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Abstract

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1 Introduction

2 Cardy-Verlinde Formula

En los sistemas termodinámicos usuales, la entrpía y la energía S, E son cantidades extensivas. Para un sistema termodinámico en un volumen finito V, la energía es funci´on de $E = E(S, V)$. Se dice que la energía E es extensiva si satisface

$$E(\lambda S, \lambda V) = \lambda E(S, V) \tag{1}$$

derivando respecto a λ

$$\frac{\partial E}{\partial \lambda S}\frac{\partial(\lambda S)}{\partial \lambda} + \frac{\partial E}{\partial \lambda V S}\frac{\partial(\lambda V)}{\partial \lambda} = E \tag{2}$$

el factor λ es un factor multiplicativo, fijando $\lambda = 1$, se obtiene

$$E = V\left(\frac{\partial E}{\partial V}\right)_S + S\left(\frac{\partial E}{\partial S}\right)_V \tag{3}$$

considerando la primera ley $dE = TdS - PdV$, se obtiene la relación de Euler

$$E = TS - PV \tag{4}$$

Para una CFT con una carga central grande la entropía y energía no son puramente extensivas. En un volumen finito la energía E de una CFT contiene una contribución de Casimir no extensiva. Entonces la energía se escribe como

$$E(S, V) = E_E(S, V) + \frac{1}{2}E_C(S, V) \tag{5}$$

donde E_E es la parte extensiva y E_C es la parte no extensiva de la energía, donde

$$E_E(\lambda S, \lambda V) = \lambda E_E(S, V) \tag{6}$$

$$E_C(\lambda S, \lambda V) = \lambda^{1-2/n} E_C(S, V) \tag{7}$$

donde n es la dimensión de la sección transversal, para un espacio tiempo de 4 dimensiones $n = 2$

$$E_C(\lambda S, \lambda V) = E_C(S, V) \tag{8}$$

La relación de Euler es

$$E(\lambda S, \lambda V) = E_E(\lambda S, \lambda V) + \frac{1}{2}E_C(\lambda S, \lambda V) = \lambda E_E(S, V) + \frac{1}{2}E_C(S, V) \tag{9}$$

derivando respecto a λ

$$\frac{\partial E}{\partial \lambda S}\frac{\partial(\lambda S)}{\partial \lambda} + \frac{\partial E}{\partial \lambda V S}\frac{\partial(\lambda V)}{\partial \lambda} = E_E \tag{10}$$

Fijando $\lambda = 1$, y considerando la primera ley se muestra que

$$E_C = 2(E + PV - TS) \tag{11}$$

La invarianza conforme implica que ER es independiente del volumen V, donde R es el radio del volumen donde la CFT se define. Entonces Verlinde plantea las siguientes expresiones generales

$$E_E = \frac{a}{4\pi R} S^{3/2}, \qquad E_C = \frac{b}{2\pi R} S^{1/2} \tag{12}$$

Verlinde destaca que a y b son coeficientes postivos arbitrarios, independientes de R y S. Con las expresiones (12) se puede mostrar que la entropía S puede escribirse como

$$S = \frac{2\pi R}{\sqrt{ab}} \sqrt{E_C(2E - E_C)} \tag{13}$$

2.1 Reissner-Nordstrom Black Hole

We will begin with an instructive example of Reissner-Nordstrom. Considering the action for Einstein-Maxwell-Anti-de Sitter, with $\Lambda = -3/l^2$

$$I = \frac{1}{16\pi} \int_{\mathcal{M}} d^4x \sqrt{-g} \left(R - F^2 + \frac{6}{l^2} \right) \tag{14}$$

The equation of motion for metric and gauge field are

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \left(R + \frac{6}{l^2} \right) = T_{\mu\nu} \tag{15}$$

$$T_{\mu\nu} = 2F^{\lambda}_{\mu} F_{\lambda\nu} - \frac{1}{2} g_{\mu\nu} F_{\sigma\gamma} F^{\sigma\gamma} \tag{16}$$

