Missing Data - Introductory notes

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These are my current notes and the themes

1 Wikipedia

Case deletion (CC) The method does not introduce any bias if the missing values are uniformly distributed.

Single imputation If one sort the data matrix according some order, *Last observation carried forward* is the method of replacing the missing value with last valid value.

The missing value can also be replaced with the mean of the other observations, however, correlations are attenuated.

Regression imputation use the other variables as predictors to replace the missing value, although precision is misleadingly augmented, hence does not reflect the statistical errors of the missing data. This problem is partially solved by multiple imputation.

Multiple imputation The multiple imputation [Rubin (1987)] is similar to bootstrapping method: Missing variables are simulated, say B times, and the desired statistics are averaged except for the standard error which is constructed by adding the variance of the imputed data and the within variance of each data set.

2 Matloff Blog post

Complete-case analysis (CC), listwise deletion Delete all record for which at least one variable is missing.

Single and multiple imputation Estimation of the distribution of missing variables conditional on the others and then sampling from that distribution. Multiple alternate matrix are generated without the NAs.

In multiple imputation, the distribution of each variable conditional and the others is fitted and in case of missing value, a sample is drawn from this distribution.

Available cases (AC), pairwise deletion Keep the observation if the missing feature is not retained for the desired measure, for example the correlation (where only 2 variables are needed). It can, nonetheless, produce correlations over 1.

2.1 MCAR: Missing Completely at Random

Let Y the variable of interest, $M \in \{0, 1\}$ denotes if Y is missing, and D the other variables than Y. This is often denoted as

$$P(M = 1|Y = s, D = t) = P(M = 1),$$

or equivalently

$$P(Y = s, D = t|M = i) = P(Y = s, D = t), i \in \{0, 1\}.$$

2.2 MAR: Missing at Random

For multiple imputation, one requires only $M \perp Y|D$, that is

$$P(M = 1|Y = s, D = t) = P(M = 1|D = t),$$

2.2.1 Conditional estimation under MAR

In practice, problems arise as D might not hold any predictive ability of the desired variable and that D might as well contain missing data. Interestingly

$$\begin{aligned}
P(Y = s|D = t, M = i) &= \frac{P(Y = s, D = t, M = i)}{P(D = t, M = i)} \\
&= \frac{P(Y = s, D = t)P(M = i|Y = s, D = t)}{P(D = t, M = i)} \\
&= \frac{P(Y = s|D = t)P(D = t)P(M = i|D = t)}{P(D = t, M = i)} \\
&= P(Y = s|t).
\end{aligned}$$

Hence if we are interested in the relationship between Y and D, that is the conditional distribution Y given D, the fact that it is missing or not will not introduce bias, hence CC and AC would perform equally well. This is ironic as MAR is meant to apply where CC and AC should not be used.

2.2.2 Unconditional estimation under MAR

Observe that

$$P(Y = s|M = 0) = \frac{P(M = 0|Y = s)}{P(M = 0)}\ P(Y = s),$$

hence our estimation of $P(Y = s)$ might still be biased with the factor of $P(M = 0|Y = s)/P(M = 0)$.

3 Missing data (Schafer and Graham 2002)

> With or without missing data, the goal of a statistical procedure should be to make valid and efficient inferences about a population of interest—not to estimate, predict, or recover missing observations nor to obtain the same results that we would have seen with complete data.

Let Y_{com} denote the complete data, and denote its partitions with observed and missing data $Y_{com} = (Y_{obs}, Y_{mis})$. If R is the random variable representing missingnes, then MAR (also called ignorable nonresponse) is defined as

$$P(R|Y_{com}) = P(R|Y_{ob}),$$

and MCAR

$$P(R|Y_{com}) = P(R).$$

Missing not at random (MNAR) or nonignorable nonresponse, is the situation when MAR is violated. Issue with MAR is, it is often unverifiable, however, only little deviation of estimates and standard errors are observed in practice.

$P(Y_{com}; \theta)$ can be interpreted as either the sampling mechanism of Y_{com} with parameter θ or the likelihood function. The following formula

$$P(Y_{obs}; \theta) = \int P(Y_{com}; \theta) dY_{mis}$$

provides a sampling distribution only when MCAR holds and is a valid likelihood function when MAR is assumed (favoring the Bayesian view). For MNAR, R and an additional parameter ξ defining the distribution of R has to be added:

$$P(Y_{obs}, R; \theta, \xi) = \int P(Y_{com}; \theta) P(R; \xi) dY_{mis}.$$

3.1 Older Methods

Listwise and Pairwise deletion Listwise deletion (case deletion or complete-case analysis) dismiss all observation with any missing values and pairwise deletion (available-case analysis) uses different sets of sample units for different parameters. Critics of AC are that the standard errors or other measures of uncertainty are difficult to assess as the parameters are computed from different sets of units.

CC analysis only works with MCAR but even if it holds, MCAR can be inefficient (e.g. with large data matrix with mild rates of missing values.

Reweighting Reweighting can eliminate bias from CC, for more details (Little and Rubin 2002, chap. 4.4). It is easy to use with univariate and monotone missing patterns.

Average imputation It replaces the missing value with the mean of the observation. This introduce bias and underestimate the standard errors. The new value is an artifact of a specific data sets and disturbes the scale of the variables. If MI is not feasible, then averagin is a reasonable choice if

> reliability is high ($\alpha > 0.7$) and each group of items to e averaged seems to form a single, well, defined domain

3.2 Single imputation

Imputation is the process of predicting the missing value conditional on the other values. It has the advantages of sharing the same dataset to all researcher working on a common project. See (Little and Rubin 2002) for shortcomings of single imputation.

Imputing unconditional means Mean substitution consists of replacing the missing value of a variable with the average accross all the other non-missing observations. Weakness are that confidence intervals $\bar{y} \pm z_\alpha \sqrt{S^2/N}$ are narrowed by overstating the number of observation N and the downward bias into S^2.

4 Data analysis using regression and multilevel/hierarchical models (Gelman and Hill 2006)

Chapter 25 of contains information about missing values.

5 Statistical analysis with missing data (Little and Rubin 2002)

The monograph describes mechanisms underlying the missingness come in several type (*mi*, *mice*, *Amelia* in R packages).

Bibliography

Gelman, Andrew, and Jennifer Hill. 2006. *Data Analysis Using Regression and Multilevel/hierarchical Models*. Cambridge University Press.

Little, RJA, and DB Rubin. 2002. "Statistical Analysis with Missing Data." Wiley.

Schafer, Joseph L, and John W Graham. 2002. "Missing Data: Our View of the State of the Art." *Psychological Methods* 7 (2). American Psychological Association: 147.