

# Math League Contest Problem Set 12113

## Team Round Problem 3

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Math League, LLC

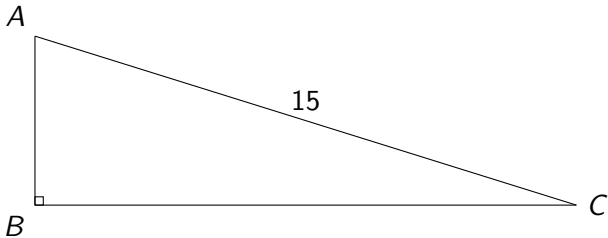
Identify our objective.

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In right-angled  $\triangle ABC$ , the hypotenuse has a length of 15. If the perimeter of  $\triangle ABC$  is  $15 + \sqrt{353}$ , what is the area of  $\triangle ABC$ ?

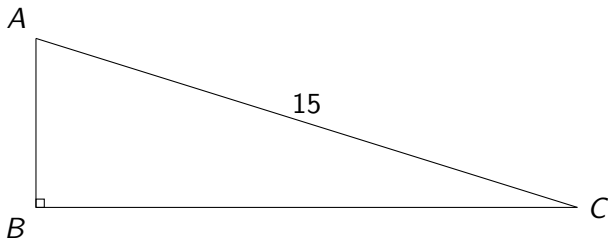
Downloaded from <http://ajph.org/> on November 10, 2014





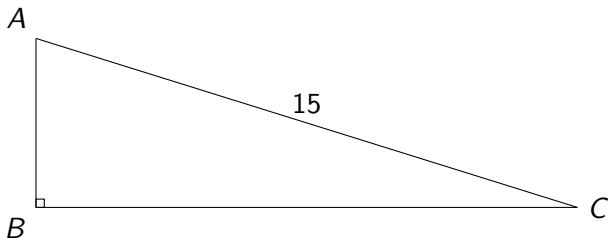


Compute the area of  $\triangle ABC$ .



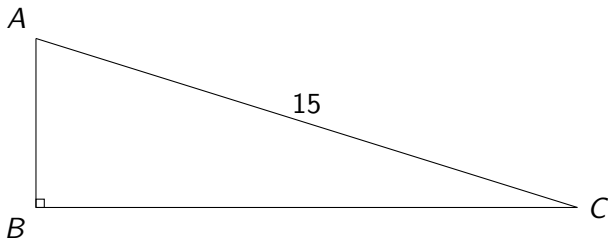
$$AB + BC + 15 = 15 + \sqrt{353}$$

Compute the area of  $\triangle ABC$ .



$$AB + BC = \sqrt{353}$$

Compute the area of  $\triangle ABC$ .

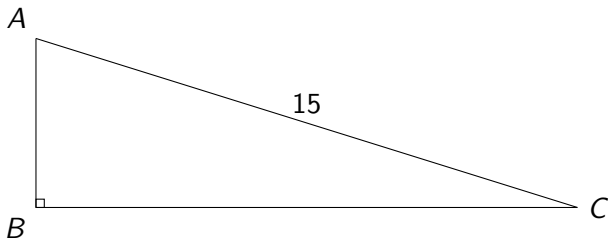


$$AB + BC = \sqrt{353}$$

Area of  $\triangle ABC$



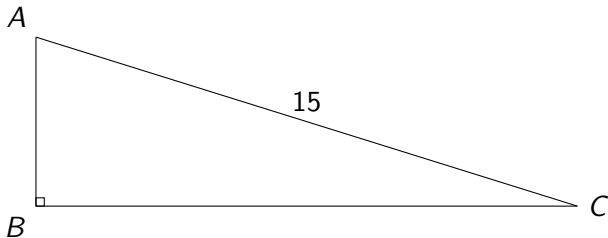
Compute the area of  $\triangle ABC$ .



$$AB + BC = \sqrt{353}$$

$$\text{Area of } \triangle ABC = \frac{1}{2} \cdot AB \cdot BC$$

Compute the area of  $\triangle ABC$ .

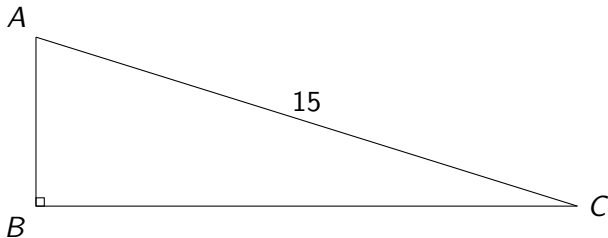


$$AB + BC = \sqrt{353}$$

$$\text{Area of } \triangle ABC = \frac{1}{2} \cdot AB \cdot BC$$

$$(AB + BC)^2$$

Compute the area of  $\triangle ABC$ .

