PCA: batch processing and online-PCA

Group name: DataFun

3.1 Preprocessing

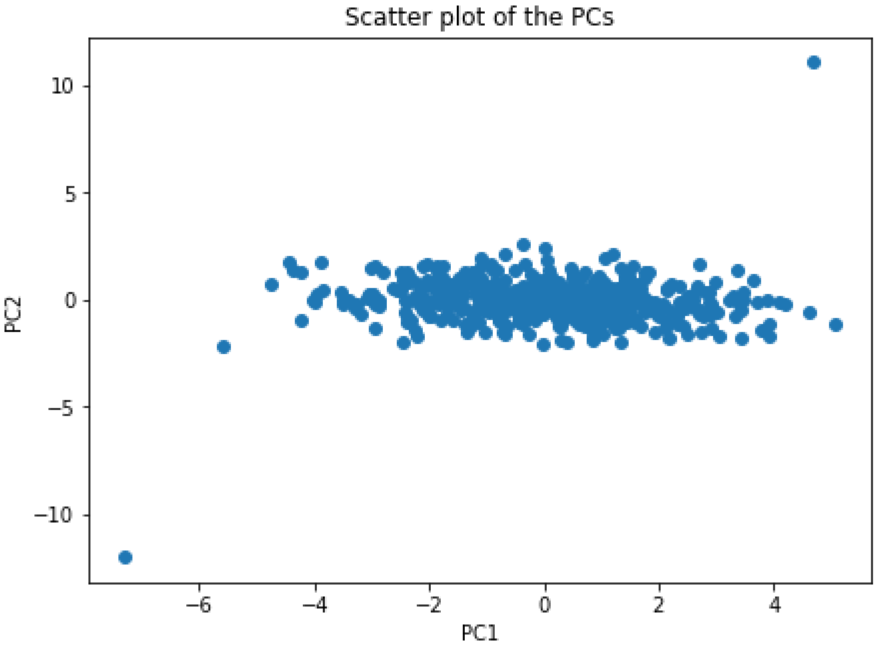
In [1]:

```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import scipy.linalg as la
```

(a) Loading the dataset, computing and plotting the first two Principal Components.

In [2]:

```
data2D = pd.read_csv("pca2.csv", sep=",")
meanVector2D = np.mean(data2D)
centeredData2D = data2D - meanVector2D
covarianceMatrix2D = np.cov(centeredData2D.T)
eigenvalues2D, eigenvectors2D = np.linalg.eig(covarianceMatrix2D)
orderedIndices2D = np.argsort(eigenvalues2D)[::-1]
orderedEigenvalues2D = eigenvalues2D[orderedIndices2D]
orderedEigenvectors2D = eigenvectors2D[orderedIndices2D]
pcaData2D = np.dot(centeredData2D, orderedEigenvectors2D)

# Plotting the results.
plt.figure(figsize=(7,5))
plt.scatter(pcaData2D[:,0], pcaData2D[:,1])
plt.title("Scatter plot of the PCs")
plt.xlabel("PC1")
plt.ylabel("PC2")
plt.show()
```

