Spatial modelling

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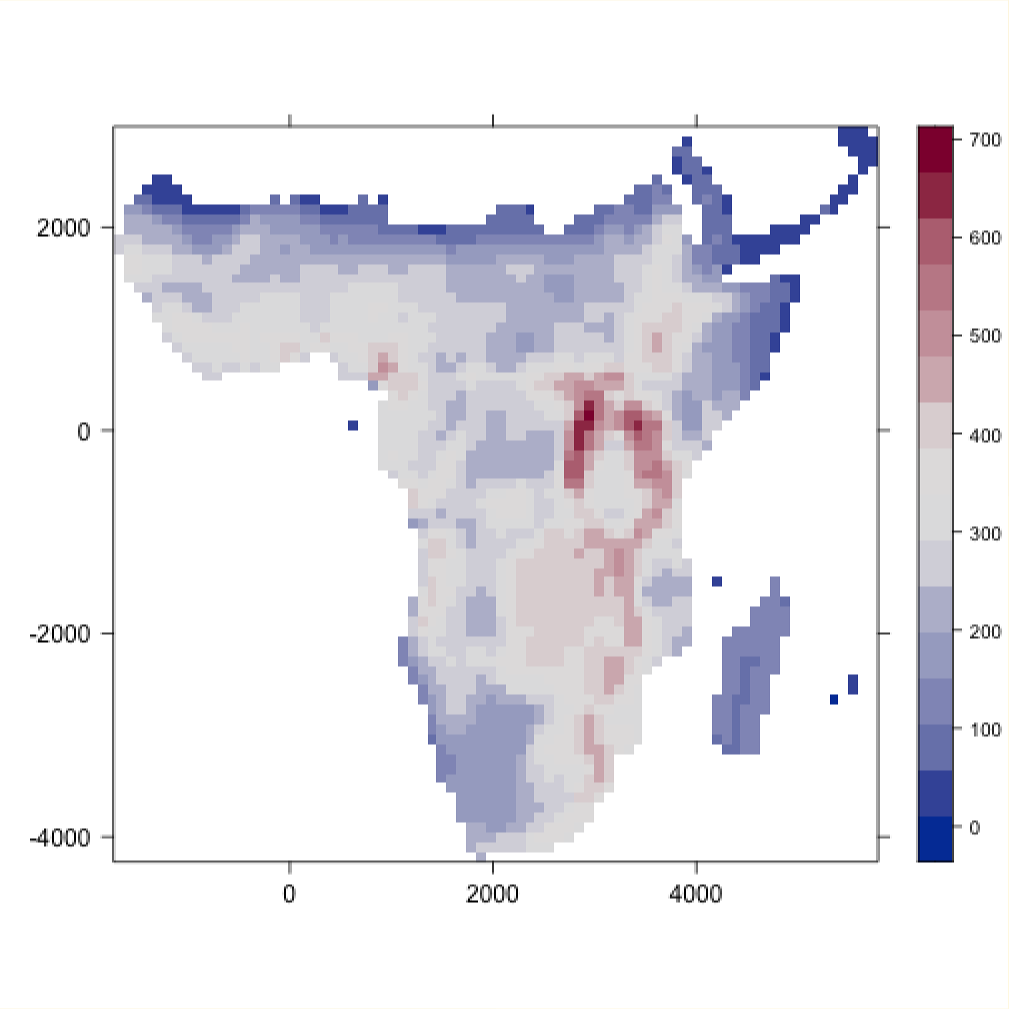
Spatial modelling tools

- The examples presented here use R
- Another excellent program with a nice GUI interface:
  - Spatial Analysis in Macroecology
  - http://www.ecoevol.ufg.br/sam/

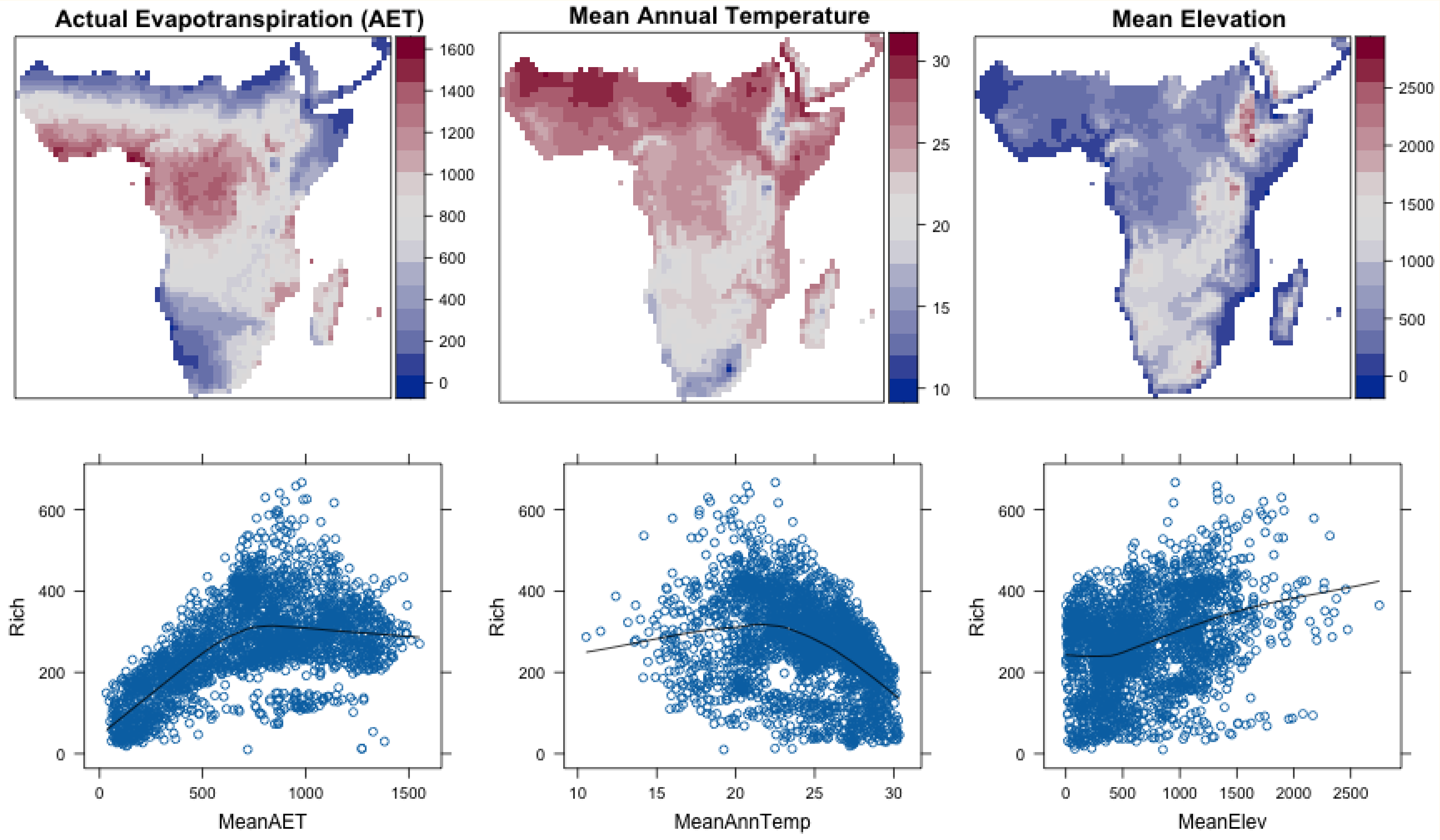
Overview

- Example data: Afrotropical bird diversity
- Naive models
- Describing spatial autocorrelation
- Accounting for spatial autocorrelation

