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# LETTER TO THE EDITOR

## THE DERIVATION OF A THICKNESS NOISE FORMULA FOR THE FAR-FIELD BY ISOM

In a report published in 1975, Isom has derived an interesting thickness noise formula for a hovering helicopter rotor blade which is valid in the far-field [1]. In this formula, the unit normal to the blade surface and the radiation direction at the emission time appear. The normal velocity of the blade surface which appears in most other formulations of the thickness noise is absent in this formula. Isom derived his result using a frame fixed to the rotating blade, thus giving the impression that it is valid for a hovering rotor or a static propeller. It will be shown here that it is in fact more general and is valid for any body in arbitrary motion. In what follows a frame fixed with respect to the undisturbed medium will be used.

The thickness noise for a body whose surface is described by $f(\mathbf{x}, t) = 0$, moving in a medium with undisturbed density $\rho_0$ is governed by the wave equation [2, 3]

$$\frac{1}{c^2}\frac{\partial^2 p'}{\partial t^2} - \nabla^2 p' = \frac{\partial}{\partial t}[\rho_0 v_n |\nabla f|\,\delta(f)]. \tag{1}$$

Here $p'(\mathbf{x}, t)$ is the acoustic pressure, $v_n = -(\partial f/\partial t)/|\nabla f|$ is the local normal velocity of the body, $\delta(f)$ is the Dirac delta function and $c$ is the speed of sound in the undisturbed medium. For a fixed observer position $\mathbf{x}$ and time $t$, the locus of points on the body which radiate sound waves that arrive at the observer simultaneously at time $t$ is a surface $\Sigma$. This surface is described by the equation $F(\mathbf{y}; \mathbf{x}, t) = f(\mathbf{y}, t - r/c) = 0$, where $r = |\mathbf{x} - \mathbf{y}|$. The solution of equation (1) is [2]

$$4\pi p'(\mathbf{x}, t) = \frac{\partial}{\partial t}\int_{F=0} \frac{\rho_0}{r}\left[\frac{v_n}{\Lambda}\right]_{\mathrm{ret}} \mathrm{d}\Sigma \tag{2}$$

where $\Lambda^2 = 1 + M_n^2 - 2M_n \cos\theta$, $M_n = v_n/c$ and $\theta$ is the angle between the outward normal to the body $f = 0$ and the radiation direction $\mathbf{r} = \mathbf{x} - \mathbf{y}$. If $\tilde{\theta}$ is the angle between the outward normal to the surface $F = 0$ and $\mathbf{r}$, then it can be shown that [2]

$$\cos\tilde{\theta} = [\cos\theta/\Lambda]_{\mathrm{ret}} - (1/c)[v_n/\Lambda]_{\mathrm{ret}}. \tag{3}$$

Solving for $[v_n/\Lambda]_{\mathrm{ret}}$ from this equation and substituting in equation (2), one obtains

$$4\pi p'(\mathbf{x}, t) = \frac{\partial}{\partial t}\int_{F=0} \frac{\rho_0 c}{r}\left[\frac{\cos\theta}{\Lambda}\right]_{\mathrm{ret}} \mathrm{d}\Sigma - \frac{\partial}{\partial t}\int_{F=0} \frac{\rho_0 c}{r}\cos\tilde{\theta}\,\mathrm{d}\Sigma. \tag{4}$$

In the far field, equation (4) can be simplified. Here the term far field refers to the observer region for which $r \gg L$, where $L$ is the length scale of the surface $F = 0$. If $\mathbf{N}$ is the unit outward normal to $F = 0$ and $\hat{\mathbf{r}}$ is the unit vector in the radiation direction, then in the far field the last integral drops out since

$$\int_{F=0} \frac{\rho_0 c}{r}\cos\tilde{\theta}\,\mathrm{d}\Sigma = \frac{\rho_0 c}{r}\hat{\mathbf{r}}.\int_{F=0} \mathbf{N}\,\mathrm{d}\Sigma = 0.$$

Therefore, in the far field

$$4\pi p'(\mathbf{x}, t) = \frac{\partial}{\partial t}\int_{F=0} \frac{\rho_0 c}{r}\left[\frac{\cos\theta}{\Lambda}\right]_{\mathrm{ret}} \mathrm{d}\Sigma. \tag{5}$$



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The following relations developed in reference [3] can now be used to write equation (5) in more familiar forms:

$$d\Sigma/\Lambda = dS/|1 - M_r| = c\, d\tau\, d\Gamma/\sin\theta, \tag{6a, b}$$

where $dS$ is the element of surface area of $f = 0$, $M_r = \mathbf{v}.\mathbf{r}/c$ is the component of the local surface Mach number in the radiation direction, $\tau$ is the source time and $d\Gamma$ is the element of the curve of intersection of the surface $f = 0$ and the collapsing sphere $g = \tau - t + r/c = 0$. Equation (5) can then be written as

$$4\pi p'(\mathbf{x}, t) = \frac{\partial}{\partial t}\int_{f=0}\left[\frac{\rho_0 c \cos\theta}{r|1 - M_r|}\right]_{\mathrm{ret}} dS = \frac{\partial}{\partial t}\int_{\substack{f=0\\g=0}} \frac{\rho_0 c^2 \cot\theta}{r}\, d\Gamma\, d\tau. \tag{7a, b}$$

Equation (7a), which was developed by Isom [1], is suitable for bodies in arbitrary subsonic motion. Equation (7b) does not have this restriction. The condition $\theta = 0$, which produces an integrable singularity, is discussed in detail in reference [3].

The above equations appear to be more suitable for numerical calculation of the thickness noise in the far field than the available expressions involving the normal velocity $v_n$, particularly for blades with a blunt leading edge. The reason is that $v_n$ varies substantially over the blade surface and the integration scheme becomes sensitive to mesh size. However, since $\cos\theta$ is of $O(1)$, this problem does not arise. It is interesting to note that, in the far field, thickness noise is equivalent to the noise from a uniform pressure distribution over the entire blade surface. This pressure has the magnitude $\rho_0 c^2$.

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