Time series analysis
Day 4.

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What will we do today

- The last class was fully dedicated to the discussion of ARIMA.

- Now we are looking to extend discussion of ARIMA by addressing:

  – Seasonality within/outside ARIMA framework
  – Heteroscedasticity in time series
  – Inclusion of independent variables in ARIMA

Linear seasonal model

- A model with non-stationary time series with <u>deterministic trend</u> and a monthly seasonal component is defined as:

$$y_t = \alpha t + s_t + w_t = \begin{cases} \alpha t + \beta_1 + w_t \\ \alpha t + \beta_2 + w_t \\ \vdots \\ \alpha t + \beta_{12} + w_t \end{cases}$$

  where $\boldsymbol{\alpha}$ is trend coefficient and $\boldsymbol{s_t}$ is a collection of seasonal terms. Effectively, seasonal terms are dummy variables ($\boldsymbol{\beta_1}$-$\boldsymbol{\beta_{12}}$). So, either model will not have an intercept (as above) or one of the seasonal terms will be dropped and an estimate for the intercept will appear in the output. The fitted models are equivalent

- The parameters for the model in equation above can be estimated by OLS or GLS.

- In principle we are free to extend the model with lags and exogenous independents (x_{1t}, x_{2t} ... x_{jt})

Extending ARMA with seasonal components

- ARMA models can be easily extended to include seasonal components:
- For instance, the model bellow specifies a simple autoregressive term for daily series, and additional two seasonal autoregressive components for weekly and monthly effects:

$$y_t = \varphi_1 y_{t-1} + \varphi_2 y_{t-week} + \varphi_3 y_{t-month} + w_t$$

(of course specifications should be in appropriate time lags)

- Depending on your model specification and data, you can define the seasonal component as AR(p) or MA(q) terms
- This is the basis of **additive seasonal ARMA/ARIMA model**
- The model implies that all coefficients in-between appropriate lags are **constrained** to zero

SARIMA

- In contrast, we can specify a **multiplicative seasonal ARIMA** model with autoregressive, moving average and differencing terms as:

$$\text{ARIMA}(p, d, q)(P, D, Q)_s$$

where ***s*** is number of periods per season.

- We use uppercase notation for the seasonal parts and lowercase notation for the non-seasonal parts of the model.

- For example, an $\text{ARIMA}(1,1,1)(1,1,1)_4$ model for quarterly data (s=4) is:

$$(1-\varphi_1 L)(1-\Phi_1 L^4)(\mathbf{1}-\mathbf{L})(\mathbf{1}-\boldsymbol{L}^{\mathbf{4}})y_t = (1+\vartheta_1 L)(1+\Theta L^{\ 4})w_t$$

(part in bold font indicates integration)

Examples of SARIMA

- AR model with a seasonal period of 12 units:

$$y_t = \varphi y_{t-12} + w_t;\ \boldsymbol{ARIMA(0,0,0)(1,0,0)_{12}}$$

- Only the value in the month of the previous year influences the current monthly value. The model is stationary when $|\alpha^{-1/12}| > 1$.

- A simple quarterly seasonal moving average model with a stochastic trend in seasonal component is defined:

$$y_t = y_{t-4} + \beta w_{t-4} + w_t;\ \boldsymbol{ARIMA(0,0,0)(0,1,1)_{4}}$$

- Naturally, the seasonal terms can be mixed with any combination of simple ARIMA(p,d,q) terms

Stochastic seasonality

- In practice, seasonality is often accounted for by the inclusion of seasonal dummy variables or by the use of seasonally adjusted data. However, there may be instances where allowing a seasonal component to drift over time is necessary. This data-generating process is termed stochastic seasonality.

- We can generalize unit root processes to:

$$y_t = y_{\boldsymbol{t-s}} + \varepsilon_t$$

where s ≥ 1.

