

Sikkim Manipal Institute of Technology
Department of Mathematics
BBA (II Sem)
Subject: Business Mathematics (MA 1205)
Second Sessional Examinations

Dur: 1 hr 30 mins

04.04.2019

Max: 50 marks

Instructions

- (i) Answer all the questions.
- (ii) Any missing or misprinted data can be assumed suitably.

1. (a) What is the expansion of $(1+x)^4$? (Write all the terms) (4)

(b) Find the middle term(s) in the expansion of $\left(\frac{x}{a} + \frac{a}{x}\right)^4$ (3)

(c) Find the last term in the expansion of $(1+x^2)^6$. (3)

2. (a) Let $A = \begin{bmatrix} 0 & 1 & 3 \\ 1 & 2 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 8 & 0 \end{bmatrix}$. Find $3A^2 - B + AB$. (6)

(b) Find the determinant of the matrix $A = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$ (4)

3. Solve the following system of equations using matrix inversion method (10)

$$x + 2y = 0; \quad y = 1; \quad x + y + 2z = 1$$

4. By looking into the Figure 1 below, answer the following questions.

(a) Is the function continuous at $x = 0$? (1)

(b) Is the function differentiable at $x = 0$? (1)

(c) In the above graphs, at how many points the derivative of the function is zero? (1)

(Don't consider the end points)

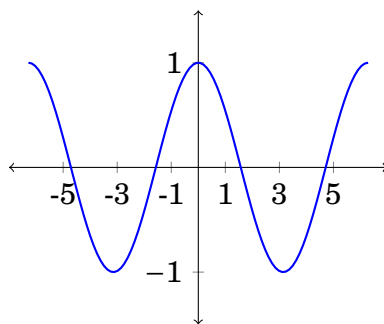


Figure 1: Graph of $\cos(x)$.

(d) State True/False: The function is discontinuous at $x = 3$. (1)

(e) State True/False: The function is not differentiable at $x = -3$. (1)

5. Check the continuity of the function (5)

$$f(x) = \begin{cases} 1 + x & \text{if } x \leq 0 \\ x^2 & \text{if } x > 0 \end{cases}$$

6. (a) Find the differentiation of the following function. (5)

$$f(x) = x \sin x + e^x \log x + 5x^5 - 1$$

(b) Find the differentiation of the following function. (5)

$$g(x) = 10 + \frac{x}{\sin x} + \frac{xe^x}{\cos x}$$

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Sessionals II Solutions

Note: Only the sketch of the solutions are given and the detailed solution is assumed to be done by students.

1.(a) $x^4 + 4x^3 + 6x^2 + 4x + 1$

1.(b) $\frac{4x^2}{a^2}$

1.(c) x^{12}

2.(a) $3A^2 - B + AB = \begin{pmatrix} 14 & 30 & 9 \\ 7 & 15 & 11 \\ 4 & 3 & 13 \end{pmatrix}$

2.(b) $\det(A) = -1$

3. The given system can be written as

$$\begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$
$$AX = b$$

Determinant of the matrix A is 2 and so inverse of the matrix exists.

The adjoint of the matrix is $\begin{pmatrix} 2 & -4 & 0 \\ 0 & 2 & 0 \\ -1 & 1 & 1 \end{pmatrix}$. And the inverse of the matrix is $\begin{pmatrix} 1 & -2 & 0 \\ 0 & 1 & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$.

And hence the solution is

$$\begin{aligned}
 X &= A^{-1}b \\
 &= \begin{pmatrix} 1 & -2 & 0 \\ 0 & 1 & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \\
 &= \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}
 \end{aligned}$$

4. (a) Yes

4. (b) Yes

4. (c) 3

4. (d) False

4. (e) False

$$6.(a) f'(x) = 25x^4 + x \cos(x) + e^x \log(x) + \frac{e^x}{x} + \sin(x)$$

$$6.(b) g'(x) = \frac{xe^x}{\cos(x)} + \frac{xe^x \sin(x)}{\cos(x)^2} + \frac{e^x}{\cos(x)} - \frac{x \cos(x)}{\sin(x)^2} + \frac{1}{\sin(x)}$$

or

$$g'(x) = \frac{\sin(x) - x \cos(x)}{\sin^2(x)} + \frac{\cos(x)xe^x + e^x \cos(x) + xe^x \sin(x)}{\cos^2(x)}$$

