SIKKIM MANIPAL INSTITUTE OF TECHNOLOGY

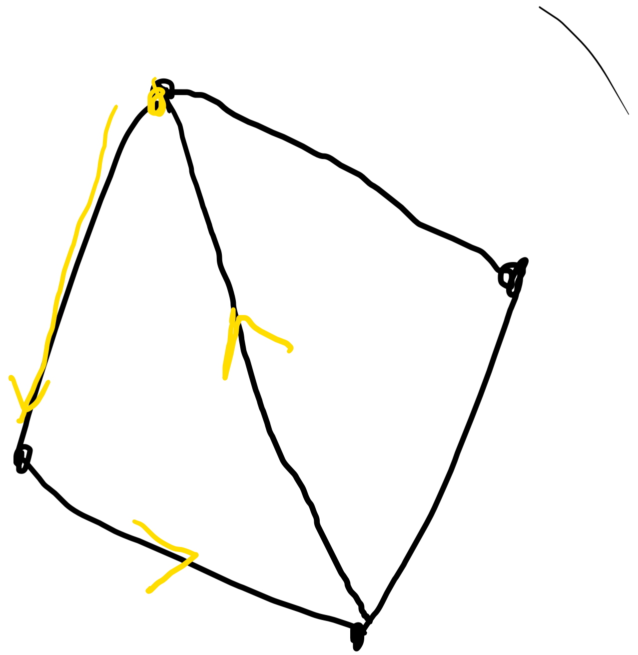
Engineering Mathematics III

Discrete Mathematics

Lecture 16

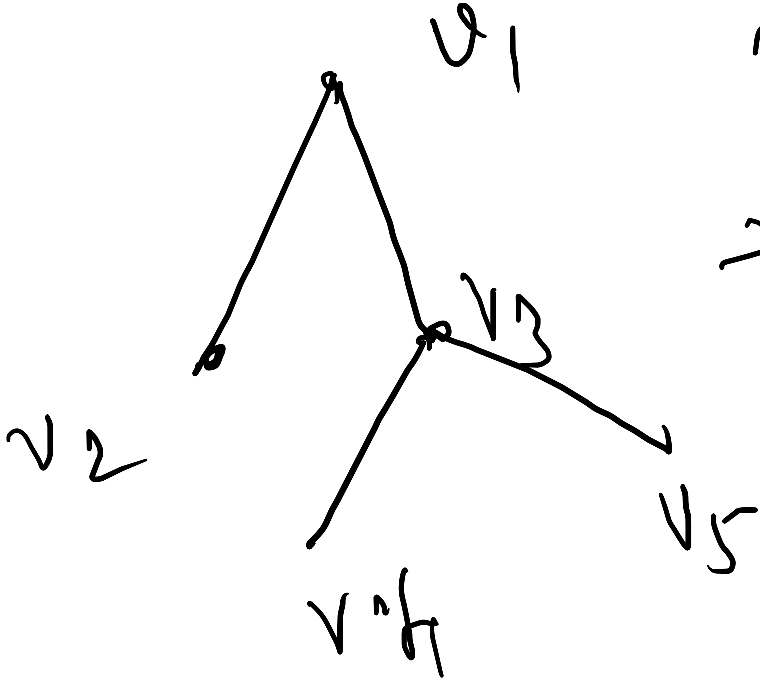
Connected Graphs, Trees & Pigeonhole Principle

This course is taught to Computer Science Engineering students in SMIT, India during Jun-Dec, 2019.



→ there is a cycle.

