SIKKIM MANIPAL UNIVERSITY

III SEMESTER B.Tech(CSE) SAMPLE QUESTIONS, SET-2

DISCRETE MATHEMATICS (LATTICE)

MA 1308

1. If X ia a nonempty set, then show that $(P(X), \preccurlyeq)$ is lattice with respect to the relation $A \preccurlyeq B \Longleftrightarrow A \subseteq B$, $A, B \in P(X)$.

   Hint: Show that $A \vee B = A \cup B$, $A \wedge B = A \cap B$.

2. In a lattice $(L, \preccurlyeq)$, show that $a \wedge (b \wedge c) = (a \wedge b) \wedge c$, for all $a, b, c \in L$.

3. In a lattice $(L, \preccurlyeq)$, show that $a \vee (b \vee c) = (a \vee b) \vee c$, for all $a, b, c \in L$.

4. In a lattice $(L, \preccurlyeq)$, show that $a \wedge (a \vee b) = a \vee (a \wedge b) = a$, for all $a, b \in L$.

5. In a lattice $(L, \preccurlyeq)$, if $a \preccurlyeq b$, show that $a \wedge b = a$, $a \vee b = b$.

6. In a lattice $(L, \preccurlyeq)$, if $a \preccurlyeq b$, $c \preccurlyeq d$, show that $a \wedge c \preccurlyeq b \wedge d$, $a \vee c \preccurlyeq b \vee d$.

7. In a lattice $(L, \preccurlyeq)$, show that $a \wedge b = a$ if and only if $a \vee b = b$.

8. In a lattice $(L, \preccurlyeq)$, show that $(a \wedge b) \vee (a \wedge c) \preccurlyeq a \wedge (b \vee c)$, for all $a, b, c \in L$.

   Hint: $a \wedge b \preccurlyeq a$, b and so, $a \wedge b \preccurlyeq a$, $b \vee c$ i.e. $a \wedge b$ is a lower bound of a and $b \vee c$. Hence, $a \wedge b \preccurlyeq a \wedge (b \vee c)$.

   Similarly, show $a \wedge c \preccurlyeq a \wedge (b \vee c)$.

   Hence, $(a \wedge b) \vee (a \wedge c) \preccurlyeq a \wedge (b \vee c)$.

9. In a lattice $(L, \preccurlyeq)$, show that $a \vee (b \wedge c) \preccurlyeq (a \vee b) \wedge (a \vee c)$, for all $a, b, c \in L$.

10. Show that in a lattice, if the meet operation is distributive over the join operation, then join operation is distributive over the meet operation.

11. Show that in a lattice, if the join operation is distributive over the meet operation, then meet operation is distributive over the join operation.

12. Show that in a distributive lattice, if the complement of an element exists, then it is unique.

13. In a distributive lattice $(L, \preccurlyeq)$, if $a \wedge b = a \wedge c$ and $a \vee b = a \vee c$, then show that $b = c$.
    Hint:

$$
\begin{aligned}
b = b \vee (a \wedge b) = b \vee (a \wedge c) &= (b \vee a) \wedge (b \vee c) \\
&= (a \vee b) \wedge (b \vee c) \\
&= (a \vee c) \wedge (b \vee c) \\
&= (a \wedge b) \vee c \\
&= (a \wedge c) \vee c \\
&= c
\end{aligned}
$$

14. Show that in a Boolean lattice $(L, \vee, \wedge, ^-)$, $\overline{a \vee b} = \overline{a} \wedge \overline{b}$, for all $a, b \in L$.

15. Show that in a Boolean lattice $(L, \vee, \wedge, ^-)$, $\overline{a \wedge b} = \overline{a} \vee \overline{b}$, for all $a, b \in L$.