OPINION PAPER

The Case for Using Mean Seismic Hazard

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Complete probabilistic seismic hazard analyses incorporate epistemic uncertainties in assumptions, models, and parameters, and lead to a distribution of annual frequency of exceedance versus ground motion amplitude (the "seismic hazard"). For decision making, if a single representation of the seismic hazard is required, it is always preferable to use the mean of this distribution, rather than some other representation, such as a particular fractile. Use of the mean is consistent with modern interpretations of probability and with precedents of safety goals and cost-benefit analysis.
[DOI: 10.1193/1.1985447]

INTRODUCTION

Estimates of earthquake ground motion hazard involve substantial epistemic uncertainty in the mean frequency of exceedance for a given ground motion or, alternatively, in the ground motion for a given mean frequency of exceedance. We believe that the mean estimate of the mean frequency of exceedance should be the standard when a single estimate is necessary. (Please refer to the Addendum for a clarification of the often misused or misunderstood definitions regarding "hazard," "frequency," and "mean." In the context of that addendum we shall adopt the common shorthand convention of "hazard" for the "frequency of exceedance" and "mean hazard" or "mean frequency of exceedance" for the "mean estimate of the frequency of exceedance.")

This epistemic uncertainty has been known and quantified in the United States since the 1970s (see, for example, McGuire 1977). In seismic hazard studies it is preferable to report the complete epistemic distribution of hazard, because this allows any effects of that uncertainty on risk mitigation decisions to be handled in an explicit, quantitative way. This reporting usually takes the form of presenting four or more hazard curves, say, three fractiles (e.g., 0.10, 0.50—or median—and 0.90) plus the mean hazard curve. This reporting position is supported by a finding of the National Research Council Panel on Seismic Hazard Analysis: "Knowledge of earthquake processes and effects in much of the United States is meager, resulting in considerable uncertainty in seismic hazard estimates. No single measure of the seismic hazard (e.g., a mean or median [estimate]) is adequate to represent this basic lack of understanding; therefore, measures of uncertainty must be transmitted as part of a PSHA [probabilistic seismic hazard analysis]." (NRC 1988) (words in brackets added for clarity).

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Earthquake Spectra, Volume 21, No. 3, pages 879–886, August 2005;

In spite of this preference, there is often a need to express seismic hazard by a single hazard curve or a single ground motion level (expressed, for example, as a peak ground acceleration or spectrum) corresponding to a given annual frequency. A single seismic hazard map of the 2% in 50-year one-second ground-motion spectral acceleration is needed for seismic design according to current building codes. For critical facilities, a requirement to design to a single ground motion level with an annual frequency of exceedance of 10^{-4} (rather than an epistemic probability distribution of such levels) is easier to implement. The reason for a single ground motion requirement relates to tradition in structural design; engineers are accustomed to designing facilities to a specific level of shaking, and to checking structural performance for that design under alternative levels of shaking.

Given the need to express seismic hazard with a single curve or value, we strongly believe that the ***mean seismic hazard*** is the preferred single curve or value to use, the mean being over the epistemic uncertainty. In particular, the mean is a better choice than a particular fractile level, such as the median estimate (Abrahamson and Bommer [2005] have recently argued to the contrary). Our preference for the mean estimate is supported by several different, although not entirely independent, arguments. They fall mainly into two broad areas: modern interpretations of probability, and precedents.

MODERN INTERPRETATIONS OF PROBABILITY

The mean estimate of the hazard is ***the*** hazard under the most widely applied modern interpretation of probability in risk analysis (see, for example, Baecher et al. 2004). Under this, the subjectivist view of probabilities, one simply does not distinguish between aleatory and epistemic uncertainties; the implication is that the mean (over the epistemic uncertainty) of the aleatory probability is numerically and operationally just the same as an application of the total probability theorem. That theorem weights each possible value of the aleatory probability or frequency of a future event by the probability that the assumptions that value is based on (or "conditioned on") is right, and then sums over all possible sets of assumptions. The result is ***the*** (unconditional) probability of the event. But this weighting is precisely the way one calculates the mean aleatory probability. Hence the mean estimate of the aleatory probability may legitimately be considered to be ***the*** probability, or ***the*** frequency of exceedance in our case. This view is reflected in modern PSHA: the logic tree defines the (epistemic) probabilities on the hypotheses (alternative models and parameter values) and the PSHA done at each end node of the tree determines the (aleatory) probability or frequency of a future event given that set of conditions.