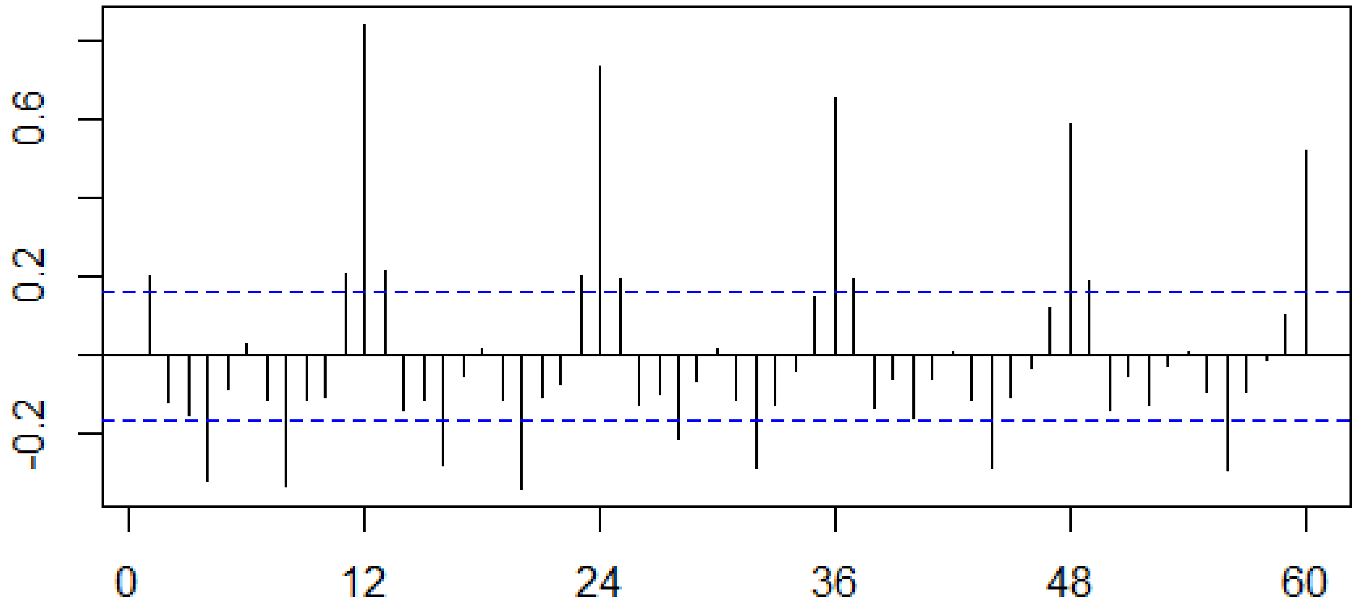
- In this case y_t is determined by its prior seasonal values plus noise.

- This type of process can be addressed with differencing. We can define the lag operator for seasonal unit roots as

$$(1 - L^s) = (1 - \mathrm{L})(1 - L + L^2 \ldots + L^{s-1}) = \Delta s(L)$$

SARIMA and autocorrelation function

- The non-stationary seasonality can be detected using autocorrelation function. The non-stationary series will exhibit pattern of slow decay at appropriate periods. The graph bellow presents non-stationary series with annual seasonal effect:



HEGY test for unit roots

- Testing for seasonal unit roots is a bit more complicated than testing for unit root. There are several tests, but we will focus on HEGY test.

- Assume we have quarterly data. For quarterly data, the seasonal difference operator results in this factorization:

$$(1 - L^4) = (1 - L)(1 + L)(1 - iL)(1 + iL)$$

- Seasonal quarterly process therefore has four possible roots, namely 1, −1, and ±i. These roots correspond to different cycles. The root 1 has a single-period cycle, the root −1 has a two-period cycle (for quarterly data a biannual cycle – twice a year); the complex roots have a cycle of four periods (annual cycle).

The problem caused by the complex roots for quarterly data is that their effects are indistinguishable from each other (so, we will test them jointly).

- Hylleberg, Engle, Granger, Yoo[1990], propose the following test regression (in this case for quarterly data):

$$\Delta_4 y_t = \sum_{i=1}^{4} \pi_i Y_{i,t-i} + \varepsilon_t$$

HEGY test for unit roots

- For the previous expression, the regressors $Y_{i,t-i}$ for $i\ =\ 1,\ldots,4$ are constructed as:

    - $Y_{1,t} = (1+L)(1+L^2)y_t = y_t + y_{t-1} + y_{t-2} + y_{t-3}$
    - $Y_{2,t} = (1-L)(1+L^2)y_t = y_t + y_{t-1} - y_{t-2} + y_{t+3}$
    - $Y_{3,t} = -(1-L)(1+L)y_t = -y_t + y_{t+2}$
    - $Y_{4,t} = Y_{3,t-1} = -(L)(1-L)(1+L)y_t = -y_{t-1} + y_{t-3}$

- Similarly to Dickey-Fuller test, the null hypothesis of seasonal integration implies that the coefficients π_i for $i\ =\ 1,\ldots,4$ are equal to zero.

- However, each π_i has a different interpretation

HEGY test - interpretation

- If only π_1 is significantly negative, then there is no non-seasonal stochastic stationary component
- If only π_2 is significant, then there is no evidence of a biannual cycle in the data
- The significance of π_3 and π_4 can be tested jointly with a Lagrange-multiplier F test, and if significant, then there is no evidence of annual unit root
- **Thus, the existence of unit roots at the zero, biannual, and annual frequencies correspond to $\boldsymbol{\pi_1 = 0}$, $\boldsymbol{\pi_2 = 0}$, and $\boldsymbol{\pi_3 = \pi_4 = 0}$, respectively**
- Deterministic terms, (intercept, trend, seasonal dummy variables) as well as lagged seasonal differences, can be added to the test regression

Additive vs. multiplicative SARIMA

- If a plot of data suggest that seasonal effect is proportional to the mean of the series, than the seasonal effect is probably multiplicative.

