Sigma: Issues, Insights, and Challenges

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INTRODUCTION

The prediction of ground-motion levels at a site is one of the key elements of seismic hazard assessment. This prediction is commonly achieved using equations derived through regression analysis on selected sets of instrumentally recorded strong-motion data, hereafter referred to as empirical ground-motion prediction equations (GMPE). Reviews and compilations of equations published to date have been presented by, among others, Campbell (1985), Joyner and Boore (1988), and Douglas (2003, 2004, 2006). These equations relate a predicted variable (Z_{pred}) characterizing the level of shaking, most commonly the logarithm of a peak ground-motion parameter (*e.g.*, PGA, PGV) or response spectral ordinate (SA, PSA, PSV, SD), to a set of explanatory variables $\{X_k\} = X_1, X_2, \ldots$ describing the earthquake source, wave propagation path, and site conditions:

$$Z_{\text{pred}} = f\left(\{X_k\}\right) \tag{1}$$

The explanatory variables $\{X_k\}$ usually include the earthquake magnitude, **M**; a factor describing the style-of-faulting of the causative event; a measure of the source-to-site distance, *R*; and a parameter characterizing the site class. Recent equations sometimes also include additional terms to characterize the location of the site with respect to the rupture plane (hanging-wall factor), to distinguish between ground motions from surface-faulting events and buried ruptures, or to include the effects of sediment depth in the case of deep alluvial basins. Other factors that are known to influence the motion (and many others that are not yet known) are not included in the equation because the information is not readily available or not predictable in advance. For instance, anisotropy effects resulting from the dynamic propagation of rupture (including directivity effects) are currently not included in predictions, although back-analyses of ground motions from past earthquakes have shown that such effects may have a strong influence on the spatial distribution of ground motions.

Even for the factors that are considered in the equation, the representation of the ground motion is very simple compared to the complexity of the physical processes involved in ground-motion generation and propagation. For instance, parameters such as site class and style-of-faulting are often characterized using very simple binning schemes, *i.e.*, the predictions for the different bins will simply be scaled by a constant factor with respect to one another. In combination with the limited number of factors considered in the functional form of the equation, these idealizations cause the values in the dataset to depart from the average value predicted by the equation in an apparently random manner. To capture the resulting dispersion, the distribution of the ground-motion residuals (δ_i) is considered; the ground-motion residuals are defined as the difference between the observed and the predicted ground motions and represent the unexplained part of the ground motion:

$$\delta_i = Z_{\text{obs}} - Z_{\text{pred}} \tag{2}$$

where Z_{obs} and Z_{pred} are the observed and predicted values of the ground-motion variable used in the regression, such as ln(PGA).

The ground-motion residual distribution is generally assumed to be normal with a mean of zero and a standard deviation σ. Thus, all models can be separated into an explained component and an unexplained component characterized by σ. For an individual observation, the normalized residual $\varepsilon_i = \delta_i / \sigma$ represents a measure of the goodness-of-fit of the equation at this particular data point:

$$Z_{\text{obs}} = Z_{\text{pred}} + \delta_i = f(X_k) + \delta_i = f(X_k) + \varepsilon_i \sigma \tag{3}$$

In the context of seismic hazard analysis, it is customary to distinguish between epistemic uncertainty (uncertainty due to incomplete knowledge and data) and aleatory uncertainty (uncertainty due to the random nature of the processes under consideration). Following the nomenclature introduced by Toro *et al.* (1997), each of these components of uncertainty is further subdivided into modeling uncertainty (uncertainty regarding the model adopted to represent the ground motion) and parametric uncertainty (uncertainty about the values of the parameters included in the model).

The scatter (σ) associated with ground-motion prediction equations is commonly interpreted as the aleatory uncertainty of the ground motion, although it is as yet unclear to what extent it represents genuine randomness (*i.e.*, intrinsic variability of the ground motion) and to which extent it reflects epistemic modeling uncertainty regarding the factors controlling the ground motion (*i.e.*, explainable variations in ground motion that have not yet been included in ground-motion models and

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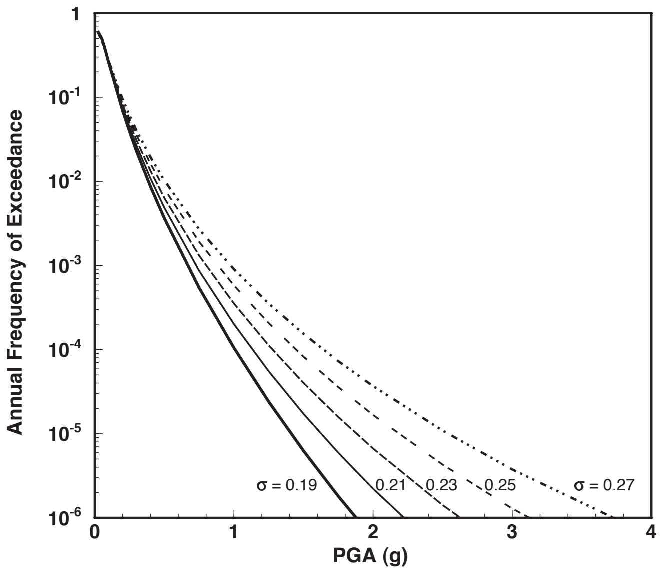


▲ **Figure 1:** Impact of the aleatory variability of ground motion (σ) on seismic hazard results. The graph shows hazard curves for PGA generated with the equation of Boore *et al.* (1997) using the original (0.23) and modified values for the standard deviation of $\log_{10}$(PGA) for a fictitious site affected by two source zones (Bommer and Abrahamson 2006).

may therefore appear as random variations). In the remainder of this article, σ is referred to as the aleatory variability of the ground motion, although it is understood that this is to some extent a convention.

The value of σ has a significant impact on the results of seismic hazard analysis, as discussed in Restrepo-Vélez and Bommer (2003) and Bommer and Abrahamson (2006), from which Figure 1 is reproduced. This figure shows the impact of σ on seismic hazard curves for PGA generated with the equation of Boore *et al.* (1997) using the original (0.23) and modified values for the standard deviation of $\log_{10}$(PGA) for a fictitious site affected by two source zones. Bommer and Abrahamson (2006) also retrace the history of the treatment of σ in probabilistic seismic hazard analysis (PSHA) and note that although some early formulations (*e.g.*, Cornell 1968) did not explicitly include σ in the hazard integral, it has since been recognized that this parameter is an integral and indispensable part of PSHA and not an optional add-on (Reiter 1990; McGuire 2004). As a result, reporting the value of the standard deviation associated with the regression has become standard practice when publishing a new GMPE, although a few exceptions persist (*e.g.*, Souriau 2006). Furthermore, the almost universal adoption of the logarithmic transform in the regressions—discussed in more detail in Campbell (1985) and Douglas and Smit (2001)—means that in most cases the value of the ground-motion parameter of interest (*e.g.*, PGA) will vary as an exponential function of any positive or negative increment in the value of σ. In other words, even small variations in the value of σ may have a significant impact on seismic hazard analysis results.

Figure 2 summarizes the values of σ for PGA and PGV GMPE published to date. This figure constitutes an update of a similar plot published in Douglas (2003) and clearly shows that the values of σ have remained stable over the past 40 years, despite an increase in the number of available records and the inclusion of additional variables in the equations. The values of σ tend to lie between 0.15 and 0.35 in $\log_{10}$ units (0.35 to 0.80 in ln units) but in some isolated cases may range as high as 0.55 $\log_{10}$ units (1.26 ln units).

In view of the lack of evolution of the values of σ and the pronounced influence this parameter has on seismic hazard analysis results at low annual frequencies of exceedance (Figure 1), the possibility of counteracting the influence of σ by truncating the ground-motion distribution has been investigated (Bommer *et al.* 2004; Strasser *et al.* 2008). Such a truncation could be operated either directly on the amplitude of the ground motion or on the distribution of ground-motion residuals. However, in view of the difficulty of justifying the choice of the truncation level, the aforementioned studies concluded that for the foreseeable future, research efforts should focus on a better understanding of the nature of σ and the issues related to the estimation of this parameter.

