SMIT SIKKIM MANIPAL UNIVERSITY

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SIKKIM MANIPAL INSTITUTE OF TECHNOLOGY

Engineering Mathematics III

Discrete Mathematics

Lecture 17

Connected Graphs: Some more problems

This course is taught to Computer Science Engineering students in SMIT, India during Jun-Dec, 2019.

Ramsey Puzzle (~~Pe~~ friendship theorem)

In a group of 6 (or) more people, there is always a group 3 mutual friends (or) 3 mutual non-friends.

Proof: First we prove that a person in the group knows 3 people (or) does not know 3 people.

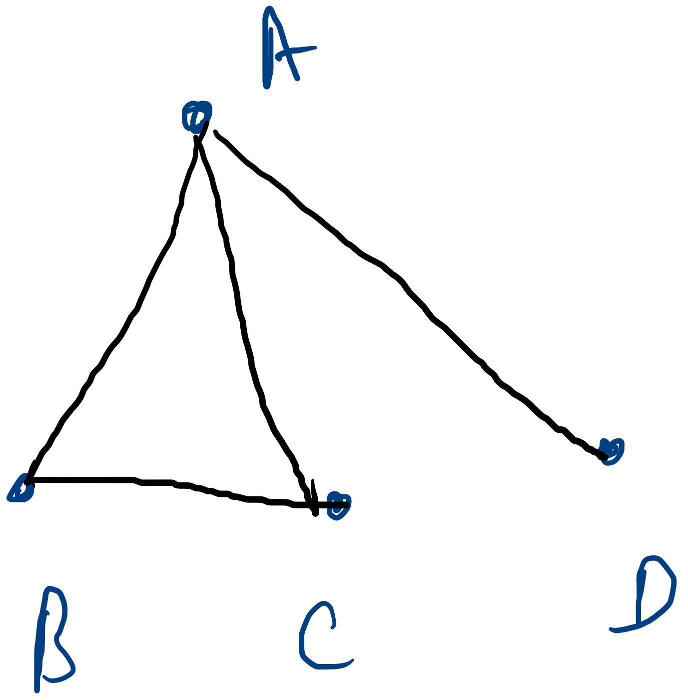
Let say A, B, C, D, E, F are the six people.

w.l.o.g., Consider A,

	no. of friends	no. nonfriend		
A	0	5	$\rightarrow \geq 3$	non-friends
	1	4	$\rightarrow \geq 3$	non-friends
	2	3	$\rightarrow \geq 3$	"
	3	2	$\rightarrow \geq 3$	friends
	4	1	$\rightarrow \geq 3$	friends
	5	0	$\rightarrow \geq 3$	friends

Case (i) A has 3 friends

w.l.o.g ~~Say~~ let B, C, D are friends



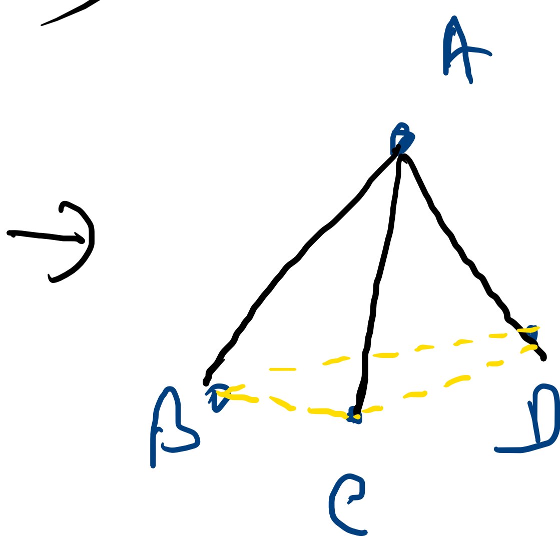
Let us connect the friend by lines and nonfriends by dotted lines.

Suppose B has a friend with any one of C & D, (Say C)