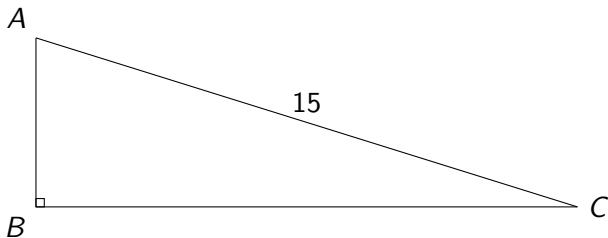


$$AB + BC = \sqrt{353}$$

$$\text{Area of } \triangle ABC = \frac{1}{2} \cdot AB \cdot BC$$

$$(AB + BC)^2 = AB^2 + 2 \cdot AB \cdot BC + BC^2$$

Compute the area of  $\triangle ABC$ .

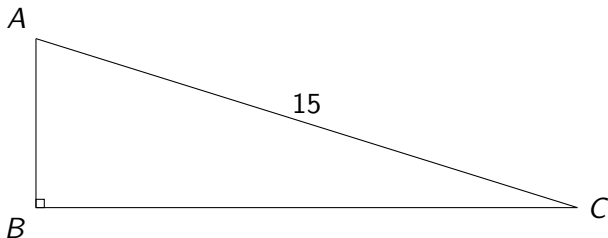


$$AB + BC = \sqrt{353}$$

$$\text{Area of } \triangle ABC = \frac{1}{2} \cdot AB \cdot BC$$

$$(\sqrt{353})^2 = AB^2 + 2 \cdot AB \cdot BC + BC^2$$

Compute the area of  $\triangle ABC$ .

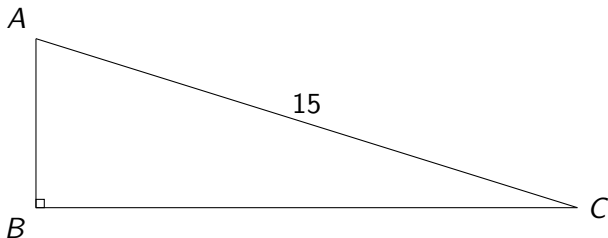


$$AB + BC = \sqrt{353}$$

$$\text{Area of } \triangle ABC = \frac{1}{2} \cdot AB \cdot BC$$

$$353 = AB^2 + 2 \cdot AB \cdot BC + BC^2$$

Compute the area of  $\triangle ABC$ .

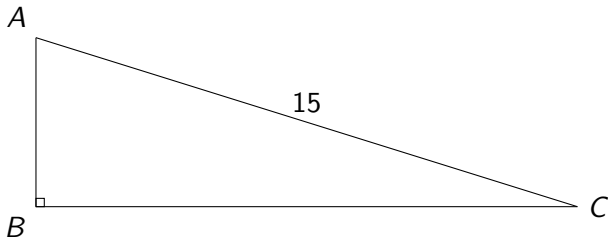


$$AB + BC = \sqrt{353}$$

$$\text{Area of } \triangle ABC = \frac{1}{2} \cdot AB \cdot BC$$

$$353 = AB^2 + (2 \cdot AB \cdot BC + BC^2)$$

Compute the area of  $\triangle ABC$ .

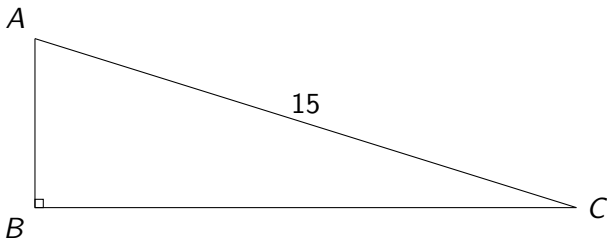


$$AB + BC = \sqrt{353}$$

$$\text{Area of } \triangle ABC = \frac{1}{2} \cdot AB \cdot BC$$

$$353 = AB^2 + (BC^2 + 2 \cdot AB \cdot BC)$$

Compute the area of  $\triangle ABC$ .



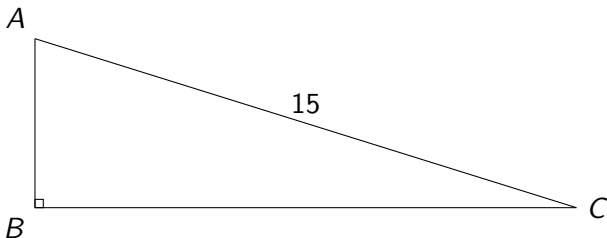
$$AB + BC = \sqrt{353}$$

$$\text{Area of } \triangle ABC = \frac{1}{2} \cdot AB \cdot BC$$

$$353 = AB^2 + BC^2 + 2 \cdot AB \cdot BC$$



Compute the area of  $\triangle ABC$ .

