QUESTION BANK

III SEMESTER B.Tech(CSE)

DISCRETE MATHEMATICS (Unit II)

MA 1308

ORDERING OF PERMUTATIONS

1. Write the algorithm for reverse Lexicographical ordering to find a permutation next to a given permutation. Use the same to find the permutation next to the permutation 632541.

2. Write the algorithm for reverse Lexicographical ordering to find a permutation next to a given permutation. Use the same to find the permutation next to the permutation 7436521.

3. Write the algorithm for Lexicographical ordering to find a permutation next to a given permutation. Use it to find the permutation next to the permutation 632541.

4. Write the algorithm for Lexicographical ordering to find a permutation next to a given permutation. Use the same to find the permutation next to the permutation 7436521.

GENERATING FUNCTIONS

5. Find the sequence $\{a_n\}$ whose generating function is given by

   (i) $G(x) = \dfrac{5}{(1-x)^2} + \dfrac{3}{1+2x}$.

   (ii) $G(x) = \dfrac{x}{(1-x)^2} + \dfrac{x}{1-x}$.

   (iii) $G(x) = \dfrac{6}{(1+x)^2} + \dfrac{1}{1-x}$.

6. Using generating functions, solve the following recurrence relations:

   (i) $a_n - 9a_{n-1} + 20a_{n-2} = 0,\ a_0 = -3,\quad a_1 = -10.$

   (ii) $a_{n+2} - 2a_{n+1} + a_n = 2^n,\ a_0 = 2,\quad a_1 = 1.$

   (iii) $a_{n+2} - 5a_{n+1} + 6a_n = 2,\ a_0 = 1,\quad a_1 = 2.$

   (iv) $a_n - 7a_{n-1} + 10a_{n-2} = 0,\ a_0 = a_1 = 3.$

MATHEMATICAL LOGIC

7. Using truth table show that $P \to (Q \vee R) \rightleftarrows (P \to Q) \vee (P \to R)$ is a tautology.

8. Using truth table show that $(P \vee (Q \wedge R)) \wedge (\sim P \wedge (\sim Q \vee \sim R))$ is a contradiction.

9. Using truth table establish the following :

   (i) $(\sim P \wedge (\sim Q \wedge R)) \vee ((Q \wedge R) \vee (P \wedge R)) \Longleftrightarrow R$.

   (ii) $(P \to (Q \vee R)) \Longleftrightarrow ((P \to Q) \vee (P \to R))$.

   (iii) $((P \vee R) \to Q) \Longleftrightarrow ((P \to Q) \wedge (R \to Q))$.

10. Without using truth table establish the following :

    (i) $(\sim P \wedge (\sim Q \wedge R)) \vee ((Q \wedge R) \vee (P \wedge R)) \Longleftrightarrow R$.

    (ii) $(P \to (Q \vee R)) \Longleftrightarrow ((P \to Q) \vee (P \to R))$.

    (iii) $((P \vee R) \to Q) \Longleftrightarrow ((P \to Q) \wedge (R \to Q))$.

11. Without using truth table establish the following :

$$(\sim (P \wedge Q) \to (\sim P \vee (\sim P \vee Q))) \Longleftrightarrow (\sim P \vee Q).$$

    Hence, show that

$$((P \vee Q) \wedge (\sim P \wedge (\sim P \wedge Q))) \Longleftrightarrow (\sim P \wedge Q).$$

12. Without using truth table, show that

$$((P \vee Q) \wedge \sim (\sim P \wedge (\sim Q \vee \sim R))) \vee ((\sim P \wedge \sim Q) \vee (\sim P \wedge \sim R))$$

    is a tautology.

13. Using truth table, check the validity of the following arguments:

    (i) $H_1 : P \to Q, \quad H_2 : \ \sim Q \to R, \quad H_3 : R \ ; \quad C : \ P$

    (ii) $H_1 : \ \sim R, \quad H_2 : \ \sim Q \vee R, \quad H_3 : \ \sim (P \wedge \sim Q) \ ; \quad C : \ \sim P$

    where, H_1, H_2 , H_3 are the premises and C is the conclusion.

14. Without using truth table show that $\sim P$ follows logically from the premises $\sim (P \wedge \sim Q)$, $\sim Q \vee R$ and $\sim R$.

15. Without using truth table show that S follows logically from the premises $P \to Q$, $P \to R$, $\sim (Q \wedge R)$ and $P \vee S$.

16. Without using truth table show that $S \vee R$ follows logically from the premises $P \vee Q$, $P \to R$, $Q \to S$.

17. Without using truth table show that $\sim S$ follows logically from the premises $P \to Q$, $\sim Q \vee R$, $\sim R$ and $\sim (\sim P \wedge S)$.

18. Without using truth table show that $R \to S$ can be derived from the premises $P \to (Q \to S)$, $\sim R \vee P$ and Q.

19. "*If there was a ball game, then travelling was difficult. If they arrived on time, then travelling was not difficult. They arrived on time. Therefore, there was no ball game.*" Without using truth table, show that these statements constitute a valid argument.

20. Without using truth table, show that the following premises constitute a valid argument:

    "*If A works hard, then either B or C will win the game. If B wins the game, then A will not work hard. If D wins the game, then C will not win the same. Therefore, if A works hard, D will not win the game.*"

21. Without using truth table, show that the following premises are inconsistent:

    (i) If Jack misses many classes through illness, then he fails high school.

    (ii) If Jacks fails high school, then he is uneducated,

    (iii) If Jack reads a lot of books, then he is not uneducated.

    (iv) Jack misses many classes through illness and reads lots of books.

22. Without using truth table, show that the following premises are inconsistent:

    (i) If it is raining then John will stay at home.

    (ii) If it is raining then he will cook for himself.

    (iii) If John stays at home then he will not cook for himself.

    (iv) It is a raining.

23. Without using truth table, show that the premises are inconsistent:

    (i) If Raj and Sameer score half centuries, then we will win the game.

    (ii) If Punit comes after fall of first wicket, then Sameer will score half century, but we can not win the game.

    (iii) Raj scored half century and Punit came after the fall of first wicket.

24. Using indirect method of proof, show that

$$((R \rightarrow \sim Q) \wedge (R \vee S) \wedge (S \rightarrow \sim Q) \wedge (P \rightarrow Q)) \Longrightarrow \sim P.$$

25. Find the principal conjunctive normal form and principal disjunctive normal form of $\sim (\sim (P \vee Q) \vee (\sim P \wedge R))$.

26. Find the principal conjunctive normal form and principal disjunctive normal form of $(\sim P \rightarrow R) \wedge (Q \rightleftarrows P)$.

27. Show that the argument "All scientists are intelligent. Neuton is a scientist. Therefore Neuton is intelligent" is a valid argument.

28. Verify the validity of the following argument: "Every living thing is a plant or an animal. John's gold fish is alive and it is not a plant. All animals have hearts. Therefore John's gold fish has a heart."

29. Show that the conclusion $(\exists x)D(x)$ follows logically from the premises $(x)(A(x) \to (B(x) \vee C(x)))$, $(\exists x)(A(x) \wedge \sim B(x))$ and $(x)(C(x) \to D(x))$.

30. Show that the conclusion $(x)(F(x) \to \sim S(x))$ follows logically from the premises $(\exists x)(F(x) \wedge S(x)) \to (y)(M(y) \to W(y))$ and $(\exists y)(M(y) \wedge \sim W(y))$.