SIKKIM MANIPAL UNIVERSITY

SIKKIM MANIPAL INSTITUTE OF TECHNOLOGY

Engineering Mathematics III

Discrete Mathematics

Lecture 34

Consistency of premises

This course is taught to Computer Science Engineering students in SMIT, India during Jun-Dec, 2019.

Recall

- **Rule P:** A premise may be introduced at any point in the derivation
- **Rule T:** A formula S can be introduced in a direction if S is tautologically implied by any one or more of the preceding formulas in the derivation.

Demonstrate that R is a valid inference from the premises $p \to Q, Q \to R$, and P.

Solution:

{1}	(1)	$P \to Q$	Rule P
{2}	(2)	P	Rule P
{1,2}	(3)	Q	Rule T, (1), (2) and $(P \to Q) \wedge P \implies Q$
{3}	(4)	$Q \to R$	Rule P
{1,2,3}	(5)	R	Rule T, (3), (4) and $(Q \to R) \wedge Q \implies R$

Lets continue...

Rule CP: If we can derive S from R and a set of premises, then we can derive $R \to S$ from the set of premises alone.

Note: Rule CP is also called as *deduction theorem*.

Show that $R \to S$ can be derived from the premises $P \to (Q \to S)$, $\neg R \lor P$ and Q.

Proof:

Instead of showing $R \to S$, we shall include the premise R as an additional premise and show S first.

Means we have that,
$P \to (Q \to S)$, $\neg R \lor P$, Q and R.

{2}	(1)	$\neg R \vee P$	Rule P
{4}	(2)	R	Rule P
{2,4}	(3)	P	Rule T, $(\neg R \vee P) \equiv R \implies P$, (2) and (3)
{1}	(4)	$P \to (Q \to S)$	Rule P
{1,2,4}	(5)	$Q \to S$	Rule T, (3), (4) and $(P \to Q)$, P \implies Q$
{3}	(6)	Q	Rule P
{1,2,3,4}	(7)	S	Rule T, (5), (6) and $P, (P \to Q) \implies Q$
{1,2,3,4}	(8)	$R \to S$	Rule CP

5.2

Checking Consistency of premises

"If there was a ball game, then travelling was difficult. If they arrived on time, then travelling was not difficult. They arrived on time. Therefore, there was no ball game."

Without using truth table, show that these statements constitute a valid argument.

Checking Consistency of premises

"If there was a ball game, then travelling was difficult. If they arrived on time, then travelling was not difficult. They arrived on time. Therefore, there was no ball game."
Without using truth table, show that these statements constitute a valid argument.

P: There was a ball game,
Q: Travelling was difficult,
R: They arrived on time

So the given paragraph can be written as

$$H_1 : P \to Q, \quad H_2 : R \to \neg Q, \quad H_3 : R \quad C : \neg P$$

Now, verify that the conclusion logically follows from H_1,, H_2 and H_3.

Questions?

Thank you