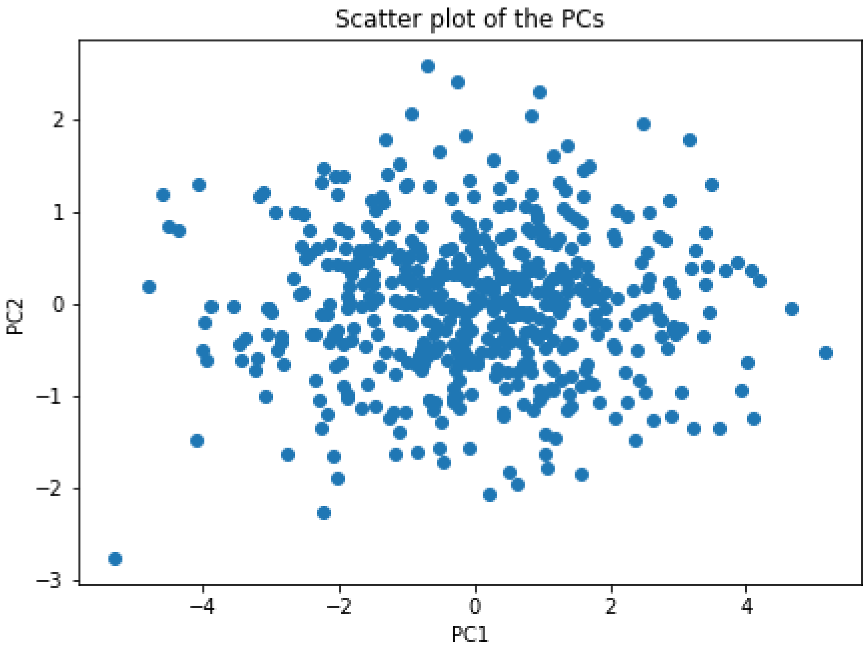


The data looks like one bigger cluster and some outliers.

(b) Removing required observations and redoing the steps.

In [3]:

```
data2D = data2D.drop([16, 156])
meanVector2D = np.mean(data2D)
centeredData2D = data2D - meanVector2D
covarianceMatrix2D = np.cov(centeredData2D.T)
eigenvalues2D, eigenvectors2D = np.linalg.eig(covarianceMatrix2D)
orderedIndices2D = np.argsort(eigenvalues2D)[::-1]
orderedEigenvalues2D = eigenvalues2D[orderedIndices2D]
orderedEigenvectors2D = eigenvectors2D[orderedIndices2D]
pcaData2D = np.dot(centeredData2D, orderedEigenvectors2D)

# Plotting the results.
plt.figure(figsize=(7,5))
plt.scatter(pcaData2D[:,0], pcaData2D[:,1])
plt.title("Scatter plot of the PCs")
plt.xlabel("PC1")
plt.ylabel("PC2")
plt.show()
```



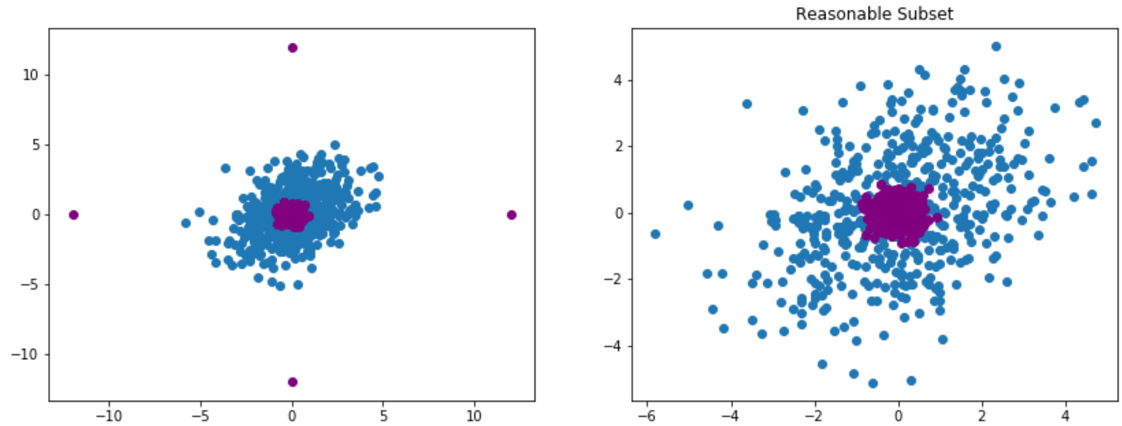
The outliers were removed the data looks like one big cluster.