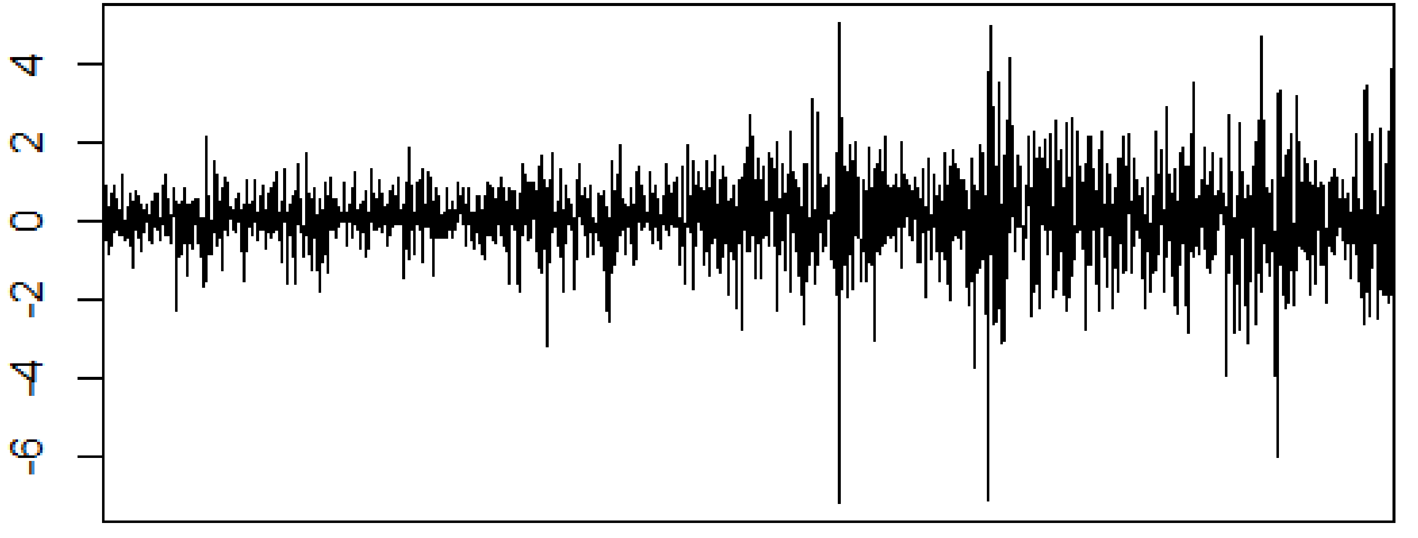
- Box, Jenkins, and Reinsel (2008) suggest starting with multiplicative SARIMA and then exploring non-multiplicative SARIMA.

- In essence, try both additive and multiplicative SARIMA and see which provides better fits and forecasts.

- Seasonal ARIMA models can potentially have a large number of parameters and combinations of terms. Therefore, it is appropriate to try out a wide range of models. Some confidence in the best-fitting model can be gained by deliberately overfitting the model by including further parameters and observing an increase in the AIC.

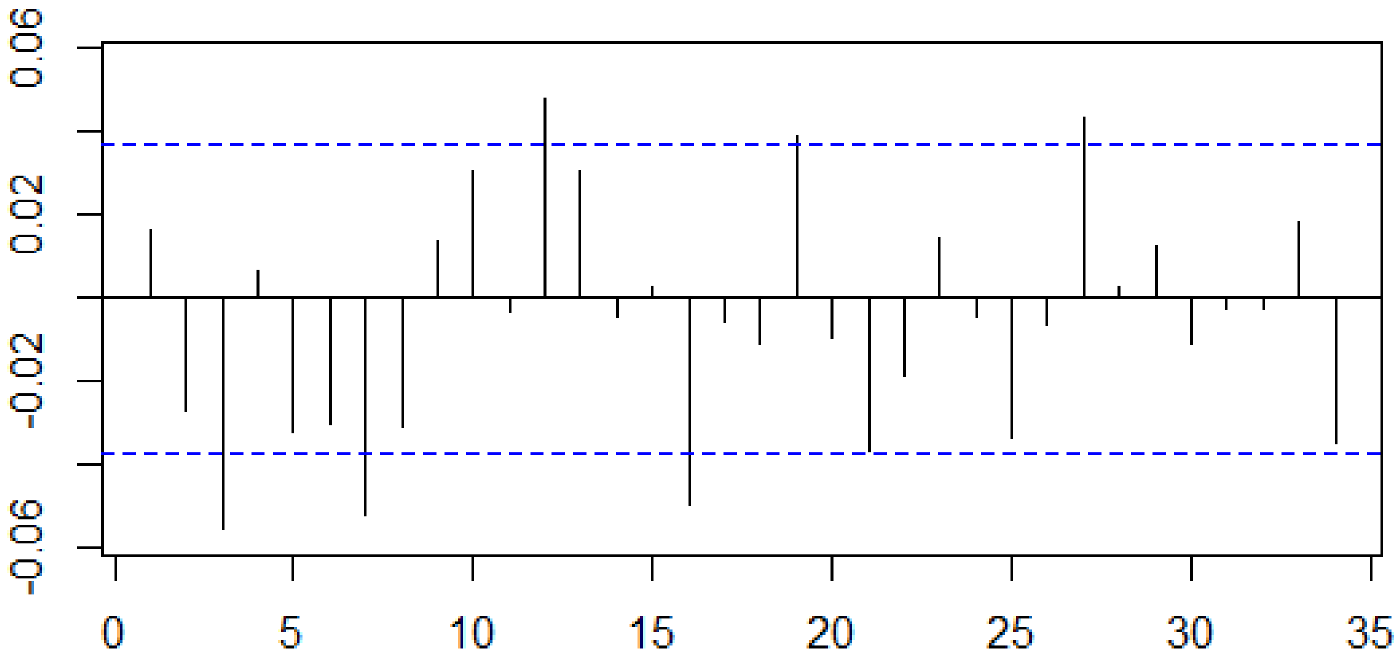
Autoregressive Conditional Heteroskedasticity: ARCH models

