CPSC 322 2022W1 Assignment 4

Make sure you follow all assignment instructions here and on Canvas. Failure to do so may result in heavy penalties.

Make sure that in your answers you clearly indicate the exact section you are answering.

Please start each question on a new page (eg. don't put your answer to Q3 on the same page as your answer to Q2).

Question 1 [15 points] Probabilities

A	B	C	p
T	T	T	0.1
T	T	F	0.1
T	F	T	0.4
T	F	F	0.3
F	T	T	0.1
F	T	F	0.2
F	F	T	0.1
F	F	F	0.1

Consider the above joint probability distribution over the three random variables A, B, C.

(a) [3 points] You are told that it contains **only one** incorrect value. What is that value? Do you have sufficient information to fix it? If that is the case, how would you fix it? If not, why not?

(b) [8 points] With the joint resulting from the previous question (after you fixed the error), compute the marginal probability distributions for P(A) and P(B). Are A and B independent? Also, what is the value of P(A=T | B=F)?

(c) [4 points] ***Without performing any calculations***, is it true that P(A) * P(B|A) * P(C|A,B) = P(C) * P(B|C) * P(A|B,C)? Why or why not?

Question 2 [20 points] Bayes' rule

You've taken your prized sports car into a repair shop for a routine maintenance check. As part of a standard set of diagnostics, the car's onboard computer is checked for logged problems. The mechanic tells you that they have good and bad news. The bad news is that the car's computer has reported that it has detected a failure in the electrical system that will definitely cause an electrical fire in the near future (thus you may treat the failure and the resulting fire as the same "thing"). The probability of the computer reporting this warning given that there is actually a failure in the electrical system is 0.95, as is the probability that the computer will correctly not report anything if the failure isn't present. The good news is that this type of failure and the consequent electric fire will only occur in one out of every ten thousand cars of this model on the road.

(a) **[12 points]** What is the actual probability that an electrical fire will occur in your car, given the computer report? (Show your calculations as well as giving the final result., Also, you may find this link useful: https://en.wikipedia.org/wiki/Law_of_total_probability)

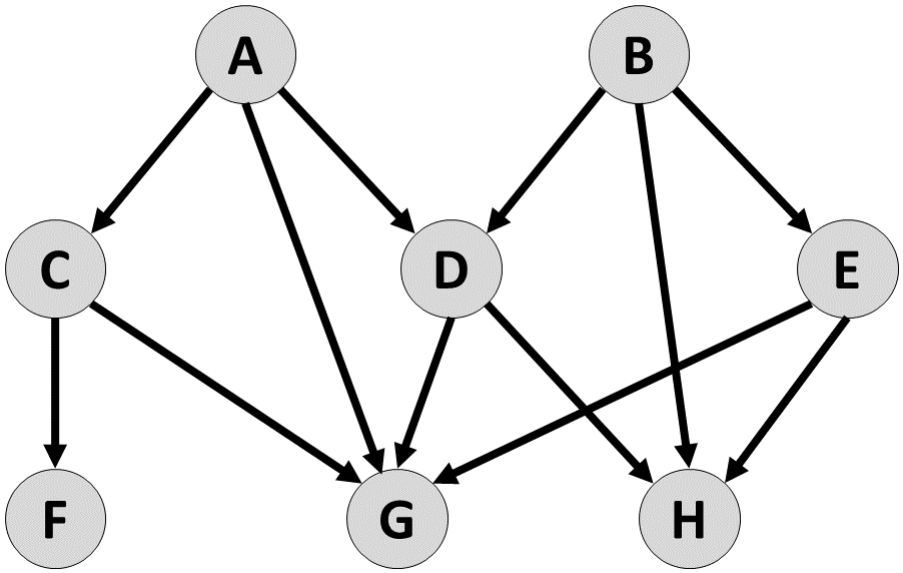
(b) **[4 points]** Repeat (and show) the calculation of part (a), but for the case where the failure occurs in one out of every hundred cars of this model.

(c) **[4 points]** Considering the computer report and your answers to parts (a) and (b), why is it good that the issue is so rare?

