

Problem set-2 for MA1201

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Lecture-5: Class Problems: (Non-exact differential equations)

1. Solve: $(x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0$, Ans: $\frac{x}{y} - 2 \log x + 3 \log y = c$.
2. Solve: $(x^2y^2 + xy + 1)ydx + (x^2y^2 - xy + 1)x dy = 0$, Ans: $xy + \log x - \frac{1}{xy} - \log y = c$.
3. Solve: $(1 + xy)ydx + (1 - xy)x dy = 0$.

Homework:

1. Solve: $(xy + 2x^2y^2)ydx + (xy - x^2y^2)x dy = 0$, Ans: $2 \log x - \frac{1}{xy} - \log y = c$.
2. Solve: $(x^3 + y^3)dx - xy^2 dy = 0$.

Lecture-6: Class Problems: (Non-exact differential equations)

1. Solve: $(y^4 + 2y)dx + (xy^3 + 2y^4 - 4x)dy = 0$, Ans: $xy + \frac{2x}{y^2} + y^2 = c$.
2. Solve: $(3xy - 2ay^2)dx + (x^2 - 2axy)dy = 0$, Ans: $x^3y - ax^2y^2 = c$.
3. Solve: $(3x^2y^4 + 2xy)dx + (2x^3y^3 - x^2)dy = 0$.

Homework:

1. Find an integrating factor and solve : $(x^2 + y^2 + 1)dx + x(x - 2y)dy = 0$,
Ans: $x - \frac{y^2}{x} - \frac{1}{x} + y = c$.

Lecture-7: Class Problems: (Leibnitz's linear differential equations)

1. Solve: $\frac{dy}{dx} + y \cot x = 4x \operatorname{cosec} x$, if $y = 0$ when $x = \frac{\pi}{2}$. Ans: $y = \left(2x^2 - \frac{\pi^2}{2}\right) \operatorname{cosec} x$.
2. Solve: $\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right) \frac{dx}{dy} = 1$, Ans: $ye^{2\sqrt{x}} = 2\sqrt{x} + c$.
3. Solve $(x + 2y^3)dy = ydx$, Ans: $x = y(y^2 + c)$.

Homework:

1. Solve: $(1 + x) \frac{dy}{dx} + 3y = \frac{1+x+x^2}{1+x}$. Ans: $y = \frac{1}{(1+x)^3} \left\{x + x^2 + \frac{2}{3}x^3 + \frac{x^4}{4} + c\right\}$.
2. Solve: $\frac{dy}{dx} + y \tan x = \sec x$. Ans: $y = \frac{1}{\sec x} \{\tan x + c\}$.

Lecture-8: Class Problems: (Bernoulli's Equations)

1. Solve: $\frac{dy}{dx} + y \tan x = y^3 \sec x$, Ans: $\frac{\cos^2(x)}{y^2} = -2 \sin(x) + c$.
2. Solve: $\frac{dz}{dx} + \frac{z}{x} \log z = \frac{z}{x} (\log z)^2$, Ans: $(x \log z)^{-1} = x^{-1} + c$.
3. Solve: $\frac{dy}{dx} + x \sin 2y = x^3 \cos^2 y$, Ans: $\tan y = \frac{1}{2}(x^2 - 1) + ce^{-x^2}$.

Homework:

1. Solve: $\frac{dy}{dx} - \frac{\tan y}{1+x} = (1+x)e^x \sec y$. Ans: $\frac{\sin y}{1+x} = e^x + c$.
2. Solve: $2 \frac{dy}{dx} = (y^3 \tan x + y \sec x)$. Ans: $-\frac{1}{y^2} = \frac{1}{\sec x + \tan x} (\sec x \tan x - x + c)$.

Problems for Remedial Class:

1. Solve: $x dy - y dx + x^2 dy = 0$.
2. Solve: $x \frac{dy}{dx} + 2y = x^2 \log x$. Ans: $y = \frac{1}{x^2} \left\{ \log x \frac{x^4}{4} - \frac{x^4}{16} + c \right\}$.
3. Solve: $x \frac{dy}{dx} + y \log y = x^3 y (\log y)^2$. Ans: $\frac{1}{\log y} = -\frac{x^3}{2} + cx$.

Note: Here c is an arbitrary constant.