- Series may also be **non-stationar**y because the variance is serially correlated (conditional heteroskedasticity), which usually results in periods of volatility, where there is a clear change in variance. This is common in financial series, but may also occur in other series such as climate records.



ARCH and ACF

- The autocorrelation function of a volatile series does not differ significantly from white noise but the series is non-stationary since the variance is different at different times



Definition of ARCH

- Assuming, for instance, AR(1) model $y_t = \varphi y_{t-1} + \epsilon_t$, series ϵ_t is first-order autoregressive conditional heteroskedastic, ARCH(1), model if:

$$\epsilon_t = \sigma_t w_t = w_t \sqrt{\alpha_0 + \alpha_1 \epsilon_{t-1}^2}$$

where w_t is white noise with zero mean and α_0 and α_1 are model parameters.

- Engle (1982) suggested that, conditional variance of ϵ_t, that is **the square of ϵ_t, could follow AR(p) process.** So for ARCH(1):

$$\epsilon_t^2 = \sigma_t^2 w_t^2$$
$$\alpha_0 + \alpha_1 \epsilon_{t-1}^2 = \sigma_t^2$$

and subtract the two equations from each other to obtain

$$\epsilon_t^2 - (\alpha_0 + \alpha_1 \epsilon_{t-1}^2) = \sigma_t^2 w_t^2 - \sigma_t^2$$

$$\boldsymbol{\epsilon_t^2 = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \nu_t}$$