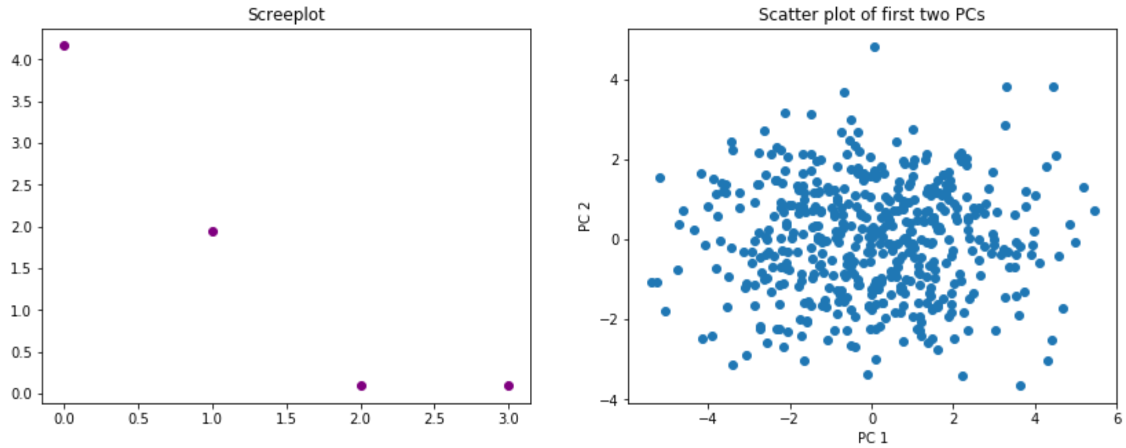
3.2 Whitening

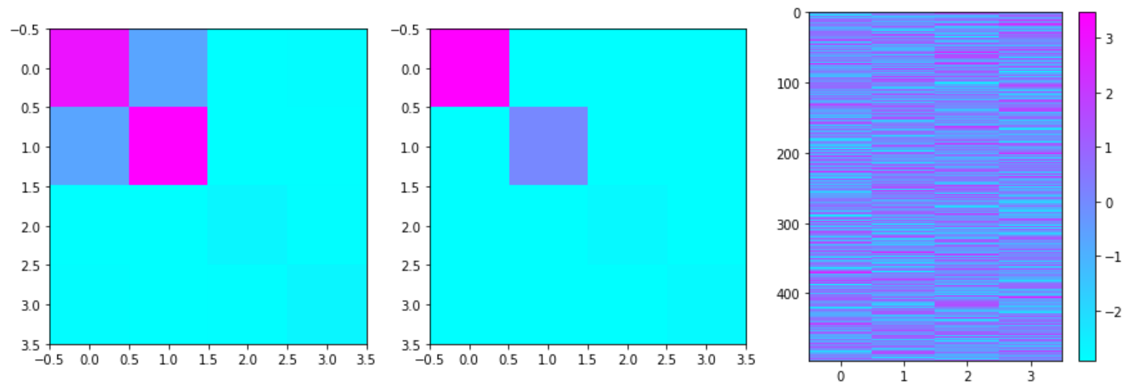
In [4]:

```
# Load and show outliners
dataWhite = pd.read_csv("pca4.csv", sep=",").as_matrix()
plt.figure(figsize=(14,5))
plt.subplot(1,2,1)
plt.scatter(dataWhite[:,0],dataWhite[:,1])
plt.scatter(dataWhite[:,2],dataWhite[:,3], color='purple')

# Remove outliners
for i in range(495):
    if (abs(dataWhite[:,2][i]) > 10 or abs(dataWhite[:,3][i]) > 10 ):
        dataWhite = np.delete(dataWhite, (i), axis=0)
```

```
# reasonable Subset
plt.subplot(1,2,2)
plt.scatter(dataWhite[:,0],dataWhite[:,1])
plt.scatter(dataWhite[:,2],dataWhite[:,3], color='purple')
plt.title("Reasonable Subset")
plt.show()

# PCA
meanVectorWhite = np.mean(dataWhite,axis=0)
centeredDataWhite = dataWhite - meanVectorWhite
covarianceMatrixWhite = np.cov(centeredDataWhite.T)
eigenvaluesWhite, eigenvectorsWhite = np.linalg.eig(covarianceMatrixWhite)
orderedIndicesWhite = np.argsort(eigenvaluesWhite)[::-1]
orderedEigenvaluesWhite = eigenvaluesWhite[orderedIndicesWhite]
orderedEigenvectorsWhite = eigenvectorsWhite[orderedIndicesWhite]
pcaDataWhite = np.dot(centeredDataWhite, orderedEigenvectorsWhite)
np.cov(pcaDataWhite)

# Screeplot
plt.figure(figsize=(14,5))
plt.subplot(1,2,1)
plt.title("Screeplot")
plt.plot(orderedEigenvaluesWhite, 'o',label="original", color='purple')
# 2 PCs represent the data well (see plot)

# Plotting against two PC.
plt.subplot(1,2,2)
plt.scatter(pcaDataWhite[:,0], pcaDataWhite[:,1])
plt.title("Scatter plot of first two PCs")
plt.xlabel("PC 1")
plt.ylabel("PC 2")
plt.show()

#Whiten
X = centeredDataWhite
E = eigenvectorsWhite
L = la.sqrtm(np.linalg.inv(np.diag(eigenvaluesWhite)))
Z = np.dot(np.dot(X,E),L)

#Heat plots
plt.figure(figsize=(15,5))
plt.subplot(1,3,1)
plt.imshow(covarianceMatrixWhite, cmap="cool")
plt.subplot(1,3,2)
plt.imshow(np.cov(pcaDataWhite.T), cmap="cool")
plt.subplot(1,3,3)
plt.imshow(Z, cmap="cool", aspect='auto')
plt.colorbar()
plt.show()
```