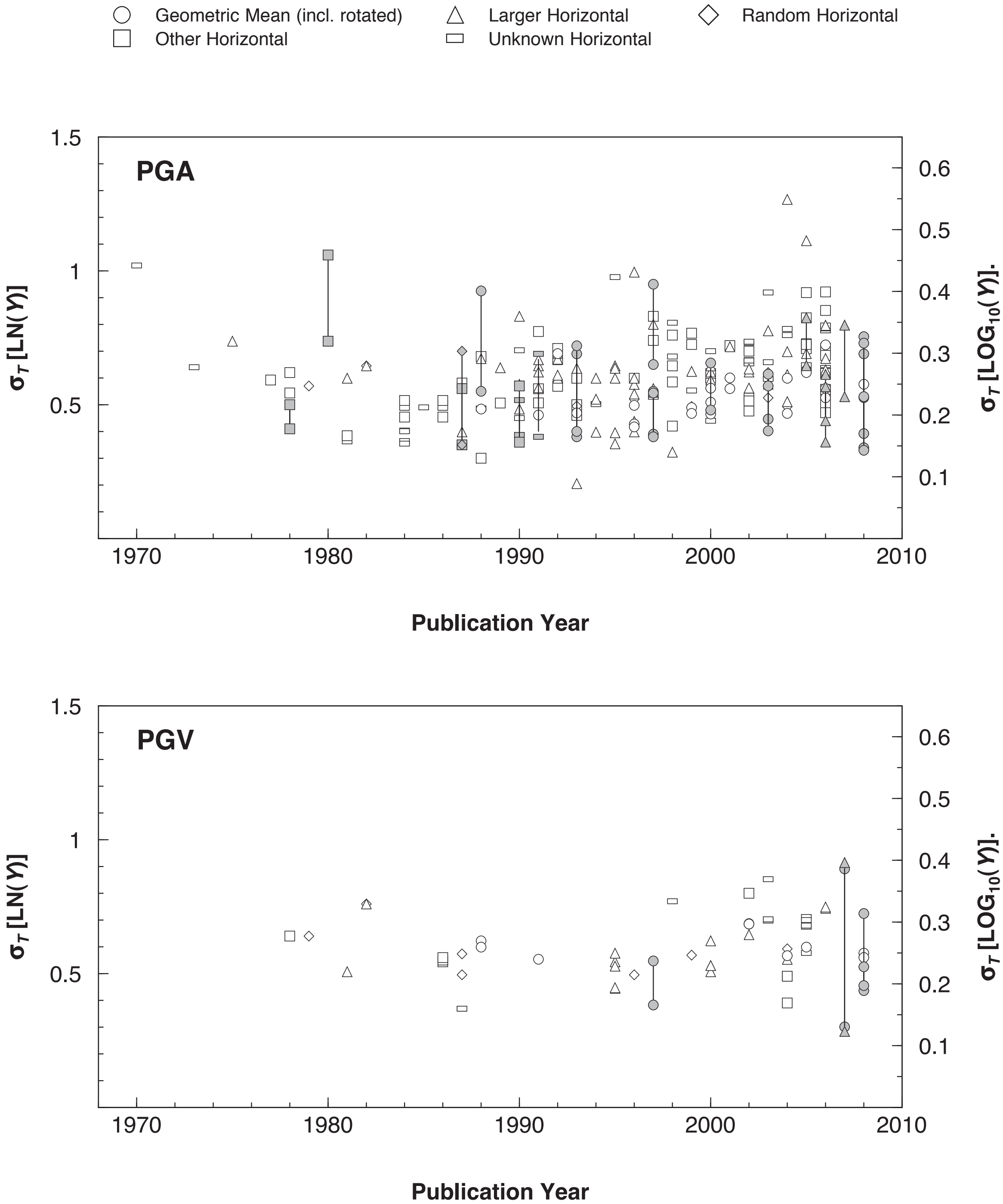
The present paper reviews the current state of knowledge regarding the estimation of σ, with a particular emphasis on data-related factors influencing the value of σ as well as possible sources of bias in the estimation of σ when the functional form of the equation is chosen and fitted. These points are discussed in light of their implications for seismic hazard analysis. In particular, an attempt is made to identify the more promising approaches for a reduction of σ.

Typically, the strong-motion datasets used in regression include events contributing several accelerograms. As a result, the vectors of explanatory variables of these records are correlated, since the values of the event-specific parameters (*e.g.*, magnitude or style-of-faulting) are the same. Furthermore, the uneven number of records contributed by the various events may cause events with a large number of records to exercise an undue influence during the regression process. In order to avoid these problems, it has become customary to separate the inter-event and intra-event components of variability, as illustrated in Figure 3.

The inter-event variability, σ_E (also denoted τ by some authors), can be interpreted as the combined ground-motion variability resulting from event-specific factors (*e.g.*, randomness in the source process) that have not been included in the predictive model. The intra-event variability, σ_A (also denoted σ by some authors), on the other hand, represents the combined ground-motion variability coming from record-specific factors (*e.g.*, randomness in the site amplification for a given site class or $V_{S,30}$ value). In empirical GMPE, the inter-event variability is generally found to be smaller than the intra-event variability.

DATA QUALITY AND SIGMA

Although the main cause for the scatter is believed to be the fact that the representation of the ground motion is very simple compared to the complexity of the physical processes involved in ground-motion generation and propagation, measure-

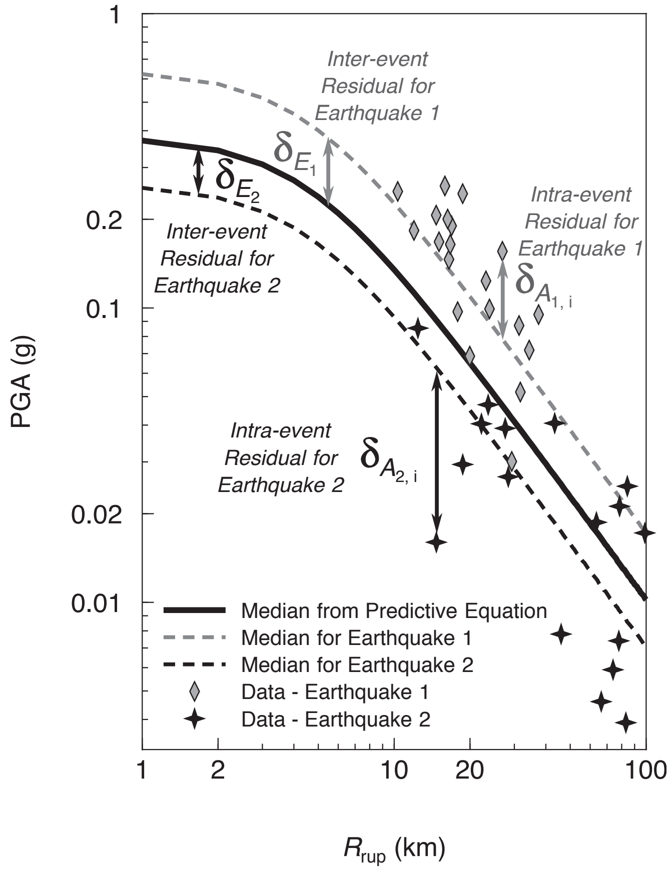


▲ **Figure 2.** Summary of σ values for PGA (*top*) and PGV (*bottom*) from GMPE published over the past 40 years. Values from heteroscedastic models (*i.e.*, models for which σ is a function of the explanatory variables) are shown using gray-shaded symbols representing the bounds of the σ values that can be obtained by considering predictor variables spanning the applicability range of the equation. All values plotted correspond to the total variability, σ_T.

ment errors and uncertainties in the values of both predicted and predictor variables might also contribute to the scatter. Additionally, the value of σ may be affected by the selection of given parameter definitions, both for the predicted variable (*e.g.*, horizontal parameter definition) and for the explanatory variables (*e.g.*, choice of distance metric).

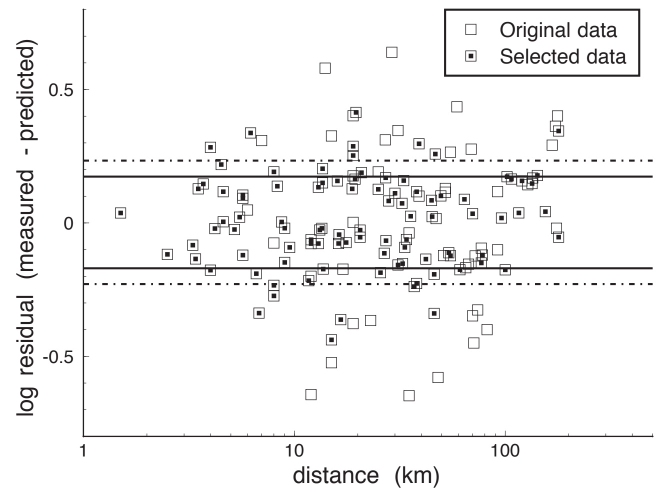
Uncertainties Regarding the Observed Ground-motion Values

The selection and processing of strong-motion data is one of the key stages of the derivation of a new empirical GMPE, to which considerable time and effort is devoted. The quality of the data used will naturally have an impact on the goodness of the fit,



▲ **Figure 3.** Inter-event and intra-event components of ground-motion variability (after Youngs *et al.* 1995). The data shown is for two events having the same magnitude, which is also the magnitude used to calculate the median of the predictive equation. The inter-event variability, σ_E, characterizes the dispersion of the inter-event residuals, δ_E (one residual per earthquake). Similarly, the intra-event variability, σ_A, characterizes the dispersion of the intra-event residuals, δ_A (several residuals per earthquake).

once a functional form has been selected. Censoring data that is considered erroneous is a common practice in statistical analysis, but in the case of strong-motion data, determining what constitutes an "erroneous" recording is not straightforward, except in the case of clear instrument malfunction (Strasser *et al.* 2008). Accelerograms exhibiting large amplitudes of ground motion that have been viewed with mistrust at the time of their recording (*e.g.*, the Pacoima Dam record of the 1971 San Fernando earthquake) have often been "rehabilitated." The isolated nature of their departure from median predictions (*i.e.*, the expected values of ground motion) may be an indication of an under-sampling of the tails of the ground-motion distribution in the set of strong-motion observations available to GMPE developers. Strasser *et al.* (2008) discuss the issue of the truncation of the distribution of ground-motion residuals and conclude that there is currently no physical basis to define a level of deviation from predictions beyond which ground motions may be dismissed, although there are reasons to believe that such a physical limit exists for the absolute amplitudes of ground motions (*e.g.*, Bommer *et al.* 2004; Andrews *et al.* 2007).



