$aRb \iff a-b$ divisible by 5

(i) R - Reflexive

Let $a \in \mathbb{Z}$

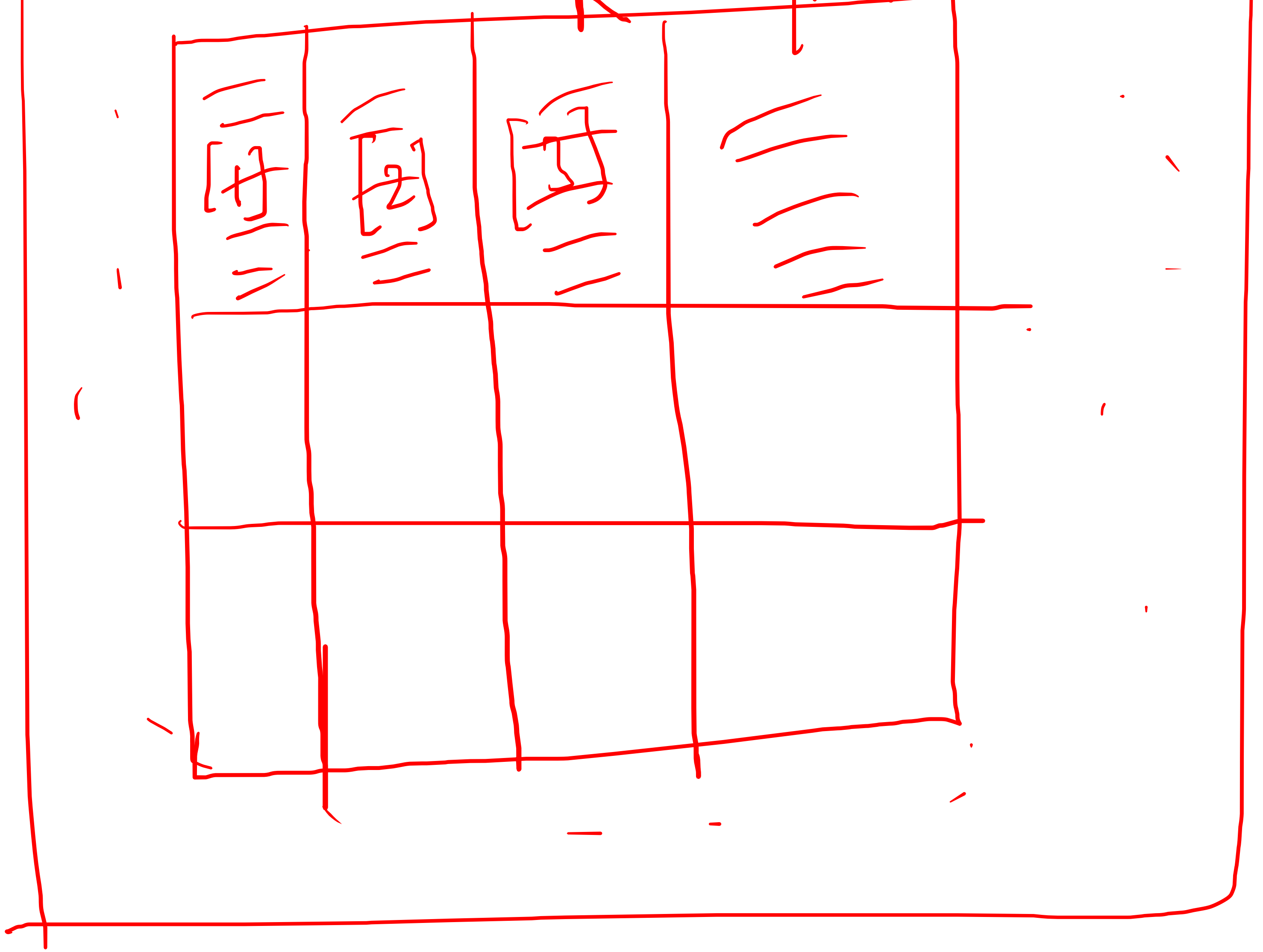
To prove: aRa i.e., $a-a$ should be divisible by 5

$a-a=0$ is divisible by 5

$\Rightarrow$ R - is reflexive

R - equivalence relation $A \times A$

(eg: $\mathbb{Z} \times \mathbb{Z}$



Find the equivalence class of $[3]$

$[a]$ = set of all elements who are related to a.

