EJERCICIO 1

Hallamos primero los nodos de Chebyshev en el intervalo [-1,1]

(%i1)   nodos_che:makelist(cos(((2·i+1)·%pi)/(2·10)),i,0,9);

(nodos_che) $\left[\cos\left(\dfrac{\pi}{20}\right), \cos\left(\dfrac{3\,\pi}{20}\right), \dfrac{1}{\sqrt{2}}, \cos\left(\dfrac{7\,\pi}{20}\right), \cos\left(\dfrac{9\,\pi}{20}\right), \cos\left(\dfrac{11\,\pi}{20}\right)\right.$

$\left., \cos\left(\dfrac{13\,\pi}{20}\right), -\dfrac{1}{\sqrt{2}}, \cos\left(\dfrac{17\,\pi}{20}\right), \cos\left(\dfrac{19\,\pi}{20}\right)\right]$

Ahora trasladamos todo esto al intervalo [0,91/11], mediante un
isomorfismo afín.

(%i2)   linsolve([−x+y=0, x+y=91/11], [x,y]);
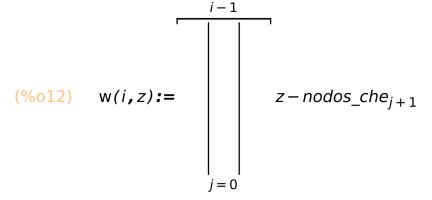
(%o2) $\left[x = \dfrac{91}{22}, y = \dfrac{91}{22}\right]$

(%i3)   f_paso(t):=(91/22)·t+91/22;

(%o3) $f_{paso}(t) := \dfrac{91}{22}\,t + \dfrac{91}{22}$

(%i4)   for i:0 thru 9 do nodos_che[i+1]:f_paso(float(nodos_che[i+1]));
(%o4)   done

(%i5)   nodos_che;
(%o5)   [8.221801772461707, 7.821890622779159,
7.061214413089811, 6.014233430740854, 4.783433469030046,
3.489293803697226, 2.25849384198642, 1.211512859637463,
0.4508366499481151, 0.05092550026556708]

EJERCICIO 2

(%i6)   f(x):=log(1+sqrt(x)) + 9;
(%o6) $f(x) := \log(1 + \sqrt{x}) + 9$

(%i7)   y:makelist(float(f(nodos_che(j+1))),j,0,9);
(y) $[\log(sqrt([(1.0)) + 1.0) + 9.0, \log(sqrt([(2.0)) + 1.0) +$
$9.0, \log(sqrt([(3.0)) + 1.0) + 9.0, \log(sqrt([(4.0)) + 1.0) + 9.0, \log($
$sqrt([(5.0)) + 1.0) + 9.0, \log(sqrt([(6.0)) + 1.0) + 9.0, \log(sqrt([$
$(7.0)) + 1.0) + 9.0, \log(sqrt([(8.0)) + 1.0) + 9.0, \log(sqrt([(9.0)) +$
$1.0) + 9.0, \log(sqrt([(10.0)) + 1.0) + 9.0]$

(%i8)   matrix_aux: matrix(
         [0,0,0,0,0,0,0,0,0,0],
         [0,0,0,0,0,0,0,0,0,0],
         [0,0,0,0,0,0,0,0,0,0],
         [0,0,0,0,0,0,0,0,0,0],
         [0,0,0,0,0,0,0,0,0,0],
         [0,0,0,0,0,0,0,0,0,0],
         [0,0,0,0,0,0,0,0,0,0],
         [0,0,0,0,0,0,0,0,0,0],
         [0,0,0,0,0,0,0,0,0,0],
         [0,0,0,0,0,0,0,0,0,0]
         );

(matrix_aux)

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

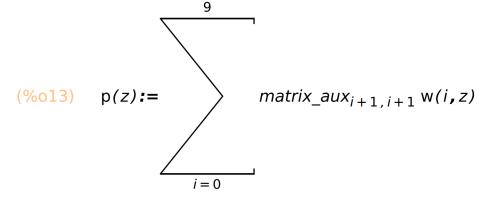(%i9)   for i:0 thru 9 do matrix_aux[i+1,1]:float(y[i+1]);

(%o9)   *done*

(%i10)   matrix_aux;

(%o10)

$\log(\sqrt{[8.221801772461707, 7.821890622779159, 7.061214413089811, 6.014233430740854, 4.78343346}$

$\log(\sqrt{[8.221801772461707, 7.821890622779159, 7.061214413089811, 6.014233430740854, 4.78343346}$

