

# Logic Tutorial 2

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# Overview

- ▶ 16:00 What's it all good for?
- ▶ 16:10 Recap
- ▶ 16:20 **Q&A**
- ▶ 16:50 Quiz
- ▶ 17:00 **Q&A** ->
- ▶ 18:00 Feierabend

# What's it all good for? – Studies

## Bachelor

- ▶ Reasoning techniques
- ▶ Logic for AI (elective)
- ▶ Prolog (elective)

## Master

- ▶ Foundations of Agents
- ▶ Master projects

Logic master Amsterdam, Munich

# What's it all good for? – Studies

## Programming paradigms

- ▶ **Imperative:** C, Java, Python, Javascript
- ▶ **Functional:** Elm, Scala, Haskell, Racket
- ▶ **Relational:** Prolog

## What's it all good for? – Studies

(img/masterproject.png){width=100%}

## What's it all good for? – Industry

- ▶ Expert systems, decision support systems
  - ▶ Law: *Neota Logic*, *Bryter*, *LegalOS*, *KnowledgeTools*
- ▶ ...

# What's it all good for? – Research

## **Symbolic AI [Explainable AI]** (*vs neural AI*)

- ▶ Probabilistic logic programming
- ▶ Neural logic programming
- ▶ Relational machine learning
  - ▶ Inductive logic programming
- ▶ Neuro-symbolic learning
- ▶ Answer set programming
- ▶ ...