$[3]$ = set of all integers who are related to 3

$= \{ x \in \mathbb{Z} : 3 - x \text{ is divisible by } 5 \}$

$3 - x = 5k$ for ~~some~~ every $k \in \mathbb{Z}$

$x = 3 - 5k$ for ~~some~~ every $k \in \mathbb{Z}$

$[3] = \{ 3 - 5k : k \in \mathbb{Z} \}$

(ii) <u>Symmetry:</u>

Let $a, b \in \mathbb{Z}$ and aRb

<u>To prove:</u> bRa (i.e. $b-a$ should be divisible by 5)

I know that, $a-b$ is divisible by 5

$\Rightarrow -(a-b)$ " " "

$\Rightarrow b-a$ " "

$\Rightarrow bRa$

$\Rightarrow R$ is a Symmetric Relation

(iii) **Transitive:**

Let $a, b, c \in \mathbb{Z}$ and aRb and bRc

$\Downarrow$ $(a-b)$ is divisible by 5

$\Downarrow$ $(b-c)$ is divisible by 5

To prove: aRc

WKT, $a-b$ is divisible by 5

$b-c$ " "

Then $a-\cancel{b}+\cancel{b}-c$ is also divisible by 5

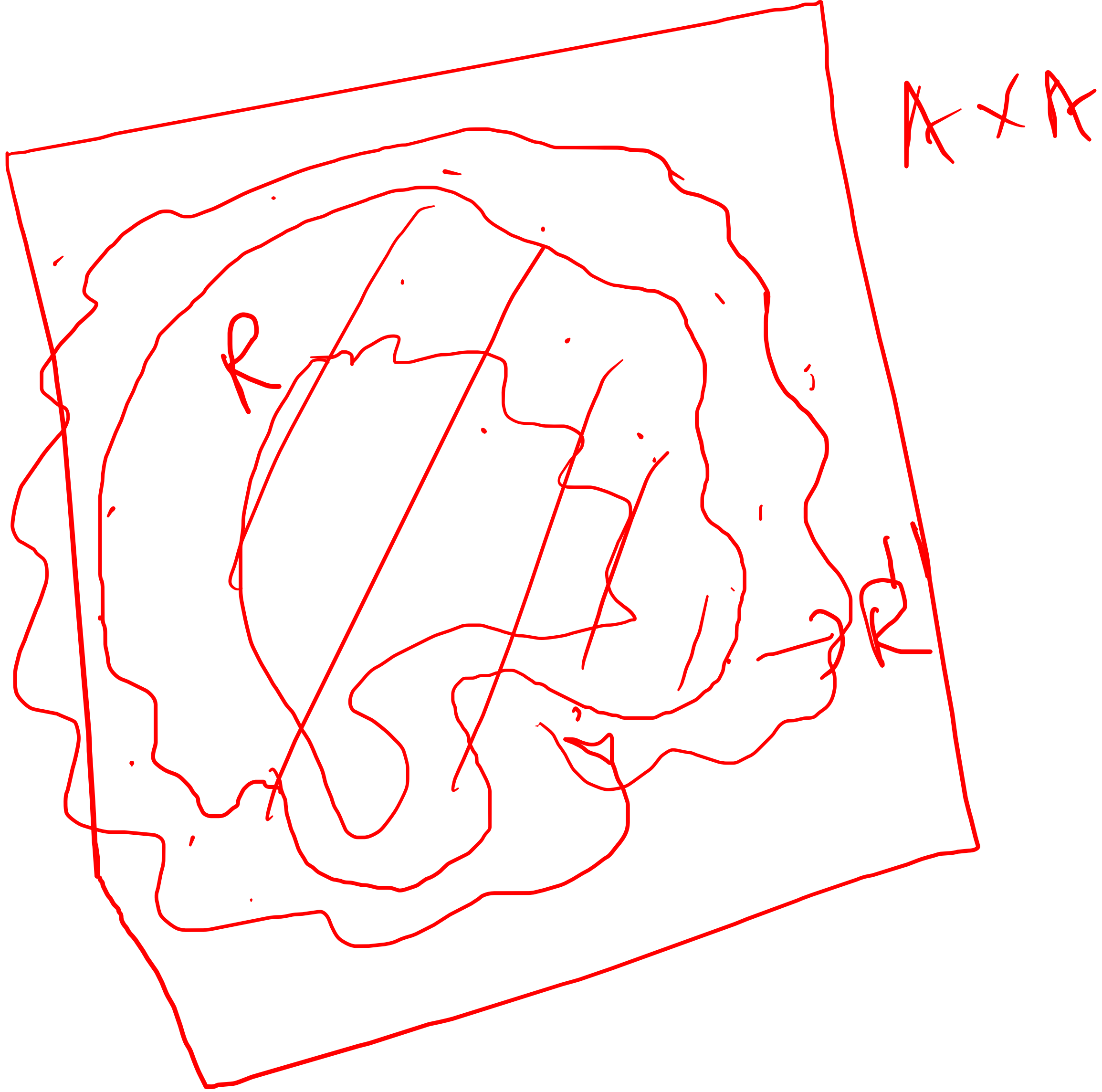
i.e., $a-c$ " "