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$\log(\sqrt{[8.221801772461707, 7.821890622779159, 7.061214413089811, 6.014233430740854, 4.78343346}$

$\log(\sqrt{[8.221801772461707, 7.821890622779159, 7.061214413089811, 6.014233430740854, 4.78343346\overline{9}}$

(%i11)   for i:1 thru 9 do(
                for j:1 thru i do(
                    matrix_aux[i+1,j+1]:float((matrix_aux[i+1,j]−matrix_aux[i,j])/(nodos_c

(%o11)   *done*

(%i12)   w(i,z):=product(z−nodos_che[j+1], j, 0, i−1);

(%o12) $w(i,z) := \prod_{j=0}^{i-1} z - nodos\_che_{j+1}$

(%i13)   p(z):=sum(matrix_aux[i+1,i+1]·w(i,z) , i, 0, 9);

(%o13) $p(z) := \sum_{i=0}^{9} matrix\_aux_{i+1,i+1} \, w(i,z)$

(%i14)   float(p(3));

(%o14) $-111.2824763640209\,(0.1286841446634289\,($

$0.1426474732261967\,(0.1676921620782877\,($

$0.2113044513818475\,(0.2908356265970551\,($

$0.4529871085306813\,(0.8616326827316089\,(2.500555437861153$

$(\log(sqrt([(2.0))+1.0)-1.0\log(sqrt([(1.0))+1.0))-$

$1.314619791262289\,(\log(sqrt([(3.0))+1.0)-1.0\log(sqrt([(2.0))$

$+1.0)))-0.5532022356918267\,(1.314619791262289\,(\log(sqrt([$

$(3.0))+1.0)-1.0\log(sqrt([(2.0))+1.0))-0.955127186509584\,($

$\log(sqrt([(4.0))+1.0)-1.0\log(sqrt([(3.0))+1.0))))-$

$0.3291143989857198\,(0.5532022356918267\,(1.314619791262289$

$(\log(sqrt([(3.0))+1.0)-1.0\log(sqrt([(2.0))+1.0))-$

$0.955127186509584\,(\log(sqrt([(4.0))+1.0)-1.0\log(sqrt([(3.0))$

$+1.0)))-0.4390237799679133\,(0.955127186509584\,(\log(sqrt([$

$(4.0))+1.0)-1.0\log(sqrt([(3.0))+1.0))-0.8124797132833868\,($

$\log(sqrt([(5.0))+1.0)-1.0\log(sqrt([(4.0))+1.0)))))-$

$0.2308084600892768\,(0.3291143989857198\,($

$0.5532022356918267\,(1.314619791262289\,(\log(sqrt([(3.0))+1.0$

$)-1.0\log(sqrt([(2.0))+1.0))-0.955127186509584\,(\log(sqrt([$

$(4.0))+1.0)-1.0\log(sqrt([(3.0))+1.0)))-0.4390237799679133$

$(0.955127186509584\,(\log(sqrt([(4.0))+1.0)-1.0\log(sqrt([(3.0)$

$)+1.0))-0.8124797132833868\,(\log(sqrt([(5.0))+1.0)-1.0\log($

$sqrt([(4.0))+1.0))))-0.2799614295374983\,($

$0.4390237799679133\,(0.955127186509584\,(\log(sqrt([(4.0))+1.0$

$)-1.0\log(sqrt([(3.0))+1.0))-0.8124797132833868\,(\log(sqrt([$

$(5.0))+1.0)-1.0\log(sqrt([(4.0))+1.0)))-0.3960490735261138$

$(0.8124797132833868\,(\log(sqrt([(5.0))+1.0)-1.0\log(sqrt([$

$(4.0))+1.0))-0.7727141256757828\,(\log(sqrt([(6.0))+1.0)-1.0$

$\log(sqrt([(5.0))+1.0))))))-0.1797463023763529\,($

$0.2308084600892768\,(0.3291143989857198\,($

$0.5532022356918267\,(1.314619791262289\,(\log(sqrt([(3.0))+1.0$

$)-1.0\log(sqrt([(2.0))+1.0))-0.955127186509584\,(\log(sqrt([$

$(4.0))+1.0)-1.0\log(sqrt([(3.0))+1.0)))-0.4390237799679133$

$(0.955127186509584\,(\log(sqrt([(4.0))+1.0)-1.0\log(sqrt([(3.0)$

EJERCICIO 3

(%i15)   u:[1,1,1,1,1,1,1,1,1,1];
(u)   **[1,1,1,1,1,1,1,1,1,1]**

(%i16)   echelon(matrix(nodos_che,u));
rat: replaced 8.221801772461707 by 62962040/7657937 = 8.221801772461696
rat: replaced 7.821890622779159 by 23498735/3004227 = 7.821890622779171
