SIKKIM MANIPAL UNIVERSITY
SIKKIM MANIPAL INSTITUTE OF TECHNOLOGY

Engineering Mathematics III

Discrete Mathematics

Lecture 15

Handshaking Lemma and related Problems

This course is taught to Computer Science Engineering students in SMIT, India during Jun-Dec, 2019.

Theorem 1 (Hand Shaking Lemma). *The sum of the degrees of the vertices of a graph G is twice the number of edges in G. Mathematically,*

$$\sum_{v \in V(G)} \deg(v) = 2 \times |E(G)|.$$

$\deg(b) = 2$

$\deg(a) = 2$

$\deg(c) = 3$

$\deg(e) = 3$

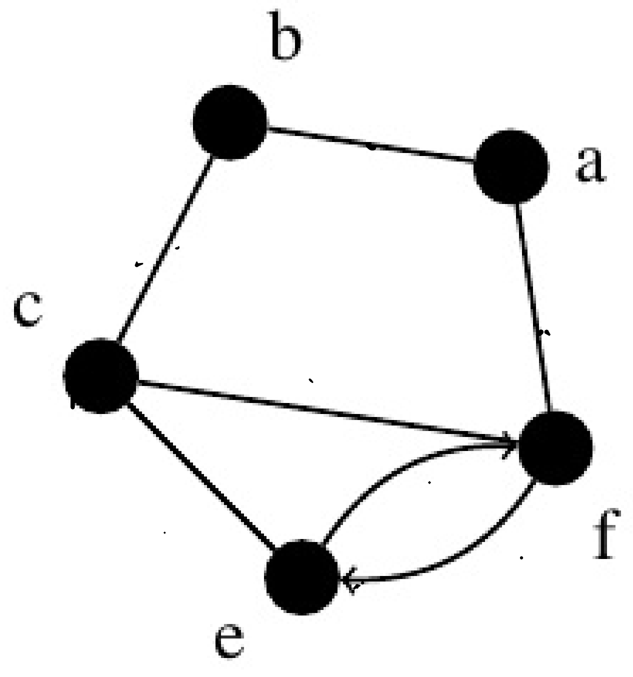
$\deg(f) = 4$

$$\sum_{v \in V(G)} \deg(v) = 2 + 2 + 3 + 3 + 4$$

$$= 14$$

$$= 2(7)$$

$$= 2\,|E(G)|$$



Proof: Let G be a graph and $V(G)$, $E(G)$ are the set of vertices and set of edges.

Each edge contributes deg - 2 to the

Sum of the all the degrees of the graph.

$\Rightarrow$ 2 (no. of edges) = Sum of all the deg of the graph

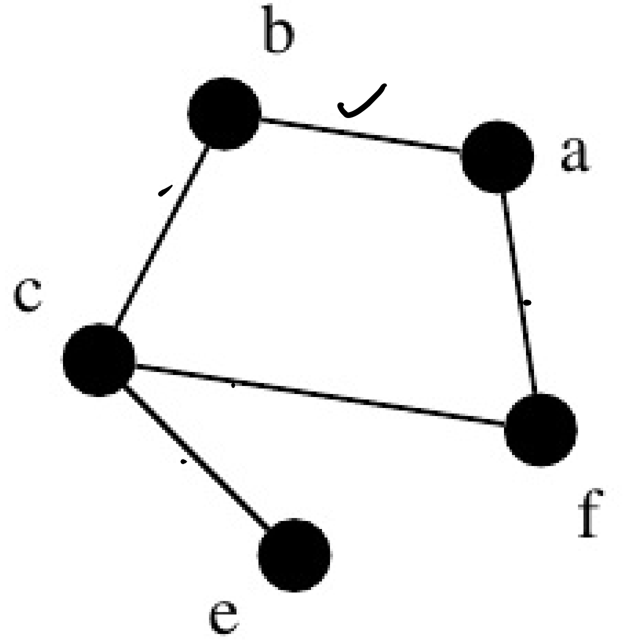
$$2\,|E(G)| = \sum_{v \in V(G)} \deg(v).$$

Hence proved

Verify Theorem 1 :

$$\sum_{v \in V(G)} \deg(v) = 2 + 2 + 3 + 1 + 2$$

$$= 10 \checkmark \longrightarrow (1)$$



No. of edges $= |E(G)| = 5$

$$2 \times |E(G)| = 2 \times 5 = 10 \checkmark \rightarrow (2)$$

From (1) and (2), $\sum_{v \in V(G)} \deg(v) = 2\,|E(G)|$.