Afrotropical bird species richness



Explanatory variables



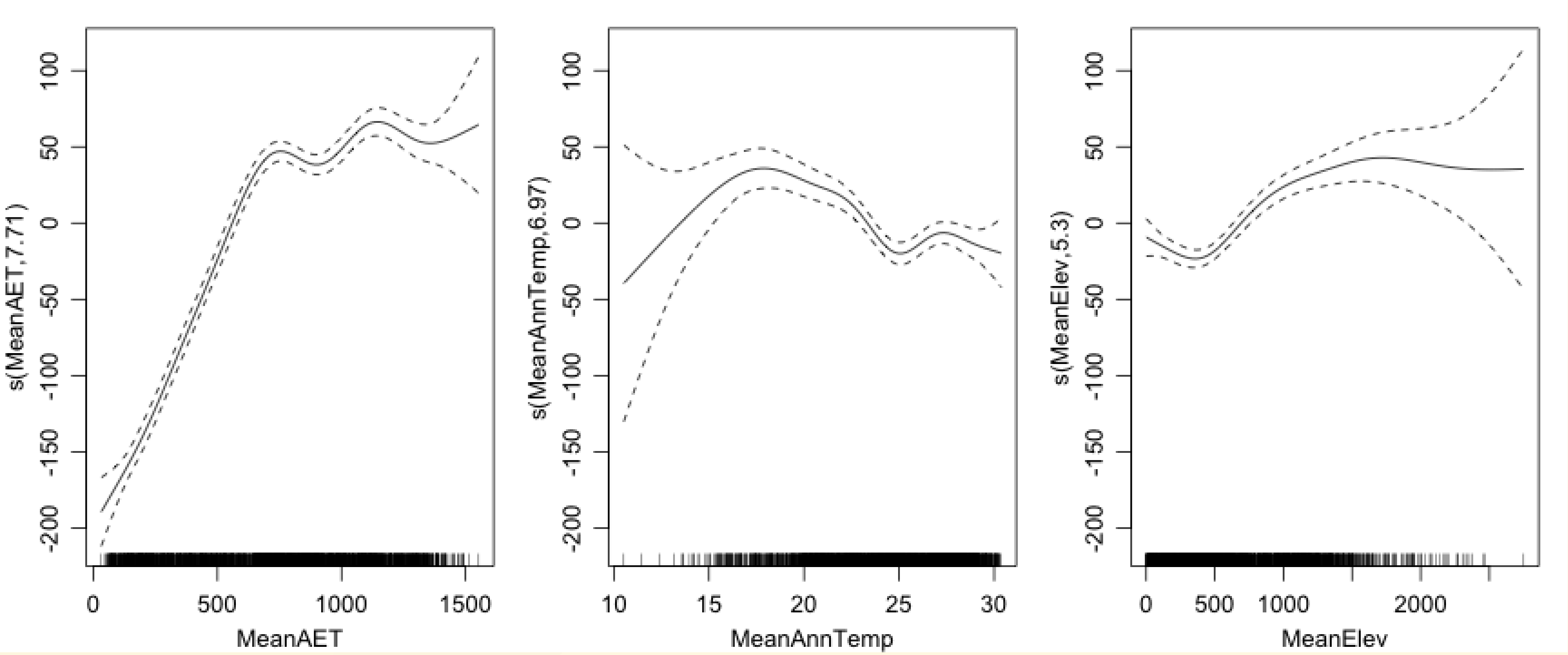
A simple linear model

Richness ~ AET + Temperature + Elevation

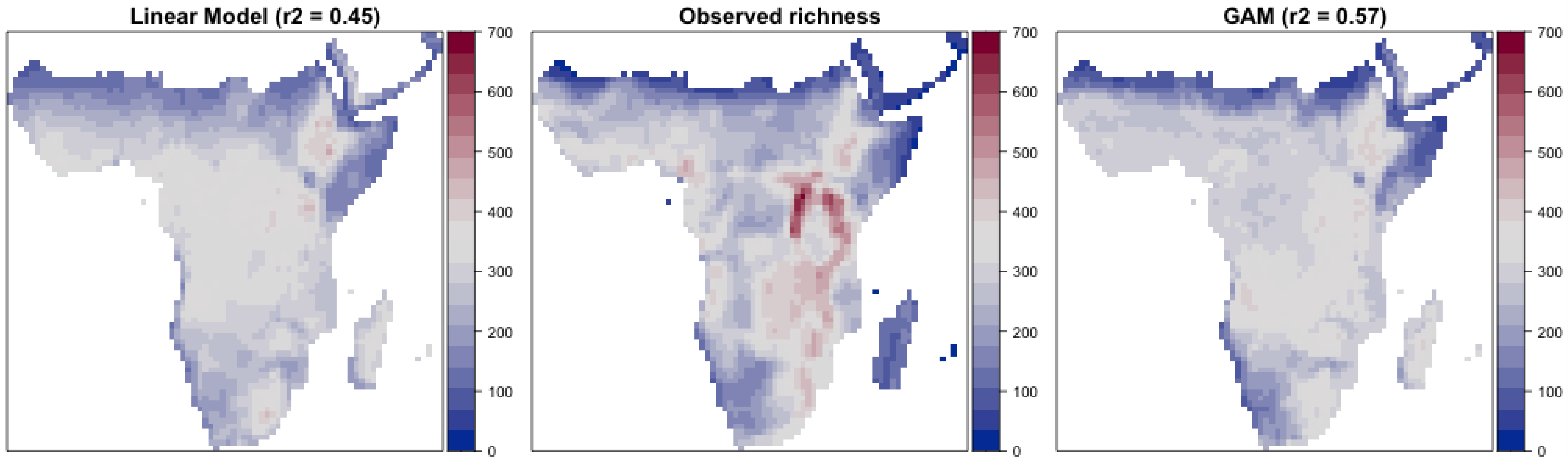
	Est	SE	t	p
(Intercept)	189.45	21.33	8.88	< 0.001
MeanAET	0.18	0.00	37.34	< 0.001
MeanAnnTemp	-4.18	0.72	-5.79	< 0.001
MeanElev	0.08	0.01	13.85	< 0.001

A simple GAM

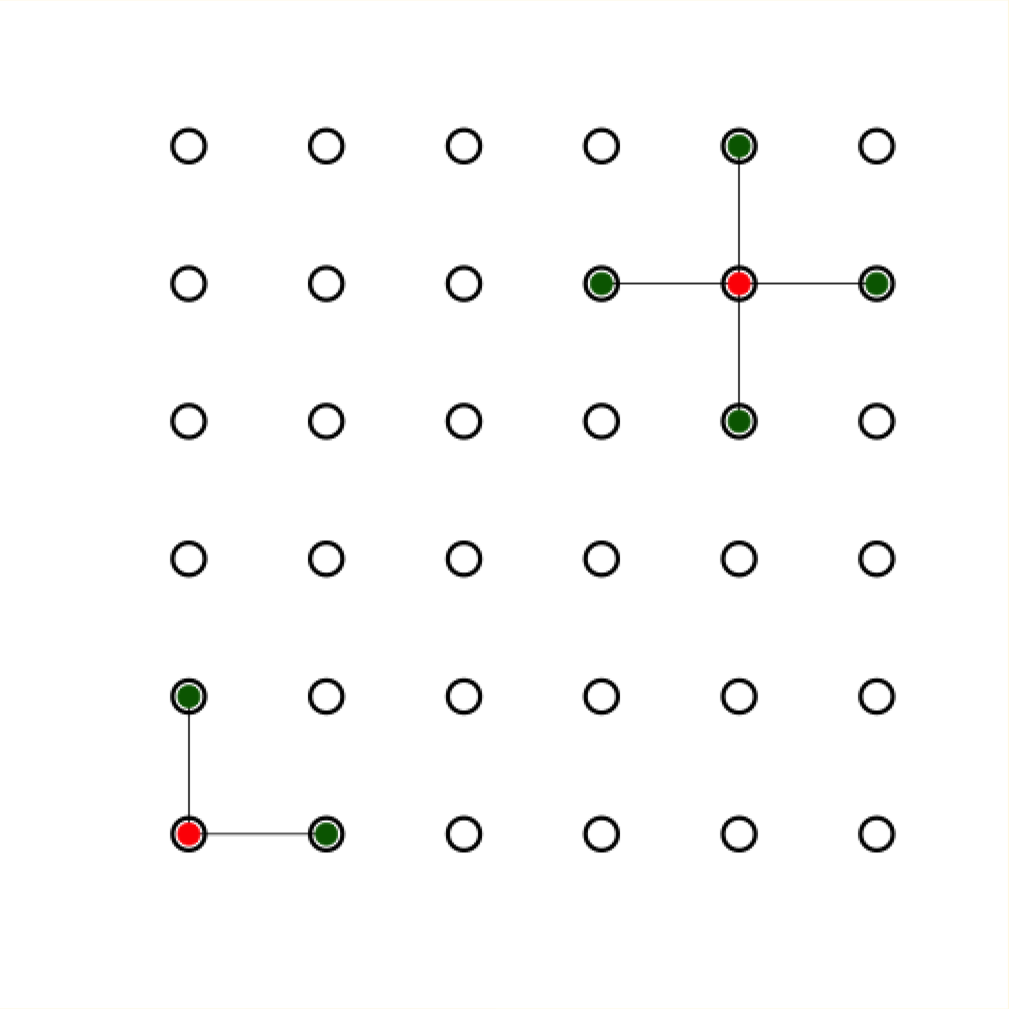
Richness ~ s(AET) +s(Temperature) + s(Elevation)



Model predictions



Neighbourhoods

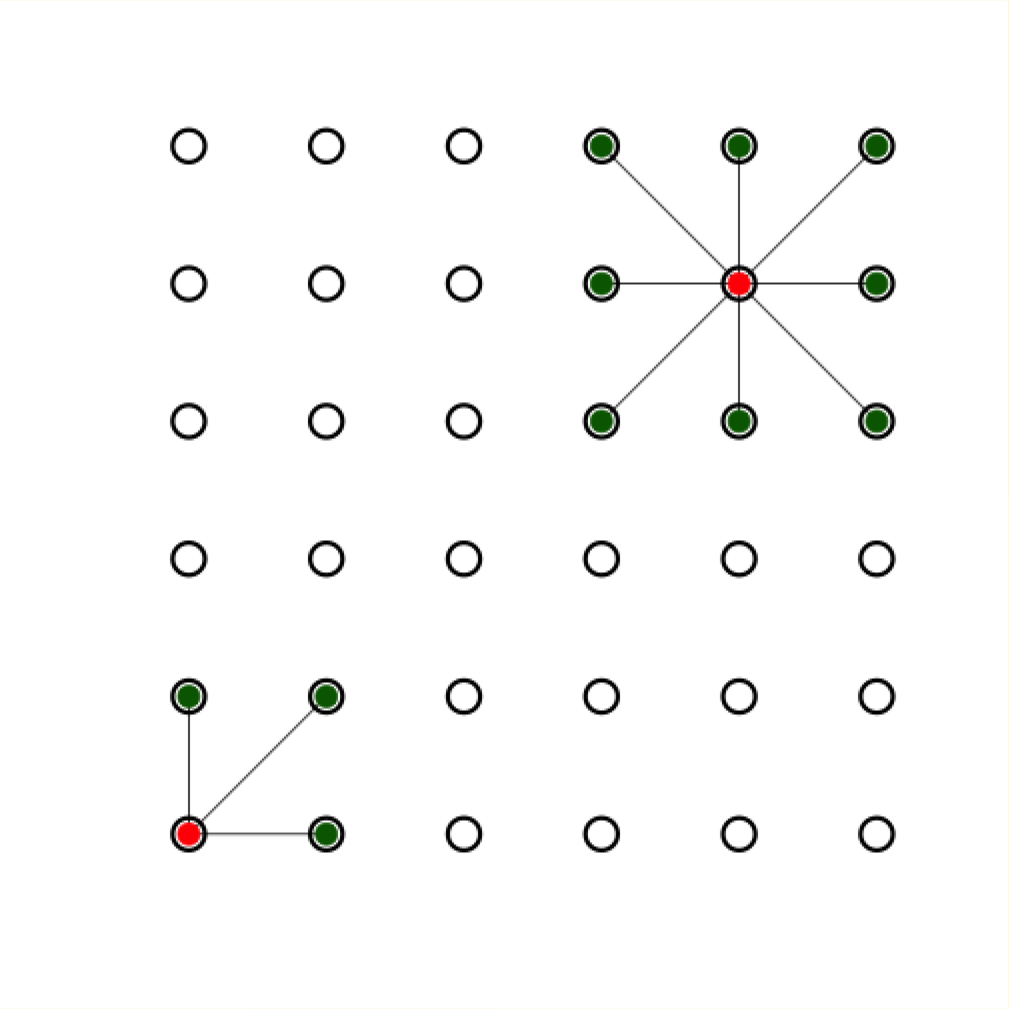


Rooks move

All cells within one step:

- vertically or
- horizontally

Neighbourhoods

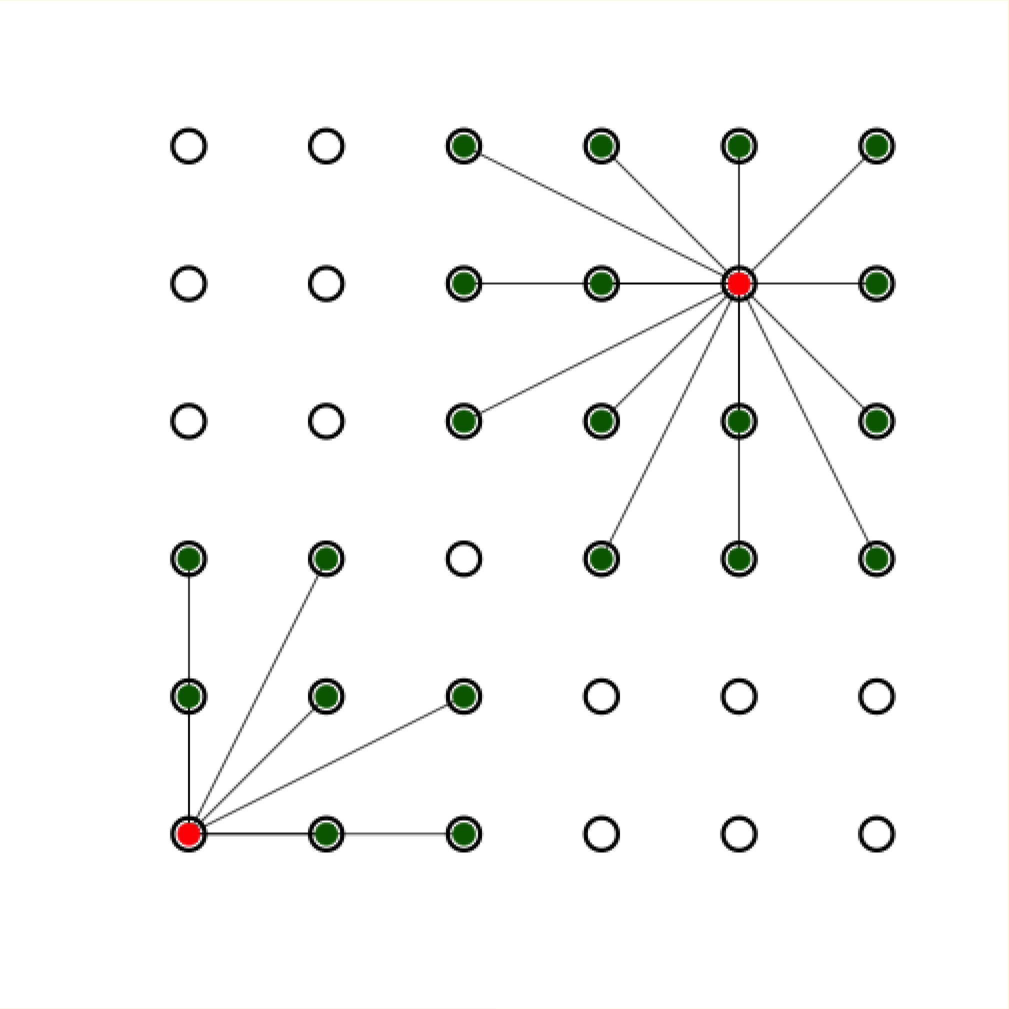


Queens move

All cells within one step:

- vertically,
- horizontally or
- diagonally

Neighbourhoods

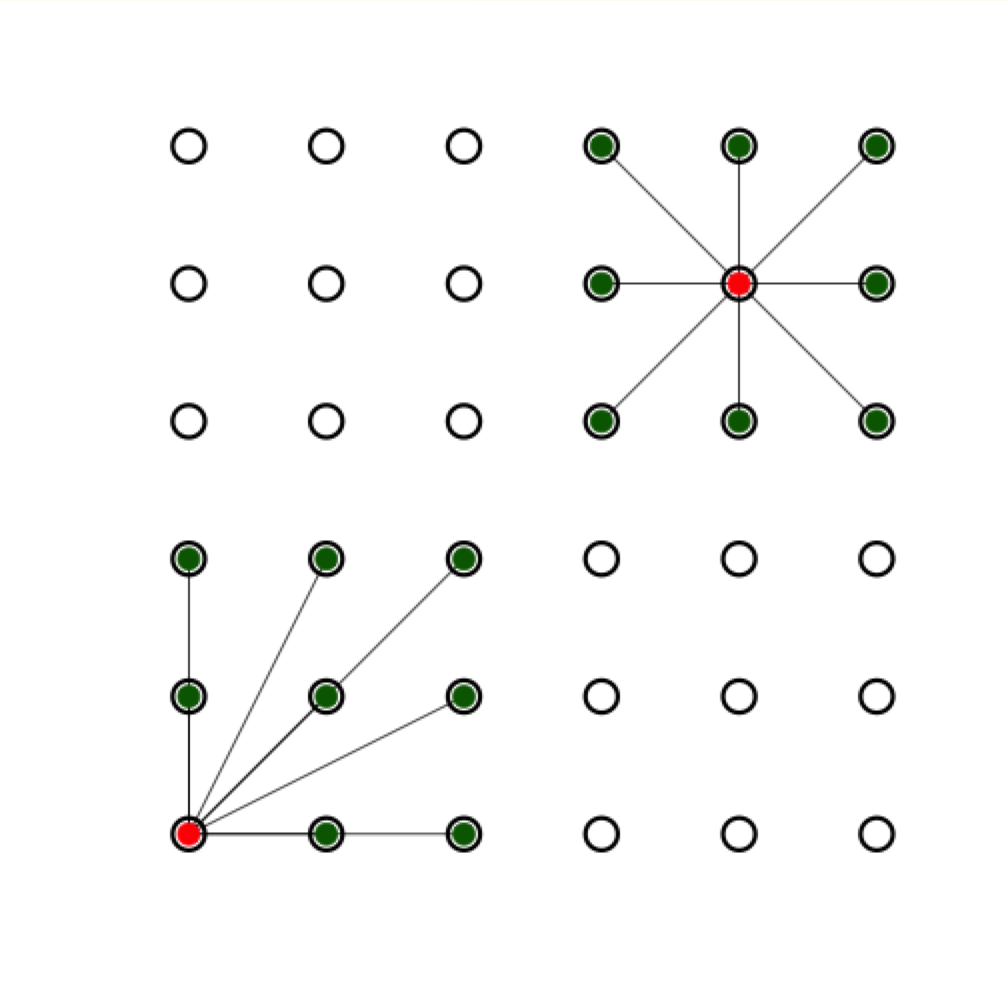


Distance based

All cells within:

- 2.4 units

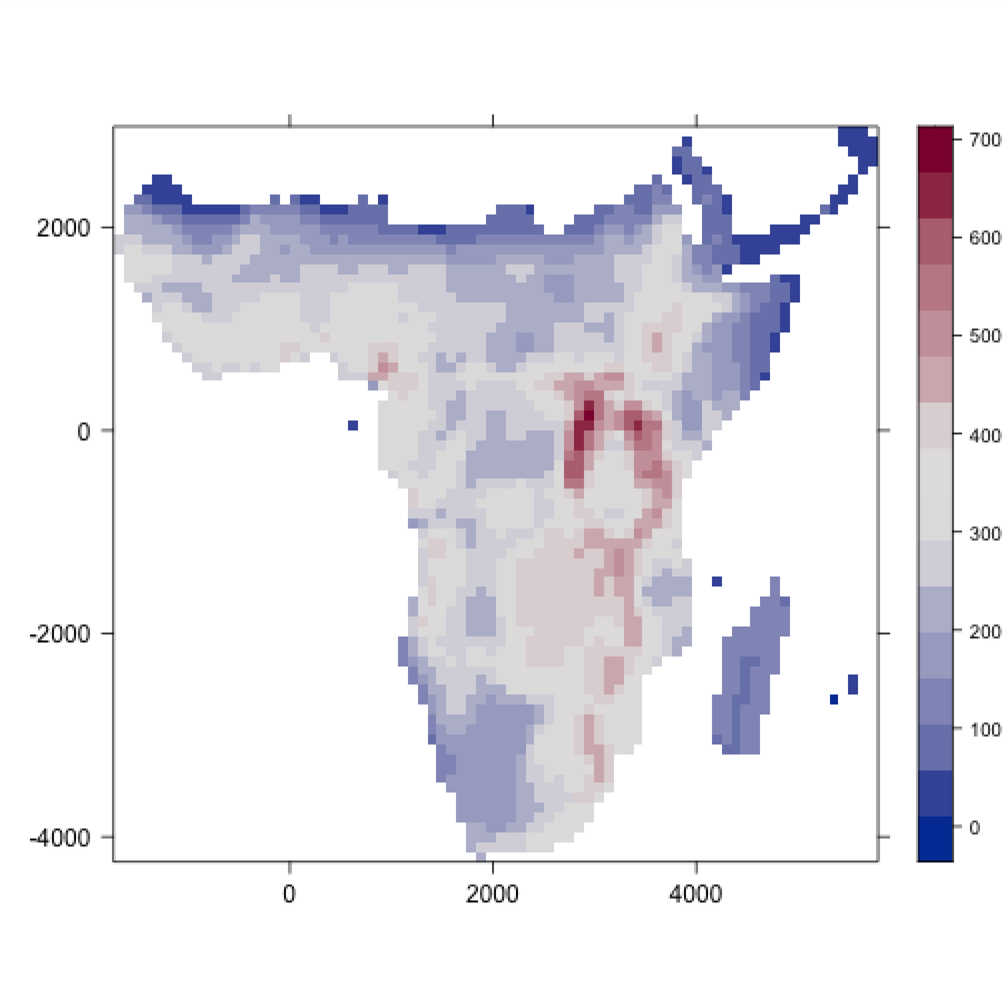
Neighbourhoods



***k* nearest**

The closest *k* cells

Spatial autocorrelation



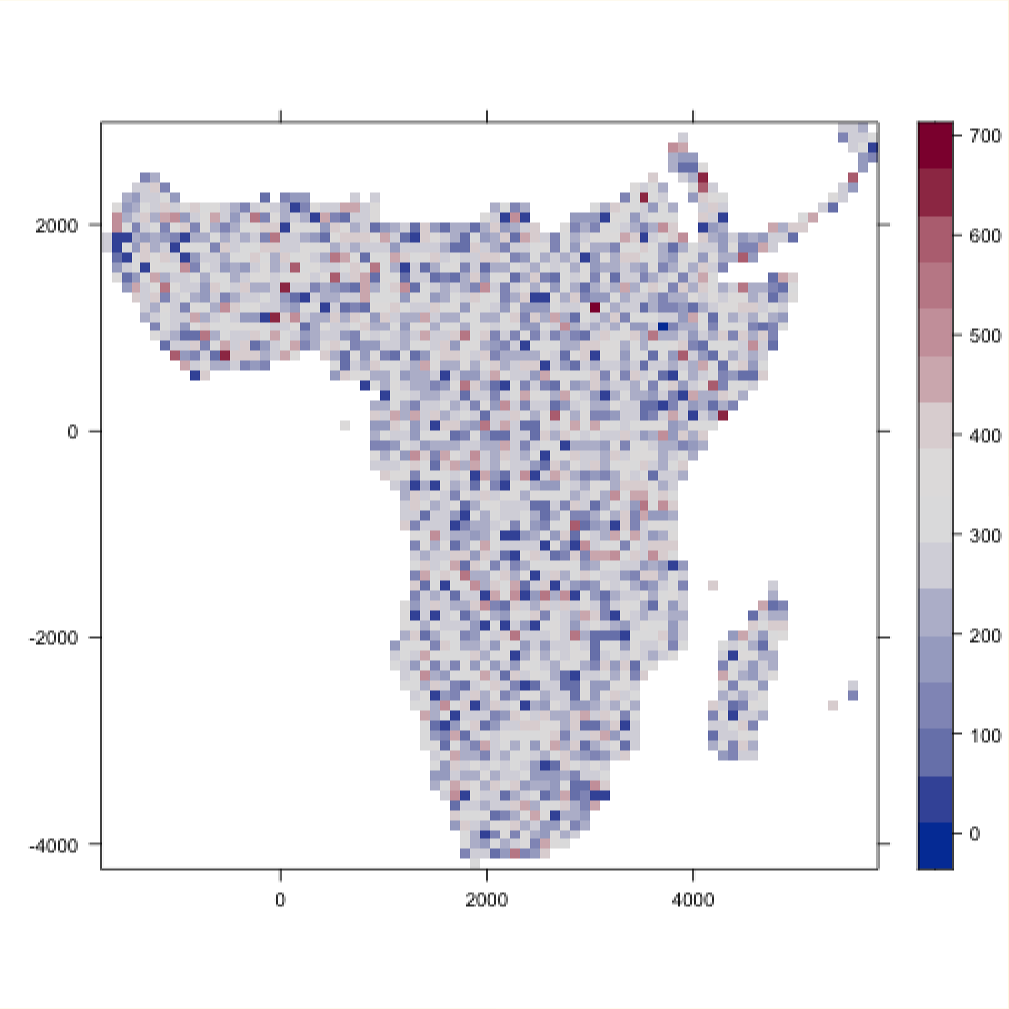
Global Moran's I

- I = 0.922
- p << 0.001

Global Geary's C

- C = 0.070
- p << 0.001

Spatial autocorrelation



Global Moran's I

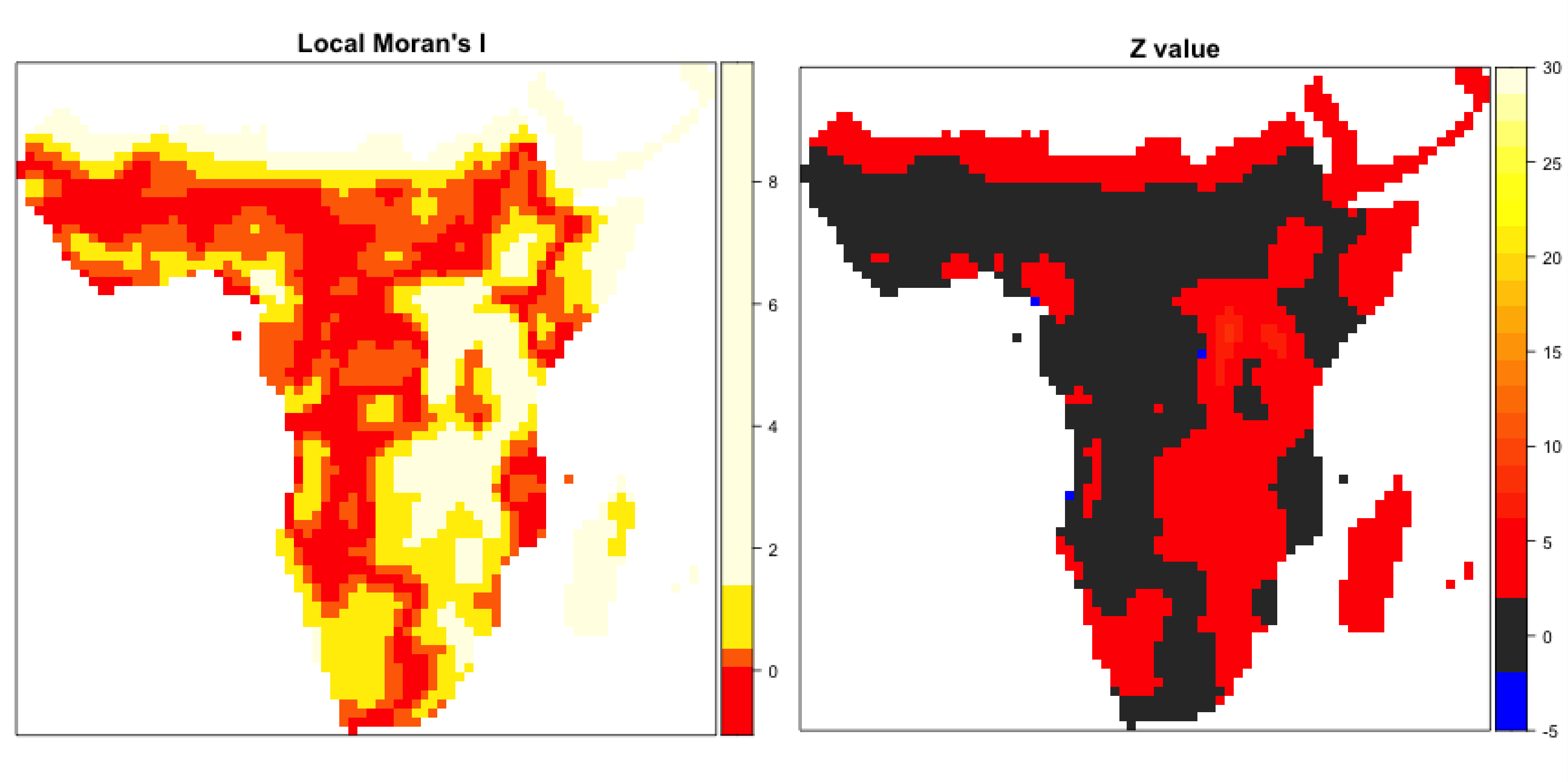
- I = -0.003
- p = 0.607

Global Geary's C

- C = 1.002
- p = 0.577

Local autocorrelation

Local indicators of spatial autocorrelation (LISA)



