

# Math League Contest Problem Set 12114

## Team Round Problem 3

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Identify our objective.

A whole number value is assigned to every letter in the English alphabet. The *score* of a word is the sum of the values of each letter in the word. Suppose the scores of the words *ERRANT*, *RERUN*, and *AUNT* are 67, 49, and 37 respectively, and the value of *N* is the average of the values of *A* and *T*. What is the value that was assigned to the letter *N*?

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Objective: Compute the value assigned to the letter *N*.

Compute the value assigned to the letter  $N$ .

Let the value of each letter be denoted by its lowercase.

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 $a =$  the value of  $A$

Compute the value assigned to the letter  $N$ .

Let the value of each letter be denoted by its lowercase.

$a$  = the value of  $A$

$b$  = the value of  $B$

Compute the value assigned to the letter  $N$ .

Let the value of each letter be denoted by its lowercase.

$a$  = the value of  $A$

$b$  = the value of  $B$

$\vdots$

Compute the value assigned to the letter  $N$ .

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$\vdots$

$$2 \cdot r + e + a + t + n$$



Compute the value assigned to the letter  $N$ .

Let the value of each letter be denoted by its lowercase.

$a$  = the value of  $A$

$b$  = the value of  $B$

$\vdots$

$$2 \cdot r + e + a + t + n = 67,$$

Compute the value assigned to the letter  $N$ .

Let the value of each letter be denoted by its lowercase.

$a$  = the value of  $A$

$b$  = the value of  $B$

$\vdots$

$$2 \cdot r + e + a + t + n = 67,$$

$$2 \cdot r + e + u + n$$

Compute the value assigned to the letter  $N$ .

Let the value of each letter be denoted by its lowercase.

$a$  = the value of  $A$

$b$  = the value of  $B$

$\vdots$

$$2 \cdot r + e + a + t + n = 67,$$

$$2 \cdot r + e + u + n = 49,$$

Compute the value assigned to the letter  $N$ .

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$$2 \cdot r + e + a + t + n = 67,$$

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$a$  = the value of  $A$

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$$2 \cdot r + e + a + t + n = 67,$$

$$2 \cdot r + e + u + n = 49,$$

$$a + t + u + n = 37.$$

Compute the value assigned to the letter  $N$ .

$$2 \cdot r + e + a + t + n = 67 \quad (1)$$

$$2 \cdot r + e + u + n = 49 \quad (2)$$

$$a + t + u + n = 37 \quad (3)$$

Compute the value assigned to the letter  $N$ .

$$2 \cdot r + e + a + t + n = 67 \quad (1)$$

$$2 \cdot r + e + u + n = 49 \quad (2)$$

$$a + t + u + n = 37 \quad (3)$$

$$(2) - (3)$$

Compute the value assigned to the letter  $N$ .

$$2 \cdot r + e + a + t + n = 67 \quad (1)$$

$$2 \cdot r + e + u + n = 49 \quad (2)$$

$$a + t + u + n = 37 \quad (3)$$

$$(2) - (3) \implies 2 \cdot r + e - (a + t) = 12$$



Compute the value assigned to the letter  $N$ .

$$2 \cdot r + e + a + t + n = 67 \quad (1)$$

$$2 \cdot r + e + u + n = 49 \quad (2)$$

$$a + t + u + n = 37 \quad (3)$$

$$2 \cdot r + e - (a + t) = 12 \quad (4)$$

Compute the value assigned to the letter  $N$ .

$$2 \cdot r + e + a + t + n = 67 \quad (1)$$

$$2 \cdot r + e + u + n = 49 \quad (2)$$

$$a + t + u + n = 37 \quad (3)$$

$$2 \cdot r + e - (a + t) = 12 \quad (4)$$

$$(1) - (4)$$

Compute the value assigned to the letter  $N$ .

$$2 \cdot r + e + a + t + n = 67 \quad (1)$$

$$2 \cdot r + e + u + n = 49 \quad (2)$$

$$a + t + u + n = 37 \quad (3)$$

$$2 \cdot r + e - (a + t) = 12 \quad (4)$$

$$(1) - (4) \implies 2 \cdot (a + t) + n = 55$$

Compute the value assigned to the letter  $N$ .

$$2 \cdot r + e + a + t + n = 67 \quad (1)$$

$$2 \cdot r + e + u + n = 49 \quad (2)$$

$$a + t + u + n = 37 \quad (3)$$

$$2 \cdot r + e - (a + t) = 12 \quad (4)$$

$$2 \cdot (a + t) + n = 55 \quad (5)$$

Compute the value assigned to the letter  $N$ .

$$2 \cdot r + e + a + t + n = 67 \quad (1)$$

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Given that the value of  $N$  is the average of the values of  $A$  and  $T$ ,

Compute the value assigned to the letter  $N$ .

$$2 \cdot r + e + a + t + n = 67 \quad (1)$$

$$2 \cdot r + e + u + n = 49 \quad (2)$$

$$a + t + u + n = 37 \quad (3)$$

$$2 \cdot r + e - (a + t) = 12 \quad (4)$$

$$2 \cdot (a + t) + n = 55 \quad (5)$$

Given that the value of  $N$  is the average of the values of  $A$  and  $T$ ,

$$n = \frac{1}{2} \cdot (a + t)$$

Compute the value assigned to the letter  $N$ .

