SIKKIM MANIPAL UNIVERSITY
II Semester BCA Degree Examination
Mathematics II
MA 1204 / BCA 201 $[13-18]$ [Credit: 4]

Time: 3 hrs Max Marks: 100

Note: Answer any **FIVE** questions selecting at least TWO from each UNIT. Any missing or mis-printed data may be assumed suitably.

UNIT I

1. (a) If $y = e^{a\sin^{-1}x}$, prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+a^2)y_n = 0$ (7)

   (b) Find the equations of tangent and normal to the curve $y^2 = 4ax$ at $(a, -2a)$ (8)

   (c) Verify Rolles' theorem for $f(x) = x^2 - 2x - 8$ on $[-1, 3]$. (5)

2. (a) Find the radius of curvature of the Folium $x^2 + y^2 = 3axy$ at the point $\left(\frac{3a}{2}, \frac{3a}{2}\right)$. (10)

   (b) If $u = \log(x^2 + y^2 + z^2)$, prove that $x\frac{\partial^2 u}{\partial y \partial z} = y\frac{\partial^2 u}{\partial z \partial x} = z\frac{\partial^2 u}{\partial x \partial y}$. (10)

3. (a) Solve $\dfrac{dy}{dx} = \dfrac{y + \sqrt{x^2+y^2}}{x}$. (10)

   (b) Solve $\left(x\sqrt{1-x^2y^2} - y\right)dy + \left(x + y\sqrt{1-x^2y^2}\right)dx = 0$ (10)

4. (a) State Euler's theorem for function of two variables. (3)

   (b) Verify Euler's theorem for $u = x^2yz - 4y^2z^2 + 2xz^3$. (7)

   (c) Find the stationary points of $x^3 + y^3 - 3axy$, $a > 0$. (10)

UNIT II

5. (a) If $3A - B = \begin{pmatrix} 3 & 4 & 4 \\ 1 & 0 & 1 \\ 1 & -1 & 1 \end{pmatrix}$ and $A + B = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 3 \end{pmatrix}$, find A and B. (5)

   (b) Solve the following system using Gaussian elimination method. (8)

   $$x + 2y + z = 1; \quad x - y + z = 0; \quad -x + z = 2.$$

   (c) Find the rank of the matrix $\begin{pmatrix} 1 & 2 & 0 & 1 \\ -1 & 1 & 1 & 3 \\ 4 & 2 & 1 & 0 \end{pmatrix}$ using elementary row transformations. (7)

6. (a) Check whether, the following system of equations solvable or not. If yes, solve using Crammar's rule. (10)

   $$x + y + z = 0; \quad x - y - z = 1; \quad 2x + y + 2z = -1$$

(b) Find the inverse the matrix $\begin{pmatrix} 2 & 1 & 1 \\ 1 & -1 & -1 \\ 2 & 1 & 2 \end{pmatrix}$ using Gauss-Jordan method. (10)

7. (a) Describe comparison test for testing the convergence of series of real numbers with an example. (4)

   (b) Test the convergence of the series $\frac{2!}{3} + \frac{3!}{3^2} + \frac{4!}{3^3} + \ldots$ using an appropriate test. (8)

   (c) Check for what value of x the series $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots$ converges. (8)

8. (a) Find the value of $f(1.02)$ for $f(x) = \sqrt{x}$ using the Taylor Series expansion upto 4 decimal places correction. (10)

   (b) Test the following series for absolute or conditional convergence. (10)

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \ldots$$

Sikkim Manipal Institute of Technology
Department of Mathematics
BCA, Degree Examination
Mathematics II
Solutions

Note: Only the sketch of the solutions are given and the detailed solution is assumed to be done by students.

