Computational benchmarking of exact methods for the bilevel discrete network design problem
Supplementary material

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Abstract

The discrete network design problem (DNDP) is a well-studied bilevel optimization problem in transportation. The goal of the DNDP is to identify the optimal set of candidate links (or projects) to be added to the network while accounting for users' reaction as governed by a traffic equilibrium. Several approaches have been proposed to solve the DNDP exactly using single-level, mixed-integer programming reformulations, linear approximations of link travel time functions, relaxations and decompositions. To date, the largest DNDP instances solved to optimality remain of small scale and existing algorithms are no match to solve realistic problem instances involving large numbers of candidate projects. In this work, we examine the literature on exact methodologies for the DNDP and attempt to categorize the main approaches employed. We introduce a new set of benchmarking instances for the DNDP and implement three solution methods to compare computational performance and outline potential directions for improvement. For reproducibility purposes and to promote further research on this challenging bilevel optimization problem, all implementation codes and instance data are provided in a publicly available repository.

Keywords: Network design problem, bilevel optimization, benchmarking

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Appendix A. Optimization formulations

We implement three solution methods for the linearized DNDP: i) the B&B algorithm of Farvaresh and Sepehri (2013) which is an extension of Leblanc (1975)—SOBB; ii) the SO-relaxation based algorithm of Wang et al. (2013) with solution interdiction cuts—SOIC, and iii) the primal-dual formulation of Fontaine and Minner (2014) as a single-level MILP (without Benders' decomposition)—MKKT. All three solution methods, SOBB, SOIC and MKKT, are implemented using the piecewise linear approximation of link travel time functions proposed by Farvaresh and Sepehri (2011). Hence, in all cases, the optimization problems solved are MILPs and LPs (TAPs within SOBB and SOIC are solved in their linearized form). This differs to the original implementation of Farvaresh and Sepehri (2013) and Wang et al. (2013), wherein outer approximation schemes are used. In addition, the method MKKT is implemented as direct MILP approach unlike the Benders' decomposition scheme proposed in Fontaine and Minner (2014).

A.1. Linearized SO-DNDP

We use the piecewise linear approximation proposed by Farvaresh and Sepehri (2011) to approximate link travel time functions $t_{ij}(x_{ij}) = T_{ij} + c_{ij}x_{ij}^{e_{ij}}$. Let $V \cup \{0\}$ be the set of support point for the piecewise linear approximation, where $V = \{1, \ldots, m-1\}$, and let $\alpha_{ij,v}$ represent the vth support point. Variables $\lambda_{ij}^L \geq 0$ and $\lambda_{ij}^R \geq 0$ are introduced to approximate the nonlinear term $x_{ij}^{e_{ij}+1} = \left(\sum_{s\in D} x_{ij,s}\right)^{e_{ij}+1}$. The linearized SO-DNDP formulation is a MILP summarized in L-SO-DNDP. This MILP formulation is the starting point of methods SOIC and SOBB.

$$
\begin{aligned}
\min \quad & \sum_{(i,j)\in A} \left(T_{ij} \sum_{s\in D} x_{ij,s} + c_{ij} \sum_{v\in V} \left(\lambda_{ij}^L \alpha_{ij,v-1}^{e_{ij}+1} + \lambda_{ij}^R \alpha_{ij,v}^{e_{ij}+1} \right) \right) && \\
\text{s.t.} \quad & \sum_{(i,j)\in A_2} y_{ij} g_{ij} \leq B && \\
& \sum_{j\in N:(i,j)\in A} x_{ij,s} - \sum_{j\in N:(j,i)\in A} x_{ji,s} = d_{is} && \forall i \in N, \forall s \in D \\
& \sum_{s\in D} x_{ij,s} = \sum_{v\in V} \left(\lambda_{ij}^L \alpha_{ij,v-1} + \lambda_{ij}^R \alpha_{ij,v} \right) && \forall (i,j) \in A \\
& \sum_{v\in V} \left(\lambda_{ij}^L + \lambda_{ij}^R \right) = 1 && \forall (i,j) \in A \\
& \sum_{s\in D} x_{ij,s} \leq y_{ij} M && \forall (i,j) \in A_2 \\
& y_{ij} \in \{0,1\} && \forall (i,j) \in A_2 \\
& x_{ij,s} \geq 0 && \forall (i,j) \in A, \forall s \in D \\
& \lambda_{ij}^L \geq 0 && \forall (i,j) \in A, \forall v \in V \cup \{0\} \\
& \lambda_{ij}^R \geq 0 && \forall (i,j) \in A, \forall v \in V \cup \{0\}
\end{aligned}
\tag{L-SO-DNDP}
$$