The metric

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega_2^2 \tag{17}$$

and the solution is

$$f(r) = 1 - \frac{\mu}{r} + \frac{Q^2}{r^2} + \frac{r^2}{l^2} \tag{18}$$

The equations for the gauge field are solved by

$$A = -\frac{Q}{r} dt, \qquad F = \frac{Q}{r^2} dr \wedge dt \tag{19}$$

the thermodynamic quantities which satisfied the first law $dE = TdS + \Phi dQ$ are

$$E = \frac{1}{2} \left(r_+ + \frac{r_+^3}{l^2} + \frac{Q^2}{r_+} \right), \quad \Phi = \frac{Q}{r_+}, \quad S = \pi r_+^2, \quad T = \frac{1}{4\pi} \left(\frac{3r_+}{l^2} + \frac{1}{r_+} - \frac{Q}{r_+^3} \right) \tag{20}$$

The conformal metric where the CFT_3 is defined, where the time-coordinate is re escalated like $t = (l/R)\tau$

$$ds^2 = -d\tau^2 + R^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{21}$$

The new thermodynamics tems are

$$E = \frac{l}{2R} \left(r_+ + \frac{r_+^3}{l^2} + \frac{Q^2}{r_+} \right), \quad \Phi = \frac{Ql}{Rr_+}, \quad S = \pi r_+^2, \quad T = \frac{l}{4R\pi} \left(\frac{3r_+}{l^2} + \frac{1}{r_+} - \frac{Q}{r_+^3} \right) \tag{22}$$

This new thermodynamics joint with the volume $V = 4\pi R$ satisfied the first law $dE = TdS - \mathcal{P}dV + \Phi dQ$. The relation of pressure with volume can be obtained from the first law or from stress tensor

$$\mathcal{P} = -\left(\frac{\partial E}{\partial V} \right)_{Q,S} = -\frac{\partial E}{\partial R} \frac{\partial R}{\partial V} = \frac{E}{2V}, \qquad \mathcal{P} = \frac{E}{2V} \tag{23}$$

2.2 CV formula with electric charge

En los sistemas termodinámicos usuales, la entrpía y la energía S, E son cantidades extensivas. Para un sistema termodinámico en un volumen finito V, la energía es funcíon de $E = E(S, V, Q)$. Se dice que la energía E es extensiva si satisface

$$E(\lambda S, \lambda V, \lambda Q) = \lambda E(S, V, Q) \tag{24}$$

derivando respecto a λ

$$\frac{\partial E}{\partial \lambda S}\frac{\partial(\lambda S)}{\partial \lambda} + \frac{\partial E}{\partial \lambda V}\frac{\partial(\lambda V)}{\partial \lambda} + \frac{\partial E}{\partial \lambda Q}\frac{\partial(\lambda Q)}{\partial \lambda} = E \tag{25}$$

el factor λ es un factor multiplicativo, fijando $\lambda = 1$, se obtiene

$$E = V\left(\frac{\partial E}{\partial V}\right)_{S,Q} + S\left(\frac{\partial E}{\partial S}\right)_{V,Q} + Q\left(\frac{\partial E}{\partial Q}\right)_{S,V} \tag{26}$$

considerando la primera ley $dE = TdS - PdV + \Phi Q$, se obtiene la relación de Euler

$$E = TS - PV + \Phi Q \tag{27}$$

Para una CFT con una carga central grande la entropía y energía no son puramente extensivas. En un volumen finito la energía E de una CFT contiene una contribución de Casimir no extensiva. Entonces la energía se escribe como

$$E(S, V, Q) = E_E(S, V, Q) + \frac{1}{2}E_C(S, V, Q) \tag{28}$$

donde E_E es la parte extensiva y E_C es la parte no extensiva de la energía, donde

$$E_E(\lambda S, \lambda V, \lambda Q) = \lambda E_E(S, V, Q) \tag{29}$$

$$E_C(\lambda S, \lambda V, \lambda Q) = \lambda^{1-2/n} E_C(S, V, Q) \tag{30}$$

donde n es la dimensión de la sección transversal, para un espacio tiempo de 4 dimensiones $n = 2$

