Math League Contest Problem Set 12128

Sprint Round Problem 29

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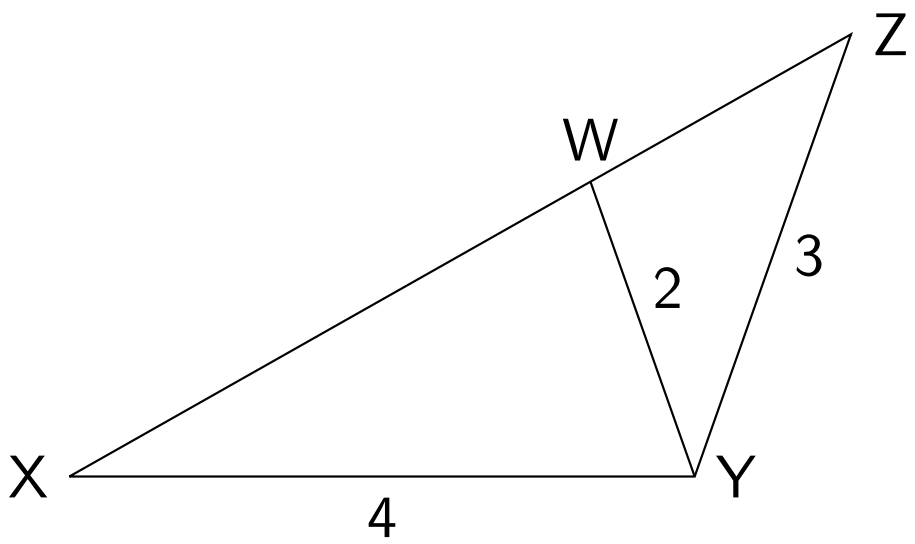
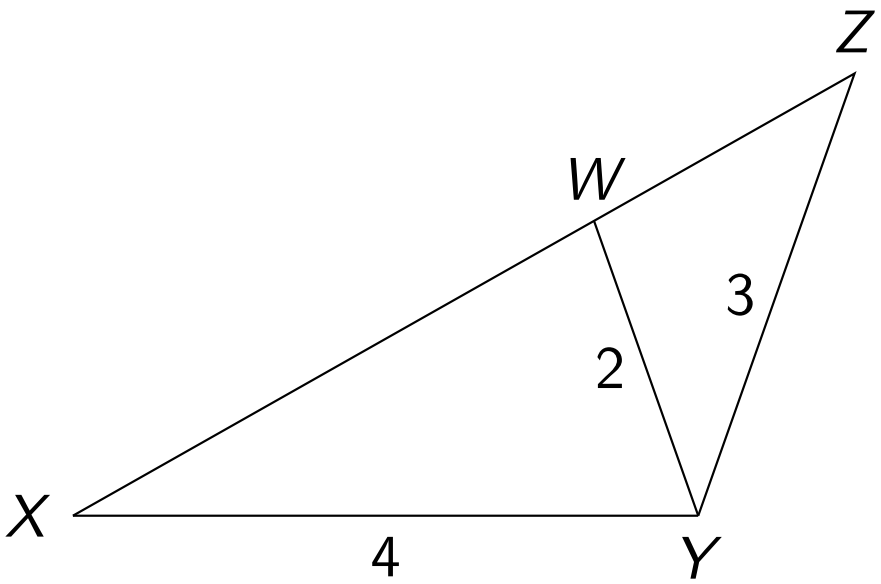
Math League, LLC

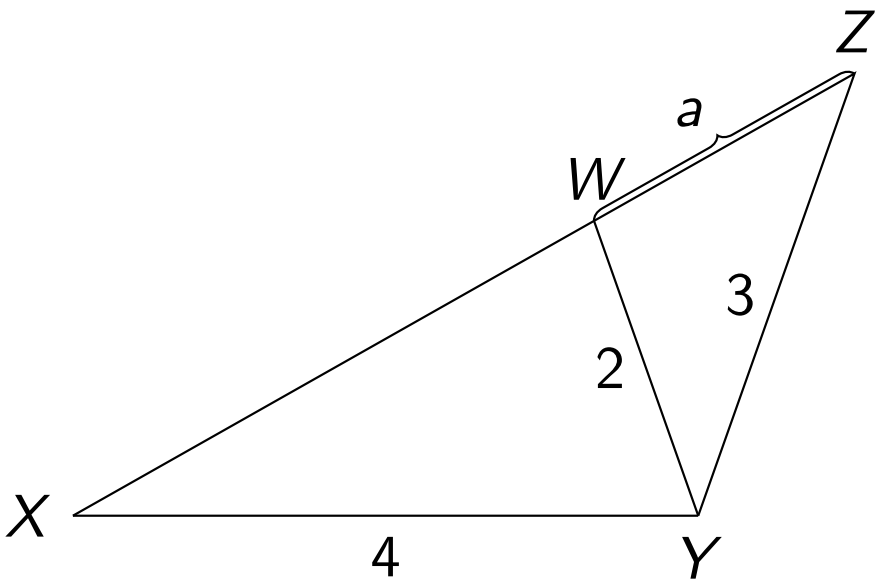
In $\triangle XYZ$, side XY has length 4 and side YZ has length 3. Point W lies on side XZ such that the length of segment YW is 2, and the length of segment XW is twice the length of segment WZ. What is the square of the length of side XZ?



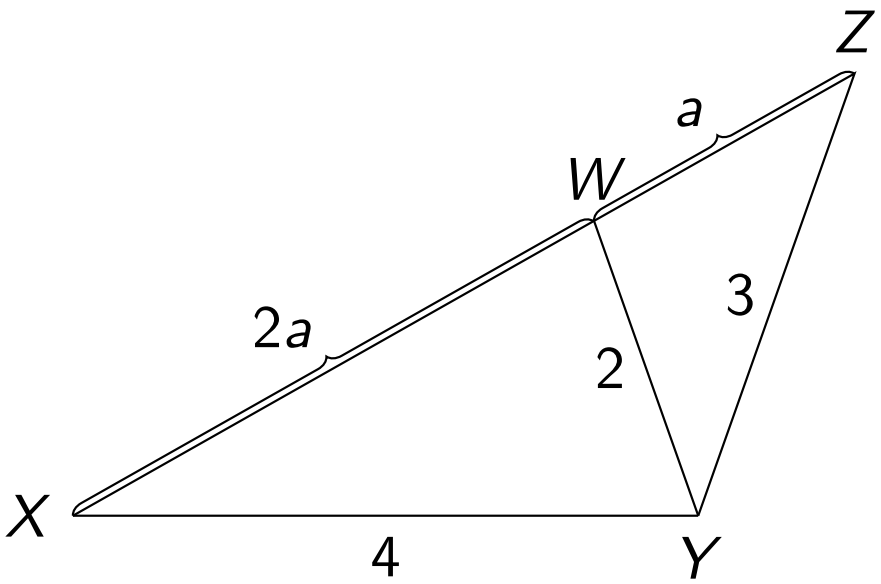
Compute XZ^2.



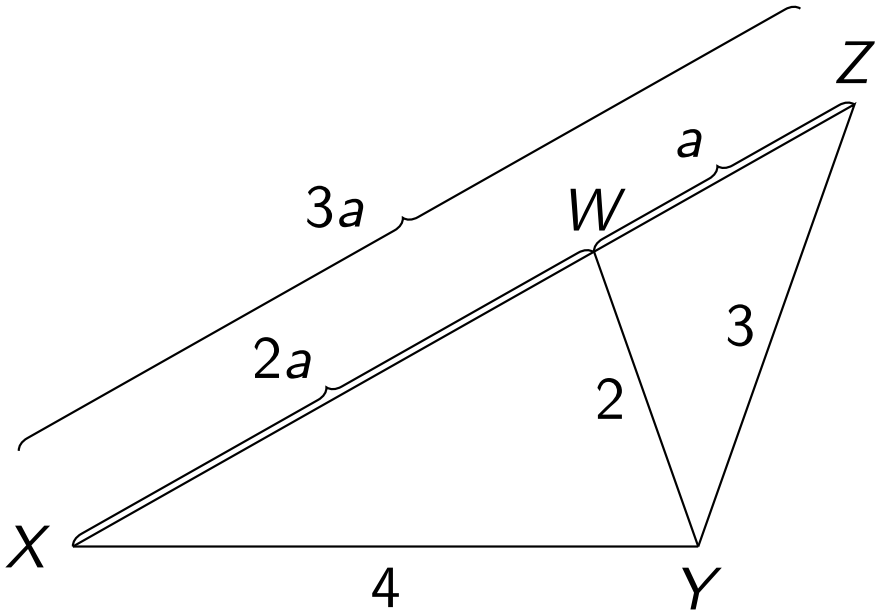
Compute XZ^2.



