Problem set-6 for MA1201

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Lecture-21: Class Problems: (Inverse Laplace Transform)

1. Find $L^{-1}\left\{\frac{s^2-3s+4}{s^3}\right\}$, **2.** Find $L^{-1}\left\{\frac{s+2}{s^2-4s+13}\right\}$, **3.** Find $L^{-1}\left\{\frac{5s+3}{(s-1)(s^2+2s+5)}\right\}$, (using partial fraction)
4. Find $L^{-1}\left\{\frac{1}{s(s^2+a^2)}\right\}$, **5.** Find $L^{-1}\left\{log\left(\frac{s+a}{s+b}\right)\right\}$, **6.** Find $L^{-1}\left\{cot^{-1}\left(\frac{s}{2}\right)\right\}$.

Homework:

1. Find $L^{-1}\left\{\frac{3s+7}{s^2-2s-3}\right\}$, 2. Find $L^{-1}\left\{\frac{s^2+5s-2}{s(s-2)(s+3)}\right\}$, *3. Find $L^{-1}\left\{tan^{-1}\left(\frac{2}{s^2}\right)\right\}$.

Lecture-22: Class Problems: (Inverse Laplace Transform and unit step function)

1. Using convolution theorem find $L^{-1}\left\{\frac{s}{(s^2+a^2)^2}\right\}$.
2. Using convolution theorem find $L^{-1}\left\{\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right\}$.
3. Using unit step function find the Laplace transform of $f(t)=\begin{cases}t-1, & 1<t<2\\ 3-t, & 2<t<3.\\ 0, & otherwise\end{cases}$

Homework:

1. Using convolution theorem find $L^{-1}\left\{\frac{1}{s^2(s^2+a^2)}\right\}$.
2. Using convolution theorem find $L^{-1}\left\{\frac{2}{(s+1)(s^2+4)}\right\}$.
3. *Using unit step function find the Laplace transform of $f(t)=\begin{cases}t^2, & 0<t<2\\ t-1, & 2<t<3.\\ 7, & t>3\end{cases}$

Lecture-23: Class Problems: (Unit step function and periodic function)

1. Find the inverse Laplace transform of $\frac{s\,e^{-\frac{s}{2}}+\pi\,e^{-s}}{s^2+\pi^2}$.
2. Find the Laplace transform of the following periodic function with period $\frac{2\pi}{\omega}$:

$$f(t)=\begin{cases}sin\omega t, & 0<t<\frac{\pi}{\omega}\\ 0, & \frac{\pi}{\omega}<t<\frac{2\pi}{\omega}\end{cases}.$$

3. Using the Laplace transform solve: $\frac{d^2y}{dt^2}+\omega^2 y=0$, given $y(0)=a, y'(0)=b$.

Homework:

1. Find the inverse Laplace transform of $\frac{e^{-bs}}{s^2(s+a)}$, $b>0$.
2. Find the Laplace transform of the following periodic function with period $2c$:

$$f(t)=\begin{cases}t, & 0<t<c\\ 2c-t, & c<t<2c\end{cases}.$$

3. Using the Laplace transform solve: $\frac{d^2y}{dt^2}+4\frac{dy}{dt}+3y=0$, given $y(0)=3, y'(0)=1$.

Lecture-24: Class Problems: (Laplace transforms)

1. Using the Laplace transform solve: $\frac{d^2y}{dt^2}+\frac{dy}{dt}=6\,cos2t$, given $y(0)=3, y'(0)=1$.
2. In an electric circuit with EMF $E(t)$, resistance R and inductance L, the current i builds up at the rate given by $L\frac{di}{dt}+Ri=E(t)$. If the switch is connected at $t=0$ and disconnected at $t=a$, then find the current at any instant.

Homework:

1. Using the Laplace transform solve: $\frac{d^2y}{dt^2}+4\frac{dy}{dt}+4y=4e^{-2t}$, given $y(0)=-1, y'(0)=4$.
2. *A simple electric circuit contains a resistance of 10 ohms and inductance of 4 henries in series with an induced EMF of $100\,sin200t$ volts. If the current $i=0$ when $t=0$, then find the current when $t=0.01$.

Problems for Remedial Class:

1. Find $L^{-1}\left\{\frac{1}{2}\,log\left(\frac{s^2+a^2}{s^2+b^2}\right)\right\}$,
2. Find the Laplace transform of $f(t)=e^{-t}\{1-u(t-2)\}$, where $u(t-2)$ is a unit step function.
3. *Using the Laplace transform solve: $\frac{d^2y}{dt^2}-6\frac{dy}{dt}+9y=6t^2e^{3t}$, given $y(0)=0, y'(0)=0$.

Note: *denotes challenging problem.