# What's it all good for? – Summer schools

~~Law and logic~~

Logic, language and information

~~Logic and formal epistemology~~

~~Contemporary logic, rationality and information~~

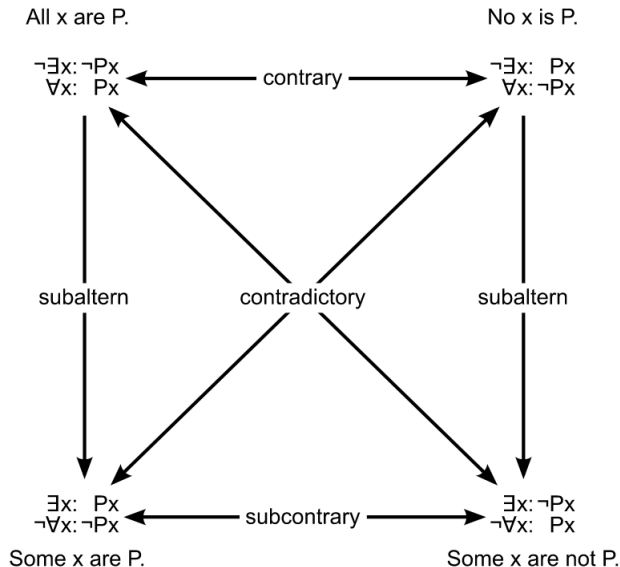
Probability and logic

Mathematical philosophy for female students

- ▶ More extensive list by UvA



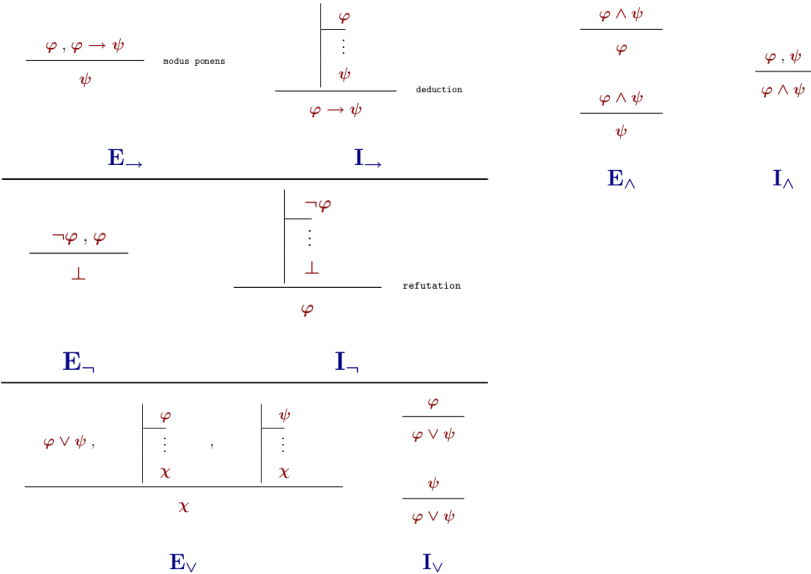
# Square of opposition



# Semantic Tableau

$\neg$	$\begin{array}{c} \neg\varphi \circ \\   \\ \varphi \circ \end{array}$	$\begin{array}{c} \circ \neg\varphi \\   \\ \varphi \circ \end{array}$
$\wedge$	$\begin{array}{c} \varphi \wedge \psi \circ \\   \\ \varphi, \psi \circ \end{array}$	$\begin{array}{c} \circ \varphi \wedge \psi \\ / \quad \backslash \\ \circ \varphi \quad \circ \psi \end{array}$
$\vee$	$\begin{array}{c} \varphi \vee \psi \circ \\ / \quad \backslash \\ \varphi \circ \quad \psi \circ \end{array}$	$\begin{array}{c} \circ \varphi \vee \psi \\   \\ \circ \varphi, \psi \end{array}$
$\rightarrow$	$\begin{array}{c} \varphi \rightarrow \psi \circ \\ / \quad \backslash \\ \circ \varphi \quad \psi \circ \end{array}$	$\begin{array}{c} \circ \varphi \rightarrow \psi \\   \\ \varphi \circ \psi \end{array}$
$\leftrightarrow$	$\begin{array}{c} \varphi \leftrightarrow \psi \circ \\ / \quad \backslash \\ \varphi, \psi \circ \quad \circ \varphi, \psi \end{array}$	$\begin{array}{c} \circ \varphi \leftrightarrow \psi \\ / \quad \backslash \\ \varphi \circ \psi \quad \psi \circ \varphi \end{array}$
$\exists$	$\begin{array}{c} \exists x\varphi(x) \circ \\   \\ \varphi(a) \circ \\ \text{For a new } a \end{array}$	$\begin{array}{c} \circ \exists x\varphi(x) \\   \\ \circ \varphi(a_1), \dots, \varphi(a_n) \\ \text{For all existing } a_1, \dots, a_n \end{array}$
$\forall$	$\begin{array}{c} \forall x\varphi(x) \circ \\   \\ \varphi(a_1), \dots, \varphi(a_n) \circ \\ \text{For all existing } a_1, \dots, a_n \end{array}$	$\begin{array}{c} \circ \forall x\varphi(x) \\   \\ \circ \varphi(a) \\ \text{For a new } a \end{array}$

# Natural deduction



# Natural deduction

$$\frac{\forall x \varphi}{(\varphi)_t^x}$$

$$\frac{\begin{array}{c|c} u & \\ \hline & (\varphi)_u^x \\ & \vdots \\ & \end{array}}{\forall x \varphi}$$

provided that no variable in  $t$  occurs bounded in  $\varphi$

for  $u$  a special symbol not used anywhere else in the proof

$E_{\forall}$

$I_{\forall}$

$$\frac{\exists x \varphi, \begin{array}{c|c} u & (\varphi)_u^x \\ \hline & \vdots \\ & \psi \end{array}}{\psi}$$

$$\frac{(\varphi)_t^x}{\exists x \varphi}$$

for  $u$  a special symbol not used anywhere in the proof

provided that no variable in  $t$  occurs bounded in  $\varphi$

$E_{\exists}$

$I_{\exists}$

$$\frac{t_1 = t_2, \varphi}{\varphi[t_1/t_2]}$$

$$\frac{t_1 = t_2, \varphi}{\varphi[t_2/t_1]}$$

$$\frac{}{t = t}$$

where  $\varphi[t_1/t_2]$  is the result of replacing, in  $\varphi$ , some occurrences of  $t_2$  by  $t_1$ , provided that

- $t_2$  contains only variables that occur freely in  $\varphi$ , and
- $t_1$  contains only variables that do not get bounded after replacement.

for any term  $t$ .