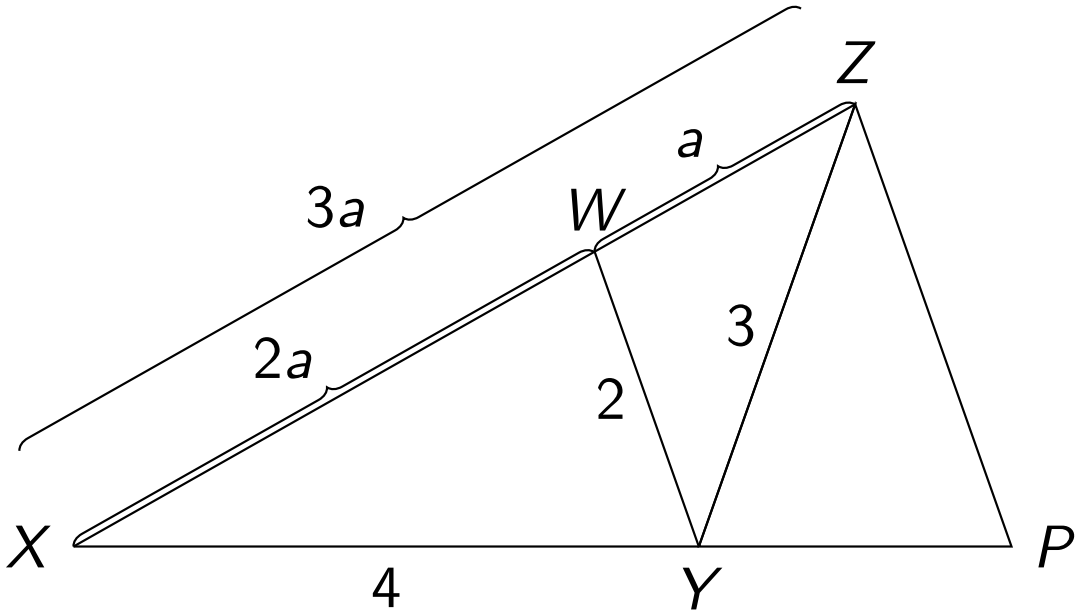
Compute XZ^2.



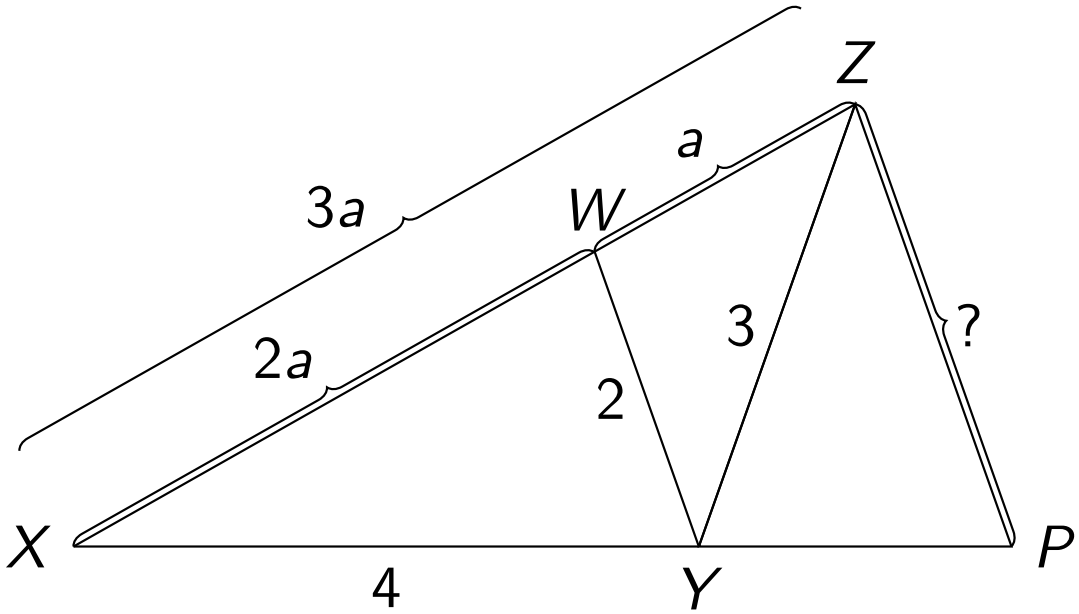
Compute XZ^2.



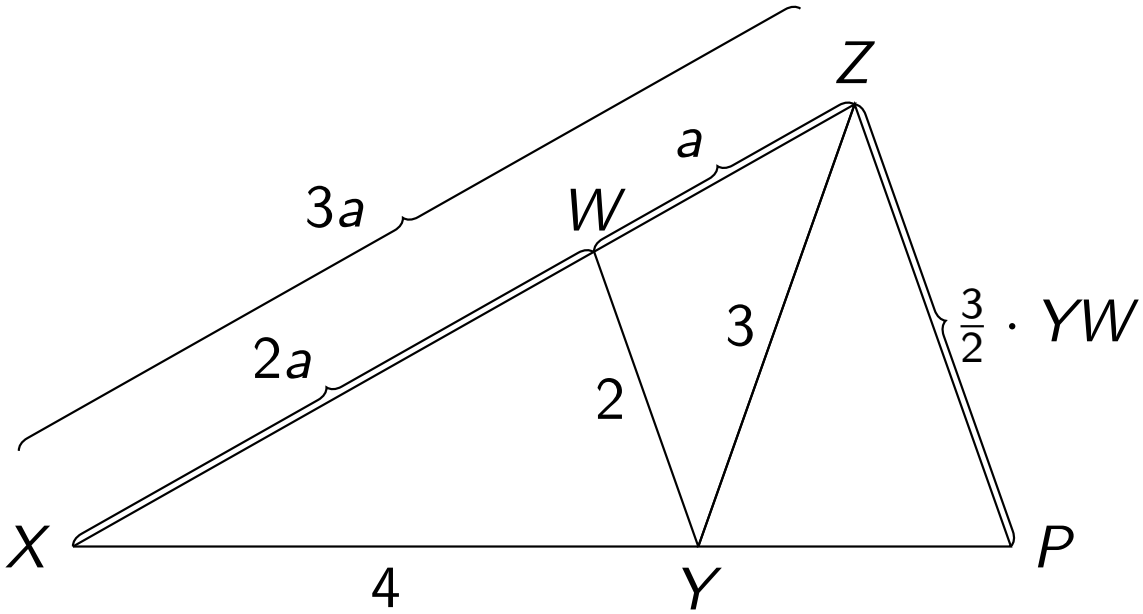
Compute XZ^2.



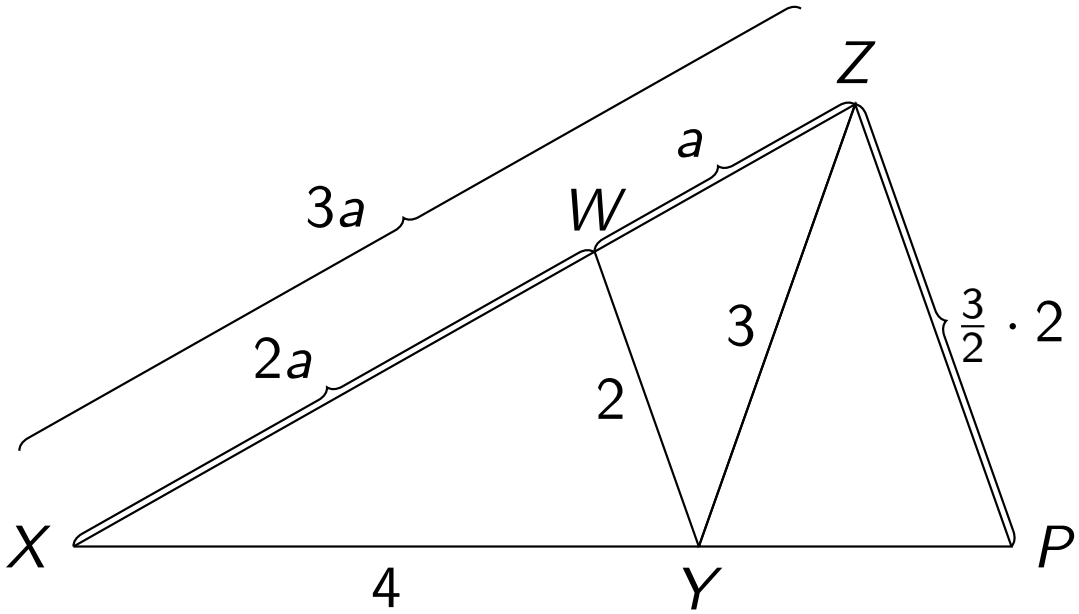
Compute XZ^2.