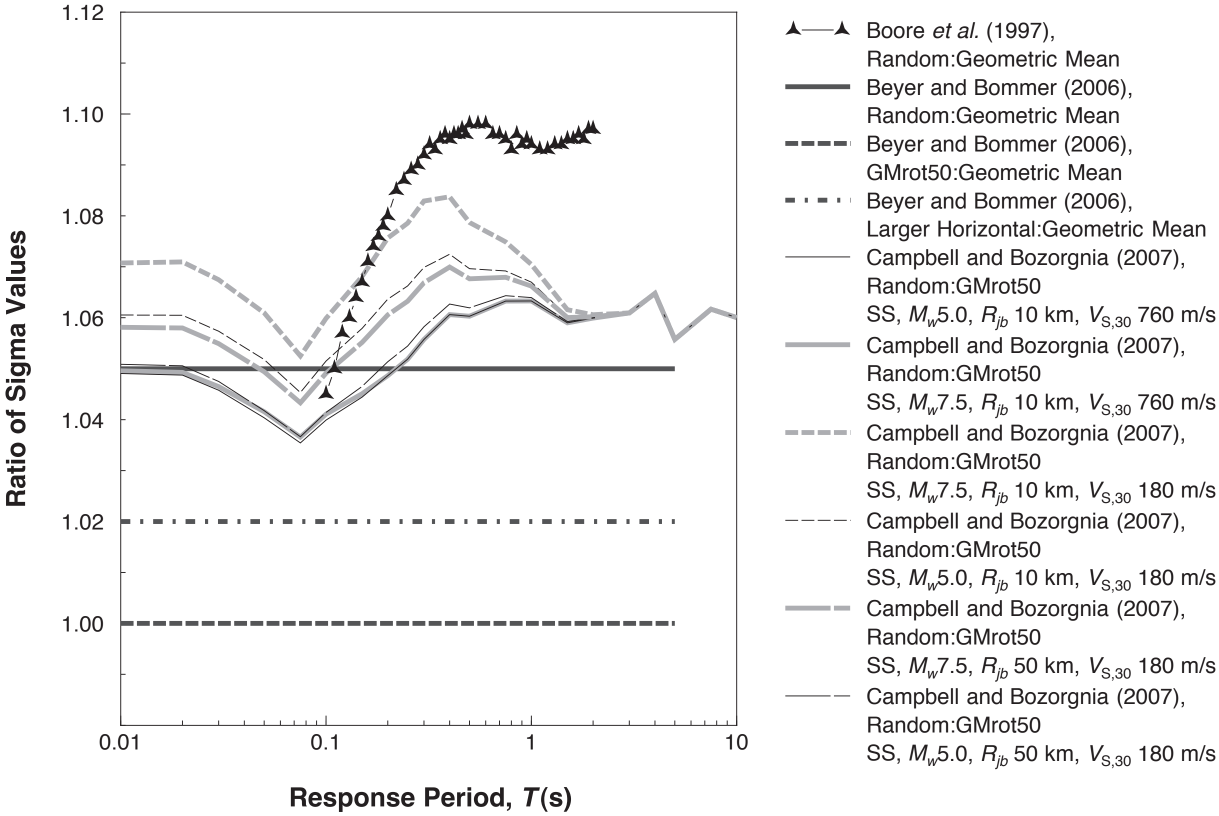
▲ **Figure 4.** PGA residuals from the Sabetta and Pugliese (1996) equation. The solid lines show the ± 1σ levels obtained from the records used to derive the equation, while the dashed lines are obtained using all available records for the same earthquakes (Bommer and Scherbaum 2005).

While ensuring that data of insufficient quality do not contaminate the regression, care should be taken not to use overly conservative selection criteria, which may lead to under-sampling of the tails of the ground-motion distribution, which in turn affects the quality of the variability estimator obtained from the sample. For example, Bommer and Scherbaum (2005) point out that the data selection procedure of Sabetta and Pugliese (1996) could have led to an underestimation of the true variability, as shown in Figure 4, which compares the σ value obtained by Sabetta and Pugliese (1996) with that obtained using all available records from the events considered. It should, however, be pointed out that it is debatable whether the records that were excluded by Sabetta and Pugliese (1996) should have been included in the regressions or not, since many of these records are associated with data quality problems, such as *S*-wave triggers and poor digitization, or with source-to-site distances in excess of 100 km (J. Douglas, personal communication, 2006).

Choice of Horizontal Component Definition

In addition to the choice of the functional form of the equation, discussed later on in the paper, the choice of the horizontal component of ground motion adopted in the regression will have an impact on the value of σ, since certain definitions are naturally more variable than others. A more general discussion of the issue of adjusting ground motions for changes of the horizontal component definition can be found in Beyer and Bommer (2006), as well as in Watson-Lamprey and Boore (2007). In particular, the latter study includes an expression for adjusting from one horizontal component definition to another with full consideration of the covariance between the original component definition and the adjustment factor.

Figure 5 compares horizontal adjustment factors for σ suggested by Boore *et al.* (1997), Beyer and Bommer (2006), and



▲ **Figure 5.** Effect of horizontal component definition on σ.

Campbell and Bozorgnia (2007). Note that the values for the random component for Boore *et al.* (1997) have been corrected with respect to the values tabulated in the original reference (see Boore 2005). These results show that adjustment factors are generally of the order of 5 to 10%.

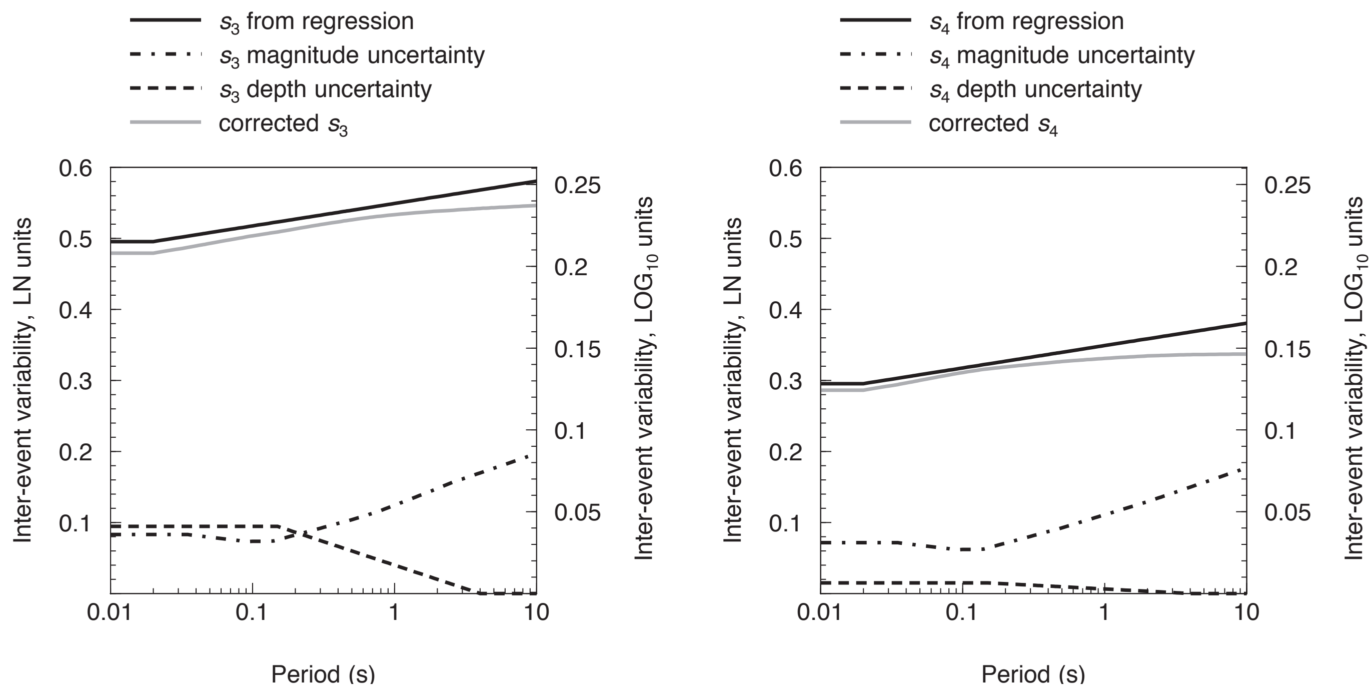
Metadata Errors

Errors in the values of the predictor variables, hereafter referred to as *metadata errors*, will contribute to the scatter through error propagation. While they are often acknowledged (*e.g.*, Joyner and Boore 1981), metadata errors are generally ignored during the fitting process. Formally, the contribution to the standard deviation ($\sigma_{Z_{\text{pred}}}$) of the predicted parameter (Z_{pred}) due to measurement error (ΔX_k) in a single predictor variable (X_k) can be estimated by:

$$\sigma^2_{Z_{\text{pred}}}(\Delta X_k) = \left(\frac{\partial f(X_1, X_2, \ldots)}{\partial X_k}\right)^2 \sigma^2_{\Delta X_k} \tag{4}$$

where $\sigma_{\Delta X_k}$ is the standard deviation of measurement error in parameter X_k. The general form for estimating the contribution to the standard deviation of measurement errors in multiple parameters, which involves the covariances of the measurement errors of these parameters taken pairwise, can be found in Abrahamson and Silva (2007). If the functional form of the model is correct, then the standard deviation calculated from the regression, which assumed no errors in the independent parameters, can be reduced if these metadata errors are identified and reduced or eliminated altogether. For instance, Rhoades (1997) has investigated the contribution of magnitude uncertainties on the values of σ determined from regression analysis, using the Joyner and Boore (1981) dataset, which includes 189 strong-motion records from 23 mostly California events. Assuming an uncertainty of 0.1 magnitude units for the events for which an estimate of M_w was available (class 1 events) and an uncertainty of 0.3 magnitude units for the seven events for which M_L has been used as a surrogate for M_w (class 2 events), Rhoades finds that the contribution of magnitude uncertainties is 57% of the inter-event variability value σ_E that is obtained when the magnitude values are considered exact, whereas the value of the intra-event variability σ_A remains almost the same. He concludes that this reduction is predominantly driven by the large magnitude uncertainties of the class 2 events. It is worth pointing out that nowadays such large magnitude uncertainties, although still encountered in practice, are seldom associated with data retained for regression analysis, whose magnitudes are generally known within 0.1 to 0.2 magnitude units, due to the widespread availability of M_w values computed using a standard technique from data coming from a standard set of digital broadband instruments (*e.g.*, the values published in the Global CMT database, http://www.globalcmt.org).