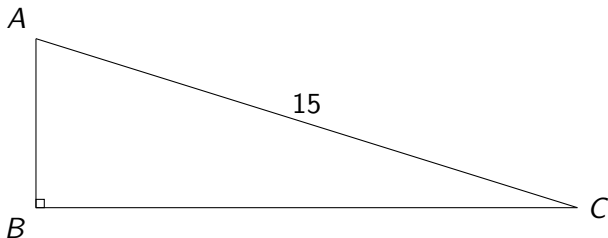


$$AB + BC = \sqrt{353}$$

$$\text{Area of } \triangle ABC = \frac{1}{2} \cdot AB \cdot BC$$

$$353 = (AB^2 + BC^2) + 2 \cdot AB \cdot BC$$

Compute the area of  $\triangle ABC$ .

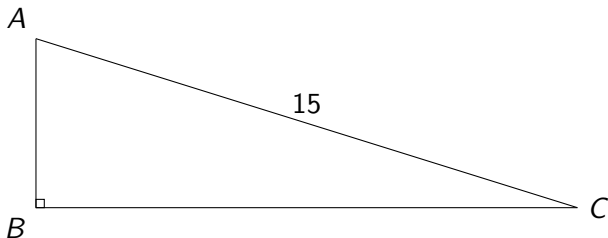


$$AB + BC = \sqrt{353}$$

$$\text{Area of } \triangle ABC = \frac{1}{2} \cdot AB \cdot BC$$

$$353 - (AB^2 + BC^2) = 2 \cdot AB \cdot BC$$

Compute the area of  $\triangle ABC$ .

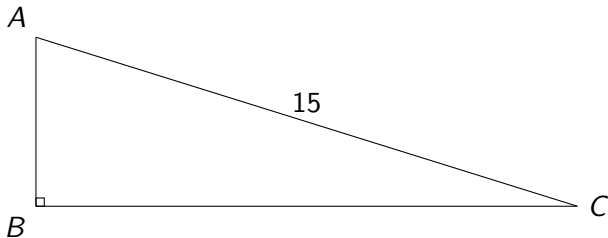


$$AB + BC = \sqrt{353}$$

$$\text{Area of } \triangle ABC = \frac{1}{2} \cdot AB \cdot BC$$

$$2 \cdot AB \cdot BC = 353 - (AB^2 + BC^2)$$

Compute the area of  $\triangle ABC$ .



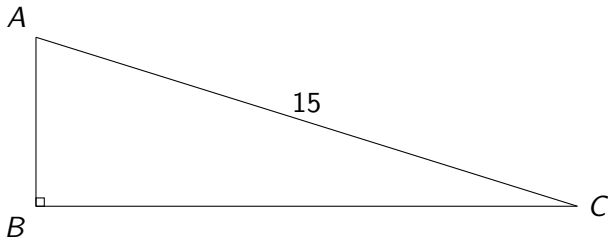
$$AB + BC = \sqrt{353}$$

$$\text{Area of } \triangle ABC = \frac{1}{2} \cdot AB \cdot BC$$

$$2 \cdot AB \cdot BC = 353 - (AB^2 + BC^2)$$

$$AB^2 + BC^2 = AC^2$$

Compute the area of  $\triangle ABC$ .

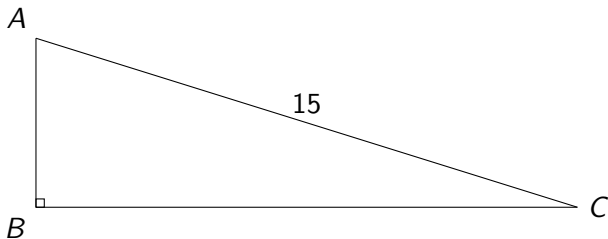


$$AB + BC = \sqrt{353}$$

$$\text{Area of } \triangle ABC = \frac{1}{2} \cdot AB \cdot BC$$

$$2 \cdot AB \cdot BC = 353 - (AB^2 + BC^2)$$

$$AB^2 + BC^2 = 15^2$$

Compute the area of  $\triangle ABC$ .

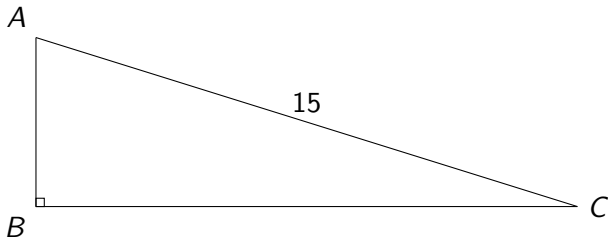
$$AB + BC = \sqrt{353}$$

$$\text{Area of } \triangle ABC = \frac{1}{2} \cdot AB \cdot BC$$

$$2 \cdot AB \cdot BC = 353 - (AB^2 + BC^2)$$

$$AB^2 + BC^2 = 225$$

Compute the area of  $\triangle ABC$ .



