SMIT SIKKIM MANIPAL UNIVERSITY

SIKKIM MANIPAL INSTITUTE OF TECHNOLOGY

Engineering Mathematics III

Discrete Mathematics

Lecture 14

Introduction to Graphs (Part 2)

This course is taught to Computer Science Engineering students in SMIT, India during Jun-Dec, 2019.

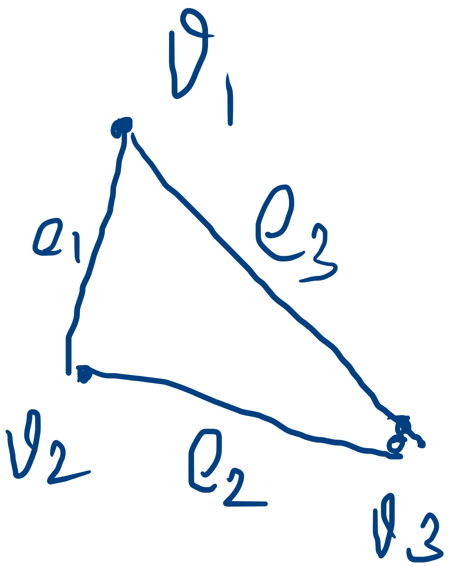
Definition 1.1 (Graph). A *graph* $G = (V, E)$ consists of nonempty set of vertices V (or nodes) and set of edges E such that each edge e_k is identified with an unordered pair of vertices (v_i, v_j).

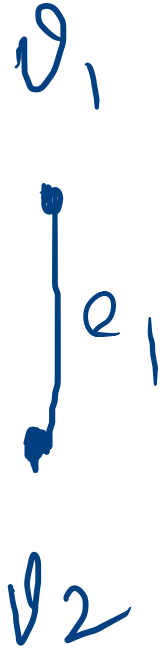
$$G = (V, E)$$

$$V = \{v_1, v_2, v_3\}$$

$$E = \{e_1, e_2, e_3\}$$

$$= \{(v_1, v_2), (v_2, v_3), (v_3, v_1)\}$$



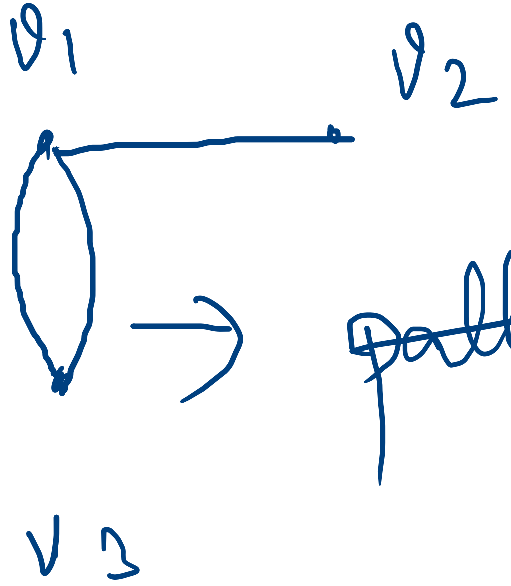


$$G = (V, E)$$

$$V = \{v_1, v_2\}$$

$$E = \{e_1\}$$

Definition 1.2 (Pseudo, Multi, Simple graphs)**.** A graph with no self loops and no parallel edges is called as *Simple Graph*. A graph with parallel edges is called as *Multi Graph* and the graph with both parallel edges and self loops is called as *Pseudo Graph or General Graph*.