A.2. Linearized KKT conditions

The linearized KKT conditions based formulation combines the primal and the dual of the follower of the DNDP. Variables $\pi_{is} \in \mathbb{R}$, $\beta_{ij} \in \mathbb{R}$, $\gamma_{ij} \in\geq 0$ and $\mu_{ij} \geq 0$ are dual variables corresponding to the constraint of the primal follower. Variable $\varphi_{ij} \geq 0$ is used to linearize the bilinear term $\mu_{ij}y_{ij}$ which appears in the KKT condition constraint. The MILP summarized in L-KKT-DNDP corresponds to method MKKT.

$$
\begin{aligned}
\min \quad & \sum_{(i,j)\in A}\left(T_{ij}\sum_{s\in D}x_{ij,s} + c_{ij}\sum_{v\in V}\left(\lambda^L_{ij}\alpha^{e_{ij}+1}_{ij,v-1} + \lambda^R_{ij}\alpha^{e_{ij}+1}_{ij,v}\right)\right) \\
\text{s.t.} \quad & \sum_{(i,j)\in A_2} y_{ij}g_{ij} \leq B \\
& \sum_{j\in N:(i,j)\in A} x_{ij,s} - \sum_{j\in N:(j,i)\in A} x_{ji,s} = d_{is} && \forall i \in N, \forall s \in D \\
& \sum_{s\in D} x_{ij,s} = \sum_{v\in V}\left(\lambda^L_{ij}\alpha_{ij,v-1} + \lambda^R_{ij}\alpha_{ij,v}\right) && \forall (i,j) \in A \\
& \sum_{v\in V}\left(\lambda^L_{ij} + \lambda^R_{ij}\right) = 1 && \forall (i,j) \in A \\
& \sum_{s\in D} x_{ij,s} \leq y_{ij}M && \forall (i,j) \in A_2 \\
& \pi_{is} - \pi_{js} + \beta_{ij} \geq -T_{ij} && \forall (i,j) \in A_1, \forall s \in D \\
& \pi_{is} - \pi_{js} + \beta_{ij} + \mu_{ij} \geq -T_{ij} && \forall (i,j) \in A_2, \forall s \in D \\
& \beta_{ij}\alpha_{ij,v} + \gamma_{ij} \geq \frac{-c_{ij}}{e_{ij}+1}\alpha^{e_{ij}+1}_{ij,v} && \forall (i,j) \in A, \forall v \in V \\
& \varphi_{ij} \leq \mu_{ij} && \forall (i,j) \in A_2 \\
& \varphi_{ij} \geq \mu_{ij} - (1-y_{ij})M_2 && \forall (i,j) \in A_2 \\
& \varphi_{ij} \leq y_{ij}M_2 && \forall (i,j) \in A_2 \\
& \sum_{(i,j)\in A} T_{ij}\sum_{s\in D}x_{ij,s} + \frac{c_{ij}}{e_{ij}+1}\sum_{v\in V}\lambda^L_{ij}\alpha^{e_{ij}+1}_{ij,v-1} + \lambda^R_{ij}\alpha^{e_{ij}+1}_{ij,v} \\
& \qquad \leq -\left(\sum_{i\in N}\sum_{s\in D}\pi_{is}d_{is} + \sum_{(i,j)\in A}\gamma_{ij} + \sum_{(i,j)\in A_2}\varphi_{ij}M\right) \\
& y_{ij} \in \{0,1\} && \forall (i,j) \in A_2 \\
& x_{ij,s} \geq 0 && \forall (i,j) \in A, \forall s \in D \\
& \lambda^L_{ij} \geq 0 && \forall (i,j) \in A, \forall v \in V \cup \{0\} \\
& \lambda^R_{ij} \geq 0 && \forall (i,j) \in A, \forall v \in V \cup \{0\} \\
& \gamma_{ij} \geq 0 && \forall (i,j) \in A \\
& \mu_{ij} \geq 0 && \forall (i,j) \in A_2 \\
& \varphi_{ij} \geq 0 && \forall (i,j) \in A_2 \\
& \beta_{ij} \in \mathbb{R} && \forall (i,j) \in A \\
& \pi_{is} \in \mathbb{R} && \forall i \in N, \forall s \in D
\end{aligned}
\tag{L-KKT-DNDP}
$$