3.3 Derivation of Oja's rule

We are moving out from the following normalization, what was introduced by Oja:

$$w_i(t+1) = \frac{w_i(t) + \epsilon y(t) x_i(t)}{\left(\sum_{j=1}^{N} [w_j(t) + \epsilon y(t) x_j(t)]^2\right)^{\frac{1}{2}}}$$

Taylor expansion with one term:

$$f(x) = f(x_0) + f'(x - x_0)(x - x_0)$$

After Taylor expanding the right side (using quotient rule):

$$\approx \frac{w_i(t)}{(\sum_{j=1}^N [w_j(t)]^2)^{\frac{1}{2}}}$$

$$+ \frac{y(t)x_i(t)(\sum_{j=1}^N [w_j(t)+\epsilon y(t)x_j(t)]^2)^{\frac{1}{2}} - (w_i(t)+\epsilon y(t)x_i(t))\frac{1}{2}(\sum_{j=1}^N [w_j(t)+\epsilon y(t)x_j(t)]^2)^{-\frac{1}{2}} \sum_{j=1}^N [2}{\sum_{j=1}^N [w_j(t)+\epsilon y(t)x_j(t)]^2}$$

$$\approx \frac{w_i(t)}{(\sum_{j=1}^N w_j(t)^2)^{\frac{1}{2}}} + \frac{y(t)x_i(t)(\sum_{j=1}^N w_j(t)^2)^{\frac{1}{2}} - \frac{1}{2}w_i(t)(\sum_{j=1}^N w_j(t)^2)^{-\frac{1}{2}} \sum_{j=1}^N 2w_j(t)y(t)x_j(t)}{\sum_{j=1}^N w_j(t)^2}\epsilon$$