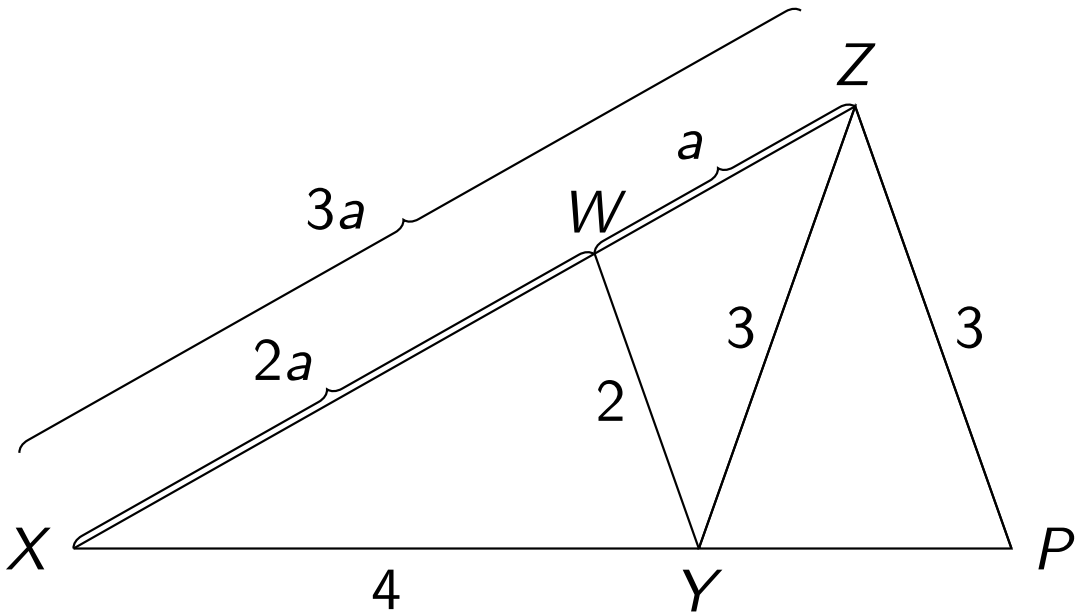
Compute XZ^2.



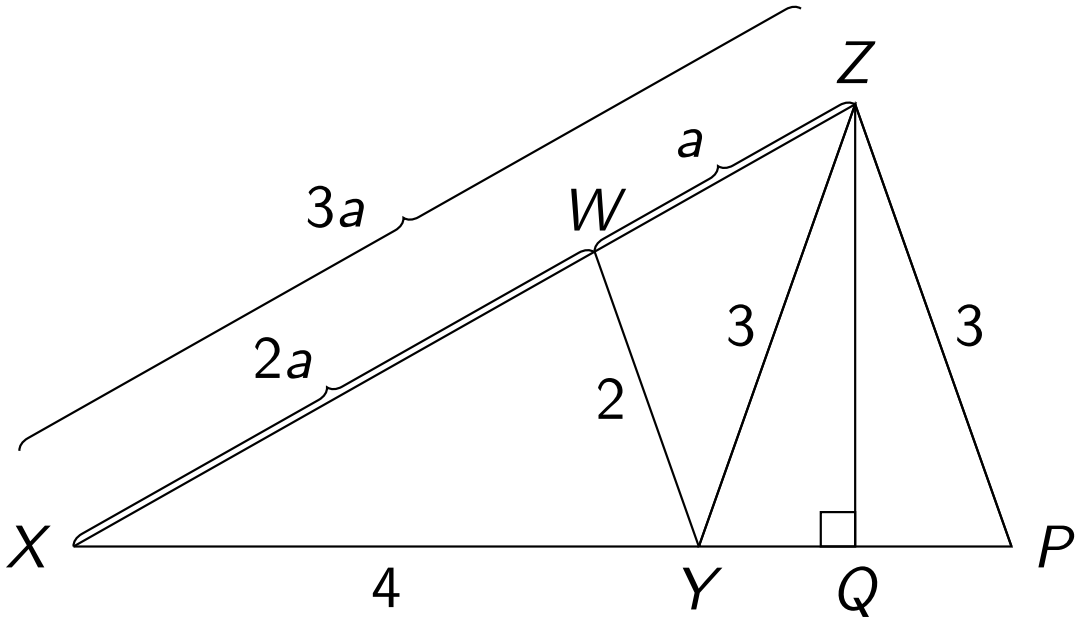
Compute XZ^2.



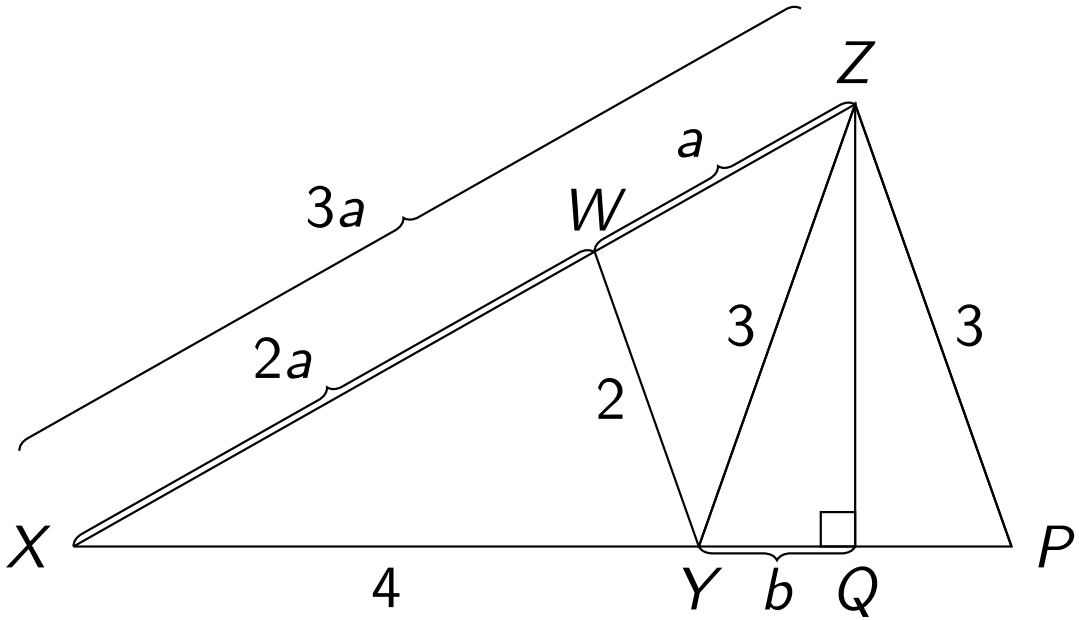
Compute XZ^2.



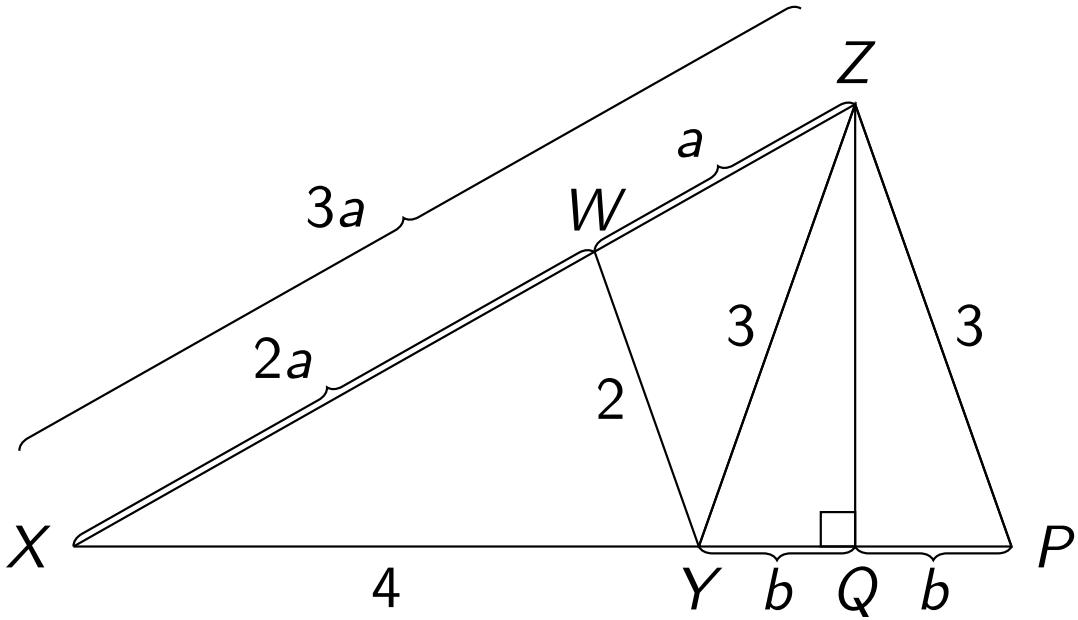
Compute XZ^2.



