Sikkim Manipal Institute of Technology
Department of Mathematics
BCA (II Sem)
Subject: Mathematics II (MA 1204)
Second Sessional Examination

Dur: 1 hr 30 mins | 03.04.2019 | Max: 50 marks

Instructions

(i) Answer all the questions.

(ii) Any missing or misprinted data may be assumed suitably.

1. Solve the following system of equations by Cramer's rule (10)

$$2x-3y+4z=-4$$
$$x+z=0$$
$$-y+4z=2$$

2. Find the inverse of the matrix using elementary row operations $\begin{pmatrix} 3 & 2 & 1 \\ 1 & 1 & 1 \\ 5 & 1 & -1 \end{pmatrix}$. (10)

3. Solve the following system using Guass-Jordan method (10)

$$x+y+z=4$$
$$2x-y+3z=1$$
$$3x+2y-z=1$$

4. (a) Test the convergence of the series $\sum_{n=1}^{\infty}\left(\frac{n}{2n+1}\right)^n$ using an appropriate test of convergence. (5)

   (b) Use Ratio test and test the convergence of the following series for different values of x. (5)

$$\sum_{n=1}^{\infty}\frac{x^{n-1}}{n3^n}$$

5. (a) Find the value of $f(0.1)$ using the Taylor series expansion if $f(x)=e^x$ (Use upto 4th order term and use $e=2.718$) (7)

   (b) State Liebnitz's Test of convergence for an infinite series. (3)

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1. Suppose Δ_x is the determinant of the matrix A after replacing the 1st column as b, then by Crammer rule,

$$x = \frac{\Delta_x}{\Delta},$$

where Δ is the determinant of the matrix A. Therefore,

$$x = \frac{\Delta_x}{\Delta} = \frac{\begin{vmatrix} -4 & -3 & 4 \\ 0 & 0 & 1 \\ 2 & -1 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & -3 & 4 \\ 1 & 0 & 1 \\ 0 & -1 & 4 \end{vmatrix}} = \frac{-10}{10} = -1.$$

Similarly,

$$y = \frac{\Delta_x}{\Delta} = \frac{\begin{vmatrix} 2 & -4 & 4 \\ 1 & 0 & 1 \\ 0 & 2 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & -3 & 4 \\ 1 & 0 & 1 \\ 0 & -1 & 4 \end{vmatrix}} = \frac{20}{10} = 2$$

and

$$z = \frac{\Delta_z}{\Delta} = \frac{\begin{vmatrix} 2 & -3 & -4 \\ 1 & 0 & 0 \\ 0 & -1 & 2 \end{vmatrix}}{\begin{vmatrix} 2 & -3 & 4 \\ 1 & 0 & 1 \\ 0 & -1 & 4 \end{vmatrix}} = \frac{10}{10} = 1 \tag{1}$$

2. The augemented matrix is

$$[A|I] = \left(\begin{array}{ccc|ccc} 3 & 2 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 5 & 1 & -1 & 0 & 0 & 1 \end{array}\right).$$

and the reduced row echelon form to find the inverse is,

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & \frac{3}{2} & \frac{1}{2} \\ 0 & 1 & 0 & 3 & -4 & -1 \\ 0 & 0 & 1 & -2 & \frac{7}{2} & \frac{1}{2} \end{array}\right).$$

Therefore, the inverse is,

$$A^{-1} = \begin{pmatrix} -1 & \frac{3}{2} & \frac{1}{2} \\ 3 & -4 & -1 \\ -2 & \frac{7}{2} & \frac{1}{2} \end{pmatrix}.$$

3. The given system can be written as

$$\underbrace{\begin{pmatrix} 1 & 1 & 1 \\ 2 & -1 & 3 \\ 3 & 2 & -1 \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} x \\ y \\ z \end{pmatrix}}_{X} \; \underbrace{=}_{=} \; \underbrace{\begin{pmatrix} 4 \\ 1 \\ 1 \end{pmatrix}}_{b}$$