where $\nu_t = \sigma_t^2 (w_t^2 - 1)$

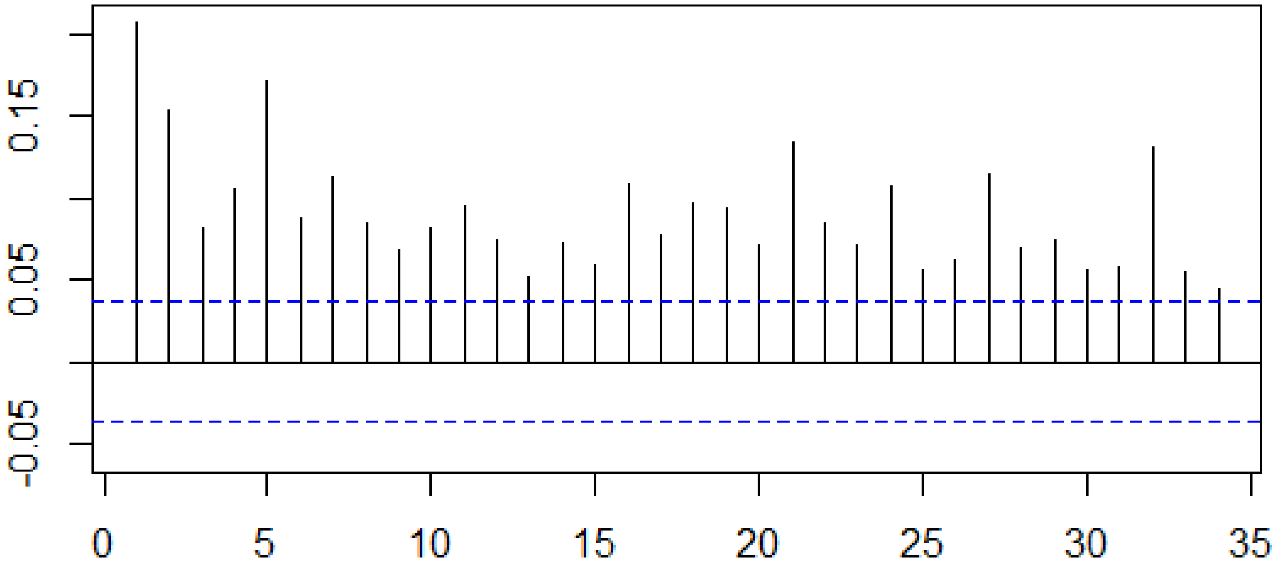
Characteristics of ARCH

- One obvious constraint is that α_1 must not be negative, or else ϵ_t^2 may be negative.
- For stationarity of ϵ_t^2, α_1 must be $0 < \alpha_1 < 1$, or for the ARCH(p) $\alpha_1 + \cdots + \alpha_p < 1$
- If condition $0 < \alpha_1 < 1$ stands, ν_t is white noise and its unconditional distribution is symmetrically distributed around zero
- A more general ARCH(p) process can be defined as:

$$\epsilon_t^2 = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \cdots . + \alpha_p \epsilon_{t-p}^2 + \nu_t$$

Detection of ARCH

- Volatility can be detected by looking at the correlogram of the squared values since the squared values are equivalent to the variance (provided the series is adjusted to have a mean of zero).



The lag length of ARCH errors

- Determining the lag of ARCH errors can be difficult. There are several possibilities.
- As ARCH ϵ_t^2 follows AR process, partial autocorrelation function may provide some indication of the appropriate lag length
- Overfitting is additional way to asses the appropriate lag length. Thus, start with ARCH(1) and continue adding lags until the last one is insignificant.
- However, in practice we tend to avoid processes higher than second order (in these cases we typically opt for GARCH model)

Lagrange multiplier test for the lag length of ARCH errors

- You can also use a Lagrange multiplier test. Estimate the best fitting autoregressive model AR(p)

- Obtain the squares of the error ε^2 and regress them on a constant and *p* lagged values: $\varepsilon^2 = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \cdots + \alpha_p\varepsilon_{t-p}^2$ where *p* is the length of ARCH lags.

- The null hypothesis is that, in the absence of ARCH components, we have $\alpha_i = 0$ for all $i = 1, \ldots p$.

$$\alpha_0 = \alpha_1 = \cdots = \alpha_p$$

- The alternative hypothesis is that, in the presence of ARCH components, at least one of the estimated α_i coefficients is not null in statistically significant manner.

Extension of ARCH

- Engle’s innovation sparked a variety of ARCH approaches The basic ARCH model addresses only the fundamental observation that volatility often varies over time, but other aspects of time varying volatility are not captured by the “vanilla” ARCH model. Some of the empirical regularities which researches seek to understand are:

1. The news (impactful new information) causes time volatility, however, many series appear to react differently to positive and negative news (bed news create more distress) - EGARCH

2. The conditional mean of observed time series often depends on the current level of volatility (e.g. level of stock prices often declines during periods of uncertainty) – ARCH-M

3. In many situations high order ARCH process is required to provide an adequate description of the time-varying volatility. However, high order processes are unwieldy and difficult to estimate precisely. The GARCH can provide a good fit with a lower-order parameterization

ARMAX

- ARIMA can be extended to include exogenous independent variables. In other words in addition to past values of y_t, we assume that values of y_t are also dependent on some explanatory variables. In terms of lag operator:

$$\varphi(L)y_t = \beta x_t + \vartheta(L)w_t$$

- We can extended the model to multiple independent variables:

$$y_t = \varphi_1 y_{t-1} + \cdots + \varphi_p y_{t-p} + \vartheta_1 w_{t-1} + \cdots + \vartheta_q w_{t-q} + \\ + \beta_1 x_{1t} + \cdots + \beta_n x_{nt} + \varepsilon_t$$

- Here β is **not** the effect on y_t when x_t increases by one. <u>The **presence of lagged values as regressors** means that β can only be interpreted conditional on the previous values of the dependent variable</u>.

ARMAX

- ARMAX can be understood in three forms:

- The **first form** is:

$$y_t = \frac{\beta}{\varphi(L)} x_t + \frac{\vartheta(L)}{\varphi(L)} w_t$$

- Here the AR coefficients get mixed up with both the covariates.

- The **second form** is:

$$y_t = \frac{\beta(L)}{\nu(L)} x_t + \frac{\vartheta(L)}{\varphi(L)} w_t$$

- By introducing $\beta(L)$ we allow for lagged effects of x_t as well as their decaying effect via the term $\nu(L)$. These models are called **transfer function models or dynamic regression models**. This is the most general expression of the ARMAX/ARIMAX model and other specifications can be understood as special cases of transfer function models

ARMAX

- This **third form** is specification of regression model with ARMA errors, defined as:

$$y_t = \beta x_t + \varepsilon_t$$

$$\varepsilon_t = \varphi_1 y_{t-1} + \cdots + \varphi_p y_{t-p} + \vartheta_1 w_{t-1} + \cdots + \vartheta_q w_{t-q} + w_t$$

- In terms of backshift operators, this model can be written as:

$$y_t = \beta x_t + \frac{\vartheta(L)}{\varphi(L)} w_t$$

- In this case, the regression coefficient has its usual interpretation

ARIMAX

- ARIMAX is extension of ARMAX to non-stationary data.

