SIKKIM MANIPAL UNIVERSITY
II Semester BBA Degree Examination
Business Mathematics
MA 1205 / BBA 201 $[13-18]$ [Credit: 4]

Time: 3 hrs | Max Marks: 100

Note: Answer any **FIVE** questions selecting at least TWO from each UNIT. Any missing or misprinted data may be assumed suitably.

UNIT I

1. (a) The sum of the 4th and 8th terms of an A.P. is 24 and the sum of the 6th and 10th terms is 34. Find the 1st term and the common difference of the A.P. (8)

   (b) Write down the 8th term in the Geometric Progression 1, 3, 9, ... (4)

   (c) If the 5th term and 12th term of an Arithmetic Progression are 30 and 65 respectively, find the sum of its 26 terms. (8)

2. (a) Solve the quadratic equation $5x^2 + 6x + 1 = 0$. (7)

   (b) Find the value of c, so that the given quadratic equation $x^2 - 2x - c = 0$ has real and equal roots. (7)

   (c) Find the number of permutations of the letters of the word 'SIKKIM'. (6)

3. (a) Expand: $(1-x)^5 + (1+x)^5$. (8)

   (b) Find the middle term in the expansion of $\left(3x - \frac{2}{3}\right)^6$. (7)

   (c) Solve the equation $\log_2(2x^2) - \log_2(5x-6) = 1$ for x. (5)

4. (a) Solve the following LPP using graphical method. (10)

$$\text{Maximize } Z = x + 3y$$

   Subject to

$$x + y \leq 50; \quad 2x + y \geq 60; \quad x, y \geq 0.$$

(b) Find the basic feasible solution by North-West corner Method. (10)

Sources	Destination D_1	D_2	D_3	Supply
S1	5	8	4	50
S2	6	6	3	40
S3	3	9	6	60
Demand	20	95	35	150

UNIT II

5. (a) For $A = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 0 & 1 \\ 4 & 2 & 1 \end{pmatrix}$, can the multiplication $B^T BA$ possible? If yes, find $B^T BA$. (4)

(b) Consider $A = \begin{pmatrix} 2 & 2 & 3 \\ 1 & 1 & 1 \\ 3 & 2 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 2 & 1 \\ 1 & -1 & 0 \end{pmatrix}$. Find the determinant of AB and $A + B$. (8)

(c) Show that $A^3 - 2A^2 - 2A + 3I_3 = 0$, for the matrix $A = \begin{pmatrix} 2 & 1 & 2 \\ 1 & -1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$. (8)

6. (a) Find the inverse of the following matrix $A = \begin{pmatrix} 1 & -1 & 1 \\ 1 & -1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$. (8)

(b) Solve the following system of equations using matrix inversion method (12)

$$x - y = 0; \quad y = 1; \quad x - y - z = 1$$

7. (a) At how many points the following curve (See Figure 1) has derivative zero? (2)

(b) Prove that the following function is discontinuous at $x = 1$. (8)

$$f(x) = \begin{cases} 2 + x^3 & \text{if } x > 1 \\ 2x - 1 & \text{if } x \leq 1 \end{cases}$$

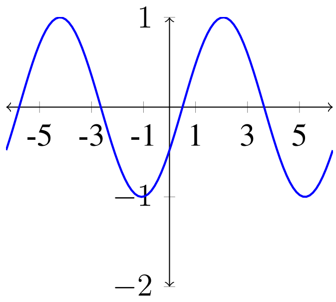


Figure 1: Graph for Question 7(a).

(c) Differentiate the following functions with respect to x. (10)

(i) $f(x) = 2 + 2x^2 + x \sin x$ (ii) $g(x) = 10 - \frac{x^3}{e^x} + x^2 e^x$

8. (a) Evaluate the following. (10)

(i) $\displaystyle\int (x + xe^x) dx$ (ii) $\displaystyle\int 2x \cos(x^2 + 1) dx$

(b) Find the area of circle $x^2 + y^2 = 1$ using integration. (10)

Sikkim Manipal Institute of Technology
Department of Mathematics
BBA (II Sem)
Subject: Business Mathematics (MA 1205)
Degree Exam Solutions

Note: Only the sketch of the solutions are given and the detailed solution is assumed to be done by students.

1.(a)

Let $a, a+d, a+2d, \ldots$ be the A.P.