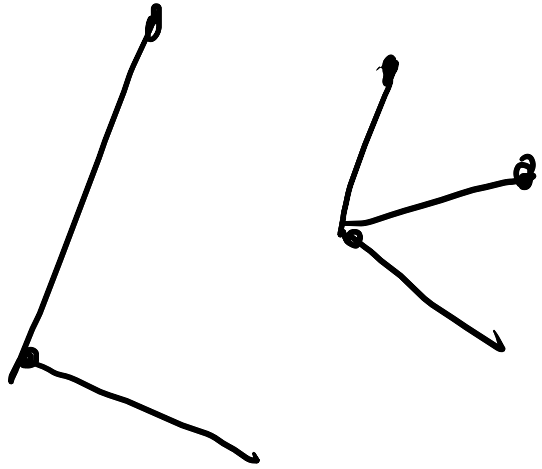
Acyclic graph



Tree

→ ~~Tree~~

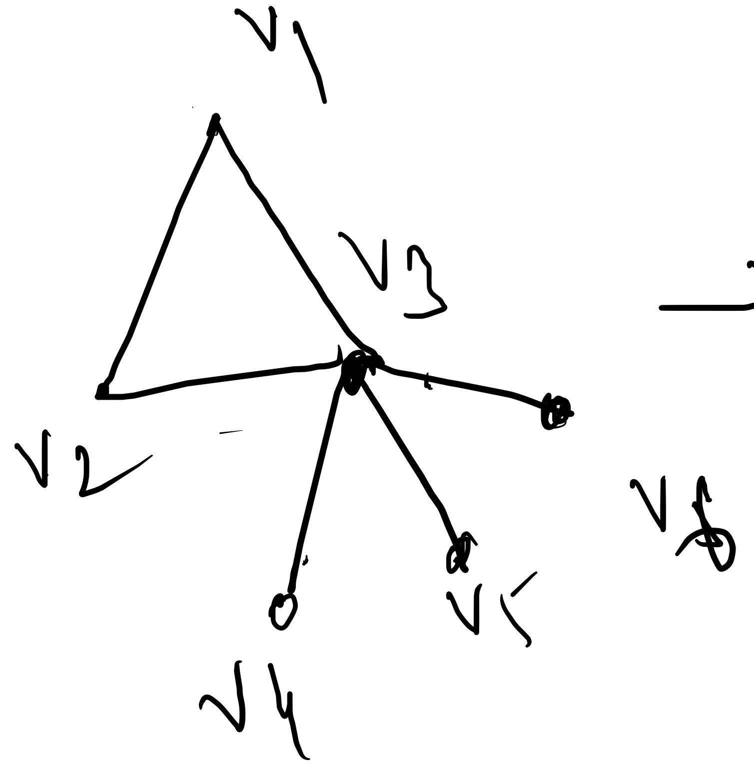
A graph without any cycles is called as acyclic graph.



$\rightarrow$ Acyclic (not tree)

