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Direction of arrival estimation analysis using a 2D antenna array

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Abstract

The direction of arrival of the incoming signals to an antenna array is one of the properties which can be obtained through the processing of the received data. This property is an important key in the development of adaptive antennas, enabling to discover the number of impinging signals to an antenna and its directions. This paper describes the most known algorithms that determine the directions of arrival using a two dimensional antenna array, analyzing its performance. Properties such as the computational efficiency and estimation accuracy using several scenarios of noise are presented.

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Keywords: Direction of arrival; planar array;

1. Introduction

The smart antenna opens up new opportunities for wireless communications. This antenna is composed by an antenna array and a signal processing system which adapts its radiation pattern to different environments, providing several advantages such the increase of the capacity, security and the coverage range combined with the reduction of interferences. The block diagram of a smart antenna system is presented in the Fig. 1.

In this system the antenna receives the signal from each element that composes the array in order to estimate the directions of arrival (DOA) signals. Applying a precise processing of the received signals, the desired and the interference signals are distinguished. After that, the system employs the beamforming techniques to create the

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desired radiation pattern of the antenna array, pointing to the preferred direction, and placing nulls in the interferences locations.

The DOA's algorithms have become a preponderant part of these systems. In the major part of applications, the spatial location of the electromagnetic waves that reach the antenna is needed, and the planar arrays must be used in order to be possible estimate the angular directions of arrival (θ, ϕ) of its signals. In this work the main 2D algorithms used are presented and a detailed comparative analysis is done.

This paper is divided into four sections. The paper starts with an introduction to the proposed topic, followed by the section describing the main 2D DOA algorithms. In the third section, the system to test the algorithms is implemented and the algorithms are analyzed, allowing presenting some conclusions in the fourth section.

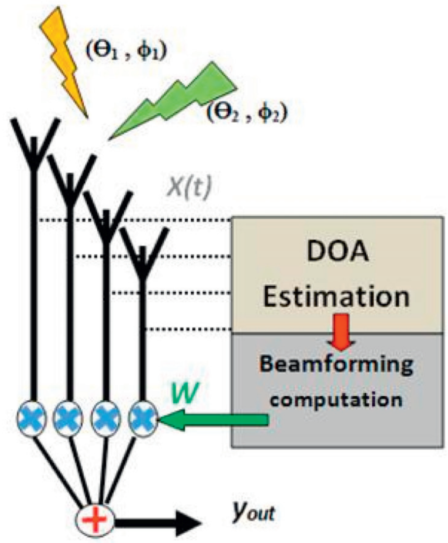


Fig. 1 Adaptive antenna array system

2. Direction of arrival algorithms

The DOA's processor consists of an important task in an adaptive antenna system, detecting the directions of arriving signals. These algorithms can be classified as Classical, Maximum likelihood or Subspace methods, which differ in the performance and in the complexity [1]. The subspace methods use the subspace of the signal and noise of the received data to determine the DOA's, combining the good estimation accuracy with an efficient computational complexity. The principal algorithms used with a two dimensional antenna array, determining the spatial angles of arrival are the 2D MUSIC (Multiple Signal Classification) and 2D ESPRIT (Estimation of Signal Parameters via Rotational Invariance Techniques), that belongs to the subspace methods [1][2].

2.1. 2D MUSIC

The MUSIC algorithm assumes that the noise subspace eigenvectors U_N are orthogonal to the array steering vectors $s(\theta, \phi)$ for the pair of angles (θ_i, ϕ_i) $i = 1 \ldots J$ that corresponds to the J angles of arrival. The MUSIC looks for the entire possible steering vectors and those that are orthogonal to the noise subspace of the received data correlation matrix [2]-[4]. When the steering vector is associated with a received signal, the MUSIC spectrum equation (1) assumes a high value.

$$P_{MUSIC}(\theta,\phi) = \frac{1}{s^H(\theta,\phi)U_N U_N^H s(\theta,\phi)} \tag{1}$$

Where H represents the conjugate transpose matrix (Hermitian).

