# Fall 2021 I609 Assignment 5

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Nov 21, 2021

## Chapter 19 Questions

### 19.3

Consider the model from Chapter 19 for the diffusion of a new behavior through a social network. Recall that for this we have a network, a behavior $B$ that everyone starts with, and a threshold $q$ for switching to a new behavior $A$ — that is, any node will switch to $A$ if at least a $q$ fraction of its neighbors have adopted $A$.
Consider the network depicted in Figure 1; suppose that each node starts with the behavior $B$, and each node has a threshold of $q = \frac{2}{5}$ for switching to behavior $A$.

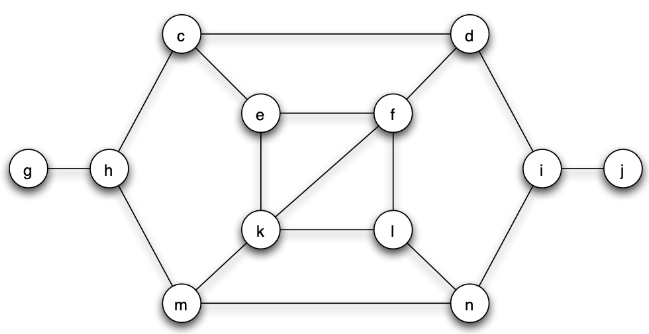


Figure 19.29: A social network in which a new behavior is spreading.

Figure 1: PS 19.3 Network

**(a)** Now, let $e$ and $f$ form a two-node set S of initial adopters of behavior $A$. If other nodes follow the threshold rule for choosing behaviors, which nodes will eventually switch to $A$?

**(b)** Find a cluster of density $1 - q = \frac{3}{5}$ in in the part of the graph outside $S$ that blocks behavior $A$ from spreading to all nodes, starting from $S$, at threshold $q$.

**(c)** Suppose you're allowed to add one node to the set $S$ of initial adopters, which currently consists of $e$ and $f$. Can you do this in such a way that the new 3-node set causes a cascade at threshold $q = 2/5$ ?

*-Next page for solutions-*

## Solutions

**(a)** Based on the spreading rule of $q = \frac{2}{5}$ and initial "infected" node set $\{e, f\} \in S$, only nodes $k$ and $l$ will be switch to brand $A$. As per Figure 2, the system iterates only one (1) time period system before the "viral" brand spreading ends. At $t = 1$, nodes $k$ and $l$ switch to $B$ since exactly $1/2$ of their neighbors belong to set $S$/have already adopted $A$. However, by time step $t = 2$, behavior/brand $A$ has reached its greatest extent. Nodes $k$ and $l$ now face the same difficulty in "infecting" nodes $m$ and $n$ (respectively) as the difficulty faced by nodes $e$ and $f$ vis-à-vis nodes $c$ and $d$.

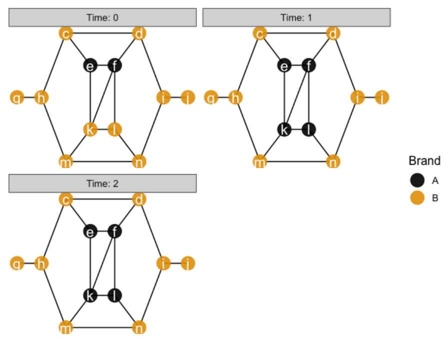


Figure 2: Lack of cascade when $q = \frac{2}{5}$

**(b)** The preceding step winks at the what the blocking cluster consists of, namely all nodes with behavior/brand $B$ at the end of Figure 1. Figure 3 helpfully maps out node's set membership. Moreover, I've taken the liberty of printing node $z$'s name and the share of $z$'s neighbors that belong to the block cluster.

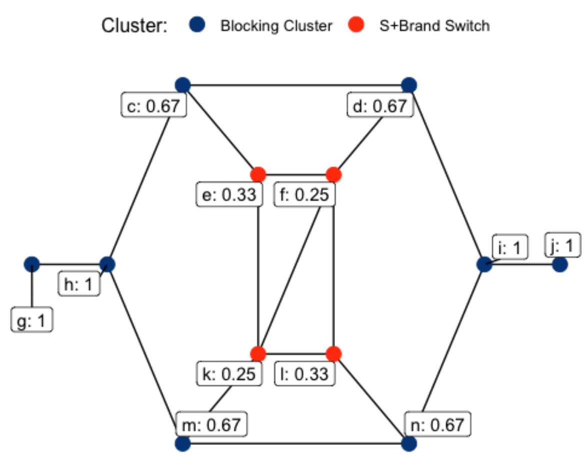


Figure 3: Clusters

**(c)** Based on Figure 3, nodes $h$ and $i$ in the Blocking Cluster appear to be the most promising candidates to ensure a cascade because they both have *all* their neighbors in the blocking cluster AND they have a sufficiently high degree to ensure spread (see nodes $g$ and $j$). "Corner" nodes $c, d, m$ and $n$ are poorly suited to facilitate spread since they won't be able to "infect" cornerstone nodes $h$ or $i$. Indeed, Figure 4 confirms the fact that adding $i$ to $S$ will ensure a cascade.

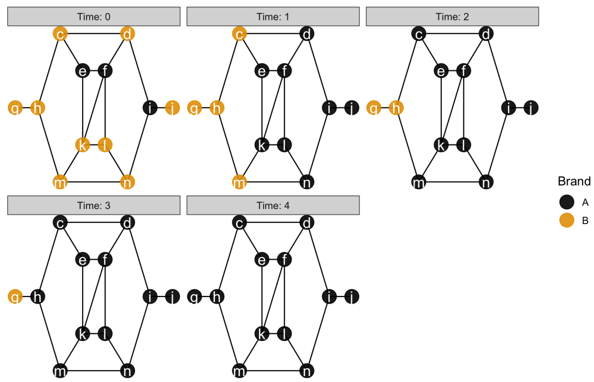


Figure 4: Cascade when $q = \frac{2}{5}$ and $i \in S$