$$E_C(\lambda S, \lambda V, \lambda Q) = E_C(S, V, Q) \tag{31}$$

La relación de Euler es

$$E(\lambda S, \lambda V, \lambda Q) = E_E(\lambda S, \lambda V, \lambda Q) + \frac{1}{2}E_C(\lambda S, \lambda V, \lambda Q) = \lambda E_E(S, V, Q) + \frac{1}{2}E_C(S, V, Q) \tag{32}$$

derivando respecto a λ y fijando $\lambda = 1$, además $2E - E_C = 2E_E$ se obtiene

$$E_C = 2(E + PV - TS - \Phi Q) \tag{33}$$

Usando las cantidades termodinámicas se puede mostrar que la energía Casimir E_C y extensiva E_E son

$$E_C = \frac{lr_+}{R}, \quad E = E_E + \frac{1}{2}E_C \rightarrow E_E = E - \frac{1}{2}E_C = \frac{Q^2 l^2 + r_+^4}{2lrR} \tag{34}$$

Teniendo ya la energía E_C y E estamso tentando a verificar la fórmula de Cardy-Verlinde

$$S = \pi\sqrt{E_C(2E - E_C)} \to \pi R\sqrt{\frac{Q^2 l^2 + r_+^4}{R^2}} \neq \pi r_+^2 \tag{35}$$

Lamentablemente no se cumple, la propuesta es separar la energía extensiva en dos partes de modo que $E_E \to E_E + E_Q$, es claro que E_Q también será extensiva, entonces

$$E = E_E + E_Q + \frac{1}{2}E_C \to 2(E - E_Q) = 2E_E + E_C \tag{36}$$

Por lo que la fórmula de Cardy Verlinde

$$S = \pi\sqrt{E_C(2(E - E_Q) - E_C)} \tag{37}$$

Analizando la energía extensiva obtenida anteriormente

$$E_E = E - \frac{1}{2}E_C \to E_E = \frac{lQ^2}{2Rr_+} + \frac{r_+^3}{2lR} \tag{38}$$

Entonces tenemos que $E_E \to E_E + E_Q$

$$E_E = \frac{r_+^3}{2lR}, \qquad E_Q = \frac{lQ^2}{2Rr_+}, \qquad E_C = \frac{lr_+}{R} \tag{39}$$

La cantidad E_Q puede ser calculada también desde la primera ley $dE_Q = \Phi Q$

$$E_Q = \int \Phi dQ = \frac{\Phi Q}{2} = \frac{lQ^2}{2Rr_+} \tag{40}$$

También puede ser leída desde la energía

$$E = \frac{l}{2R}\left(r_+ + \frac{r_+^3}{l^2}\right) + \frac{lQ^2}{2Rr_+} \tag{41}$$

y la formula de Cardy-Verlinde se satisface

$$S = \pi\sqrt{E_C(2(E - E_Q) - E_C)} = \pi r_+^2 \tag{42}$$

3 R-charged to RN BH

We are interested in a truncated version of $D = 4$, $N = 8$ gauged supergravity and use the conventions of [2, 2, 3] for the action and equations of motion:

$$I[g_{\mu\nu}, \vec{\phi}, A_i] = \frac{1}{16\pi}\int_{\mathcal{M}} d^4x\sqrt{-g}\Big[R - \frac{1}{2}(\partial\vec{\phi})^2 - \frac{1}{4}\sum_i^4 X_i^{-2}F_i^2 + \frac{1}{l^2}\sum_{i<j}^4 X_i X_i\Big] \tag{43}$$

where the three dilatonic scalars are written in the compact form $\vec{\phi} = (\phi_1, \phi_2, \phi_3)$, $\{A_i\}$ are the four 1-form gauge potentials, and the four $\{X_i\}$, which satisfy $X_1X_2X_3X_4 = 1$, can be written in terms of the scalars as

$$X_i = e^{-\frac{1}{2}\vec{a}_i\cdot\vec{\phi}}, \qquad \vec{a}_i \cdot \vec{a}_j = 4\delta_{ij} - 1 \tag{44}$$