$\Rightarrow aRc$

$10 + 15$

5 5

$\frac{25}{5} = \frac{10}{5} + \frac{15}{5}$


A × A
R
R'

R – is reflexive,
symmetry
and transitive

$\Rightarrow$ R is an equivalence relation.

$$R = \{(1,2), (2,1), (2,3), (3,4)\}$$

$$M_R = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 1 & 0 & 0 \\ 2 & 1 & 0 & 1 & 0 \\ 3 & 0 & 0 & 0 & 1 \\ 4 & 0 & 0 & 0 & 0 \end{array}$$

$$W_0 = M_R = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Look 1st column and identify the indices where '1' is there

$\{2\}$

Look at the 1st row and identify the indices of cols having '1'

$\{2\}$

$$W_1 = \begin{bmatrix} \quad \end{bmatrix}$$

The indices of 3^{rd} col where '1' are there.

$$\{1,2\}$$

The indices of 3^{rd} row where '1' are there

$$\{4\}$$

$\Rightarrow$ Include '1' in the places $\{1,2\} \times \{4\} = \{(1,4),(2,4)\}$

$$W_3 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 1 & 1 & 1 \\ 2 & 1 & 1 & 1 & 1 \\ 3 & 0 & 0 & 0 & 1 \\ 4 & 0 & 0 & 0 & 0 \end{array}$$

The indices of 4th col where 1's are there in W_3.

$\{1, 2, 3\}$

The indices of 4th row where 1's are there in W_3,

$\{ \}$

$\therefore$ There nothing to be included for W_4 newly.

$$W_4 = W_3 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\therefore$ the transitive Closure is,

$$R^+ = \{(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4), (3,4)\}$$

$$\begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

There include ~~other elements~~ 1 in place where ~~the~~ entries are $\{2\} \times \{2\} = \{(2,2)\}$

in ~~the~~ m

$$W_1 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 1 & 0 & 0 \\ 2 & 1 & 1 & 1 & 0 \\ 3 & 0 & 0 & 0 & 1 \\ 4 & 0 & 0 & 0 & 0 \end{array}$$

$$w_Q = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 1 & 1 & 0 \\ 2 & 1 & 1 & 1 & 0 \\ 3 & 0 & 0 & 0 & 1 \\ 4 & 0 & 0 & 0 & 0 \end{array}$$

The indices where the 2nd col has '1' are $\{1, 2\}$

The indices where the 2nd row has '1' are $\{1, 2, 3\}$

Include in the phrase mentioned by

$$\{1, 2\} \times \{1, 2, 3\} = \{(1,1), (1,2), (1,3), (2,1), (2,2), (2,3)\}$$