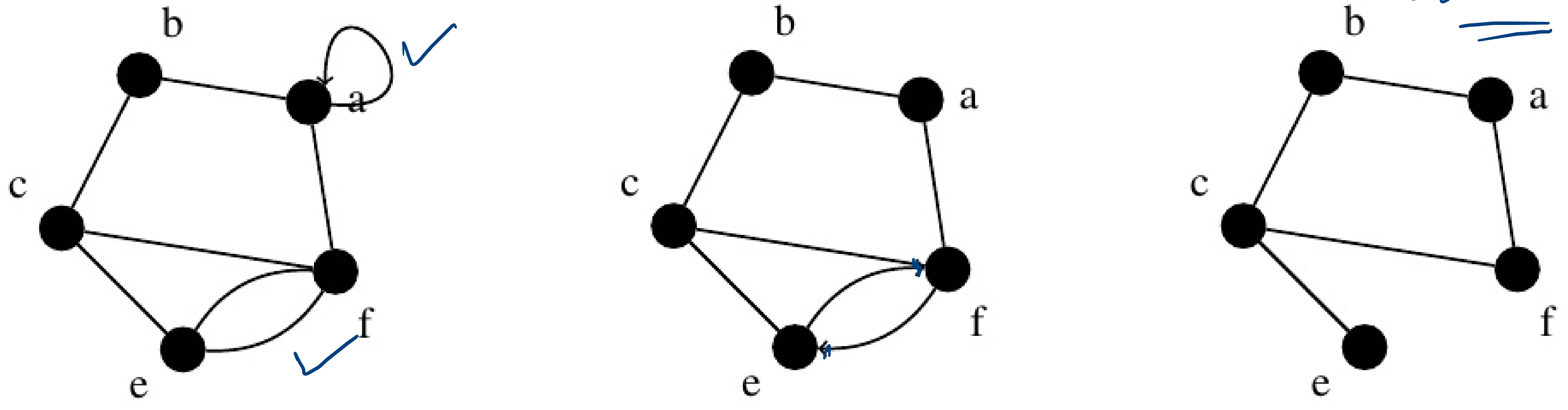


→ ~~pallel~~ parallel edge

finite graph:

Number of vertices & number of edges are finite.

Throughout the course



(a) Pseudo Graph (General Graph)

(b) Multi Graph

(c) Simple Graph

Figure 1: Example of Graphs

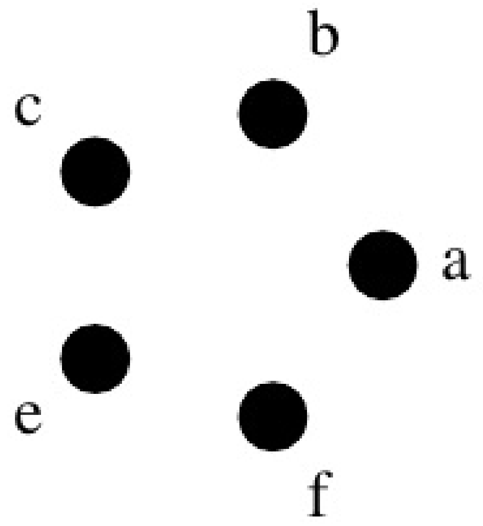


Figure 2: Example of a null graph

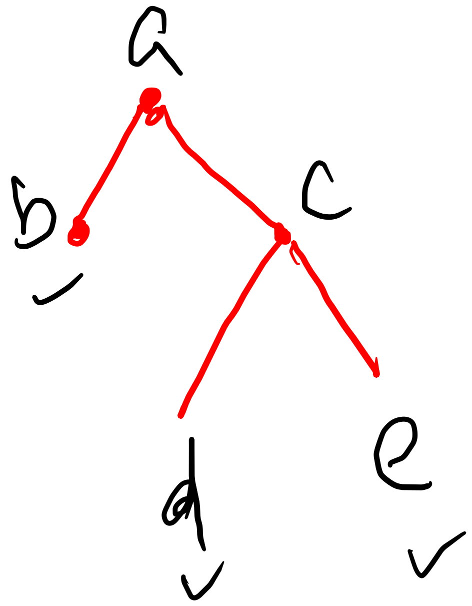
Definition 1.3 (Degree of a vertex)**.** The degree of a vertex v in a graph G is the number of edges incident to the vertex. We denote it as $\deg(v)$. A vertex is said to be odd or even according to its degree is odd or even.

