



## Final Assessment Test - April 2022

Programme : Integrated M.Tech.	Semester : Win 2021-22
Course : Applications of Differential and Difference Equations	Code : MAT2002
	Slot : B2+TB2
Faculty : Dr. David Raj Micheal	Class ID : CH2021222300226
Time : 180 Minutes	Max.Marks : 120

### Instruction to Candidates:

- (i) Non-programmable Calculators are allowed.
- (ii) Any misprinted values can be assumed suitably.
- (iii) This question paper contains 12 questions and 3 page(s).

### Part – A ( $10 \times 10 = 100$ ) Answer any TEN Questions

1. Find the Fourier series expansion of [10]

$$f(x) = \begin{cases} 0 & 0 \leq x \leq \pi \\ \cos x & \pi \leq x \leq 2\pi \end{cases}.$$

2. Determine the first two harmonics of the Fourier series for the following data: [10]

$x$	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	$\pi$	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$
$y$	1.98	1.30	1.05	1.30	-0.88	-0.25

3. Find  $P$  such that  $P^{-1}AP = D$ , where  $D$  is a diagonal matrix for [10]

$$A = \begin{bmatrix} -1 & 1 & 0 \\ 0 & 2 & -1 \\ 0 & 0 & 3 \end{bmatrix}.$$

4. (a) Let  $A = \begin{bmatrix} 3 & 3 & 0 \\ 0 & 2 & 0 \\ 1 & 1 & 1 \end{bmatrix}$  and  $x^3 - 6x^2 + 11x - 6 = 0$  be the characteristic equation [5]  
of  $A$ . Find  $A^{-1}$  using Cayley-Hamilton theorem.

- (b) Use Laplace transform to solve [5]

$$y'' + 4y' + 2y = u(t - 2), \quad y(0) = 0 = y'(0),$$

where  $u$  is an unit step function.

5. Solve  $y'' - 2y' = e^x \sin x + 5$  using the method of undetermined coefficients. [10]

**Solution:**  $y = A + Be^{2x} - \frac{1}{2}e^x \sin x$

6. Solve  $(3x + 2)^2 y'' + 3(3x + 2)y' - 36y = 3x^2 + 4x + 1$ . [10]

**Solution:**  $y = c_1(3x + 2)^2 + c_2(3x + 2)^{-2} + \frac{1}{108} ((3x + 2)^2 \log(3x + 2) + 1)$

7. A particle is moving along a plane curve, the co-ordinate  $(x, y)$  at time  $t$  is given by, [10]

$$\begin{aligned} \frac{dy}{dt} + x - 2y &= \cos 2t \\ \frac{dx}{dt} + 2x - y &= \sin 2t \end{aligned}$$

for  $t > 0$ . If at  $t = 0$ ,  $x = 1$  and  $y = 0$ , use Laplace transform to find the curve  $(x(t), y(t))$  on which the particle is moving.

8. Find the power series solution about  $x = 0$  of the following differential equation [10]

$$(x^2 + 2x - 1)y'' + 3y' = 0.$$

9. (a) Find the Eigen functions of the Strum-Liouville problem [5]

$$y'' + \lambda y = 0, \quad y(0) = 0, \quad y(\pi) = 0$$

and verify their orthogonality.

- (b) Use Convolution theorem to find the inverse Z-transform of  $\left(\frac{z}{z-a}\right)^2$  and hence deduce for  $\left(\frac{2z}{2z-1}\right)^2$ . [5]

10. Find the Z-transform of the following:

(a)  $2n + 4 \sin \frac{n\pi}{2} - 4a^4$  [3]

(b)  $e^{-2n} \cos n\theta$  [3]

(c)  $\frac{n}{(n+2)!}$  [4]

11. Solve the recurrence relation [10]

$$a_n = 4a_{n-1} - 4a_{n-2} + (n+1)2^n,$$

given that  $a_0 = 1$  and  $a_1 = 2$ .

12. Use Z-transform to solve the difference equation [10]

$$u_{n+2} - 4u_{n+1} + 3u_n = 5^n.$$

**Programme** : Integrated M.Tech.  
**Exam Date & Time** : 30/08/2022 9.00 AM to 10.30 AM  
**Room Number** : DB-108  
**Course Code** : MAT2002  
**Course Name** : Applications of Differential and Difference Equations  
**Class Id & Slot** : CH2021222300226 B2+TB2  
**Total QP Alloted** : 9  
**Faculty & Emp. Id** : Dr. David Raj Micheal (51942)

**Programme** : Integrated M.Tech.  
**Exam Date & Time** : 30/08/2022 9.00 AM to 10.30 AM  
**Room Number** : DB-203  
**Course Code** : MAT2002  
**Course Name** : Applications of Differential and Difference Equations  
**Class Id & Slot** : CH2021222300226 B2+TB2  
**Total QP Alloted** : 36  
**Faculty & Emp. Id** : Dr. David Raj Micheal (51942)

**Programme** : Integrated M.Tech.  
**Exam Date & Time** : 30/08/2022 9.00 AM to 10.30 AM  
**Room Number** : DB-204  
**Course Code** : MAT2002  
**Course Name** : Applications of Differential and Difference Equations  
**Class Id & Slot** : CH2021222300226 B2+TB2  
**Total QP Alloted** : 25  
**Faculty & Emp. Id** : Dr. David Raj Micheal (51942)