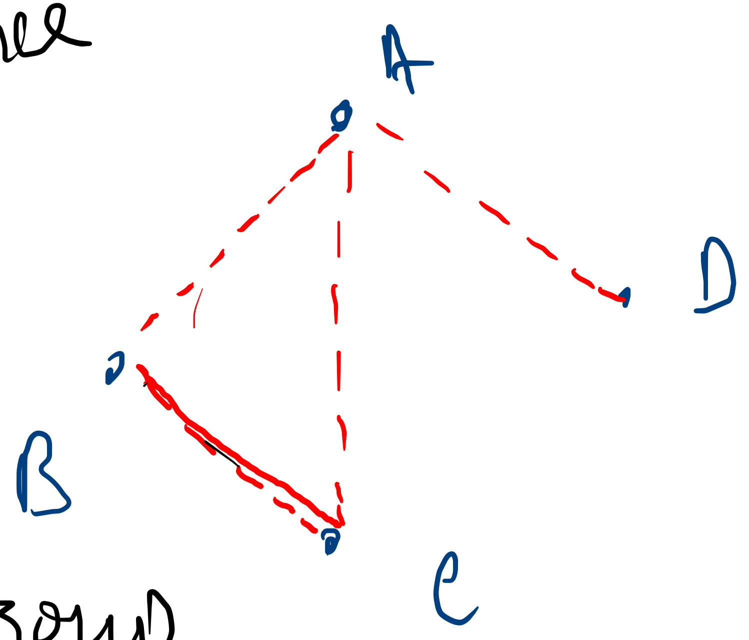
⇒ A, B, C know each other.

Suppose none of two of B, C, D are friends.

⇒ B, C, D forms ~~an~~ a non-acquaintance group



Case (ii) A has 3 non-acquaintance

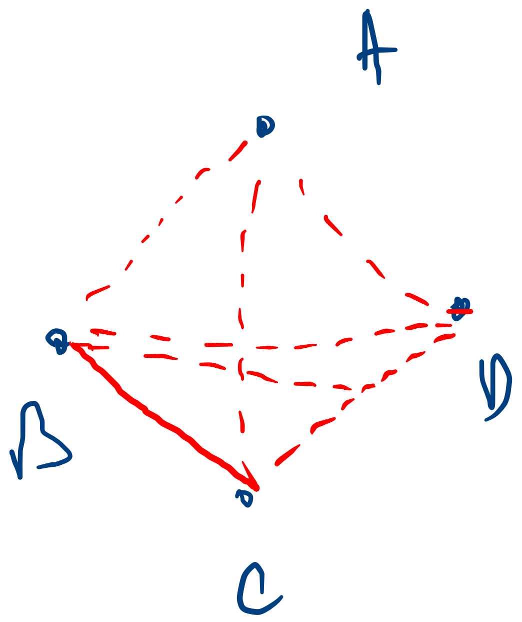


Suppose one of the B, C, D does not know each other, say B, C

$\Rightarrow$ A, B, C form an non-acquaintence group.

Suppose only one of the pair B, C, know each other

then B, A, D form non-acquaintance



Suppose all the pairs in B, C, D, know each other

B, C, D forms a mutual friends group.

2. Let G be a (p, q) graph whose vertices are of degree k or $k+1$. If G has t number of vertices of degree k, then show that $t = p(k+1) - 2q$.

Hint: $G(p, q)$ = A graph with p-vertices and q edges.

Use Handshaking lemma.

$$t(k) + (p-t)(k+1) = 2q$$

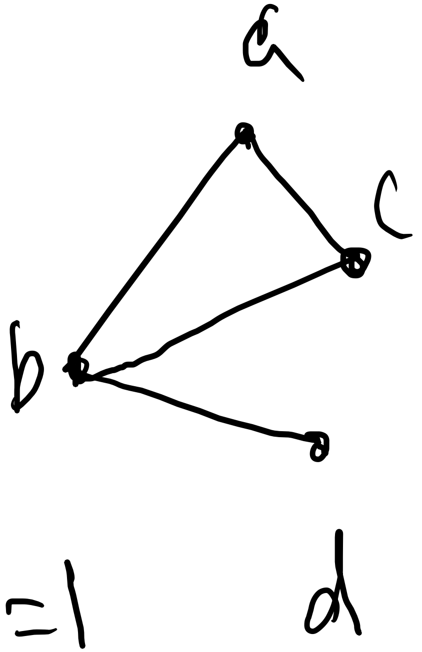
$$\Rightarrow \boxed{t = p(k+1) - 2q}$$

Minimum & maximum Degree:

$$\delta(G) = \text{minimum deg of vertices}$$

$$= \min\{\deg(v) : v \in V(G)\}$$

$$\Delta(G) = \max\{\deg(v) : v \in V(G)\}$$



$$\delta(G) = 1$$

$$\Delta(G) = 3$$

If G is a graph with p-vertices and $\delta(G) \geq \frac{p-1}{2}$, then show that G is connected. Give an example to show that the converse is not true.