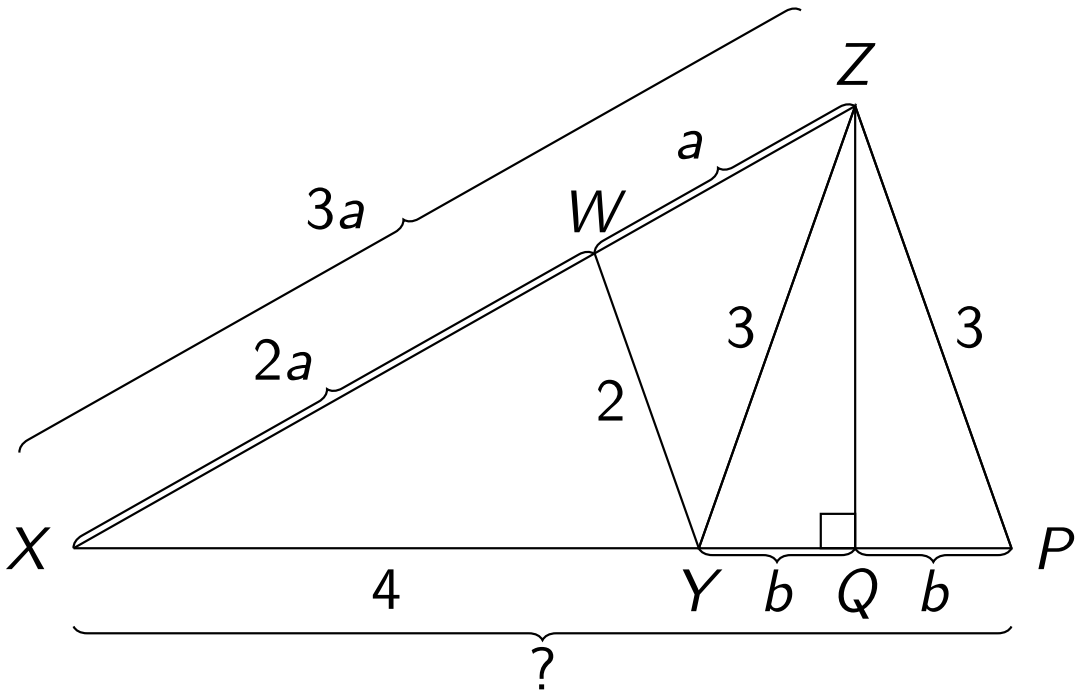
Compute XZ^2.



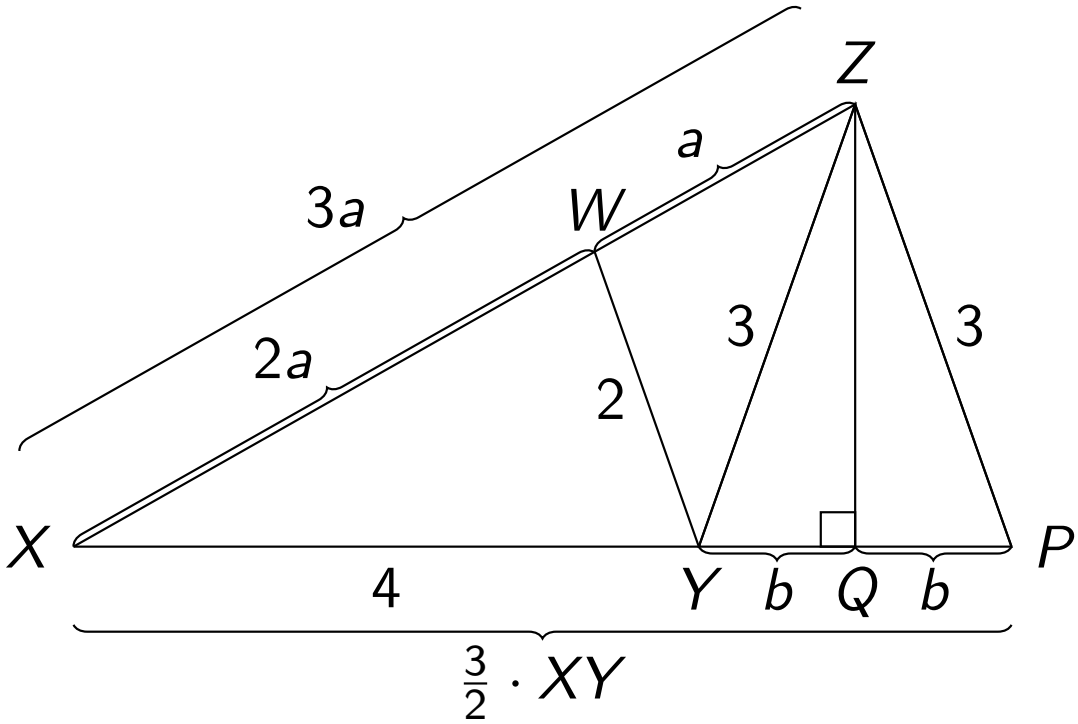
Compute XZ^2.



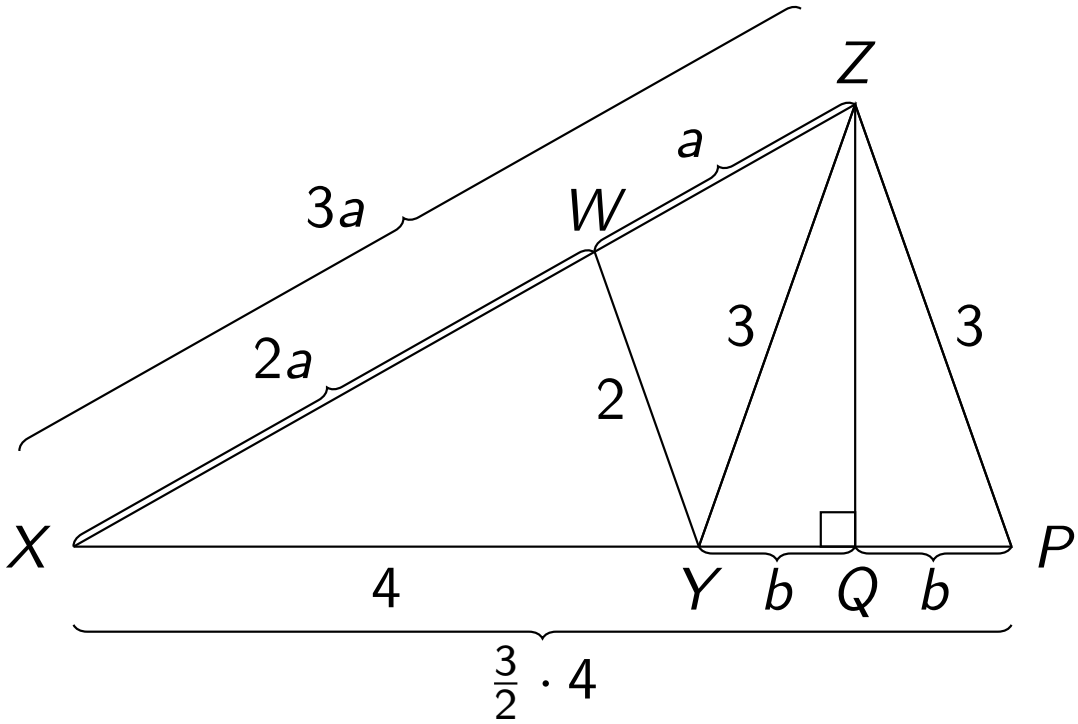
Compute XZ^2.



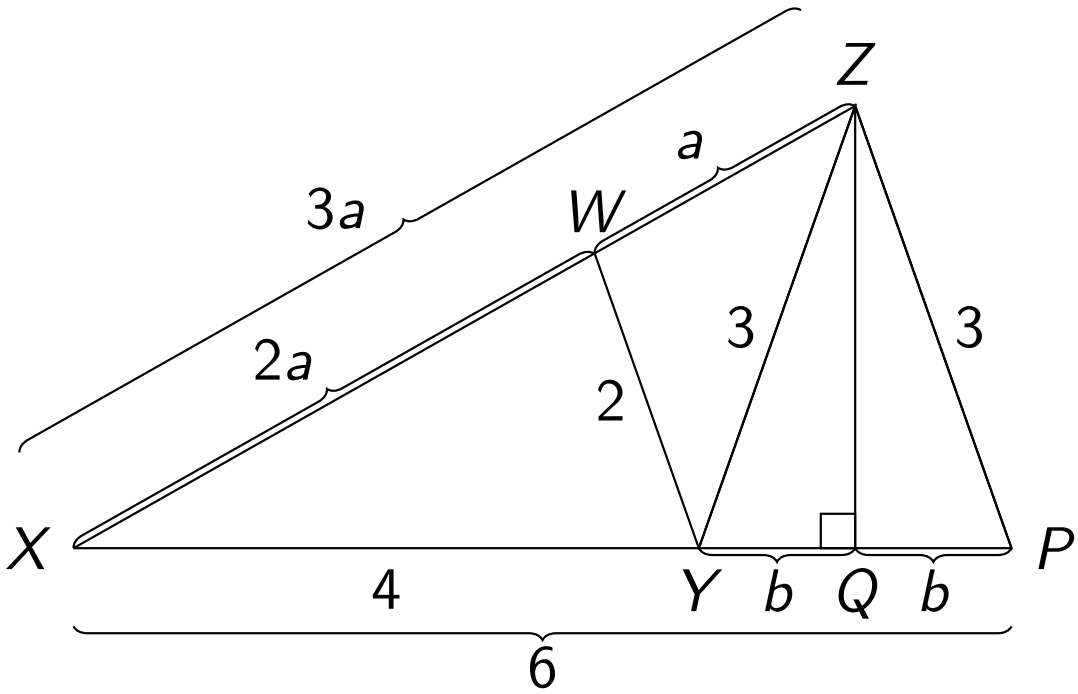
Compute XZ^2.



