OPINION PAPER

Probability and Uncertainty in Seismic Hazard Analysis

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In the current practice of probabilistic seismic hazard analysis (PSHA) using logic trees, it is common to use the mean hazard curve to determine ground motions for engineering design. We present the case against the use of the mean hazard curve and explain why this practice should be discontinued and, where necessary, removed from regulations.

The identification and quantification of uncertainties is integral to modern seismic hazard analysis. In probabilistic seismic hazard studies, the variability of the earthquake magnitude, earthquake location, and ground motion level (expressed as the number of logarithmic standard deviations above the logarithmic mean) are considered explicitly in the computation of the hazard. In major seismic hazard projects, the scientific uncertainty in the models of the distributions of earthquake magnitude, location, and ground motion are also considered using logic trees (Kulkarni et al. 1984, Coppersmith and Youngs 1986, Reiter 1990, Bommer et al. 2005).

The inherent variability considered directly in the hazard computation is called the aleatory variability, and the scientific uncertainty in the models of the earthquake occurrence and ground motion is called the epistemic uncertainty. The terms randomness and uncertainty have also been used for aleatory variability and epistemic uncertainty, respectively; however, the former terms are now commonly used interchangeably. As a result, they are often mixed up when used in hazard analysis. The terms "aleatory variability" and "epistemic uncertainty" are used to provide an unambiguous terminology. This is not simply semantics: distinguishing between the two types of uncertainty is fundamental to the way that they are dealt with in the hazard calculations and how uncertainty is handled in decision making on the basis of the hazard analysis.

In application, the key difference is that aleatory variability leads to the shape of the hazard curve and the epistemic uncertainty leads to alternative hazard curves. There is no dilemma regarding the inclusion of the aleatory variability in the hazard calculations, particularly the variability associated with ground-motion prediction equations: a "hazard curve" calculated using only median values from the equations and neglecting the standard deviation has little meaning and cannot be considered a genuine hazard curve.

The hazard analyst does, however, have control over the branches of the logic tree and the weights assigned to these, and hence over the degree to which epistemic uncer-

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tainty is incorporated into the hazard calculations. Once a logic tree has been set up, the hazard calculations are performed following each of the possible routes across the branches: there will be a hazard estimate at each branch tip, each with an associated relative weight obtained by multiplying the weights on the branches included in that particular calculation. From these pairs of hazard estimates and weights, hazard curves can be drawn for any fractile (confidence level), and for engineering applications the question that then obviously arises is which of the curves should be used?

For design, as opposed to assessment (in which all the curves may be used), the selection of a design ground motion involves selecting an annual exceedance frequency (or return period) and a fractile (i.e., which hazard curve is used). These two decisions can be interpreted as deciding what level of safety is required (the exceedance frequency) and how sure one wants to be that this level of safety is being achieved (the fractile); the latter is a decision regarding epistemic uncertainty. These are both decisions that should ultimately be taken by the owner and/or regulator, within the context of the associated risk, rather than by the seismologist or the engineer, although there is an onus on these earthquake professionals to provide the decision maker with the appropriate information to facilitate these choices.

The most common approach in current practice is to use the mean hazard curve and we believe that this is the least appropriate choice. At the return periods that have generally been considered for engineering projects, in the range of 500 to 2,500 years, the implications of using the mean hazard curve have not been particularly serious, but this is not the case when hazard estimates are pushed to much lower annual frequencies of exceedance. In the PEGASOS project to assess seismic hazard at nuclear power plant sites in Switzerland (Abrahamson et al. 2002), annual frequencies as low as 10^{-7} were considered, and for the Yucca Mountain project (Stepp et al. 2001), the hazard estimates were taken to annual frequencies of exceedance of 10^{-8}. At these very long return periods, the mean hazard curve tends to climb across the fractile curves and can result in very high design ground motions (Figure 1). That the use of the mean hazard curve can lead to very high design motions is, of course, not in itself a valid reason to adopt a different hazard curve. We believe, however, that there are strong arguments to be made for abandoning the practice of using the mean hazard curve.

A fundamental issue in deciding how to treat the epistemic uncertainty in the hazard is the interpretation of what the weights on the logic tree branches represent. Vick (2002) describes this as the "*the distinction between probability as a measure of stable frequency on the one hand and an expression of belief on the other*." If one favors the first interpretation and assumes that the weights are frequency-based probabilities of the alternative models being correct, then the mean value of the hazard can be appropriate. In most cases, however, the weights on logic tree branches represent our relative confidence in the alternative models, in other words, our degree of belief in one model with respect to another. In our opinion, to describe these indicators of relative merit as "subjective probabilities" compounds the confusion surrounding this issue. Typically the weights on the branches are developed considering the strengths and weaknesses of alternative models in terms of their applicability to the specific project under consideration. Based on these evaluations, the weights are determined to reflect the relative merits of the alter-