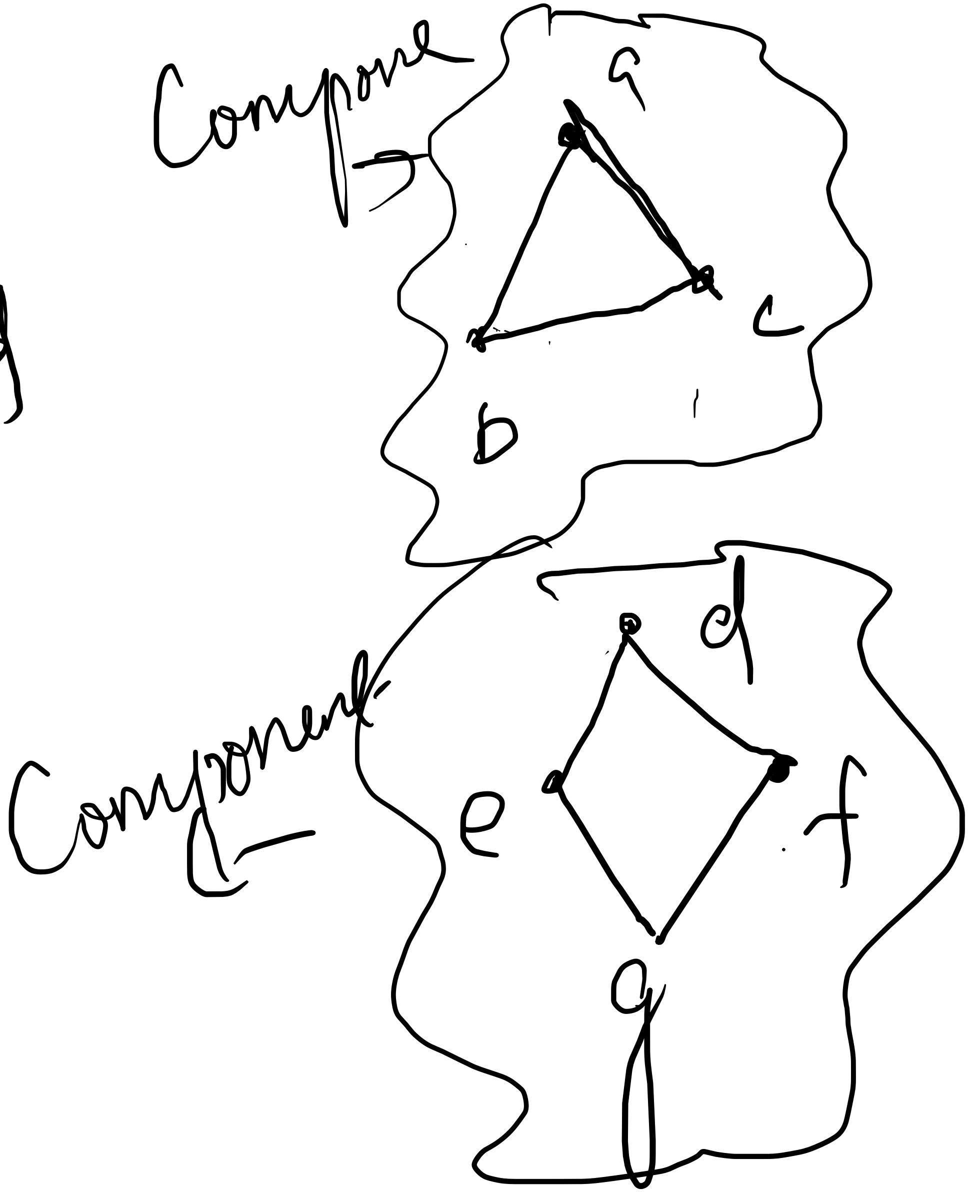
Proof:

Suppose G is not connected

There exist atleast two ~~d~~ connected components.

given $\delta(G) \geq \frac{p-1}{2}$

$\Rightarrow$ there is no vertex is each component has degree less than $\frac{p-1}{2}$.



That is all the vertices have deg greater than or equal to $\frac{p-1}{2}$

$\Rightarrow$ Each Component has $\frac{p-1}{2}+1$ vertices.

$\Rightarrow$ Two Component has $\left(\frac{p-1}{2}+1\right)+\left(\frac{p-1}{2}+1\right)$ vert-ices

$\Rightarrow$ " " has $p+1$ vertices.

this is a Contradiction to the graph has only p vertices.

$\Rightarrow$ Our assumption is wrong.

That G has to ~~be~~ be connected.

Hence the proof.

$\_\_\_\_\_\_ \times \_\_\_\_\_\_$

Prove the converse is not true;

Converse: If $G^{(p,q)}$ is connected then $\delta(G) \geq \frac{p-1}{2}$

Exercise.