- In essence, ARIMAX requires differencing of both y_t and x_t before fitting the model with ARMA errors.

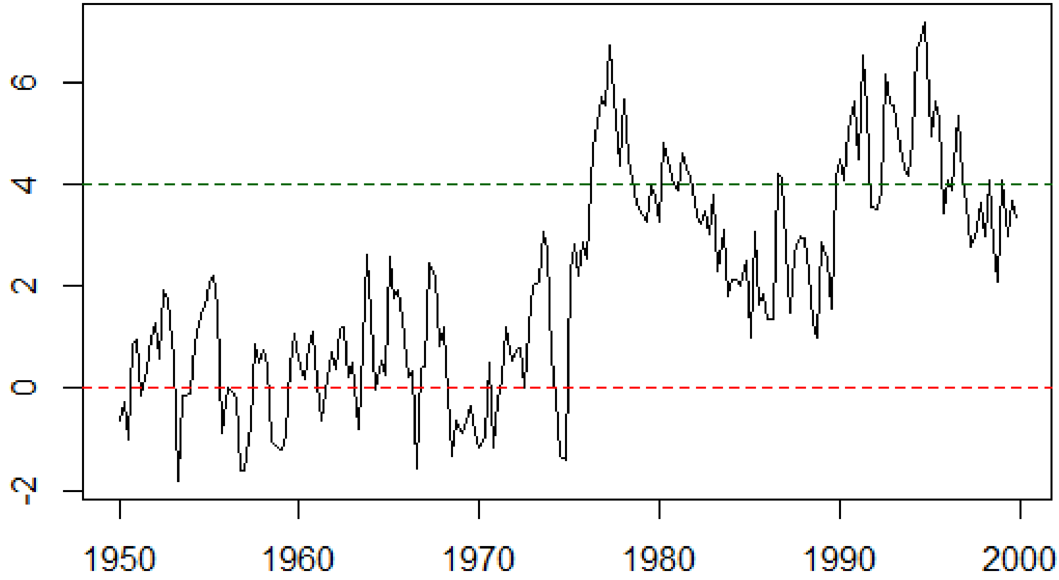
$$\varphi(L)\Delta y_t = \beta\Delta x_t + \vartheta(L)w_t$$

- Differencing of all variables is necessary otherwise estimation may not be consistent and can lead to spurious regression. What about interpretation of such a model?

- This also leads us to the concept of **cointegration**, which we will address in the next class

Regime changes

- ARMAX models can be extended to include regime changes



- Typically we account for the regime changes by including dummy variables

Endogeneity

- In ARIMAX modeling we were able to include exogenous variables. What should we do if we have endogenous variables?

- **The exogenous variables are those that are determined outside the system while those that are determined inside the system are endogenous variables. An exogenous change is one that comes from outside the model and is unexplained by the model**

- The presence of endogenous variables violates assumption of exogeneity $E\left[\varepsilon_i x_{jk}\right] = 0 \; \forall \; i, j$

- **Endogeneity is caused by**:
  - Omitted variable
  - Measurement error in independent variable
  - Causality (reverse/reciprocal/ feed-forward causality, simultaneity)

Causality as a cause of endogeneity

- The causality is the main reason why we may expect the problem of endogeneity in time series. Endogeneity means you got the causation wrong.

(we will omit the discussion with respect to the other sources of endogeneity)

The relationship among the variables may be endogenous in one of two senses:

1. Changes in one of the variables may have a delayed effect on another

2. The relationship may be contemporaneous in that changes to the system of equations (shocks or innovations) may change both or several variables at the same time. This arises because the shocks to one variable are correlated with the shocks to another variable (**simultaneity)**

Simultaneity in more details

- One situation in which the assumption that the error term is uncorrelated with each of the independent variables in a regression equation is guaranteed to be violated is when there is simultaneity.

- Simultaneity arises when one or more of the independent variables, is jointly determined with the dependent variable, typically through an equilibrium mechanism. Suppose that the equilibrium relation between X and Y is expressed by the following simultaneous equations:

$$y_t = \beta_0 + \beta_1 x_t + u_t$$
$$x_t = \alpha_0 + \alpha_1 y_t + v_t$$

- If we try to estimate any of the equations by substituting any of independent variables (e.g. x) as expressed in their equation form (e.g. $\alpha_0 + \alpha_1 y_t + v_t$), this will lead to simultaneity bias.