Given that

$$a+3d+a+7d=24$$

$$2a+10d=24$$

$$\boxed{a+5d=12} \rightarrow (1) \qquad — [2]$$

and $a+5d+a+9d=34$

$$\Rightarrow \quad 2a+14d=34$$

$$\boxed{a+7d=17} \rightarrow (2) \qquad — [2]$$

From (1) & (2),

$$\begin{array}{r} \cancel{a}+5d=12 \\ -\cancel{a}-7d=-17 \\ \hline -2d=-5 \end{array}$$

$$\boxed{d=5/2} \qquad — [2]$$

and (1) $\Rightarrow$, $a+5(5/2)=12$

$$a=12-\frac{25}{2}$$

$$\boxed{a=-1/2} \qquad — [2]$$

1.(b) The general term is $T_n = ar^{n-1}$ [1]

and from the given G.P, $a = 1,, r = 3$ [1]

Hence,

$$T_8 = ar^{8-1}$$
$$= 1 \times 3^7$$
$$= 2187$$

Therefore $T_8 = 2187$ [2]

1.(c) Given that $a + 4d = 30$ and $a + 11d = 65$ — [1]

Therefore,
$$\begin{array}{r} a + 4d = 30 \\ -a - 11d = -65 \\ \hline -7d = -35 \end{array}$$ — [2]

$$\boxed{d = 5}$$

$$a + 4(5) = 30 \Rightarrow \boxed{a = 10}$$ — [1]

— [2]

Now,
$$S_n = \frac{n}{2}(2a + (n-1)d)$$
$$S_{26} = \frac{26}{2}(2(10) + (26-1)5)$$
$$= 13(20 + 25 \times 5)$$
$$= 13(145) = 1885$$ — [2]

2.(a)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ — [2]

Here $a=5 \quad b=6 \quad c=1$

$$\therefore \quad x = \frac{-6 \pm \sqrt{6^2 - 4 \times 5 \times 1}}{2(5)}$$ — [2]

$$= \frac{-6 \pm \sqrt{16}}{10}$$

$$= \frac{-6 \pm 4}{10}$$

$$= \frac{-6+4}{10} \text{ (or) } \frac{-6-4}{10}$$

$$= \frac{-2}{10} \text{ (or) } \frac{-10}{10}$$ — [3]

$$= -1/5 \text{ (or) } -1.$$

2.(b) The given quadratic equation has real and equal roots if $b^2 - 4ac = 0$. [3]
That is,

$$\begin{aligned} b^2 - 4ac &= 4 - 4 \times 1 \times (-c) = 0 \\ \Longrightarrow \quad & 4(1+c) = 0. \end{aligned}$$ [3]

Therefore, $c = -1$. [1]

2.(c) There are 6 letters in the word 'SIKKIM' of which 'K' and 'I' are repeated two times each. [3]
Therefore, the number of permutations is $\frac{6!}{2!2!} = 180$ [3]

3.(a) Note that $(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \cdots + b^n$ [2]

and so,

$(1-x)^5 = -x^5 + 5\,x^4 - 10\,x^3 + 10\,x^2 - 5\,x + 1$ [2]

$(1+x)^5 = x^5 + 5\,x^4 + 10\,x^3 + 10\,x^2 + 5\,x + 1$ [2]

Therefore,

$(1-x)^5 + (1+x)^5 = 10\,x^4 + 20\,x^2 + 2$ [2]

3.(b)

The middle term is the 4th term and we know that

$$T_{r+1} = \binom{n}{r} a^{n-r} b^r. \quad [2]$$

Substituting $a = 3x$, $b = -\frac{2}{3}$, $n = 6$ and $r = 4$, we get,

$$\begin{aligned} T_4 &= \tbinom{6}{4} (3x)^{6-4} \left(-\tfrac{2}{3}\right)^4 \\ &= \tbinom{6}{2} \times 9x^2 \times \tfrac{2^4}{3^4} \\ &= \tfrac{6\times5}{1\times2} \times 9x^2 \times \tfrac{2^4}{3^4} \\ &= \tfrac{80}{3} x^2 \text{ (or) } 26.67x^2 \end{aligned} \quad [5]$$

