Sikkim Manipal Institute of Technology
Department of Mathematics
B.Tech Mechanical Engineering (IV Sem)
Subject: Numerical Methods (MA 1405)
Second Sessional Examination

Dur: 1 hr 30 mins | 03.04.2019 | Max: 50 marks

Instructions

(i) Answer all the questions.

(ii) Any missing or misprinted data may be assumed suitably.

1. Using Picard's Method find the solution up to third approximation for $\frac{dy}{dx} = y+x$, $y(0) = 1$. Hence find $y(0.1)$ correct to 3 decimal places. (10)

2. Using Taylor's series method find $y(0.1)$ correct to 3 decimal places for $\dfrac{dy}{dx} = 1+3xy$, $y(0) = 1$ (10)

3. Using LU decompostion method solve the following system of linear equations:

$$\begin{aligned} 2x + y + z &= 3 \\ x + 3y + z &= -2 \\ x + y + 4z &= -6 \end{aligned}$$

4. Using Gauss-seidal method find the solution correct to 3 decimal places of the following system of equations:

$$\begin{aligned} 8x + 2y - 2z &= 8 \\ x - 8y + 3z &= -4 \\ 2x + y + 9z &= 12 \end{aligned}$$

5. Using Power method with at least six iterations, find numerically largest eigenvalue and the corresponding eigenvector of the following matrix:

$$\begin{pmatrix} 10 & -2 & 1 \\ -2 & 10 & -2 \\ 1 & -2 & 10 \end{pmatrix}$$

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1. The approximations are,
$y^{(1)}(x) = \frac{1}{2}x^2 + x + 1$
$y^{(2)}(x) = \frac{1}{6}x^3 + x^2 + x + 1$
$y^{(3)}(x) = \frac{1}{24}x^4 + \frac{1}{3}x^3 + x^2 + x + 1$
Therefore, $y(0.1) = 1.110$

2. Taylor series expansion of $y(x)$ around $x = 0$ is

$$y(x) = y(0) + \frac{x}{1!}y'(0) + \frac{x^2}{2!}y''(0) + \frac{x^3}{3!}y'''(0) + \frac{x^4}{4!}y^{iv}(0) + \dots$$

and

$y'(x) = 1 + 3xy \implies y'(0) = 1$

$y''(x) = 3xy\prime + 3y \implies y''(0) = 3$

$y'''(x) = 3xy' + 3y \implies y'''(0) = 6$

Therefore,

$$\begin{aligned} y(x) &= 1 + x + 3\frac{x^2}{2} + 6\frac{x^3}{3!} + \dots \\ &= 1 + x + \frac{3x^2}{2} + x^3 + \dots \end{aligned}$$

Note that, $0.1^4 = 0.0001 \simeq 0$ (correct to 3 decimal places) implies that it is enough to consider until 3rd order term to compute $y(0.1)$, correct to three decimal places. Hence,

$$y(0.1) = 1 + (0.1) + \frac{3(0.1)^2}{2} + (0.1)^3 = 1.116$$

3. The given system is

$$\begin{pmatrix} 2 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \\ -6 \end{pmatrix}$$
$$AX = b$$

Note that, A is diagonally dominant and hence

$$A = LU$$

where $L = \begin{pmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ \frac{1}{2} & \frac{1}{5} & 1 \end{pmatrix}$ and $U = \begin{pmatrix} 2 & 1 & 1 \\ 0 & \frac{5}{2} & \frac{1}{2} \\ 0 & 0 & \frac{17}{5} \end{pmatrix}$. Therefore, $AX = b$ can be written as $LUX = b$ and we solve

$$UX = q, \quad Lq = b.$$

Solving $Lq = b$:

$$\begin{pmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ \frac{1}{2} & \frac{1}{5} & 1 \end{pmatrix} \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \\ -6 \end{pmatrix} \Longrightarrow q = \begin{pmatrix} 3 \\ -\frac{7}{2} \\ -\frac{34}{5} \end{pmatrix} = \begin{pmatrix} 3 \\ -3.5 \\ -6.8 \end{pmatrix}$$

Solving $UX = q$:

$$\begin{pmatrix} 2 & 1 & 1 \\ 0 & \frac{5}{2} & \frac{1}{2} \\ 0 & 0 & \frac{17}{5} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ -3.5 \\ -6.8 \end{pmatrix} \Longrightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \\ -2 \end{pmatrix}$$

Hence, $x = 3$, $y = -1$ and $z = -2$.

4. The iteration table is

Iter	$x^{(n)}$	$y^{(n)}$	$z^{(n)}$
0	0	0	0
1	1.00	0.625	1.042
2	1.104	1.029	0.974
3	0.986	0.988	1.004
4	1.004	1.002	0.999
5	0.999	1.00	1.00
6	1.00	1.00	1.00
7	1.00	1.00	1.00

(OR)

Iter	$x^{(n)}$	$y^{(n)}$	$z^{(n)}$
0	1	0.5	1.333
1	1.208	1.151	0.9370
2	0.947	0.970	1.015
3	1.011	1.007	0.997
4	0.998	0.999	1.001
5	1	1	1
6	1	1	1

5. The table gives the approximate eigenvalue at each iteration and the corresponding eigenvector.

Iter	EigenValue	(x,y,z)
0	1	(1,1,1)
1	9.666	$(1,0.667,1)$
2	10.448	$(1,0.276,1)$
3	11.238	$(1,-0.119,1)$
4	11.924	$(1,-0.462,1)$
5	12.446	$(1,-0.723,1)$
6	12.804	$(1,-0.902,1)$
7	13.033	$(1,-1.017,1)$
8	13.174	$(1,-1.087,1)$
9	13.258	$(1,-1.129,1)$
10	13.306	$(1,-1.153,1)$
11	13.334	$(1,-1.167,1)$
12	13.35	$(1,-1.175,1)$
13	13.36	$(1,-1.18,1)$
14	13.366	$(1,-1.183,1)$
15	13.368	$(1,-1.184,1)$
16	13.370	$(1,-1.185,1)$
17	13.370	$(1,-1.185,1)$