\,[\text{sccm}]\,, \tag{2.7-7}$$

where P is the vacuum chamber pressure in torr, T is the backflowing neutral gas temperature in K, M_a is the gas mass in AMU, and A is the open area in m^2. The total flow rate into the thruster is then

$$Q_{\text{total}} = Q_{\text{injected}} + Q_{\text{ingested}}\,. \tag{2.7-8}$$

References

[1] J. M. Lafferty, *Foundations of Vacuum Science and Technology*, New York: John Wiley and Sons, 1998.

[2] A. Ross, *Vacuum Technology*, Amsterdam, Holland: Elsevier, 1990.

[3] G. Lewin, *Fundamentals of Vacuum Science and Technology*, New York: McGraw-Hill, 1965.

[4] P. Clausing, "The Flow of Highly Rarefied Gases Through Tubes of Arbitrary Length," *Journal of Vacuum Science and Technology*, vol. 8, pp. 636–646, 1971.

Homework Problems

1. Assume that the ion charge density in a one-dimensional (1-D) accelerator gap between two grids varies as $\rho = \rho_o x / d$, and that a voltage V_o is applied to the electrodes bounding the gap.
   a. Find the potential and electric field as a function of position in the gap.
   b. Find the force on each of the grids.
   c. Find the total electrostatic force between the ions and the grids.
2. A mission under study desires to deliver a 800-kg payload through 8 km/s of Δv. The spacecraft has 3 kW of electric power available for propulsion. The mission planners want to understand the trade-offs for different thrusters and operating conditions, and they want you to make plots of propellant mass and trip time required versus specific impulse for the following cases. Assume xenon is the propellant.
   a. Ion thruster case: The ion thruster can run at full power from 1 kV to 2 kV. For all throttle conditions, assume the following parameters are constant: total efficiency of 55%, propellant utilization of 85%, beam divergence angle of 12 deg, and double-to-single ion current ratio of 10%.
   b. Hall thruster case: The Hall thruster can run at full power from 300 V to 400 V. For all throttle conditions, assume the following parameters are constant: total efficiency of 45%, propellant utilization of 85%, beam divergence angle of 25 deg, and double-to-single ion current ratio of 15%.
3. Derive Eq. (2.4-7) for the mass utilization efficiency correction due to double ions.
4. Derive the thrust correction factor and the resulting thrust equation accounting for the presence of triply ionized atoms. Assuming 10% doubles, what is the error in the calculated thrust if 5% actually present triples have been neglected?
5. Mission planners have two candidate ion and Hall thrusters to place on a spacecraft and want to understand how they compare for thrust-to-power ratio and performance. The xenon ion thruster has a total power of 5 kW, a 1200-V, 3.75-A beam with 10% double ions, a total efficiency of 65%, and a mass utilization efficiency of 86%. The Hall thruster has a total power of 5 kW, a 300-V discharge voltage and a 12.5-A beam with 10% double ions, a total efficiency of 50%, and an input xenon gas flow of 19 mg/s.

a. What is the thrust-to-power ratio (usually expressed in mN/kW) for each thruster?

b. What is the Isp for each engine?

c. For a 1000-kg spacecraft, what is the fuel mass required to achieve a 5-km/s delta-v?

d. What is the trip time to expend all of the fuel for each thruster type if the thrusters are on 90% of the time?

6. The thrust correction factor for multiply ionized species is based on the current of charges in the beam (see Eq. (2.3-12) for singles and doubles).

   a. Derive an expression for the number of atoms of each ionized species in the beam for a given value of I^{++}/I^{+} and I^{+++}/I^{+}.

   b. If $I^{++}/I^{+} = 10\%$ and $I^{+++}/I^{+} = 5\%$, what are the actual percentages of the number of atoms of each species in the beam compared to the total beam current?

7. An ion thruster is being tested in a vacuum chamber with a measured xenon pressure of 1×10^{-5} torr at room temperature (300°C). The thruster grids have a 25-cm grid diameter and a Clausing factor of 0.5.

   a. If the thruster is producing a 3-A beam with 15% double ions with a total of 50 sccm of xenon gas flow into the thruster, what is the mass utilization efficiency neglecting gas ingestion?

   b. What is the mass utilization efficiency including the effects of ingestion?