Logic Tutorial 2

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Overview

- 16:00 What’s it all good for?
- 16:10 Recap
- 16:20 **Q&A**
- 16:50 Quiz
- 17:00 **Q&A**
- 18:00 Feierabend

What’s it all good for? – Studies

Bachelor

- Reasoning techniques
- Logic for AI (elective)
- Prolog (elective)

Master

- Foundations of Agents
- Master projects

Logic master Amsterdam, Munich

What's it all good for? – Studies

Programming paradigms

- **Imperative:** C, Java, Python, Javascript
- **Functional:** Elm, Scala, Haskell, Racket
- **Relational:** Prolog

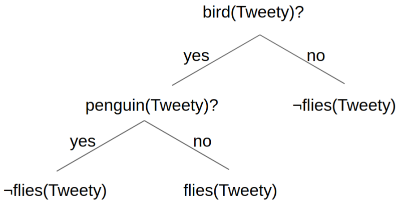
What’s it all good for? – Studies

bird(x) ~> flies(x)
penguin(x) ~> ¬flies(x)
penguin(x) --> bird(x)

flies(Tweety)?

penguin(Tweety) **penguin(x) ~> ¬flies(x)**	¬flies(Tweety)

penguin(Tweety) penguin(x) --> bird(x) **bird(x) ~> flies(x)**	flies(Tweety)



What’s it all good for? – Industry

- Expert systems, decision support systems
  - Law: *Neota Logic, Bryter, LegalOS, KnowledgeTools*
- …

What's it all good for? – Research

Symbolic AI [Explainable AI] *(vs neural AI)*

- Probabilistic logic programming
- Neural logic programming
- Relational machine learning
  - Inductive logic programming
- Neuro-symbolic learning
- Answer set programming
- …

What's it all good for? – Summer schools

~~Law and logic~~

Logic, language and information

~~Logic and formal epistemology~~

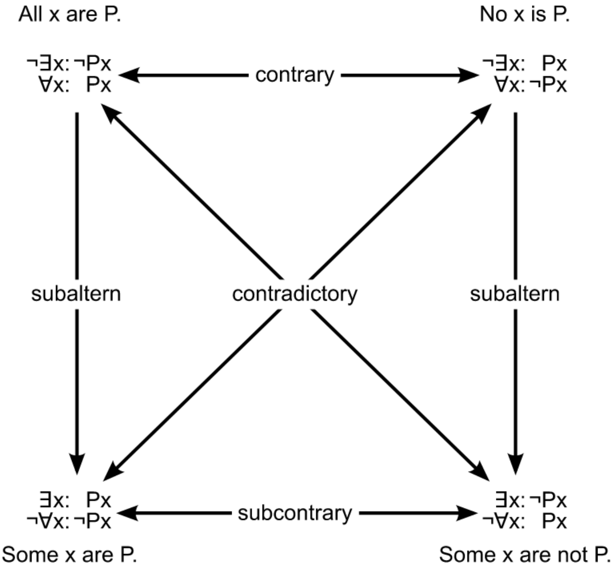
~~Contemporary logic, rationality and information~~

Probability and logic

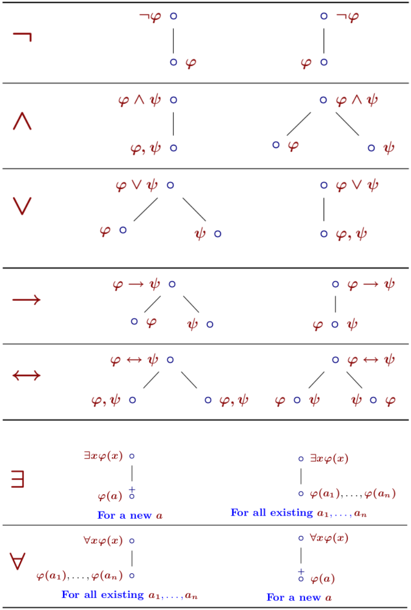
Mathematical philosophy for female students

- More extensive list by UvA

Square of opposition



Semantic Tableau



Natural deduction

$$\frac{\varphi\ ,\ \varphi \to \psi}{\psi}\ \texttt{modus ponens}$$

$$\frac{\begin{array}{|l} \varphi \\ \hline \vdots \\ \psi \end{array}}{\varphi \to \psi}\ \texttt{deduction}$$

$\mathbf{E}_{\to}$ $\mathbf{I}_{\to}$

$$\frac{\varphi \wedge \psi}{\varphi} \qquad \frac{\varphi \wedge \psi}{\psi}$$

$$\frac{\varphi\ ,\ \psi}{\varphi \wedge \psi}$$

$\mathbf{E}_{\wedge}$ $\mathbf{I}_{\wedge}$

$$\frac{\neg\varphi\ ,\ \varphi}{\bot}$$

$$\frac{\begin{array}{|l} \neg\varphi \\ \hline \vdots \\ \bot \end{array}}{\varphi}\ \texttt{refutation}$$

