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CPSC 322 Assignment 4

Question 1 [15 points] Probabilities

a)

p(A=T, B=F, C=T) = 0.4 is wrong. We have the necessary information to fix it, since the sum of all probabilities should be 1, and without this probability it is already 1, so it should be 0.

b)

Marginal distributions of A and B: p(A=T)=0.5, p(A=F) = 0.5, p(B=T)=0.5, p(B=F) = 0.5
A and B are not independent because p(A=T,B=T)=0.2, so it is not equal to p(A=T)*p(A=B)
p(A=T|B=F) = 0.3/0.5 =0.6

c)

Yes, P(A) * P(B|A) * P(C|A,B) = P(C) * P(B|C) * P(A|B,C) is true because both are the result of applying the chain rule to the same joint distribution, the only difference is the chosen ordering.

Question 2 [20 points] Bayes' rule

a. Using W=1 to denote a warning and W=0 for no warning, and F=1 and F=0 for a failure or not of the electrical system:

   1. Given that $P(F = 1|W = 1) = \frac{P(W=1|F=1)*P(F=1)}{P(W=1)}$ we need to start by finding P(W=1):

$$P(W = 1) = P(W = 1|F = 0) * P(F = 0) + P(W = 1 | F = 1) \cdot P(F = 1)$$
$$P(W = 1) = 0.05 * 0.9999 + 0.95 * 0.0001$$
$$P(W = 1) = 0.0501$$

   2. Finally calculate P(F=1|W=1):

$$P(F = 1|W = 1) = \frac{P(W=1|F=1)*P(F=1)}{P(W=1)}$$
$$P(F = 1|W = 1) = \frac{0.95*0.0001}{0.0501}$$
$$P(F = 1|W = 1) = 0.0019$$

b. We will have to do the same thing using P(F=1)=0.01:

   1. Find P(W=1):

$$P(W = 1) = P(W = 1|F = 0) * P(F = 0) + P(W = 1 | F = 1) \cdot P(F = 1)$$
$$P(W = 1) = 0.05 * 0.99 + 0.95 * 0.01$$
$$P(W = 1) = 0.059$$

   2. Finally calculate P(F=1|W=1):

$$P(F = 1|W = 1) = \frac{P(W=1|F=1)*P(F=1)}{P(W=1)}$$
$$P(F = 1|W = 1) = \frac{0.95*0.01}{0.059}$$
$$P(F = 1|W = 1) = 0.16$$

c. Because these are calculating the probability of having a fire given that you have had a warning (the prior probability). Therefore, one directly affects the other.

Question 3 [25 points] Bayesian/Belief networks

a. Applying the JPD we have to store 17.781.120 values because:

$$4^2 * 15 * 6^2 * 7^2 * 42 = 17781120$$

We can get fewer values to store by using the belief network given:

- P(A)
  - |Domain| = 4
  - Values to store: 3
- P(B)
  - |Domain| = 15
  - Values to store: 14
- P(C|A)
  - 24 values, but given that |domain(A)| = 4 we can save 1 value from each row
  - Values to store: 20
- P(D|A,B)
  - 240 values but again can save 1 per row = 60 values saved
  - Values to store: 180
- P(E|B)
  - 105 values, can save 15 values
  - Values to store: 90
- P(F|C)
  - 36 values, can save 6 values
  - Values to store: 30
- P(G|C,A,D,E)
  - 4.704 values, can save 672 values
  - Values to store: 4.032
- P(H|B,D,E)
  - 17.640 values, can save 420 values
  - Values to store: 17.220

Therefore, 17.220 + 4.032 + 30 + 90 + 180 + 20 + 14 + 3 = 21.589. Saving 17.759.531 values.

b. Given that in part a we saved:
   a. 1
   b. 1
   c. |Domain(A)|
   d. |Domain(A)|*|Domain(B)|
   e. |Domain(B)|
   f. |Domain(C)|
   g. |Domain(C)|*|Domain(A)|*|Domain(D)|*|Domain(E)|
   h. |Domain(B)|*|Domain(D)|*|Domain(E)|

Therefore we are saving 2+a+(a*b)+b+c+(a*c*d*e)+(b*d*e)

Which, when rearranged yields: 2+a+b+c+ab+bde+acde

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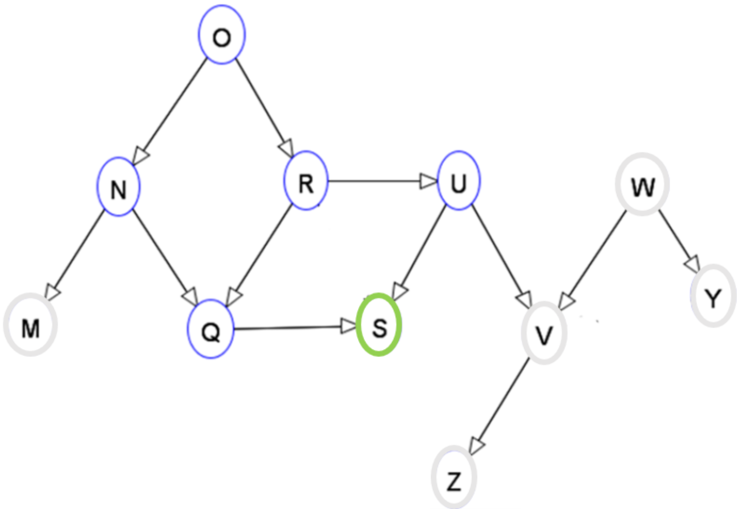
Question 4 [40 points] Variable Elimination

1)

W and Y are conditionally independent of S given Q, and the nodes Z, M and V can be pruned because they are unobserved leaf nodes. In fact, W and Y can also be pruned as unobserved leaf nodes. Hence, the result simpler network will look like the following picture:



The factors that we must have to express this network are expressed as follows: f(O), f(N,O), f(R,O), f(Q,N,R), f(U,R), f(S,Q,U)
And after assigning Q=F we will get:
$f_1(O),\ f_2(N,O),\ f_3(R,O),\ f_4(N,R),\ f_5(U,R),\ f_6(S,U)$

2)

First of all, our alphabetical elimination ordering will be N,O,R,U, so we will express the factors as the following formula:
$\Sigma_U f_6(S,U)\,\Sigma_R f_5(U,R)\,\Sigma_O f_1(O)\,f_3(R,O)\ \Sigma_N f_2(N,O)\,f_4(N,R)$
And the steps are:
$f_7(O,R)\ =\ \Sigma_N f_2(N,O)\,f_4(N,R)$

$f_8(R)\ =\ \Sigma_O f_1(O)\,f_3(R,O)\,f_7(O,R)$

$f_9(U)\ =\ \Sigma_R f_5(U,R)\,f_8(R)$

$f_{10}(S)\ =\Sigma_U f_6(S,U)\,f_9(U)$
And their corresponding tables:

O	R	$f_7(O,R)$
T	T	0.84
T	F	0.74
F	T	0.76
F	F	0.66

R	$f_8(R)$
T	0.4668
F	0.3062

U	$f_9(U)$
T	0.41562
F	0.35738

S	$f_{10}(S)$
T	0.41641
F	0.35659

Once we have all the factors, we can compute our normalization term, which is going to be: $\Sigma_S f_{10}(S) = 0.41641 + 0.35659 = 0.773$ and with this we can finally compute our conditional probability distribution given Q=F:

S	$P(S \mid Q = F)$
T	0.5387
F	0.4613