Module 4: Multi-faceted Models

5.11.19

1

Main topics

- Introduction of multiple facets
- Models with at least one nested facet
- Models with a fixed facet

Describing the crossed model

- Two or more facets are involved in the design. For example, 50 students' responses on the same four items were scored by 2 raters (p x i x r).
- The facets can be random or fixed.
- The universe to be generalized is defined as conditions from all those facets.

Describing the model: equations

Take the p x i x r as an example:

- Observed score

$X_{pir} = \mu$	[grand mean]
$+ \mu_p - \mu$	[person effect]
$+ \mu_i - \mu$	[item effect]
$+ \mu_r - \mu$	[rater effect]
$+ \mu_{pi} - \mu_p - \mu_i + \mu$	[pi interaction]
$+ \mu_{pr} - \mu_p - \mu_r + \mu$	[pr interaction]
$+ \mu_{ir} - \mu_i - \mu_r + \mu$	[ir interaction]
$+ X_{pir} - \mu_{pi} - \mu_{pr} - \mu_{ir} + \mu_p + \mu_i + \mu_r - \mu$	[pir interaction effect confounded with residual]

Variance

$$\sigma^2 (X_{pir}) = \sigma_p^2 + \sigma_i^2 + \sigma_r^2 + \sigma_{pi}^2 + \sigma_{pr}^2 + \sigma_{ir}^2 + \sigma_{pir,e}^2$$

Estimating variance components Using p x i x r design as an example

	Expected MS	Est. G Study V Components
Person(p)	$EMS_p = \sigma_{pir,e}^2 + n_i\sigma_{pr}^2 + n_r\sigma_{pi}^2 + n_i n_r \sigma_p^2$	$\sigma_p^2 = (EMS_p - EMS_{pi} - EMS_{pr} + EMS_{pir,e}) / n_i \times n_r$
Item(i)	$EMS_i = \sigma_{pir,e}^2 + n_r\sigma_{pi}^2 + n_p\sigma_{ir}^2 + n_p n_r \sigma_i^2$	$\sigma_i^2 = (EMS_i - EMS_{pi} - EMS_{ir} + EMS_{pir,e}) / n_p \times n_r$
rater(r)	$EMS_r = \sigma_{pir,e}^2 + n_i\sigma_{pr}^2 + n_p\sigma_{ir}^2 + n_p n_i \sigma_r^2$	$\sigma_r^2 = (EMS_r - EMS_{pr} - EMS_{ir} + EMS_{pir,e}) / n_p \times n_i$
Int.(pi)	$EMS_{pi} = \sigma_{pir,e}^2 + n_r\sigma_{pi}^2$	$\sigma_{pi}^2 = (EMS_{pi} - EMS_{pir,e}) / n_r$
Int.(pr)	$EMS_{pr} = \sigma_{pir,e}^2 + n_i\sigma_{pr}^2$	$\sigma_{pr}^2 = (EMS_{pr} - EMS_{pir,e}) / n_i$
Int.(ir)	$EMS_{ir} = \sigma_{pir,e}^2 + n_p\sigma_{ir}^2$	$\sigma_{ir}^2 = (EMS_{ir} - EMS_{pir,e}) / n_p$
pir, e	$EMS_{pir,e} = \sigma_{pir,e}^2$	$\sigma_{pir,e}^2 = EMS_{pir,e}$

Now your turn: what about abs. and rel. G measurement error?

- absolute measurement error includes:
- relative measurement error includes:

Specifics related to multi-faceted crossed models

- Programming and calculating
  - Going over one example to understand how to manually calculate the VCs for D studies based on the G study results
  - Running the urGENOVA
- Conceptual issues
  - Dealing with negative values of EVCs
  - Introducing a facet that is nested or fixed

Now the programming time

- Genova to run typical designs
- urGenova when your data involve missing values

Mini-project 2: working example (a)

- Use data on Shavelson and Webb (p. 43) to run a G study. Then conduct the following D studies:
  - p x o x i, O=1 and i=20
  - p x o x i, O=2 and i=10

Mini-project 2: working example (b)

- Use the data again, but make two missing values (the first two scores in the first row). Use the urGenova to conduct the G study. Then use the excel file or Genova to perform the D studies listed above.

