

Problem set-1 for MA1201

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Lecture-1: Class Problems:

1. Definitions: order, degree and solution of a ordinary differential equation. Examples.
2. Find the differential equations for
 - i) $y = A e^x + B e^{-x}$, ii) $y = A \log_e x + B$, iii) $y = A \cos x + B \sin x$,
iv) $y = (A \cos x + B \sin x)e^x$, where A and B are arbitrary constants.
3. Find the differential equation of all straight lines passing through the origin.
4. Find the differential equation of a system of concentric circles having the centre at the origin.

Homework:

1. Find the differential equations for
 - i) $y = A \sin x + B \cos x + x \sin x$, ii) $xy = A$, where A and B are arbitrary constants.
2. Find the differential equation of a system of all the circles touching x-axis at the origin.
3. Find the differential equation of all circles of radius 4 with centre at (A, B) , where A and B are arbitrary constants.

Lecture -2: Class Problems: (Separation of variables method):

1. Solve $(1 + y^2)dx + (1 + x^2)dy = 0$.
2. Solve $x(1 + y^2)dx + y(1 + x^2)dy = 0$.
3. Solve $y dx + (1 + x^2) \tan^{-1} x dy = 0$.
4. Solve $e^{x-y} dx + e^{y-x} dy = 0$.

(Homogeneous Differential Equations)

5. Solve $(x^2 - y^2)dx = xy dy$
6. Solve $x dy - y dx = \sqrt{x^2 + y^2} dx$, Ans: $\sinh^{-1} \left(\frac{y}{x} \right) = \log|x| + \log c$.

Homework:

1. Solve $\sec^2 x \tan y dx + \sec^2 y \tan x dy = 0$.
2. Solve $\log \left(\frac{dy}{dx} \right) = ax + by$.
3. Solve $\frac{dy}{dx} = \frac{y(y+x)}{x(y-x)}$.
4. Solve $\frac{dy}{dx} = \frac{y}{x} + \tan \left(\frac{y}{x} \right)$, Ans: $\sin \left(\frac{y}{x} \right) = cx$, where c is an arbitrary const.

Lecture -3: Class Problems:

(Non-homogeneous Differential Equations)

1. Solve $(2x + y - 3)dy = (x + 2y - 3)dx$, Ans: $\log \left| \frac{X+Y}{X-Y} \right| - \frac{1}{2} \log \left| \frac{X^2-Y^2}{X^2} \right| = \log|cX|$, where $X = x - 1, Y = y - 1$.
2. Solve $(x + y + 1)dx - (2x + 2y + 1)dy = 0$, Ans: $\frac{2}{3}(x + y) - \frac{1}{9} \log|3x + 3y + 2| = x + c$.
3. Solve $\frac{dy}{dx} = \frac{2x-y+1}{6x-5y+4}$, Hints: $h = -\frac{1}{4}, k = \frac{1}{2}$.

Homework:

1. Solve $\frac{dy}{dx} = \frac{y+x-2}{y-x-4}$, Hints: $h = -1, k = 3$.
2. Solve $(4x - 6y - 1) dx + (3y - 2x - 2)dy = 0$.

Lecture -4: Class Problems: (Exact Differential Equations)

1. Solve $\left[\left(1 + \frac{1}{x} \right) y + \cos y \right] dx + [x + \log x - x \sin y] dy = 0$, Ans: $xy + y \log x + x \cos y = c$
2. Solve $y e^{xy} dx + (x e^{xy} + 2y) dy = 0$ Ans: $y^2 + e^{xy} = c$
3. Solve $[\cos x \tan y + \cos(x + y)]dx + [\sin x \sec^2 y + \cos(x + y)]dy = 0$.
Ans: $\sin x \tan y + \sin(x + y) = c$.
4. Solve $(x^2 - ay)dx = (ax - y^2)dy$.

Homework:

1. Solve $(x^2 + y^2 - a^2)xdx + (x^2 - y^2 - b^2)ydy = 0$.
2. Solve $(\sec x \tan x \tan y - e^x)dx + \sec x \sec^2 y dy = 0$.

Problems for Remedial Class:

1. Find the particular solution of $\cos y dx + (1 + 2 e^{-x}) \sin y dy = 0$, when $x = 0, y = \frac{\pi}{4}$.
Ans: $\sec y (e^x + 2) = 3\sqrt{2}$.
2. Solve $x^3 dx - y^3 dy = 3xy (ydx - xdy)$, Ans: $\sqrt{y^2 - x^2} = c(x^2 + y^2)$.
3. Solve $(6x - 5y + 4)dy + (y - 2x - 1)dx = 0$.