$$2 \cdot r + e + a + t + n = 67 \quad (1)$$

$$2 \cdot r + e + u + n = 49 \quad (2)$$

$$a + t + u + n = 37 \quad (3)$$

$$2 \cdot r + e - (a + t) = 12 \quad (4)$$

$$2 \cdot (a + t) + n = 55 \quad (5)$$

Given that the value of  $N$  is the average of the values of  $A$  and  $T$ ,

$$n = \frac{1}{2} \cdot (a + t) \implies 2 \cdot n = a + t.$$

Compute the value assigned to the letter  $N$ .

$$2 \cdot r + e + a + t + n = 67 \quad (1)$$

$$2 \cdot r + e + u + n = 49 \quad (2)$$

$$a + t + u + n = 37 \quad (3)$$

$$2 \cdot r + e - (a + t) = 12 \quad (4)$$

$$2 \cdot (a + t) + n = 55 \quad (5)$$

Given that the value of  $N$  is the average of the values of  $A$  and  $T$ ,

$$n = \frac{1}{2} \cdot (a + t) \implies 2 \cdot n = a + t.$$

Substituting  $2 \cdot n$  for  $(a + t)$  in (5), we have that



Compute the value assigned to the letter  $N$ .

$$2 \cdot r + e + a + t + n = 67 \quad (1)$$

$$2 \cdot r + e + u + n = 49 \quad (2)$$

$$a + t + u + n = 37 \quad (3)$$

$$2 \cdot r + e - (a + t) = 12 \quad (4)$$

$$2 \cdot (a + t) + n = 55 \quad (5)$$

Given that the value of  $N$  is the average of the values of  $A$  and  $T$ ,

$$n = \frac{1}{2} \cdot (a + t) \implies 2 \cdot n = a + t.$$

Substituting  $2 \cdot n$  for  $(a + t)$  in (5), we have that

$$2 \cdot 2 \cdot n + n = 55.$$

Compute the value assigned to the letter  $N$ .

$$2 \cdot r + e + a + t + n = 67 \quad (1)$$

$$2 \cdot r + e + u + n = 49 \quad (2)$$

$$a + t + u + n = 37 \quad (3)$$

$$2 \cdot r + e - (a + t) = 12 \quad (4)$$

$$2 \cdot (a + t) + n = 55 \quad (5)$$

Given that the value of  $N$  is the average of the values of  $A$  and  $T$ ,

$$n = \frac{1}{2} \cdot (a + t) \implies 2 \cdot n = a + t.$$

Substituting  $2 \cdot n$  for  $(a + t)$  in (5), we have that

$$4 \cdot n + n = 55.$$

Compute the value assigned to the letter  $N$ .

$$2 \cdot r + e + a + t + n = 67 \quad (1)$$

$$2 \cdot r + e + u + n = 49 \quad (2)$$

$$a + t + u + n = 37 \quad (3)$$

$$2 \cdot r + e - (a + t) = 12 \quad (4)$$

$$2 \cdot (a + t) + n = 55 \quad (5)$$

Given that the value of  $N$  is the average of the values of  $A$  and  $T$ ,

$$n = \frac{1}{2} \cdot (a + t) \implies 2 \cdot n = a + t.$$

Substituting  $2 \cdot n$  for  $(a + t)$  in (5), we have that

$$(4 + 1) \cdot n = 55.$$

Compute the value assigned to the letter  $N$ .

$$2 \cdot r + e + a + t + n = 67 \quad (1)$$

$$2 \cdot r + e + u + n = 49 \quad (2)$$

$$a + t + u + n = 37 \quad (3)$$

$$2 \cdot r + e - (a + t) = 12 \quad (4)$$

$$2 \cdot (a + t) + n = 55 \quad (5)$$

Given that the value of  $N$  is the average of the values of  $A$  and  $T$ ,

$$n = \frac{1}{2} \cdot (a + t) \implies 2 \cdot n = a + t.$$

Substituting  $2 \cdot n$  for  $(a + t)$  in (5), we have that

$$5 \cdot n = 55.$$

Compute the value assigned to the letter  $N$ .

$$2 \cdot r + e + a + t + n = 67 \quad (1)$$

$$2 \cdot r + e + u + n = 49 \quad (2)$$

$$a + t + u + n = 37 \quad (3)$$

$$2 \cdot r + e - (a + t) = 12 \quad (4)$$

$$2 \cdot (a + t) + n = 55 \quad (5)$$

Given that the value of  $N$  is the average of the values of  $A$  and  $T$ ,

$$n = \frac{1}{2} \cdot (a + t) \implies 2 \cdot n = a + t.$$

Substituting  $2 \cdot n$  for  $(a + t)$  in (5), we have that

$$\frac{1}{5} \cdot 5 \cdot n = \frac{1}{5} \cdot 55.$$

Compute the value assigned to the letter  $N$ .

$$2 \cdot r + e + a + t + n = 67 \quad (1)$$

$$2 \cdot r + e + u + n = 49 \quad (2)$$

$$a + t + u + n = 37 \quad (3)$$

$$2 \cdot r + e - (a + t) = 12 \quad (4)$$

$$2 \cdot (a + t) + n = 55 \quad (5)$$

Given that the value of  $N$  is the average of the values of  $A$  and  $T$ ,

$$n = \frac{1}{2} \cdot (a + t) \implies 2 \cdot n = a + t.$$

Substituting  $2 \cdot n$  for  $(a + t)$  in (5), we have that

$$n = \boxed{11}.$$

Review the key concepts we used.

# Key Concepts

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## ■ Elimination



Review the key concepts we used.

# Key Concepts

- Elimination
- Substitution