Therefore, the augmented matrix is

$$\left[\begin{array}{c|c} A & b \end{array}\right] = \left(\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 2 & -1 & 3 & 1 \\ 3 & 2 & -1 & 1 \end{array}\right).$$

And the reduced row echelon form to solve the system is,

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 0 & 1 & -\frac{1}{3} & \frac{7}{3} \\ 0 & 0 & 1 & 2 \end{array}\right). \tag{2}$$

Therefore, from Equation (2),

$$z = 2; \quad y - \frac{1}{3}z = \frac{7}{3}; \quad x + y + z = 4.$$

Further solving the above system of equations we get,

$$x = -1, \quad y = 3, \quad \text{and} \quad z = 2$$

4. (a) The given series is of positive terms and $u_n = \left(\frac{n}{2n+1}\right)^n$. Now,

$$\begin{aligned} \lim_{n\to\infty} (u_n)^{\frac{1}{n}} &= \lim_{n\to\infty} \left[\left(\frac{n}{2n+1}\right)^n\right]^{\frac{1}{n}} \\ &= \lim_{n\to\infty} \frac{n}{2n+1} \\ &= \lim_{n\to\infty} \frac{1}{2+\frac{1}{n}} \\ &= \frac{1}{2} < 1. \end{aligned}$$

Therefore, by Root test, the given series converges.

4. (b) Let $u_n = \frac{x^{n-1}}{n3^n}$ and so,

$$
\begin{aligned}
\frac{u_{n+1}}{u_n} &= \frac{\frac{x^{n+1-1}}{(n+1)3^{n+1}}}{\frac{x^{n-1}}{n3^n}} \\
&= \frac{x^n}{(n+1)3^{n+1}} \times \frac{n3^n}{x^{n-1}} \\
&= \frac{x^{n-1} \times x}{n(1+\frac{1}{n})3^n \times 3} \times \frac{n3^n}{x^{n-1}} \\
&= \frac{x}{(1+\frac{1}{n}) \times 3}
\end{aligned}
$$

and so,

$$\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = \frac{x}{3}.$$

Hence, by D'Alembert's Ratio test, The given series converges if $\frac{x}{3} < 1$. That is if, $x < 3$. And the given series diverges if $\frac{x}{3} > 1$. That is, if $x > 3$.

5. (a) Since 0.1 is closed to 0, to find the value of $f(0.1)$, we write the Taylor series expansion of the function f, around $x = 0$. Therefore,

$$f(x) = f(0) + \frac{x}{1!}f'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0) + \frac{x^4}{4!}f^{iv}(0) + \dots$$

Now,

$$
\begin{aligned}
f(x) = e^x &\implies f(0) = e^0 = 1 \\
f'(x) = e^x &\implies f'(0) = e^0 = 1 \\
f''(x) = e^x &\implies f''(0) = e^0 = 1 \\
f'''(x) = e^x &\implies f'''(0) = e^0 = 1 \\
f^{iv}(x) = e^x &\implies f^{iv}(0) = e^0 = 1
\end{aligned}
$$

So, considering $f(x)$ upto 4th order, we get,

$$\begin{aligned}
f(x) &= f(0) + \frac{x}{1!}f'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0) + \frac{x^4}{4!}f^{iv}(0) \\
&= 1 + \frac{x}{1!}1 + \frac{x^2}{2!}1 + \frac{x^3}{3!}1 + \frac{x^4}{4!}1
\end{aligned}$$

And hence,

$$f(0.1) = 1 + \frac{(0.1)}{1!}1 + \frac{(0.1)^2}{2!}1 + \frac{(0.1)^3}{3!}1 + \frac{(0.1)^4}{4!}1 = 1.1052.$$

5.(b) **Liebnitz's Test:** Let $\sum_{n=1}^{\infty}(-1)^{n-1}u_n$ is an alternating series of real numbers. Then, if

(i) $\lim_{n\to\infty} u_n = 0$, and

(ii) $u_n - u_{n+1} > 0$,

the given series converges. Otherwise, does not converges.

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