SIKKIM MANIPAL UNIVERSITY
SIKKIM MANIPAL INSTITUTE OF TECHNOLOGY

Engineering Mathematics III

Discrete Mathematics

Lecture 20

Theory of Inference & Predicate Calculus

This course is taught to Computer Science Engineering students in SMIT, India during Jun-Dec, 2019.

Premises & Conclusion

In logic, an argument requires a set of (at least) two declarative sentences (or "propositions") known as the **premises** or premisses along with another declarative sentence (or "proposition") known as the **conclusion**.

Validity of a formula

Let p and q be two statement formulas. We define ``q is a valid conclusion of the premise p" if and only if $p \to q$ is a tautology.

Rules of inference

Rule P:

A premise can be inserted at any point in the derivation

Rule T:

If the formula q is tautologically implied by any one or more of the previous formulas in a derivation, then q can be inserted in the derivation.

Rule CP:

Rule CP stands for rule of conditional proof. It is also known ad *deduction theorem*. It states that, if we can derive a formula q from p and a set of premises then we can derive $p \to q$ from the set of the premises alone.

Predicate Calculus

Predicate calculus.

predicate

Dog is an animal

Cat is an animal.

M: ~~is~~ an animal → predicate.

Dog is M ≡ M(Dog)

Cat is M ≡ M(Cat)

2 is an integer

3 is an integer

I : an integer → predicate

2 is an integer : $I(2)$

3 is an integere : $I(3)$

Quantifiers

There exists a student in the class who have scored 50/50.

↓

Quantifier.

↑

For all students in the class 1 mark will be added as bonus there exists : $\exists$

For all : $\forall$

belongs to ($\in$)

Let x be a flower,

~~Rose~~ x is red : R : ~~Red~~

$R(\cancel{rose}\ x)$

There is a flower which is red.

$$\boxed{\exists x,\ x \in R(x)}$$

for all x, if x is a dog then x is animal.

A : an animal

D : a dog

$\forall x$, if x is dog, $x \in A(x)$ ✓

$$\Rightarrow \boxed{\forall x, \; D(x) \to A(x)}$$

For all x, if x is a cat then x is an animal.

A : animal $\qquad$ $A(x)$: x is an animal

C : cat $\qquad$ $C(x)$: x is a cat

Symbolic form:

$$\forall x, \; C(x) \rightarrow A(x).$$

Every rose is red.

quantifier

variable

predicate,

x: rose,

R: Red,

$R(x)$: x is Red

$$\forall x,\ R(x)$$

All dogs are animals

x: dog

A: Animal,

$\forall x,\ A(x)$

1) For all x, if x is a rose then x is red.

$R(x)$: x is a rose

$R'(x)$: x is red

$$\boxed{\forall x,\ R(x) \Rightarrow R'(x)}$$

2) Some roses are Red.

$R(x)$: x is a Rose

$R'(x)$: x is red

$$\exists x,\ R(x) \rightarrow R'(x)$$

(3) All dogs are animals.

$A(x)$: x is animal

$D(x)$: x is a dog

$$\forall x,\ D(x) \rightarrow A(x)$$