Problem set-5 for MA1201

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Lecture-17: Class Problems: (Laplace Transform)

1. Using linear property of Laplace transform, find the Laplace transform of following functions

   i) $f(t) = \sin 2t \cos 3t$.　　　Ans:　$\dfrac{2s^2-10}{(s^2+25)(s^2+1)}$.

   ii) $f(t) = \sin^2 2t$.　　　　　Ans:　$\dfrac{1}{2}\left(\dfrac{1}{s} - \dfrac{s}{s^2+16}\right)$.

   iii) $f(t) = e^{2t} + 4t^3 - 2\sin 3t + 3\cos 3t$.

   iv) $f(t) = (\sin t - \cos t)^2$.

Homework:

1. Using linear property of Laplace transform, find the Laplace transform of following functions:

   i) $f(t) = \cos^2 3t$,　$* ii) f(t) = \sin^3 2t$.

Lecture-18: Class Problems: (Laplace Transform)

1. Using first shifting property, find the Laplace transform of following functions

   i) $f(t) = e^{-3t}(2\cos 5t - 3\sin 5t)$　　　　　　　Ans:　$\dfrac{2s-9}{(s+3)^2 +5^2}$.

   ii) $f(t) = e^{3t}\cos^2 t$　　　　　　　　　　　　Ans:　$\dfrac{1}{2}\left(\dfrac{1}{s-3} + \dfrac{s-3}{(s-3)^2+4}\right)$.

   iii) $f(t) = e^{4t}\sin 2t \cos t$　　　　Ans:　$\dfrac{1}{2}\left(\dfrac{3}{(s-4)^2+3^2} + \dfrac{1}{(s-4)^2+1}\right)$.

2. If $L\left\{\dfrac{\sin t}{t}\right\} = \tan^{-1}\left(\dfrac{1}{s}\right)$, find $L\left\{\dfrac{\sin at}{t}\right\}$.　　Ans: $\tan^{-1}\left(\dfrac{a}{s}\right)$ (Using Change of scale prop)

3. Using transform of integral, find $L\left\{\int_0^t e^{3t}\cos 2t \, dt\right\}$.

Homework:

1. Using first shifting property, find the Laplace transform of following functions:

   i) $f(t) = e^{3t}(4\cos 2t + 3\sin 3t)$

   ii) $f(t) = e^{-3t}\sin^2 3t$

   *iii) $f(t) = e^{3t}\sin 3t \cos t$:

Lecture-19: Class Problems: (Laplace Transform)

1. Find $L\{t\cos at\}$,　**2.** Find $L\{t^2\sin at\}$,　**3.** Find $L\{t^3 e^{-3t}\}$,　**4.** Find $L\left\{\dfrac{\cos at - \cos bt}{t}\right\}$.

5. Find $L\{t\sin^4 t\}$.

Homework:

1. Find $L\{t e^{-t}\sin 3t\}$, **2.** Find $L\{t^2\cos^2 t\}$, **3.** Find $L\left\{\dfrac{1-e^t}{t}\right\}$, *4. Find $L\left\{\dfrac{1-\cos t}{t^2}\right\}$.

Lecture-20: Class Problems: (Laplace transforms)

1. Find $L\left\{\int_0^t \dfrac{\sin t}{t} dt\right\}$　　　　　　　　　Ans: $\dfrac{1}{s}\left(\dfrac{\pi}{2} - \tan^{-1} s\right)$

2. Find $\left\{\int_0^\infty t e^{-2t}\sin t \, dt\right\}$　　　　　　　Ans: $\dfrac{4}{25}$

3. Show that $\left\{\int_0^\infty \dfrac{e^{-t}-e^{-3t}}{t} dt\right\} = \log 3$.

4. Find $L\left\{\int_0^t \dfrac{e^t \sin t}{t} dt\right\}$

Homework:

1. Find $\left\{\int_0^\infty \dfrac{\cos at - \cos bt}{t} dt\right\}$

Problems for Remedial Class:

1. Find $L\left\{\dfrac{\cos 2t - \cos 3t}{t}\right\}$, **2.** Find $L\{t^2 e^{-2t}\cos t\}$, **3.** Find $L\left\{\dfrac{e^{-t}\sin t}{t}\right\}$, **4.** Find $L\{t e^{-t}\cos ht\}$.

Note:　*denotes challenging problem.