# Final Assessment Test II - April 2022

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|---|---|---|---|---|---|
| Programme | : | Integrated M.Tech. I Year | Semester | : | Win 2021-22 |
| Course | : | Applications of Differential and Difference Equations | Code | : | MAT2002 |
| | | | Slot | : | B2+TB2 |
| Faculty | : | Dr. David Raj Micheal | Class ID | : | CH2021222300226 |
| Time | : | 180 Minutes | Max.Marks | : | 120 |

**Instruction to Candidates:**

(i) Non-programmable Calculators are allowed.

(ii) Any misprinted values can be assumed suitably.

(iii) This question paper contains 12 questions and 2 page(s).

## Part – A $(10 \times 10 = 100)$

### Answer any TEN Questions

**Q1.** **Module:** 01 **CO:** 01 **Level:** Easy **BL:** K2 **Hots:** No **Marks:** 10

Find the Fourier series expansion of

$$f(x) = \begin{cases} 0 & 0 \le x \le \pi \\ \cos x & \pi \le x \le 2\pi \end{cases}.$$

**Q2.** **Module:** 01 **CO:** 01 **Level:** Medium **BL:** K2 **Hots:** No **Marks:** 10

Determine the first two harmonics of the Fourier series for the following data:

| $x$ | $0$ | $\frac{\pi}{3}$ | $\frac{2\pi}{3}$ | $\pi$ | $\frac{4\pi}{3}$ | $\frac{5\pi}{3}$ |
|---|---|---|---|---|---|---|
| $y$ | 1.98 | 1.30 | 1.05 | 1.30 | −0.88 | −0.25 |

**Q3.** **Module:** 02 **CO:** 02 **Level:** Easy **BL:** K2 **Hots:** No **Marks:** 10

Find $P$ such that $P^{-1}AP = D$, where $D$ is a diagonal matrix for

$$A = \begin{bmatrix} -1 & 1 & 0 \\ 0 & 2 & -1 \\ 0 & 0 & 3 \end{bmatrix}.$$

**Q4.** **Module:** 02 **CO:** 02 **Level:** Easy **BL:** K2 **Hots:** No **Marks:** 10

(a) Let $A = \begin{bmatrix} 3 & 3 & 0 \\ 0 & 2 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ and $x^3 - 6x^2 + 11x - 6 = 0$ be the characteristic equation of $A$. Find $A^{-1}$ using Cayley-Hamilton theorem. [5]

(b) Use Laplace transform to solve [5]

$$y'' + 4y' + 2y = u(t-2), \quad y(0) = 0 = y'(0),$$

where $u$ is an unit step function.

**Q5. Module: 03 CO: 03 Level: Hard BL: K2 Hots: No Marks: 10**

Solve $y'' - 2y' = e^x \sin x + 5$ using the method of undetermined coefficients.

**Q6. Module: 03 CO: 03 Level: Medium BL: K2 Hots: No Marks: 10**

Solve $(3x+2)^2 y'' + 3(3x+2)y' - 36y = 3x^2 + 4x + 1$.

**Q7. Module: 04 CO: 03 Level: Hard BL: K2 Hots: Yes Marks: 10**

A particle is moving along a plane curve, the co-ordinate $(x, y)$ at time $t$ is given by,

$$\frac{dy}{dt} + x - 2y = \cos 2t$$
$$\frac{dx}{dt} + 2x - y = \sin 2t$$

for $t > 0$. If at $t = 0$, $x = 1$ and $y = 0$, use Laplace transform to find the curve $(x(t), y(t))$ on which the particle is moving.

**Q8. Module: 05 CO: 04 Level: Hard BL: K2 Hots: Yes Marks: 10**

Find the power series solution about $x = 0$ of the following differential equation equation

$$(x^2 + 2x - 1)y'' + 3y' = 0.$$

**Q9. Module: 05,06 CO: 04,05 Level: Hard BL: K2 Hots: Yes Marks: 10**

(a) Find the Eigen functions of the Strum-Liouville problem [5]

$$y'' + \lambda y = 0, \quad y(0) = 0, \quad y(\pi) = 0$$

and verify their orthogonality.

(b) Use Convolution theorem to find the inverse Z-transform of $\left(\frac{z}{z-a}\right)^2$ and hence deduce for $\left(\frac{2z}{2z-1}\right)^2$. [5]

**Q10. Module: 06 CO: 05 Level: Easy BL: K2 Hots: No Marks: 10**

Find the Z-transform of the following:

(a) $2n + 4\sin\frac{n\pi}{2} - 4a^4$ [3]

(b) $e^{-2n}\cos n\theta$ [3]

(c) $\frac{n}{(n+2)!}$ [4]

**Q11. Module: 07 CO: 05 Level: Medium BL: K2 Hots: No Marks: 10**

Solve the recurrence relation

$$a_n = 4a_{n-1} - 4a_{n-2} + (n+1)2^n,$$

given that $a_0 = 1$ and $a_1 = 2$.

**Q12. Module: 07 CO: 05 Level: Easy BL: K2 Hots: No Marks: 10**

Use Z-transform to solve the difference equation

$$u_{n+2} - 4u_{n+1} + 3u_n = 5^n.$$