$E_{=}$

$I_{=}$

## Q & A

excalidraw

## Q&A - “Moving in” the negation

$$\neg \forall x: \neg \exists y: Rxy$$

$$\exists x: \neg \neg \exists y: Rxy$$

$$\exists x: \exists y: Rxy$$

$$\neg \exists x P(x)$$

$$\forall x \neg P(x)$$

# Q&A - Syllogism

all humans are mortal  
socrates is a human

~~~~~  
socrates is mortal

there is a mortal human

$\exists x: \text{human}(x) \wedge \text{mortal}(x)$

$\forall x: \text{human}(x) \rightarrow \text{mortal}(x)$

$\text{human}(\text{Socrates})$

~~~~~  
 $\text{mortal}(\text{Socrates})$



$\forall x: \text{human}(x) \rightarrow \text{mortal}(x), \text{human}(\text{Socrates})$

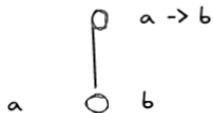
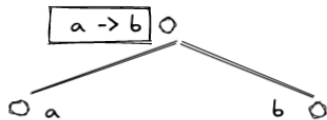
$\text{mortal}(\text{Socrates})$

$\text{human}(\text{Socrates}) \rightarrow \text{mortal}(\text{Socrates})$

$\text{human}(\text{Socrates})$      $\otimes \text{human}(\text{Socrates})$

$\text{mortal}(\text{Socrates})$      $\otimes \text{mortal}(\text{Socrates})$

## Q&A - (Counter)examples in a semantic tableau



$\neg a, b$   
 $\neg a, \neg b$

$b, a$   
 $b, \neg a$

$a \wedge \neg b$

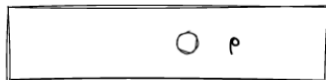


# Q&A - Proving validity, invalidity, satisfiability in a semantic tableau

proving that  $p$  is valid:

all branches close  $\rightarrow$  valid

open branch  $\rightarrow$  counterexample to the validity



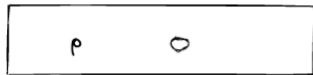
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proving that  $p$  is invalid:

all branches close  $\rightarrow$  invalid

open branch  $\rightarrow$  counterexample to the invalidity

= example for the satisfiability



## Q&A - Order of rule application in semantic tableaux

1. eliminate operators  $\wedge \vee \rightarrow \neg$
2. eliminate existence
3. eliminate all quantifiers

$$\textcircled{Ax:} Dx \wedge Ix, \quad \underline{\textcircled{Ex:} Ix \vee Cx} \quad \circ$$

$$\underline{Ax: Dx \wedge Ix}, \quad Ia \vee Ca \quad \circ$$

$$Da \wedge Ia, Ia \vee Ca \quad \circ$$

## Q&A - Natural deduction

$$\neg \exists x: Px \models Ax: \neg Px$$

1	$\neg \exists x: Px$	
2	$u$	
3	$P(u)$	assumption
4	$\exists x: Px$	E-introduction (3)
	introduction	
5	$\neg P(u)$	$\neg$ -introduction (1, 4)
6	$Ax: \neg Px$	A-introduction (2, 5)

# Q&A - Examples for the natural deduction rules for predicate logic

existence introduction

$$\begin{array}{|l} Pab \wedge \forall x: Px \rightarrow Qx \\ \hline \exists y: Pyb \wedge \forall x: Px \rightarrow Qx \end{array}$$

all elimination

$$\begin{array}{|l} Qb \\ \forall x: Rxa \\ \hline Rba \end{array}$$

all introduction

$$\begin{array}{|l} a \quad (a \text{ is a new variable!}) \\ \dots \\ Rab \vee a \\ \hline \forall x Rxb \vee a \end{array}$$

existence elimination

$$\begin{array}{|l} \exists x Px \\ Pa \\ \hline \dots \\ Qa \wedge Rc \\ \hline \exists x: Qx \wedge Rc \end{array}$$

# Quiz

- ▶ Tahook

# Feedback

Anonymous feedback form:

▶ [linktr.ee/davidpomerenke](https://linktr.ee/davidpomerenke)