SMIT SIKKIM MANIPAL UNIVERSITY
SIKKIM MANIPAL INSTITUTE OF TECHNOLOGY

Engineering Mathematics III

Discrete Mathematics

Lecture 10

Properties of Lattice

This course is taught to Computer Science Engineering students in SMIT, India during Jun-Dec, 2019.

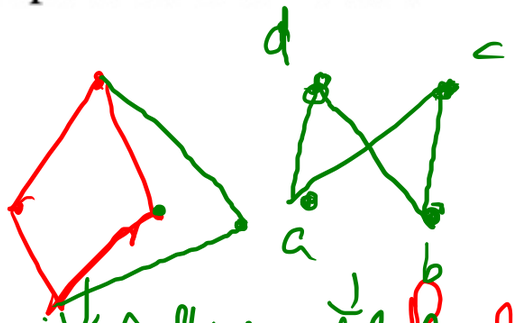
$(P(X), \leq)$

3.20. If X ia a nonempty set, then show that $(P(X), \preccurlyeq)$ is lattice with respect to the relation

$A \preccurlyeq B \iff A \subseteq B, A, B \in P(X)$.

if and only if

$A \leq B$.



is lattice not lattice

Prove that : which is a poset (ex)

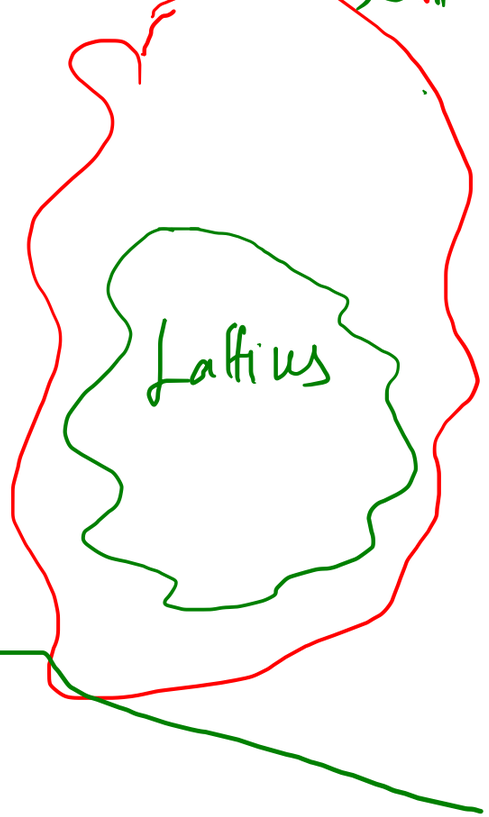
Let $A, B \in P(X)$

Consider $A \cup B$,

Note that $A \subseteq A \cup B$, $B \subseteq A \cup B$

$\Rightarrow A \cup B$ is an upper bound

Let C be an arbitrary upper bound of A and B



Let C be an arbitrary upper bound of A and B.

$$\left.\begin{array}{l} A \subseteq C \\ \text{and } B \subseteq C \end{array}\right\} \Rightarrow A \cup B \subseteq C \Rightarrow A \cup B \leq C$$

$\Rightarrow A \cup B$ is the lub of A and B.

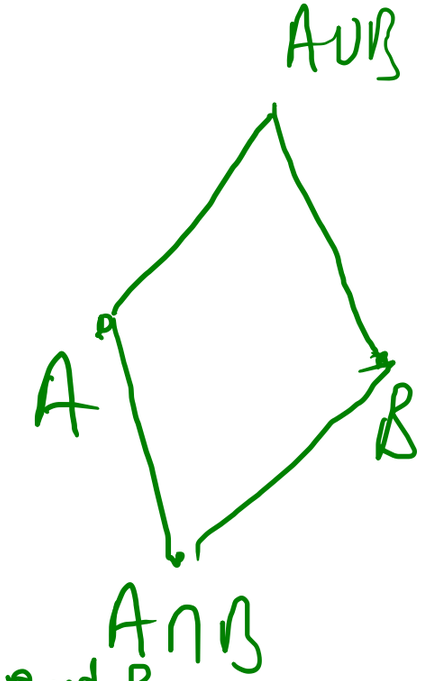
Proof of $A \cap B$ is the glb of A and B:

Let $A, B \in \mathcal{P}(X)$, $\left.\begin{array}{l} A \cap B \subseteq A \\ A \cap B \subseteq B \end{array}\right\} \Rightarrow A \cap B$ is a lower bound of A & B.