Alternatively, and perhaps still more popular today among those concerned with the philosophical bases of probability in its application, one may adopt a decision-theoretic perspective. In this case, probabilities are deemed to reflect, in effect, the personal probability values that a decision maker is prepared to act upon. As a simple example, if one's decision about how to bet on a particular poker hand is the same before the cards are shuffled (pure aleatory variability) as they are after five cards are dealt face down on the table (pure epistemic uncertainty), then our equality of preference between the two cases implies that we are assigning them the same probability and *acting* as if there is no difference between aleatory and epistemic uncertainties. In that case, as described above, ***the*** probability of an event (that has been defined in turn by probabilities and conditional probabilities of events segregated for whatever practical, operational reasons into epistemic and aleatory) is the average aleatory probability. The averaging, again, is over the epistemic probabilities. The formal basis for this interpretation lies in decision theory (e.g., Savage 1954, Raiffa 1968) which has the use of probability firmly in mind. The theory is in turn based on one or more sets of intuitive, quite attractive preference axioms; the most critical here is the decomposition axiom, which accepts the compounding of two-stage lotteries. In our case the first "lottery" is over the quantities having epistemic uncertainty and the second is over the possible future earthquake events and their random ground motion. Under the decision-theoretic perspective, if these preference axioms are accepted, a decision maker's personal probabilities can be inferred from his choices between simple pairs of lotteries prepared and proffered to him by a "probability elicitor." It can be argued, of course, that when a PSHA is conducted, the hazard analyst may not know the decision maker whose preferences should in principle underlie this construction. In this view, it is implicit that the decision maker (e.g., the building owner or policy-setting official), lacking technical knowledge to the contrary, will adopt as his own probabilities those provided by the earthquake science community through PSHAs.

Both of these common underlying bases for probabilities conclude that the mean hazard is ***the*** hazard. The invariance of the mean hazard to whether uncertainty is treated as aleatory or epistemic is of practical importance in seismic hazard analysis because the distinction in specific applications may not always be clear-cut. It may even depend on how the stochastic model of seismic hazard is formulated (see USNRC 1997; Veneziano 1995, 2003). For example, most probabilistic seismic hazard studies treat the scatter (around a predictive equation) in multi-site ground motion observations as aleatory, or event-to-event variability. Some studies, however, apportion a fraction of this scatter to epistemic uncertainty reflecting a fixed (but uncertain) site-specific effect (e.g., PG&E 1988). Studies by Anderson and Brune (1999) argue that, under some special conditions, much of the scatter in ground motions may be epistemic. These might include cases where the large motions come from repeated occurrences of characteristic events with the same slip distribution and dynamics, the same path, and the same site effects, these factors being fixed (and unknown, but potentially knowable, i.e., epistemic) at any one site, while different at different sites. One advantage of using the mean frequency of exceedance (over epistemic uncertainties) is that this mean is invariant to whether scatter in ground motion observations is treated as epistemic or aleatory.

An extreme assumption with respect to the epistemic-aleatory distinction is the Laplacian view that the world is completely deterministic. Even the apparent randomness of events such as the outcome of a coin toss results solely from our inability to derive appropriate physical models and parameters with which to predict future outcomes. The implication of this view is that *all* uncertainty is epistemic. Even under this assumption, however, it is pragmatically convenient, e.g., with respect to communication among experts and to properly maintaining certain kinds of stochastic dependence that arise, to "treat the world as if some uncertainties are not due to limited knowledge but are actually random" (Baecher et al. 2004).

The important thing in PSHA is to make sure all important sources of uncertainty (whether aleatory or epistemic) are included, to document clearly how uncertainties have been treated, and to adopt decision methods that are insensitive to alternative valid interpretations. When a single number is required, the use of the mean hazard uniquely accomplishes this objective. Any fractile-based criterion will not. For example, consider the ground motion scatter discussion above. Transporting a portion of the aleatory variance to the epistemic variance will lower the median estimate of the hazard, raise the ratio of the 0.9 fractile to the median, and modify all fractiles. But as discussed above, it will not change the mean hazard.

Before leaving the subject of preference for the mean hazard and the benefits of its invariance to the aleatory-epistemic distinction, we should point to cases in which this distinction may be important to maintain. It has been noted that society sometimes treats aleatory and epistemic uncertainties differently when judging failures (see, for example, Corotis 2003, and references therein.) This may or may not be "rational" behavior. There are, however, cases in which seismic decisions depend on the amount of epistemic uncertainty. One example is when we wish to account for possible future changes in seismic hazard estimates when setting design criteria today (see, for example, McGuire 1987; Veneziano 1995, 2003). In such cases, designing to a level higher than required by a mean hazard criterion may be more cost-effective from the point of view of the facility owner, to avoid the future cost of retrofits that may be mandated as a result of a revised perception of the hazard. The option to design to levels higher than those currently required by regulations is *always* available to owners; regulated design levels are minimum requirements developed to protect the health and safety of the public from undue risk. As a second example, the decision maker may define his preferences in a manner that reflects different degrees of risk aversion when faced with epistemic rather than aleatory uncertainty. Recall the poker hand discussion. This comparatively little studied decision problem (see Paté-Cornell and Fischbeck 1991, for an earthquake engineering example, and references) also requires a full representation of the epistemic uncertainties in hazard if optimal decisions are to be made. This is not a criticism of the mean as a single decision tool, but a reason that the *entire* epistemic distribution should be taken into account in certain decision-making circumstances. The use of a fractile hazard will not avoid these complex problems.

