EJERCICIO 1

Hallamos primero los nodos de Chebyshev en el intervalo [-1,1]

(%i1)
```
nodos_che:makelist(cos(((2·i+1)·%pi)/(2·10)),i,0,9);
```
(nodos_che) $$\left[\cos\left(\frac{\pi}{20}\right),\cos\left(\frac{3\,\pi}{20}\right),\frac{1}{\sqrt{2}},\cos\left(\frac{7\,\pi}{20}\right),\cos\left(\frac{9\,\pi}{20}\right),\cos\left(\frac{11\,\pi}{20}\right),\cos\left(\frac{13\,\pi}{20}\right),-\frac{1}{\sqrt{2}},\cos\left(\frac{17\,\pi}{20}\right),\cos\left(\frac{19\,\pi}{20}\right)\right]$$

Ahora trasladamos todo esto al intervalo [0,91/11], mediante un isomorfismo afín.

(%i2)
```
linsolve([−x+y=0, x+y=91/11], [x,y]);
```
(%o2) $$\left[x=\frac{91}{22},y=\frac{91}{22}\right]$$

(%i3)
```
f_paso(t):=(91/22)·t+91/22;
```
(%o3) $$f_{paso}(t):=\frac{91}{22}\,t+\frac{91}{22}$$

(%i4)
```
for i:0 thru 9 do nodos_che[i+1]:f_paso(float(nodos_che[i+1]));
```
(%o4) *done*

(%i5)
```
nodos_che;
```
(%o5) [8.221801772461707, 7.821890622779159, 7.061214413089811, 6.014233430740854, 4.783433469030046, 3.489293803697226, 2.25849384198642, 1.211512859637463, 0.4508366499481151, 0.05092550026556708]

EJERCICIO 2

(%i6)
```
f(x):=log(1+sqrt(x)) + 9;
```
(%o6) $$f(x):=\log(1+\sqrt{x})+9$$

(%i7)
```
y:makelist(float(f(nodos_che(j+1))),j,0,9);
```
(y) [log(sqrt([(1.0))+1.0)+9.0, log(sqrt([(2.0))+1.0)+9.0, log(sqrt([(3.0))+1.0)+9.0, log(sqrt([(4.0))+1.0)+9.0, log(sqrt([(5.0))+1.0)+9.0, log(sqrt([(6.0))+1.0)+9.0, log(sqrt([(7.0))+1.0)+9.0, log(sqrt([(8.0))+1.0)+9.0, log(sqrt([(9.0))+1.0)+9.0, log(sqrt([(10.0))+1.0)+9.0]

(%i8)
```
matrix_aux: matrix(
 [0,0,0,0,0,0,0,0,0,0],
 [0,0,0,0,0,0,0,0,0,0],
 [0,0,0,0,0,0,0,0,0,0],
 [0,0,0,0,0,0,0,0,0,0],
 [0,0,0,0,0,0,0,0,0,0],
 [0,0,0,0,0,0,0,0,0,0],
 [0,0,0,0,0,0,0,0,0,0],
 [0,0,0,0,0,0,0,0,0,0],
 [0,0,0,0,0,0,0,0,0,0],
[0,0,0,0,0,0,0,0,0,0]
);
```

(matrix_aux)

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(%i9)
```
for i:0 thru 9 do matrix_aux[i+1,1]:float(y[i+1]);
```

(%o9) *done*

(%i10)
```
matrix_aux;
```

(%o10)

$$\begin{bmatrix} \log(\sqrt{[8.221801772461707,7.821890622779159,7.061214413089811,6.014233430740854,4.78343346} \\ \log(\sqrt{[8.221801772461707,7.821890622779159,7.061214413089811,6.014233430740854,4.78343346} \\ \log(\sqrt{[8.221801772461707,7.821890622779159,7.061214413089811,6.014233430740854,4.78343346} \\ \log(\sqrt{[8.221801772461707,7.821890622779159,7.061214413089811,6.014233430740854,4.78343346} \\ \log(\sqrt{[8.221801772461707,7.821890622779159,7.061214413089811,6.014233430740854,4.78343346} \\ \log(\sqrt{[8.221801772461707,7.821890622779159,7.061214413089811,6.014233430740854,4.78343346} \\ \log(\sqrt{[8.221801772461707,7.821890622779159,7.061214413089811,6.014233430740854,4.78343346} \\ \log(\sqrt{[8.221801772461707,7.821890622779159,7.061214413089811,6.014233430740854,4.78343346} \\ \log(\sqrt{[8.221801772461707,7.821890622779159,7.061214413089811,6.014233430740854,4.78343346} \\ \log(\sqrt{[8.221801772461707,7.821890622779159,7.061214413089811,6.014233430740854,4.78343346} \end{bmatrix}$$

