Engineering Mathematics III

Discrete Mathematics

Lecture 25

Normal Forms: DNF & CNF (Part 2)

This course is taught to Computer Science Engineering students in SMIT, India during Jun-Dec, 2019.

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Books Name:- Discrete Mathematical Structures with Applications to Computer Science

What is DNF?

Disjunctions of Conjunctions

For eg: $(p \wedge q) \vee (q \wedge r) \vee (\neg q \wedge r)$

What is CNF?

Conjunctions of Disjunctions

For eg: $(p \vee q) \wedge (q \vee r) \wedge (\neg q \vee r)$

1) $P \wedge (P \rightarrow Q) \equiv P \wedge (\neg P \vee Q)$ → CNF

$\equiv (P \wedge \neg P) \vee (P \wedge Q)$ → DNF

$(P \wedge \neg P) \equiv \bot$ ↓ $\qquad (P \wedge Q) \equiv \bot$ ↓

2) $\neg (P \vee Q) \rightleftarrows (P \wedge Q)$

$\equiv (\neg(P \vee Q) \rightarrow (P \wedge Q)) \wedge ((P \wedge Q) \rightarrow \neg(P \vee Q))$

$\equiv ((P \vee Q) \vee (P \wedge Q)) \wedge (\neg(P \wedge Q) \vee \neg(P \vee Q))$

$\equiv ((P \vee Q) \vee P) \wedge ((P \vee Q) \vee Q) \wedge ((\neg P \vee \neg Q) \vee (\neg P \wedge \neg Q))$

$$\equiv (P \vee Q \vee P) \wedge (P \vee Q \vee Q) \wedge (\neg P \vee \neg Q \neg P)$$

$$\wedge (\neg P \vee \neg Q \vee \neg Q)$$

Sum

product

This is the required CNF

$$\equiv (P \vee Q) \wedge (P \vee Q) \wedge (\neg P \vee \neg Q) \wedge (\neg P \vee \neg Q)$$

Min terms: Let p and q are the only two atomic statements in the statement formula

minterms ← $p \wedge q$, $\neg p \wedge q$, $p \wedge \neg q$, $\neg p \wedge \neg q$

p	q	$p \wedge q$	$p \vee q$	$\neg p \vee q$	$\neg p \wedge \neg q$
T	T	T	T	T	F
T	F	F	T	F	F
F	T	F	T	T	F
F	F	F	F	T	T

$(p \vee q) \wedge p$

$(p \wedge p) \vee (p \wedge q)$

$p \vee (p \wedge q)$

Max - terms:

Dual of min-terms are called as max-terms.

$P \vee Q$, $\neg P \vee Q$, $P \vee \neg Q$, $\neg P \vee \neg Q$.

What is principal Disjunctive Normal form (PDNF)

In a DNF, if

Then that DNF is a PDNF.

1) The Commutative term does not present (eg) $(P \wedge Q) = (Q \wedge P)$

2) All the ~~terms~~ Conjunctions should be min terms.

3) None of the formulae should have a variable and its negation.

PCNF: (Self study)

Problem: Obtain a PDNF of $\neg P \vee Q$.

$$\neg P \vee Q \equiv (\neg P \wedge T) \vee (Q \wedge T)$$

$$\equiv (\neg P \wedge (Q \vee \neg Q)) \vee (Q \wedge (P \vee \neg P))$$

$$\equiv (\neg P \wedge Q) \vee (\neg P \wedge \neg Q) \vee (P \wedge Q) \vee (\neg P \wedge Q)$$

This is the required PDNF.

prob: obtain the PDNF

$(P \wedge Q) \vee (\neg P \wedge R) \vee (Q \wedge R)$

Soln: exercise

Prob: Show that the following (without using truth table)

(a) $P \vee (P \wedge Q) \equiv P$

(b) $P \vee (\neg P \wedge Q) \equiv P \vee Q$.

Hint: Use PDNF (or) PCNF.