

Logic Tutorial 2

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Overview

- ▶ 16:00 What's it all good for?
- ▶ 16:10 Recap
- ▶ 16:20 **Q&A**
- ▶ 16:50 Quiz
- ▶ 17:00 **Q&A** ->
- ▶ 18:00 Feierabend

What's it all good for? – Studies

Bachelor

- ▶ Reasoning techniques
- ▶ Logic for AI (elective)
- ▶ Prolog (elective)

Master

- ▶ Foundations of Agents
- ▶ Master projects

Logic master Amsterdam, Munich

What's it all good for? – Studies

Programming paradigms

- ▶ **Imperative:** C, Java, Python, Javascript
- ▶ **Functional:** Elm, Scala, Haskell, Racket
- ▶ **Relational:** Prolog

What's it all good for? – Studies

(img/masterproject.png){width=100%}

What's it all good for? – Industry

- ▶ Expert systems, decision support systems
 - ▶ Law: *Neota Logic*, *Bryter*, *LegalOS*, *KnowledgeTools*
- ▶ ...

What's it all good for? – Research

Symbolic AI [Explainable AI] (*vs neural AI*)

- ▶ Probabilistic logic programming
- ▶ Neural logic programming
- ▶ Relational machine learning
 - ▶ Inductive logic programming
- ▶ Neuro-symbolic learning
- ▶ Answer set programming
- ▶ ...

What's it all good for? – Summer schools

~~Law and logic~~

Logic, language and information

~~Logic and formal epistemology~~

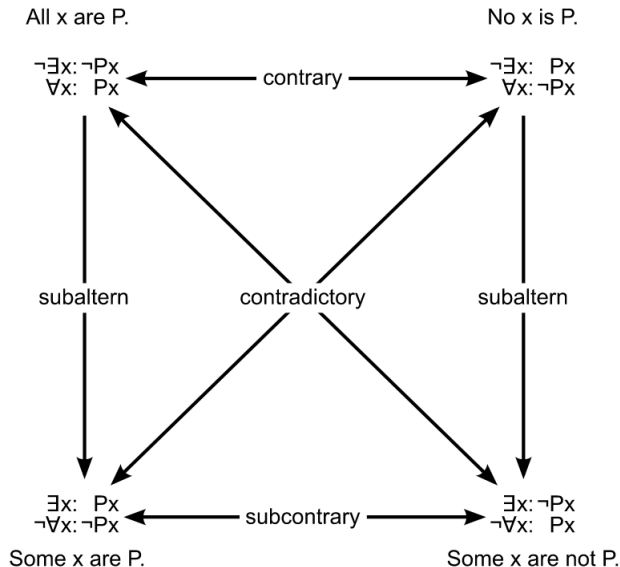
~~Contemporary logic, rationality and information~~