Question 1.5. An undirected graph has 10 vertices labeled 1,2,...10 and 37 edges. Vertices 1,3,5,7, 9 have degree 8 and vertices 2,4,6,8 have degree 7. What is the degree of vertex 10? [CMI 2015]

By Hand Shaking Lemma,

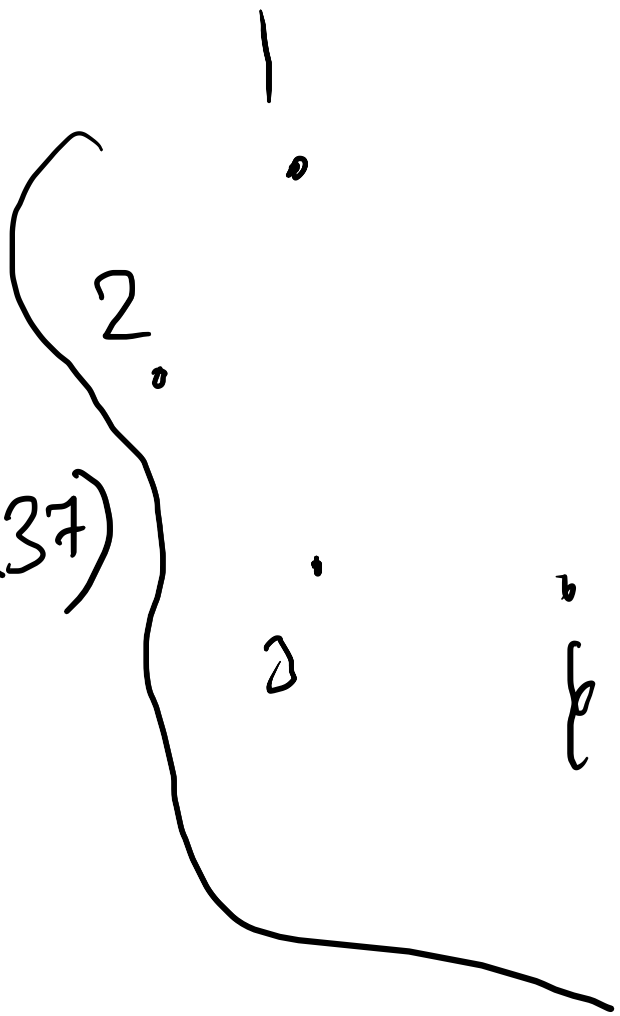
$$\sum_{v \in V(G)} \deg(v) = 2(\text{no of edges})$$

$\downarrow$

$$\Rightarrow 5 \times 8 + 4 \times 7 + \deg(\text{vertex } 10) = 2(37)$$

$$40 + 28 + \deg(v_{10}) = 74$$

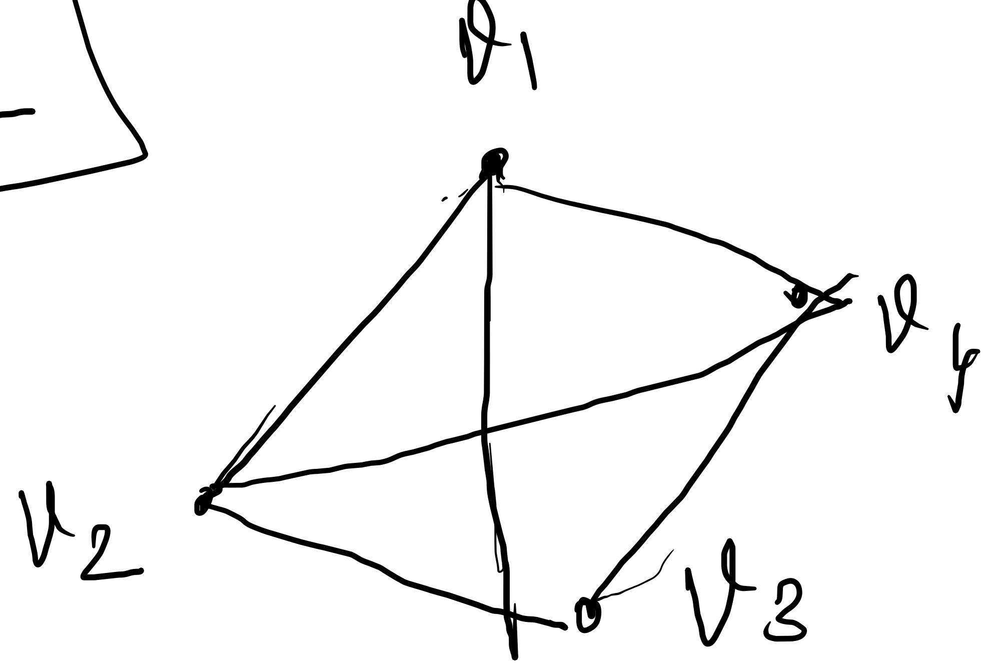
$$\boxed{\deg(v_{10}) = 6}$$



Question 1.8. What is the maximum number of edges in a $n-$node undirected graph without self loops? [GATE 2002]

without parallel edges

$\boxed{nC_2}$



$3+2$

$+1$

$=6$

Question 1.6. Prove that the number odd vertices in a graph is even.

Intuitive

Sum of deg } = even

Sum of ~~odd~~ even degrees } = even

Sum of odds deg } = k

Proof:

Let G be a graph and $V(G)$ be the vertex set.