(%i11)
```
for i:1 thru 9 do(
    for j:1 thru i do(
        matrix_aux[i+1,j+1]:float((matrix_aux[i+1,j]−matrix_aux[i,j])/(nodos_c
```

(%o11) *done*

(%i12)
```
w(i,z):=product(z−nodos_che[j+1], j, 0, i−1);
```

(%o12) $$w(i,z):=\prod_{j=0}^{i-1} z-nodos\_che_{j+1}$$

(%i13)
```
p(z):=sum(matrix_aux[i+1,i+1]·w(i,z) , i, 0, 9);
```

(%o13) $$p(z):=\sum_{i=0}^{9} matrix\_aux_{i+1,i+1}\, w(i,z)$$

(%i14) float(p(3));

(%o14) −111.2824763640209 (0.1286841446634289 (
0.1426474732261967 (0.1676921620782877 (
0.2113044513818475 (0.2908356265970551 (
0.4529871085306813 (0.8616326827316089 (2.500555437861153

(log(sqrt([(2.0))+1.0)−1.0 log(sqrt([(1.0))+1.0))−

1.314619791262289 (log(sqrt([(3.0))+1.0)−1.0 log(sqrt([(2.0))

+1.0)))−0.5532022356918267 (1.314619791262289 (log(sqrt([

(3.0))+1.0)−1.0 log(sqrt([(2.0))+1.0))−0.955127186509584 (
log(sqrt([(4.0))+1.0)−1.0 log(sqrt([(3.0))+1.0))))−
0.3291143989857198 (0.5532022356918267 (1.314619791262289

(log(sqrt([(3.0))+1.0)−1.0 log(sqrt([(2.0))+1.0))−

0.955127186509584 (log(sqrt([(4.0))+1.0)−1.0 log(sqrt([(3.0))

+1.0)))−0.4390237799679133 (0.955127186509584 (log(sqrt([

(4.0))+1.0)−1.0 log(sqrt([(3.0))+1.0))−0.8124797132833868 (
log(sqrt([(5.0))+1.0)−1.0 log(sqrt([(4.0))+1.0)))))−
0.2308084600892768 (0.3291143989857198 (
0.5532022356918267 (1.314619791262289 (log(sqrt([(3.0))+1.0

)−1.0 log(sqrt([(2.0))+1.0))−0.955127186509584 (log(sqrt([

(4.0))+1.0)−1.0 log(sqrt([(3.0))+1.0)))−0.4390237799679133

(0.955127186509584 (log(sqrt([(4.0))+1.0)−1.0 log(sqrt([(3.0)

)+1.0))−0.8124797132833868 (log(sqrt([(5.0))+1.0)−1.0 log(

sqrt([(4.0))+1.0))))−0.2799614295374983 (
0.4390237799679133 (0.955127186509584 (log(sqrt([(4.0))+1.0

)−1.0 log(sqrt([(3.0))+1.0))−0.8124797132833868 (log(sqrt([

(5.0))+1.0)−1.0 log(sqrt([(4.0))+1.0)))−0.3960490735261138

(0.8124797132833868 (log(sqrt([(5.0))+1.0)−1.0 log(sqrt([
(4.0))+1.0))−0.7727141256757828 (log(sqrt([(6.0))+1.0)−1.0

log(sqrt([(5.0))+1.0))))))−0.1797463023763529 (
0.2308084600892768 (0.3291143989857198 (
0.5532022356918267 (1.314619791262289 (log(sqrt([(3.0))+1.0

)−1.0 log(sqrt([(2.0))+1.0))−0.955127186509584 (log(sqrt([

(4.0))+1.0)−1.0 log(sqrt([(3.0))+1.0)))−0.4390237799679133

(0.955127186509584 (log(sqrt([(4.0))+1.0)−1.0 log(sqrt([(3.0)

EJERCICIO 3

(%i15)
```
u:[1,1,1,1,1,1,1,1,1,1];
```
(u) $[1,1,1,1,1,1,1,1,1,1]$

(%i16)
```
echelon(matrix(nodos_che,u));
```
rat: replaced 8.221801772461707 by 62962040/7657937 = 8.221801772461696
rat: replaced 7.821890622779159 by 23498735/3004227 = 7.821890622779171
rat: replaced 7.061214413089811 by 112705745/15961241 = 7.061214413089898
rat: replaced 6.014233430740854 by 104903429/17442527 = 6.014233430740854
rat: replaced 4.783433469030046 by 24474710/5116557 = 4.783433469030053
rat: replaced 3.489293803697226 by 16843861/4827298 = 3.489293803697223
rat: replaced 2.25849384198642 by 23245051/10292280 = 2.25849384198642
rat: replaced 1.211512859637463 by 76914252/63486121 = 1.211512859637463
rat: replaced 0.4508366499481151 by 11638904/25816233 = 0.4508366499481151
rat: replaced 0.05092550026556708 by 4593355/90197543 = 0.05092550026556708
(%o16)