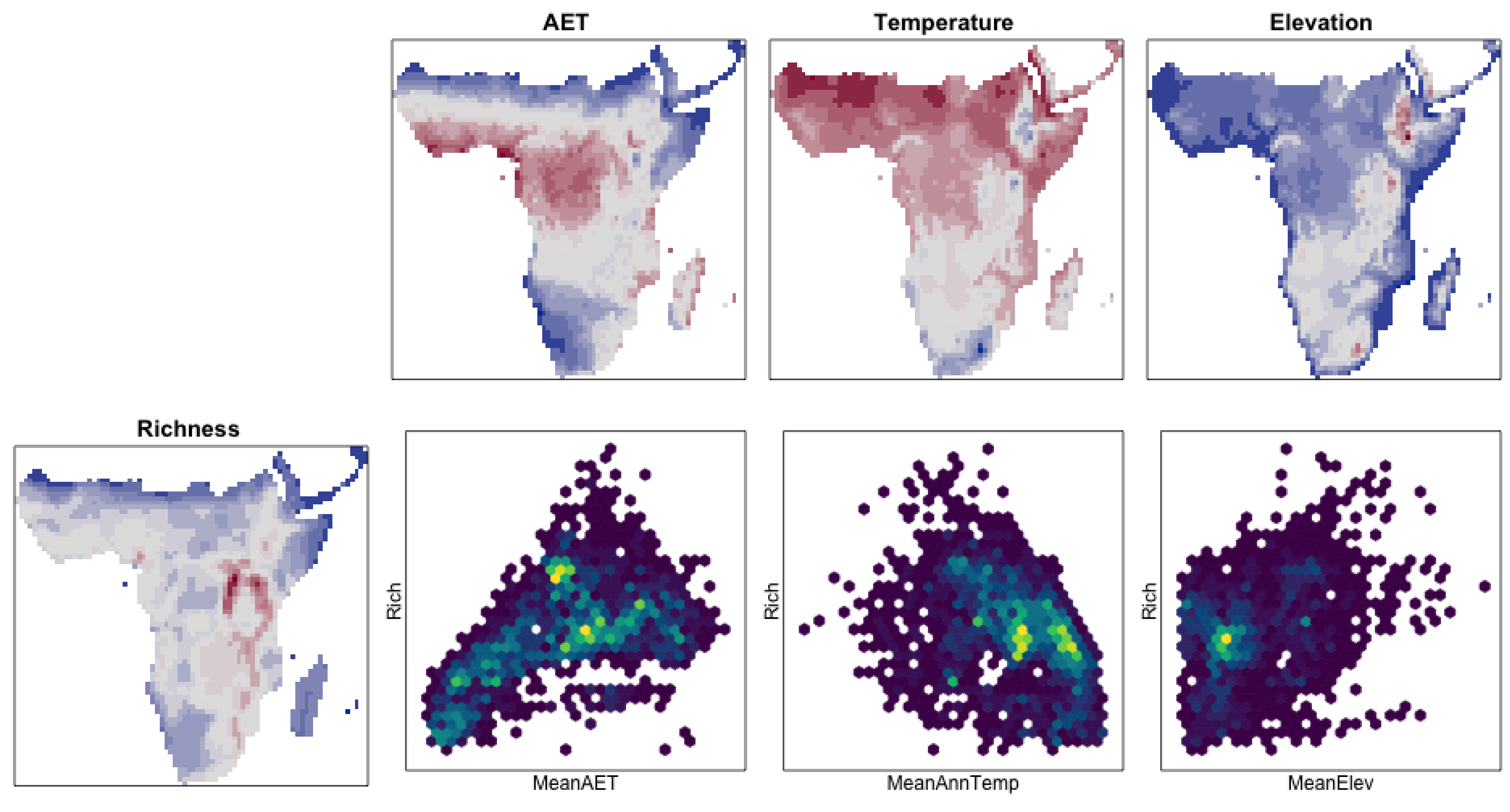
Effects of Spatial Autocorrelation

- Data points **not independent**
- Degrees of freedom reduced:
  - **standard errors and significance testing affected**
- Not equally weighted :
  - **parameter estimation affected**

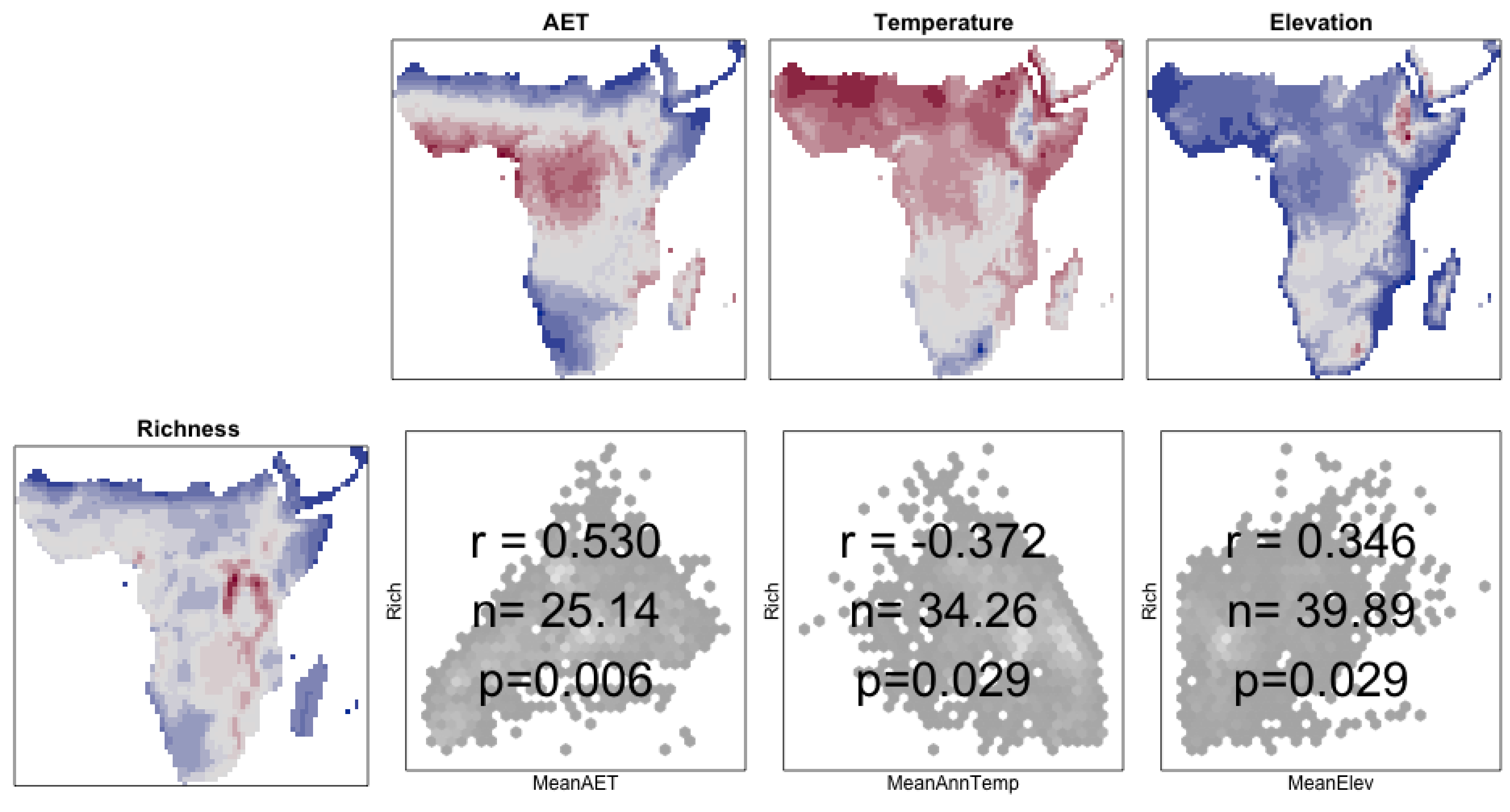
Dealing with Spatial Autocorrelation

- Modify the degrees of freedom in significance testing
- Account for autocorrelation in models:
  - Simultaneous autoregressive models
  - Generalised least squares
  - Eigenvector filtering
  - Geographically weighted regression

Degrees of freedom correction



Degrees of freedom correction

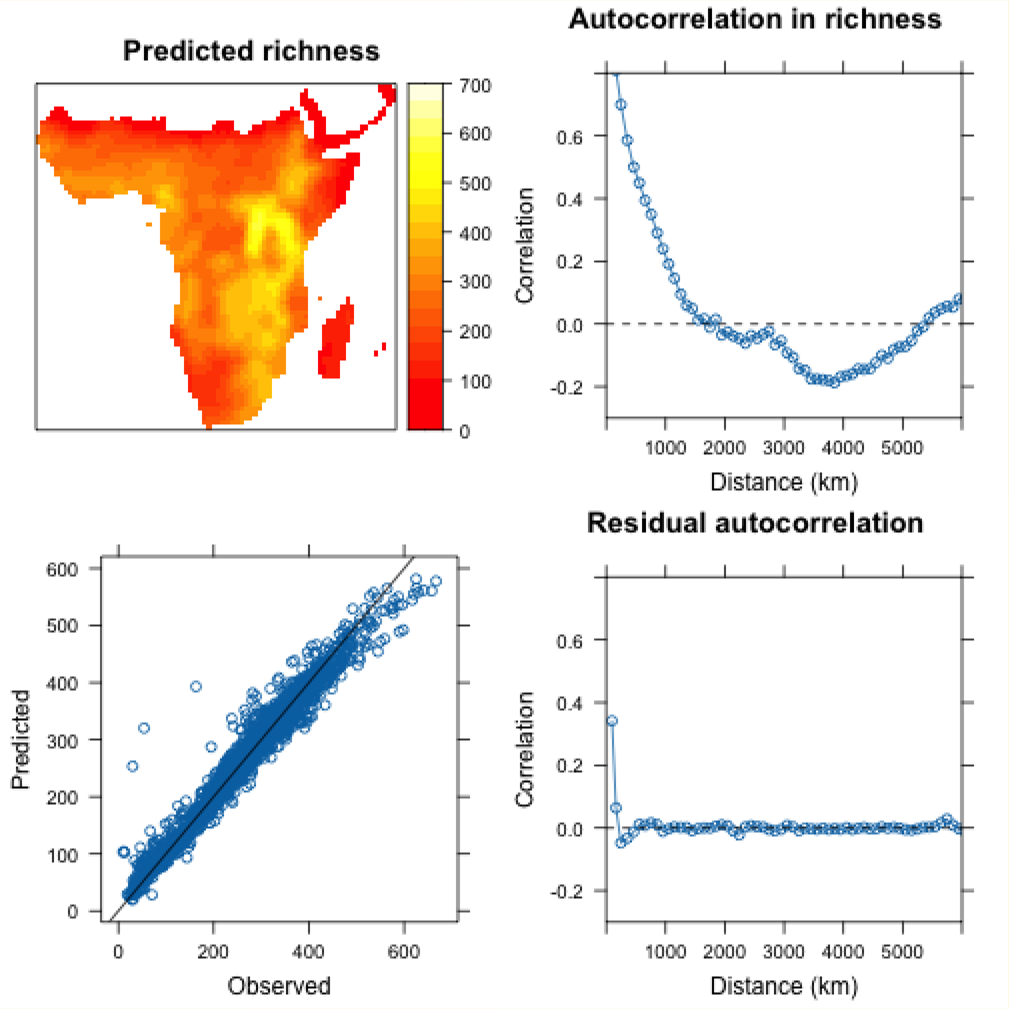


Spatial Autoregression

●	●	●	●	
●	●	●	●	$bx_1 + \frac{1}{2}bx_2$
●	●	●	●	$\frac{1}{2}bx_1 + bx_2 + \frac{1}{2}bx_3$
●	●	●	●	$\frac{1}{2}bx_2 + bx_3 + \frac{1}{2}bx_4$
●	●	●	●	$\frac{1}{2}bx_3 + \frac{1}{2}bx_4$
x_1	x_1	x_3	x_4	

The best fit for coefficient b is influenced by the neighbour and weightings schemes.