Simultaneous equations

- Simultaneous equation approach is developed in the 1940s and 1950s

- As above, simultaneous equation representation of the relationships among variables would have one equation for each variable. Each endogenous variable would be a function of the other and (possibly) past values of each variable

- Although the theoretical interest of an analyst may be on just a single equation, statistics requires that all equations be considered, otherwise inferences can be biased and inefficient - inferences can be made only with reference to the system as a whole

- *The simultaneous equations can be perceived as a subset of more general structural equation models.*
- *Typically, these type of problems are solved using instrumental variables approach or related methods (however, we will not discuss these approaches in this course)*

Conditions for estimating simultaneous equations

- The models require that choices be made about the inclusion or exclusion of different variables and lagged values to ensure identification. Two methods are common:

1. restricting “predetermined” or lagged endogenous variables as exogenous variables (here, “theory” is used to restrict the parameter space of the model parameters)

2. the classification of variables as either endogenous or exogenous.

Structural form

- The structural form is formulated as:

$$\boldsymbol{y_{1,t}} = \alpha_1 \boldsymbol{y_{2,t}} + \varphi_{11}\, y_{1,t-1} + \varphi_{12} y_{2,t-1} + w_{1,t}$$
$$\boldsymbol{y_{2,t}} = \alpha_2 \boldsymbol{y_{1,t}} + \varphi_{21} y_{1,t-1} + \varphi_{22} y_{2,t-1} + w_{2,t}$$

with:

$$w_{i,t} = N\left(0, \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix}\right)$$

- The **simultaneity** in the model results from the fact that each variable depends on the <u>contemporaneous value</u> of the other variables in the model.
- The **dynamics** arise from the <u>lagged values</u>.

- We refer to the variables $y_{1,t}$ and $y_{2,t}$ as the endogenous variables.

- Variables $y_{1,t-1}$ and $y_{2,t-1}$ can be either exogenous variables or lagged endogenous variables whose values are already known. Thus, endogenous variables act both as dependent and independent variables, while exogenous variables act purely as independent variables.

Simultaneity bias

- To be able to estimate this system of equations, we must substitute one of the equations into the other, because at least one of the equations is necessary to determine the other. So, for $y_{1,t}$, we get:

$$y_{1,t} = \alpha_1\left(\boldsymbol{\alpha_2 y_{1,t} + \varphi_{21} y_{1,t-1} + \varphi_{22} y_{2,t-1} + w_{2,t}}\right) \\ + \varphi_{11} y_{1,t-1} + \varphi_{12} y_{2,t-1} + w_{1,t}$$

$$y_{1,t} = \alpha_1\alpha_2 y_{1,t} + \alpha_1\varphi_{21} y_{1,t-1} + \alpha_1\varphi_{22} y_{2,t-1} + \alpha_1 w_{2,t} \\ + \varphi_{11} y_{1,t-1} + \varphi_{12} y_{2,t-1} + w_{1,t}$$

- Moving $\boldsymbol{\alpha_1\alpha_2 y_{1,t}}$ to the left side and collecting terms, we get:

$$y_{1,t}(1-\alpha_1\alpha_2) = (\alpha_1\varphi_{21} + \varphi_{11}) y_{1,t-1} + \\ (\alpha_1\varphi_{22} + \varphi_{12}) y_{2,t-1} + \alpha_1 w_{2,t} + w_{1,t}$$

- Dividing with $(1-\alpha_1\alpha_2)$, we get <u>reduced form</u>:

$$y_{1,t} = \underbrace{\frac{\alpha_1\varphi_{21} + \varphi_{11}}{1-\alpha_1\alpha_2}}_{\boldsymbol{\Pi_{11}}} y_{1,t-1} + \underbrace{\frac{\alpha_1\varphi_{22} + \varphi_{12}}{1-\alpha_1\alpha_2}}_{\boldsymbol{\Pi_{12}}} y_{2,t-1} + \underbrace{\frac{\alpha_1 w_{2,t} + w_{1,t}}{1-\alpha_1\alpha_2}}_{\boldsymbol{\varepsilon}}$$

$$y_{1,t} = \boldsymbol{\Pi_{11}}\ y_{1,t-1} + \boldsymbol{\Pi_{12}}\ y_{2,t-1} + \boldsymbol{\varepsilon}$$

Simultaneity bias

- So, the result of our substitution of one of the equations into the other was

$$y_{1,t} = \frac{\alpha_1\varphi_{21} + \varphi_{11}}{1-\alpha_1\alpha_2} y_{1,t-1} + \frac{\alpha_1\varphi_{22} + \varphi_{12}}{1-\alpha_1\alpha_2} y_{2,t-1} + \frac{\alpha_1 w_{2,t} + w_{1,t}}{1-\alpha_1\alpha_2}$$

(Analogous transformation can be performed for $y_{2,t}$)

- Obviously, due to $\frac{\alpha_1 w_{2,t}+w_{1,t}}{1-\alpha_1\alpha_2}$ $y_{1,t}$ depends on $w_{2,t}$. Nevertheless, $w_{2,t}$ is residual from equation for $y_{2,t}$, and thus correlated to $y_{2,t}$.