Probability and logic

Mathematical philosophy for female students

- ▶ More extensive list by UvA

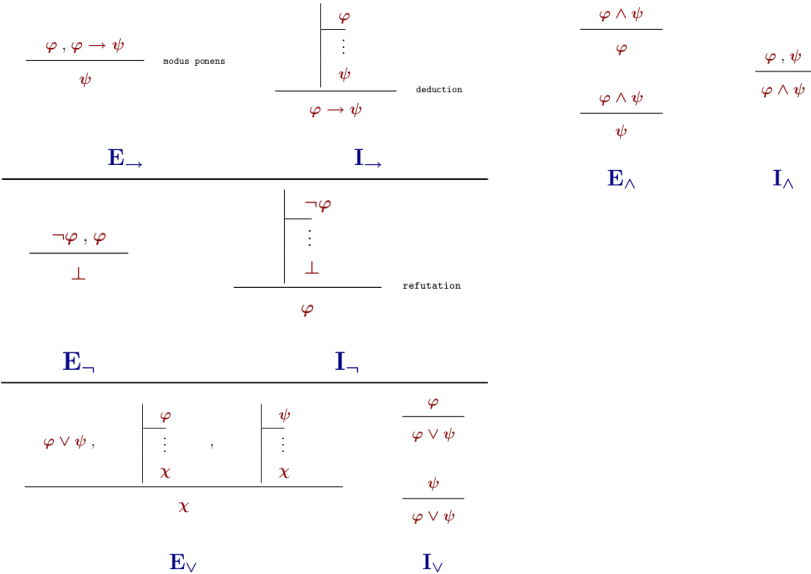
Square of opposition



Semantic Tableau

\neg	$\begin{array}{c} \neg\varphi \circ \\ \\ \varphi \circ \end{array}$	$\begin{array}{c} \circ \neg\varphi \\ \\ \varphi \circ \end{array}$
\wedge	$\begin{array}{c} \varphi \wedge \psi \circ \\ \\ \varphi, \psi \circ \end{array}$	$\begin{array}{c} \circ \varphi \wedge \psi \\ / \quad \backslash \\ \circ \varphi \quad \circ \psi \end{array}$
\vee	$\begin{array}{c} \varphi \vee \psi \circ \\ / \quad \backslash \\ \varphi \circ \quad \psi \circ \end{array}$	$\begin{array}{c} \circ \varphi \vee \psi \\ \\ \circ \varphi, \psi \end{array}$
\rightarrow	$\begin{array}{c} \varphi \rightarrow \psi \circ \\ / \quad \backslash \\ \circ \varphi \quad \psi \circ \end{array}$	$\begin{array}{c} \circ \varphi \rightarrow \psi \\ \\ \varphi \circ \psi \end{array}$
\leftrightarrow	$\begin{array}{c} \varphi \leftrightarrow \psi \circ \\ / \quad \backslash \\ \varphi, \psi \circ \quad \circ \varphi, \psi \end{array}$	$\begin{array}{c} \circ \varphi \leftrightarrow \psi \\ / \quad \backslash \\ \varphi \circ \psi \quad \psi \circ \varphi \end{array}$
\exists	$\begin{array}{c} \exists x\varphi(x) \circ \\ \\ \varphi(a) \circ \\ \text{For a new } a \end{array}$	$\begin{array}{c} \circ \exists x\varphi(x) \\ \\ \circ \varphi(a_1), \dots, \varphi(a_n) \\ \text{For all existing } a_1, \dots, a_n \end{array}$
\forall	$\begin{array}{c} \forall x\varphi(x) \circ \\ \\ \varphi(a_1), \dots, \varphi(a_n) \circ \\ \text{For all existing } a_1, \dots, a_n \end{array}$	$\begin{array}{c} \circ \forall x\varphi(x) \\ \\ \circ \varphi(a) \\ \text{For a new } a \end{array}$

Natural deduction



Natural deduction

$$\frac{\forall x \varphi}{(\varphi)_t^x}$$

$$\frac{\begin{array}{c|c} u & \\ \hline & \vdots \\ & (\varphi)_u^x \end{array}}{\forall x \varphi}$$

provided that no variable in t occurs bounded in φ

for u a special symbol not used anywhere else in the proof

E_{\forall}

I_{\forall}

$$\frac{\exists x \varphi, \begin{array}{c|c} u & (\varphi)_u^x \\ \hline & \vdots \\ & \psi \end{array}}{\psi}$$

$$\frac{(\varphi)_t^x}{\exists x \varphi}$$

for u a special symbol not used anywhere in the proof

provided that no variable in t occurs bounded in φ

E_{\exists}

I_{\exists}

$$\frac{t_1 = t_2, \varphi}{\varphi[t_1/t_2]}$$

$$\frac{t_1 = t_2, \varphi}{\varphi[t_2/t_1]}$$

$$\frac{}{t = t}$$

where $\varphi[t_1/t_2]$ is the result of replacing, in φ , some occurrences of t_2 by t_1 , provided that

- t_2 contains only variables that occur freely in φ , and
- t_1 contains only variables that do not get bounded after replacement.

for any term t .