$$\begin{bmatrix} 1 & \frac{35990366441939}{37830452108616} & \frac{172618698949613}{200990458858328} & \frac{803343850365973}{1098217082675080} & \frac{18742578727327}{32214886649628} & \frac{18427}{43419} \\ 0 & 1 & \frac{85235207155279138305}{29370050786477266157} & \frac{885866126080291595289}{160478719616632863895} & \frac{26982580808327044218}{3138301066145290363} & \frac{75083058}{6344744} \end{bmatrix}$$

(%i17)
```
a:transpose(matrix(nodos_che,u));
```
(a)

$$\begin{bmatrix} 8.221801772461707 & 1 \\ 7.821890622779159 & 1 \\ 7.061214413089811 & 1 \\ 6.014233430740854 & 1 \\ 4.783433469030046 & 1 \\ 3.489293803697226 & 1 \\ 2.25849384198642 & 1 \\ 1.211512859637463 & 1 \\ 0.4508366499481151 & 1 \\ 0.05092550026556708 & 1 \end{bmatrix}$$

(%i18)
```
recta:invert(transpose(a).a).transpose(a).y;
```
(recta)

$$\begin{bmatrix} 0.0116894094916073\,(0.05092550026556708\,(\log(\sqrt{[8.221801772461707,7.821890622779159,7.06121} \\ 0.3000000000000002\,(\log(\sqrt{[8.221801772461707,7.821890622779159,7.061214413089811,6.0142334} \end{bmatrix}$$

(%i19) `float(recta);`

(%o19)

$$\begin{bmatrix} 0.0116894094916073\ (0.05092550026556708\ (\log(\sqrt{[8.221801772461707, 7.821890622779159, 7.0612} \\ 0.3000000000000002\ (\log(\sqrt{[8.221801772461707, 7.821890622779159, 7.061214413089811, 6.0142334} \end{bmatrix}$$

(%i20) `lista:makelist([nodos_che[i], y[i]], i, 1, 10);`

(lista)
$$[[8.221801772461707, \log(\mathit{sqrt}([(1.0))+1.0)+9.0],[$$
$$7.821890622779159, \log(\mathit{sqrt}([(2.0))+1.0)+9.0],[$$
$$7.061214413089811, \log(\mathit{sqrt}([(3.0))+1.0)+9.0],[$$
$$6.014233430740854, \log(\mathit{sqrt}([(4.0))+1.0)+9.0],[$$
$$4.783433469030046, \log(\mathit{sqrt}([(5.0))+1.0)+9.0],[$$
$$3.489293803697226, \log(\mathit{sqrt}([(6.0))+1.0)+9.0],[$$
$$2.25849384198642, \log(\mathit{sqrt}([(7.0))+1.0)+9.0],[$$
$$1.211512859637463, \log(\mathit{sqrt}([(8.0))+1.0)+9.0],[$$
$$0.4508366499481151, \log(\mathit{sqrt}([(9.0))+1.0)+9.0],[$$
$$0.05092550026556708, \log(\mathit{sqrt}([(10.0))+1.0)+9.0]]$$

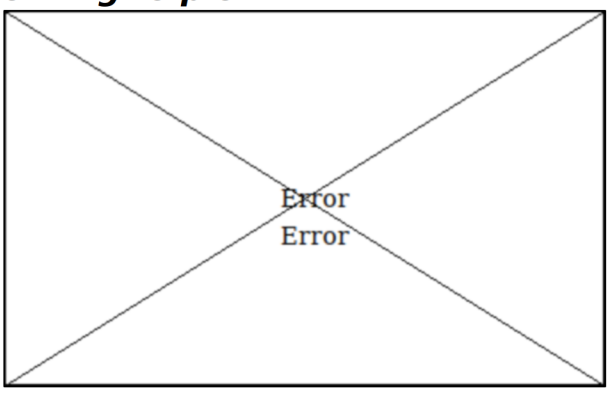
(%i21) `wxplot2d([[discrete, lista],recta[1,1]·t+recta[2,1]],[t,0,91/11],[style,points,lines`

Warning: none of the points have numerical values.

plot2d: expression evaluates to non-numeric value everywhere in plotting rang

plot2d: nothing to plot.

(%t21)



(%o21)