SMIT SIKKIM MANIPAL UNIVERSITY
SIKKIM MANIPAL INSTITUTE OF TECHNOLOGY

Engineering Mathematics III

Discrete Mathematics

Lecture 9

Lattice (Part 2)

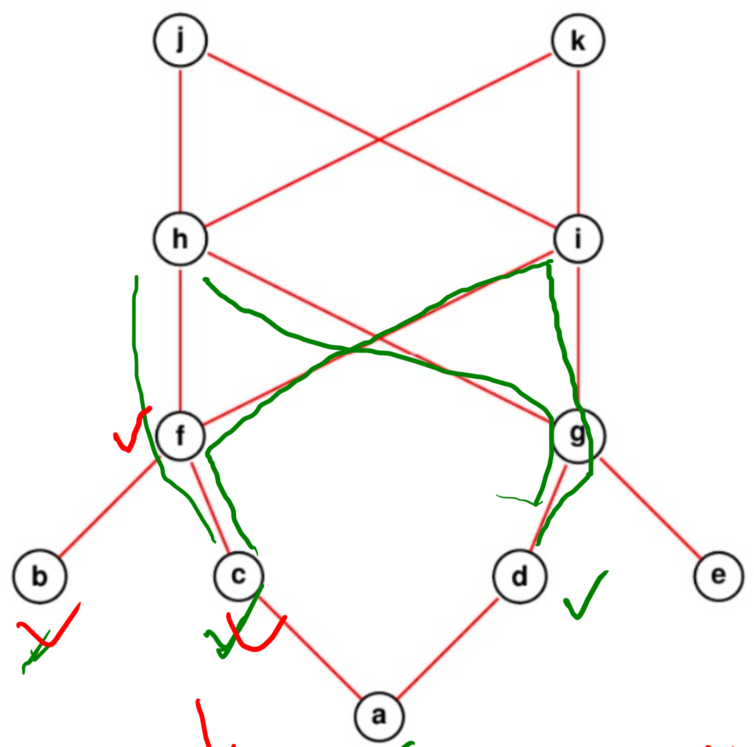
This course is taught to Computer Science Engineering students in SMIT, India during Jun-Dec, 2019.

Least upper bound

Least Upper Bound

Let $(A, \leq)$ be a poset. An element $c \in A$ is called as **least upper bound** of two elements a and b if

- c is an upper bound of a and b and
- if there exists an upper bound d of a and b then $c \leq d$.