The MUSIC algorithm can be implemented with all arrays shapes, is easy to comprehend but is computationally heavier. This DOA algorithm involves an extensive search through all possible steering vectors to determine the locations of the power spectral peaks and to estimate the DOA, which is computationally intensive. The estimation error is strongly affected by these steering vectors, which are used to compute the MUSIC function.

The MUSIC algorithm depends on the knowledge of the array steering vector, and the array response must be known. In addition, possible errors in the calibration of the array may be critical degrading the estimation accuracy.

2.2. 2D ESPRIT

The ESPRIT makes use of the property of shift invariance of the arrays, while the precise steering vectors of MUSIC are not required, suppressing problems with the array calibration [5]. Also, the ESPRIT algorithm solves the problem of MUSIC with a huge reduction in computational requirements imposing constraints to the array structure.

This algorithm supposes that an antenna array is composed of two identical sub-arrays at least, that may overlap [5]-[7]. The similar pair of elements of each sub-array is assumed to have an equal physical separation. This property leads to a displacement invariance and leads to a rotational invariance of signal subspaces spanned by the data vectors associated to the sub-arrays.

The ESPRIT algorithm follows three well defined steps [5]-[7]:

a) *Signal subspace estimation*

$$\text{Computation of the } U_S \tag{2}$$

b) *Solve the invariance equation*

$$\begin{aligned} K_{u1}U_s\Upsilon_u &= K_{u2}U_s \\ K_{v1}U_s\Upsilon_v &= K_{v2}U_s \end{aligned} \tag{3}$$

where K_{u1}, K_{u2}, K_{v1} and K_{v2} represent the two pairs of transforming selection matrices, while Υ_u and Υ_v are the real-valued matrices [7].

c) *DOA estimation*

$$\begin{aligned} &\lambda_i \quad i = 1\ldots d \rightarrow \text{eigenvalues of } \Upsilon_u + j\Upsilon_v \\ &u_i = 2\tan^{-1}(\mathrm{Re}\{\lambda_i\}) \\ &v_i = 2\tan^{-1}(\mathrm{Im}\{\lambda_i\}) \\ &\phi_i = \arg(u_i - jv_i) \quad \theta_i = \sin^{-1}(\|u_i - jv_i\|) \end{aligned} \tag{4}$$

where θ_i and ϕ_i are the DOA angular information.

3. System and Results

The simulated DOA estimator system receives an input signal x(t) from a planar antenna array. This signal is composed of several signals with different directions that collide to the antenna, plus noise. The received data is after processed to estimate the angles of arrival of each signal.

Considering a $M \times N$ planar array antenna, with spacing between elements of d_1 and d_2, in rows and columns respectively. Assume also that there are J signals $s(t)$ arriving to the antenna from different directions (θ,ϕ). Therefore the input signal at each array element (m, n) is the addition of the J signals contributions and of noise n(t):

$$x_{mn}(t)=\sum_{i=1}^{J}s_i(t)e^{j\frac{2\pi}{\lambda}[u_i d_1(m-1)+v_i d_2(n-1)]}+n_{nm}(t)$$
$$u_i=\sin\theta_i \quad v_i=\cos\theta_i\sin\phi_i \tag{5}$$
$$m=1,\dots,M \quad n=1,\dots,N \quad i=1,\dots,J$$

where λ is the wavelength.

The steering matrix A, of every signal arriving to the planar array represent the phase delays that a wave will take relating to each element of the array, and can be represented:

$$A=C_u\otimes C_v \tag{6}$$

$$C_u=\begin{bmatrix}1 & e^{j\frac{2\pi d_1}{\lambda}(2-1)u_i} & \cdots & e^{j\frac{2\pi d_1}{\lambda}(M-1)u_i}\end{bmatrix}^T \tag{7}$$

$$C_v=\begin{bmatrix}1 & e^{j\frac{2\pi d_2}{\lambda}(2-1)v_i} & \cdots & e^{j\frac{2\pi d_2}{\lambda}(N-1)v_i}\end{bmatrix}^T \tag{8}$$

where the $\otimes$ is the Kronecker product, C_u and C_v are the steering vectors in *x* and *y* direction.

