Reg.No	
Name	

VIT®

Vellore Institute of Technology

(Deemed to be University under section 3 of UGC Act, 1956)

Final Assessment Test II - April 2022

Programme	:	Integrated M.Tech. I Year	Semester	:	Win 2021-22
Course	:	Applications of Differential and Difference Equations	Code	:	MAT2002
			Slot	:	B2+TB2
Faculty	:	Dr. David Raj Micheal	Class ID	:	CH2021222300226
Time	:	180 Minutes	Max.Marks	:	120

Instruction to Candidates:

(i) Non-programmable Calculators are allowed.

(ii) Any misprinted values can be assumed suitably.

(iii) This question paper contains 12 questions and 3 page(s).

Part – A $(10 \times 10 = 100)$
Answer any TEN Questions

Q1. **Module:** 01 **CO:** 01 **Level:** Easy **BL:** K2 **Hots:** No **Marks:** 10

Find the Fourier series expansion of

$$f(x) = \begin{cases} 0 & 0 \le x \le \pi \\ \cos x & \pi \le x \le 2\pi \end{cases}.$$

Q2. **Module:** 01 **CO:** 01 **Level:** Medium **BL:** K2 **Hots:** No **Marks:** 10

Determine the first two harmonics of the Fourier series for the following data:

x	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$
y	1.98	1.30	1.05	1.30	-0.88	-0.25

Q3. **Module:** 02 **CO:** 02 **Level:** Easy **BL:** K2 **Hots:** No **Marks:** 10

Find P such that $P^{-1}AP = D$, where D is a diagonal matrix for

$$A = \begin{bmatrix} -1 & 1 & 0 \\ 0 & 2 & -1 \\ 0 & 0 & 3 \end{bmatrix}.$$

Q4. Module: 02 CO: 02 Level: Easy BL: K2 Hots: No Marks: 10

(a) Let $A = \begin{bmatrix} 3 & 3 & 0 \\ 0 & 2 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ and $x^3 - 6x^2 + 11x - 6 = 0$ be the characteristic equation of A. Find A^{-1} using Cayley-Hamilton theorem. [5]

(b) Use Laplace transform to solve [5]

$$y'' + 4y' + 2y = u(t-2), \quad y(0) = 0 = y'(0),$$

where u is an unit step function.

Q5. Module: 03 CO: 03 Level: Hard BL: K2 Hots: No Marks: 10

Solve $y'' - 2y' = e^x \sin x + 5$ using the method of undetermined coefficients.

Q6. Module: 03 CO: 03 Level: Medium BL: K2 Hots: No Marks: 10

Solve $(3x+2)^2 y'' + 3(3x+2)y' - 36y = 3x^2 + 4x + 1$.

Q7. Module: 04 CO: 03 Level: Hard BL: K2 Hots: Yes Marks: 10

A particle is moving along a plane curve, the co-ordinate (x, y) at time t is given by,

$$\frac{dy}{dt} + x - 2y = \cos 2t$$
$$\frac{dx}{dt} + 2x - y = \sin 2t$$

for $t > 0$. If at $t = 0$, $x = 1$ and $y = 0$, use Laplace transform to find the curve $(x(t), y(t))$ on which the particle is moving.

Q8. Module: 05 CO: 04 Level: Hard BL: K2 Hots: Yes Marks: 10

Find the power series solution about $x = 0$ of the following differential equation equation

$$(x^2 + 2x - 1)y'' + 3y' = 0.$$

Q9. Module: 05,06 CO: 04,05 Level: Hard BL: K2 Hots: Yes Marks: 10

(a) Find the Eigen functions of the Strum-Liouville problem [5]

$$y'' + \lambda y = 0, \quad y(0) = 0, \quad y(\pi) = 0$$

and verify their orthogonality.

(b) Use Convolution theorem to find the inverse Z-transform of $\left(\frac{z}{z-a}\right)^2$ and hence deduce for $\left(\frac{2z}{2z-1}\right)^2$. [5]

Q10. Module: 06 CO: 05 Level: Easy BL: K2 Hots: No Marks: 10

Find the Z-transform of the following:

(a) $2n + 4\sin\frac{n\pi}{2} - 4a^4$ [3]

(b) $e^{-2n}\cos n\theta$ [3]

(c) $\frac{n}{(n+2)!}$ [4]

Q11. Module: 07 CO: 05 Level: Medium BL: K2 Hots: No Marks: 10

Solve the recurrence relation

$$a_n = 4a_{n-1} - 4a_{n-2} + (n+1)2^n,$$

given that $a_0 = 1$ and $a_1 = 2$.

Q12. Module: 07 CO: 05 Level: Easy BL: K2 Hots: No Marks: 10

Use Z-transform to solve the difference equation

$$u_{n+2} - 4u_{n+1} + 3u_n = 5^n.$$