8. Calcular las funciones masa de probabilidad de las variables $Y = X + 2$ y $Z = X^2$, siendo X una variable aleatoria con distribución:

$$P(X=-2)=\frac{1}{5},\quad P(X=-1)=\frac{1}{10},\quad P(X=0)=\frac{1}{5},\quad P(X=1)=\frac{2}{5},\quad P(X=2)=\frac{1}{10}.$$

¿Cómo afecta el cambio de X a Y en el coeficiente de variación?

X v.a. $\qquad Y = X + 2 \qquad Z = X^2$

x_i	p_i
-2	1/5
-1	1/10
0	1/5
1	2/5
2	1/10

a) $Y = X + 2 \qquad \Sigma \equiv \{-2, -1, 0, 1, 2\}$

$\Sigma' \equiv \{0, 1, 2, 3, 4\}$

y_i	p_i'
0	1/5
1	1/10
2	1/5
3	2/5
4	1/10

$p_i' = P(Y = y) = \sum_{x / h(x) = y} p_i =$

$= P(X = y - 2) \quad \forall y = 0, 1, 2, \ldots y.$

b) $Z = X^2 \qquad \Sigma'' \equiv \{0, 1, 4\}$

z_i	p_i''
0	1/5
1	1/2
4	3/10
	1

$p_i'' = P(Z = 0) =$

$= \sum_{x / x^2 = 0} P(X = x) = P(x = 0) = \frac{1}{5}$

$P(Z = 1) = P(X = -1) + P(X = 1) = \frac{1}{2}$

$P(Z = 4) = P(x = -2) + P(x = 2) =$

$= 3/10$

Sabemos lo siguiente sobre la varianza y la esperanza.

$Y = aX + b \Rightarrow E[Y] = aE[X] + b$

$Var[Y] = a^2 \, Var[X] \Rightarrow \sigma_y = a\sigma_x$

En este caso $a = 1$ y $b = 2$

$E[Y] = E[X] + 2$

$\sigma_y = \sigma_x$

$$CV[Y] = \frac{\sigma_y}{E[Y]} = \frac{\sigma_x}{E[X]+2}$$

Vamos a calcular $E[X]$

$$E[X] = \sum_i x_i \, P(X = x_i)$$

$$E[X] = -\frac{2}{5} + \frac{-1}{10} + \frac{2}{5} + \frac{2}{10} = \frac{1}{10}$$

$$Var[X] = E[X^2] - E[X]^2$$

$$E[X^2] = \frac{4}{5} + \frac{1}{10} + \frac{2}{5} + \frac{4}{10} = \frac{17}{10}$$

$$Var[X] = \frac{17}{10} - \left(\frac{1}{10}\right)^2 = \frac{169}{100} = Var[Y]$$

$$\sigma_x = +\sqrt{\frac{169}{100}} = \frac{13}{10} = \sigma_y$$

$$E[Y] = \frac{1}{10} + 2 = \frac{21}{10}$$

$$CV(X) = \frac{13/10}{1/10} = 13 \qquad CV(Y) = \frac{13/10}{21/10} = \frac{13}{21}$$

$$\boxed{CV(Y) = \frac{1}{21} CV(X)}$$