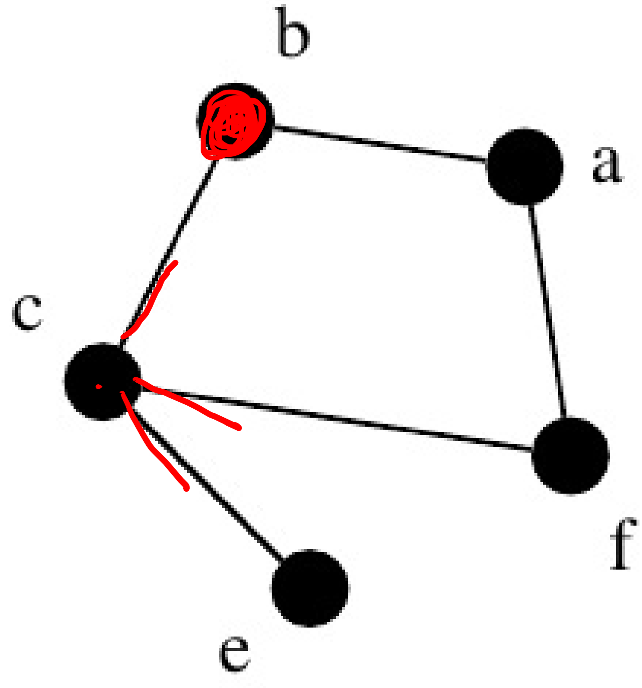
$dg(b) = 2$

$dg(c) = 3$

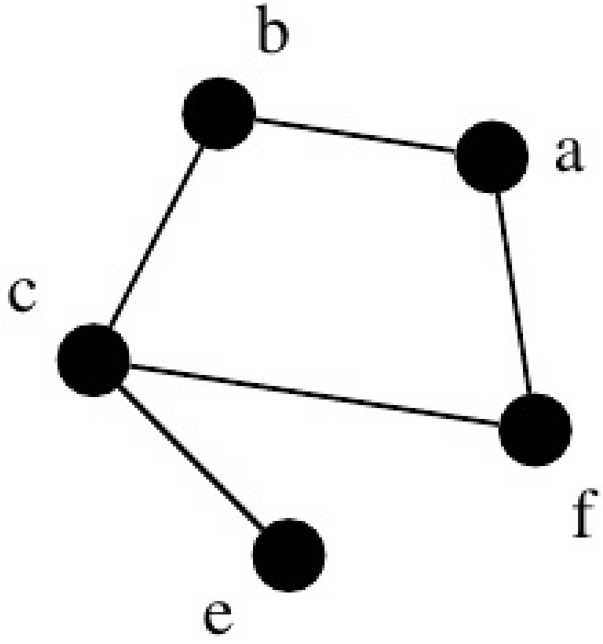
$dg(e) = 1$

$\longrightarrow$ pendent vertex.



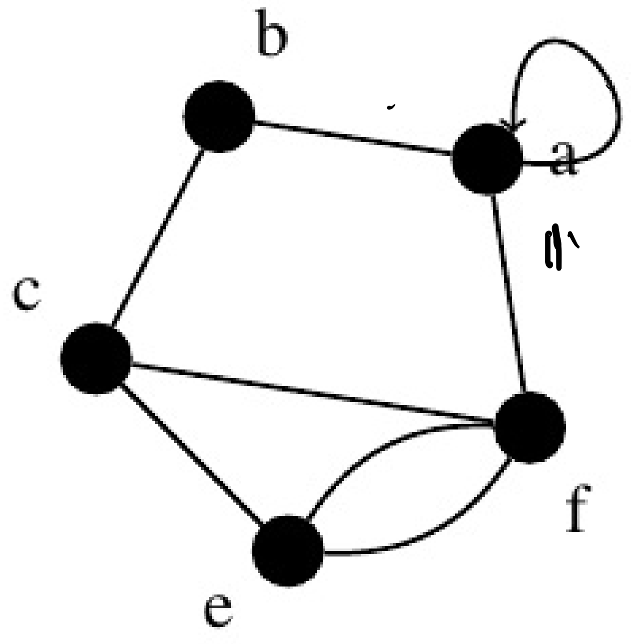


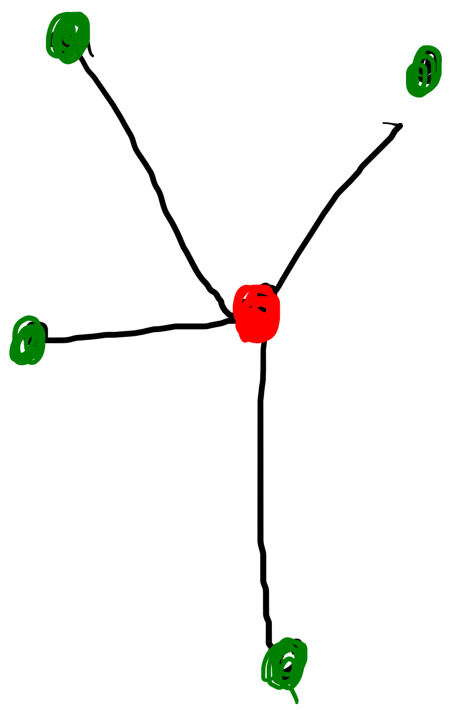
Verify Theorem 1 :

