SMIT SIKKIM MANIPAL UNIVERSITY
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Engineering Mathematics III

Discrete Mathematics

Lecture 5

Partial Order relations, Covering, Partial Order set & Problems

This course is taught to Computer Science Engineering students in SMIT, India during Jun-Dec, 2019.

Anti Symmetric

A relation R is said to be anti-symmetric if for all $(a,b) \in R$

$a \neq b \quad aRb \Rightarrow b \not R a$ unless $a=b$.

$A = \mathbb{N}$ — R — $aRb \iff a|b$ (a divides b)

$a \overline{) \, b \,} \, x$

$\underline{}$

$\underline{0}$

Partial ordering Relation:

R - is said to be partial ordering relation,

if (i) R is Reflexive

(ii) R is antisymmetry

(iii) R is transitive

~~$\mathbb{Z} \setminus \{0\}$~~ $A = \mathbb{N}$ $aRb \Leftrightarrow a \mid b$.

Examine R ~~if~~ for partial ordering relation.

Soln:

(i) Reflexive: Let $a \in \mathbb{N}$

To prove: aRa i.e., $a \mid a$

WKT $a \mid a$ because $a = 1.a + 0$

$\Rightarrow$ R is reflexive

(ii) Anti-Symmetric

Let $a, b \in \mathbb{N}$, $a \neq b$.
and assume that aRb.

TP: $b \not R a$ i.e, b does not divide a.

WKT,
$aRb \Rightarrow$ a is smaller than b
$\Rightarrow$ b does not divide a
$\Rightarrow$ R is an anti-symmetric

(iii) Transitive :

Let $a, b, c \in \mathbb{N}$ and assume that aRb and bRc

TP: aRc i, $a|c \Rightarrow c = q \cdot a + 0$

Wki, $a|b \Rightarrow b = q_1 a + 0$

$b|c \Rightarrow c = q_2 b + 0$

$\Rightarrow c = q_2 b$

$= q_2(q_1 a)$

$= (q_1 q_2) a + 0$

$\Rightarrow a|c$

$\Rightarrow R$ - is ~~fan~~ transitive

Therefore

R - is a partial ordering relation

Let A be any set. Then consider the power set of A. $(\mathcal{P}(A))$

Define R on $\mathcal{P}(A)$ such that ARB iff $A \subseteq B$

Soln:

(i) Reflexive

Let $A \in \mathcal{P}(A)$

$\boxed{\text{Let } a \in A, \Rightarrow a \in A}$

$\Rightarrow A \subseteq A \Rightarrow ARA$

$\Rightarrow$ R is reflexive

(ii) Antisymmetric

Let $A, B \in \mathcal{P}(A)$ and $A \subseteq B$, $A \neq B$

TP: $B \nsubseteq A$

$A \subseteq B$ & $A \neq B \Rightarrow$

$\exists\, x \in B$ such that

$x \notin A$

$\Rightarrow B \nsubseteq A$

$\Rightarrow R$ is antisymmetric

(c) Transitive:

Let $A, B, C \in P(A)$, $A \subseteq B$, $B \subseteq C$

TP: $A \subseteq C$

$A \subseteq B \Rightarrow \forall a \in A, a \in B$

$B \subseteq C \Rightarrow \forall b \in B, b \in C$

$\Rightarrow$ Let $a \in A$, $\Rightarrow a \in C$

$\Rightarrow A \subseteq C$

$\Rightarrow R$ is ~~not~~ transitive

$\Rightarrow R$ is a partial ordering relation.

lower bound

$$A_1 \subseteq A_2 \subseteq A_3 \subseteq A_4 \subseteq \cdots \subseteq A_n$$

upper bound

bigger element

or

smallest element

$$B_1 \subseteq B_2 \subseteq B_3 \subseteq \cdots$$

Chain