# Should we give Solow another chance? Testing human capital proxies

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**Abstract.** In this paper, we tested an augmented Solow model with human capital on cross-sectional data with various proxy variables for human capital. We found that the assumed output elasticities of the production factors coincide with the real data in the periods between 1960 and 2000. In later periods, the estimated impact of physical capital was overestimated. We have identified the source of this overestimation in the imperfect selection of the proxy variable for human capital. We tested the Solow model with four different proxy variables for human capital on two samples of countries in the period 1990-2015. We found that using the proportion of people over the age of 15 with at least some tertiary education and the average length of schooling, our estimates are very close to the predicted values from the article by Mankiw, Romer, and Weil [15].

**Keywords:** Solow model, Human capital, Cross-sectional data

**JEL classification:** O47, I25

## 1 Introduction

Mankiw, Romer and Weil (hereinafter referred to as 'MRW') “show that an augmented Solow model that includes accumulation of human as well as physical capital provides an excellent description of the cross-country data“ [15]. In this paper, we have estimated the basic Solow model based on the MRW approach over a moving 25-year period between 1960-2017. We have found that the impact of physical capital is significantly overestimated and increases over time.

The abovementioned authors dealt with the problem of overestimated impact of physical capital by incorporating human capital, which they identified with an education measured by the share of enrolled students in secondary school in the working age population [15]. This choice of a proxy variable for human capital has been criticized by several authors [9], [16]. For the basic replication of the model, we therefore chose data on the proportion of people older than 15 years who completed at least some education at a secondary school. We estimated the augmented Solow model on a moving 25-year period in the years 1960-2015. The output elasticity of human capital oscillates around the predicted value of 0.3 during all periods. The impact of physical capital is slightly lower in the first half of the period compared to the predicted values and in the second half it is slightly overestimated in both samples of countries. As a potential source of overestimation of the effect of physical capital, we identified an imperfect choice of a proxy variable for human capital. It is possible that with the increase in automation of many activities in the last two decades, the share of the secondary school population no longer captures a significant part of the human capital used in production. Based on this hypothesis, we estimated the augmented Solow model in the period 1990-2015 with various proxy variables for education. We have come to a conclusion that our estimates are closest to the MRW predictions using the proportion of people over the age of 15 with at least some tertiary education and the average length of schooling.

## 2 Review of literature

In 1956, Robert Solow introduced his theoretical model of long-term economic growth [17]. According to the Solow model, whether a country is rich or poor depends on its savings rate and population growth. The accumulation of household savings creates investments that result in new physical capital. Solow presented several possible extensions of the model, one of which was exogenous technological progress, which multiplies the influence of production factors on the output of the economy.

Mankiw, Romer, and Weil took Solow's work seriously and derived testable hypotheses from the Solow's “textbook” model, which they verified by using standard econometric techniques [15]. After the incorporation of human capital, they found out that the extended Solow model well describes the cross-sectional data in the years 1960-1985.

The MRW paper provoked a lot of discussion and was criticized mainly for the chosen econometric framework. According to Islam, the chosen approach based on cross-sectional data is inappropriate due to an omitted variable bias and due to a non-control for country specific shocks [12]. He dealt with these problems using dynamic panel data models but did not experience similarly convincing results as MRW. The issue with the endogeneity of explanatory variables was addressed by Caselli et al. [6]. Based on the GMM estimator, they estimated a “textbook” and an augmented Solow model. Likewise, as with Islam, the resulting estimates did not agree with the basic predictions of the Solow model.

Several authors have investigated the robustness of the Solow model using various samples of countries divided into groups by per capita income [4], [18]. Brumm focused exclusively on the analysis of cross-sectional data and did not confirm the robustness of the augmented Solow model. Similar conclusions were reached by Temple, who analyzed panel data in addition to cross-sectional data and showed that "estimated technology parameters and convergence rates are highly sensitive to measurement error" [18].

Although MRW showed that the extended Solow model describes data well for large samples of countries, the results for the OECD group were not so convincing. According to MRW, this implies a greater distance from the steady state in OECD countries [15]. This problem was addressed in more detail by Nonneman and Vanhoudt, who saw the reasons for these shortcomings in the excessive similarity of the countries, or in the possible non-inclusion of an important variable in the model [16]. Based on this observation, authors added an endogenous accumulation of technological know-how to the augmented Solow model, which was then able to explain up to 3/4 of the total differences in per capita income. Another approach was taken by Canarella and Pollard [5]. For their estimates, they used a newer version of the Penn World Table database, which has undergone several methodological adjustments. They found that, compared to MRW estimates, the explanatory power of the Solow model has increased significantly in OECD countries.