Appendix B. Detailed numerical results

Detailed numerical results of the numerical experiments conducted in Section 4.3 of the paper in tables B.1-B.4. Details on the benchmark instances used are provided in Section 4.2.

All methods are implemented in Python. All MILPs and LPs are solved using CPLEX 12.8 MIP solver. Convex TAPs are solved using the Pyomo module and IPOPT solver. All solution methods were tested and implemented on the same Windows 7 machine with 16Gb of RAM and a CPU of 2.7Ghz, in a single-thread mode with a time limit of 10 minutes. The upper bound on link flows $\bar{x}_{ij}$ was set to $1e^5$ and this value is also used for M. The number of segments used in the piece-wise linear approximations of link travel time functions is $m = 100$. A scaling factor of $1e^{-3}$ is used to scale travel demand and link capacities as it was found to improve computational performance.

To measure the quality of the approximated solutions, the flow pattern corresponding to the best (lowest leader objective value) $\boldsymbol{y}$ solution among all three methods is calculated by solving the TAP as a convex problem.

For reproducibility purposes, all implementation codes and benchmarking instances are publicly available at the repository https://github.com/davidrey123/DNDP.

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		MKKT			SOIC			SOBB			
Instance	$B_{\%}$	UB	Gap	Time	UB	Gap	Time	UB	Gap	Time	TAP
SF_DNDP_10_1	25	6293.8	0.0	26.1	6287.9	0.0	8.5	6287.9	0.0	26.6	6227.9
	50	5718.4	0.0	155.5	5712.8	0.0	126.9	5712.8	0.0	265.2	5680.2
	75	5300.2	0.0	71.5	5294.1	0.0	64.2	5294.1	0.0	374.9	5294.0
SF_DNDP_10_2	25	6519.1	0.0	22.5	6514.1	0.0	16.1	6514.1	0.0	34.2	6509.7
