Team Control Number

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Problem Chosen

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2014 Mathematical Contest in Modeling (MCM) Summary Sheet
(Attach a copy of this page to each copy of your solution paper.)

Abstract

Traffic rule has a great impact on the efficiency and safety of a traffic system. Thus to build a model to evaluate the rule is of great importance. In most countries, there exists such a rule: unless overtaking another car, drivers are supposed to drive on the right-most lane. In our passage, we are trying to develop a model to evaluate this rule from two aspects: the traffic flow and the safety. We construct a blank situation that is without the right-most rule. Using our model, we compare the safety and the traffic flow under the two situation, with the right-most rule and without the right-most rule.

To evaluate the safety, we build a model to calculate the safety distance. If the distance between two car is less than the safety distance. The collision will happen. The less the probability of the collision is, the safer. So we use the probablity of the collision to evaluate the

title

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Contents

1 Introduction

In bidirectional traffic, right- and left-hand traffic requires vehicles keep either to the right or the left side of the road, respectively.[1] The first right-hand traffic law in United State dates back to 1792, applied to the Philadelphia and Lancaster Turnpike. [2]

The right-hand traffic rule derives regulations on multi-lane freeways, which often requires drivers to drive in the right-most lane unless they are passing another vehicle, in which case they move one lane to the left, pass, and return to their former travel lane. The overtaking lane provides redundant convenience and safety for vehicles to pass others, but lowers freeway's traffic flow capability.

Different models may exist that provide a better tradeoff among traffic flow, safety and convenience. This paper considers these three criteria, introduces a different freeway traffic model and analyses performances of the two models.

2 Performance Evaluation

Performance of a traffic rule is determined by the three creteria below:

Safety xxxxx

Traffic Flow xxxxxxxxx

Convenience xxxxxxx

3 Safety Factor

Assumptions

The value of acceleration subjects to normal distribution, $\mu = \frac{max(a)}{2}$, $\sigma = \frac{1}{4}\mu$, for $p(0 < a_2 < \frac{1}{4}\mu)$ and $p(\frac{7}{4}\mu < a_2 < 2\mu)$ is very small.

3.1 Safe Distance Model

On the highway, if the car in front suddenly slows down, it is likely that a car crash happens. We build this model to calculate the safe distance between two adjacent cars. That is, if the adjacent $d \leq l$ and the car in front suddenly slows down,the accident will not happen.

There are two possible situations that the accident will happen:

- The accident happens after the car in front stopped. That is, when

$$\frac{v_2}{a_2} \geq \frac{v_1}{a_1} + t_r$$

Parameter	Description
a_1	the acceleration of the car in behind
v_1	the velocity of the car in front
a_2	the acceleration of the car in front
v_1	the velocity of the car in front
t_r	the driver's reaction time
t	the total time whole process
l	the safe distance
d	a constant which is the real distance between the car in front and the car in behind.

Table 1: Model parameter

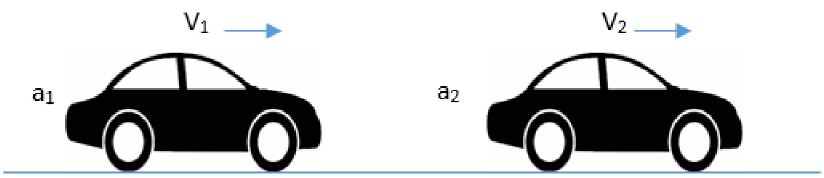

Figure 1:

We have:

$$\begin{aligned} \frac{v_1^2 - v_0^2}{2a_1} + v_1 t_r - l &= \frac{v_2^2 - v_0^2}{2a_2} && (1) \\ \frac{v_2 - v_0}{a_2} &= t && (2) \\ \frac{v_1 - v_0}{a_1} &= t - t_r && (3) \end{aligned}$$

- The collision happens before the car in front stopped. That is, when

$$\frac{v_2}{a_2} > \frac{v_1}{a_1} + t_r$$

We have:

$$v_1 t_r + \frac{v_1^2}{2a_1} - l = \frac{v_2^2}{2a_2} \quad (4)$$

Solve this equation array, we have

$$l(a_1, a_2, v_1, v_2) = \begin{cases} \dfrac{(a_1 a_2 t_r^2 + 2a_1 t_r v_2 + v_1^2 - 2v_1 v_2 + v_2^2)}{2(a_1 - a_2)} & \dfrac{v_2}{a_2} \geq \dfrac{v_1}{a_1} + t_r \\ v_2 t_r + \dfrac{v_1^2}{2a_1} - \dfrac{v_2^2}{2a_2} & \dfrac{v_2}{a_2} < \dfrac{v_1}{a_1} + t_r \end{cases}$$

To calculate l_{max}, firstly, we are going to find the extrema,

$$\begin{cases} \dfrac{\partial l}{\partial a_1} = 0 \\ \dfrac{\partial l}{\partial a_2} = 0 \\ \dfrac{\partial l}{\partial v_1} = 0 \\ \dfrac{\partial l}{\partial v_2} = 0 \end{cases}$$

This equation array has no solution, which indicates that we can get the l_{max} at the boundary of domain.

l reaches the maximum when a_1 equals to a_{min}, v_1 equals to v_{1max}, v_2 equals to v_{2min}. (If the front car drives with the lowest speed, the rear car drives with the highest speed, and at the same time, rear car's acceleration is the minimum, when emergency happens, the braking distance is the longest.)