PRECEDENTS

There are important precedents for expressing safety criteria for the performance of facilities in terms of mean frequencies of occurrence of undesirable effects. We provide two examples. The U.S. Nuclear Regulatory Commission quantifies safety goals related to nuclear power reactor accidents and fatalities (core damage, release of radioactivity, and immediate and latent cancers in terms of tolerable *mean* estimates of frequency of occurrence of these accidents (Okrent 1987; USNRC 1998, 2001). This decision to not use fractile estimates was based on both theoretical grounds, as discussed above, and practical grounds, as will be discussed below. "The Commission has adopted the use of mean estimates for purposes of implementing the quantitative objectives of this safety goal policy (i.e., the mortality risk objectives). Use of the mean estimates comports with the customary practices of cost-benefit analyses and it is the correct usage for purposes of mortality risk comparisons." (USNRC 2001). Further, the U.S. Army Corps of Engineers risk-based procedure for flood damage assessment incorporates epistemic uncertainties and uses the mean estimate of the annual frequency of exceedance in assessing, for example, flood-induced levee failures (USACE 1996). This policy has been reviewed and independently recommended by a National Research Council committee (NRC 2000, Baecher 2005, Stedinger 2005).

Determining the frequency of accidents or fatalities due to seismic events, whether in nuclear power plants or in more conventional facilities, requires integration (over ground motion level) of the probability of the accident given a level (a "fragility curve") times the frequency of that level. Both this (conditional) system failure probability and this seismic frequency are subject to significant epistemic uncertainties. To obtain a fractile estimate or the entire distribution of the induced epistemic uncertainty in the accident frequency in this case is a very complex computation (requiring, for example, information about the *joint* distribution of epistemic uncertainty in the seismic hazard curve values at multiple ground motion levels). In contrast, obtaining the *mean* estimate of the accident frequency requires only a simple integration of the mean estimates of the seismic frequency and accident probability curves. Thus a very practical use of the mean hazard curve is to make mean estimates of system-level accident frequencies. Further, the system safety problem may be broadened in definition to include multiple facilities (e.g., all U.S. operating power reactors), multiple years (e.g., frequency per 50 years), or multiple causes (seismic- and tornado-induced building collapses). The mean frequency of the total number of accidents in such cases is simply the sum of the mean frequencies of each contributor (facility, year, or cause). This mean frequency can be useful in comparisons with a total system safety goal or with an observed total system failure rate. Determining a fractile estimate of the total frequency of such problems would be extremely burdensome, especially given the subtle correlations in the epistemic uncertainties. In all such system failure frequency analysis problems, having a single fractile estimate of the seismic hazard frequencies will not provide useful input to either a mean or a fractile estimate of the system accident frequency.

An alternative or supplementary approach to system assessment and design is via cost-benefit or risk analysis, typically ending in expected (mean) economic loss comparisons (see, for example, Lave and Balvanyos 1998, USNRC 1998, Deierlein 2004). In the simplest terms, if the cost of a facility seismic accident or failure is C, then the annual expected or mean risk due to such a failure is the expectation of $[p_f \times C]$, where p_f is the annual probability of seismic failure. The implication is that the mean risk depends on the mean of p_f or, as discussed above, on the mean estimate of the hazard curve. Some argue that even if seismic criteria in codes and standards are set in the simpler terms of a frequency of accident or frequency of ground motion exceedance, there is implicit in such criteria an underlying, unspecified expected societal risk–based objective. This argument also supports the use of the mean seismic frequency in such criteria. Again we see that the mean is sufficient and a fractile estimate curve is not.