Recently, Klügel *et al.* (2006) have suggested that measurement errors in magnitude have a large impact on the standard deviation. This conclusion was based on an assumption that there is a very large uncertainty of the magnitude (standard deviation of 0.4 magnitude units). Klügel *et al.* (2006) justified their use



▲ **Figure 6.** Effect of metadata corrections on the coefficients s_3 and s_4 determining the value of the inter-event variability σ_E in the Abrahamson and Silva (2008) model. The s_3 and s_4 coefficient define the σ_E value for M_w 5.0 and 7.0, respectively. The values shown assume an uncertainty on magnitude $\sigma_m = 0.12$ and an uncertainty on depth $\sigma_{depth} = 3$ km.

of this large standard deviation of the magnitude estimate by the approach they proposed for scenario-based risk calculations. In this approach, Klügel *et al.* (2006) grouped the earthquakes into broad magnitude bins to reduce the number of scenarios to be considered and based the magnitude uncertainty of 0.4 units on the width of these broad magnitude bins. However, the measurement error of magnitudes in ground-motion databases is a seismological issue based on regional and teleseismic recordings of earthquakes; it is not dependent on how earthquake magnitudes are grouped in an engineering application. Therefore, the results given in Klügel *et al.* (2006) showing a large impact on the estimated standard deviation due to measurement errors in magnitude are not credible. It is also important to note that $\sigma_{\Delta X_k}$ in Equation 4 is the standard deviation of the measurement errors in the independent parameter, X_k, for the data used in the regression analysis. It does not represent the measurement error of historical earthquakes used in seismicity catalogs that are not included in the strong-motion database used for the regression analysis.

Abrahamson and Silva (2007, 2008) have evaluated the impact of measurement errors in magnitude, distance, average shear-wave velocity over the top 30 m ($V_{S,30}$), and depth-to-top of rupture, all of which are included as explanatory variables in their model. Figure 6 shows the correction to the inter-event standard deviation to account for measurement errors in magnitude and depth-to-top of rupture. This correction is small at short periods but becomes significant at longer periods. Similarly, Boore and Atkinson (2008) derived separate values of σ depending on whether the mechanism of the causative earthquake is known or has to be inferred. Again, the difference in the σ values is small.

Another practice that has become increasingly more common is to reconcile the need for a large dataset with the requirement for reasonably homogeneous data by constraining the individual terms of the equation using selected data subsets rather than the whole dataset. Data that either are known to show a significantly different behavior for the term under consideration, or for which the relevant parameters are unavailable, are excluded from the calculations. For instance, Campbell and Bozorgnia (2008) only used records for which sediment depth was available to constrain the term reflecting basin effects. This might have an impact on the value of σ estimated from the residual distributions, since these residuals are strictly speaking derived with respect to different predictive models (with and without sediment depth). Similarly, Atkinson and Boore (2003) excluded data from the K-NET network for the computation of σ, as these data had been found to be associated with larger high-frequency site amplifications than the remainder of the data, which in the opinion of Atkinson and Boore (2003) could lead to artificially inflated values of σ when the equation is used in regions other than Japan.

ESTIMATION OF SIGMA

Once all data-related issues have been resolved and a functional form has been chosen, the value of σ will essentially depend on the method used for the regression analysis. This statement is particularly true for strong-motion datasets, since very little control (in the statistical sense) is possible on the nature of the dataset, and particularly on the distribution of the number of records contributed by individual earthquakes and stations.

Strong-motion datasets used in the derivation of GMPE are generally of very unbalanced nature (*i.e.*, the numbers of records contributed by individual events are very uneven), and this will affect the correlation structure of the dataset and thus the estimation of the individual components of variability.

Components of Variability

Correlations in the explanatory variables may affect the estimation of σ if they are not appropriately considered in the regression process. Such correlations need to be acknowledged first during the selection of the dataset and second in the selection and fitting of the functional form. The strong-motion datasets available to researchers for the selection of accelerograms to be included in regression are generally strongly correlated in terms of the magnitude and distance distribution of the data. For earlier datasets including predominantly accelerograms recorded on analog instruments, the data at longer distances were often almost exclusively contributed by the larger events in the dataset (*e.g.*, Fukushima and Tanaka 1990). With the advent of fully digital strong-motion networks, this issue has become less important over the past decade or so, as now even small events are commonly recorded out to 100 km. Nevertheless, correlations in terms of the distribution of the data in magnitude-distance may still be present and need to be addressed.

Two-step regression techniques (*e.g.*, Joyner and Boore 1981) were introduced to prevent these correlations from biasing the regression. Biases may also be avoided by using a one-stage maximum-likelihood technique based on the random effects approach (Brillinger and Preisler 1984, 1985; Abrahamson and Youngs 1992). In these approaches, the total variability σ_T is separated into the inter-event variability σ_E and the intra-event variability σ_A, these three parameters being related through the following expression:

$$\sigma_T = \sqrt{\sigma_E^2 + \sigma_A^2} \tag{5}$$

The intra-event variability can further be separated into a site-to-site component σ_S, and the record-to-record variability σ_R (also denoted σ_O by some authors) remaining after contributions from source and site have been accounted for:

$$\sigma_A = \sqrt{\sigma_S^2 + \sigma_R^2} \tag{6}$$

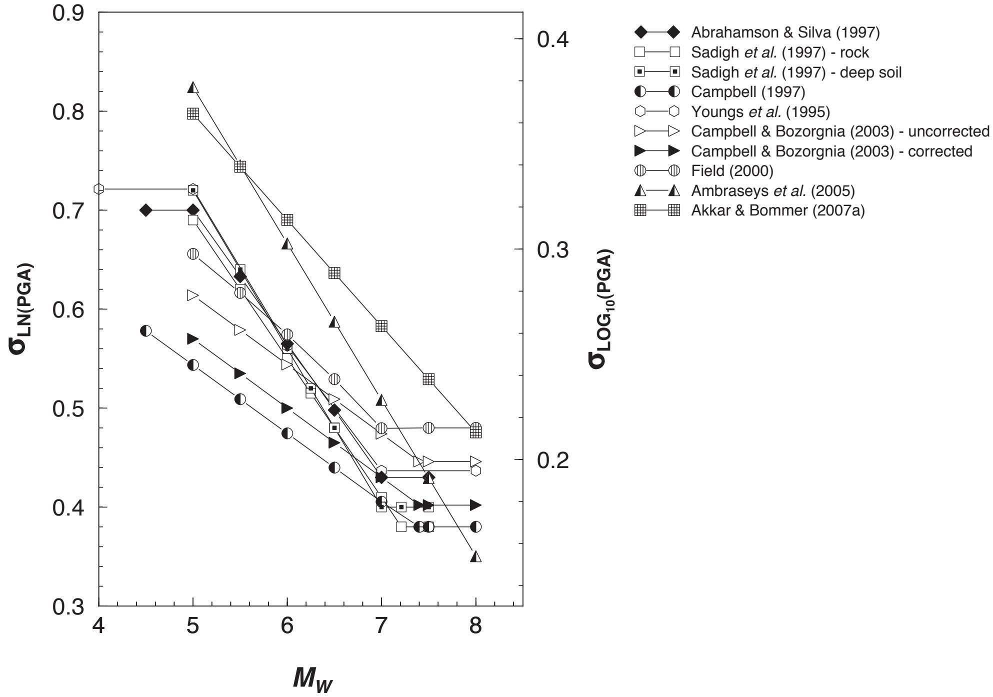
In practice, however, the analysis is generally performed on the lumped intra-event variability σ_A, because the number of data points from different events recorded at a single site is usually small (Joyner and Boore 1993). One exception is the study of Chen and Tsai (2002), who take advantage of the large number of accelerograms recorded on the dense strong-motion network installed in Taiwan to derive an equation with three components of variability. It is also worth noting that the problem of estimating three components of variability is significantly more complex than that of estimating only two components, since it is no longer possible to organize the data in mutually independent blocks of correlated data ("1-way classification"; McCulloch and Searle 2001, 126), leading to a block-diagonal covariance matrix, which can easily be inverted. The three-component case implies a "2-way classification" (by earthquake and by station), leading to non-zero off-diagonal blocks in the covariance matrix, whose inversion therefore becomes more difficult.

In addition to Chen and Tsai (2002), a number of studies considering the variability of ground motions recorded at the same station may be found in the literature (Sen 1990; Niazi and Bozorgnia 1991; Bindi *et al.* 2006; Luzi *et al.* 2006). These studies are based on strong-motion datasets with a sufficient number of repeated recordings at the same station to warrant consideration of this variability. Unlike Chen and Tsai (2002), however, these studies do not attempt to estimate all three components of variability but instead derive estimates of the variability of ground motions recorded at the same station (intra-station variability) versus the variability of ground motions from different recording sites (inter-station variability), possibly due to the numerical difficulties associated with the 2-way classification. The estimates of inter-station variability can then be compared with those of inter-event variability in an attempt to identify the main cause of variability. Alternatively, the relative influence of event-specific and site-specific effects can be investigated using analysis of variance, as proposed by Douglas and Gehl (2008). It should be noted that the data used in these studies usually come from seismic sequences including foreshocks and aftershocks, and therefore these findings might not be generally applicable. In particular, aftershock events are generally associated with postseismic slip and therefore are expected to differ from similar-size mainshock events for parameters characterizing the rupture process, such as stress drop. Abrahamson and Silva (2008) checked the values of intra-event variability obtained from mainshock data and from aftershock data and found no difference between these values. The total variability obtained from datasets including a large proportion of aftershock data may, however, still be different from that obtained from datasets restricted to mainshock data, because including the aftershock data is likely to increase the relative contribution of small-magnitude events.