1(a)

$$y = e^{a\sin^{-1}x}$$

$$\Rightarrow y_1 = \frac{a}{\sqrt{1-x^2}} e^{a\sin^{-1}x}$$

$$= \frac{ay}{\sqrt{1-x^2}} \quad \text{— [1]}$$

$$\Rightarrow (1-x^2) y_1^2 = a^2 y^2$$

Diff. w.r.to x, we get,

$$\left.\begin{aligned} &-2x y_1^2 + (1-x^2)\cdot 2y_1 y_2 = a^2 2y\cdot y_1 \\ &\Rightarrow \boxed{(1-x^2)y_2 - x y_1 - a^2 y = 0} \end{aligned}\right\} \text{— [2]}$$

Now by Leibnitz's Rule, we get,

$$y_{n+2}(1-x^2) + {}^nC_1 y_{n+1}(-2x) + {}^nC_2 y_n(-2) - (y_{n+1}x + {}^nC_1 y_n) - a^2 y_n = 0 \quad \text{— [2]}$$

$$\Rightarrow y_{n+2}(1-x^2) - y_{n+1} 2nx - n(n-1) y_n - (y_{n+1}x + n y_n) - a^2 y_n = 0$$

$$\Rightarrow \boxed{(1-x^2)y_{n+2} - (2n+1)x y_{n+1} - (n^2+a^2)y_n = 0} \quad \text{— [2]}$$

1(b)

$y^2 = 4ax$

Diff. w.r.to x, we get

$2y \cdot \frac{dy}{dx} = 4a$

$\frac{dy}{dx} = \frac{\cancel{4}^{2}a}{\cancel{2}y}$

$\boxed{\frac{dy}{dx} = 2 \cdot a/y}$ — [1]

∴ Slope of the tangent is,

$\left.\frac{dy}{dx}\right|_{(a,-2a)} = \frac{2 \cdot a}{-2a} = -1$ — [1]

And the tangent passes through $(a,-2a)$

Therefore,

$y - y_1 = m(x - x_1)$ gives,

$y + 2a = (-1)(x - a)$

$y + 2a = -x + a$

$\Rightarrow \boxed{x + y + a = 0}$ → [3]

Slope of the normal is $\frac{-1}{\text{slope of tangent}} = 1$

$\Rightarrow y + 2a = 1(x - a)$

$\boxed{-x + y + 3a = 0}$ — [3]

is the eqn of normal.

1(c)

$$f(x) = x^2 - 2x - 8 \quad \text{on } [-1, 3]$$

Since $f(x)$ is a polynomial

(i) f is continuous on $[-1, 3]$

(ii) f is differentiable on $(-1, 3)$

(iii) $f(a) = f(-1) = (-1)^2 - 2(-1) - 8$
$= 1 + 2 - 8 = -5$

$f(b) = f(3) = 3^2 - 2(3) - 8$
$= 9 - 6 - 8$
$= -5$

– [3]

So, $f(a) = f(b)$.

Suppose Rolle's thm true then
$f'(c) = 0$ for some $c \in (-1, 3)$.

That is,
$2x - 2 = 0$
$\Rightarrow x = 1$ and $1 \in (-1, 3)$.

– [2]

Hence Rolle's thm is verified.

2(a)

$$\text{Radius of curvature} = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}$$

Given that $x^2+y^2 = 3axy$

Diff. w.r.to x,

$$2x + 2y\cdot\frac{dy}{dx} = 3a\left(x\frac{dy}{dx} + y\cdot 1\right)$$

$$2x + 2y\cdot\frac{dy}{dx} - 3ax\cdot\frac{dy}{dx} = 3ay \rightarrow (1)$$

$$\boxed{\frac{dy}{dx} = \frac{3ay-2x}{2y-3ax}}$$ — [3]

Diff (1) w.r.to x again,

$$2 + 2y\cdot\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^2 - 3ax\frac{d^2y}{dx^2} - 3a\frac{dy}{dx} = 3a\frac{dy}{dx}$$

$$\boxed{\frac{d^2y}{dx^2} = \frac{6a\,dy/dx - 2\left(\frac{dy}{dx}\right)^2 - 2}{2y-3ax}}$$ — [3]