Question 3 [25 points] Bayesian/Belief networks

Consider a probabilistic problem that can be represented by the Bayesian Network given below. Let the variables have domain sizes as follows:

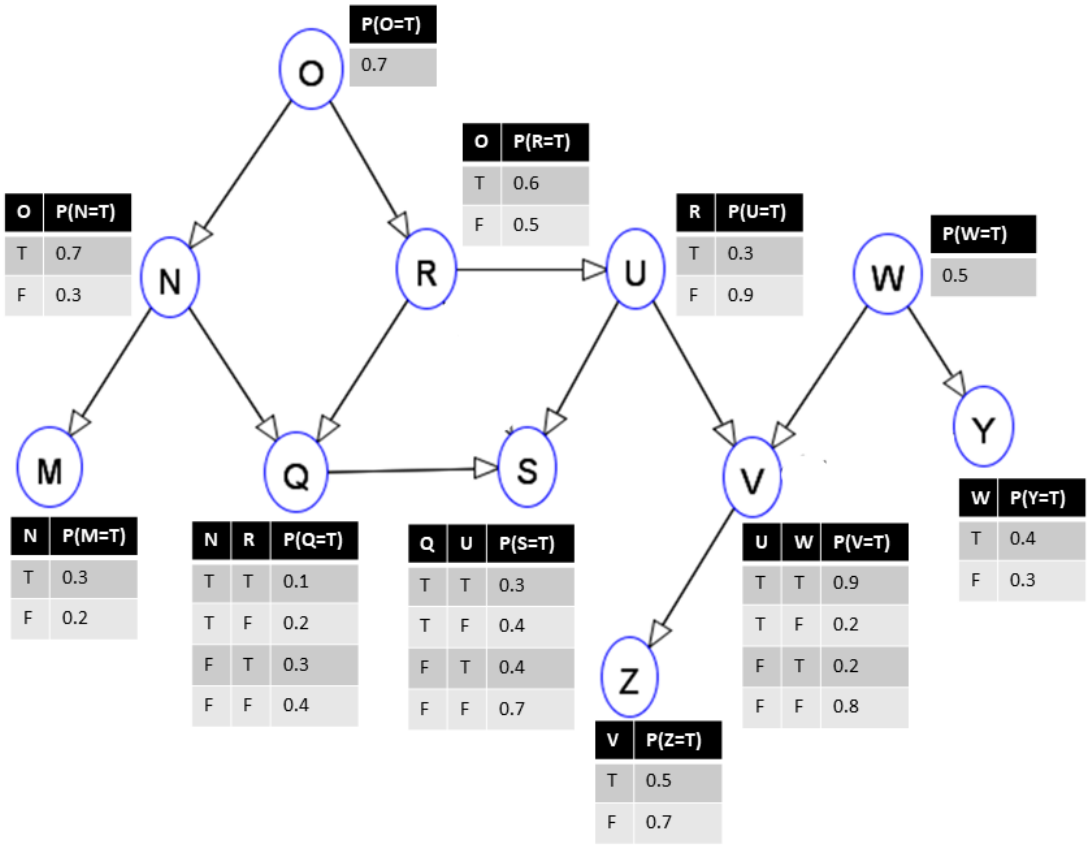
- A and D have 4 values each
- B has 15 values
- C and F have 6 values each
- E and G have 7 values each
- H has 42 values



a) Compute the **representational savings** ***(i.e. how many fewer values you need to store)*** by using this belief network instead of the joint distribution for this problem. Show your work.

b) Suppose A had *a* values, B had *b* values, C had *c* values, and so on. What would be the representational savings now?

Question 4 [40 points] Variable Elimination



Carry out variable elimination (VE) on this network to compute P(S|Q=F).

1. [10 marks] Indicate which nodes can be pruned (justifying each pruning step) before performing VE, and list the factors (with their values) that VE needs to initially create. **(Leave any factor operations to the next part.)**
2. [30 marks] Perform VE in order to compute P(S|Q=F). Use an alphabetical elimination ordering. Show your work:
    - Show how the factors and the summations should be ordered at each step. Refer to the multi-slide example covered in class as an example.
    - Include tables (with values) of any new factors you create.