The literature on the selection of a proxy variable for human capital is not as broad as the one on testing the Solow model. Knowles and Owen have shown in a wide sample of countries that human capital in the form of health has a greater impact on per capita income than human capital in the form of education has [13]. In cross-sectional data from 50 countries, Hanushek and Woessmann pointed out that the impact of quality of education (measured by international student tests) has a significantly greater impact on economic growth than variables reflecting the quantity of education (average number of years of education) [10]. Breton criticized this approach and estimated the augmented dynamic Solow model on panel data with various proxy variables for human capital [3]. Author focused mainly on comparing the variable on the quality of education from Hanushek and Woessmann's article with education expenditures. He found that improvements in student tests only increase GDP growth rates for countries with low average school attendance (less than 7.5 years) in contrast to spending on education, which, according to Breton, increases growth even in countries with longer average school attendance.

To summarize: using a cross-sectional approach, the results of the augmented Solow model are largely consistent with the basic predictions, with a significant impact of both physical and human capital. Using panel data, the results seem much more sensitive to the sample of countries used and to the econometric techniques chosen.

## 3 Methodology and data

As already mentioned, analysis of the Solow model based on panel data does not provide results similar to the basic predictions, and the impact of human capital is rarely statistically significant. A panel approach in examining various proxy variables for human capital in the augmented Solow model has been applied in e.g. Hojdan [11]. The robust significant impact of human capital has not been confirmed in international PISA tests either.

Criticism of the MRW model estimates using OLS was largely associated with omitted variable bias. The omission of other variables is addressed by Breton, who argues that other variables such as institutions, culture, or policy variables determine the amount of national income through the accumulation of physical and human capital [2]. Several studies that have examined the robustness of determinants of economic growth confirm this hypothesis [14], [7]. Authors such as Islam, using panel data and various estimators tried to deal with other econometric issues such as assumption of homogeneity of production functions across countries [12]. Although panel data models are better able to deal with econometric problems, it should be emphasized that the Solow model is a model of long-term economic growth. The selection of the length of periods when testing the Solow model is in our view essential. The discrepancy between estimates on different types of data may be the result of the rigidity of school systems, which can only change very slowly. Based on this consideration, we therefore chose to use the cross-sectional approach with OLS estimator for our analysis.

Data on the real GDP at chained PPPs (in 2011 US $) were obtained from the Penn World Table 9.1. For the share of investment in GDP, we used data from the United Nations on Gross fixed capital formation as % of GDP (the problem here is that they are available only since 1970). Like MRW, we divided the real GDP by data on the population aged 15-64, which we obtained from the World Development Indicators database. From these data, we also calculated a population growth. As a primary source of data on education, we used the Barro and Lee database from which we gathered data on the proportion of people older than 15 years who completed at least some secondary and tertiary attendance [1]. From the same dataset, we have drawn the data on the average length of education. The human capital index from Penn World Table 9.1 was chosen as another variable. This indicator is composed as the number of years of education and the estimates of returns for individual years of schooling.

In this paper, we used two samples of countries. Like MRW, we used groups of countries called non-oil and intermediate countries. The first sample is for countries where oil extraction is not the dominant sector. In the case of the intermediate group, countries with a population of less than 1 million were removed from the non-oil sample (referring to the period when the MRW paper was published). Some data were not available, especially for some African countries, resulting in an imperfect replication of samples from the MRW article[1].

## 4 Empirical results

In the first part of this chapter, we tested the predictions of the Solow model about the output elasticities of production factors over a moving 25-year period from 1960 to the present. We found that the augmented Solow model with human capital, coincides with the basic predictions during all periods. We have found that the augmented Solow model with human capital is relatively consistent with the fundamental prediction during all periods. In the second part, we tested various proxy variables for human

[1] Datasets we worked with, including .do files in the Stata, can be found at: https://www.dropbox.com/s/lkm6dx5lsprnqz4/Hojdan%20%282020%29%20edamba.rar?dl=0

capital. The human capital index has the largest estimated impact on GDP per worker. In contrast, models with a share of people with tertiary education and an average number of years of schooling are closest to the predictions of the Solow model.

### 4.1 Testing the Solow model over time

As a starting point for our analysis, we chose to test the basic Solow model without human capital. The main parameter on the basis of which we can verify the validity of the model is the so-called output elasticity of physical capital α. Essentially, it indicates how the output of the economy will increase if we involve another unit of physical capital in the production process. The general rule and also the MRW prediction for the Solow model is a value of α about 0.3.