Let C be any arbitrary lower bound of A and B.

$$\left.\begin{array}{l} C \subseteq A \\ C \subseteq B \end{array}\right\} \Rightarrow C \subseteq A \cap B \Rightarrow C \leq A \cap B$$

$\Rightarrow A \cap B$ is the glb of A and B.



Ex: Prove that Set of all natural numbers ≤ 10, forms a lattice w.r.to, the ~~st~~ relation divides.

Construct the Hasse diagram.

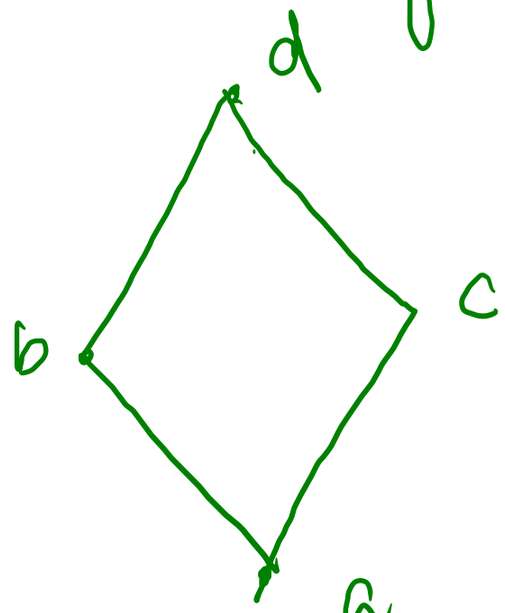
. **Idempotent Law:** In a lattice $(L, \preccurlyeq)$, show that $a \wedge a = a$ if and only if $a \vee a = a$.

meet

and

join

Proof in Exercise.



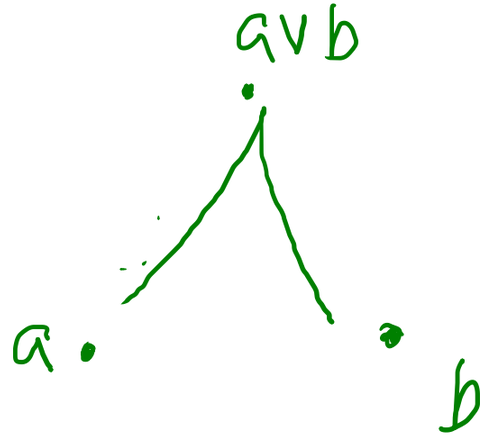
a is a upper bound of a

3.21. In a lattice $(L, \preccurlyeq)$, for any $a, b \in L$, show that $a \preccurlyeq a \vee b$ and $a \wedge b \preccurlyeq a$.

$a \vee b$ is the lub of a and b,

$\Rightarrow a \leq a \vee b$

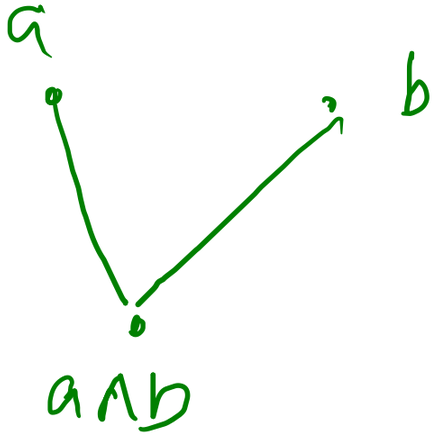
$b \leq a \vee b$



But $a \wedge b$ be the glb of a and b,

$a \wedge b$ is glb$(a, b) \Rightarrow a \wedge b \leq a$

$a \wedge b \leq b$.



3.22. In a lattice $(L, \preccurlyeq)$, if $a \preccurlyeq b$, show that $a \wedge b = a$, $a \vee b = b$.

Exercise.

proof:

Let $a \leq b$.

$a \leq b$, $b \leq b$ $\Rightarrow$ b is a upper bound of a and b.

Let c be any arbitrary upper bound of a and b.

$a \leq c$, $b \leq c$ $\Rightarrow$ $b \leq c$ $\Rightarrow$ b is the lub of a and b.

3.23. In a lattice $(L, \preccurlyeq)$, show that $a \wedge b = a$ if and only if $a \vee b = b$.