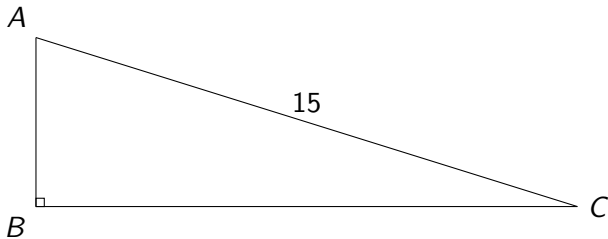
$$AB + BC = \sqrt{353}$$

$$\text{Area of } \triangle ABC = \frac{1}{2} \cdot AB \cdot BC$$

$$2 \cdot AB \cdot BC = 353 - 225$$

$$AB^2 + BC^2 = 225$$

Compute the area of  $\triangle ABC$ .



$$AB + BC = \sqrt{353}$$

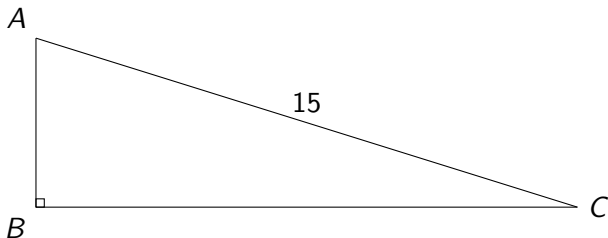
$$\text{Area of } \triangle ABC = \frac{1}{2} \cdot AB \cdot BC$$

$$2 \cdot AB \cdot BC = 128$$

$$AB^2 + BC^2 = 225$$



Compute the area of  $\triangle ABC$ .



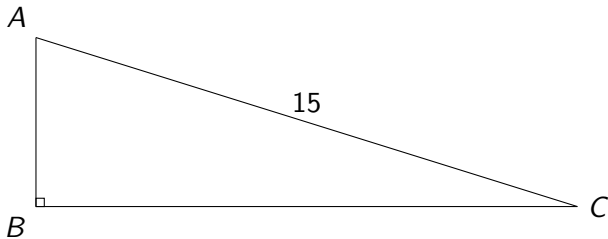
$$AB + BC = \sqrt{353}$$

$$\text{Area of } \triangle ABC = \frac{1}{2} \cdot AB \cdot BC$$

$$\frac{1}{2} \cdot 2 \cdot AB \cdot BC = \frac{1}{2} \cdot 128$$

$$AB^2 + BC^2 = 225$$

Compute the area of  $\triangle ABC$ .

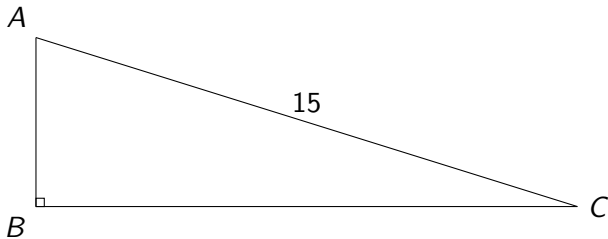


$$AB + BC = \sqrt{353}$$

$$\text{Area of } \triangle ABC = \frac{1}{2} \cdot AB \cdot BC$$

$$AB \cdot BC = 64$$

$$AB^2 + BC^2 = 225$$

Compute the area of  $\triangle ABC$ .

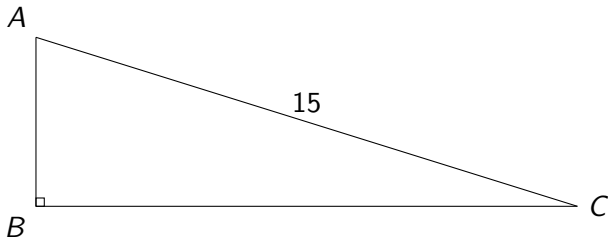
$$AB + BC = \sqrt{353}$$

$$\text{Area of } \triangle ABC = \frac{1}{2} \cdot 64$$

$$AB \cdot BC = 64$$

$$AB^2 + BC^2 = 225$$

Compute the area of  $\triangle ABC$ .



$$AB + BC = \sqrt{353}$$

$$\text{Area of } \triangle ABC = \boxed{32}$$

$$AB \cdot BC = 64$$

$$AB^2 + BC^2 = 225$$

Review the key concepts we used.

# Key Concepts

Review the key concepts we used.

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## ■ Perimeter and Area of a Triangle

Review the key concepts we used.

## Key Concepts

- Perimeter and Area of a Triangle
- Algebraic Manipulation

Review the key concepts we used.

## Key Concepts

- Perimeter and Area of a Triangle
- Algebraic Manipulation
- Pythagorean Theorem