Problem set-4 for MA1201

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Lecture-13: Class Problems: (Method of variation parameters)

1. Solve: $\frac{d^2y}{dx^2} + y = cosec\, x.$ *Ans:* $y = c_1 \cos x + c_2 \sin x - x \cos x + \sin x \log|\sin x|.$
2. Solve: $\frac{d^2y}{dx^2} + 4y = \tan 2x.$ *Ans:* $y = c_1 \cos 2x + c_2 \sin 2x - \frac{1}{4}\cos 2x \log|\sec 2x + \tan 2x|.$
3. Solve: $\frac{d^2y}{dx^2} + y = \sec x.$

Homework:

1. Solve: $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = e^x \tan x.$
2. Solve: $\frac{d^2y}{dx^2} + y = \tan x.$

Lecture-14: Class Problems: (Cauchy-Euler's homogeneous linear equation)

1. Solve: $x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 4y = 2x.$
2. Solve: $x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 4y = x^2.$ *Ans:* $y = c_1x^{-1} + c_2x^4 - \frac{1}{6}x^2.$
3. Solve: $x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 4y = 2\log x.$ *Ans:* $y = c_1x^{-1} + c_2x^4 - \frac{1}{2}\log x + \frac{3}{8}.$

Homework:

1. Solve: $x^2\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + 6y = x^2.$ *Ans:* $y = c_1x^2 + c_2x^3 - (\log x)\, x^2.$
2. *Solve: $x^3\frac{d^3y}{dx^3} + 2x^2\frac{d^2y}{dx^2} + 2y = 10\left(x + \frac{1}{x}\right).$

Lecture-15: Class Problems: (Simultaneous Linear differential equations)

1. Solve: $\begin{matrix} \frac{dx}{dt}+y=\sin t \\ \frac{dy}{dt}+x=\cos t \end{matrix}$ *Ans:* $\begin{matrix} x=c_1e^t+c_2e^{-t} \\ y=\sin t-c_1e^t+c_2e^{-t} \end{matrix}.$
2. Solve: $\begin{matrix} \frac{dx}{dt}-2y=t \\ \frac{dy}{dt}+2x=0 \end{matrix}$

Homework:

1. Solve: $\begin{matrix} \frac{dx}{dt}+y=\sin t \\ \frac{dy}{dt}+x=\cos t \end{matrix}$ *given that* $x = 2, y = 0,$ *when* $t = 0.$

   Ans: $x = \frac{1}{2}(e^{-t} - e^t), y = \sin t + \frac{1}{2}(e^t + e^{-t}).$
2. Solve: $\begin{matrix} \frac{dx}{dt}+2x+3y=0 \\ \frac{dy}{dt}+3x+2y=2e^{-t} \end{matrix}.$

Lecture-16: Class Problems: (Laplace transforms)

1. Define Laplace transform of a function $f(t)$. Using the same, find the Laplace transform of following functions:

   i) $f(t) = e^{2t},\ t \geq 0,$ ii) $f(t) = \begin{cases} \frac{t}{2}, & 0 < t < 2 \\ 1, & t > 2 \end{cases}$, iii) $f(t) = \begin{cases} e^t & 0 < t < 1 \\ 1, & t > 1 \end{cases}.$

Homework:

1. Define Laplace transform of a function $f(t)$. Using the same, find the Laplace transform of following functions:

   i) $f(t) = Sin(2t),\ t \geq 0.$

   ii) $f(t) = Sinh(2t),\ t > 0.$

   *iii) $f(t) = |t-1| + |t+1|,\ t \geq 0.$

Problems for Remedial Class:

1. Solve : $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - 4y = \sin(3\log x).$
2. Solve by the method of variation parameters: $\frac{d^2y}{dx^2} + 9y = cosec\, 3x.$
3. Using the definition, find the Laplace transform of $f(t) = t + |t-2|,\ t \geq 0.$

Note: *denotes challenging problem.