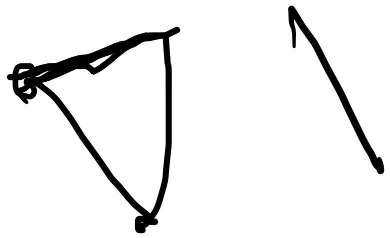
G

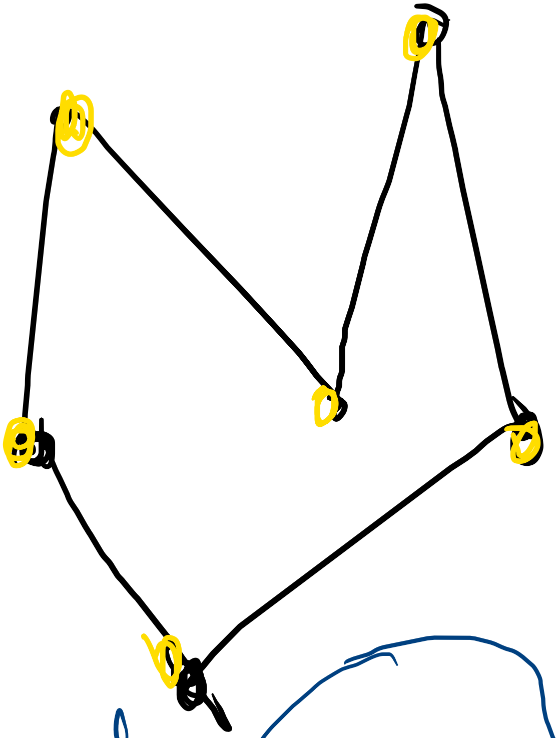
Connected graph:-



→ ~~A graph so that any two vertices can be connected~~

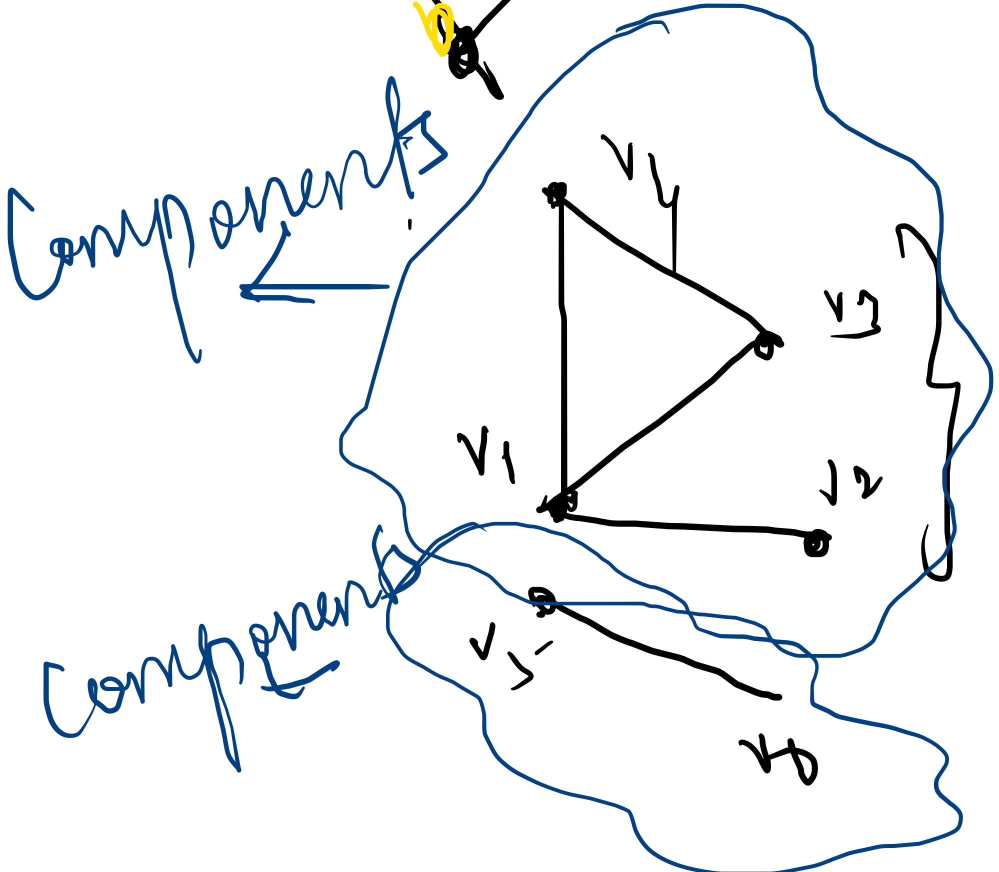
If there a path b/w any two vertices of a graph then that graph is called the Connected.