a b

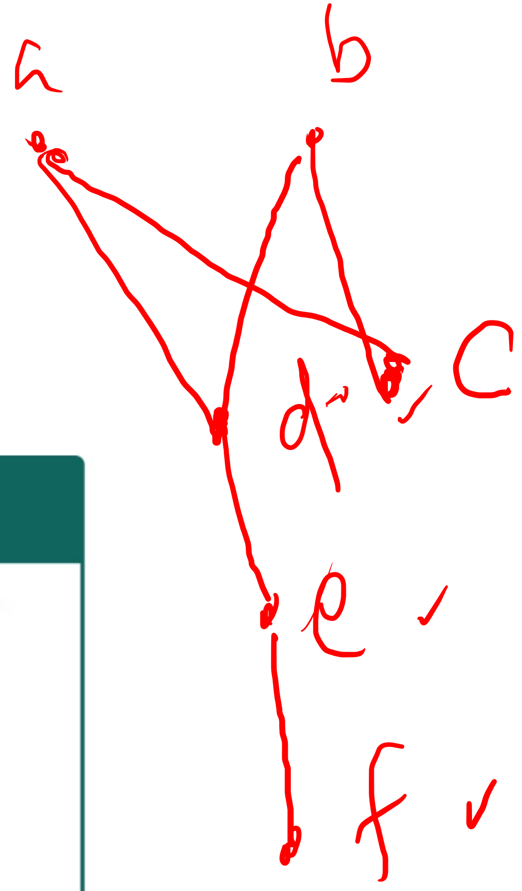
C

$a \leq b$

$b \leq c$

No y ∃ $y \leq f$ + y is an upper bound

Greatest Lower Bound

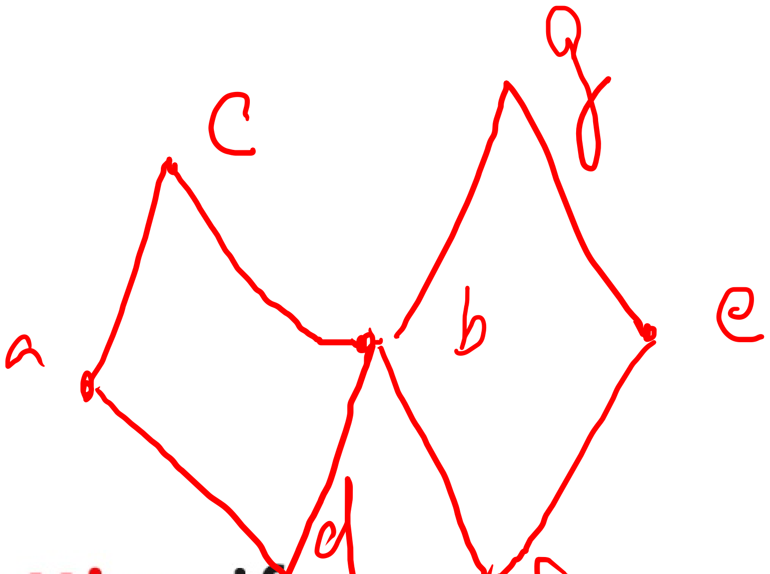


Greatest Lower Bound

An element $c \in A$ is said to be a **greatest lower bound** of a and b if

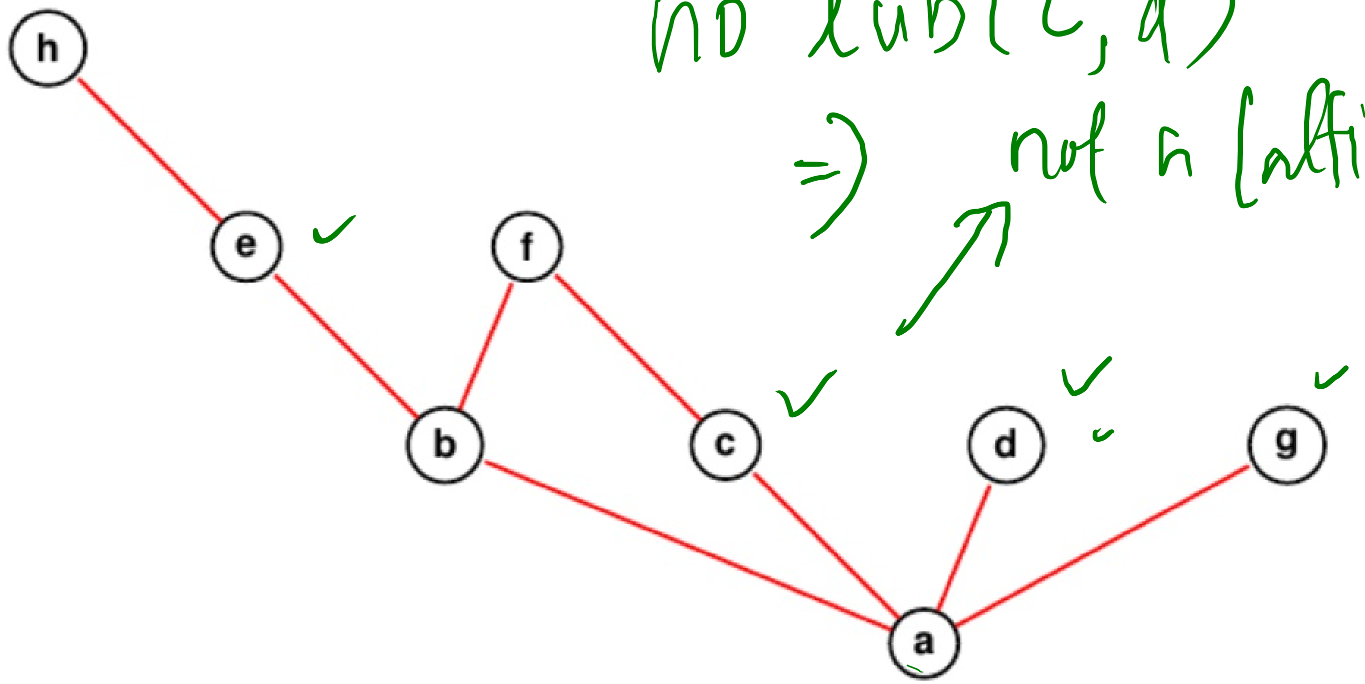
- c is an lower bound of a and b and
- if there exists a lower bound d of a and b then $d \leq c$.