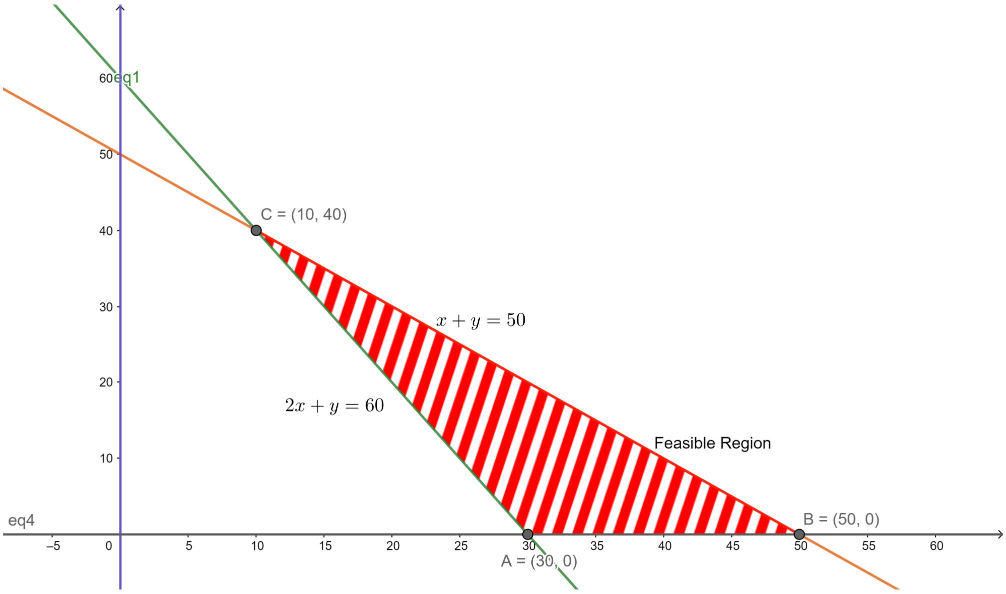
3.(c) $\log_2 \left(\dfrac{2x^2}{5x-6} \right) = 1$ [2]

Therefore, $\dfrac{2x^2}{5x-6} = 2^1$ [1]

That is, $x^2 - 5x + 6 = 0$. [2]

Solving this, we get $x = 2, 3$ are the solutions.

4.(a)



[5]

Finding intersection Point:

Solving $x + y = 50$ and $2x + y = 60$ we get the intersection point is $(10, 40)$. [2]
And, $Z = x + 3y$ gives

$$\begin{aligned} Z(30, 0) &= 30 \\ Z(10, 40) &= 130 \\ Z(50, 0) &= 50 \end{aligned}$$ [2]

Since the maximum among the above is 130, the maximized value is 130 and it is attained at $(10, 40)$. [1]

4.(b)

Plant	Distribution Centres			
	D_1	D_2	D_3	Supply
S_1	5 20	8 30	4	(50)
S_2	6	6 40	3	(60)
S_3	3	9 25	6 35	(60)
Demand	(20)	(95)	(35)	150

[7]

Therefore,

$$\text{the transportion cost} = 20 \times 5 + 30 \times 8 + 40 \times 6 + 25 \times 9 + 35 \times 6 = 1015$$ [3]

5.(a) Yes. $B^T BA$ can be multiplied [1]

$$B^T B = \begin{pmatrix} 17 & 8 & 5 \\ 8 & 4 & 2 \\ 5 & 2 & 2 \end{pmatrix}$$ [2]

$$B^T BA = \begin{pmatrix} 61 & 8 & 5 \\ 28 & 4 & 2 \\ 19 & 2 & 2 \end{pmatrix}$$ [1]

5.(b)

$$AB = \begin{pmatrix} 5 & 1 & 4 \\ 2 & 1 & 2 \\ 3 & 3 & 5 \end{pmatrix}$$ [2]

$\det(AB) = 3$ [2]

$$A + B = \begin{pmatrix} 2 & 2 & 4 \\ 2 & 3 & 2 \\ 4 & 1 & 1 \end{pmatrix}$$ [2]

$\det(A + B) = -26$ [2]

5.(c)

$$A^2 = \begin{pmatrix} 5 & 1 & 7 \\ 1 & 2 & 2 \\ 0 & 0 & 1 \end{pmatrix}$$ [2]

$$A^3 = A^2A = \begin{pmatrix} 11 & 4 & 18 \\ 4 & -1 & 6 \\ 0 & 0 & 1 \end{pmatrix}$$ [2]

$$A^3 - 2A^2 - 2A + 3I_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$ [4]

Hence, verified.

6.(a) $\det(A) = 1$ [2]

and hence the inverse of A exists.

$$\text{Adjoint of } A = \begin{pmatrix} -1 & 2 & 1 \\ -1 & 1 & 1 \\ 1 & -1 & 0 \end{pmatrix}$$ [4]

Inverse of $A = \dfrac{1}{\det(A)}\,\mathrm{adj}(A)$ [1]

Therefore,

$$A^{-1} = \begin{pmatrix} -1 & 2 & 1 \\ -1 & 1 & 1 \\ 1 & -1 & 0 \end{pmatrix}$$ [1]

6.(b) The given system can be written as

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \qquad [2]$$

$$A \qquad X \quad = \quad b$$

Therefore, if A^{-1} exists,

$$X = A^{-1}b \qquad [1]$$

<u>**Finding A^{-1}:**</u>

$\det(A) = -1$ [2]

