AMS - Solar Modulation of Cosmic Rays

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Description of the Problem

This internship had the purpose of constructing a one-dimensional computational model to understand the influence of the Sun on the flux of cosmic rays across the heliosphere, doing so by solving the Parker's Transport Equation, numerically.

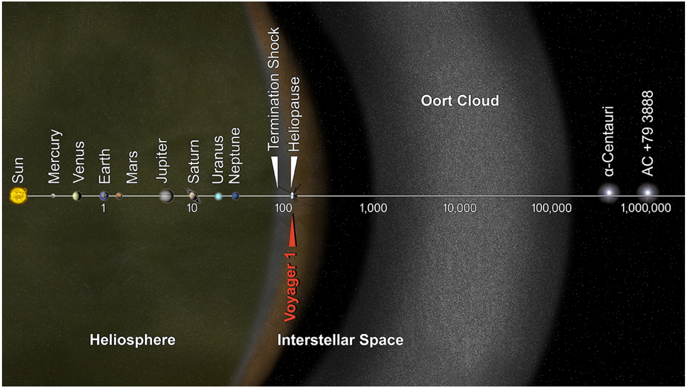


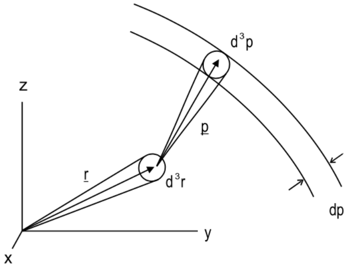
Figura 1: Picture of the Heliosphere and the insterstellar medium - scaled

Parker Equation

Parker's Transport Equation is written in the following way:

$$\frac{\partial f}{\partial t} + \underbrace{\mathbf{V}\cdot\nabla f}_{\text{convection and drift}} - \underbrace{\nabla\cdot(\mathbf{K}\cdot\nabla f)}_{\text{diffusion}} - \underbrace{\frac{1}{3}(\nabla\cdot\mathbf{V})\frac{\partial f}{\partial lnp}}_{\text{adiabatic cooling}} = \underbrace{q}_{sources}$$

In (1), V**(r)** stands for the Solar wind, and **K** is a 3x3 tensor which represents the diffusion phenomena. There is no full analytical solution to this equation, so we had to solve it by using finite difference schemes.



Graphic representation of an infinitesimal element on the $r - |p|$ space

Parker Equation (1-D spacial simplification)

In the equation above, if one assumes a spherical symmetric geometry, with $f = f(r)$, in a steady-state ($\partial/\partial t = 0$), and the source term zero, then it is very easy to obtain:

$$V\frac{\partial f}{\partial r} - \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^3 k\frac{\partial f}{\partial r}\right) - \frac{1}{3r^2}\frac{\partial}{\partial r}\left(r^2 V\right)\frac{\partial f}{\partial lnp} = 0 \tag{1}$$

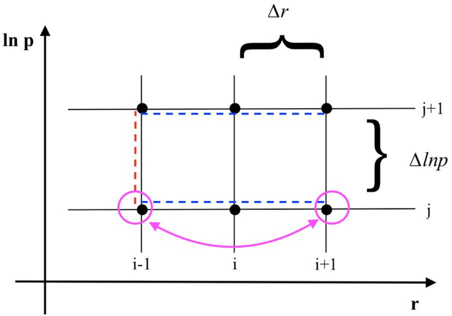
where k now became, after the simplification, a phenomenological diffusion coefficient that can be a function only of the radial coordinate r and momentum p. Finally, (2) can be readily rewritten in a simpler form:

$$k\frac{\partial^2 f}{\partial r^2} + \left(-V + \frac{2k}{r} + \frac{\partial k}{\partial r}\right)\frac{\partial f}{\partial r} + \left(\frac{2V}{3r} + \frac{1}{3}\frac{\partial V}{\partial r}\right)\frac{\partial f}{\partial lnp} = 0 \tag{2}$$

being the equation to solve.