	50	5758.2	0.0	57.6	5753.4	0.0	25.8	5753.4	0.0	113.8	5756.8
	75	5084.3	0.0	34.0	5080.9	0.0	15.2	5080.9	0.0	171.3	5088.5
SF_DNDP_10_3	25	6281.8	0.0	73.4	6275.7	0.0	37.0	6275.7	0.0	70.2	6287.8
	50	5474.7	0.0	291.2	5468.5	0.0	518.0	5468.5	0.0	450.3	5448.4
	75	5086.2	0.0	304.2	5080.5	1.1	600.0	5080.5	2.3	600.0	5087.8
SF_DNDP_10_4	25	6138.1	0.0	71.2	6130.2	0.0	24.0	6130.2	0.0	75.7	6059.4
	50	5693.4	0.0	70.7	5685.7	0.0	20.8	5685.7	0.0	136.7	5626.4
	75	5532.9	0.0	25.1	5526.8	0.0	65.4	5526.8	0.0	336.0	5504.4
SF_DNDP_10_5	25	5910.0	0.0	83.9	5905.0	0.0	50.3	5905.0	0.0	81.6	5900.9
	50	5335.1	0.0	320.4	5328.6	0.5	600.0	5328.6	0.9	600.0	5359.0
	75	5179.2	0.0	293.3	5175.9	2.0	600.0	5175.9	4.4	600.0	5111.8
SF_DNDP_10_6	25	5825.0	0.0	64.8	5816.7	0.0	36.4	5816.7	0.0	71.2	5823.6
	50	5180.3	0.0	297.1	5173.6	0.0	566.9	5173.6	0.0	500.5	5152.0
	75	4803.5	0.0	260.2	4798.6	0.0	458.7	4798.6	1.6	600.0	4810.4
SF_DNDP_10_7	25	5910.0	0.0	67.1	5905.0	0.0	42.1	5905.0	0.0	73.4	5900.9
	50	5655.3	0.0	104.4	5650.1	2.6	600.0	5650.1	3.0	600.0	5650.4
	75	5603.2	0.0	94.1	5597.8	3.6	600.0	5597.8	4.3	600.0	5593.9
SF_DNDP_10_8	25	5910.0	0.0	31.6	5905.0	0.0	19.3	5905.0	0.0	40.3	5900.9
	50	5390.5	0.0	150.0	5385.2	0.0	145.8	5385.2	0.0	346.0	5366.5
	75	5195.5	0.0	89.9	5189.2	0.0	492.0	5189.2	2.4	600.0	5189.5
SF_DNDP_10_9	25	6373.7	0.0	63.0	6367.7	0.0	48.8	6367.7	0.0	39.9	6335.5
	50	5380.2	0.0	70.0	5374.4	0.0	23.3	5374.4	0.0	69.9	5377.4
	75	4972.3	0.0	77.4	4967.7	0.0	79.8	4967.7	0.0	367.4	4952.0
SF_DNDP_10_10	25	6379.8	0.0	68.1	6370.2	0.0	56.1	6370.2	0.0	54.8	6349.7
	50	5510.5	0.0	188.3	5504.2	0.0	206.8	5504.2	0.0	256.1	5505.2
	75	5164.1	0.0	308.8	5161.6	0.2	600.0	5161.6	2.5	600.0	5180.8