A convenient choice [3] for $\vec{a}_i$ is

$$\vec{a}_1 = (1,1,1), \qquad \vec{a}_2 = (1,-1,-1), \qquad \vec{a}_3 = (-1,1,-1), \qquad \vec{a}_4 = (-1,-1,1) \tag{45}$$

for which the action and potential can be rewritten as

$$I[g_{\mu\nu}, \vec{\phi}, A_i] = \frac{1}{16\pi}\int_{\mathcal{M}} d^4x\sqrt{-g}\left[R - \frac{1}{2}(\partial\vec{\phi})^2 - \frac{1}{4}\sum_i e^{\vec{a}_i\cdot\vec{\phi}}F_i^2 - V(\vec{\phi})\right] \tag{46}$$

where the potential is

$$V(\vec{\phi}) = -\frac{2}{l^2}(\cosh\phi_1 + \cosh\phi_2 + \cosh\phi_3) \tag{47}$$

There exist then an exact static hairy black hole solution in this theory [6] with the metric

$$ds^2 = -\left(\prod_{i=1}^4 H_i\right)^{-1/2} f dt^2 + \left(\prod_{i=1}^4 H_i\right)^{1/2}[f^{-1}d\rho^2 + \rho^2(d\theta^2 + \sin^2\theta d\phi^2)] \tag{48}$$

$$f(\rho) = 1 - \frac{\eta}{\rho} + \frac{\rho^2}{l^2}\prod_{i=1}^4 H_i(\rho)\,, \qquad H_i(\rho) = 1 + \frac{q_i}{\rho}, \qquad \prod_{i=1}^4 H_i(\rho) = H_1(\rho)H_2(\rho)H_3(\rho)H_4(\rho) \tag{49}$$

gauge potentials and $\{X_i\}$

$$A_i = (1 - H_i^{-1})\frac{\sqrt{q_i(q_i+\eta)}}{q_i}dt = \frac{\sqrt{q_i(q_i+\eta)}}{\rho + q_i}dt\,, \qquad X_i = \left(\prod_{j=1}^4 H_j\right)^{1/4} H_i^{-1} \tag{50}$$

and the three scalar field are

$$\phi_1(r) = \frac{1}{2}\ln\left(\frac{H_1H_2}{H_3H_4}\right), \quad \phi_2(r) = \frac{1}{2}\ln\left(\frac{H_1H_3}{H_2H_4}\right), \quad \phi_3(r) = \frac{1}{2}\ln\left(\frac{H_1H_4}{H_2H_3}\right) \tag{51}$$

Si fijamos el caso particular en donde $q \equiv q_1 = q_2 = q_3 = q_4$ es fácil chequear que en este caso $\phi_1 = \phi_2 = \phi_3 = 0$ y $V(0) = -\frac{2}{l^2}(\cosh 0 + \cosh 0 + \cosh 0) = -\frac{6}{l^2}$ Es decir la acción 46 se reduce a

$$I[g_{\mu\nu}, \vec{\phi}, A_i] = \frac{1}{16\pi}\int_{\mathcal{M}} d^4x\sqrt{-g}\left[R - \frac{1}{4}\sum_i F_i^2 + \frac{6}{l^2}\right] \tag{52}$$

Ya que al ser todas las cargas iguales $F_1 = F_2 = F_3 = F_4 \equiv F$ por lo que la acción se transforma en

$$I = \frac{1}{16\pi}\int_{\mathcal{M}} d^4x\sqrt{-g}\left(R - F^2 + \frac{6}{l^2}\right) \tag{53}$$

La que es la misma acción que 14

$$f(r) = \frac{g(\rho)}{H(\rho)^2}, \qquad \frac{dr^2}{f(r)} = \frac{H(\rho)^2}{g(\rho)}d\rho^2, \qquad r^2 = H^2\rho^2 \tag{54}$$