→ Cycle

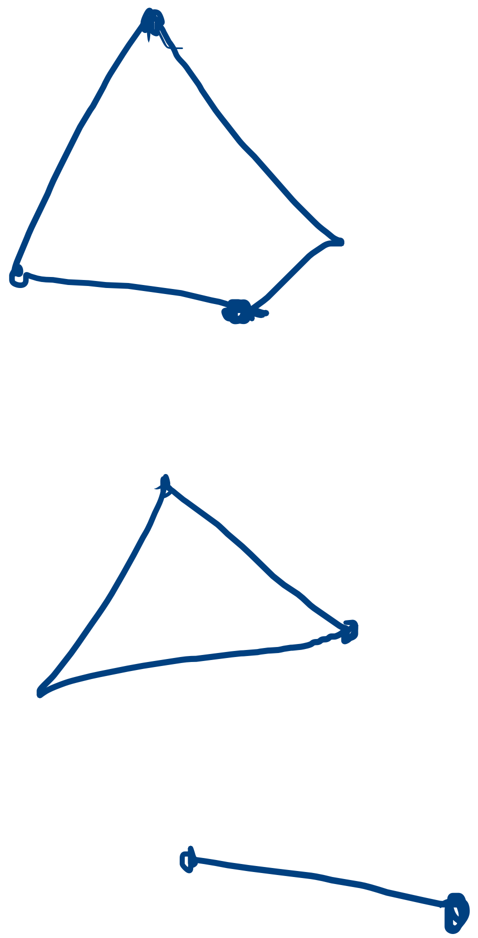
→ Connected graph



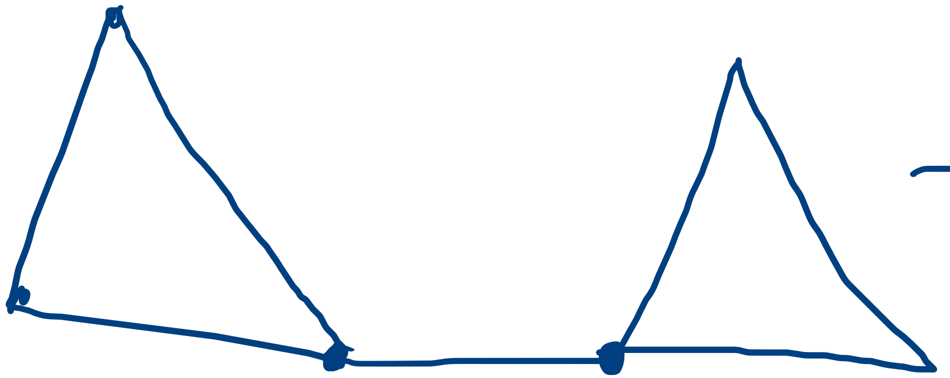
→ not Connected (disconnected

why? (there is no path
b/w v_4 and v_5)

How many components does this graph has?



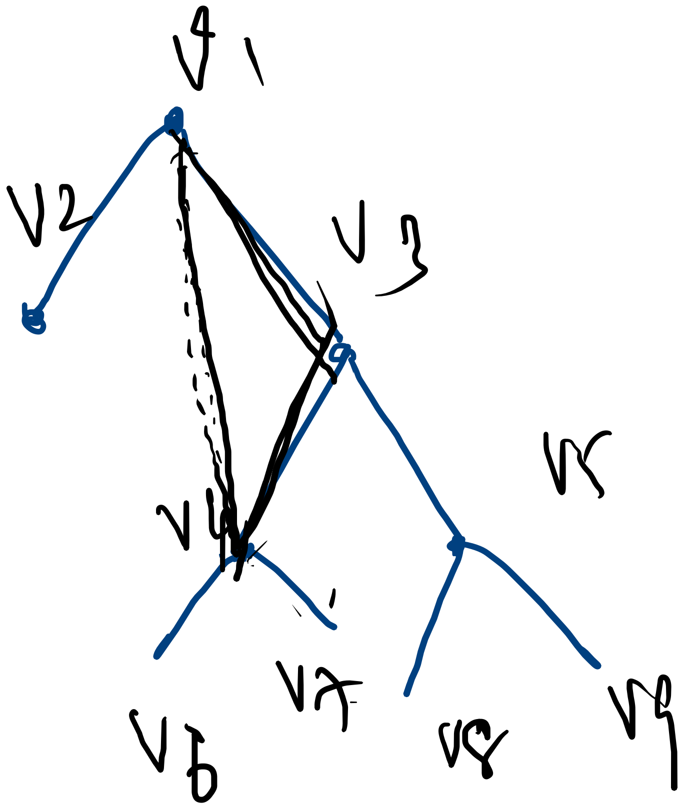
— three.

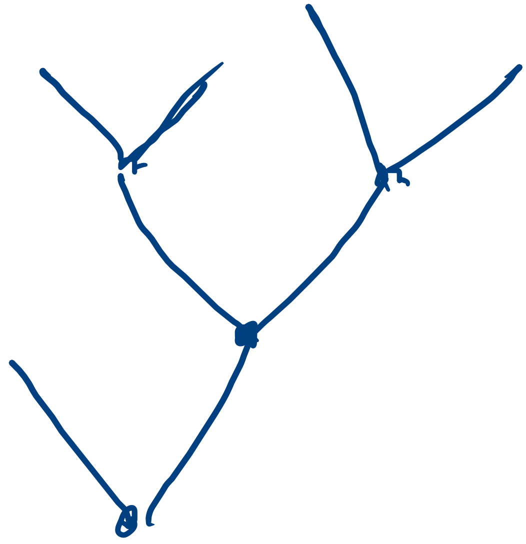


→ not a tree.

Tree

A Connected graph is a tree if there is always only one path b/w any two vertices.





T

The following are equivalent

(i) G is a tree

(ii) G is a connected and acyclic graph.

Proof:

(i) $\Rightarrow$ (ii) Let G be a tree.

$\Rightarrow$ G is connected and any two vertices have unique path b/w them.

