13.5

3) $\int_{-\pi/2}^{\pi/2} |\text{sen}(x)|^3\,dx = \frac{4}{3}$.

$$\int_{-\pi/2}^{\pi/2} |\text{sen}(x)|^3\,dx = \int_{-\pi/2}^{\pi/2} |\text{sen}^3(x)|\,dx$$

Sabemos que:

$$|\text{sen}^3(x)| = \begin{cases} -\text{sen}^3(x) & \text{si } -\frac{\pi}{2} < x < 0 \\ \text{sen}^3(x) & \text{si } 0 \leq x < \frac{\pi}{2} \end{cases}$$

Por tanto:

$$\int_{-\pi/2}^{\pi/2} |\text{sen}^3(x)|\,dx = \int_{-\pi/2}^{0} -\text{sen}^3(x)\,dx + \int_{0}^{\pi/2} \text{sen}^3(x)\,dx =$$

$$= -\int_{-\pi/2}^{0} \text{sen}^3(x)\,dx + \int_{0}^{\pi/2} \text{sen}^3(x)\,dx$$

Calculemos la integral indefinida $\int \text{sen}^3(x)\,dx$:

$$\int \text{sen}^3(x)\,dx = \int \text{sen}^2(x)\cdot \text{sen}(x)\,dx = \int (1-\cos^2(x))\,\text{sen}(x)\,dx =$$

$$= -\int (1-\cos^2(x))(-\text{sen}(x))\,dx = \begin{bmatrix} u = \cos(x) \\ du = -\text{sen}(x)dx \end{bmatrix} =$$

$$= -\int (1-u^2)\,du = -\left(\int 1\,du - \int u^2\,du\right) = -\left(u - \frac{u^3}{3}\right) + C =$$

$$= \frac{u^3}{3} - u + C = \frac{u^3-3u}{3} + C = \frac{\cos^3(x) - 3\cos(x)}{3} + C$$

Por tanto:

$$\int_{-\pi/2}^{\pi/2} |\operatorname{sen}^3(x)|\,dx = -\int_{-\pi/2}^{0} \operatorname{sen}^3(x)\,dx + \int_{0}^{\pi/2} \operatorname{sen}^3(x)\,dx =$$

$$= -\left[\frac{\cos^3(x) - 3\cos(x)}{3}\right]_{-\pi/2}^{0} + \left[\frac{\cos^3(x) - 3\cos(x)}{3}\right]_{0}^{\pi/2} =$$

$$= -\left(\frac{-2}{3} - 0\right) + \left(0 - \left(\frac{-2}{3}\right)\right) = \frac{2}{3} + \frac{2}{3} = \boxed{\frac{4}{3}}$$