The metric is

$$ds^2 = -\frac{g(\rho)}{H(\rho)^2}dt^2 + H(\rho)^2[g(\rho)^{-1}d\rho^2 + \rho^2 d\Omega_2^2] \tag{55}$$

the metric functions are

$$g(\rho) = 1 - \frac{\eta}{\rho} + \frac{\rho^2}{l^2}H^4, \qquad H(\rho) = 1 + \frac{q}{\rho} \tag{56}$$

Reduciendo la solución a RN en coordenadas de branas que se relaciona las cantidades μ y Q mostradas en 18 con *eta* y q de la siguiente forma.

$$\mu = \eta + 2q, \qquad Q^2 = q(\eta + q) \tag{57}$$

Choosing for simplicity $\rho_+ \equiv \rho$, we have the following thermodynamics quantities

$$S = \pi(\rho_+ + q)^2, \qquad M = \frac{\eta + 2q}{2}, \qquad \Phi = \frac{\sqrt{q(\eta + q)}}{\rho_+ + q}, \qquad Q = \sqrt{q(\eta + q)} \tag{58}$$

Where Q represents the physics charge and M the total energy

$$T = \frac{3\,\rho_+^4 + 8\,q\rho_+^3 + \left(l^2 + 6\,q^2\right)\rho_+^2 - q^4}{4\rho_+\,(\rho_+ + q)^2\,l^2\pi} \tag{59}$$

which satisfy the first law

$$dM = TdS + \Phi dQ \tag{60}$$

Space-time for Cardy-Verlinde analysis

We construct the holographic stress tensor

$$\tau^{ab} \equiv \frac{2}{\sqrt{-h}}\frac{\delta I}{\delta h_{ab}} \tag{61}$$

The boundary metric at $\rho = \infty$. The boundary stress tensor $\langle \tau_{\mu\nu} \rangle$ is defined up to scale factor ρ/l,

$$\gamma_{ab}dx^a dx^b = -dt^2 + l^2(d\theta^2 + \sin\theta^2 d\varphi^2) \tag{62}$$

The boundary stress tensor is an operator but for simplicity we will consider $\langle \tau_{ab} \rangle \equiv \tau_{ab}$. Is easy to show $\gamma^{ab}\tau_{ab} = 0$

$$\tau_{tt} = \frac{\eta + 2\,q}{8\pi l^2}, \qquad \tau_{\theta\theta} = \frac{(\eta + 2\,q)}{16\pi}, \qquad \tau_{\phi\phi} = \frac{(\eta + 2\,q)\sin^2\theta}{16\pi} \tag{63}$$

From the metric (62) we see that the gas of particles lived on the volume $V = l^2 \int \sin\theta\; d\theta d\varphi = 4\pi l^2$. Defining the time-like velocity $u^a = (\partial_t)^a$, which is normalized to: $u^a u_a = -1$, the stress tensor components describe a perfect gas of particles with state equation: $\varrho = 2p$

$$\tau_{ab} = (\varrho + p)u_a u_b + p\gamma_{ab} \tag{64}$$

where the density and pressure are

$$\varrho = \frac{\eta + 2\,q}{8\pi l^2}, \qquad p = \frac{\eta + 2q}{16\pi l^2} \tag{65}$$

the perfect gas of particles are confined in a fixed volume $V = 4\pi l^2$. In order to have a CFT_{1+2} defined on variable spatial volume we need impose a conformal transformation

$$\frac{R^2}{l^2}\gamma_{ab}dx^a dx^b = -\frac{R^2}{l^2}dt^2 + R^2(d\theta^2 + \sin^2\theta d\varphi^2) \tag{66}$$

