d) Sea $B = \{v_1, \dots, v_n\}$ una base de V y $B^* = \{\varphi_1, \dots, \varphi_n\}$ su base dual. Calcula la matriz de $\varphi \otimes \psi$ en la base B relacionándola con las coordenadas de φ y ψ en la base B^*. Demuestra que $B' = \{\varphi_i \otimes \varphi_j \,/\, i, j = 1, \dots, n\}$ es una base de $\mathcal{B}(V)$. Describe la base B' cuando $V = \mathbb{R}^n$ y $B = B_u$.

$\mathcal{B} = \{v_1 \dots v_n\} \quad \mathcal{B}^* = \{\varphi_1, \dots, \varphi_n\}$

$$\mathcal{M}(\varphi \otimes \psi, \mathcal{B}) = \begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{pmatrix}$$

$\varphi = a_1 \varphi_1 + \dots + a_n \varphi_n$

$\psi = b_1 \psi_1 + \dots + b_n \psi_n$

$$m_{11} = (\varphi \otimes \psi)(v_1, v_1) = \varphi(v_1)\,\psi(v_1) = a_1 \overset{1}{\varphi_1(v_1)} + \cancel{\dots} + \overset{0}{a_n \cancel{\varphi_n}(v_1)} + b_1 \overset{1}{\psi_1(v_1)} + b_n \overset{0}{\cancel{\psi_n}(v_1)} = a_1 b_1$$

$$m_{12} = (\varphi \otimes \psi)(v_1, v_2) = \varphi(v_1)\,\psi(v_2) = a_1 b_2$$

$$m_{ij} = (\varphi \otimes \psi)(v_i, v_j) = \varphi(v_i)\,\psi(v_j) = a_i b_j$$

$$\mathcal{M}(\varphi \otimes \psi, \mathcal{B}) = \begin{pmatrix} a_1 b_1 & a_1 b_2 & a_1 b_3 \\ a_2 b_1 & a_2 b_2 & a_2 b_3 \\ a_3 b_1 & a_3 b_2 & a_3 b_3 \end{pmatrix}$$