DISCUSSION AND SUMMARY

There are compelling theoretical and practical reasons for using the mean seismic hazard, if a single measure (over epistemic uncertainties) is needed. In rebuttal to this position it is sometimes said that the mean is "overly sensitive" to extreme interpretations (e.g., Abrahamson and Bommer 2005), and that one or a few interpretations or individuals can dominate (or even manipulate) the mean estimate of the hazard. This may occur in some situations, but it does not constitute a problem with use of the mean. Instead, it underscores the difficult task at hand. The appropriate action by the hazard analyst (be it an individual or complex set of experts and integrators [USNRC 1997]) is to examine the validity of those extreme interpretations and to include or exclude them (or adjust the weights assigned to them) based on their consistency with accepted theory and data. In multi-expert studies, feedback mechanisms should be incorporated from the beginning to assess the validity of interpretations and exclude some, if necessary. The appropriate response is to not allow those interpretations to stand and then find a way to recommend design ground motions that avoid those invalid opinions. Most public-safety decision making requires taking into account a low-probability, high-consequence situation (which, in the seismic hazard sense, is a high seismic hazard curve that is assigned low weight). Ignoring such situations by using a fractile level that is completely insensitive to extreme interpretations may well result in ineffective decisions.

An additional Abrahamson and Bommer (2005) rebuttal to using the mean hazard is that it is based on assigned models that are neither mutually exclusive nor collectively exhaustive, so the associated weights cannot be treated as probabilities. It is true that the process of quantifying epistemic uncertainty in the inputs for a detailed PSHA is not an easy task. The analyst or expert must go from a few alternative models (which often share some common elements and do not span the range of possible states of nature) to a broad, inclusive, and sufficiently fine-grained representation of the uncertainty in the model or parameter under consideration. The final models or parameters need not be collectively exhaustive in the strict sense (of a continuous distribution). They should be viewed only as points in the expert's discretized representation of uncertainty in the models or parameters, with some alternative models perhaps representing variations of the same original model. Further, if one takes the position that these subjectively derived weights cannot be viewed and manipulated as probabilities, then *any* probabilistic manipulation of them—such as computing a fractile—is as meaningless as the calculation of the mean. We would then be left with a family of alternative hazard curves and no meaningful way to summarize them or represent them in a more manageable form.

Epistemic uncertainties in seismic hazard analysis generally involve alternative models that will be validated or replaced in the future: alternative ground motion estimation equations, alternative mean recurrence intervals of large magnitude earthquakes on specific faults, etc. We will make the best seismic risk mitigation decisions today if we account for these uncertainties in a complete and transparent way. The mean seismic hazard estimate (over epistemic uncertainties) is a convenient and effective way to account for all valid interpretations, properly weighted by their assessed probabilities, and to synthesize the seismic hazard into a single curve that is best suited for modern regulatory and economic decision making.

ADDENDUM

The first paragraph of this opinion paper already brings up two points of terminology that deserve clarification, as they continue to cause confusion in this subject. First, the word “hazard” may be used in its lay sense of a threat to a site (e.g., a severe ground motion level), in its technical sense of the quantitative chance of a given ground motion level being equaled or exceeded, or, if appropriate, in an intentionally inclusive way that includes both senses of the word. The first paragraph—including the quote from the NRC—can be read the latter way because the principles discussed apply “in either direction” of the axes of a conventional seismic hazard curve. The remainder of this opinion paper, however, generally restricts the term “hazard” to the likelihood or “frequency” definition, unless the context makes it clear that the broader inclusive sense may be used. Second, the word “frequency” is used here in this field’s most common, but strictly incorrect, sense. A frequency (as in “the observed frequency of magnitude 7-plus events in California in the 1900s was 3/100 or 0.03 per annum”) is modeled in PSHA as an aleatory random variable because the numerator of that ratio for a future time period is a random number. The mean of the frequency (as in “the long-term or mean frequency of magnitude 7-plus events in California is 0.043 per year as estimated from geological slip rates”) is not. (Incidentally, the mean annual frequency is not strictly speaking the same as the annual probability, but the two are numerically close for values of 0.1 or less.) Therefore, technically, we should have used mean frequency or mean rate in the first paragraph. When, however, one is given an epistemic uncertainty distribution on the mean frequency, and it becomes necessary to report a single value as an estimate, one candidate is the mean of this distribution or, to be precise, “the mean estimate of the mean annual frequency.” This awkward phrase, involving a double use of “mean,” will be replaced in this note, as in the title, by simply “mean frequency,” the “mean” referring to the type of estimate, as in “mean vs. median.” By implication, “frequency” is being used here (as in the opening sentence) for the correct term “mean frequency.” In doing so we bow to convention in the current PSHA field to avoid distracting from the objective of this paper, while footnoting both our concern for the confusion it may cause those from other fields reading our literature and our sympathy for those who use “mean frequency” in the strict sense and are misunderstood.

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(Received 13 May 2005; accepted 19 May 2005)