Thus,

$$l_{max}(a_2) = \begin{cases} \dfrac{(a_{min}a_2t_r^2 + 2a_{min}t_rv_{min} + v_{max}^2 - 2v_{max}v_{min} + v_{min}^2)}{2(a_{min} - a_2)} & \dfrac{v_{min}}{a_2} \geq \dfrac{v_1}{a_1} + t_r \\ v_{min}t_r + \dfrac{v_{max}^2}{2a_{min}} - \dfrac{v_{min}^2}{2a_2} & \dfrac{v_2}{a_2} < \dfrac{v_1}{a_1} + t_r \end{cases}$$

l **Distribution** From the result above, we can get $a_2(l)$

$$a_2(l) = \begin{cases} -\dfrac{v_{max}^2 - 2v_{max}v_{min} + 2a_{min}t_rv_{max} + v_{min}^2 - 2a_{min}t_rv_{min} - 2la_{min}}{a_{min}t_r^2 + 2l}, \\ \quad l \leq \dfrac{v_{max}^2 - v_{max}v_{min} + 2v_{max}a_{min}t_r - a_{min}v_{min}t_r}{2a_{min}} \\ \dfrac{a_{min}v_{min}^2}{v_{max}^2 + 2a_{min}t_rv_{max} - 2la_{min}}, \quad l > \dfrac{v_{max}^2 - v_{max}v_{min} + 2v_{max}a_{min}t_r - a_{min}v_{min}t_r}{2a_{min}} \end{cases}$$

Safety Factor α For every car, it has a unique safety factor α, the closer the distance between two adjacent cars is, the smaller the α is, vice versa.

$$\alpha(l) = \int_0^l f(x)dx.$$

We can see that , when a_2 = 0, that is, the front car has the lowest deceleration. Since $l(a)$ is a monotone increasing function, at this time,l is minimal. In other word, it is of little possibility that the distance is less than the l, that is, $\alpha(l) \to 0$

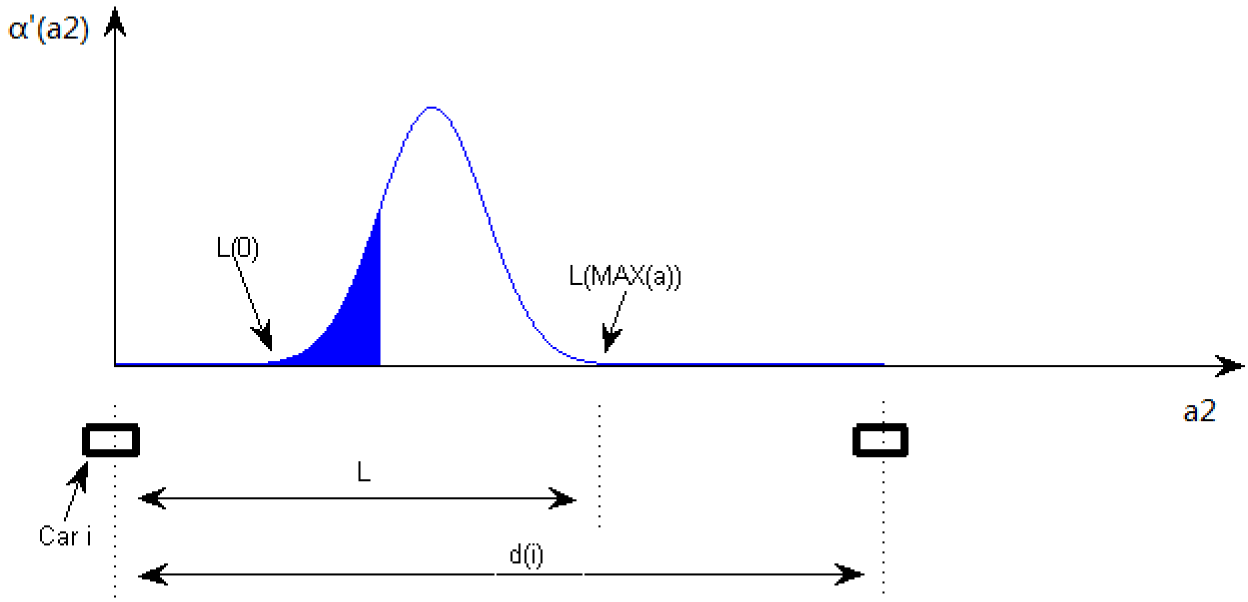

Figure 2: safety factor distribution

4 Traffic Flow Factor

Since the layout of the cars on the freeway is complicated, to simplify, we use a special model to discuss the relationship between the safety and traffic flow under the situation with and without the right-most rule.

Assumption

$$J^* = J_{supply} - J_{need} \tag{5}$$

$$J^* = \begin{cases} 0, \\ \bar{l} - \bar{l}_{max} \end{cases}$$

$$\beta = A\beta_1 + B\beta_2$$

Parameter	Discription
k	The traffic density
k_j	The traffic density under the traffic jam condition
k_m	The traffic density when the the traffic flow is the highest.
v_f	The velocity when the traffic density is small
Q	Traffic flow

Table 2: Model parameter

Parameter	Meaning
J	traffic flow
J_1	the supply traffic flow
J_2	the demand traffic flow
s	the lenth of a car
l	the safe distance
$\bar{v}$	the average velocity

Table 3: Model parameter

5 Convenience Factor

6 Model Analysis

7 Conclusions

8 Discussion

8.1 Strengths

8.2 Weaknesses

References

[1] Draper, Geoff (1993). "Harmonised Headlamp Design for Worldwide Application". Motor Vehicle Lighting. Society of Automotive Engineers. pp. 23-36.

[2] Weingroff, Richard. "On The Right Side of the Road". United States Department of Transportation. Retrieved 10 January 2014.

[3] Left is right on the road', Mick Hamer New Scientist, 25 December 1986 - 1 January 1987 No 1540/1541, p.16.