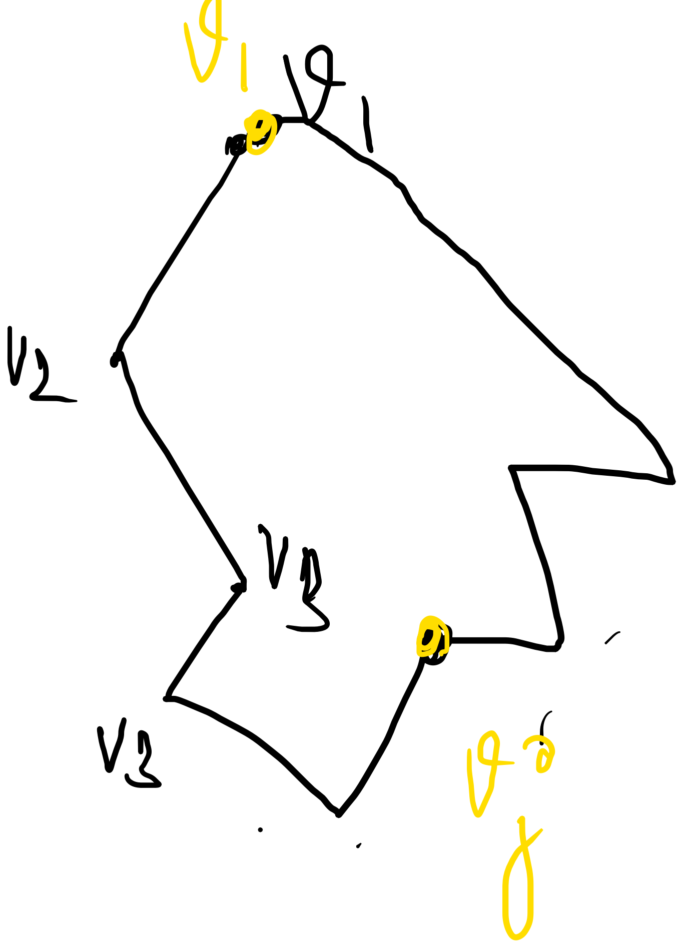
TP: G does not have a cycle

Suppose G has a cycle

Let $v_1, v_2, \ldots v_k = v_1$ forms a cycles

Let v_i be a vertex in the cycle

and v_j be another vertex in cycles



$\Rightarrow$ there exist two paths b/w v_i and v_j

which is a contradiction to G is a tree

$\Rightarrow$ G cannot have a cycle $\Rightarrow$ G is acyclic

(ii) ⇒ (i) Let G be a connected graph
and G be acyclic graph

Let v_i and v_j have a path P

TP: There is only one path b/w v_i & v_j

Suppose there is another path b/w v_i & v_j

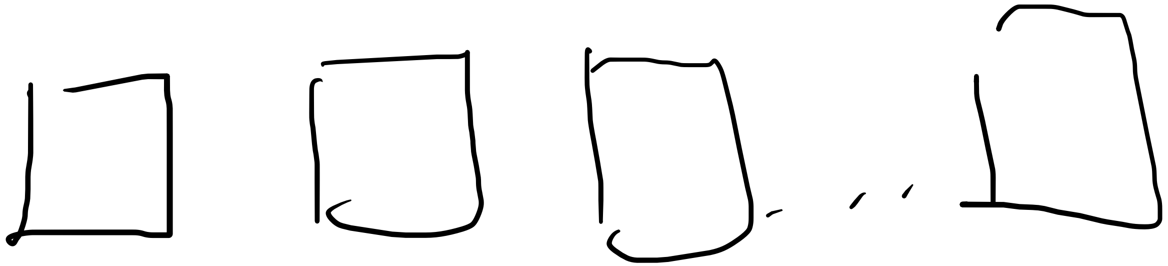
Then the two paths makes a cycle
which is a contraction to G-acyclic
⇒ there is only one path b/w v_i and v_j.

This is true for any i, and j, $1 \le i, j \le$ no of vertices of the graph

$\Rightarrow$ G is a tree.

Hence proved.

