$$G(x) = \frac{5}{(1-x)^2} + \frac{3}{1+2x}$$

n - rational

$$\boxed{(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots}$$

$$G(x) = 5(1-x)^{-2} + 3(1+2x)^{-1}$$

$$= 5\left[1 + (-2)(-x) + \frac{(-2)(-2-1)}{2!}(-x)^2 + \frac{(-2)(-2-1)(-2-2)}{3!}(-x)^3 + \frac{(-2)(-2-1)(-2-2)(-2-3)}{4!}(-x)^4 + \cdots\right]$$

$$+ 3\left[1 + (-1)(2x) + \frac{(-1)(-1-1)}{2!}(2x)^2 + \frac{(-1)(-1-1)(-1-2)}{3!}(2x)^3\right.$$

$$+\frac{(-1)(-1-1)(-1-2)(-1-3)}{4!}(2x)^4+\cdots\Bigg]$$

$$= 5\left[1+2x+3x^2+4x^3+5x^4+\cdots\right] + 3\left[1-2x+4x^2-8x^3+16x^4+\cdots\right]$$

$$= 8+4x+27x^2-4x^3+73x^4+\cdots$$

Therefore the corresponding sequence is,

$$8,\ 4,\ 27,\ -4,\ 73,\ \ldots$$

$\cancel{1, 1, 1, 1, \ldots}$

$$\boxed{a_n = 2a_{n-1} - 1}, \quad n \geq 1, \quad a_0 = 1$$

multiply x^n on both sides,

$$a_n x^n = 2a_{n-1} x^n - x^n, \quad n \geq 1,$$

Sum all terms,

$$\sum_{n=1}^{\infty} a_n x^n = 2 \sum_{n=1}^{\infty} a_{n-1} x^n - \sum_{n=1}^{\infty} x^n$$

$$-a_0 + a_0 + a_1 x^1 + a_2 x^2 + \dots = 2\left[a_0 x^1 + a_1 x^2 + a_2 x^3 + \dots\right]$$

$$- \left[x + x^2 + x^3 + \dots\right]$$

$$-a_0 + A(x) = 2x\left[\frac{a_0 \cancel{x}}{\cancel{x}} + \frac{a_1 x^{\cancel{2}}}{\cancel{x}} + \dots\right]$$

$$- \left[-1 + 1 + \underbrace{x + x^2 + \dots}\right]$$

$$\boxed{A(x) = \sum_{n=0}^{\infty} a_n x^n}$$

$$-a_0 + A(x) = 2x \cdot A(x) - \left[-1 + \frac{1}{1-x}\right]$$

$$(1-2x)\, A(x) = a_0 + 1 - \frac{1}{1-x}$$

$$(1-2x)A(x) = 1 + 1 - \frac{1}{1-x}$$

$$(1-2x)\,A(x) = 2 - \frac{1}{1-x}$$

$$A(x) = \frac{2}{1-2x} - \frac{1}{(1-x)(1-2x)}$$

$$\frac{1}{(1-x)(1-2x)} = \frac{A}{1-x} + \frac{B}{1-2x}$$

$$1 = A(1-2x) + B(1-x)$$

$x = 1$,

$$\Rightarrow 1 = A(-1) \Rightarrow \boxed{A = -1}$$

$x = \frac{1}{2} \Rightarrow 1 = B\left(1 - \frac{1}{2}\right)$

$$\boxed{B = 2}$$

$$\Rightarrow \frac{1}{(1-x)(1-2x)} = \frac{-1}{1-x} + \frac{2}{1-2x}$$

$$A(x) = \frac{\cancel{2}}{1-\cancel{2}x} - \left[\frac{-1}{1-x} + \frac{\cancel{2}}{\cancel{1}-2x} \right]$$

$$= \frac{1}{1-x}$$

$$= (1-x)^{-1}$$

$$\boxed{A(x) = 1 + x + x^2 + x^3 + \cdots}$$

Therefore, $\boxed{a_n = 1}$, the seq is, 1, 1, 1, 1, ...

(i) $a_n - 9a_{n-1} + 20a_{n-2} = 0,\ a_0 = -3,\quad a_1 = -10.$ $\quad n \geq 2$.

$$\sum_{n=2}^{\infty}\left(a_n x^n - 9a_{n-1}x^n + 20a_{n-2}x^n\right) = 0$$

$$\sum_{n=2}^{\infty} a_n x^n - 9\underline{\sum_{n=2}^{\infty} a_{n-1}x^n} + 20\sum_{n=2}^{\infty} a_{n-2}x^n = 0$$

First term:

$$\sum_{n=2}^{\infty} a_n x^n = a_2 x^2 + a_3 x^3 + \cdots$$

$$= \left(\underline{a_0 + a_1 x} + a_2 x^2 + a_3 x^3 + \cdots\right) - \left(a_0 + a_1 x\right)$$

$$= A(x) - a_0 - a_1 x$$

$$= A(x) + 3 + 10x$$

Second term:

$$\sum_{n=2}^{\infty} a_{n-1} x^n = \left(a_1 x^2 + a_2 x^3 + \cdots\right)$$

$$= x\left(a_1 x + a_2 x^2 + \cdots\right)$$

$$= x\left(-a_0 + a_0 + a_1 x + a_2 x^3 + \cdots\right)$$

$$= x\left(3 + A(x)\right)$$

$$= 3x + xA(x)$$