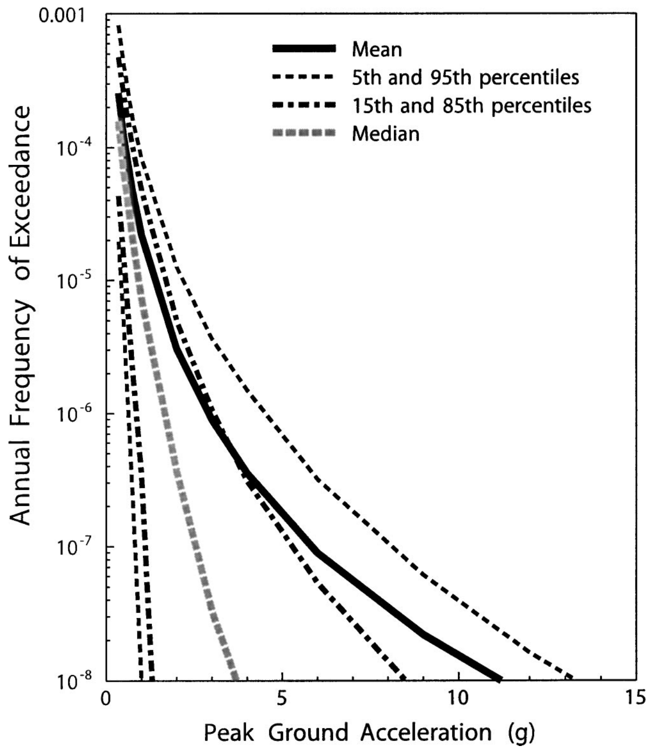

Figure 1. Seismic hazard curves from the Yucca Mountain project (I. Wong, personal communication, 2003).

native models; weights developed in such a manner are not frequency-based probabilities. There are four main features of frequency-based probabilities:

1. They sum to unity
2. The events for which they are defined include all possibilities (i.e., an exhaustive set)
3. The events for which they are defined are mutually exclusive
4. They represent the relative rate at which alternative events occur

Let us now examine logic-tree branch weights with regard to these features.

1. The weights on logic trees, for branches at any node, sum to unity (although this is not strictly necessary), which contributes to the common interpretation that they are frequency-based probabilities.
2. In general, however, the branches do not include all of the possible alternatives, although there are cases where this condition can be assumed to be met. For example, if a logic tree branch is for the slip rate of a fault, then it is possible to sample the entire range of physical possibilities in the logic tree. However, for most branches—and particularly for those representing ground mo-

tion models—the input is composed of models rather than scalars and as such will generally represent the range of proposed models and not the range of possible models.

3. In many cases, the options on the branches of a logic tree will not be mutually exclusive; this is particularly likely to be the case for branches representing different ground-motion prediction equations.
4. If we have two competing models and we consider that model 1 has twice the merit of model 2, then we would give weights of two-thirds and one-third to models 1 and 2, respectively. This is not the same as saying the results of model 1 will happen twice as often as model 2; rather, we are saying that we have more confidence in model 1 than in model 2. This is a subtle, but vitally important distinction. If one accepts that the branch weights are not frequency-based probabilities, then computing a weighted average of the hazard values does not give the expected value in the strict (i.e., frequency-based) statistical sense.

Leaving to one side the meaning of the branch weights, there is another—and even more compelling—reason not to use the mean hazard, which is simply that it is inherently unstable. Invariably, the mean hazard curve tends to correspond to higher and higher fractiles of the hazard as the return period increases, a result of the typical skew to the higher side of the set of alternative hazard curves from a PSHA (Figure 1). The fact that the mean hazard curve will generally yield rather high ground motions is sometimes defended as the logical outcome of including unlikely (but feasible) models in the logic tree that produce extreme consequences, in this case, very high accelerations. If one accepts our assertion that the weights are not frequency-based probabilities, the meaning of multiplying these extreme consequences by the very small weights that may be assigned to them is not clear. Unlikely cases for various parameters in the hazard analysis are often included in the setup of the logic tree in order to capture the full epistemic uncertainty; the small weights generally assigned to these models reflect the fact that they are considered to be unlikely. The problem is that when the mean hazard curve is used, at least for long return periods, this unlikely case can become the basis for design.

The mean hazard curve is highly sensitive to the most severe of the alternative models considered in the analysis, and at longer return periods these models will push the mean hazard curve to higher fractiles. This drift of the mean hazard curve across the fractiles is neither systematic nor predictable, hence for a given return period, different hazard analyses will produce mean values corresponding to different fractiles. As a result of the variation of the implied confidence level with return period, use of the mean hazard effectively removes the possibility of selecting the basis for the design motion by making explicit decisions, as described earlier, regarding the level of safety required and the degree of confidence that this level of safety is being achieved.

The use of the mean hazard curve blurs the distinction between aleatory variability (to which frequency-based probabilities are associated) and epistemic uncertainty (which are statements of belief rather than frequency-based probabilities). We believe that the distinction is valuable and useful because it provides a statement of the accuracy with which the hazard can be computed and gives an indication of the range for revisions in the foreseeable future.

Our case against the use of the mean hazard curve should not be interpreted as an argument to adopt the median hazard curve in its place. We believe that the hazard curve taken as the basis for design should be chosen on the basis of the fractile that reflects the desired degree of confidence that the safety level implied by the selected annual frequency of exceedance (or return period) is being achieved in light of the uncertainty in the estimation of the hazard. For critical projects, the 85th fractile may be considered appropriate, whereas for others, values as low as the median may be deemed suitable.

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