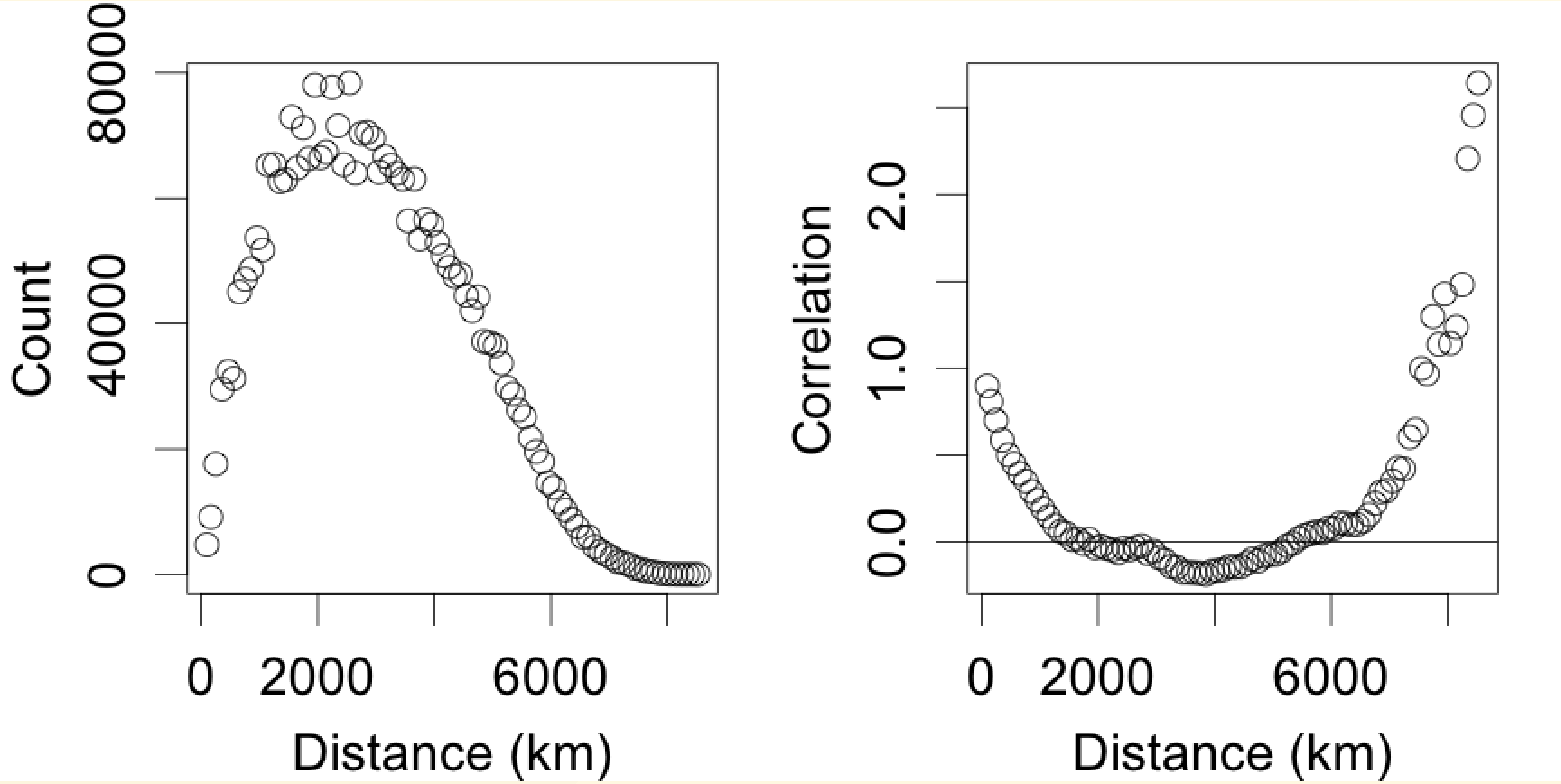
Spatial Autoregresssion



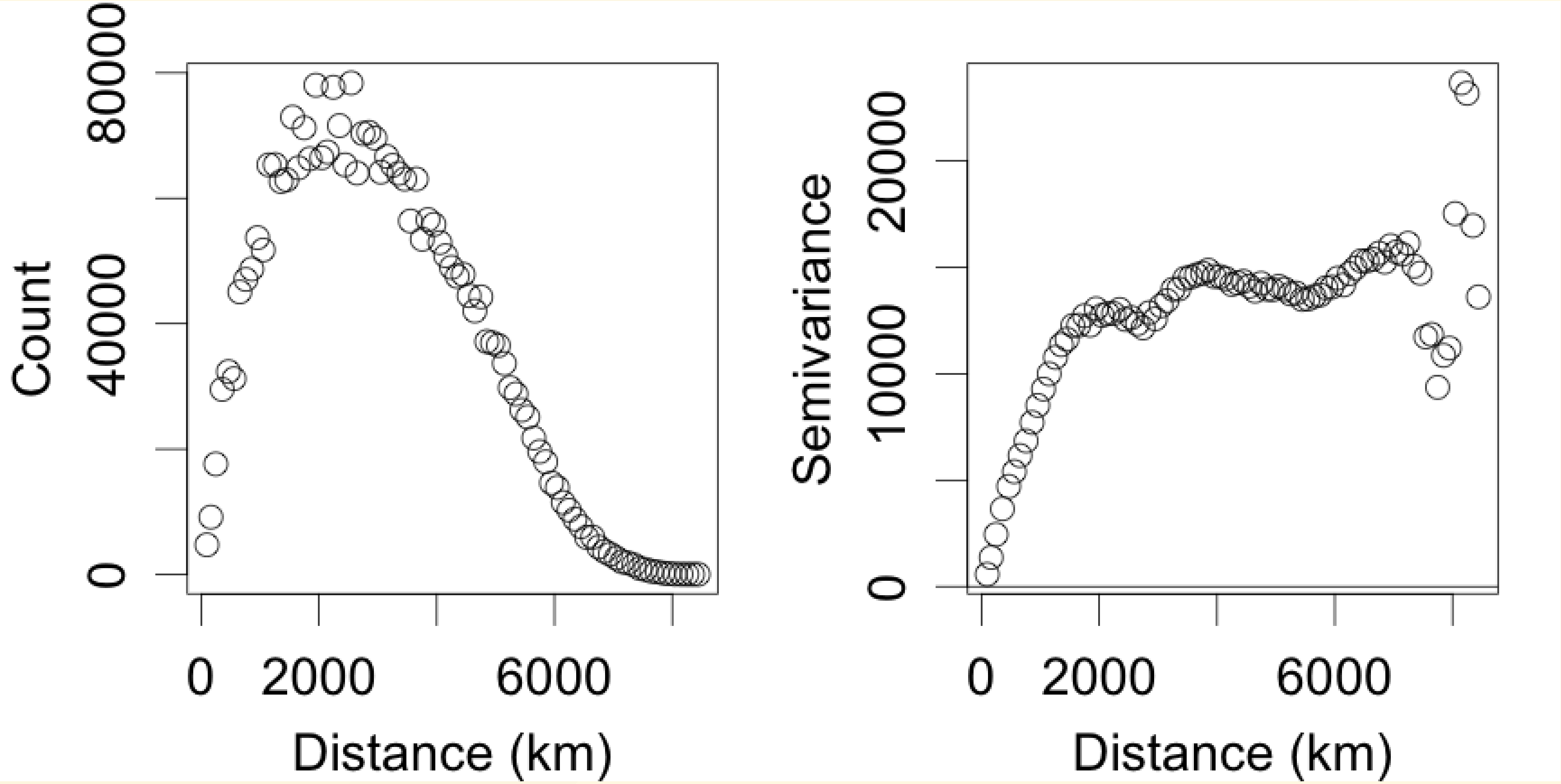
Spatial Autoregresssion

```
Call:lagsarlm(formula = Rich ~ MeanAETScaled + MeanAnnTempScaled +
    MeanElevScaled, data = figDat@data, listw = figDat.lw, type = "lag")

Residuals:
        Min          1Q      Median          3Q         Max
-266.301618   -9.585387   -0.050679    9.997008  105.697309

Type: lag
Coefficients: (asymptotic standard errors)
                  Estimate Std. Error z value  Pr(>|z|)
(Intercept)        3.03299    0.70394  4.3086 1.643e-05
MeanAETScaled      2.84545    0.48197  5.9037 3.553e-09
MeanAnnTempScaled  1.34439    0.65707  2.0460   0.04075
MeanElevScaled     5.77947    0.69995  8.2570 2.220e-16

Rho: 0.98408, LR test value: 6043.7, p-value: < 2.22e-16
Asymptotic standard error: 0.0025867
    z-value: 380.44, p-value: < 2.22e-16
Wald statistic: 144730, p-value: < 2.22e-16

Log likelihood: -11459.21 for lag model
ML residual variance (sigma squared): 462.9, (sigma: 21.515)
Number of observations: 2484
Number of parameters estimated: 6
AIC: 22930, (AIC for lm: 28972)
LM test for residual autocorrelation
test value: 497.74, p-value: < 2.22e-16
```

