SMIT SIKKIM MANIPAL UNIVERSITY
SIKKIM MANIPAL INSTITUTE OF TECHNOLOGY

Engineering Mathematics III

Discrete Mathematics

Lecture 3

Equivalence Relations

This course is taught to Computer Science Engineering students in SMIT, India during Jun-Dec, 2019.

Recall...

Cartesian Product

Let A and B be two sets. The **Cartesian Product** of A and B, denoted $A \times B$, is the set of all ordered pairs of the form (a, b) where $a \in A$ and $b \in B$.

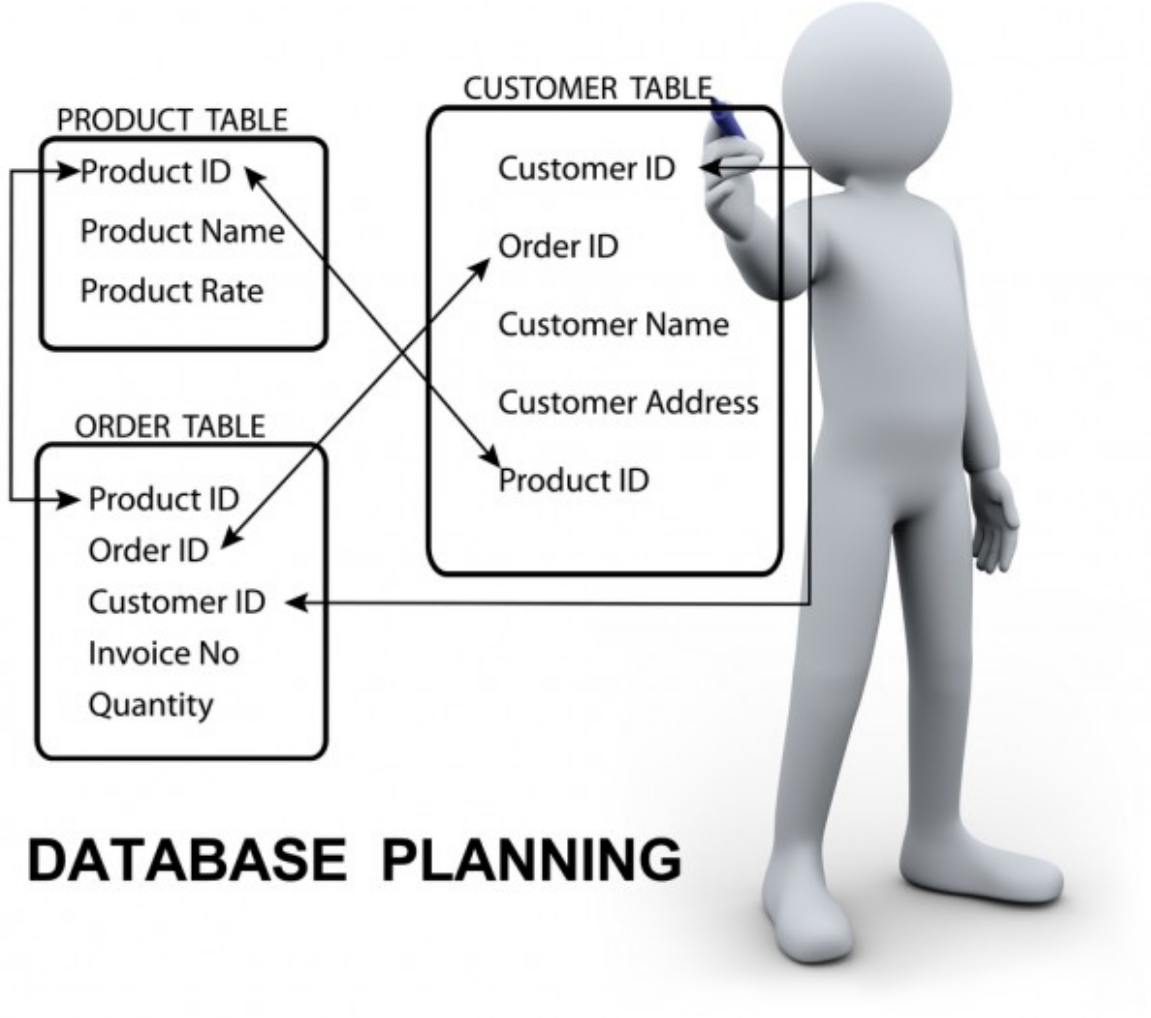
For example,

$$\{a, b\} \times \{1, 2, 3\} = \{(a, 1), (a, 2), (a, 3), (b, 1), ($$

Definition

A binary relation (also known as relation) from A to B is a subset of $A \times B$.

- So, relation is nothing but an intuitive formulation, so that one can say who is related with whom from the set A to the set B.
- So, if a in A is related to b in B, we say (a, b) is in the relation $\mathcal{R}$, and denoted it as $(a, b) \in R$.
- And similarly, when $(a, b) \in \mathcal{R}$, we say that a and b are related.



Picture credit BRITTANI SPONAUGLE

Representations of Relation

Example

Let us say, we have actor names in the set A and the movie names in the set B. Our relation is defined such that we say an actor from set A is related to a movie in set B, if he acted in that movie.