Now, $$\left.\frac{dy}{dx}\right|_{\left(\frac{3a}{2},\frac{3a}{2}\right)} = \frac{3a\cdot\frac{3a}{2} - 2\cdot\frac{3a}{2}}{2\cdot\frac{3a}{2} - 3a\cdot\frac{3a}{2}}$$

$$= \frac{\cancel{3a/2}\,(3a-2)}{\cancel{3a/2}\,(2-3a)} = -1$$ — [1]

and $$\left.\frac{d^2y}{dx^2}\right|_{\left(3a/2,\,3a/2\right)} = \frac{6a(-1) - 2(-1)^2 - 2}{2\cdot\frac{3a}{2} - 3a\cdot\frac{3a}{2}}$$

$$= \frac{-6a-4}{\frac{3a}{2}(2-3a)}$$

$$= \frac{-2(3a+2)}{2-3a}\times\frac{2}{3a}$$ — [1]

$$= \frac{-4}{3a}\times\frac{2+3a}{2-3a}$$

$$\therefore \text{Radius of curvature} = \frac{\left[1+(-1)^2\right]^{3/2}}{\frac{-4}{3a}\times\left(\frac{2+3a}{2-3a}\right)}$$

$$= \frac{2^{3/2}\,3a(2-3a)}{-4(2+3a)}$$

$$= \frac{\cancel{2}\sqrt{2}\times 3a}{-\cancel{4}\,2}\;\frac{2-3a}{2+3a}\times\frac{2-3a}{2-3a}$$

$$\boxed{R = \frac{3a(2-3a)^2}{(4-9a^2)\sqrt{2}}}$$ — [2]

2(b)

$$u = \log(x^2+y^2+z^2)$$

Then $\frac{\partial u}{\partial x} = \frac{1}{x^2+y^2+z^2} \times 2x$ | $\frac{\partial^2 u}{\partial z \partial x} = \frac{-4xz}{(x^2+y^2+z^2)^2}$

[3] $\frac{\partial u}{\partial y} = \frac{2y}{x^2+y^2+z^2}$ | $\frac{\partial^2 u}{\partial x \partial y} = \frac{-4xy}{(x^2+y^2+z^2)^2}$ — [:

$\frac{\partial u}{\partial z} = \frac{2z}{x^2+y^2+z^2}$ | $\frac{\partial^2 u}{\partial y \partial z} = \frac{-4yz}{(x^2+y^2+z^2)^2}$

Therefore,

$$y\frac{\partial^2 u}{\partial z \partial x} = \frac{-4xyz}{x^2+y^2+z^2} \rightarrow (1)$$ — [1]

$$x\frac{\partial^2 u}{\partial y \partial z} = \frac{-4xyz}{x^2+y^2+z^2} \rightarrow (2)$$ — [1]

$$z\frac{\partial^2 u}{\partial x \partial y} = \frac{-4xyz}{x^2+y^2+z^2} \rightarrow (3)$$ — [1]

From (1), (2) and (3),

$$\boxed{x\frac{\partial^2 u}{\partial y \partial z} = y\frac{\partial^2 u}{\partial z \partial x} = \frac{z\,\partial^2 u}{\partial x \partial y}}.$$ — [1]

3(a)
$$\frac{dy}{dx} = \frac{y + \sqrt{x^2 + y^2}}{x}$$

Substitute $y = ux$.

Then $\dfrac{dy}{dx} = u + x\dfrac{du}{dx}$ gives [3]

$$x\frac{du}{dx} = \sqrt{1 + u^2}.$$ [4]

To solve this, we get

$$\frac{du}{\sqrt{1 + u^2}} = \frac{1}{x}dx$$

Now integrating on both sides, we get,

$$\sinh^{-1} u = c + \log x \implies u = \sinh(c + \log x)$$

[3]

Hence the solution is

$$y = x \sinh(c + \log x)$$

3(b)