To test the prediction of the model on real data, it is necessary to derive its econometric form according to the MRW model:

$$\ln\left(\frac{Y(t)}{L(t)}\right) = c + \frac{\alpha}{1-\alpha}\left(\ln(s_k) - \ln(n+g+\delta)\right) + \epsilon \tag{1}$$

where $Y/L$ is GDP per worker, $s_k$ is the savings rate (in our case the average share of gross fixed capital formation in GDP), $n$ is population growth (an average growth of the labor force), $g$ expresses technological progress and $\delta$ is a depreciation of capital. Like MRW, we assume that $g + \delta = 0.05$. The constant $c$ expresses the initial state of technology in the countries and $\epsilon$ expresses the random component. We estimated this econometric model on samples of two countries using constrained regression on cross-sectional data. We then calculated the average values of the parameter α and the corresponding confidence intervals. We repeated this process 34 times for both samples of countries and on all possible 25-year periods (the length of the periods was chosen according to MRW) between 1960 and 2017. The average elasticities are shown in Figure 1. First, we can notice that the estimated parameter α based on our data slightly

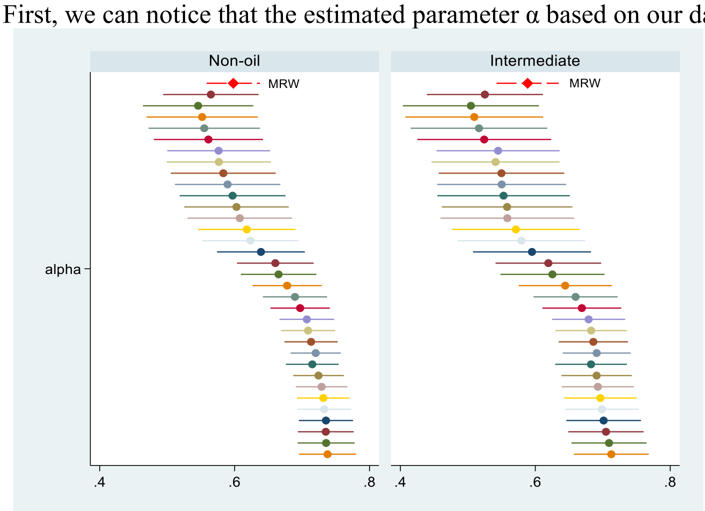


**Fig. 1:** Evolution of the estimated output elasticity of physical capital over time. The first point corresponds to the MRW estimates. Estimates are displayed from the top to the bottom in the chronological order. *Source: Author's calculations*

differs from the parameter estimated by MRW (first and second observations from above). This is mainly due to the corrections that databases have undergone over the last decades. Based on our results, we came to the same conclusion as MRW that the basic Solow model significantly overestimates the impact of physical capital. As we can see, this overestimation increased over time to a level close to 0.8. This large difference from the predicted value implies omitting an important variable in the model.

MRW have tackled this problem by extending the Solow model to another factor of production - human capital. As with the basic model, in this case we can derive an econometric equation for the extended Solow model in the form, where we can calculate the output elasticities after estimates:

$$\ln\left(\frac{Y(t)}{L(t)}\right) = c + \frac{\alpha}{1-\alpha-\beta}(ln(s_k) - \ln(n+g+\delta)) + \frac{\beta}{1-\alpha-\beta}(\ln(s_h) - \ln(n+g+\delta)) + \epsilon \quad (2)$$

The parameter $\beta$ symbolizes the output elasticity of human capital and the variable $s_h$ shows the rate of investment in human capital according to the MRW model (in our case we used the share of the population older than 15 years who completed at least some secondary school attendance). We estimated the effects of individual variables using constrained regression and then we calculated the average output elasticities $\alpha$ and $\beta$. In this case, we repeated this process 32 times on all possible 25-year periods between 1960 and 2015. The average elasticities and the corresponding 95% confidence intervals are shown in Figure 2. The average values of β oscillate very close to the value predicted by MRW in all time periods and in both samples of countries. With the output elasticity of physical capital, our estimates are slightly underestimated in the first half of the periods under review whereas in the second half we may observe a more significant overestimation.