Numerical Approach I

For getting the solution for $f(r,p)$, the Crank-Nicolson method was used. Let f_i^j represent the value of the function in the coordinates $r_i = r_0 + i\Delta_r$ and $lnp_j = lnp_0 + j\Delta_{lnp}$, with $i \in 1,\dots,N$ and $j \in 1,\dots,M$

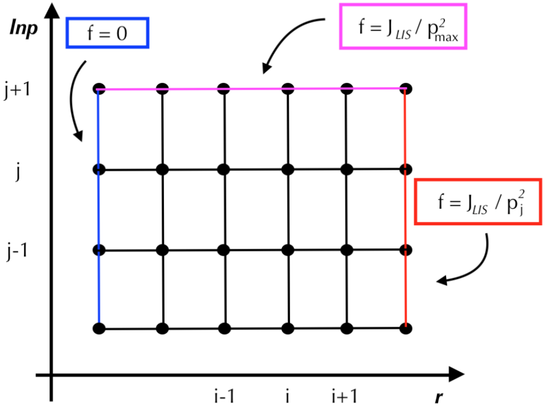


$$\frac{\partial f}{\partial lnp} = \frac{\left(f_i^{j+1} - f_i^j\right)}{\Delta_{lnp}}$$

$$\frac{\partial f}{\partial r} = \frac{1}{2}\left(\frac{f_{i+1}^{j+1} - f_{i-1}^{j+1}}{2\Delta_r} + \frac{f_{i+1}^{j} - f_{i-1}^{j}}{2\Delta_r}\right)$$

$$\frac{\partial^2 f}{\partial r^2} = \frac{1}{2}\left(\frac{f_{i+1}^{j+1} - 2f_i^{j+1} + f_{i-1}^{j+1}}{\Delta_r^2} + \frac{f_{i+1}^{j} - 2f_i^{j} + f_{i-1}^{j}}{\Delta_r^2}\right)$$

Numerical Approach II



keeping in mind the relationship

$j_T = p^2 f$

Numerical Approach III

$$\left.\begin{aligned}
\frac{\partial f}{\partial lnp} &= \frac{\left(f_i^{j+1} - f_i^j\right)}{\Delta_{lnp}} \\
\frac{\partial f}{\partial r} &= \frac{1}{2}\left(\frac{f_{i+1}^{j+1} - f_{i-1}^{j+1}}{2\Delta_r} + \frac{f_{i+1}^{j} - f_{i-1}^{j}}{2\Delta_r}\right) \\
\frac{\partial^2 f}{\partial r^2} &= \frac{1}{2}\left(\frac{f_{i+1}^{j+1} - 2f_i^{j+1} + f_{i-1}^{j+1}}{\Delta_r^2} + \frac{f_{i+1}^{j} - 2f_i^{j} + f_{i-1}^{j}}{\Delta_r^2}\right)
\end{aligned}\right\} k\frac{\partial^2 f}{\partial r^2} + \left(-V + \frac{2k}{r} + \frac{\partial k}{\partial r}\right)\frac{\partial f}{\partial r} + \ldots$$

$$\Leftrightarrow$$

$$\boxed{\begin{aligned}
&\left(\frac{f_{i+1}^{j+1} - 2f_i^{j+1} + f_{i-1}^{j+1}}{\Delta_r^2} + \frac{f_{i+1}^{j} - 2f_i^{j} + f_{i-1}^{j}}{\Delta_r^2}\right)\frac{k}{2} + \\
&\frac{1}{2}\left(-V + \frac{2k}{r} + \frac{\partial k}{\partial r}\right)\left(\frac{f_{i+1}^{j+1} - f_{i-1}^{j+1}}{2\Delta_r} + \frac{f_{i+1}^{j} - f_{i-1}^{j}}{2\Delta_r}\right) + \cdots
\end{aligned}}$$

Numerical Approach IV

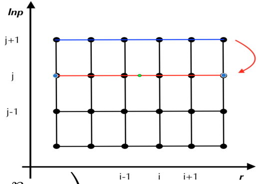
Then, setting $\varphi_1 = k$, $\varphi_2 = -V + \frac{2k}{r} + \frac{\partial k}{\partial r}$ and $\varphi_3 = \frac{2}{3}\frac{V}{r} + \frac{1}{3}\frac{\partial k}{\partial r}$ (3) becames:

$$f_{i-1}^j \underbrace{\left(\frac{\varphi_1}{2\Delta_r^2} - \frac{\varphi_2}{4\Delta_r}\right)}_{\eta_1} + f_i^j \underbrace{\left(-\frac{\varphi_1}{\Delta_r^2} - \frac{\varphi_3}{\Delta_{lnp}}\right)}_{\eta_2} + f_{i+1}^j \underbrace{\left(\frac{\varphi_1}{2\Delta_r^2} + \frac{\varphi_2}{4\Delta_r}\right)}_{\eta_3} =$$