3(b)

$$\left(x\sqrt{1-x^2y^2} - y\right)dy + \left(x + y\sqrt{1-x^2y^2}\right)dx = 0$$

$$P = x + y\sqrt{1-x^2y^2} \qquad \frac{\partial P}{\partial y} = \sqrt{1-x^2y^2} - \frac{x^2y^2}{\sqrt{1-x^2y^2}}$$

$$Q = x\sqrt{1-x^2y^2} - y \qquad \frac{\partial Q}{\partial x} = \sqrt{1-x^2y^2} - \frac{x^2y^2}{\sqrt{1-x^2y^2}}$$

$\boxed{\frac{\partial Q}{\partial x} = \frac{\partial P}{\partial y}}$ ∴ Given diff. eqn is exact. — [3]

$\frac{\partial u}{\partial x} = P$.

and $\frac{\partial v}{\partial y} = Q$

$\Rightarrow u = \int P\,dx$

$= \int \left(x + y\sqrt{1-x^2y^2}\right)\partial x$

$= \frac{x^2}{2} + y^2\int\sqrt{\left(\frac{1}{y}\right)^2 - x^2}\,dx$

$u = \frac{x^2}{2} + y^2\left\{\frac{x}{2}\sqrt{\frac{1}{y^2}-x^2} + \frac{1}{2y^2}\sin^{-1}(xy)\right\} + \phi(y)$

$$\boxed{u = \frac{x^2}{2} + \frac{xy^2}{2}\sqrt{1/y^2 - x^2} + \frac{1}{2}\sin^{-1}(xy) + \phi(y)}$$

→ [3]

Diff. w.r.to y partially,

$\frac{\partial u}{\partial y} = x + \frac{\partial}{\partial y}\left(\frac{xy^2}{2}\sqrt{1/y^2 - x^2} + \frac{1}{2}\sin^{-1}(xy)\right) + \phi'(y)$

Now Comparing with 'Q' we get,

$\boxed{\phi(y) = \sin^{-1}(xy)}$ → [2]

Hence, the soln is,

$$\boxed{x^2 - y^2 + xy\sqrt{1-x^2y^2} + \sin^{-1}(xy) = C}$$

→ [2]

4(a)

If u is a homogeneous function of two variable x and y ~~and then~~ of degree n, then

$$\boxed{x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nu.}$$ — [3]

4(b)

$$u = x^2yz - 4y^2z^2 + 2xz^3$$

Replacing x, y, z by xt, yt and zt respectively.

$$u = t^4(x^2yz - 4y^2z^2 + 2xz^3)$$ —[2]

Therefore u is homogeneous of degree 4.

Now, $\frac{\partial u}{\partial x} = 2xyz + 2z^3$

$\frac{\partial u}{\partial y} = -8yz^2 + x^2z$

$\frac{\partial u}{\partial z} = x^2y - 8y^2z + 6xz^2$

— [2]

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = x(2xyz + 2z^3) + y(-8yz^2 + x^2z) + z(x^2y - 8y^2z + 6xz^2)$$ —[1]

$$= 2x^2yz + 2xz^3 - 8y^2z^2 + x^2yz + x^2yz - 8y^2z^2 + 6xz^3$$

$$= 4x^2yz + 8xz^3 - 16y^2z^2$$

$$= 4(x^2yz + 2xz^3 - 4y^2z^2)$$ — [2]

$$= 4u.$$

Hence, Euler's theorem is verified.