Notes that

$$\sum_{v \in V(G)} \deg(v) = \text{sum of all degrees}$$

$$= \text{sum of even deg} + \text{sum of odd deg}$$

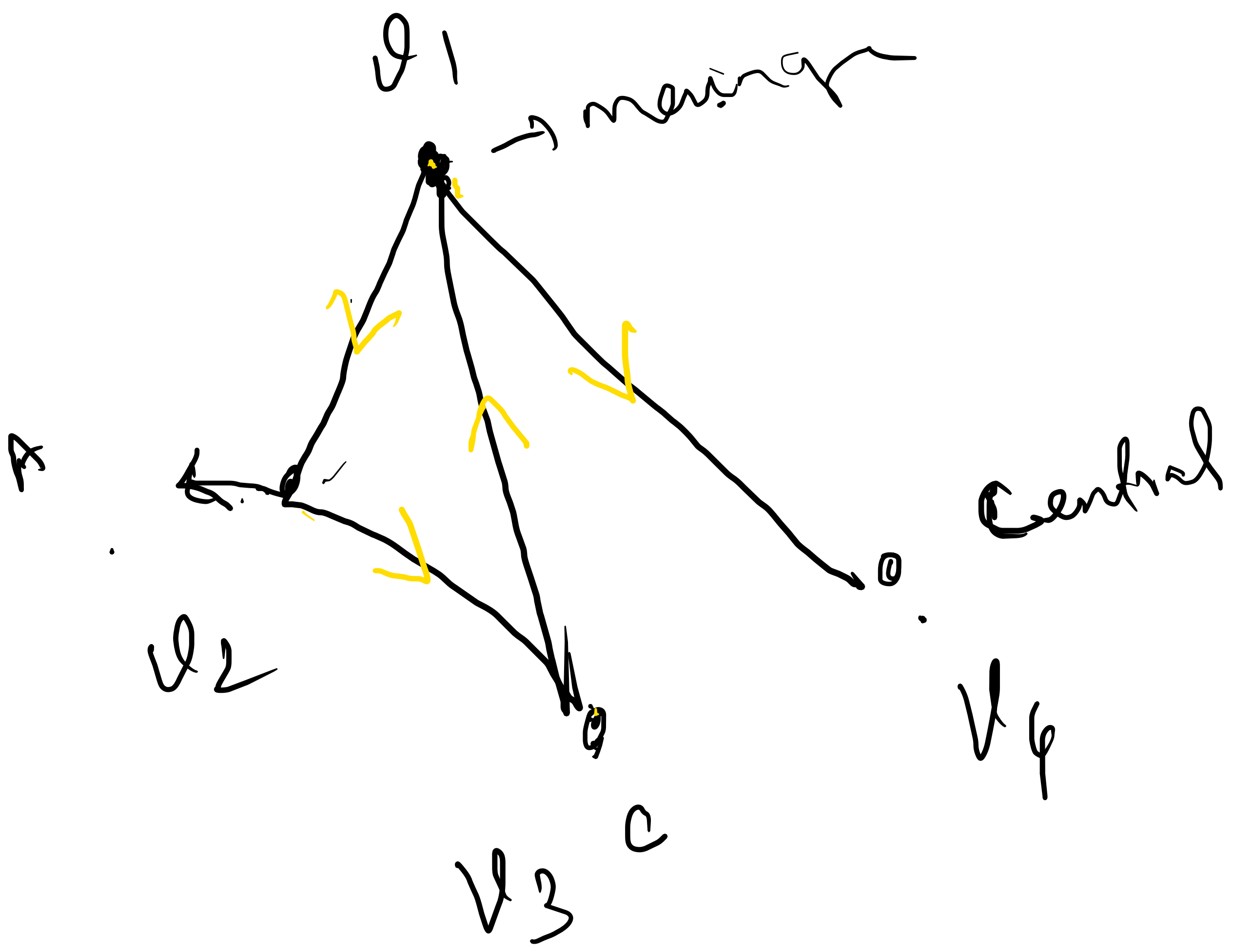
$\{\because$ By hand shaking lemma $\}$

$$2 \times \text{no. of edges} = \sum_{\substack{v \in V(G) \\ \deg(v) \text{ is even}}} \deg(v) + \sum_{\substack{v \in V(G) \\ \deg(v) \text{ is odd}}} \deg(v)$$

$$\text{even} = \text{even} + \sum_{\substack{v \in V(G) \\ \deg(v) \text{ is odd}}} \deg(v)$$

$$\boxed{\text{even} = \sum_{\substack{v \in V(G) \\ \deg(v) \text{ is odd}}} \deg(v)}$$

Hence proved.



Walk — closed / open walk

Circuit

trail

Cycle

path

$v_1 - v_2 - v_3 - v_1 - v_4 - v_1 - v_3 - v_2$

↳ walk

Every cycle is a walk