So if,

$$A = \{\text{Tony Stark}, \text{Steve Rogers}, \text{Bruce Banner}, \text{Peter Parker}\}$$

and

$$B = \{\text{Spider Man}, \text{Iron Man}, \text{Captain America}, \text{Avengers Infinity War}, \text{Avengers Endgame}\}$$

So, the relation

$$\mathcal{R} = \left\{ \begin{array}{l} (\text{Tony Stark}, \text{Iron Man}), (\text{Tony Stark}, \text{Avengers Infinity War}), (\text{Tony Stark}, \text{Avengers Endgame}), \\ (\text{Steve Rogers}, \text{Captain America}), (\text{Steve Rogers}, \text{Avengers Infinity War}), \\ (\text{Steve Rogers}, \text{Avengers Endgame}), \\ (\text{Bruce Banner}, \text{Avengers Infinity War}), (\text{Bruce Banner}, \text{Avengers Endgame}), \\ (\text{Peter Parker}, \text{Spider Man}), (\text{Peter Parker}, \text{Avengers Infinity War}), \\ (\text{Peter Parker}, \text{Avengers Endgame}) \end{array} \right\}$$

	Spider Man	Ironman	Captain America	Avengers Infinity war	Avengers Endgame
Tony Stark		✓		✓	✓
Steve Rogers			✓	✓	✓
Bruce Banner				✓	✓
Peter Parker	✓			✓	✓

Table 1: Tabular form for a relation

	Spiderman	Ironman	Captain America	Avengers Infinity war	Avengers Endgame
Tony Stark	0	1	0	1	1
Steve Rogers	0	0	1	1	1
Bruce Banner	0	0	0	1	1
Peter Parker	1	0	0	1	1

Figure 1: Matrix Form of Relation

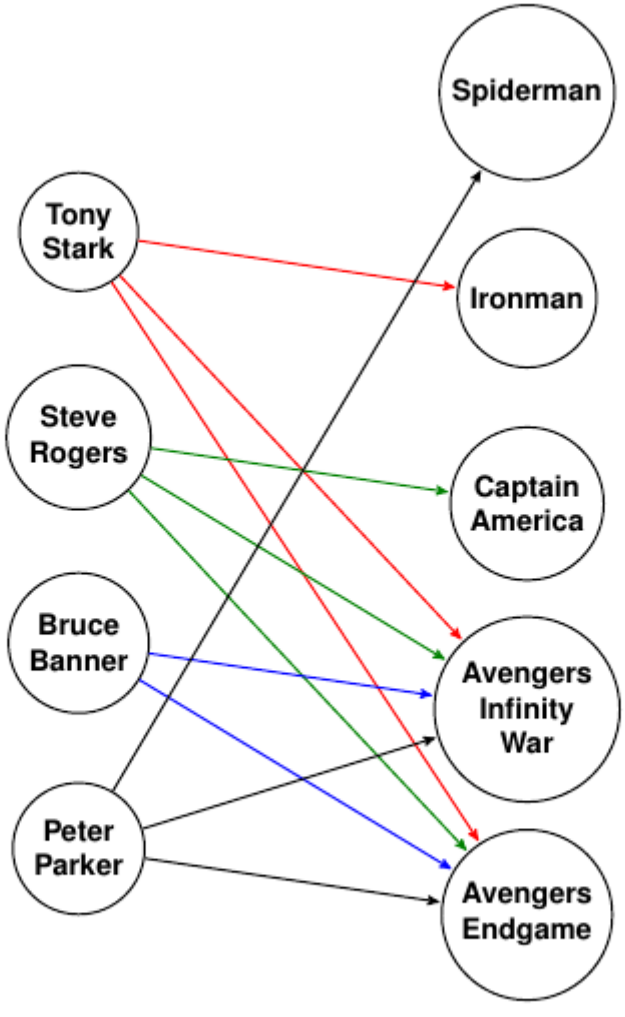


Figure 2: Graphical Representation of Relations

Reflexive Relation

Symmetric Relation

Anti-symmetric Relation

Transitive Relation

Equivalence Relation

Problems

Problem

Let $\mathbb{Z}$ be the set of all integers. Define $\mathcal{R}$ on $\mathbb{Z} \times \mathbb{Z}$ such that $a \mathcal{R} b \iff (a - b)$ is divisible by 5, $a, b \in \mathbb{Z}$. Show that $\mathcal{R}$ is an equivalence relation on $\mathbb{Z}$. Find the equivalence class of 3.

Problem

Let $\mathbb{Z}$ be the set of all integers. Define $\mathcal{R}$ on $\mathbb{Z} \times \mathbb{Z}$ such that $a \mathcal{R} b \iff (a + b)$ is even, $a, b \in \mathbb{Z}$. Show that $\mathcal{R}$ is an equivalence relation on $\mathbb{Z}$.

Problem

Let $\mathbb{N}$ be the set of all positive integers. Define $\mathcal{R}$ on $\mathbb{N} \times \mathbb{N}$ such that $a\mathcal{R}b \iff |a+b|+2$ is a prime, $a, b \in \mathbb{N}$. Examine whether $\mathcal{R}$ is an equivalence relation on $\mathcal{N}$.

Problem

Using Warshall's algorithm, find the transitive closure of the relation

$$\mathcal{R} = \{(1,2),(2,1),(2,3),(3,4)\}$$

on the set $A = \{1,2,3,4\}$.

Questions?

Thank you