4(c)

$$f(x,y) = x^3 + y^3 - 3axy$$

$$\left.\begin{aligned} \frac{\partial f}{\partial x} &= 3x^2 - 3ay \\ \frac{\partial f}{\partial y} &= 3y^2 - 3ax \end{aligned}\right\} \quad - [4]$$

$$\frac{\partial f}{\partial x} = 0 \Rightarrow 3x^2 - 3ay = 0$$

$$\Rightarrow x^2 = ay$$

$$\Rightarrow \boxed{y = x^2/a} \longrightarrow (1) \quad - [1]$$

$$- [1]$$

$$\frac{\partial f}{\partial y} = 0 \Rightarrow 3y^2 - 3ax = 0$$

$$\left.\begin{aligned} &3\left(\frac{x^2}{a}\right)^2 - 3ax = 0 \quad (\text{from } (1)) \\ &3x^4 - 3a^3x = 0 \\ &x(x^3 - a^3) = 0 \end{aligned}\right\} \quad - [2]$$

$$\Rightarrow x = 0 \text{ (or) } x = a.$$

Therefore the stationary points are,

$$\boxed{(0,0) \text{ and } (a,a) \quad \text{from } (1)} \quad - [2] \quad a^3 \neq a$$

5(a) $A = \begin{pmatrix} 1 & \frac{3}{4} & 1 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{4} & 1 \end{pmatrix}$ [3]

$B = \begin{pmatrix} 0 & -\frac{7}{4} & -1 \\ -\frac{1}{4} & \frac{3}{4} & \frac{1}{2} \\ \frac{1}{2} & \frac{7}{4} & 2 \end{pmatrix}$ [2]

5(b) The given system can be written as

[2]

$$\begin{bmatrix} 1 & 2 & 1 \\ 1 & -1 & 1 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$$
$$A \quad X = b$$

Therefore, $\begin{bmatrix} A & b \end{bmatrix} = \left[\begin{array}{rrr|r} 1 & 2 & 1 & 1 \\ 1 & -1 & 1 & 0 \\ -1 & 0 & 1 & 2 \end{array}\right]$

Row reduced echelon form of $\begin{bmatrix} A & b \end{bmatrix}$ is $\begin{bmatrix} 1 & 0 & 5 & 5 \\ 0 & 1 & 4 & 5 \\ 0 & 0 & 6 & 7 \end{bmatrix}$ [3]

So, the system can be re-written as [1]

$$x + 5z = 5; \quad y + 4z = 5; \quad 6z = 7.$$

Hence the solution is [2]

$$x = -\frac{5}{6}; \quad y = \frac{1}{3}; \quad z = \frac{7}{6}$$

5(c)

The echelon form of the given matrix is

$$\begin{pmatrix} 1 & 2 & 0 & 1 \\ 0 & 3 & 1 & 4 \\ 0 & 0 & 3 & 4 \end{pmatrix}$$ [5]

Hence the rank is 3 [2]

6(a) The given system can be written as

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 2 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} \qquad [1]$$
$$A \quad X \quad = \quad b$$

Now $\Delta = \det(A) = -2$. Hence the system is solvable. [2]

The solution by Crammer rule is

$$x = \frac{\Delta_1}{\Delta}; \quad x = \frac{\Delta_2}{\Delta}; \quad x = \frac{\Delta_3}{\Delta}$$

where Δ_i is the determinant of the matrix obtained by replacing the ith column of A by b.

$$\Delta_1 = \begin{vmatrix} 0 & 1 & 1 \\ 1 & -1 & -1 \\ -1 & 1 & 2 \end{vmatrix} = -1 \qquad [2]$$

$$\Delta_2 = \begin{vmatrix} 1 & 0 & 1 \\ 1 & 1 & -1 \\ 2 & -1 & 2 \end{vmatrix} = -2 \qquad [2]$$

$$\Delta_3 = \begin{vmatrix} 1 & 1 & 0 \\ 1 & -1 & 1 \\ 2 & 1 & -1 \end{vmatrix} = 3 \qquad [2]$$

Hence,

$$x = \frac{-1}{-2} = \frac{1}{2}; \quad x = \frac{-1}{-2} = 1; \quad x = \frac{-3}{2} \qquad [1]$$

6(b) $$\left[\begin{array}{c|c} A & I \end{array}\right] = \left[\begin{array}{ccc|ccc} 2 & 1 & 1 & 1 & 0 & 0 \\ 1 & -1 & -1 & 0 & 1 & 0 \\ 2 & 1 & 2 & 0 & 0 & 1 \end{array}\right]$$ [2]

