SMIT SIKKIM MANIPAL UNIVERSITY
SIKKIM MANIPAL INSTITUTE OF TECHNOLOGY

Engineering Mathematics III

Discrete Mathematics

Lecture 23

Problems on Equivalence of Formulae

This course is taught to Computer Science Engineering students in SMIT, India during Jun-Dec, 2019.

1. Show that $(p \rightarrow q) \equiv (\neg p \vee q)$

Exercise.

p, q

↓ ↓

2 2 = 2^2

p q r

↓ ↓

2 × 2 × 2 = 2^3

$p_1, p_2, \ldots, p_n$

↓

2 × 2 × ... × 2 = 2^n

2. Show that $(P \wedge \neg P) \vee Q$ is equivalent to Q.

independent Component.

Exercise.

3. Show that $(p \rightarrow (q \rightarrow r)) \equiv (p \rightarrow (\neg q \vee r)) \equiv ((p \wedge q) \rightarrow r)$

p	q	r	$q \rightarrow r$	$p \rightarrow (q \rightarrow r)$
T	T	T		
T	T	F		
T	F	T		
T	F	F		
F	T	T		
F	T	F		
F	F	T		
F	F	F		

Exercise

$q \rightarrow r \equiv (\neg q \vee r)$

$p \rightarrow (q \rightarrow r)$

$\equiv (\neg p \vee (q \rightarrow r))$

$\equiv (\neg p \vee (\neg q \vee r))$

$\equiv$

$$p \to (q \to r) \equiv p \to (\neg q \vee r)$$

Soln:

$$p \to (q \to r) \equiv p \to (\neg q \vee r)$$

$$\equiv \neg(p \wedge q) \vee r$$

$$\equiv (p \wedge q) \to r$$

TP: $p \to (q \to r) \equiv (p \wedge q) \to r$

$$p \to (q \to r) \equiv p \to (\neg q \vee r)$$

$$\equiv \neg p \vee (\neg q \vee r)$$

$$\equiv (\neg p \vee \neg q) \vee r$$

Problem

Write an equivalent formula for $p \wedge (q \leftrightarrow r)$ which does not contains bi-conditional.

$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

$\equiv (\neg p \vee q) \wedge (\neg q \vee p)$

$\leftrightarrow \equiv \rightleftarrows$

$\longrightarrow$ CNF

$p \wedge (q \leftrightarrow r) \equiv p \wedge [(p \rightarrow r) \wedge (r \rightarrow q)]$

Problem

Write an equivalent formula for $p \wedge (q \leftrightarrow r)$ which contains neither bi-conditional nor conditionals.

Soln:

$$p \wedge (q \leftrightarrow r) \equiv p \wedge ((q \rightarrow r) \wedge (r \rightarrow q))$$

$$\equiv p \wedge ((\neg q \vee r) \wedge (\neg r \vee q))$$

$$\equiv p \wedge (\neg q \vee r) \wedge (\neg r \vee q)$$ (CNF

conjections

disjuction

Problem

Write an equivalent formula for $p \wedge (q \leftrightarrow r) \vee (r \leftrightarrow p)$ which ~~does not~~ (i) doesnot contains bi-conditional.

Soln: (ii) Contains neither conditionals nor bi-conditional.

(i) $p \wedge (q \leftrightarrow r) \vee (r \leftrightarrow p) \equiv p \wedge \big((q \to r) \wedge (r \to q)\big) \vee \big((r \to p) \wedge (p \to r)\big)$

(ii) (i) $\Rightarrow \equiv p \wedge \big((\neg q \vee r) \wedge (\neg r \vee q)\big) \vee \big((\neg r \vee p) \wedge (\neg p \vee r)\big)$