SIKKIM MANIPAL UNIVERSITY
SIKKIM MANIPAL INSTITUTE OF TECHNOLOGY

Engineering Mathematics III

Discrete Mathematics

Lecture 5

Partial Order relations, Covering, Partial Order set & Problems

This course is taught to Computer Science Engineering students in SMIT, India during Jun-Dec, 2019.

Anti Symmetric

A relation R is said to be anti-symmetric if for all $(a, b) \in R$

$aRb \Rightarrow b \not R a$ unless $a = b$.

$\cancel{a \neq b}$

$A = \mathbb{N}$ — R — $aRb \Leftrightarrow a|b$ (a divides b)

$a \overline{) b}$

0

Partial ordering Relation:

R – is said to be partial ordering relation,

if (i) R is Reflexive

(ii) R is anti-symmetry

(iii) R is transitive

$\cancel{\mathbb{Z}\setminus\{0\}}$ $A = \mathbb{N}$ $aRb \Leftrightarrow a \mid b$.

Examine R for partial ordering relation.

Soln:

(i) **Reflexive:** Let $a \in \mathbb{N}$

To prove: aRa i.e., $a \mid a$

WKT $a \mid a$ because $a = 1.a + 0$

$\Rightarrow$ R is reflexive

(ii) **Anti-symmetric**

Let $a, b \in \mathbb{N}$, $a \neq b$.
and assume that aRb.

TP: $b \not{R}\, a$ i.e., b does not divide a.

WKT, $aRb \Rightarrow$ a is smaller than b

$\Rightarrow$ b does not divide a

$\Rightarrow$ R is an anti-symmetric

(iii) Transitive :

let $a, b, c \in \mathbb{N}$ and assume that aRb and bRc

TP: aRc ie, $a|c \Rightarrow c = q \cdot a + 0$

wki, $a|b \Rightarrow b = q_1 a + 0$

$b|c \Rightarrow c = q_2 b + 0$

$\Rightarrow c = q_2 b$

$= q_2 (q_1 a)$

$= (q_1 q_2) a + 0$

$\Rightarrow a|c$

$\Rightarrow R$ - is ~~fa~~ transitive

Therefore R - is a partial ordering relation

Let A be any set. Then consider the power set of A. $(P(A))$

Define R on $P(A)$ such that ARB iff $A \subseteq B$

Soln:

(i) Reflexive

Let $A \in P(A)$

$$\boxed{\text{Let } a \in A \Rightarrow a \in A}$$

$\Rightarrow A \subseteq A \Rightarrow ARA$

$\Rightarrow$ R is reflexive

(ii) Antisymmetric

Let $A, B \in P(A)$ and $A \subseteq B$, $A \neq B$

TP: $B \not\subseteq A$

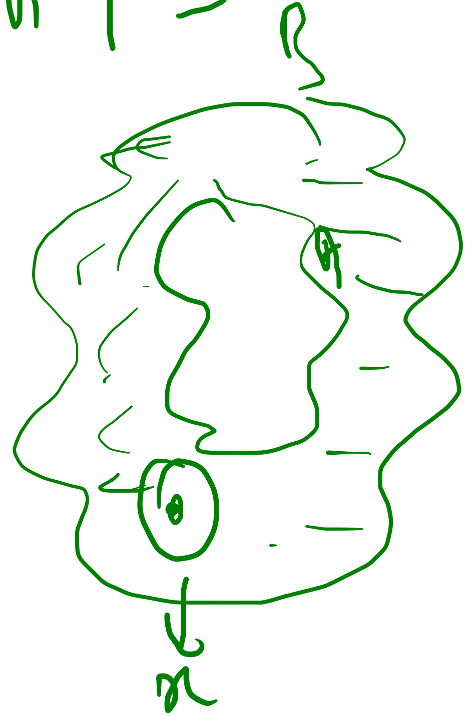
$A \subseteq B$ & $A \neq B \Rightarrow$

$\exists\, x \in B$ such that

$x \notin A$

$\Rightarrow B \not\subseteq A$

$\Rightarrow$ R is antisymmetric



(c) Transitive:

Let $A, B, C \in P(A)$, $A \subseteq B$, $B \subseteq C$

TP: $A \subseteq C$

$A \subseteq B \Rightarrow \forall a \in A, a \in B$

$B \subseteq C \Rightarrow \forall b \in B, b \in C$

$\Rightarrow$ Let $a \in A$, $\Rightarrow a \in C$

$\Rightarrow A \subseteq C$

$\Rightarrow R$ is ~~not~~ transitive

$\Rightarrow R$ is a partial ordering relation.

lower bound / or smallest element

$A_1 \subseteq A_2 \subseteq A_3 \subseteq A_4 \subseteq \dots \subseteq A_n$

upper bound / bigger element

$B_1 \subseteq B_2 \subseteq B_3 \subseteq \dots$

Chain