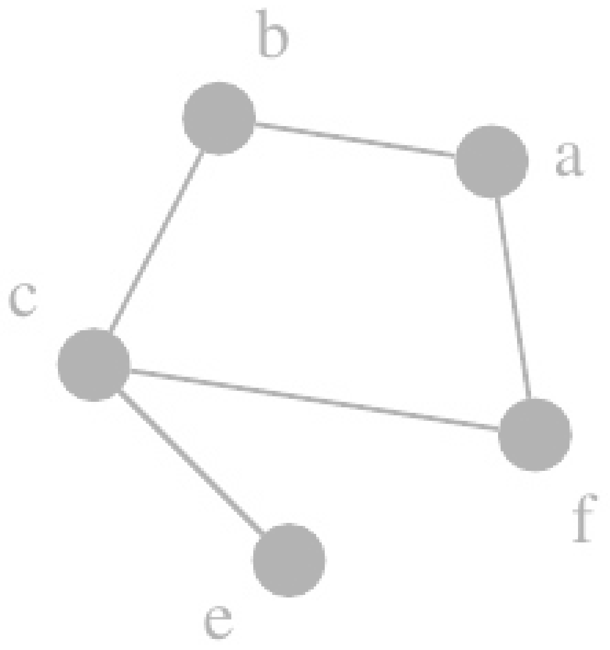


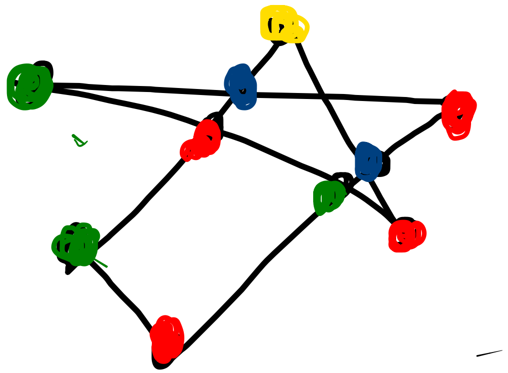
$\deg(a) = 4$

$\deg(b) = 2$







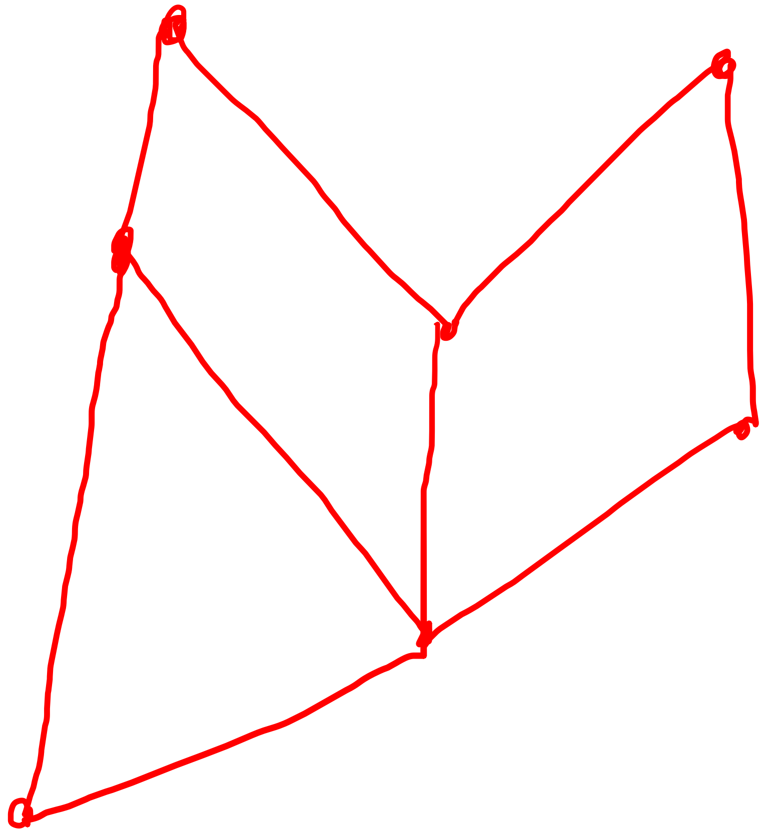


$\deg(c) = 3$

$\deg(e) = 3$

$\deg(f) = 4$

Ex:



How many different colours are needed so that the adjacent vertices will not have same color?

Which of the following is not a degree sequence of ~~any~~ simple graph? [GATE 2010]

A. 7, 6,5,4,4,3,2,1 ✓

B. 6,6,6,3,3,2,2

C. 7,6,6,4,4,3,2,2

D. 8,7,7,6,4,2,1,1

$5, \underbrace{5, 2, 2, 1, 1}_{-1}$

$4, \underbrace{1, 1, 0, 0}_{-1}$

$0, 0, -1, -1$

$G = (V, E)$, $\#(V) = n$

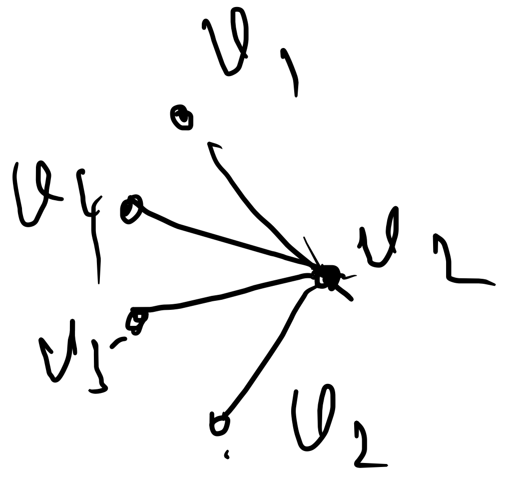
$7, \underbrace{6, 5, 4, 4, 3, 2, 1}_{-1}$

$5, \underbrace{4, 3, 3, 2, 1}_{-1}, 0$

$3, 2, 2, 1, 0, 0$

$1, 1, 0, 0, 0$

$\Rightarrow 0, 0, 1, 0$



(i) If you end up in all the 1's and 0's then simple graph exist

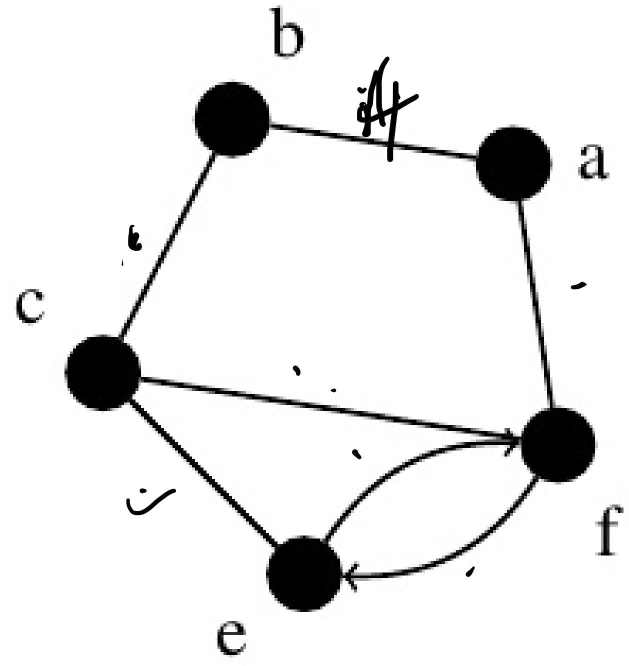
(ii) you reach neg deg $\Rightarrow$ no simple graph.

(iii) there may not enough vertices, to remove the edges,

Theorem 1 (Hand Shaking Lemma)**.** *The sum of the degrees of the vertices of a graph G is twice the number of edges in G. Mathematically,*

$$\sum_{v \in V(G)} \deg(v) = 2 \times |E(G)|.$$

$14 = 2(7)$



Verify Theorem 1 :