Because of the normalization in the previous step we know: $\sum_{j=1}^N w_j(t)^2 = 1$, therefore:

$$= w_i(t) + \epsilon[y(t)x_i(t) - y(t)w_i(t)\sum_{j=1}^N w_j(t)x_j(t)]$$

With the definition of y: $y(t) = \sum_{j=1}^N w_j(t)x_j(t)$:

$$w_i(t+1) \approx w_i(t) + \epsilon y(t)[x_i(t) - w_i(t)y(t)]$$

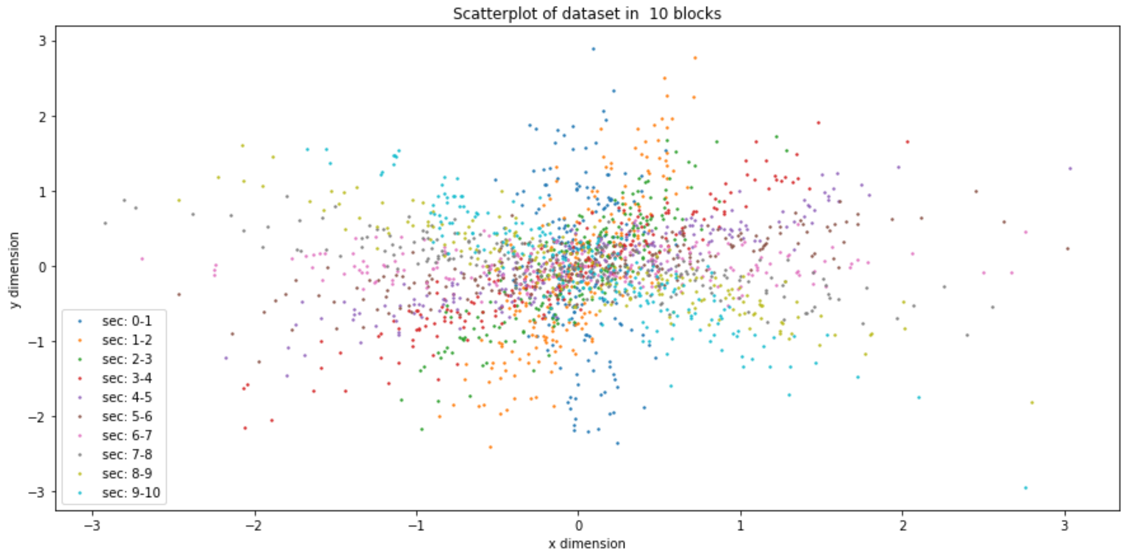
This formula is what we wanted to achieve.

3.4

1) Make a scatter plot of the data and indicate the time index by the color of the datapoints

In [5]:

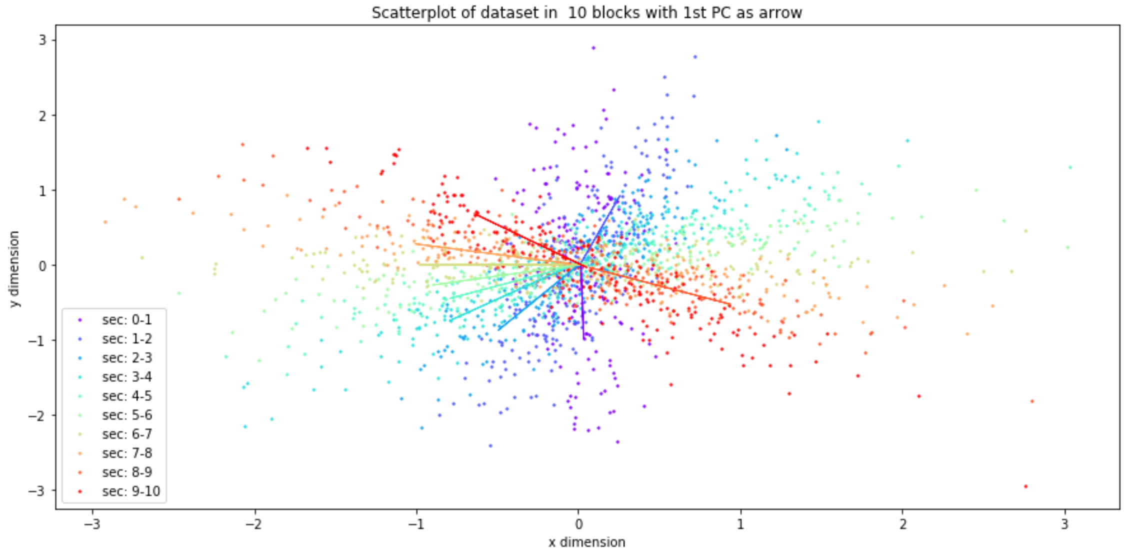
```
import numpy as np
import matplotlib.pyplot as plt
from matplotlib.mlab import PCA
import matplotlib.cm as cm

dataSet = np.loadtxt('./data-onlinePCA.txt', delimiter=',',
                    skiprows=1,
                    usecols=(1,2))

# divide the data into 10 sets covering each one second
dataBlocks = dataSet.reshape(10,200,2)
# get the mean of each set (in both dimensions)
dataMean = np.mean(dataBlocks, axis=1)


plt.figure(figsize=(15,7))
for i in range(0, dataBlocks.shape[0]):
    strLabel = 'sec: ' + str(i) + '-' + str(i+1)
    plt.scatter(dataBlocks[i,:,0], dataBlocks[i,:,1], s=2, label=strLabel)
plt.legend()
plt.title('Scatterplot of dataset in  10 blocks')
plt.xlabel('x dimension')
plt.ylabel('y dimension')
plt.show()
```