Pigeonhole principle



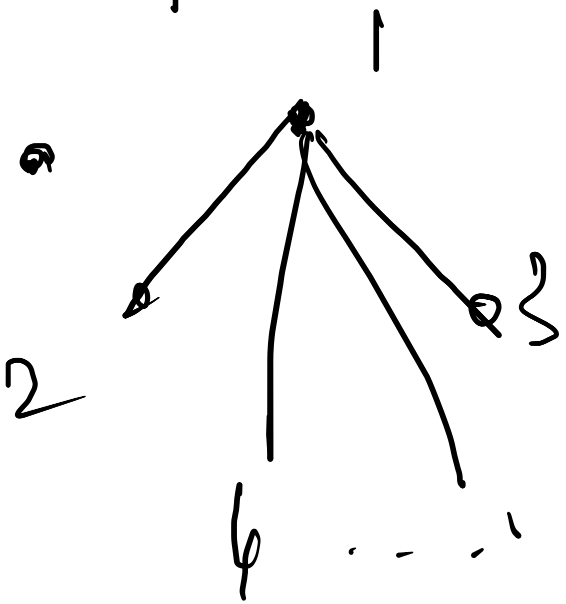
If there are n pigeons and $n-1$ pigeonholes then atleast one pigeonhole contains two or more pigeon.

Question 1.9. In a party with atleast two people, there are alteast two people who have same number of friends.

Proof: Let 0, 1, 2, ... N are the number of friends and N - no. of people in the party.

suppose if a person does not have anybody as friend.

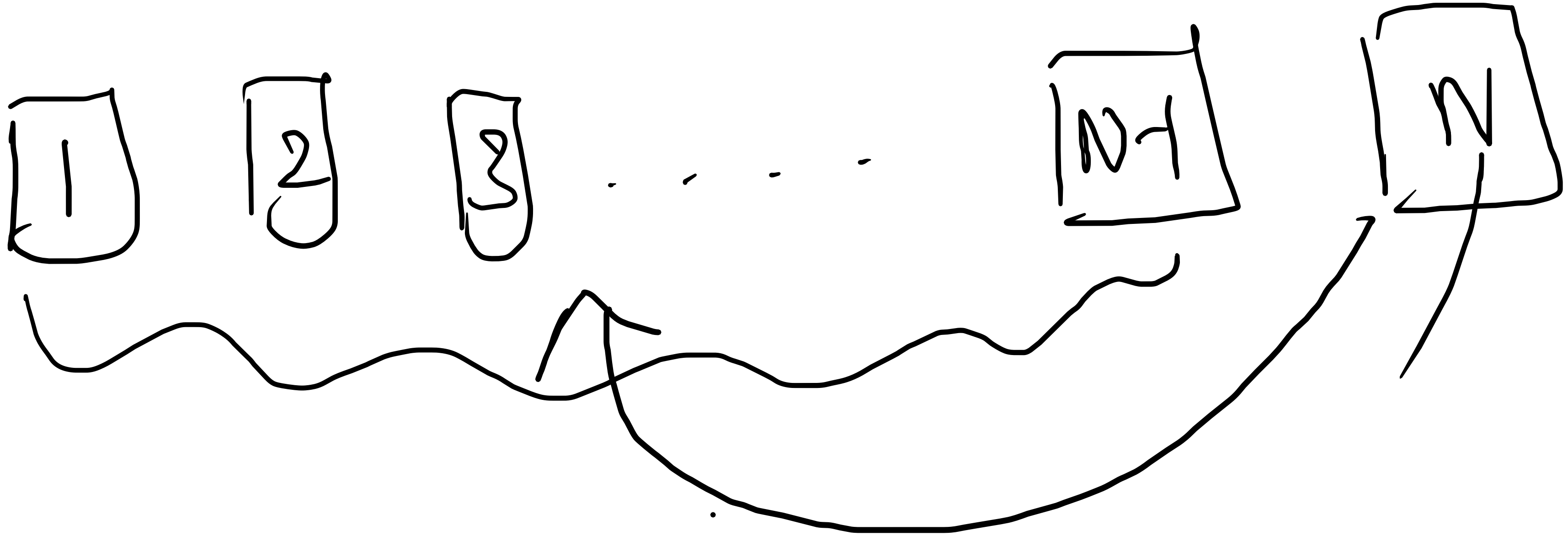
There is always one person who is friend of everybody. (?)



$\Rightarrow$ the seq of number of friends has to be 1, 2, ... N-1

Pigeons = people

pigeonholes = no of friends they have



⇒ The nth person should occupy one of the pigeonholes 1, 2, ... N-1.

⇒ there are atleast two people have same of friends.