VIT®
Vellore Institute of Technology
(Deemed to be University under section 3 of UGC Act, 1956)

Final Assessment Test - April 2022

Programme : Integrated M.Tech.	Semester : Win 2021-22
Course : Applications of Differential and Difference Equations	Code : MAT2002
	Slot : B2+TB2
Faculty : Dr. Dhanasekar, Dr. V. Parthiban, Dr. Manivannan, Dr. David Raj Micheal,	Class Nbr : CH2021222300226, 227,228, 229
Time : 180 Minutes	Max.Marks : 120

Part − A ($5 \times 10 = 50$)
Answer all the Questions

1. Find the Fourier series expansion of $f(x) = \begin{cases} 0 & 0 \le x \le \pi \\ \cos x & \pi \le x \le 2\pi \end{cases}$. [10]

2. Determine the first two harmonics of the Fourier series for the following data: [10]

x	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$
y	1.98	1.30	1.05	1.30	−0.88	−0.25

3. Find P such that $P^{-1}AP = D$, where D is a diagonal matrix for $A = \begin{bmatrix} -1 & 1 & 0 \\ 0 & 2 & -1 \\ 0 & 0 & 3 \end{bmatrix}$. [10]

ERB

4. (a) Let $A = \begin{bmatrix} 3 & 3 & 0 \\ 0 & 2 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ and $x^3 - 6x^2 + 11x - 6 = 0$ be the characteristic equation of A. [5]
   Find A^{-1} using Cayley-Hamilton theorem.

   (b) i. Subpart 01 [3]
        ii. Subpart 02 [2]

5. Solve $y'' - 2y' = e^x \sin x + 5$ using the method of undetermined coefficients. [10]

Part − B ($5 \times 10 = 50$)
Answer any FIVE Questions

6. Solve $(3x + 2)^2 y'' + 3(3x + 2)y' - 36y = 3x^2 + 4x + 1$. [10]

7. A particle is moving along a plane curve, the co-ordinate (x, y) at time t is given by, [10]

$$\frac{dy}{dt} + x - 2y = \cos 2t$$
$$\frac{dx}{dt} + 2x - y = \sin 2t$$

for $t > 0$. If at $t = 0$, $x = 1$ and $y = 0$, use Laplace transform to find the curve $(x(t), y(t))$ on which the particle is moving.

8. Find the power series solution about $x = 0$ of the following differential equation equation [10]

$$(x^2 + 2x - 1)y'' + 3y' = 0.$$

9. (a) Find the Eigen functions of the Strum-Liouville problem [5]

$$y'' + \lambda y = 0, \quad y(0) = 0, \quad y(\pi) = 0$$

and verify their orthogonality.

(b) Use Convolution theorem to find the inverse Z-transform of $\left(\frac{z}{z-a}\right)^2$ and hence deduce [5] for $\left(\frac{2z}{2z-1}\right)^2$.

10. Find the Z-transform of the following:
   (a) $2n + 4\sin\frac{n\pi}{2} - 4a^4$ [3]
   (b) $e^{-2n}\cos n\theta$ [3]
   (c) $\frac{n}{(n+2)!}$ [4]

11. Solve the recurrence relation [10]

$$a_n = 4a_{n-1} - 4a_{n-2} + (n+1)2^n,$$

given that $a_0 = 1$ and $a_1 = 2$.

12. Use Z-transform to solve the difference equation [10]

$$u_{n+2} - 4u_{n+1} + 3u_n = 5^n.$$

2

Programme	: Integrated M.Tech.
Exam Date & Time	: 30/08/2022 9.00 AM to 10.30 AM
Room Number	: DB–108
Course Code	: MAT2002
Course Name	: Applications of Differential and Difference Equations
Class Id & Slot	: CH2021222300226 & B2+TB2
Total QP Alloted	: 9+2
Faculty & Emp. Id	: Dr. David Raj Micheal (51942)
Faculty Mobile No	: +91 99999 00000

Programme	: Integrated M.Tech.
Exam Date & Time	: 30/08/2022 9.00 AM to 10.30 AM
Room Number	: DB–203
Course Code	: MAT2002
Course Name	: Applications of Differential and Difference Equations
Class Id & Slot	: CH2021222300226 & B2+TB2
Total QP Alloted	: 36+2
Faculty & Emp. Id	: Dr. David Raj Micheal (51942)
Faculty Mobile No	: +91 99999 00000

Programme	: Integrated M.Tech.
Exam Date & Time	: 30/08/2022 9.00 AM to 10.30 AM
Room Number	: DB–204
Course Code	: MAT2002
Course Name	: Applications of Differential and Difference Equations
Class Id & Slot	: CH2021222300226 & B2+TB2
Total QP Alloted	: 25+2
Faculty & Emp. Id	: Dr. David Raj Micheal (51942)
Faculty Mobile No	: +91 99999 00000