$\mathbf{E}_{\neg}$ $\mathbf{I}_{\neg}$

$$\frac{\varphi \vee \psi\ ,\quad \begin{array}{|l} \varphi \\ \hline \vdots \\ \chi \end{array}\quad ,\quad \begin{array}{|l} \psi \\ \hline \vdots \\ \chi \end{array}}{\chi}$$

$$\frac{\varphi}{\varphi \vee \psi} \qquad \frac{\psi}{\varphi \vee \psi}$$

$\mathbf{E}_{\vee}$ $\mathbf{I}_{\vee}$

Natural deduction

$$\frac{\forall x\, \varphi}{(\varphi)^x_t}$$

provided that no variable in t occurs bounded in φ

$\mathbf{E}_\forall$

$$\frac{\begin{array}{l|l} u & \\ \hline & \vdots \\ & (\varphi)^x_u \end{array}}{\forall x\, \varphi}$$

for u a special symbol not used anywhere else in the proof

$\mathbf{I}_\forall$

$$\frac{\exists x\, \varphi\ , \quad \begin{array}{l|l} u & (\varphi)^x_u \\ \hline & \vdots \\ & \psi \end{array}}{\psi}$$

for u a special symbol not used anywhere in the proof

$\mathbf{E}_\exists$

$$\frac{(\varphi)^x_t}{\exists x\, \varphi}$$

provided that no variable in t occurs bounded in φ

$\mathbf{I}_\exists$

$$\frac{t_1 = t_2\ ,\ \varphi}{\varphi_{[t_1/t_2]}}$$

$$\frac{t_1 = t_2\ ,\ \varphi}{\varphi_{[t_2/t_1]}}$$

where $\varphi_{[t_1/t_2]}$ is the result of replacing, in φ, some ocurrences of t_2 by t_1, provided that

- t_2 contains only variables that occurr freely in φ, and
- t_1 contains only variables that do not get bounded after replacement.

$\mathbf{E}_=$

$$\frac{}{t = t}$$

for any term t.

$\mathbf{I}_=$

Q & A

excalidraw

Q&A - “Moving in” the negation

$\neg\forall\ x\!: \neg E\ y\!: Rxy$

$E\ x\!: \neg\neg E\ y\!: Rxy$

$E\ x\!: E\ y\!: Rxy$

$\neg E\ x\ P(x)$

$A\ x\ \neg P(x)$

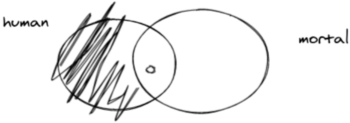
Q&A - Syllogism

all humans are mortal
socrates is a human
____
socrates is mortal

there is a mortal human

$\boxed{\exists x\text{: human}(x) \wedge \text{mortal}(x)}$

∀x: human(x) -> mortal(x)
human(Socrates)
____
mortal(Socrates)



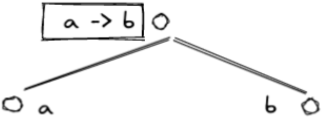
∀x: human(x) -> mortal(x), human(Socrates) ○ mortal(Socrates)

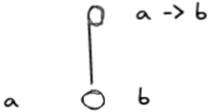
human(Socrates) -> mortal(Socrates) ○

human(Socrates) human(Socrates)

mortal(Socrates) mortal(Socrates)

Q&A - (Counter)examples in a semantic tableau





$a \wedge \neg b$

$\neg a, b$
$\neg a, \neg b$

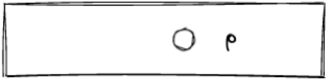
b, a
$b, \neg a$

Q&A - Proving validity, invalidity, satisfiability in a semantic tableau

proving that p is valid:

all branches close -> valid

open branch -> counterexample to the validity



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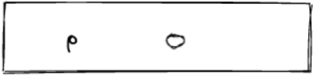
proving that p is invalid:

all branches close -> invalid

open branch -> counterexample to the invalidity
= example for the satisfiability



Q&A - Order of rule application in semantic tableaux

1. eliminate operators $\wedge$ $\vee$ -> $\neg$
2. eliminate existence
3. eliminate all quantifiers

$Ax: Dx \wedge Ix, \quad Ex: Ix \vee Cx \quad \circ$

$Ax: Dx \wedge Ix, \quad Ia \vee Ca \quad \circ$

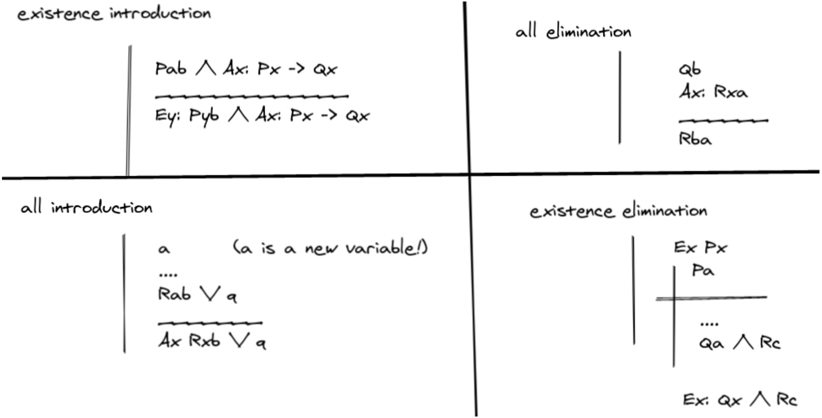
$Da \wedge Ia, \; Ia \vee Ca \quad \circ$

Q&A - Natural deduction

$\neg \exists x: Px \models \forall x: \neg Px$

1	$\neg \exists x: Px$	
2	u	
3	$P(u)$	assumption
4	$\exists x: Px$	$\exists$-introduction (3)
	introduction	
5	$\neg P(u)$	$\neg$introduction (1, 4)
6	$\forall x: \neg Px$	$\forall$-introduction (2, 5)

Q&A - Examples for the natural deduction rules for predicate logic



Quiz

- Tahook

Feedback

Anonymous feedback form:

- linktr.ee/davidpomerenke