The entire input signal X(t) can be summarized in the following formula:

$$X(t)=\sum_{i=1}^{J}s_i(t)A_i+N(t) \tag{9}$$

In this work, the signal X(t) is generated according to (9). This data is the sum of diverse signals with different directions (θ,ϕ), the signal of interest, interferences and noise. The signal X(t) is then processed by a DOA algorithm to determine the angles of arrival and the number of signals.

The DOA's algorithms were performed with the MATLAB [8] enabling the analysis of its performance when two signals with directions $(\theta,\phi)=(15°,65°)$ and $(\theta,\phi)=(50°,100°)$ arrives to a 4×4 planar antenna array with 0.5λ element spacing.

Applying the 2D MUSIC algorithm, a two-dimensional grid is created, in the range which the angles vary $\theta\in[0,90]$ $\phi\in[0,360]$, and the function (1) for each point of the grid is evaluated. The angle grid (θ,ϕ) was divided the interval $\theta\in[0,90]$ in 150 uniformly spaced points and 250 in the interval $\phi\in[0,360]$. The result is illustrated in the Fig. 2, representing the MUSIC spectrum in which are presented the peaks in the position of incident signals.

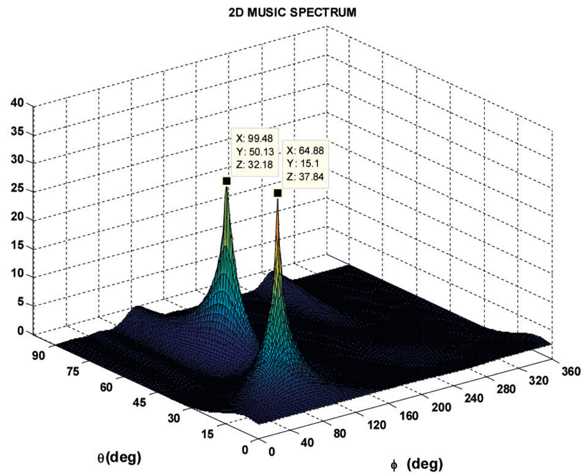


Fig. 2 Output of the 2D MUSIC estimation algorithm

To identify with more accuracy the peaks of the Fig. 2, a MATLAB function was developed, to find out the correct maximum values, based on the zero gradient locals. The output is presented in the Fig. 3. In this figure is possible to see the two well defined peaks. Finally, the result of the 2D MUSIC algorithm is that the two incident signals that collide to the array are arriving from (15.1º, 64.88º) and (50.13º, 99.48º).

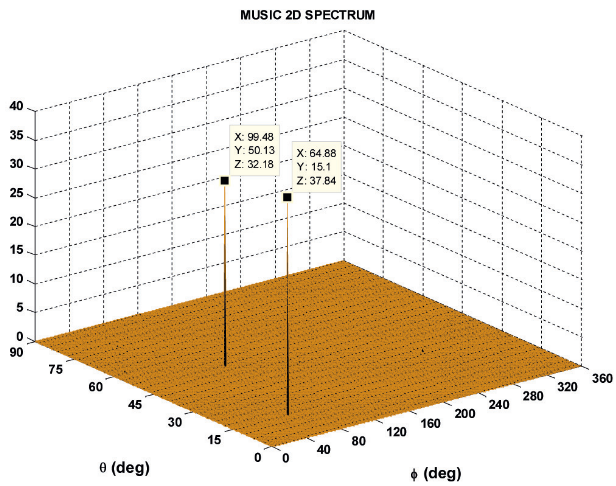


Fig. 3 Detected peaks using the 2D MUSIC spectrum

Applying the same received signal, X(t) to the ESPRIT algorithm the obtained estimations of the incoming signals directions was $(\theta,\phi) = (15.00^{\circ}, 64.91^{\circ})$ and $(49.95^{\circ}, 100.01^{\circ})$. The two 2D DOA algorithms implemented MUSIC and ESPRIT, present estimation results very similar to the original pretended angles.