Correlogram

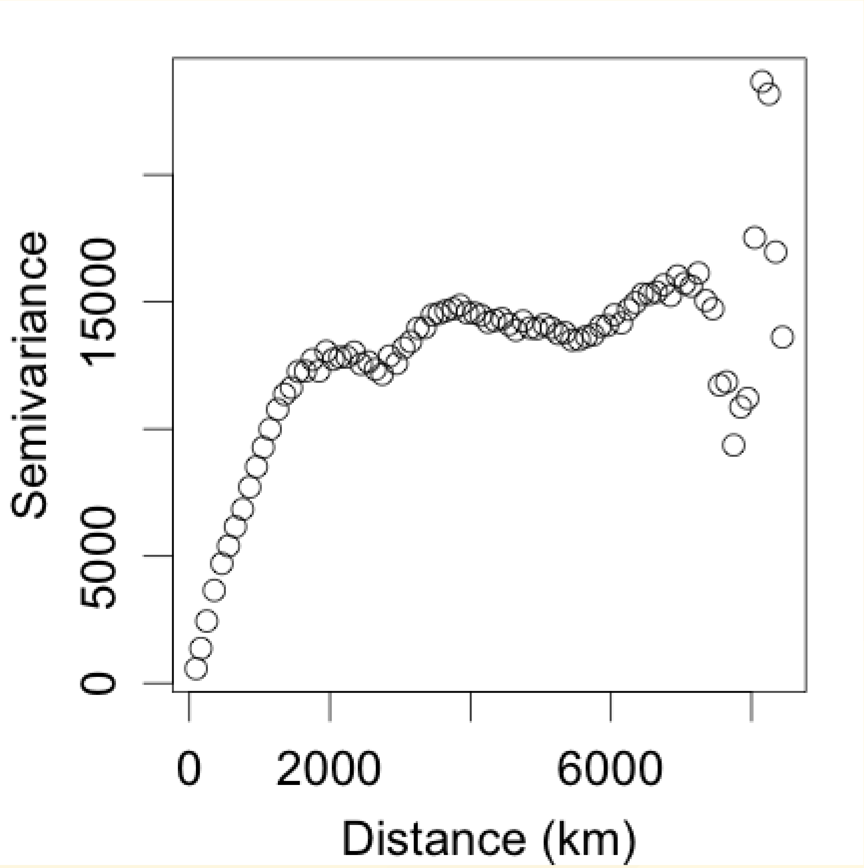


Variogram



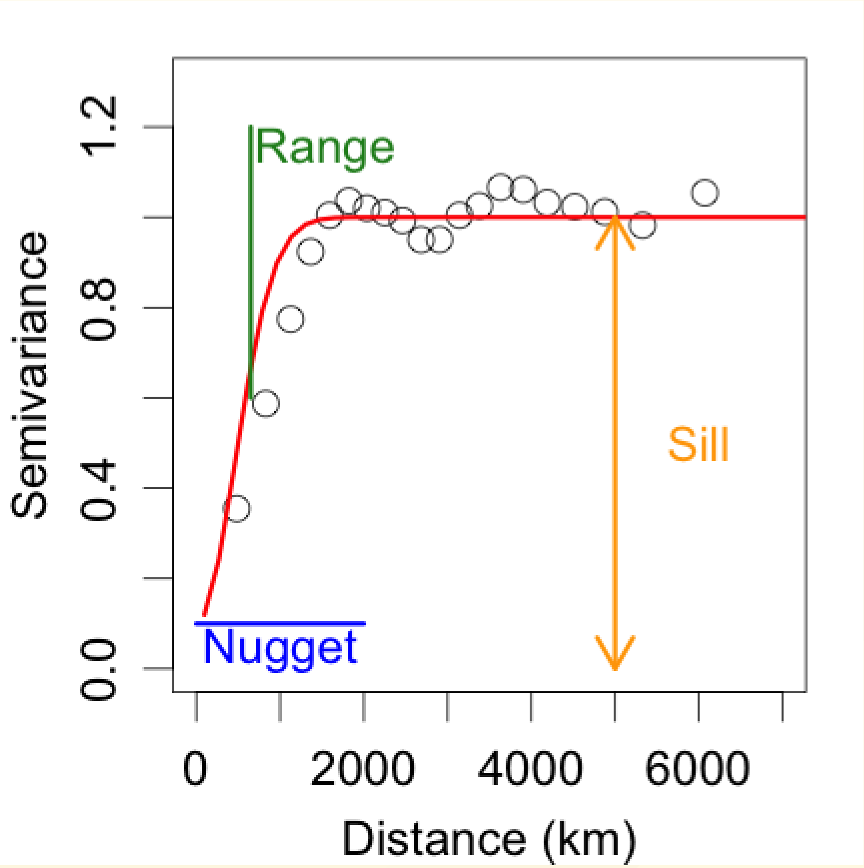
Count
0
40000
80000
0
2000
6000
Distance (km)
Semivariance
0
10000
20000
0
2000
6000
Distance (km)

Generalised Least Squares



- Model variance as a function of **distance**
- Generate a covariance **matrix**

Generalised Least Squares



- Different shapes:
  - Exponential
  - Spherical
  - Linear
- Parameters

Generalised Least Squares

```
Generalized least squares fit by REML
  Model: Rich ~ MeanAETScaled + MeanAnnTempScaled + MeanElevScaled
  Data: figDat
       AIC      BIC    logLik
  24676.89 24705.97 -12333.44

Correlation Structure: Gaussian spatial correlation
 Formula: ~e_centre_behr + n_centre_behr
 Parameter estimate(s):
 range nugget
 650.0    0.1

Coefficients:
                      Value Std.Error   t-value p-value
(Intercept)       199.67323 16.755430 11.916927  0.0000
MeanAETScaled      17.65050  3.038337  5.809265  0.0000
MeanAnnTempScaled -27.53775  5.663121 -4.862645  0.0000
MeanElevScaled      3.59893  4.321932  0.832712  0.4051

 Correlation:
                  (Intr) MnAETS MnAnTS
MeanAETScaled     0.048
MeanAnnTempScaled 0.141  0.047
MeanElevScaled    0.156  0.079  0.936

Standardized residuals:
        Min          Q1         Med          Q3         Max
-2.35080108  0.09376345  0.73813223  1.30659392  4.48588628

Residual standard error: 97.85917
Degrees of freedom: 2484 total; 2480 residual
```

Stationarity and isotropy

Is the same process happening in:

- different locations (stationarity)?
- different directions (isotropy)?

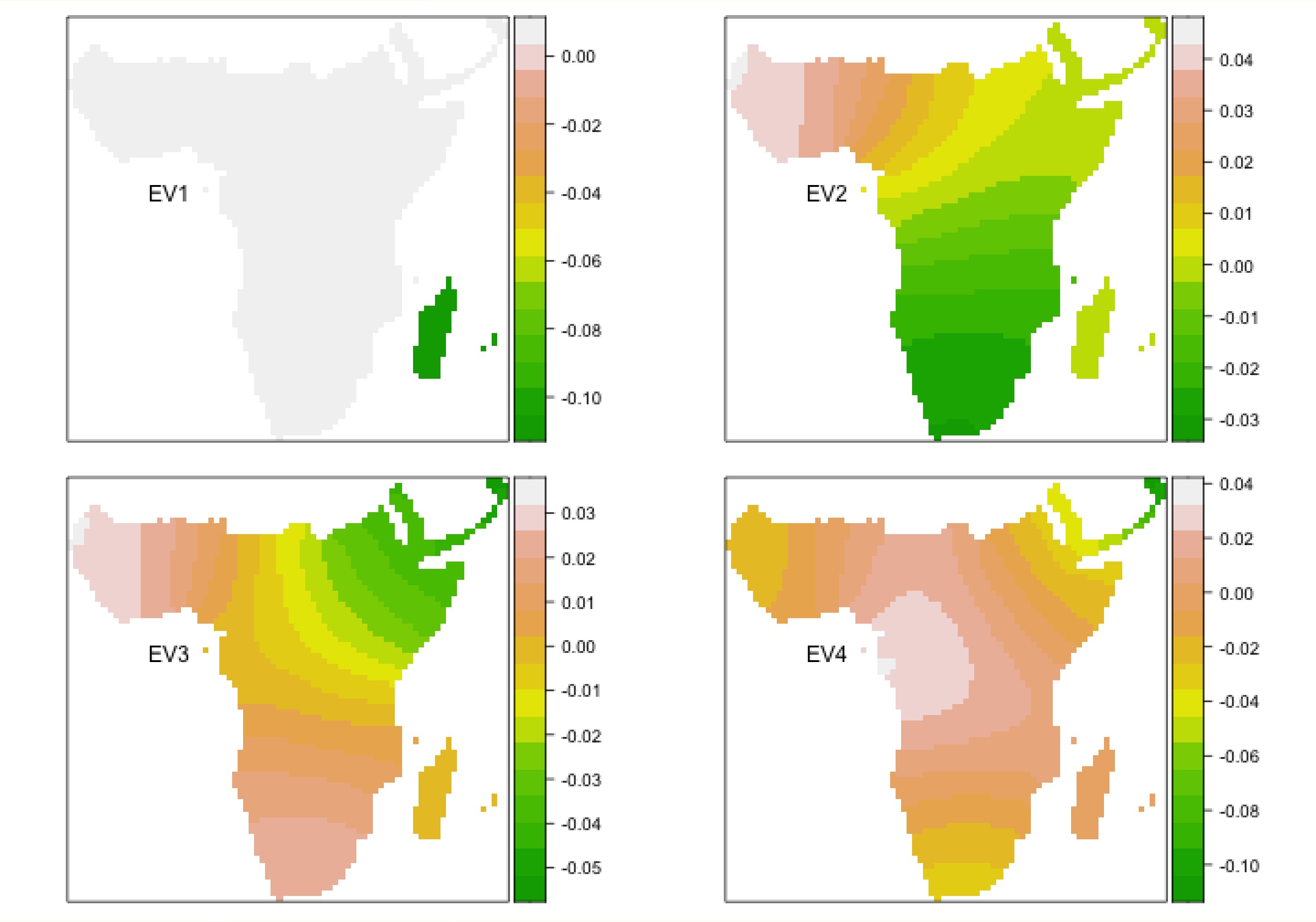
Is the problem in:

- the spatial structure of autocorrelation?
- differences in the actual relationship?