Homoscedastic vs. Heteroscedastic Sigma

In most equations, the scatter is assumed to be homoscedastic, *i.e.*, independent of the variables included in the equation. However, several authors have found trends relating the scatter to one or more explanatory variables and therefore suggested heteroscedastic models for the scatter, in which σ depends on the predictor variables (in most cases, magnitude).

As summarized in Figure 7, a number of studies have found a decrease of the scatter with increasing magnitude (Sadigh *et al.* 1997; Campbell and Bozorgnia 2003; Youngs *et al.* 1995; Abrahamson and Silva 1997, 2008; Ambraseys *et al.* 2005; Akkar and Bommer 2007b; Bommer *et al.* 2007). Youngs *et al.* (1995) investigate the issue of the magnitude dependence of the PGA variability and find that this effect is more pronounced for the inter-event term than for the intra-event term. While they find that the explanations suggested by other authors, such as nonlinear site effects (Chin and Aki 1991) or a shift of the predominant period of the motion to longer periods with increas-



▲ **Figure 7:** Selection of published models of magnitude-dependent σ_T for horizontal components of PGA.

ing magnitude (Beresnev *et al.* 1994) cannot be discounted entirely, Youngs *et al.* (1995) suggest that the decrease in inter-event variability at larger magnitudes could also be related to a decrease of the variability of stress drop at larger magnitudes. Errors in the location and magnitude determination of smaller events, in particular aftershocks, might also contribute to the trend, alongside potential biases introduced by the relative scarcity of data from large-magnitude earthquakes.

Boore *et al.* (1994) examine the magnitude dependence of the residuals of their equations for PGA and 5% damped PSV. Their results in terms of PGA are consistent with the findings of Youngs *et al.* (1995): the data exhibit a decrease in variance with increasing magnitude that is particularly pronounced at magnitudes below 6.0. However, no significant dependence is found for the response spectral ordinates, which the authors suggest could be related to the scarcity of records from small magnitude ($M_w < 6.0$) events in the response spectral data set. Douglas and Smit (2001) find a significant dependence (at the 5% significance level) of σ on M_S for a dataset combining predominantly European, western North American, and Taiwanese data, with larger magnitude earthquakes associated with smaller variability than smaller earthquakes. This dependence on magnitude is also a feature of the equations for Europe and the Middle East derived by Ambraseys *et al.* (2005) and Akkar and Bommer (2007a, 2007b). On the other hand, Midorikawa and Ohtake (2004) derive equations for PGA and PGV based on a large sample of Japanese data and find only a mild dependence of variability on magnitude, whereas they find a significant dependence on distance in the near-source region ($R_{jb} \le 50$ km), with σ decreasing closer to the source. These findings are in contrast with the conclusions of Blume (1980), who divided a dataset of 816 records into 10 distance bins, for which he computed individual estimates of σ and found that the value of σ decreases with distance from the source. Most other GMPE that have been published since, however, have ignored the dependence of σ on distance or found that it could be neglected.

The inconsistencies in the conclusions regarding the relative influence of predictor variables on σ point to the fact that the interaction between the physical processes involved in the generation and propagation of ground motions is complex, and that this complexity is maintained in their impact on the value of σ. In particular, at short source-to-site distances, it is not straightforward to decouple the issue of magnitude-dependence of σ from that of the effects of soil nonlinearity. Due to the degradation of the strength and stiffness of the soil under cyclic loading, large-amplitude ground motions are amplified less than small-amplitude motions. This can lead to a dependence of the value of σ on the amplitude of the ground motion (*e.g.*, Campbell 1997; Campbell and Bozorgnia 2003), which would indicate that the use of the logarithmic transformation in the functional form of the equations may not be strictly correct (Douglas and Smit 2001). Since ground motions also scale with magnitude, this dependence of σ on the level of ground motion can also be translated into a dependence on magnitude, as has been done for instance by Campbell and Bozorgnia (2003).

Values of σ on Rock vs. Values of σ on Soil

Consideration of the nonlinear behavior of surficial geological deposits is one of the aspects of the selection of a functional form for the GMPE that may have an influence on the value of σ. A number of studies have derived separate equations for ground motions on rock and on soil by splitting the underlying dataset in two and carrying out separate regression analyses (Huo and Hu 1991; Crouse and McGuire 1996; Sarma and

Srbulov 1998; Sadigh *et al.* 1997). These studies generally find that the σ value for soil sites is lower than that for rock sites. One exception is Sadigh *et al.* (1997), who found larger σ values for deep soil sites than for rock sites. This might be due to the fact that deep soil sites are generally located in basins, and therefore 3D-geometry effects might be increasing the variability more than soil nonlinearity decreases it. Furthermore, in the Sadigh *et al.* (1997) model, the σ values are assumed to be magnitude-dependent, with different slopes for deep soil (–0.16) and rock (–0.14) motions, which means that nonlinearity effects are already partially accounted for when comparing the rock and soil σ values corresponding to the same magnitude.

Another approach to include the effect of soil nonlinearity in ground-motion prediction consists in explicitly including a term modeling the amplification of surface ground motions with respect to their bedrock value (*e.g.*, Abrahamson and Silva 1997, 2008). This approach requires the use of a nonlinear fitting procedure in order to obtain an unbiased estimate of the coefficients. Abrahamson and Silva (2008), followed by Campbell and Bozorgnia (2008), also take the step of propagating the uncertainties related to the amplification to the values of σ, which requires consideration of the correlation between ground motions at different response periods (Baker and Cornell 2006). An important point to note is that even when nonlinearity effects are accounted for in the regression, differences in the variability of the residual subsets associated with different site classes are still observed. This is illustrated in Figure 8 for four equations recently derived as part of the Next Generation of Attenuation (NGA) project (Abrahamson and Silva 2008; Boore and Atkinson 2008; Campbell and Bozorgnia 2008; Chiou and Youngs 2008).

In each plot, the symbols correspond to the value of the sample standard deviation of the intra-event residuals, δ_A, which have been divided into subsets according to their National Earthquake Hazards Reduction Program (NEHRP) site class. In view of the limited number of recordings associated with site class A or E stations, these have been lumped together with site class B and site class D recordings, respectively. Despite differences in the underlying datasets, these plots show a very consistent pattern, with the values for NEHRP site class C generally lying above those for NEHRP site classes D and E; the values for NEHRP classes A and B show a slightly more erratic behavior, which is likely to be a consequence of the relatively small size of this subset. A good agreement across models among the values corresponding to a given subset is also observed. Overall, this pattern would seem to indicate that the level of variability for ground motions recorded on similar site conditions is fairly stable with respect to the addition or subtraction of data and differences in the modeling of the median ground motion. Since the calculation of the sample standard deviations shown in Figure 8 ignores the magnitude-dependence of σ_A, the values for the Chiou and Youngs (2008) and Abrahamson and Silva (2008) equations are found to be slightly larger as a result of the consideration of a larger number of small-magnitude events.

When the uncertainties related to soil nonlinearity effects are propagated to σ, its value depends both on PGA and on $V_{S,30}$, resulting in distance-dependence when a fixed magnitude value and a given $V_{S,30}$ value are considered. This is illustrated in Figure 9 using the equations of Abrahamson and Silva (2008) and Chiou and Youngs (2008). The values plotted are for PGA, a magnitude $M_w = 7.0$, and $V_{S,30} = 270$ m/s. This figure shows that consideration of nonlinearity effects results in a substantial decrease in σ at short source-to-site distances ($R_{\mathrm{rup}} \leq 20$ km), with values up to 20% lower than the value at $R_{\mathrm{rup}} = 50$ km.

The distance-dependence is stronger for the Abrahamson and Silva (2008) model than for the Chiou and Youngs (2008) equations, as the former constrained the nonlinear terms of the equations by considering a 1-D site response model, leading to more pronounced nonlinearity than obtained from direct regression on the data by the latter authors.

Trade-offs in the Estimation of σ and μ

The value of σ and the values of the coefficients required to estimate the median ground motion are determined jointly in the regression, with the value of σ often being used as a gauge for the level of confidence attached to the mean value of the regression parameter, hereafter called μ. Such an interpretation is based on the case of a normal distribution, for which the confidence in the estimated values of the mean is given by the sample standard error of the mean, which is defined as the sample standard deviation divided by the square root of the sample size:

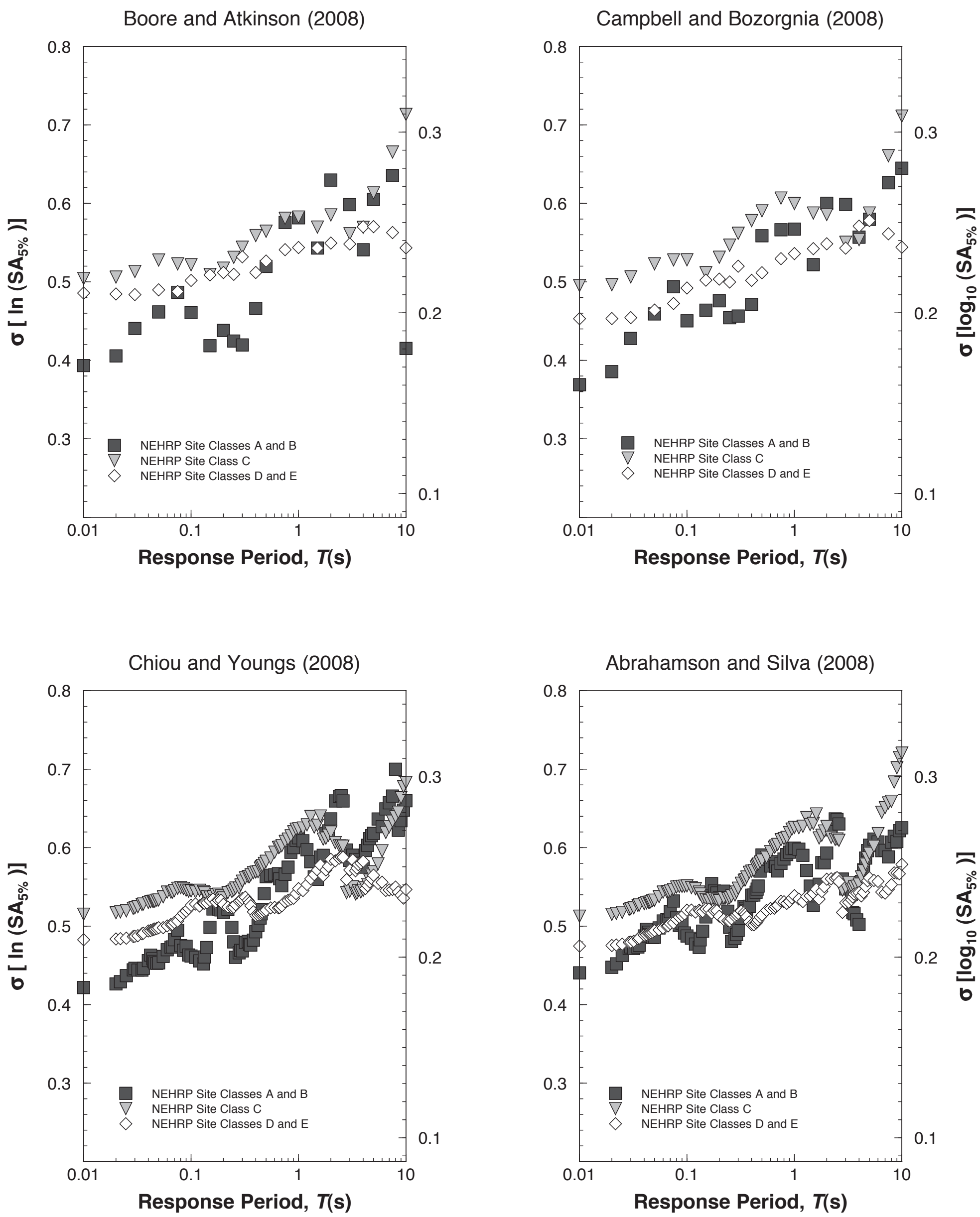
$$SE_{\mu} = \frac{\sigma}{\sqrt{N}} \tag{7}$$

In this simple case, the confidence in the estimate of the mean increases (*i.e.*, *SE* decreases) when the sample size increases or when the sample standard deviation can be reduced. In the case of strong-motion datasets, however, this is complicated by the correlations that exist between the records coming from the same event or station. Assuming a 1-way classification separating the inter-event and intra-event variability, with the usual assumption that the inter-event and intra-event residuals follow independent normal distributions, the sample standard error on the mean is given by the following expression:

$$SE_{\mu} = \sqrt{\frac{\sigma_E^2}{N_{EQ}} + \frac{\sigma_A^2}{N_{REC}}} \tag{8}$$

where σ_E and σ_A are the sample inter-event and intra-event components of variability, and N_{EQ} and N_{REC} are the number of events and the number of records, respectively. The expression in Equation 8 shows that the relative values of N_{EQ} and N_{REC} may have an impact on the overall confidence.

Another issue pertaining to the trade-off in the estimation of μ and σ is the potential contamination of the residual distribution that may occur if data following a slightly different distribution are added to increase the total number of data points. This is illustrated schematically in Figure 10. Let us assume that the true distribution of the data is characterized by a normal distribution with parameters μ_1 and σ_1. The dataset is supplemented by data that follow a normal distribution with param-



▲ **Figure 8.** Comparison between intra-event variability estimated through regression and sample standard deviations of intra-event residuals binned by NEHRP site class, for the NGA equations of Boore and Atkinson (2008), Campbell and Bozorgnia (2008), Chiou and Youngs (2008), and Abrahamson and Silva (2008).

eters $\mu_2 > \mu_1$ and $\sigma_2 \approx \sigma_1$. As a result of the shift in the mean, the best-fitting normal distribution to the overall dataset, characterized by parameters μ_3 and σ_3, will be broader than any of the subset distributions.

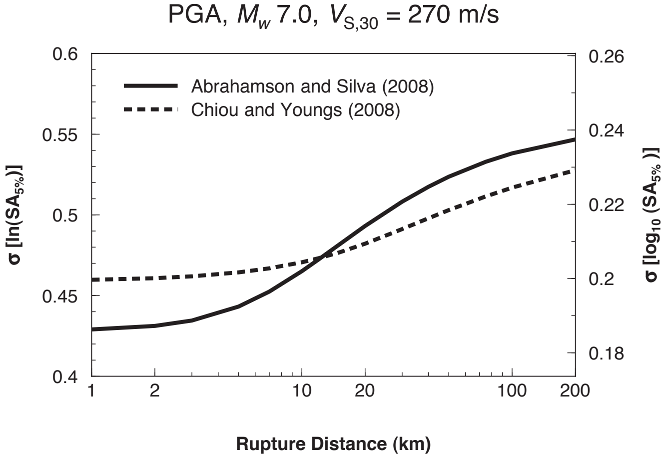
In practice, potential contaminants are difficult to detect in view of the numerous factors that may cause subsets of the data to differ from one another. Additionally, contaminant detection is hampered by the great variety of criteria adopted to exclude recordings from regression datasets. These criteria seek to exclude data that are thought to behave differently from the rest of the data, or that include contributions to the aleatory variability that may not be relevant for the intended application of the model (*e.g.*, variability due to 3-D basin structure for estimation of ground motions on flat-layered rock).

CAN SIGMA BE REDUCED?

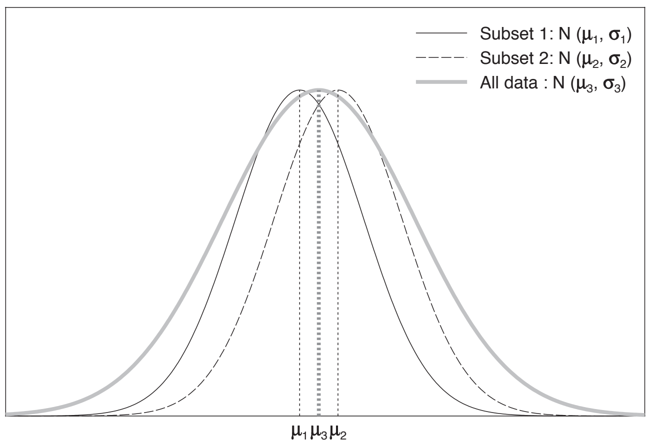
Ultimately, any investigation into the nature of σ and the issues associated with its estimation aims at the identification of methods to reduce the value of this parameter. This last section explores the options available to GMPE developers regarding the reduction of σ.

Additional Parameters

Since the prime cause of the large values of scatter seems to lie in the use of rather simple models to describe complex phenomena, a legitimate question to ask is whether the inclusion of additional terms in the predictive model leads to a reduction of the aleatory variability. Such a reduction has been observed by



▲ **Figure 9.** Distance-dependence of σ_T as a result of the inclusion of soil nonlinearity effects.



▲ **Figure 10.** Schematic illustration of trade-off between σ and μ.

Lussou *et al.* (2001) and Pousse *et al.* (2005) when they included a site classification in their equations based on Japanese K-NET data. It should be noted, however, that the decrease in σ was fairly modest, and the final values of σ obtained were still large compared to those published for other equations. Conversely, Abrahamson and Litehiser (1989) and Ambraseys and Bommer (1991) found no improvement in the value of σ_T when adding a site term to their functional form for the prediction of PGA. Similarly, equations including individual site terms for each station considered (*e.g.*, Molas and Yamazaki 1995) are not associated with noticeably lower values of σ_T than the bulk of the GMPE that have been published so far.

Douglas and Smit (2001) discuss the concept of pure error (Draper and Smith 1981, 33–42) applied to ground-motion predictive equations. For a given set of records, pure error analysis provides a lower bound on the standard deviation possible by fitting any functional form, no matter how complex, to the data once a set of explanatory variables has been selected. Douglas and Smit (2001) concluded that the values of σ then found in empirical predictive equations (including magnitude, distance, and a site term) were about the best achievable without including additional explanatory variables.

Equations that have been published since, and in particular the equations published as a result of the recent NGA project (Abrahamson and Silva 2008; Boore and Atkinson 2008; Campbell and Bozorgnia 2008; Chiou and Youngs 2008, Idriss 2008), include a number of additional terms, without, however, reducing the values of σ when compared with the previous generation of equations (Abrahamson and Silva 1997; Boore *et al.* 1997; Campbell 1997; Campbell and Bozorgnia 2003; Sadigh *et al.* 1997; Idriss 1991). In fact, in some cases, the σ value has increased compared to previous models, as illustrated in Figure 11. The additional parameters were added to improve the median estimates for source/site combinations that are important for engineering application but were not well-constrained by the data, such as locations affected by hanging-wall effects. Since there are few data for these cases, there is little impact on the computed standard deviation.