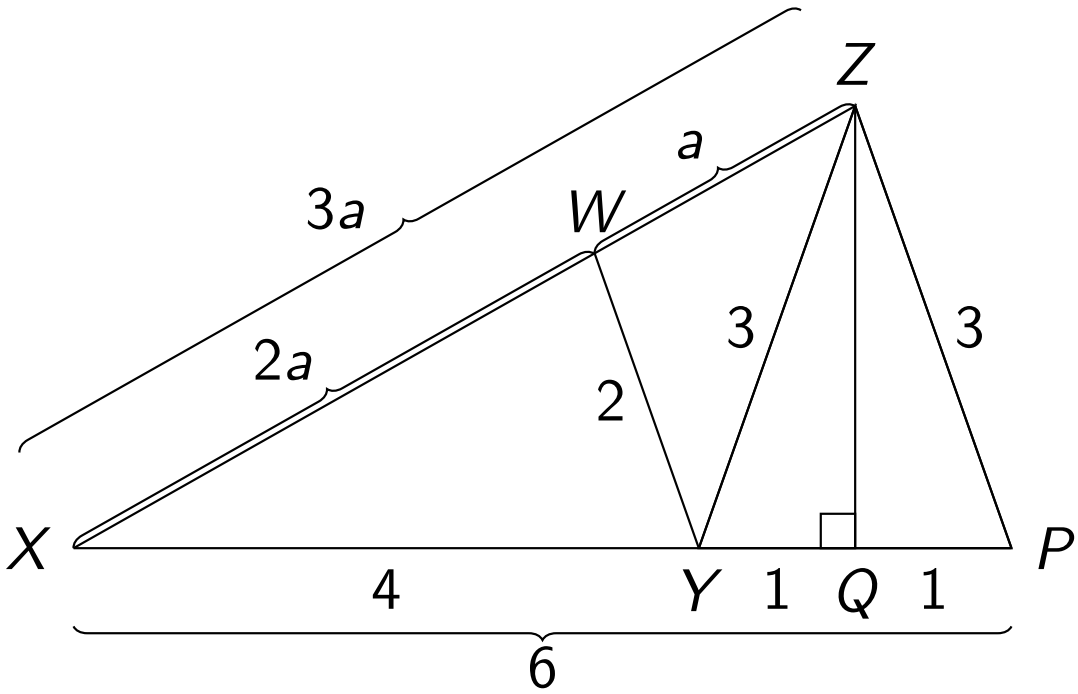
Compute XZ^2.



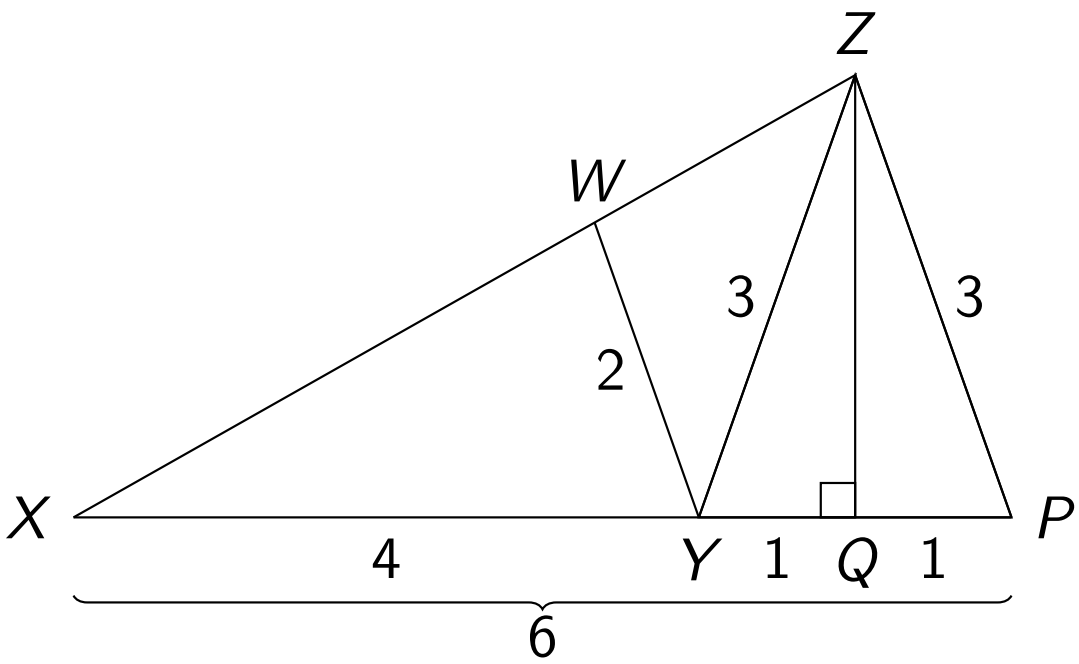
Compute XZ^2.



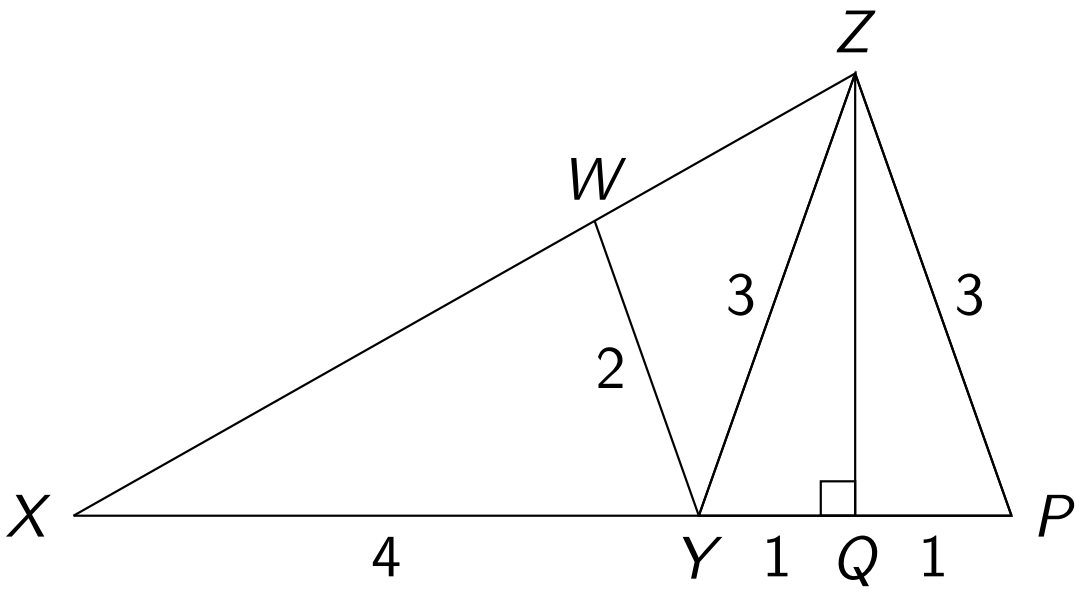
Compute XZ^2.



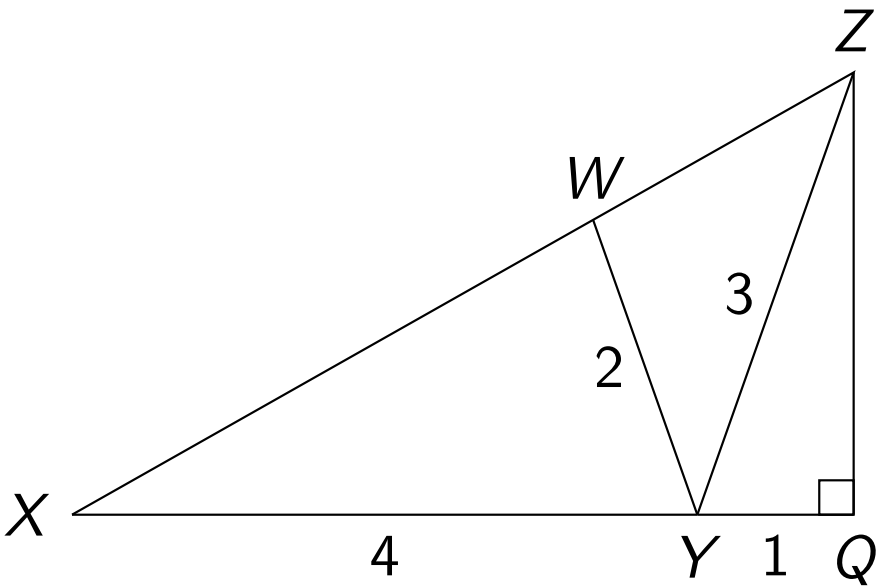
Compute XZ^2.