Eigenvector filtering

- Take the **eigendecomposition** of a spatial weights model
- Use the **eigenvectors** as variables in the model
- Use a selection process to identify and include only important eigenvectors

Eigenvector filtering



- First four eigenvector filters
- Independent components of spatial patterning

Eigenvector filtering

```
lm(Rich ~ MeanAET + MeanAnnTemp + MeanElev
```

	Est	SE	t	p
(Intercept)	189.45	21.33	8.88	< 0.001
MeanAET	0.18	0.00	37.34	< 0.001
MeanAnnTemp	-4.18	0.72	-5.79	< 0.001
MeanElev	0.08	0.01	13.85	< 0.001

Eigenvector filtering

```
lm(Rich ~ ... + Re(spEV1) + Re(spEV2) + Re(spEV3) + Re(spEV4)
```

	Est	SE	t	p
(Intercept)	79.95	33.00	2.42	1.5e-02
MeanAET	0.18	0.01	31.45	< 0.001
MeanAnnTemp	0.11	1.14	0.09	9.3e-01
MeanElev	0.08	0.01	12.71	< 0.001
Re(spEV1)	1617.43	77.65	20.83	< 0.001
Re(spEV2)	-964.81	129.21	-7.47	< 0.001
Re(spEV3)	813.41	95.80	8.49	< 0.001
Re(spEV4)	147.25	100.29	1.47	1.4e-01

Eigenvector filtering

```
lm(Rich ~ ... + MeanElev + Re(spEV1) + Re(spEV2) + Re(spEV3)
```

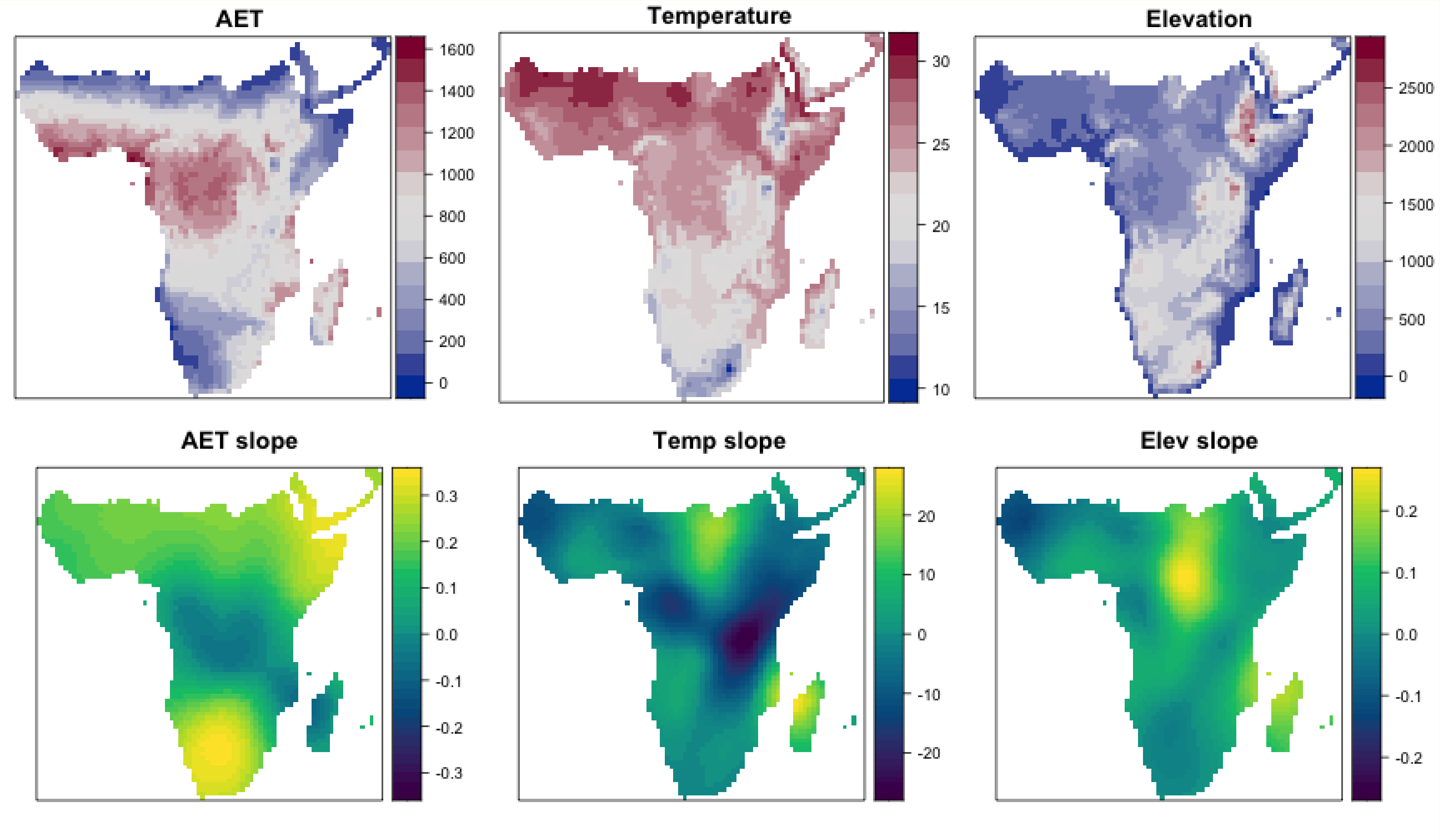
	Est	SE	t	p
(Intercept)	58.55	29.62	1.98	4.8e-02
MeanAET	0.19	0.00	43.67	< 0.001
MeanAnnTemp	0.74	1.06	0.70	4.8e-01
MeanElev	0.08	0.01	13.78	< 0.001
Re(spEV1)	1610.70	77.53	20.78	< 0.001
Re(spEV2)	-1031.06	121.10	-8.51	< 0.001
Re(spEV3)	847.10	93.03	9.11	< 0.001

Geographically weighted regression

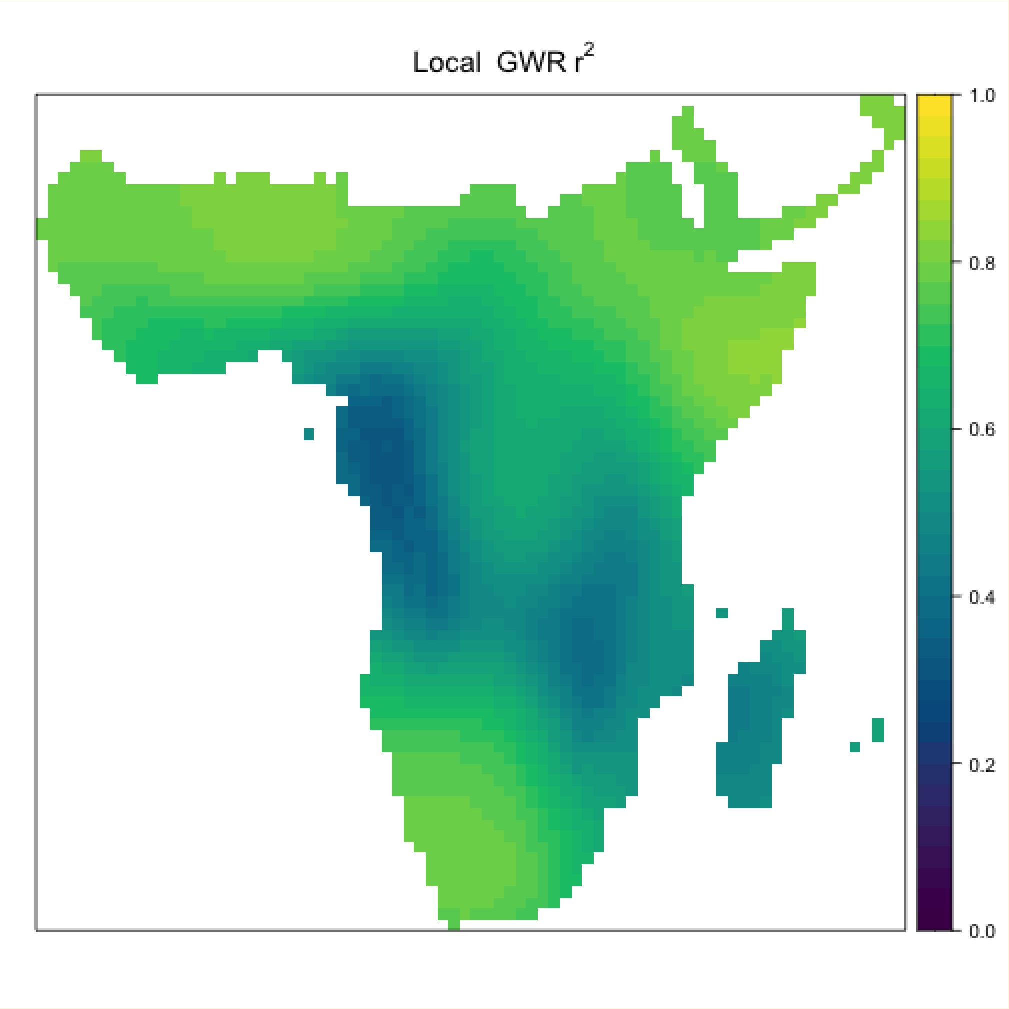
Fit a model for **every cell**:

- Define a local **region size** and a **weighting function**
- Fit a weighted regression for each cell using the weights
- Look at how coefficients **vary in space**
- Possibly serious statistical issues!

Geographically weighted regression



Geographically weighted regression



- Local r^2 values
- Variation in the explanatory power of the model

Problems

- Profusion of packages: sf, sp, spdep, mgcv, ncf, gstat, nlme, spgwr
- Different data structures
- Sometimes poor documentation
- Speed of calculation (= size of dataset)
- Memory hungry
- Too many options