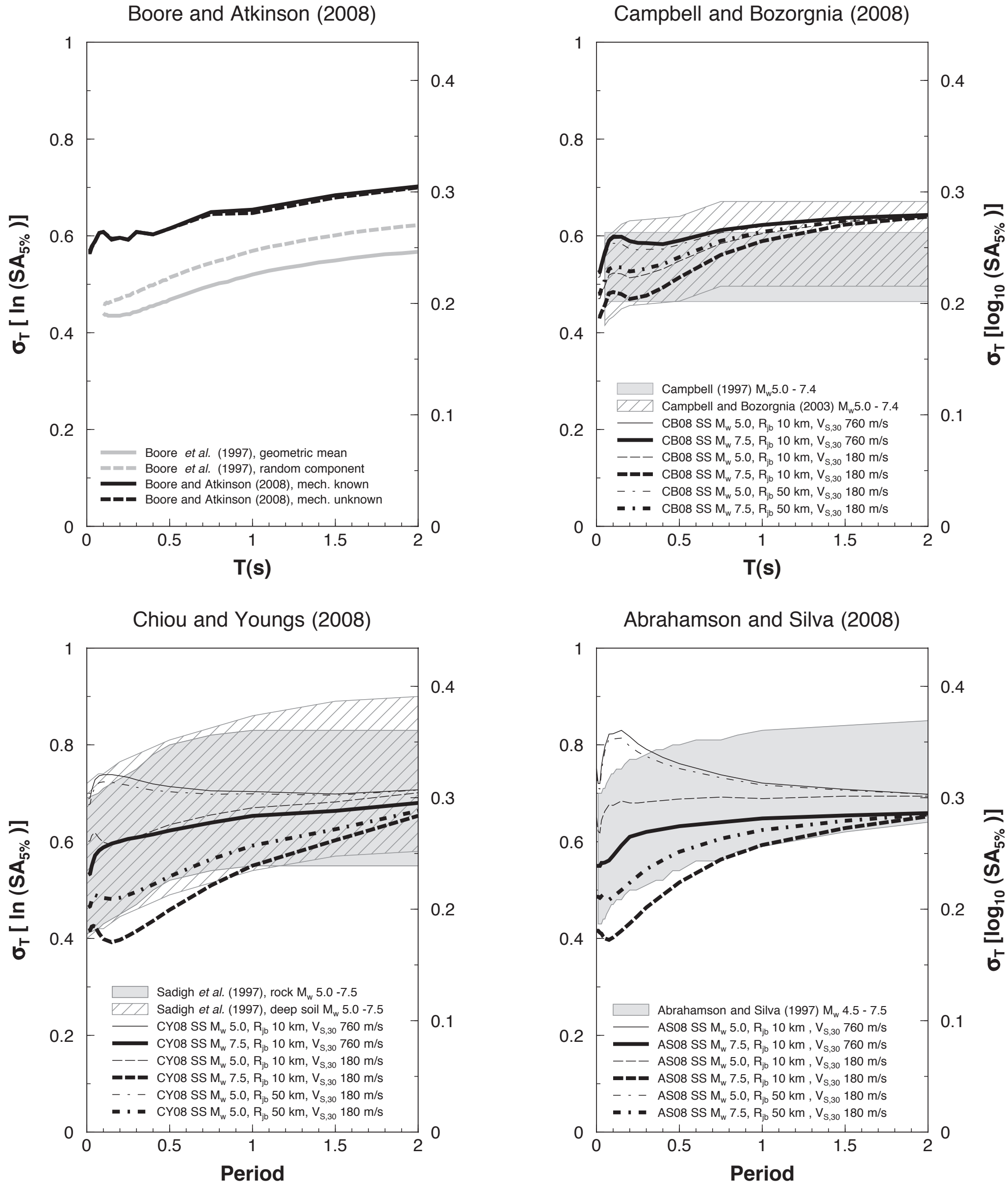
A likely explanation for this stationarity or increase of σ is the trade-off that exists between the reduction of the value of the residuals gained by a more accurate prediction of the median ground motion (*i.e.*, a larger proportion of the ground-motion value is explained) and the additional parametric uncertainty introduced by adding explanatory variables whose values may be unknown for part of the strong-motion dataset.

Larger Datasets

Since adding new terms to the equation does not seem to be promising as long as the metadata required cannot be satisfactorily constrained, another question to ask is whether σ values may be reduced by considering larger datasets. A considerably larger number of accelerograms are now available to GMPE developers than were a few decades ago, but no noticeable diminution in the value of σ seems to have occurred (Figure 2). In datasets derived for a given region, indigenous data are often supplemented by allogenous data (*i.e.*, data from other regions) in order to obtain a better distribution of the data (*e.g.*, in magnitude-distance space) and thus ensure the stability of the regression. Although the additional recordings may help to provide better constraints on some of the terms in the regression, they also increase the potential for contamination of the "true" target distribution, as discussed previously, since it is generally assumed, rather than proven, that these additional recordings exhibit a behavior similar to that to be modeled.

A striking example of this is the treatment of the large set of strong-motion data recorded during the 1999 Chi-Chi, Taiwan, earthquake and its larger aftershocks, which contributes more than half the recordings in the dataset made available to the developers of the NGA project. In view of recognized differences in the attenuation structure between Taiwan and the western United States, as well as in the spectral scaling of aftershocks, some developers chose to exclude all data from the aftershock sequence (Boore and Atkinson 2008), while others included a separate attenuation term in the regression (Abrahamson and Silva 2008).

Concerns about contamination by allogenous data have led some GMPE developers to prefer smaller datasets from a restricted geographical region (*e.g.,* Marin *et al.* 2004; Mahdavian 2006). A



▲ **Figure 11.** Total aleatory variability values of the final NGA models of Abrahamson and Silva (2008), Boore and Atkinson (2008), Campbell and Bozorgnia (2008), and Chiou and Youngs (2008) compared with the aleatory variability values associated with previous GMPE derived by the same developer teams. All NGA values are for the rotated geometric mean horizontal component of motion GM_{rot50} (Boore *et al.* 2006). The values plotted for the Abrahamson and Silva (2008), Chiou and Youngs (2008), and Campbell and Bozorgnia (2008) models are for selected scenarios, assuming the causative event occurs on a vertical strike-slip fault and is associated with surface rupture.

recent study by Liu and Tsai (2005) derived PGA and PGV equations for the whole of Taiwan, as well as for three subregions (CHY, IWA, and NTO). A noticeable reduction of σ was found for only one of the subregion regressions (CHY, $\sigma_T[\ln(\text{PGA})] = 0.637$ versus 0.687 for the whole of Taiwan), whereas the others showed almost no reduction (NTO, $\sigma_T[\ln(\text{PGA})] = 0.685$) or an increase (IWA, $\sigma_T[\ln(\text{PGA})] = 0.703$) in variability. Similarly, Douglas (2007) investigated the regional dependence of response spectra and found that σ values for equations derived using data from small geographical areas are generally higher than those derived using datasets combining several regions.

In view of the limitations on sample size caused by the restriction to a small geographic area, the stability of the regression is sometimes ensured by extending the magnitude range of the data to lower magnitudes. However, recent work has shown that ground-motion amplitudes scale differently at small and large magnitudes (*e.g.*, Bommer *et al.* 2007). Although it is in general desirable to expand the magnitude range considered for the prediction of median ground-motions, these differences in scaling between small and large magnitudes need to be addressed in the estimation of σ. Furthermore, a comparison of predictions from regional equations derived for parts of Europe with those obtained using recently derived pan-European equations (Ambraseys *et al.* 2005; Akkar and Bommer 2007a, 2007b) shows that in many cases, the latter exhibit a better fit to the data than the former. This is consistent with the finding that recent equations for ground motions from shallow crustal events perform satisfactorily even outside their intended region of application (*e.g.*, Stafford *et al.* 2008).

Source-site Specific Estimation

Yet another approach to gain control over the value of σ is to investigate in more detail the behavior of the individual components of variability by examining the variability associated with a single event or a single station (*e.g.*, Niazi and Bozorgnia 1991; Ordaz and Reyes 1999; Jain *et al.* 2000; Atkinson 2006). Such studies have generally been motivated by recognized differences in ground motions from a given source or at a given site. One exception is the Niazi and Bozorgnia (1991) study, which took advantage of the data recorded on the dense Lotung array to investigate the relative contributions of variability among stations and variability among events to the total variability. They found that the variation among stations was significantly lower than that among earthquakes for each station, but note that this was to be expected in view of the uniform site characteristics of the stations considered, which are located on a ring of 200-m radius.

Atkinson (2006) compared the total standard deviation of recordings from single stations in the Los Angeles basin with the standard deviation obtained using data from all the stations. She found that the single-station standard deviation was 10% smaller than the standard deviation for the total dataset. Furthermore, she found that by restricting the data to recordings corresponding to a single source zone (Landers), the variability could be reduced by 40% with respect to the estimate obtained in the multiple-source, multiple-station case. These findings are potentially very useful for seismic hazard analysis, since assessments are made for specific sites, and the seismic source zones dominating the hazard can often also be reasonably well-identified. However, the reduction of σ for an individual site or source-site combination for which there is not a database of ground-motion recordings would carry a penalty in terms of increased epistemic uncertainty on the median ground motion. If an analyst wishes to apply an existing GMPE at a new site, but with a reduced variability, σ_{SUB}, corresponding to the hypothetical subset of motions that could occur at the site, it would be necessary to add branches to the logic-tree to reflect the fact that the median of the subset, μ_{SUB}, is unknown in this application. For instance, let us assume that the broad distribution in Figure 10 (gray line) characterizes the original ground-motion residual distribution from a predictive equation derived using data from multiple sources and multiple sites. Applying the σ reduction for a single site (or a single source-site combination) is conceptually equivalent to isolating the contribution to the overall ground-motion distribution of the (hypothetical) subset of ground motions corresponding to that specific site (or source-site combination). In this example, and assuming that $\sigma_{SUB} = \sigma_1 = \sigma_2$, this contribution could be represented by either of the distributions labeled subset 1 and subset 2 (black lines) or any other distribution with a standard deviation equal to σ_{SUB}. Numerical modeling could be used to obtain constraints on the value of μ_{SUB}, but in the absence of such constraints, the total uncertainty (aleatory plus epistemic) is not necessarily reduced, and the mean hazard may remain unaffected.