Lattice



A partially ordered set is said to be a **lattice** if every two elements in the set have a (unique) least upper bound and a (unique) greatest lower bound.

Verify the following for lattice.



no lub(c, d)

⇒ not a lattices

chain = {a, b, e, h}

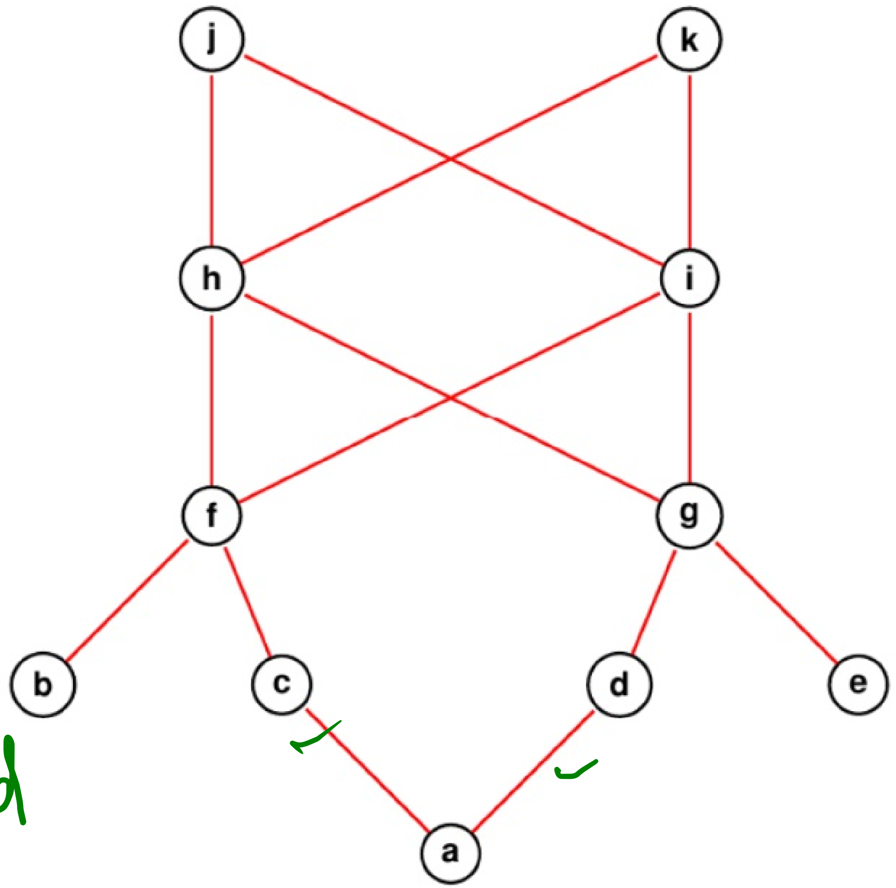
Antichain = {g, d, c, e}

What is antiChain?

In a subset of a poset if no two distinct ~~ele~~ elements are comparable then the subset is called as antichain.

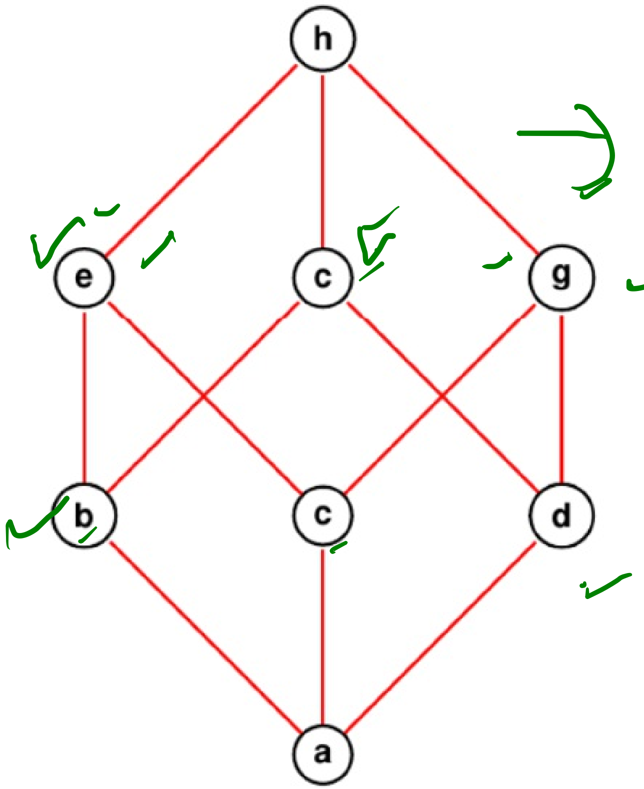
Another Reason:

b and c does not have a lower bound



(a)

c and d has no. lub
⇒ not a lattice

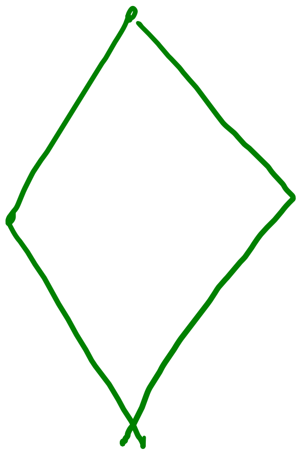


(b)

→ Lattices

Figure 4

<u>Remark:</u> Every finite chain is a lattice

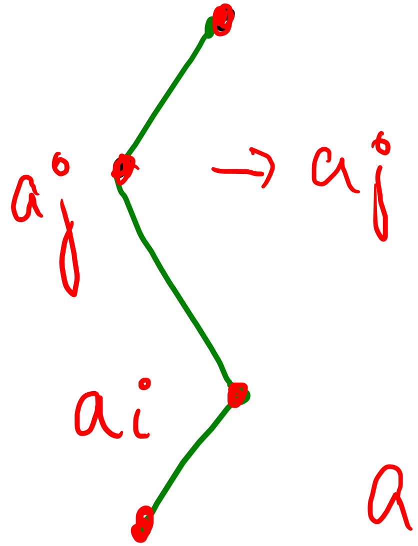


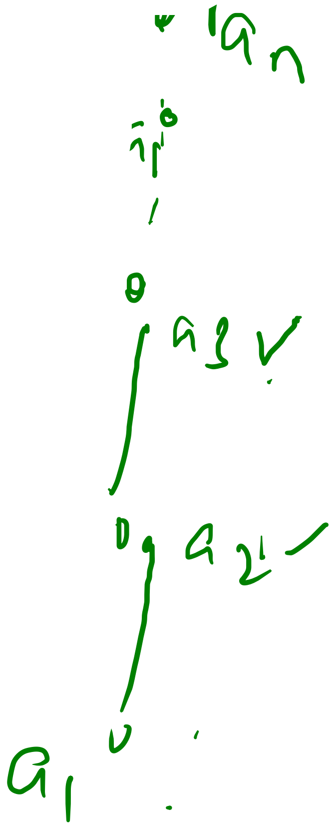
$a_1 \le a_2 \le \cdots \le a_n$

Let $i, j \in \{1, \ldots, n\}$, $i \ne i$

$lub(a_i, a_j) = \cancel{a_{j+1}}$ a_j

$glb(a_i, a_j) = \cancel{a_{i-1}}$ a_i





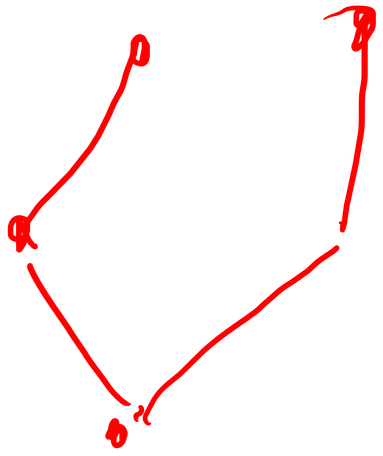
$lub(a_i, a_j) = a_j$, $a_i \le a_j$, $a_j \le a_j$

$$\left.\begin{array}{l} a_i \le a_j \\ a_j \le a_j \end{array}\right\} \Rightarrow lub(a_i, a_j) = a_j.$$

Idempotent Law: In a lattice $(L, \preccurlyeq)$, show that $a \wedge a = a$ if and only if $a \vee a = a$.

glb

lub



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