$$\underbrace{f_{i-1}^{j+1}\left(-\frac{\varphi_1}{2\Delta_r^2} + \frac{\varphi_2}{4\Delta_r}\right)_+ f_i^{j+1}\left(\frac{\varphi_1}{\Delta_r^2} - \frac{\varphi_3}{\Delta_{lnp}}\right) + f_{i+1}^{j+1}\left(-\frac{\varphi_1}{2\Delta_r^2} - \frac{\varphi_2}{4\Delta_r}\right)}_{\eta_4}$$

which can be written in a matricial form so that

$$A\mathbf{x}^{j+1} = B\mathbf{x}^j$$



with A and B being tridiagonal matrices

$$\begin{pmatrix} \eta_2 & \eta_3 & 0 & \dots & 0 \\ \eta_1 & \eta_2 & \eta_3 & 0 & \dots \\ 0 & \eta_1 & \eta_2 & \eta_3 & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \dots & \dots & \eta_2 & \eta_3 \end{pmatrix} \begin{pmatrix} f_2^j \\ f_3^j \\ \vdots \\ \vdots \\ f_{N-1}^j \end{pmatrix} = \begin{pmatrix} \eta_4 \\ \eta_4 \\ \vdots \\ \vdots \\ \eta_4 - \eta_3 f_N^j \end{pmatrix}$$

Force Field Solution

An easy way of modulate the flux throughout the heliosphere is using an approximation of modulation theory called Force-Field solution (Gleeson and Axford, 1968). It producre the so-called modulation potencial, ϕ, which can be used to get a solution:

$$\frac{j(r,p)}{E^2 - E_0^2} = \frac{j_{LIS}(E+\Phi)}{(E+\Phi)^2 - E_0^2} \tag{3}$$

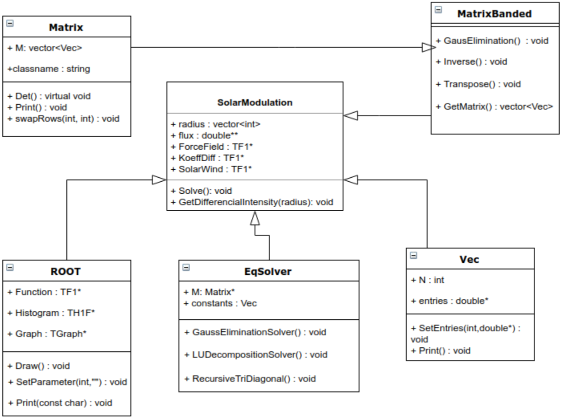
with $\Phi = Ze\phi$, Z the atomic number and e the unit charge. This modulation potencial comes from the fact that the diffusion coefficient k could be written as $k(r,p) = \beta k_1(r) k_2(p)$, which allows to get:

$$\phi(r) = \int_r^{r_b} \frac{V(r')}{3k_1(r')} dr' = \int_p^{p_b} \frac{\beta k_2(p')}{p'} dp'$$

This is valid in a variety of the phase space corresponding to $dp/dr = pV/3k(r,p)$

C++ Class and Methods

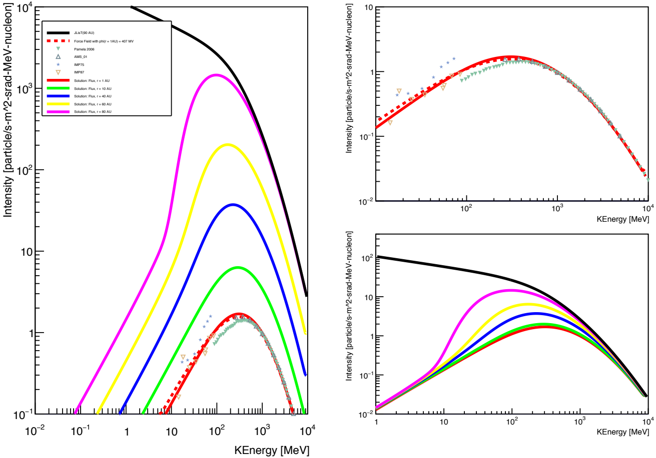
To solve the problem , we chose to develop a C++ code , in a scheme of classes to obtain the numerical solution of Parker's Transport Equation . Also it had the possibility to alter the problem's parameters like Solar Wind Speed and the Diffusion Coefficient.