rat: replaced 7.061214413089811 by 112705745/15961241 = 7.0612144130898
rat: replaced 6.014233430740854 by 104903429/17442527 = 6.0142334307408
rat: replaced 4.783433469030046 by 24474710/5116557 = 4.783433469030053
rat: replaced 3.489293803697226 by 16843861/4827298 = 3.489293803697223
rat: replaced 2.25849384198642 by 23245051/10292280 = 2.25849384198642
rat: replaced 1.211512859637463 by 76914252/63486121 = 1.21151285963746
rat: replaced 0.4508366499481151 by 11638904/25816233 = 0.4508366499481
rat: replaced 0.05092550026556708 by 4593355/90197543 = 0.05092550026555
(%o16)

$$\begin{bmatrix} 1 & \dfrac{35990366441939}{37830452108616} & \dfrac{172618698949613}{200990458858328} & \dfrac{803343850365973}{1098217082675080} & \dfrac{18742578727327}{32214886649628} & \dfrac{18427\ldots}{43419\ldots} \\ 0 & 1 & \dfrac{85235207155279138305}{29370050786477266157} & \dfrac{885866126080291595289}{160478719616632863895} & \dfrac{2698258080832704 4218}{3138301066145290363} & \dfrac{75083058\ldots}{63447441\ldots} \end{bmatrix}$$

(%i17)   a:transpose(matrix(nodos_che,u));

(a)

$$\begin{bmatrix} 8.221801772461707 & 1 \\ 7.821890622779159 & 1 \\ 7.061214413089811 & 1 \\ 6.014233430740854 & 1 \\ 4.783433469030046 & 1 \\ 3.489293803697226 & 1 \\ 2.25849384198642 & 1 \\ 1.211512859637463 & 1 \\ 0.4508366499481151 & 1 \\ 0.05092550026556708 & 1 \end{bmatrix}$$

(%i18)   recta:invert(transpose(a).a).transpose(a).y;
(recta)

$$\begin{bmatrix} 0.0116894094916073\,(0.05092550026556708\,(\log(\sqrt{[8.221801772461707, 7.821890622779159, 7.06121\ldots \\ 0.3000000000000002\,(\log(\sqrt{[8.221801772461707, 7.821890622779159, 7.061214413089811, 6.0142334\ldots \end{bmatrix}$$
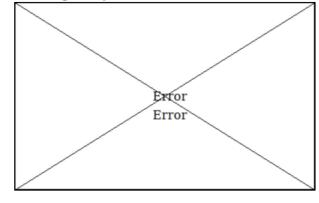
(%i19)   float(recta);

(%o19)

$0.0116894094916073 \, (0.05092550026556708 \, (\log(\sqrt{[8.221801772461707, 7.821890622779159, 7.06121}$

$0.3000000000000002 \, (\log(\sqrt{[8.221801772461707, 7.821890622779159, 7.061214413089811, 6.0142334}$

(%i20)   lista:makelist([nodos_che[i], y[i]], i, 1, 10);

(lista) $[[8.221801772461707, \log(sqrt([(1.0)) + 1.0) + 9.0], [$
$7.821890622779159, \log(sqrt([(2.0)) + 1.0) + 9.0], [$
$7.061214413089811, \log(sqrt([(3.0)) + 1.0) + 9.0], [$
$6.014233430740854, \log(sqrt([(4.0)) + 1.0) + 9.0], [$
$4.783433469030046, \log(sqrt([(5.0)) + 1.0) + 9.0], [$
$3.489293803697226, \log(sqrt([(6.0)) + 1.0) + 9.0], [$
$2.25849384198642, \log(sqrt([(7.0)) + 1.0) + 9.0], [$
$1.211512859637463, \log(sqrt([(8.0)) + 1.0) + 9.0], [$
$0.4508366499481151, \log(sqrt([(9.0)) + 1.0) + 9.0], [$
$0.05092550026556708, \log(sqrt([(10.0)) + 1.0) + 9.0]]$

(%i21)   wxplot2d([[discrete, lista],recta[1,1]·t+recta[2,1]],[t,0,91/11],[style,points,lines

Warning: none of the points have numerical values.
plot2d: expression evaluates to non-numeric value everywhere in plotting range
plot2d: nothing to plot.

(%t21)

Error
Error

(%o21)