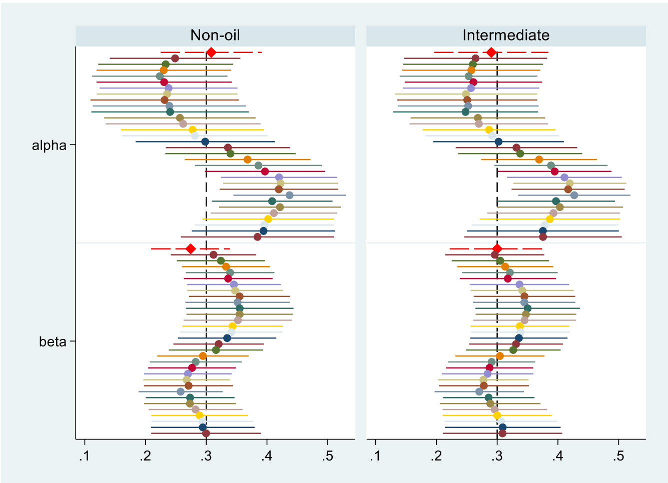


**Fig. 2:** Evolution of the estimated output elasticities of production factors over time. The first period: 1960-1985 (top), the last period: 1990 - 2015. The reference line shows MRW predictions for elasticities of production factors. *Source: Author's calculations*

Based on these results, we may conclude that the augmented Solow model relatively well describes real data during all the examined periods and on both samples of countries. Our main hypothesis is that the overvaluation of physical capital in the second half of the study period is caused by an imperfect proxy variable for human capital. It is possible that with the increase in automation of many activities in the last two decades, the share of the secondary school population no longer captures a significant part of human capital used in production. In light of this, we assume that by using other auxiliary indicators that reflect, for example, higher education, we can explain part of the differences in GDP per worker attributed to physical capital (based on the standard assumption that physical and human capital are positively correlated).

### 4.2 Testing Proxy Variables for Human Capital

When testing proxy variables for human capital in the augmented Solow model, we chose a standard framework based on MRW. The explained variable in the regression is the real GDP per worker in 2015. The explanatory variables are the average share of gross fixed capital in the country's GDP in the years 1990-2015 and the average growth rates of the population aged 15-64 in the same period (increased by $g + \delta = 0.05$). As a proxy for human capital, we tested 4 variables:

1. Share of the population older than 15 years that completed at least some education at secondary school
2. Share of the population older than 15 years that completed at least some education at tertiary school
3. Average length of schooling (in years)
4. Human capital index (based on years of schooling and returns to education)

We estimated the augmented Solow model on a non-oil sample using OLS without restriction (in the first part of Table 1) and with restriction (in the second part of Table 1). Models 1 to 4 differ only in the proxy variable used for human capital. The largest estimated impact of the proxy variable has an index of human capital from the Penn World Table database. In this case, the calculated output elasticity $\beta$ with a value of 0.509 is significantly overestimated compared to the predictions. This overestimation may be due to the endogeneity of this explanatory variable. It is reasonable to assume that in countries with a better institutional environment, educational returns are significantly higher. Thus, this variable appears to be highly correlated with other growth determinants that are not included in the model, which overestimates the estimated effect. In models 2 and 3, which use tertiary school and average years of schooling variables as proxies, the calculated elasticities of physical and human capital are closest to the values predicted by the augmented Solow model. These results are in line with our hypothesis that human capital over the last years needs to be measured by indicators that also reflect higher education than secondary school.

In Table 2 we can see the estimates of the extended Solow model for countries from the intermediate group The extended Solow model in this case explains the differences in GDP per worker (based on adjusted R-squared) worse than in the previous sample. In this group of countries, small countries with less than 1 million inhabitants are absent. The Solow model can well describe large differences in per capita income

***Tab. 1:*** *Estimates of the augmented Solow model for non-oil countries with different proxies for human capital (1990-2015).*

| *Dep. var.:* *Real GDP p. c. 2015 (PPP)* | (1) secondary | (2) tertiary | (3) years | (4) human |
|---|---|---|---|---|
| Observations | 88 | 88 | 88 | 92 |
| ln $(I/GDP)$ | 1.130*** | 0.823*** | 1.010*** | 0.928*** |
| | (0.279) | (0.242) | (0.243) | (0.218) |
| ln $(n+g+\delta)$ | -3.207*** | -2.475*** | -2.431*** | -1.532*** |
| | (0.495) | (0.436) | (0.457) | (0.457) |
| ln $(school)$ | 0.808*** | 0.602*** | 1.329*** | 2.513*** |
| | (0.126) | (0.063) | (0.148) | (0.234) |
| $Constant$ | 9.647*** | 10.796*** | 8.826*** | 7.803*** |
| | (1.340) | (1.039) | (1.162) | (1.075) |
| Adjusted R-squared | 0.733 | 0.809 | 0.797 | 0.832 |
| *Restricted regression:* | | | | |
| $\ln(I/GDP) - \ln(n+g+\delta)$ | 1.302*** | 0.969*** | 1.019*** | 0.885*** |
| | (0.273) | (0.238) | (0.236) | (0.228) |
| $\ln(school) - \ln(n+g+\delta)$ | 0.947*** | 0.667*** | 1.344*** | 1.952*** |
| | (0.110) | (0.057) | (0.115) | (0.153) |
| $Constant$ | 6.805*** | 8.615*** | 8.642*** | 10.893*** |
| | (0.254) | (0.273) | (0.276) | (0.395) |
| $Implied\ \alpha$ | 0.401*** | 0.368*** | 0.303*** | 0.231*** |
| | (0.060) | (0.062) | (0.056) | (0.052) |
| $Implied\ \beta$ | 0.292*** | 0.253*** | 0.400*** | 0.509*** |
| | (0.043) | (0.035) | (0.044) | (0.045) |
| Adjusted R-squared | 0.722 | 0.801 | 0.800 | 0.816 |