$\operatorname{adj}(A) = \begin{pmatrix} -1 & -1 & 0 \\ 0 & -1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$ [3]

Therefore, $A^{-1} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & -1 \end{pmatrix}$ [1]

And so,

$$X = A^{-1}b = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 1 & -1 & -1 \end{pmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} \qquad [2]$$

Hence,

$$x = 1, \quad y = 1, \quad z = -1 \qquad [1]$$

7.(a) 4 Points [2]

7.(b)

Note that,

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} 2x - 1 = 1$$ [3]

and

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} 2 + x^3 = 3$$ [3]

and hence

$$\lim_{x\to 1^-} f(x) \neq \lim_{x\to 1^+} f(x)$$ [1]

Therefore, f is not continuous at $x = 1$. [1]

7.(c) (i)

$$f'(x) = 4x + \frac{d}{dx}\left(x \sin x\right)$$ [1]

$$\frac{d}{dx}\left(x \sin x\right) = x\cos x + \sin x$$ [3]

Thefore,

$$f'(x) = 4x + x\cos x + \sin x$$ [1]

7.(c) (ii)

$$g'(x) = -\frac{d}{dx}\left(\frac{x^3}{e^x}\right) + \frac{d}{dx}\left(x^2e^x\right)$$ [1/2]

$$\begin{aligned}\frac{d}{dx}\left(\frac{x^3}{e^x}\right) &= \frac{e^x 3x^2 - x^3e^x}{(e^x)^2} \\ &= \frac{3x^2 - x^3}{e^x} \text{ (or)} \\ &= \frac{x^2(3-x)}{e^x}\end{aligned}$$ [2]

Similarly,

$$\frac{d}{dx}\left(x^2e^x\right) = x^2e^x + 2xe^x$$ [2]

Therefore,

$$g'(x) = -\frac{3x^2 - x^3}{e^x} + x^2e^x + 2xe^x$$

or [1/2]

$$g'(x) = -\frac{x^2(3-x)}{e^x} + x^2e^x + 2xe^x$$

8.(a) (i)

$$\int (x + xe^x)dx = \frac{x^2}{2} + \int xe^x dx$$ [2]

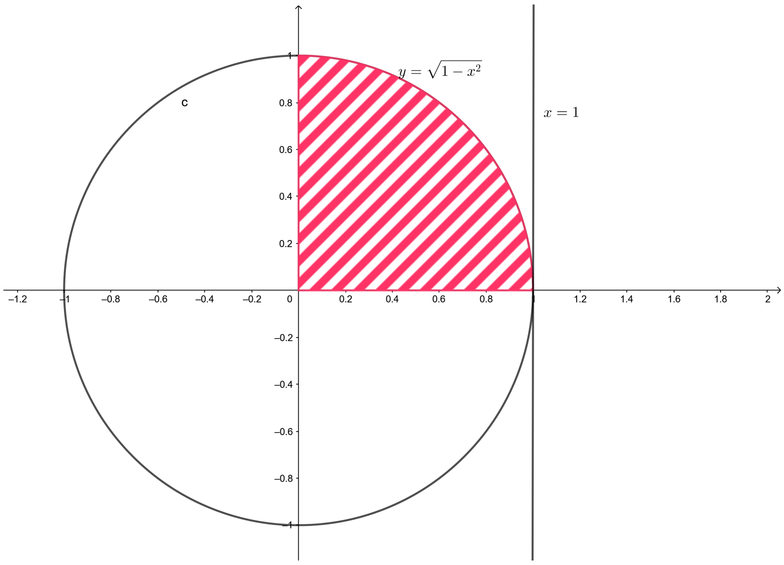
taking $u = x$ and $dv = e^x$ we get $du = dx$ and $v = e^x$ [1]

$$\int (x + xe^x)dx = \frac{x^2}{2} + xe^x - e^x + C$$ [2]

8.(a) (ii)

$$\text{Let } u = x^2 + 1$$
$$du = 2xdx \qquad [2]$$
$$\text{Therefore, } \int 2x\cos(x^2+1)dx = \int \cos(u)du \qquad [1]$$
$$= \sin(u) + C \qquad [1]$$
$$= \sin(x^2+1) + C \qquad [1]$$

8.(b)



[5]

Required Area $= 4\int_0^1 y\,dx$ — [2]

$= 4\int_0^1 \sqrt{1-x^2}\,dx$

$= 4 \left. \frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}(x) \right|_0^1$ — [2]

$= 4 \left[\frac{1}{2}\sin^{-1}(1) \right]$

$= \cancel{4} \times \frac{1}{\cancel{2}} \frac{\pi}{\cancel{2}}$

$= \pi$ sq. units. — [1]