2) Determine the principal components (using batch PCA) and plot the first PC for each of the 10 blocks in the same plot as the original data

In [6]:

```
colors = cm.rainbow(np.linspace(0, 1, 10))

plt.figure(figsize=(15,7))
for i in range(0, dataBlocks.shape[0]):
    strLabel = 'sec: ' + str(i) + '-' + str(i+1)
    batchPCA = PCA(dataBlocks[i], standardize=False)                    #calc the PC
A
    plt.scatter(dataBlocks[i,:,0], dataBlocks[i,:,1], s=2, label=strLabel,
color=colors[i])
    plt.arrow(dataMean[i,0], dataMean[i,1], batchPCA.Wt[0,0], batchPCA.Wt[0,1],
head_width=0, head_length=0, color=colors[i])    #plot the first eigenvector
plt.legend()
plt.title('Scatterplot of dataset in  10 blocks with 1st PC as arrow')
plt.xlabel('x dimension')
plt.ylabel('y dimension')
plt.show()
```

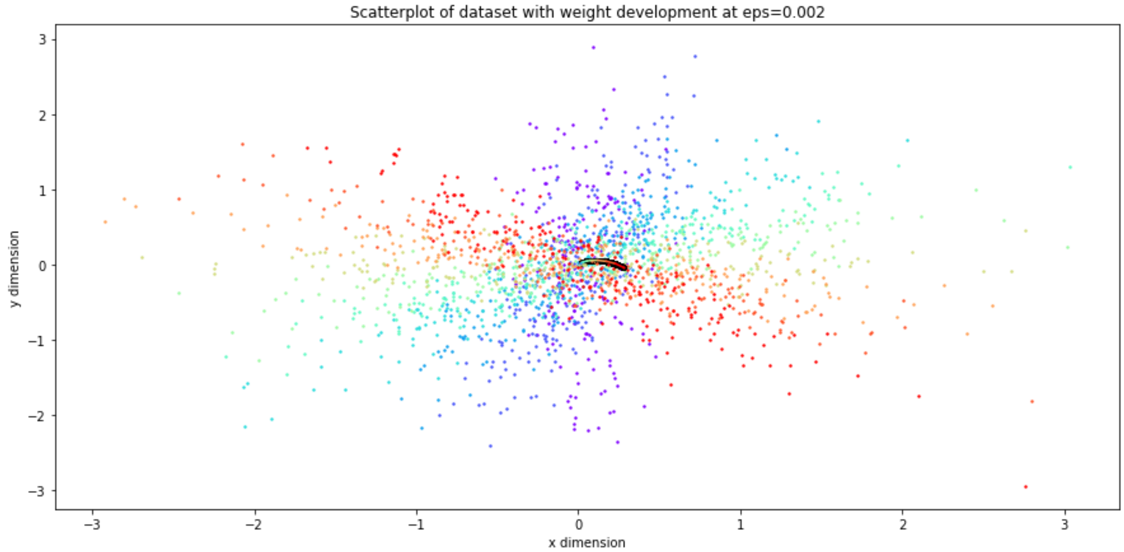


3) Implement Oja's rule and apply it with a learning-rate parameter ε ∈ {0.002, 0.04, 0.45} to the dataset