Class Scheme used to solve the problem

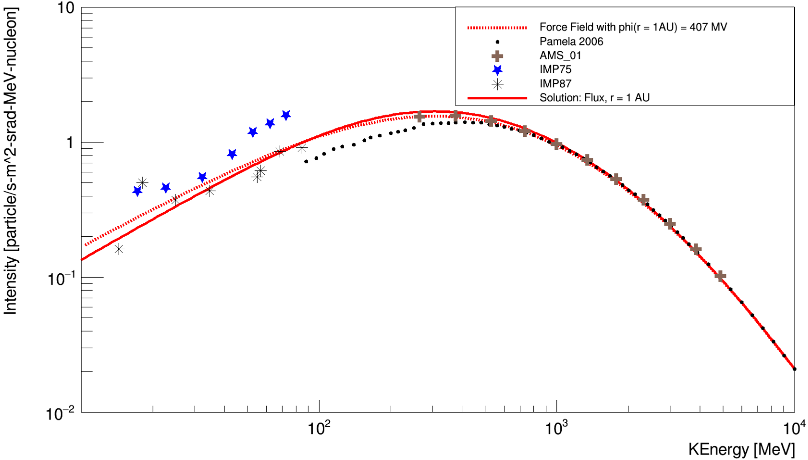
Results (One-Dimension)

Solar Wind Speed 400 km/s and diffusion coefficient $k = 4.38 \times 10^{18} (m^2/s)$ and Modulation Potential $\phi = 407 MV$



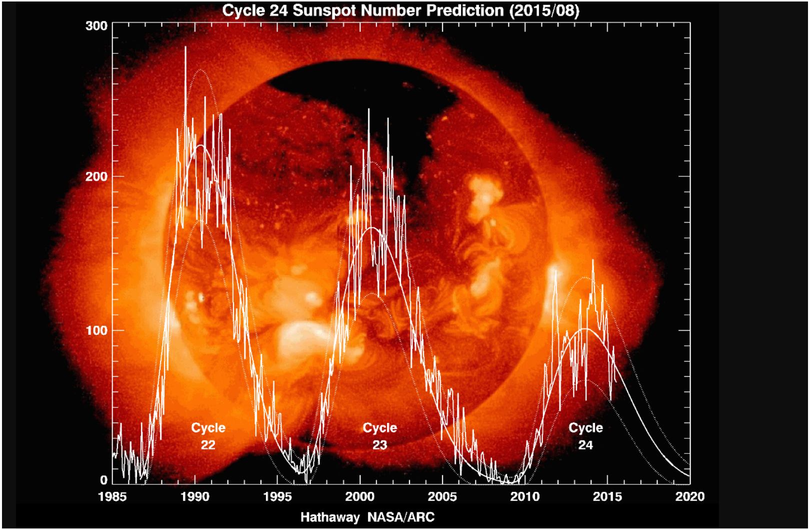
1D Resolution of Parker's Transport Equation and data from detectors

Conclusions



1 AU and Data from different years

Conclusions



Solar Cycle

References

[1] Adriani, O. et al, (2013), "Time Dependence of the Proton Flux Measured by Pamela during the 2006 JULY–2009 December Solar minimum"

[2] Alcaraz, J. , et al (2000) , "Cosmic protons - AMS Collaboration"

[3] Caballero-Lopez, R. A. and Moraal, H. (2004) "Limitations of the force field equation to describe cosmic ray modulation"

[4] Ilya G. Usoskin, Galina A. Bazilevskaya, and Gennady A. Kovaltsov, (2011), "Solar modulation parameter for cosmic rays since 1936 reconstructed from ground-based neutron monitors and ionization chambers"

[5] Moraal, H (2011), "Cosmic-Ray Modulation Equations"

[6] Usoskin, I. G. et al (2005) , "Heliospheric modulation of cosmic rays: Monthly reconstruction for 1951–2004"

[7] Webber, W. R. and Lockwood, J. A. (2001) , "Voyager and Pioneer spacecraft measurements of cosmic ray intensities in the outer heliosphere: Toward a new paradigm for understanding the global solar modulation process"