Back to conceptual issues

- Negative values in EVCs
- Nested facet
- Fixed facet

Quick tips for facets

- For nested facet, VC (x:y) = main (y) + interaction (xy)
- When a fixed facet of f involved (completely fixed), VC (x) = VC (x) + VC (xf)/n_f

Two approaches to dealing with negative values for estimating VCs

- Approach proposed by Cronbach et al. (1972) – getting rid of the theoretical impossibility
  - Set the negative estimate to 0
  - Use 0 in the calculations to generate the new VC estimates
- Approach proposed by Brennan (1983) – producing unbiased estimates for variance components
  - Continue using the negative values to calculate the new VC estimates
  - Set the negative values to 0 in the final report

Why negative values for VCs?

- Model mis-specification
- Sampling errors when too small sample size of the examinees or conditions in the facets

About the nested model

- Two or more facets are involved in the design. For example, different sets of four items were given to 50 students and their responses on different sets of four items were scored by two different raters (i:r:p).
- The facets can be random or fixed effect.
- The universe to be generalized is defined as conditions from those facets.
  - Exercise to explore the nested facet

More related to the nested facet

- Unbalanced design
  - Examples: (1) in a r:p design, 2 raters for Stu 1, 3 raters for Stu 2, 4 raters for Stu 3, …; (2) in the general exams, Joe has four questions, Diane has three questions.
- Person nested within class/group/site
  - The persons are nested data in nature
  - Programming with Genova or urGenova depending on the data files

Introducing the fixed facet

- Definition: the facet that researchers are not interested in generalizing to an *infinite* universe.
- Estimating
  - At the EFFECT line, indicate the sampling size that you would like to interpret the test scores.
  - In the output, you will notice some interesting pattern. For example, the fixed facet doesn't account for the measurement error.
  - So what is the rules for the estimated VCs?
    * [let's play with examples]

Introducing the fixed facet: two ways to deal with the fixed facet

- Conduct one G study using the mixed model
- Conduct separate G study for each of the levels of the fixed facet

More about fixed facet: Sampling all vs. some levels

- Let's work on this with our mini-project 2

19

Background information: finite universe correction

- Also called "finite population correction" introduced by Cochran (see 1977, Sampling techniques, pp. 23-25)

2.6 THE FINITE POPULATION CORRECTION

For a random sample of size n from an infinite population, it is well known that the variance of the mean is σ^2/n. The only change in this result when the population is finite is the introduction of the factor $(N-n)/N$. The factors $(N-n)/N$ for the variance and $\sqrt{(N-n)/N}$ for the standard error are called the *finite population corrections* (fpc).

- Applied when n/N >5%

20

Sharing: what design(s) will you use in your G and D studies?

- Based on the data file that you have, now you need to nail down:
  - what facets are you interested in investigating?
  - what design will you use for your G study? why?
  - what designs will you use for your D studies? why?

*Note: if you don't have your own data, I have two data files for you to choose.

21

Programing: now work on your own G and D studies

- Clarify your research focus
- Run your codes and decide what you will highlight in your output

22

Planning: what design(s) will you use in your G and D studies?

- Based on the data file that you have, now you need to nail down:
  - what facets are you interested in investigating?
  - what design will you use for your G study? why?
  - what designs will you use for your D studies? why?

*Note: if you don't have your own data, I have two data files for you to choose.

23

Leftover - please pay attention when reading Brennan et al.

When making sense of results, we look for:

- CI of the observed scores (p. 166)
- large person variance
- main source of measurement error so that we can identify how to efficiently reduce the measurement error
  - Numbers
  - Graphs
- small standard errors