After demonstrate the estimation results of the two DOA algorithms, an execution time analysis was done, comparing the mean runtime of both estimators, over a set of 50 simulations.

The Fig. 4 contains the progress of the execution time over the 50 simulations, and is presented also the line of mean value, for each DOA estimator. Is clearly noted the enormous difference in terms of runtime of the two algorithms. The MUSIC presents a mean execution time of 5.33 seconds while the ESPRIT has a mean execution time of 0.86560 milliseconds.

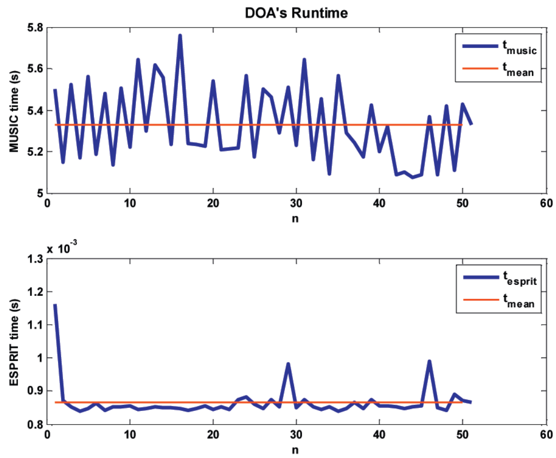


Fig. 4 Variation of the DOA's runtime over 50 simulations

This difference in terms of execution time was expected, since the computational requirements is one of the main drawbacks of MUSIC relating to ESPRIT, as is mostly affected due the number of computations of the PMUSIC function, 150x250 in this case. If this number reduces, the accuracy of the MUSIC estimation decreases, so using this algorithm a tradeoff between accuracy and runtime should be considered.

The last parameter that was analyzed is the variation of the estimation error of the algorithms with the signal–to-noise ratio (SNR).

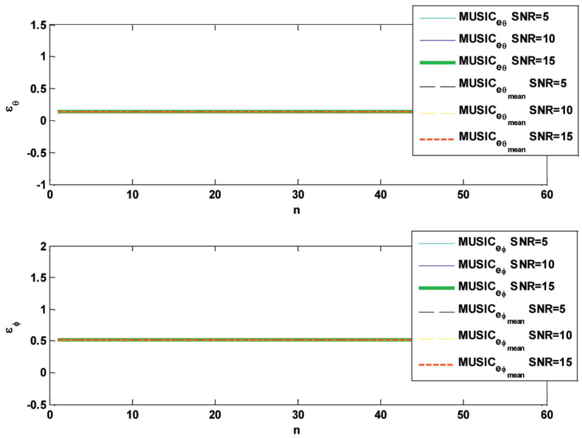


Fig. 5 Variation of the estimation error (θ and ϕ components) with the SNR, over 50 simulations using the MUSIC algorithm

The Fig. 5 shows the maximum estimation error of θ and ϕ component, and the mean of estimation error for different values of SNR (5, 10 and 15dB) over 50 simulations. The results show that the error is constant and equal to 0.1342º in θ estimation component and 0.5181º on the ϕ angle. These errors are influenced by the angle grid interval as shown the Fig. 6.