Table B.1: Budget sensitivity experiment. Methods SOBB, SOIC and MKKT are implemented for instances `SF_DNDP_10` for a budget B equal to $B_{\%}$ = 25%, 50% and 75% of the total cost $\sum_{(i,j)\in A_2} g_{ij}$. The time limit is 10 minutes. UB is the upper bound upon termination, Gap is the relative optimality gap upon termination in %, Time is the solve time, and TAP is the network travel time obtained by solving the convex TAP with $\boldsymbol{y}$ solution corresponding to the lowest upper bound solution.

Instance	$B_{\%}$	MKKT			SOIC			SOBB			TAP
		UB	Gap	Time	UB	Gap	Time	UB	Gap	Time	
SF_DNDP_20_1	25	5208.0	0.07	600.0	5196.1	0.04	600.0	5204.7	0.04	600.0	5181.5
	50	4348.7	0.07	600.0	4291.0	0.05	600.0	4291.0	0.05	600.0	4281.9
	75	3938.6	0.06	600.0	3936.8	0.05	600.0	3936.8	0.06	600.0	3908.9
SF_DNDP_20_2	25	5023.0	0.04	600.0	5017.0	0.03	600.0	5017.0	0.00	600.0	5030.4
	50	4119.4	0.04	600.0	4113.9	0.02	600.0	4113.9	0.02	600.0	4114.7
	75	3934.5	0.06	600.0	3928.3	0.05	600.0	3928.3	0.05	600.0	3922.0
SF_DNDP_20_3	25	5240.1	0.07	600.0	5233.2	0.05	600.0	5233.2	0.04	600.0	5237.7
	50	4347.4	0.07	600.0	4318.2	0.05	600.0	4318.2	0.05	600.0	4308.8
	75	4032.9	0.06	600.0	4030.0	0.05	600.0	4030.0	0.06	600.0	4040.0
SF_DNDP_20_4	25	5140.3	0.07	600.0	5134.7	0.04	600.0	5134.7	0.03	600.0	5127.1
	50	4326.9	0.07	600.0	4321.8	0.05	600.0	4321.8	0.05	600.0	4311.1
	75	4006.9	0.06	600.0	4003.6	0.04	600.0	4003.6	0.06	600.0	3985.7
SF_DNDP_20_5	25	5023.0	0.04	600.0	5017.0	0.03	600.0	5017.0	0.01	600.0	5030.4
	50	4557.7	0.08	600.0	4553.0	0.07	600.0	4550.1	0.07	600.0	4544.3
	75	4349.3	0.07	600.0	4346.3	0.07	600.0	4346.3	0.07	600.0	4336.6
SF_DNDP_20_6	25	5097.2	0.07	600.0	5093.8	0.04	600.0	5093.8	0.03	600.0	5104.7
	50	4247.6	0.07	600.0	4243.0	0.04	600.0	4243.0	0.06	600.0	4228.8
	75	4029.7	0.06	600.0	4027.3	0.06	600.0	4027.3	0.07	600.0	4007.0
SF_DNDP_20_7	25	5091.6	0.04	600.0	5086.1	0.03	600.0	5086.1	0.02	600.0	5095.5
	50	4371.5	0.06	600.0	4365.2	0.04	600.0	4365.2	0.05	600.0	4382.5
	75	4240.3	0.07	600.0	4234.8	0.06	600.0	4234.8	0.06	600.0	4245.8
SF_DNDP_20_8	25	4960.3	0.07	600.0	4956.3	0.04	600.0	4956.3	0.03	600.0	4953.9
	50	4066.8	0.05	600.0	4061.5	0.03	600.0	4061.5	0.04	600.0	4057.5
	75	3897.5	0.05	600.0	3894.5	0.05	600.0	3894.5	0.05	600.0	3887.6
SF_DNDP_20_9	25	5193.4	0.04	600.0	5187.6	0.03	600.0	5187.6	0.01	600.0	5196.9
	50	4266.1	0.06	600.0	4260.8	0.04	600.0	4260.8	0.04	600.0	4229.5
	75	3989.6	0.06	600.0	3984.5	0.04	600.0	3984.5	0.05	600.0	3964.6
SF_DNDP_20_10	25	5023.0	0.07	600.0	4991.8	0.04	600.0	5017.0	0.03	600.0	5026.4
	50	4441.3	0.08	600.0	4429.5	0.06	600.0	4420.4	0.06	600.0	4424.2
	75	4171.4	0.07	600.0	4168.2	0.06	600.0	4168.2	0.07	600.0	4160.7

Table B.2: Budget sensitivity experiment. Methods SOBB, SOIC and MKKT are implemented for instances SF_DNDP_20 for a budget B equal to $B_{\%}$ = 25%, 50% and 75% of the total cost $\sum_{(i,j)\in A_2} g_{ij}$. The time limit is 10 minutes. UB is the upper bound upon termination, Gap is the relative optimality gap upon termination in %, Time is the solve time, and TAP is the network travel time obtained by solving the convex TAP with $\boldsymbol{y}$ solution corresponding to the lowest upper bound solution.