Considering $\sqrt{-\gamma}\gamma^{ab}\tau_{bc} = \sqrt{-\gamma'}\gamma'^{ab}\tau'_{bc}$ we can show $\tau_{ab} \to \frac{l}{R}\tau_{ab}$. At the same time we need transform the time part of the metric by $t\frac{R}{l} = \tau$. Them, the conformal plus time coordinate transformation give us

$$ds^2 = -d\tau^2 + R^2(d\theta^2 + \sin^2\theta d\varphi^2) \tag{67}$$

$$\tau_{\tau\tau} = \frac{l^3}{R^3}\tau_{tt}, \qquad \tau_{\theta\theta} \to \frac{l}{R}\tau_{\theta\theta}, \quad \tau_{\varphi\varphi} \to \frac{l}{R}\tau_{\varphi\varphi} \tag{68}$$

From (62) the stress tensor components are

$$\tau_{\tau\tau} = \frac{(\eta + 2\,q)l}{8\pi R^3}, \quad \tau_{\theta\theta} = \frac{l\,(\eta + 2\,q)}{16\pi R}, \quad \tau_{\phi\phi} = \frac{(\eta + 2\,q)\sin^2\theta}{16\pi R} \tag{69}$$

The volume $V = 4\pi R^2$ where the gas is confined. The pressure and density according to: $\tau_{ab} = (\varrho + p)u_a u_b + p\gamma_{ab}$, are

$$\tau_{\tau\tau} = \varrho = \frac{l(\eta + 2q)}{2VR}, \qquad \frac{\tau_{\theta\theta}}{R^2} = p = \frac{l(\eta + 2q)}{4VR} \tag{70}$$

From the state equation $\varrho = 2p$ we have the energy and pressure of conformal perfect gas which is defined in the space-time (67)

$$\varrho = 2p \;\to\; E = \frac{l}{R}\left(\frac{\eta + 2q}{2}\right), \qquad p = \frac{E}{2V} \tag{71}$$

According to first law (60) the intensive quantities like temperature and electric potential are re-scaled to: $T \to \frac{l}{R}T$, $\Phi \to \frac{l}{R}\Phi$

$$T = \frac{3\,\rho_+^4 + 8\,q\rho_+^3 + \left(l^2 + 6\,q^2\right)\rho_+^2 - q^4}{4\rho_+\,\left(\rho_+ + q\right)^2 \pi\; l\; R}, \qquad \Phi = \frac{l\sqrt{q(\eta + q)}}{R(\rho_+ + q)} \tag{72}$$

$$S = \pi(\rho_+ + q)^2, \qquad Q = \sqrt{q(\eta + q)} \tag{73}$$

The first law for the gas with volume variable

$$dE = TdS + \Phi dQ - pdV \tag{74}$$

Cardy-Verlinde formula

The Cardy-Verlinde formula show that the energy can be separated in the following form

$$E = E_q + E_E + \frac{1}{2}E_C \tag{75}$$

Where E_Q and E_E are the electric part of the energy and the extensive part, plus the sub-extensive part E_C which is called Casimir energy. Under this consideration we can separate the energy

$$E = \frac{l\eta}{2R} + E_q, \qquad E_q = \frac{ql}{R} \tag{76}$$

La cantidad E_q puede ser calculada también desde la primera ley $dE_Q = \Phi Q$, where

$$dQ = \frac{\eta + 2q}{2\sqrt{q(q + \eta)}}dq \tag{77}$$

$$E_q = \int \Phi dQ = \int \frac{l(\eta + 2q)}{2R(\rho_+ + q)}dq = \frac{ql}{R} \tag{78}$$

and from that we define the pressure

$$\overline{p} = \frac{E - E_Q}{2V} \tag{79}$$

Using the formula for Casimir energy

$$E_C = 2(E + \overline{p}V - TS - \Phi Q) = \frac{l\rho_+}{R} \tag{80}$$

and

$$E_Q = \frac{lq}{R}, \qquad E_E = \frac{l(\eta - \rho_+)}{2R} \tag{81}$$

and the entropy can be written in the following form

$$S = \pi R \sqrt{E_C[2(E - E_Q) - E_C]} \tag{82}$$

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