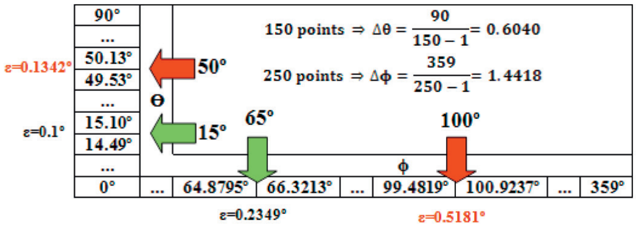
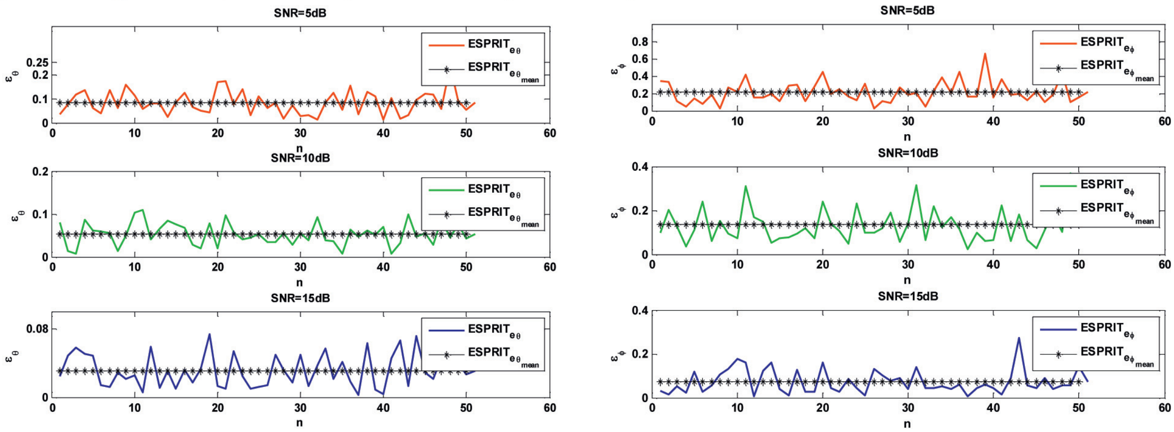


Fig. 6 Angle grid interval of MUSIC algorithm

The Fig. 7 a) and b) presents the performance of the estimation error along the *n* simulations, using the ESPRIT DOA algorithm, changing the SNR from 5dB to 15dB. Also, in these Figures are presented the mean values. According to the figures is possible to see that the estimation error in the ESPRIT algorithm reduces when the SNR increase from 5dB to 15dB. For the θ angles, Fig. 7 a) the mean value is 0.08296º (SNR=5dB), 0.05279º (SNR=10dB) and 0.03104º (SNR=15dB). In terms of the ϕ angles, Fig. 7 b), the mean values are 0.2123º (SNR=5dB), 0.1336º (SNR=10dB) and 0.07256º (SNR=15dB).



a) b)

Fig. 7 Variation of the estimation error with the SNR, over 50 simulations, using the ESPRIT algorithm: *a)* θ component *b)* ϕ component

4. Conclusions

In this paper the MUSIC and ESPRIT algorithms were applied using a two dimensional antenna array to estimate the directions of arrival of incoming signals. These algorithms were implemented in MATLAB and then a meticulous analysis was made in terms of execution time and accuracy of the estimate the DOA's, varying the SNR.

Results reveal that although the MUSIC algorithm is simpler to understand, it requires higher computational effort in relation to ESPRIT. This is expressed by the example presented in this paper, in which the runtime of the MUSIC is 6157 times higher than the ESPRIT algorithm. Also, the MUSIC algorithm requires a tradeoff between the accuracy and execution time. Moreover, the estimation error of MUSIC is constant when the SNR changes, and is dependent on the grid resolution.

The ESPRIT algorithm is more complex than MUSIC. However, it presents lower runtimes and less estimation errors. For the same conditions the estimation errors are dependent on the SNR used. Therefore, depending on the complexity of the system to be developed, the real-time requirements of some applications, and the parallelizability character of the algorithms, it should take into account the need to use more effective processing units, such as

FPGA (field-programmable gate array) or GPUs (graphics processing unit), improving the performance of computation times.

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