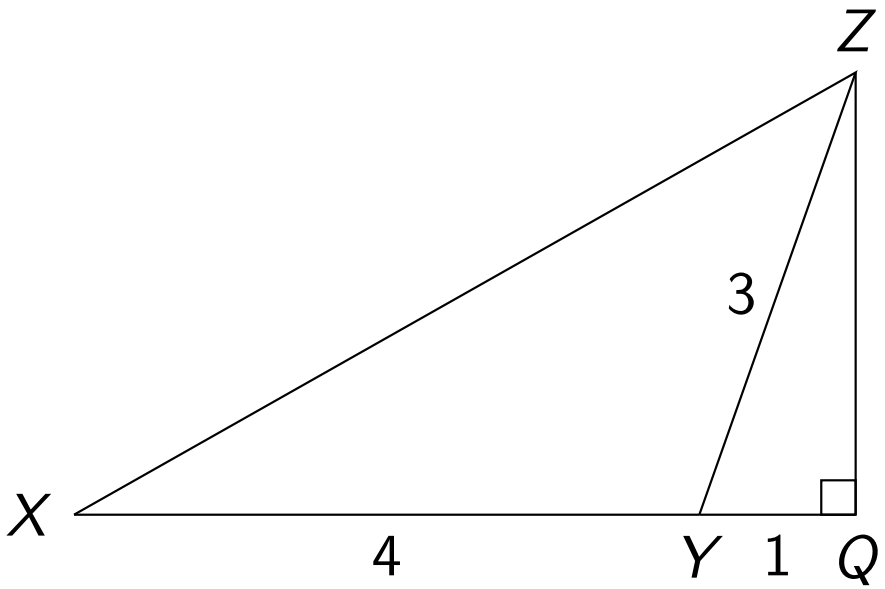
Compute XZ^2.



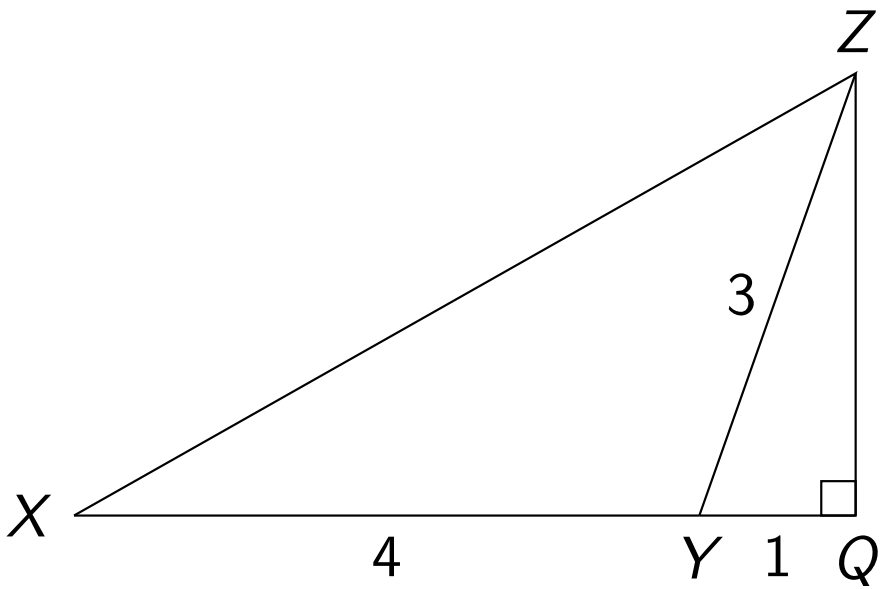
Compute XZ^2.



Compute XZ^2.

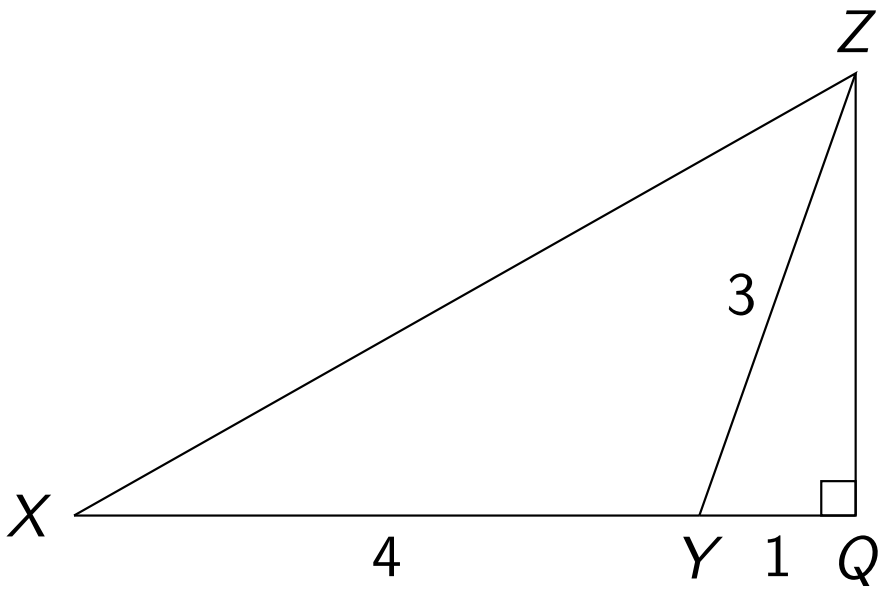


Compute XZ^2.



By Pythagorean Theorem,

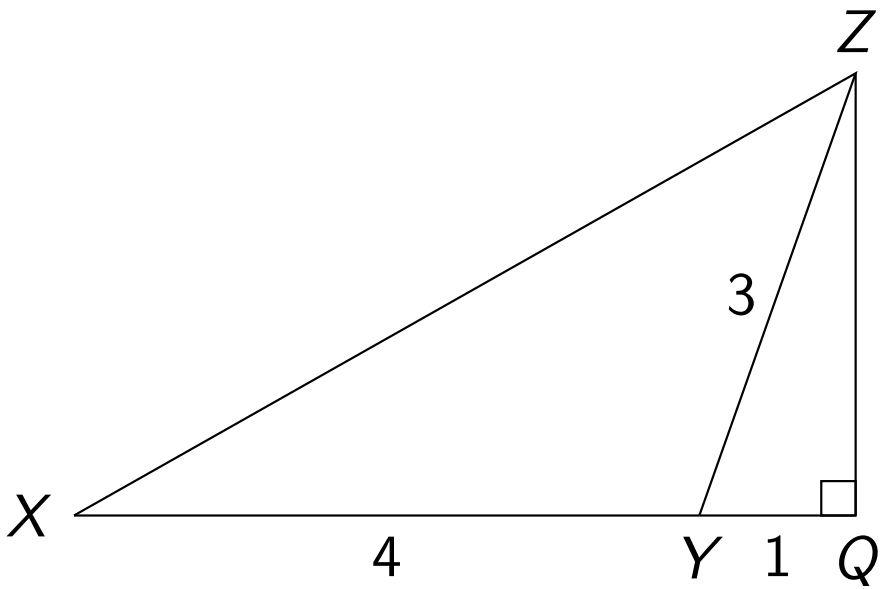
$$XZ^2 = XQ^2 + QZ^2$$



By Pythagorean Theorem,

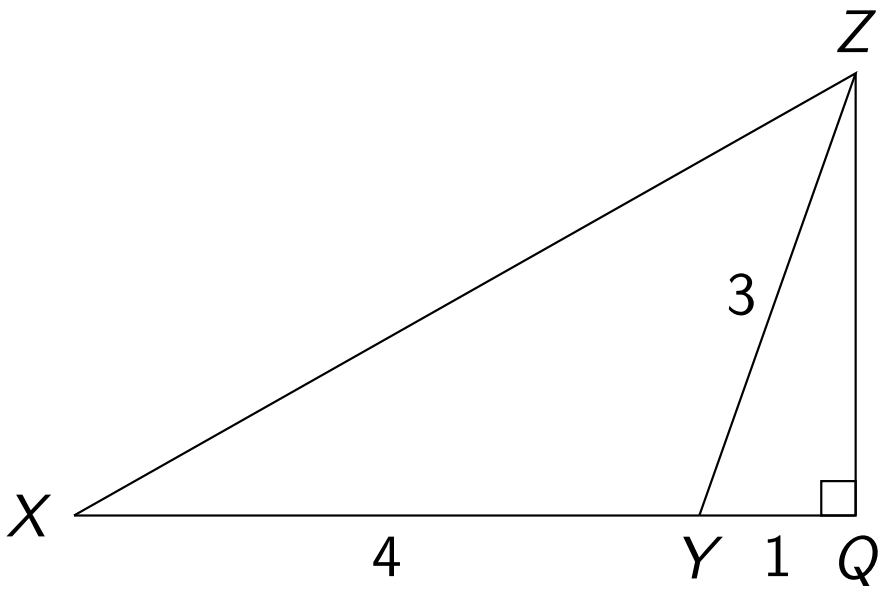
$$XZ^2 = XQ^2 + (YZ^2 - YQ^2)$$

Compute XZ^2.