Instance	$D_{\%}$	MKKT			SOIC			SOBB			TAP
		UB	Gap	Time	UB	Gap	Time	UB	Gap	Time	TAP
SF_DNDP_10_1	50	1687.4	0.0	105.7	1685.1	0.2	600.0	1685.1	0.4	600.0	1691.0
	100	5718.4	0.0	149.9	5712.8	0.0	129.2	5712.8	0.0	268.8	5680.2
	150	21066.9	0.0	61.2	21049.5	0.0	13.2	21049.5	0.0	64.6	21000.9
SF_DNDP_10_2	50	1731.4	0.0	108.4	1728.8	0.7	600.0	1728.8	0.8	600.0	1731.0
	100	5758.2	0.0	57.7	5753.4	0.0	25.9	5753.4	0.0	114.2	5756.8
	150	20971.6	0.0	32.3	20950.6	0.0	8.5	20950.6	0.0	52.8	20886.8
SF_DNDP_10_3	50	1658.1	0.0	64.0	1656.2	0.0	232.8	1656.2	0.0	304.4	1662.1
	100	5474.7	0.0	292.8	5468.5	0.0	517.4	5468.5	0.0	452.1	5448.4
	150	18408.8	0.0	90.9	18394.7	0.0	16.4	18394.7	0.0	93.1	18360.3
SF_DNDP_10_4	50	1672.0	0.0	48.3	1669.7	0.0	105.3	1669.7	0.0	272.9	1669.1
	100	5693.4	0.0	70.3	5685.7	0.0	20.5	5685.7	0.0	135.9	5626.4
	150	21137.3	0.0	21.6	21122.5	0.0	5.9	21122.5	0.0	47.8	21082.2
SF_DNDP_10_5	50	1743.1	0.0	94.8	1741.2	1.1	600.0	1741.2	1.1	600.0	1733.4
	100	5335.1	0.0	324.2	5328.6	0.5	600.0	5328.6	0.9	600.0	5359.0
	150	15511.3	0.0	62.3	15499.1	0.0	8.3	15499.1	0.0	53.3	15458.0
SF_DNDP_10_6	50	1705.1	0.0	155.2	1702.7	0.8	600.0	1702.7	0.4	600.0	1701.6
	100	5180.3	0.0	310.7	5173.6	0.0	573.9	5173.6	0.0	520.3	5152.0
	150	15239.3	0.0	144.3	15221.4	0.0	16.7	15221.4	0.0	66.8	15193.2
SF_DNDP_10_7	50	1761.1	0.0	46.8	1759.3	1.4	600.0	1759.3	1.5	600.0	1764.9
	100	5655.3	0.0	104.2	5650.1	2.6	600.0	5650.1	3.1	600.0	5650.4
	150	18964.7	0.0	106.5	18947.7	0.0	48.8	18947.7	0.0	209.4	18930.3
SF_DNDP_10_8	50	1736.5	0.0	61.8	1734.3	0.6	600.0	1734.3	0.7	600.0	1736.8
	100	5390.5	0.0	151.6	5385.2	0.0	148.4	5385.2	0.0	350.9	5366.5
	150	17301.3	0.0	64.6	17282.2	0.0	11.3	17282.2	0.0	110.2	17234.6
SF_DNDP_10_9	50	1727.3	0.0	101.5	1724.7	0.5	600.0	1724.7	0.1	600.0	1724.1
	100	5380.2	0.0	72.2	5374.4	0.0	23.3	5374.4	0.0	69.1	5377.4
	150	17687.1	0.0	43.5	17673.6	0.0	8.6	17673.6	0.0	43.9	17674.8
SF_DNDP_10_10	50	1714.5	0.0	93.4	1712.6	0.3	600.0	1712.6	0.4	600.0	1704.5
	100	5510.5	0.0	187.5	5504.2	0.0	207.1	5504.2	0.0	253.9	5505.2
	150	17667.8	0.0	32.8	17652.2	0.0	8.7	17652.2	0.0	49.7	17578.5

Table B.3: Demand sensitivity experiment. Methods SOBB, SOIC and MKKT are implemented for instances `SF_DNDP_10` for a budget B equal to 50% of the total cost $\sum_{(i,j)\in A_2} g_{ij}$ and a demand of $D_{\%}$ = 50%, 100% and 150% of the base demand. The time limit is 10 minutes. UB is the upper bound upon termination, Gap is the relative optimality gap upon termination in %, Time is the solve time, and TAP is the network travel time obtained by solving the convex TAP with $\boldsymbol{y}$ solution corresponding to the lowest upper bound solution.

