Computing Gradients for the Logistic Regression

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Linear function: $f(x) = xW + b$.

Logistic Regression function: $\text{Sigmoid}(f(x))$.

Log-loss: $-(y \log f(x) + (1-y) \log(1 - f(x)))$

Mean Log-loss: $\text{mean}(-(y \log f(x) + (1-y) \log(1 - f(x))))$

Recall the following:

$$\text{Sigmoid}(x)' = \left(\frac{1}{1+e^{-x}}\right)' = \text{Sigmoid}(x)(1 - \text{Sigmoid}(x)) \tag{1}$$

We now calculate partial derivatives of the loss function with respect to weights and a bias.

Let $h = \text{Sigmoid}(f(x))$.

We have:

$$\begin{aligned}
(-(y \log h + (1-y)\log(1-h)))' &= -\frac{y}{h} \times h' - \frac{1-y}{1-h} \times (-h') && (2)\\
&= h' * \frac{(1-y)h - y(1-h)}{h(1-h)} && (3)\\
&= h(1-h)f'(x) \times \frac{(1-y)h - y(1-h)}{h(1-h)} && (4)\\
&= f'(x) \times (h - y) && (5)\\
&= (xW + b)' \times (h - y) && (6)
\end{aligned}$$

Then the derivative w.r.t. W is

$$\boxed{\frac{1}{n}\sum_{i=0}^{n} x \cdot (\text{Sigmoid}(f(x_i)) - y_i)} \tag{7}$$

and the derivative w.r.t. b is

$$\boxed{\frac{1}{n}\sum_{i=0}^{n} \text{Sigmoid}(f(x_i)) - y_i} \tag{8}$$