Using row operations,

$$\left[\begin{array}{c|c} A & I \end{array}\right] \simeq \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & 1 & 0 & \frac{4}{3} & -\frac{2}{3} & -1 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right]$$ [7]

Hence, $A^{-1} = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & 0 \\ \frac{4}{3} & -\frac{2}{3} & -1 \\ -1 & 0 & 1 \end{bmatrix}$ [1]

If $\sum_{n=1}^{\infty} u_n$ and $\sum_{n=1}^{\infty} v_n$ be series of positive terms and

7(a) $$\lim_{n\to\infty} \frac{u_n}{v_n} = l, \text{ a finite}$$ [4]

then both the series converge or diverge together.

7(b) $$u_n = \frac{(n+1)!}{3^n}$$ [3]

$$\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = \lim_{n\to\infty} \frac{(n+2)!}{3^{n+1}} \times \frac{3^n}{(n+1)!} = \lim_{n\to\infty} \frac{n+2}{3} = \infty > 1$$ [3]

Therefore, by Ratio test, the given series diverges [2]

7(c) The given series can be written as

$$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{x^n}{n}$$

and [2]

$$u_n = \frac{x^n}{n}$$

(i) The given series is an alternating series [1]

(ii) $u_n - n_{n+1} = \dfrac{x^n}{n} - \dfrac{x^{n+1}}{n+1} = \dfrac{x^n[(n+1)-nx]}{n(n+1)} > 0$ if $0 < x < 1$. [2]

(iii) $\lim\limits_{n\to\infty} u_n = \lim\limits_{n\to\infty}\dfrac{x^n}{n} = 0$ if $-1 < x < 1$. [2]

Hence by Liebnitz's test the given series converges if $0 < x < 1$. [1]

8(a) Taylor series expansion of $f(x)$ around $x = a$ is given by

$$f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots \qquad [2]$$

Expanding $f(x)$ using taylor series around $x = 1$:

Note that $(1.2 - 1) = 0.2$ and $0.2^6 \simeq 0$ (correct to 4 decimal places). So, we find until 5th degree.

$$\begin{aligned}
f(x) &= \sqrt{x} & f(1) &= 1 \\
f'(x) &= \tfrac{1}{2\sqrt{x}} & f'(1) &= \tfrac{1}{2} \\
f''(x) &= -\tfrac{1}{4x^{\frac{3}{2}}} & f''(1) &= -\tfrac{1}{4} \\
f'''(x) &= \tfrac{3}{8x^{\frac{5}{2}}} & f'''(1) &= \tfrac{3}{8} \\
f^{iv}(x) &= -\tfrac{15}{16x^{\frac{7}{2}}} & f^{iv}(1) &= -\tfrac{15}{16} \\
f^{v}(x) &= \tfrac{105}{32x^{\frac{9}{2}}} & f^{v}(1) &= \tfrac{105}{32}
\end{aligned}$$

[4]

Therefore, we get,

$$f(x) = \frac{7}{256}(x-1)^5 - \frac{5}{128}(x-1)^4 + \frac{1}{16}(x-1)^3 - \frac{1}{8}(x-1)^2 + \frac{1}{2}x + \frac{1}{2} \qquad [2]$$

Hence, $f(1.2) = 1.0954$ [2]

8(b) The given series can be written as

$$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n}$$

and [2]

$$u_n = \frac{1}{n}$$

(i) The given series is an alternating series [1]

(ii) $u_n - n_{n+1} = \dfrac{1}{n} - \dfrac{1}{n+1} = \dfrac{1}{n(n+1)} > 0$ [2]

(iii) $\lim\limits_{n\to\infty} u_n = \lim\limits_{n\to\infty} \dfrac{1}{n} = 0$ [1]

Hence by Liebnitz's test the given series converges. [1]

Also, it is clear that the absolute series $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges. [2]

Hence the given series conditionally converges. [1]