SMIT SIKKIM MANIPAL UNIVERSITY
SIKKIM MANIPAL INSTITUTE OF TECHNOLOGY

Engineering Mathematics III

Discrete Mathematics

Lecture 19

Mathematical Logic: WFF, Substitution Instances, Equivalence of formulas

This course is taught to Computer Science Engineering students in SMIT, India during Jun-Dec, 2019.

Conditionals and Biconditionals

If I study well, I will score 90% in exam

p: I study well

q: I will score 90% in exam

$p \rightarrow q$ ✓ $q \leftarrow p$ ✓

$(\Rightarrow)$

$p \rightarrow q$ — Conditionals (read as p implies q)

$p \leftrightarrow q$ → Bi conditionals.

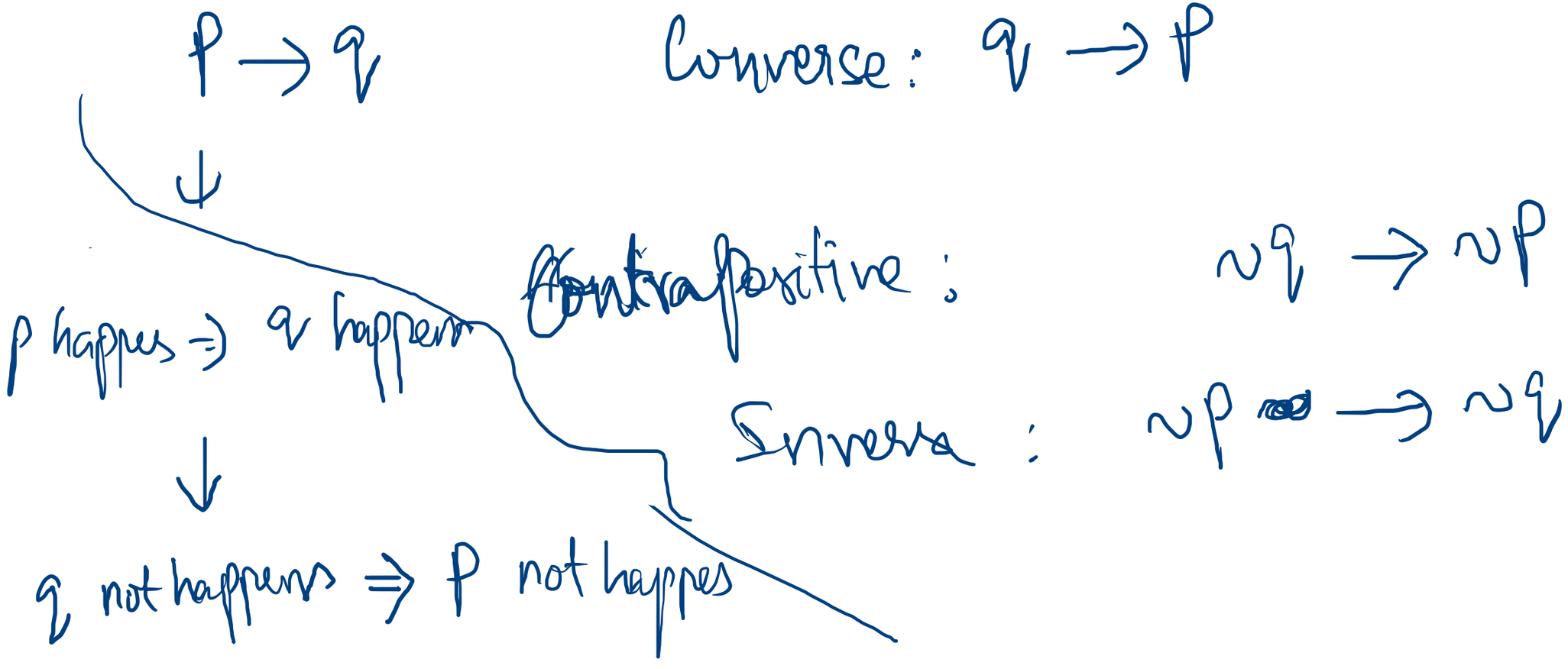
Question 3.1. Write the following statement in symbolic form: The hut will be destroyed if there is a cyclone.

p : The hut will be destroyed

q : there is a cyclone

$q \rightarrow p$

Converse, Contrapositive, Inverse



3.1. Give the converse, contrapositive and inverse of the following implications: "If it rains today, I will go the college tomorrow"

Converse:

If I go to the college tomorrow, then it rains today

Contrapositive:

If I dont go to the College tomorrow, then It does not rain today.

Inverse:

If it does not rain today, then I will not go to the college tomorrow.

p : It rains Today

q : I will go to the college tomorrow.

original: $p \to q$

Converse: $q \to p$

Contrapositive: $\sim q \to \sim p$

Inverse: $\sim p \to \sim q$

Logical Equivalence

p , $p \vee p$ are equivalent statements

$p \vee q$, $q \vee p$ are equivalent statement.

p	p	$p \vee p$
T	T	T
F	F	F

$p \equiv p \vee p$

Truth table of $p \rightarrow q$

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Truth table $p \leftrightarrow q$

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

PT. $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

Soln:

p	q	$p \leftrightarrow q$	$p \rightarrow q$	$q \rightarrow p$	$(p \rightarrow q) \wedge (q \rightarrow p)$
T	T	T ✓	T	T	T ✓
T	F	F ✓	F	T	F ✓
F	T	F ✓	T	F	F ✓
F	F	T ✓	T	T	T ✓

∴ from the truth table,

$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

Well Formed Formula

A well-formed formula can be produced by using the following rules:

Rule 1 A statement variable itself is a well-formed formula

Rule 2 If p is a well-formed formula then $\bar{p}$ is also a well-formed formula

Rule 3 If p and q are two well-formed formulas then $p \vee q$, $p \wedge q$, $p \rightarrow q$, and $p \leftrightarrow q$ are also well-formed formulas.

Rule 4 A string of symbols consisting of the statement of the statement variables, connectives, and parentheses is said to be a well-formed formula, if and only if it can be produced by applying rules 1, 2, and 3 finitely many times.

Example

$p \vee q$, $\neg p \vee q$, $\overline{p \vee q}$,

$\overline{(q \to p)} \vee (p \wedge q)$

3.1. There are two restaurants next to each other. One has a sign that says,

"Good food is not cheap"

and the other has a sign that says,

"Cheap food is not good".

Are the signs saying the same thing? Verify using mathematical logic.

p: Good food

q: Cheap food

$p \rightarrow \sim q$ ✓

$q \rightarrow \sim p$ ✓

- Exercise