- If one perform OLS to the original structural equation

$$\boldsymbol{y_{1,t}} = \alpha_1 \boldsymbol{y_{2,t}} + \varphi_{11}\, y_{1,t-1} + \varphi_{12} y_{2,t-1} + w_{1,t}$$

it will lead to called <u>simultaneity bias </u>because $w_{1,t}$ incorporates $w_{2,t}$ and:

$$Cov(y_{2,t}, w_{1,t}) \neq 0$$

(Analogous problem is inherent to $y_{2,t}$ as $Cov(y_{1,t}, w_{2,t}) \neq 0$)

Reduced form

- As we saw earlier we can reformulate the structural form into reduced form:

$$y_{1,t} = \Pi_{11} y_{1,t-1} + \Pi_{12} y_{2,t-1} + \varepsilon_{1,t}$$
$$y_{2,t} = \Pi_{21} y_{1,t-1} + \Pi_{22} y_{2,t-1} + \varepsilon_{2,t}$$

- Reduced form expresses the y variables **solely in terms of exogenous variables**. This equation can be estimated **consistently** by OLS.

- However, the reduced form parameters are not parameters of interest. We are really interested in the structural model coefficients, which must then be derived from the reduced form

- In structural form we had six parameters:

$$y_{1,t} = \alpha_1 y_{2,t} + \varphi_{11}\, y_{1,t-1} + \varphi_{12} y_{2,t-1} + w_{1,t}$$
$$y_{2,t} = \alpha_2 y_{1,t} + \varphi_{21} y_{1,t-1} + \varphi_{22} y_{2,t-1} + w_{2,t}$$

- and we need to recover the six-parameter model from the four-parameter model above (in this particular case).

Identification

- We can retrieve the estimates of structural parameters from the reduced form coefficients. In this case it is said that the model is **identified**. To **identify** the model we must make certain assumptions .

- **Which assumptions we make affects our inferences about the parameters in the structural model and our description of the dynamics.**

- Assume that want to get an estimate of α_1 for first equation. A consistent estimate of α_1 requires that there is no feedback from second equation for $y_{2,t}$ into first equation that would correlate with their parameters. Mathematically, this condition is $\alpha_2 = 0$ so that $\mathrm{E}\left[y_{2,t}, w_{1,t}\right]$ would then equal zero.

- In addition, to know if an OLS estimate of the parameter α_1 is consistent, we also need to know if the estimate of $\Pi_{21} = 0$, so that there is no feedback in the system of equations that would invalidate the OLS estimates of α_1 via the past values of this variable

Restrictions of simultaneous equation models

- Assume three scenarios:
  1. $\varphi_{21} \neq 0$ but $\alpha_2 = 0$
  2. $\varphi_{21} = -\alpha_2\varphi_{11}$, so that $\Pi_{21} = 0$ but $\alpha_2 \neq 0$
  3. $\varphi_{21} = 0$ and $\varphi_{11} = 0$ but $\alpha_2 \neq 0$
- These restrictions will result:
  1. Here there is no feedback, but the value of Π_{21} would be φ_{21}. Thus,we are assuming that only the past values of $y_{1,t}$ matter for predicting $y_{2,t}$.
  2. The coefficients for the past and the present values of $y_{1,t}$ cancel each and are not useful for predicting $y_{1,t}$or $y_{2,t}$. Only the contemporaneous correlation of the two variables describes their dynamics. However as $\alpha_2 \neq 0$ the estimate is inconsistent.
  3. The past values of $y_{1,t}$ have no predictive value in either equation—all of the explanatory power of $y_{1,t}$ for $y_{2,t}$ is in terms of the contemporaneous values.

Validity of identification assumptions

- The formal identification of a dynamic simultaneous equation model requires that the exact true lag length be known for each variable; otherwise, identification assumptions may not hold. However, the true lag lengths of the variables are not known a priori

- Identification restrictions on parameters used in SEQ models are typically not based on theory and thus may lead to incorrect conclusions

  - What is to say that some lagged variables would not be in each equation?
  - Does restricting the dynamics for identification make sense?
  - If a variable affects one equation in the system of equations, what is to say that it does (or does not) affect another?

Vector autoregressive models

- The choice of restrictions is primarily driven by the need to achieve identification for consistent estimation. **As argued by Sims (1980), such exclusion restrictions are often not theoretically justified and are often not well supported by empirical analysis**

- However, this is not necessary. We can analyze just the reduced form – without the possibly incorrect restrictions necessary to identify the structural model. In this case, we focus on the dynamic relationships of the variables and allow for a myriad of possible contemporaneous relationships

- This is the approach assumed in <u>**vector autoregressive models**</u>