1. Let $\mathbb{Z}$ be the set of all integers. Define R on $\mathbb{Z} \times \mathbb{Z}$ such that $aRb \Leftrightarrow (a-b)$ *is divisible by* 5; $a, b \in \mathbb{Z}$. Show that R is an equivalence relation on $\mathbb{Z}$. Find the equivalence class of 3.

2. Let $\mathbb{Z}$ be the set of all integers. Define R on $\mathbb{Z} \times \mathbb{Z}$ such that $aRb \Leftrightarrow (a+b)$ *is even*; $a, b \in \mathbb{Z}$. Show that R is an equivalence relation on $\mathbb{Z}$.

   [Hint: For Transitivity, $aRb,\ bRc \Rightarrow (a+b),(b+c)$ *are even*
   $$\Rightarrow (a+b)+(b+c) \text{ is even}$$
   $$\Rightarrow (a+c)+2b \text{ is even}$$
   $$\Rightarrow (a+c) \text{ is even}$$
   $$\Rightarrow aRc$$]

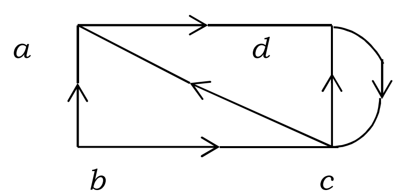
3. Let $\mathbb{N}$ be the set of all positive integers. Define R on $\mathbb{N} \times \mathbb{N}$ such that $aRb \Leftrightarrow \{|a-b|+2\}$ *is a prime*; $a, b \in \mathbb{N}$. Examine whether R is an equivalence relation on $\mathbb{N}$.

   [Hint: Not Transitive. Take $a=1, b=2, c=3$]

4. Using Warshall's algorithm, find the transitive closure of the relation $R = \{(1,2),\ (2,1),\ (2,3),(3,4)\}$ on the set $A = \{1,2,3,4\}$.

5. Using Warshall's algorithm, find the transitive closure of the relation $R = \{(1,4),\ (2,1),\ (2,3),(3,1),(3,4),(4,3)\}$ on the set $A = \{1,2,3,4\}$.

6. Using Warshall's algorithm, find the transitive closure of the relation R which is given by the following directed graph:



7. Let $\mathbb{N}$ be the set of all positive integers. Define $\preccurlyeq$ on $\mathbb{N} \times \mathbb{N}$ such that $a \preccurlyeq b \Leftrightarrow a \text{ divides } b$; $a, b \in \mathbb{N}$. Examine whether $\preccurlyeq$ is a partial order relation on $\mathbb{N}$.

8. Let $\mathbb{Z}$ be the set of all integers. Define $\preccurlyeq$ on $\mathbb{Z} \times \mathbb{Z}$ such that for $a, b \in \mathbb{Z}$ $a \preccurlyeq b \Leftrightarrow b = a^m$; *for some* $m \in \mathbb{N}$. Show that $\preccurlyeq$ is a partial order relation on $\mathbb{Z}$.

9. Let $\mathbb{Z}$ be the set of all integers. Define $\preccurlyeq$ on $\mathbb{Z} \times \mathbb{Z}$ such that for $a, b \in \mathbb{Z}$ $a \preccurlyeq b \Leftrightarrow a^3 - b^3$ *is non* $-$ *negative*. Show that $\preccurlyeq$ is a partial order relation on $\mathbb{Z}$.