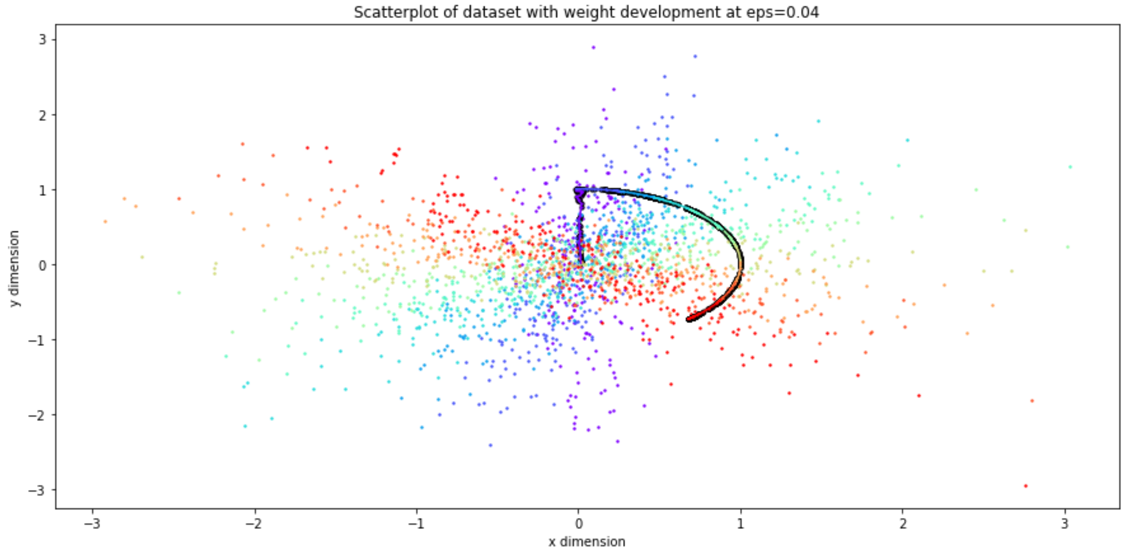
In [7]:

```
eps = [0.002, 0.04, 0.45]
for e in eps:

    #init required arrays
    plt.figure(figsize=(15,7))
    wArray = np.zeros((2000,2))

    #we initialize w with the mean of the first datablock - could be initialized
 with other values
    w = np.array([dataMean[0,0], dataMean[0,1]])

    # for each datapoint apply Oja's rule successively
    for i in range(0, dataSet.shape[0]):
        x = dataSet[i]
        y = np.dot(w.T, x)
        dW = e*y*(x-y*w)
        w = w + dW
        wArray[i] = w

    #plot the weights and show their development
    wArray = wArray.reshape(10,200,2)
    for i in range(0, dataBlocks.shape[0]):
        plt.scatter(dataBlocks[i,:,0], dataBlocks[i,:,1], s=2, color=colors[i])
        plt.plot(wArray[i,:,0], wArray[i,:,1], color=colors[i], linestyle='-')
        plt.scatter(wArray[i,:,0], wArray[i,:,1], s=10, facecolor=colors[i], edg
ecolor='k')
    plt.title('Scatterplot of dataset with weight development at eps=' + str(e))
    plt.xlabel('x dimension')
    plt.ylabel('y dimension')
    plt.show()
```

Scatterplot of dataset with weight development at eps=0.002



Scatterplot of dataset with weight development at eps=0.04



Scatterplot of dataset with weight development at eps=0.45