$E_{=}$

$I_{=}$

Q & A

excalidraw

Q&A - “Moving in” the negation

$$\neg \forall x: \neg \exists y: Rxy$$

$$\exists x: \neg \neg \exists y: Rxy$$

$$\exists x: \exists y: Rxy$$

$$\neg \exists x P(x)$$

$$\forall x \neg P(x)$$

Q&A - Syllogism

all humans are mortal
socrates is a human

~~~~~  
socrates is mortal

there is a mortal human

$\exists x: \text{human}(x) \wedge \text{mortal}(x)$

$\forall x: \text{human}(x) \rightarrow \text{mortal}(x)$

$\text{human}(\text{Socrates})$

~~~~~  
 $\text{mortal}(\text{Socrates})$



$\forall x: \text{human}(x) \rightarrow \text{mortal}(x), \text{human}(\text{Socrates})$

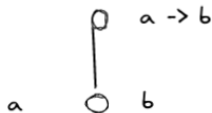
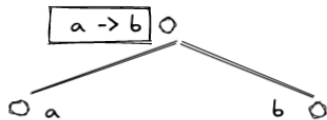
$\text{mortal}(\text{Socrates})$

$\text{human}(\text{Socrates}) \rightarrow \text{mortal}(\text{Socrates})$

$\text{human}(\text{Socrates})$ $\otimes \text{human}(\text{Socrates})$

$\text{mortal}(\text{Socrates})$ $\otimes \text{mortal}(\text{Socrates})$

Q&A - (Counter)examples in a semantic tableau



$\neg a, b$
 $\neg a, \neg b$

b, a
 $b, \neg a$

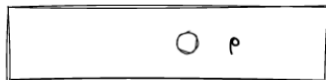
$a \wedge \neg b$

Q&A - Proving validity, invalidity, satisfiability in a semantic tableau

proving that p is valid:

all branches close \rightarrow valid

open branch \rightarrow counterexample to the validity

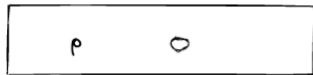


proving that p is invalid:

all branches close \rightarrow invalid

open branch \rightarrow counterexample to the invalidity

= example for the satisfiability



Q&A - Order of rule application in semantic tableaux

1. eliminate operators $\wedge \vee \rightarrow \neg$
2. eliminate existence
3. eliminate all quantifiers

$$\textcircled{Ax:} Dx \wedge Ix, \quad \underline{\textcircled{Ex:} Ix \vee Cx} \quad \circ$$

$$\underline{Ax: Dx \wedge Ix}, \quad Ia \vee Ca \quad \circ$$

$$Da \wedge Ia, Ia \vee Ca \quad \circ$$

Q&A - Mock exam 2016 (Incognito Wiki) - 7 a)

$$\neg \exists x: Px \models Ax: \neg Px$$

1	$\neg \exists x: Px$	
2	u	
3	$P(u)$	assumption
4	$\exists x: Px$	E-introduction (3)
	introduction	
5	$\neg P(u)$	\neg -introduction (1, 4)
6	$Ax: \neg Px$	A-introduction (2, 5)

Q&A - Examples for the natural deduction rules for predicate logic

existence introduction

$$\begin{array}{|l} Pab \wedge \forall x: Px \rightarrow Qx \\ \hline \exists y: Pyb \wedge \forall x: Px \rightarrow Qx \end{array}$$

all elimination

$$\begin{array}{|l} Qb \\ \forall x: Rxa \\ \hline Rba \end{array}$$

all introduction

$$\begin{array}{|l} a \quad (a \text{ is a new variable!}) \\ \dots \\ Rab \vee a \\ \hline \forall x Rxb \vee a \end{array}$$

existence elimination

$$\begin{array}{|l} \exists x Px \\ Pa \\ \hline \dots \\ Qa \wedge Rc \\ \hline \exists x: Qx \wedge Rc \end{array}$$

Quiz

- ▶ Tahook

Feedback

Anonymous feedback form:

- ▶ linktr.ee/davidpomerenke