Standard errors in parentheses; *** p<0.01, ** p<0.05, * p<0.1 *Source: Author's calculations*

between countries. However, if we choose a sample from similarly rich countries, its success is less convincing. As in the previous case, the output elasticities came closest to the predicted values in models 2 and 3 (we consider the approximation to the value of 0.3 for physical capital to be a more important factor, as we can also compare this value with national accounts).

To summarize: the output elasticities of physical and human capital are almost identical to the predictions of MRW's extended Solow model using proxy variables for tertiary education and for the average length of schooling.

## Conclusion

In this paper, we tested the augmented Solow model with human capital on cross-sectional data at different times and with different proxy variables for human capital. We found that MRW predictions of output elasticities of production factors coincide with real data in the period between 1960-2000. In later periods, the influence of

***Tab. 2:*** *Estimates of the augmented Solow model for intermediate countries with different proxies for human capital (1990-2015).*

| *Dep. var.:* *Real GDP p. c. 2015 (PPP)* | (5) secondary | (6) tertiary | (7) years | (8) human |
|---|---|---|---|---|
| Observations | 70 | 70 | 70 | 73 |
| $\ln(I/GDP)$ | 1.091*** | 0.653** | 0.988*** | 0.874*** |
| | (0.299) | (0.264) | (0.263) | (0.227) |
| $\ln(n+g+\delta)$ | -2.752*** | -2.294*** | -2.174*** | -1.425*** |
| | (0.502) | (0.427) | (0.458) | (0.440) |
| $\ln(school)$ | 0.854*** | 0.572*** | 1.345*** | 2.520*** |
| | (0.166) | (0.072) | (0.183) | (0.256) |
| $Constant$ | 8.824*** | 11.086*** | 8.427*** | 7.787*** |
| | (1.551) | (1.101) | (1.300) | (1.144) |
| Adjusted R-squared | 0.674 | 0.767 | 0.749 | 0.818 |
| *Restricted regression:* | | | | |
| $\ln(I/GDP) - \ln(n+g+\delta)$ | 1.230*** | 0.862*** | 0.966*** | 0.737*** |
| | (0.278) | (0.252) | (0.248) | (0.236) |
| $\ln(school) - \ln(n+g+\delta)$ | 0.972*** | 0.637*** | 1.314*** | 1.910*** |
| | (0.137) | (0.067) | (0.140) | (0.167) |
| $Constant$ | 6.934*** | 8.795*** | 8.758*** | 11.063*** |
| | (0.290) | (0.289) | (0.288) | (0.407) |
| $Implied\ \alpha$ | 0.384*** | 0.345*** | 0.295*** | 0.202*** |
| | (0.064) | (0.072) | (0.061) | (0.058) |
| $Implied\ \beta$ | 0.304*** | 0.255*** | 0.401*** | 0.524*** |
| | (0.049) | (0.041) | (0.049) | (0.050) |
| Adjusted R-squared | 0.671 | 0.755 | 0.752 | 0.797 |

Standard errors in parentheses; *** p<0.01, ** p<0.05, * p<0.1. *Source: Author's calculations*

physical capital is overestimated compared to the predictions of the Solow model. We have identified imperfect choice of a proxy variable for human capital as a possible source of overestimation of this impact. It is possible that with the increase in automation of a large number of tasks in the last two decades, the share of the secondary school population no longer captures a significant part of the human capital used in production. Subsequently, we estimated the augmented Solow model with four proxy variables for human capital for the period 1990-2015. We found that using the proportion of people over the age of 15 with at least some tertiary education and the average length of schooling, our estimates are very close to the predicted values from the MRW article.

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