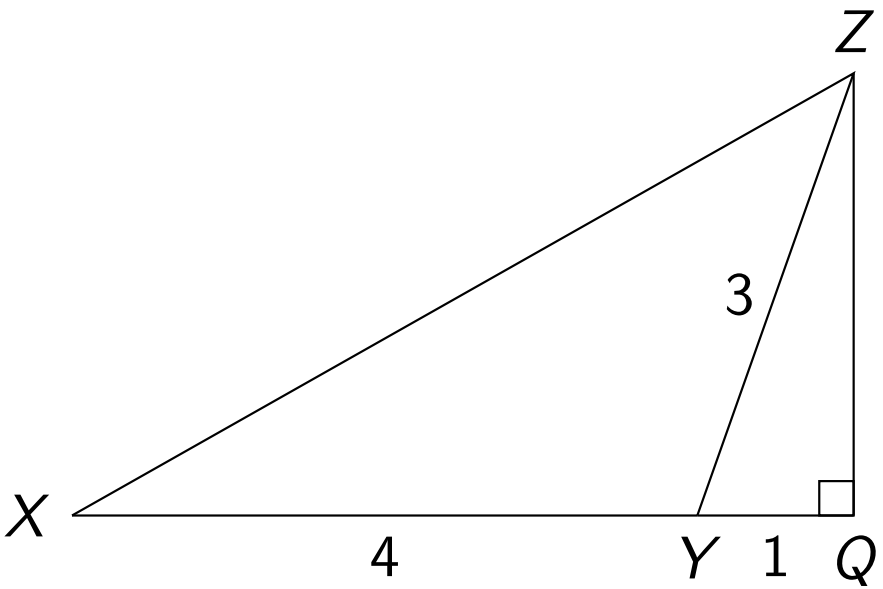
$$XZ^2 = XQ^2 + (YZ^2 - YQ^2)$$

Compute XZ^2.



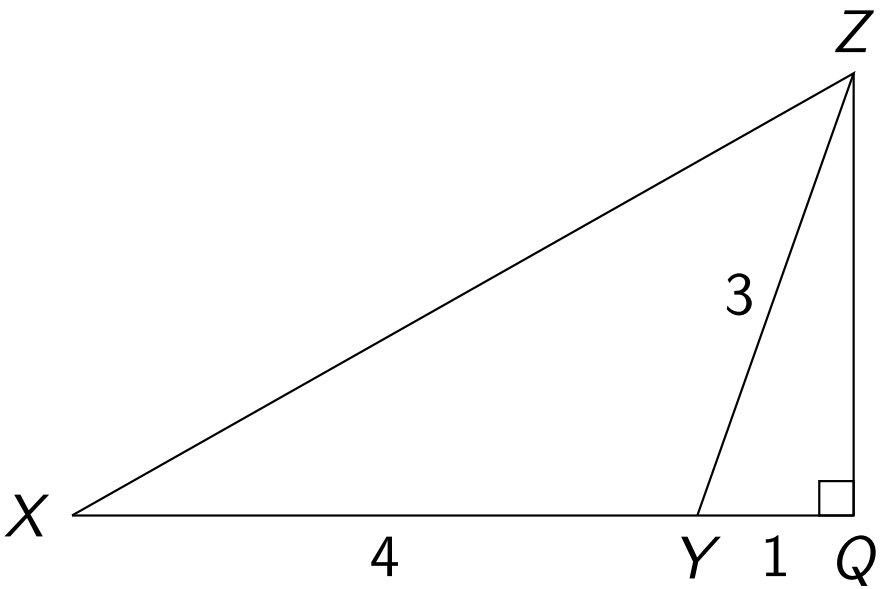
$$XZ^2 = (XY + YQ)^2 + (YZ^2 - YQ^2)$$

Compute XZ^2.



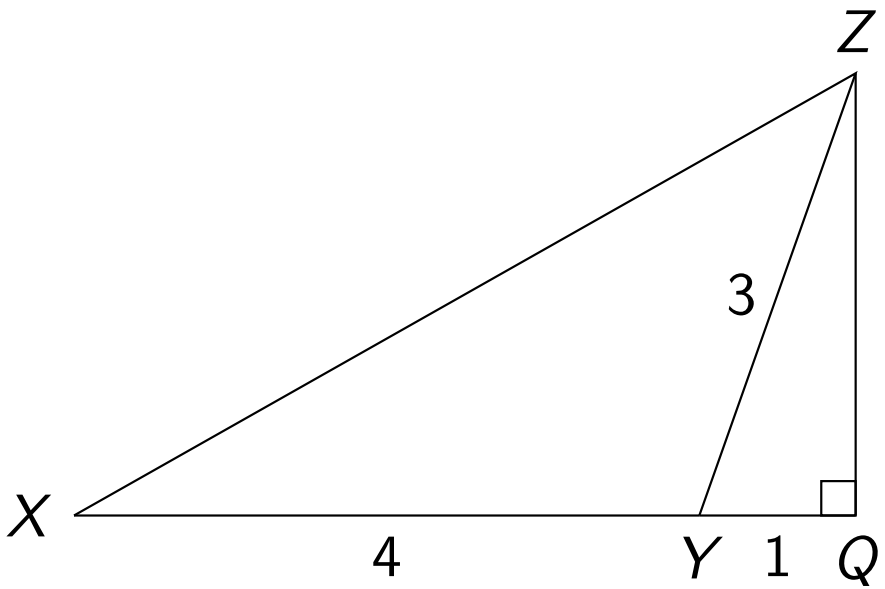
$$XZ^2 = (4 + YQ)^2 + (YZ^2 - YQ^2)$$

Compute XZ^2.



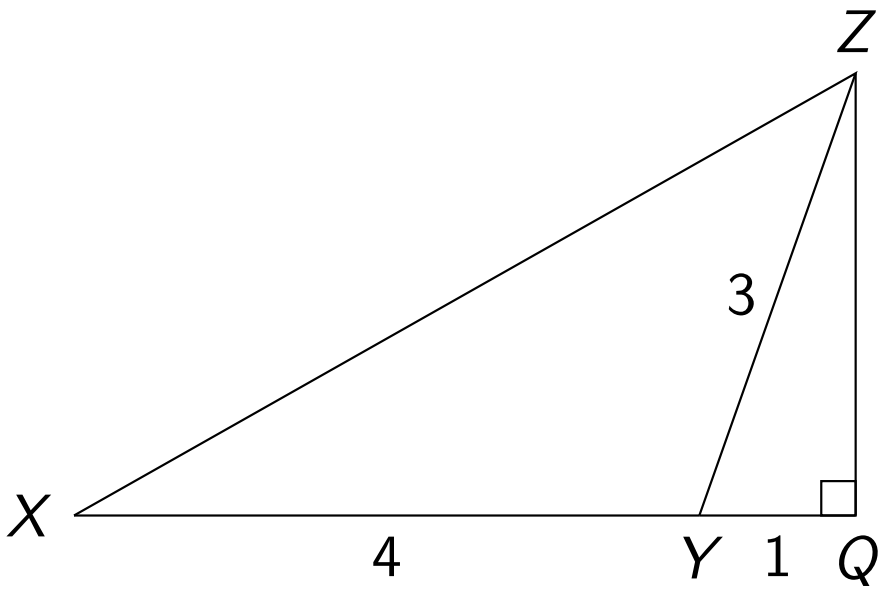
$$XZ^2 = (4+1)^2 + (YZ^2 - 1^2)$$

Compute XZ^2.