Instance	$D_{\%}$	MKKT UB	MKKT Gap	MKKT Time	SOIC UB	SOIC Gap	SOIC Time	SOBB UB	SOBB Gap	SOBB Time	TAP
SF_DNDP_20_1	50	1539.4	0.03	600.0	1538.1	0.03	600.0	1538.1	0.03	600.0	1536.6
	100	4348.7	0.07	600.0	4291.0	0.05	600.0	4291.0	0.05	600.0	4281.9
	150	11352.1	0.04	600.0	11221.3	0.00	168.4	11221.3	0.00	313.6	11192.5
SF_DNDP_20_2	50	1562.4	0.02	600.0	1560.4	0.02	600.0	1560.4	0.02	600.0	1556.9
	100	4119.4	0.04	600.0	4113.9	0.02	600.0	4113.9	0.02	600.0	4114.7
	150	10306.5	0.00	561.8	10294.3	0.00	82.2	10294.3	0.00	403.9	10241.5
SF_DNDP_20_3	50	1588.1	0.04	600.0	1585.9	0.03	600.0	1585.9	0.04	600.0	1583.8
	100	4347.4	0.07	600.0	4318.2	0.05	600.0	4318.2	0.05	600.0	4308.8
	150	10578.9	0.00	381.7	10570.9	0.00	57.7	10570.9	0.00	320.9	10489.4
SF_DNDP_20_4	50	1558.4	0.02	600.0	1556.4	0.02	600.0	1556.4	0.02	600.0	1560.5
	100	4326.9	0.07	600.0	4321.8	0.05	600.0	4321.8	0.05	600.0	4311.1
	150	10970.3	0.00	416.3	10962.3	0.00	22.7	10962.3	0.00	158.7	10911.7
SF_DNDP_20_5	50	1589.5	0.02	600.0	1586.2	0.02	600.0	1585.9	0.02	600.0	1586.9
	100	4557.7	0.08	600.0	4553.0	0.07	600.0	4550.1	0.07	600.0	4544.3
	150	11645.3	0.02	600.0	11635.4	0.00	600.0	11635.4	0.01	600.0	11622.6
SF_DNDP_20_6	50	1551.8	0.03	600.0	1549.8	0.02	600.0	1549.8	0.03	600.0	1545.2
	100	4247.6	0.07	600.0	4243.0	0.04	600.0	4243.0	0.06	600.0	4228.8
	150	10515.4	0.00	492.5	10504.7	0.00	107.9	10504.7	0.00	446.1	10471.8
SF_DNDP_20_7	50	1530.6	0.01	600.0	1528.9	0.01	600.0	1528.9	0.02	600.0	1528.8
	100	4371.5	0.06	600.0	4365.2	0.04	600.0	4365.2	0.05	600.0	4382.5
	150	11916.4	0.02	600.0	11905.8	0.00	600.0	11905.8	0.01	600.0	11885.7
SF_DNDP_20_8	50	1532.9	0.02	600.0	1530.9	0.02	600.0	1530.9	0.02	600.0	1534.9
	100	4066.8	0.05	600.0	4061.5	0.03	600.0	4061.5	0.04	600.0	4057.5
	150	10066.0	0.02	600.0	10054.4	0.00	249.9	10054.4	0.00	600.0	10040.2
SF_DNDP_20_9	50	1523.8	0.03	600.0	1521.5	0.02	600.0	1521.5	0.03	600.0	1520.4
	100	4266.1	0.06	600.0	4260.8	0.04	600.0	4260.8	0.04	600.0	4229.5
	150	11536.3	0.00	345.7	11526.3	0.00	60.7	11526.3	0.00	278.4	11473.1
SF_DNDP_20_10	50	1546.9	0.02	600.0	1545.2	0.02	600.0	1545.2	0.02	600.0	1546.2
	100	4441.3	0.08	600.0	4429.5	0.06	600.0	4420.4	0.06	600.0	4424.2
	150	11416.3	0.04	600.0	11402.8	0.01	600.0	11403.4	0.01	600.0	11384.2

Table B.4: Demand sensitivity experiment. Methods SOBB, SOIC and MKKT are implemented for instances SF_DNDP_20 for a budget B equal to 50% of the total cost $\sum_{(i,j)\in A_2} g_{ij}$ and a demand of $D_{\%}$ = 50%, 100% and 150% of the base demand. The time limit is 10 minutes. UB is the upper bound upon termination, Gap is the relative optimality gap upon termination in %, Time is the solve time, and TAP is the network travel time obtained by solving the convex TAP with $\boldsymbol{y}$ solution corresponding to the lowest upper bound solution.