Replacing the "general" multiple-source, multiple-station aleatory variability by its source-site-specific equivalent is akin to dropping the ergodic assumption, which has in the past been held responsible for inflated estimates of ground motion in seismic hazard analysis (*e.g.*, Anderson and Brune 1999). It is, however, unclear how the reduction factors found by Atkinson (2006) for the Landers–Los Angeles basin case may be adapted to other source-site configurations, since it is difficult to assess how favorable or unfavorable the source-site configuration examined is in terms of reducing the value of σ. Atkinson's results are fairly stable across the stations examined, but this could be related to the fact that the source-site azimuths are fairly similar across the dataset. Since a dependency of ground-motion variability on azimuth is to be expected, based for instance on results to account for the effects of ground-motion directivity (*e.g.*, Somerville *et al.* 1997; Abrahamson 2000), further investigations are required to examine the influence of azimuth on the reduction factors from multiple-source, multiple-station σ values to source-site-specific σ values. A recent study (Lin *et al.* unpublished) makes use of the plentiful data recorded on the dense Taiwan Strong Motion Instrumentation Program (TSMIP) network in Taiwan to investigate the influence of path effects on the value of sigma. By examining the residuals of a dataset of 7,722 three-component records from 77 crustal earthquakes, the authors find that the variability of the difference between pairs of intra-event residuals tends to increase as the similarity of the source-to-site paths decreases. The authors conclude that a reduction of σ of up to 30% could be achieved if the path effect were included in the functional form of predictive equations. Similarly, Morikawa *et al.* (forthcoming) investigated the variability of ground motions from events occurring in narrow geographical areas at selected K-NET and KiK-Net stations. By considering the residuals with respect to the Kanno *et al.* (2006) model, the authors find that applying a source-site-specific correction factor may significantly reduce the observed dispersion (from 0.3–0.4 to 0.15–0.2 $\log_{10}$ units). For regions without dense recording networks, however, such investigations are hampered by the limited number of empirical multiple-source, multiple-station datasets available. Numerical simulations could represent an attractive alternative, but are likely to be associated with other types of uncertainty, as discussed below.

Insights from Numerical Simulations vs. Empirical Data

While the focus of this paper is predominantly on the estimation of σ based on empirical data, it is important to acknowledge the insights that may be offered by numerical simulations. GMPE are generally based on simulations when empirical strong-motion data are scarce. For instance, predictive equations derived for central and eastern North America (*e.g.*, Toro *et al.* 1997; Atkinson and Boore 1995, 2006) have been based on the results of numerical simulations. However, since the variability in the results of the simulations mainly reflects the parametric aleatory variability of the input, estimates of variability obtained from numerical simulations only reflect part of the aleatory variability observed in empirical ground-motion predictions.

Hutchings *et al.* (2007), for example, used a physics-based numerical simulation to compute the ground motions for a given magnitude, location, and site (*e.g.*, a source-site-specific ground-motion model). They developed distributions of the suite of source parameters used in their model and then computed the ground motion by sampling these distributions. The variability of the resulting ground motions is the parametric aleatory variability only; they did not include the modeling aleatory variability, so they are missing part of the total aleatory variability. As a result, the variability estimates are expected to underestimate the variability observed in nature; however, the underestimation resulting from ignoring the modeling variability may be balanced by the fact that the simulations usually consider the marginal distributions of the parameters, rather than their joint probability distribution. By using marginal distributions, some of the results may correspond to physically impossible input parameter combinations, or the probability of rare combinations may be overestimated. Constraining the joint distribution of inputs for numerical simulations is a key factor in improving the estimates of the standard deviations from numerical simulations.

Another potential use for numerical simulations is as a benchmark tool for the procedures used to estimate the various components of variability; in particular, to quantify the impact of the dataset structure (*e.g.*, how uniformly the records are distributed in the space of predictor variables and how balanced the dataset is) on these estimates. This knowledge could then be used to select, from the extensive strong-motion database now available to GMPE developers, regression datasets that are statistically better-behaved than the ones currently used. This would obviously entail a considerable effort in addition to the already sizeable task of compiling a dataset that is associated with good-quality metadata and well-distributed with respect to the predictor variables. However, the crucial impact of the value of σ on the results of seismic hazard analysis might make this effort worthwhile.

CONCLUSIONS

Assessing the variability of ground motions, σ, is an inescapable reality in the ground-motion prediction process, since the appropriate characterization of ground motions needs to acknowledge the large degree of scatter associated with these motions. The value of σ has remained fairly stable over the past 40 years, with values typically lying between 0.10 and 0.40 $\log_{10}$ units (about 0.23 to 0.92 ln units) and most commonly falling in the range of 0.15 to 0.35 $\log_{10}$ units (0.35 to 0.80 ln units). The resistance of σ to any efforts made to reduce its value is a matter of great concern to ground-motion prediction in general and seismic hazard analysis in particular, in view of the significant impact the value of σ has on hazard estimates. This impact has sometimes led to the temptation of ignoring σ altogether, but it is now accepted that the inclusion of σ, rather than being an option that can be switched on and off, is an integral part of any seismic hazard assessment process (Bommer and Abrahamson 2006)—in other words, σ is here to stay.

So, is there anything that can be done about reducing its value? The scope for a reduction in the value of σ comes mainly from the observation that in practice, σ includes contributions from factors other than the intrinsic variability of ground motions. While σ is generally interpreted as representing aleatory variability, it must be borne in mind that this randomness is referred to the ground-motion model under consideration and therefore, some of the randomness may only be apparent. In other words, some contributions to σ may reflect uncertainties in the estimation process, rather than intrinsic variability of the ground motions. As discussed in the present article, these contributions can be related to two main sources of uncertainties, namely issues regarding the quality of the data used in regression and issues regarding the ground-motion model specification and fitting. In both cases, there is some hope to achieve reductions in σ, although the process may be labor-intensive.

First, contributions to σ that are related to insufficient data quality may be reduced through careful selection and reappraisal of the data. The global strong-motion databank is now sufficiently rich in data to allow the use of more stringent quality criteria in the selection of the recordings to be included in regression analyses and thus to avoid including poor-quality accelerograms and poorly documented records that could lead to artificial inflation of the value of σ. However, data selection criteria based on quality need to be reconciled with the requirement of having a dataset that is well-distributed across the space of the predictor variables, which means that in some cases it may be preferable to include some lower-quality recordings rather than to risk underestimating the true value of σ through overly restrictive data selection. In particular, a practice to be avoided is the exclusion of records associated with large residual values, unless the ground-motion values can be proven to be clearly erroneous (*e.g.*, resulting from instrument malfunction or a digitization error). The use of robust regression methods (*e.g.*, maximum-likelihood approaches) also allows a reduction of the influence of large residual values on the estimation of σ, compared to the ordinary least-squares case.

Second, there is reason to believe that σ can be reduced if the physical processes governing the behavior of ground motions are better understood and modeled. In this context, better control of sigma can be achieved by considering the components of variability associated with source, path, and site characteristics. The consideration of the inter- and intra-event components of variability is becoming increasingly routine in the derivation of ground-motion prediction equations. Other components, such as inter- and intra-site variability, have been investigated, but their applicability remains limited in view of the nature of the strong-motion data available, which include few stations that have recorded multiple events. A better grasp of the physical processes involved in ground-motion generation and propagation may lead to the inclusion of additional terms in the functional form of the equation. While this will generally lead to an improved estimate of the median ground motion, it will not necessarily result in a reduction of σ, as there is a penalty to be paid in terms of parametric uncertainty for the newly added parameters. This means that if the objective is to reduce the value of σ, rather than to have the most accurate estimate of the median, adding new parameters is only worthwhile as long as these parameters can be reliably estimated. The use of numerical simulations might help in estimating these parameters, and more generally in filling data gaps, but similarly carries a penalty in terms of modeling and parametric epistemic uncertainty. This points to a trade-off between the accuracy with which the median ground motion can be estimated and potential reductions of σ. Ground-motion modelers have hitherto concentrated efforts on the first of these items; in view of the influence σ has on results of seismic hazard analysis, it might be worth concentrating more on the second point, and investigating the derivation of ground-motion models in which the accuracy of the median is partly sacrificed in favor of a lower σ value.

Finally, among the large number of physical factors that have been found to influence the value of σ, some may not be relevant or applicable to a specific project. There is therefore some scope for reducing the value of σ to be used in a particular application by tailoring it to that specific situation. This can be achieved through the use of heteroscedastic models, which allow σ to vary with selected predictor variables of the ground-motion model (magnitude, distance, $V_{S,30}$) and thus enable the consideration of physical processes such as soil nonlinearity effects or the decrease of stress drop variability with increasing magnitude. Similarly, procedures have been recently developed (Atkinson 2006; Lin *et al.* unpublished) to tailor σ to a specific source-site configuration. This is potentially the most promising approach to reduce σ, with observed reductions of 10% to 40% in some cases, but again carries a penalty of increased epistemic uncertainty on the median ground motion. Also, for a general application of the findings of these studies, the sensitivity of the reduction in σ to the source-site configuration considered needs to be assessed, possibly by numerical simulations.

In conclusion, the prospects of reducing σ are not as hopeless as they might seem at first glance. Previous attempts at reducing σ through the use of larger datasets and the inclusion of additional terms in the functional form of predictive equations may have met with little success because the additional uncertainties introduced had not been considered. Approaches to reduce σ that bear these additional uncertainties in mind are admittedly quite labor-intensive, but the effort involved is likely to be worthwhile in the context of seismic hazard analysis.

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