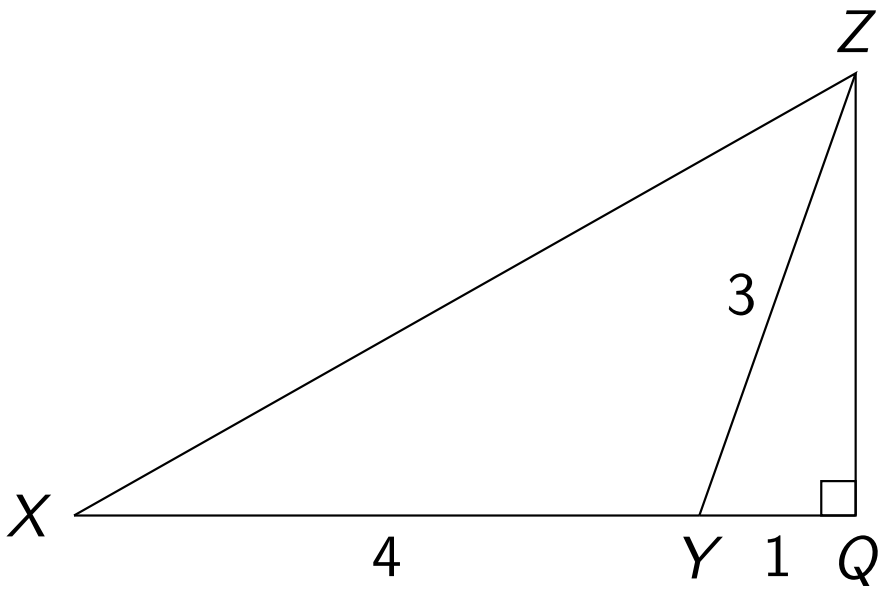
$$XZ^2 = (4+1)^2 + (3^2 - 1^2)$$

Compute XZ^2.



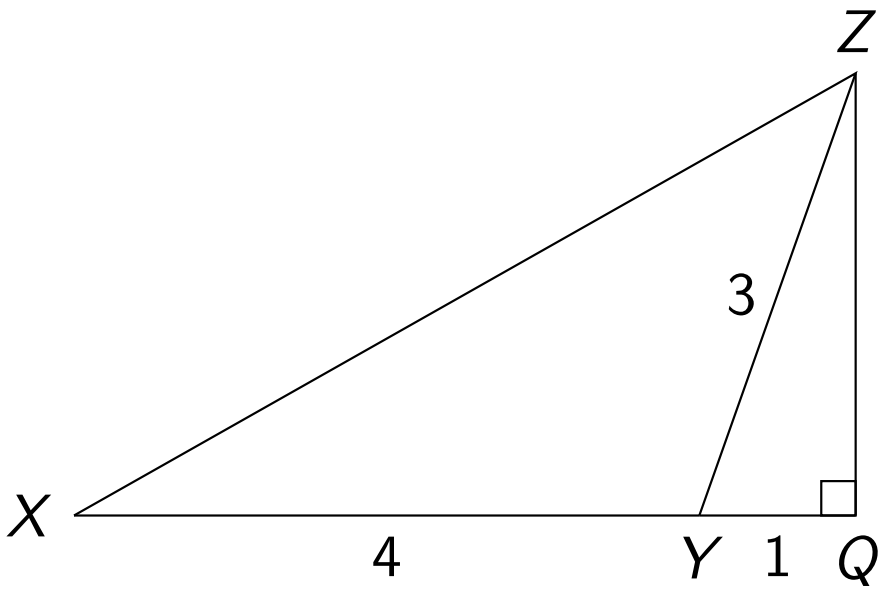
$$XZ^2 = 5^2 + (3^2 - 1^2)$$

Compute XZ^2.



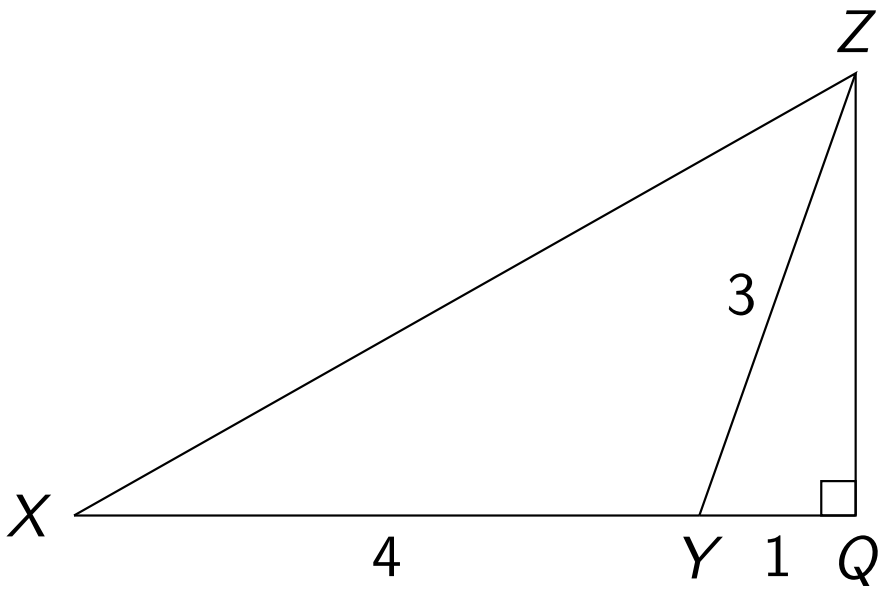
$$XZ^2 = 25 + (3^2 - 1^2)$$

Compute XZ^2.



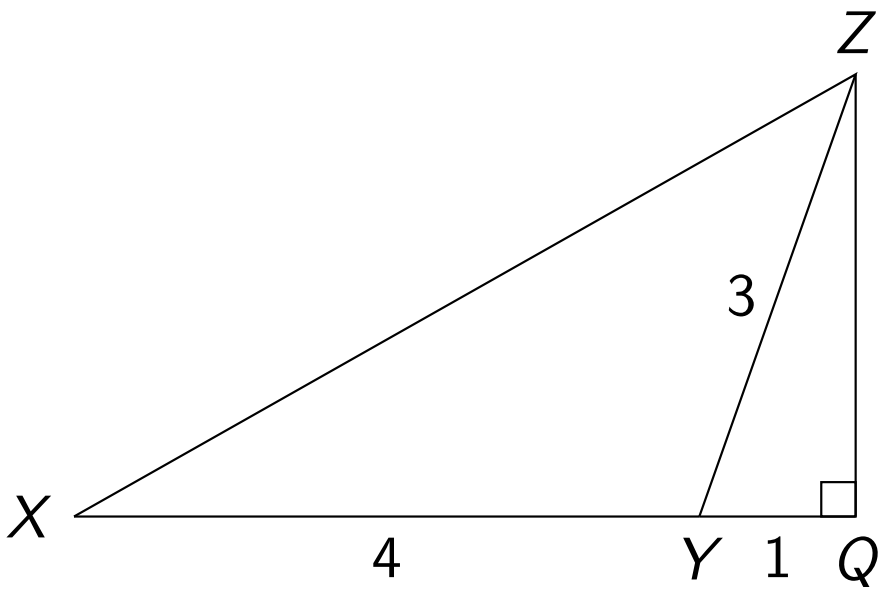
$$XZ^2 = 25 + (9 - 1^2)$$

Compute XZ^2.



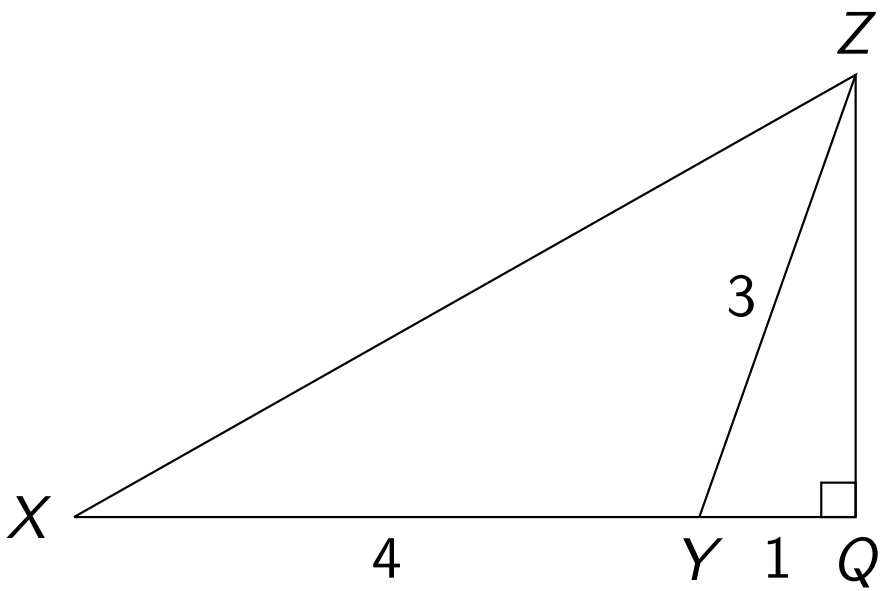
$$XZ^2 = 25 + (9 - 1)$$

Compute XZ^2.



$$XZ^2 = 25 + 8$$

Compute XZ^2.



$$XZ^2 = \boxed{33}$$

Key Concepts

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- Similar Triangles

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- Properties of Isosceles Triangles

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- Similar Triangles
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