02427 Advanced Time Series Analysis

Computer exercise 3

The topics of this exercise is an investigation of some properties of simulated solutions to stochastic differential equations (SDE), and estimation of parameters in SDEs.

Part 1: Simulation and discretization of diffusion processes

The Bonhoeffer-Van der Pol equations give a two-dimensional simplification of a famous four-dimensional system of ordinary differential equations proposed by Hodgkin and Huxley, in order to describe the firing of a single neuron. The equations are

$$\frac{dx_t^1}{dt} = \theta_3 \left(x_t^1 + x_t^2 - \frac{1}{3}(x_t^1)^3 + \theta_4 \right) \tag{1a}$$

$$\frac{dx_t^2}{dt} = -\frac{1}{\theta_3}(x_t^1 + \theta_2 x_t^2 - \theta_1) \tag{1b}$$

where x_t^1 is the negative potential over the membrane, x_t^2 is the permeability of the membrane, and $\theta_1, \theta_2, \theta_3$ and θ_4 denote the physical parameters. When $(\theta_1, \theta_2, \theta_3, \theta_4) = (0.7, 0.8, 3.0, -0.34)$ the system has a limit cycle, which describes the periodic slow charging and fast decharging that have been observed through experiments.

The effects of imperfections in the membrane and firing of the surrounding neurons can be simulated by incorporating additive noise with a small standard deviation in (1a). This leads to a two-dimensional system of Itô stochastic diffential equation

$$dX_t^1 = \theta_3 \left(X_t^1 + X_t^2 - \frac{1}{3}(X_t^1)^3 + \theta_4 \right) dt + \sigma dW_t \tag{2a}$$

$$dX_t^2 = -\frac{1}{\theta_3} \left(X_t^1 + \theta_2 X_t^2 - \theta_1 \right) dt \tag{2b}$$

where W_t is a standard Wiener process and $\sigma > 0$ is the incremental standard deviation of the noise.

In order to be able to simulate (2), it is necessary to discretize it, i.e. to consider finite steps in the Wiener process ΔW_n instead of the infinitesimal dW_t. It can be shown that the so called Euler-Marayama approximation to (2) is given by the two-dimensional stochastic difference equation

$$Y_{n+1}^1 = Y_n^1 + \theta_3 \left(Y_n^1 + Y_n^2 - \frac{1}{3}(Y_n^1)^3 + \theta_4 \right) \Delta + \sigma \Delta W_{n+1}^1 \tag{3a}$$

$$Y_{n+1}^2 = Y_n^2 - \frac{1}{\theta_3} \left(Y_n^1 + \theta_2 Y_n^2 - \theta_1 \right) \Delta \tag{3b}$$

where Δ is a suitably chosen small time interval, $\Delta W_{n+1} \in N(0, \Delta)$ and $(Y_n^1, Y_n^2), n = 1, ..., N$ is a discrete approximation to (X_t^1, X_t^2) in the time interval $0 \leq t \leq T$.

Question 1a

Let $\Delta = 2^{-9}$, $\theta_1 = 0.7$, $\theta_2 = 0.8$, $\theta_3 = 3.0$, $\theta_4 = -0.34$ og $\sigma = 0$. Simulate (3) in the time interval $0 \le t \le T = 100$ with initial conditions $Y_0^1 = -1.9$ and $Y_0^2 = 1.2$.

Plot the realizations of Y_k^1 and Y_k^2 and make a phaseplot of (Y_k^1, Y_k^2). Repeat for $\sigma = 0.10, 0.20, 0.30$ and 0.40. Comment on the effect of adding noise to the equations.

Question 1b

The Euler-Marayama method is an example of a *weak method*, implying that it only gives an approximation of (functionals of) the moments of X_t^1, X_t^2, whereas a strong method approximates the whole distribution. However, a weak method can give some essential visual information about the performance of the stochastic dynamic system in question. A histogram might for example indicate the form and support of the density function of the asymptotically stable, stationary solution.

Let $\sigma = 0.10$ and simulate (2) using the approximation (3) with the same parameter values as given above (you may reuse the results from question 1 if you like). Partition the phase plane in $100\times$ 100 equal cells. Count the number of trajectories that passes through each cell, and make a three-dimensional plot (the units on the axes have no essential meaning in this case).

Which extra information does the plot contain, compared to the standard phase-plot?

Repeat for $\sigma = 0.20, 0.30$ and 0.40 (again using the results from question 1 if you like).

Hints

You can download a Matlab and R script for making the 2D histogram plot.

Part 2: Modelling a building using CTSM-R

In this exercise you will model parts of a building belonging to University of Basque Country (UPV/EHU). The building can be seen on Google maps by clicking this link.[1] Alternatively the part of the building we are interested in is shown on Figure 1. The building consists of 3 wings each of which have 4 floors, but in this exercise we will only focus on the second floor of the east-most wing. A north facing floor plan of this is shown in Figure 2. The 4 red devices indicate placements of temperature sensors, numbered 1 to 4 in the data as shown in the floor plan. The heating of the of the rooms is done through 2 circuits of radiators, one for the northern and one for the southern part. Thus it is not possible distinguish between the heating carried out in room 1 and 2, and similarly for room 3 and 4.

The data provided contains the variables shown in Table 1. All of the data have been sampled in a 1 hour resolution between 22/12 – 2014 and 30/04 – 2015. Some of the measurements were missing and have been replaced by linear interpolation between neighbouring points.

[1] `https://www.google.com/maps/place/Campus+De+Bizkaia+-+Campus+of+Biscay/@43.3302647,-2.9712513,166a,35y,39.43t/data=!3m1!1e3!4m5!3m4!1s0xd4e5b02f7a785bf:0x9a217c9d1cd7ecd0!8m2!3d43.3309433!4d-2.9678921`

Table 1: Description of data contained in `Exercise3.RData`.

Name	Description	Type	Unit
`date`	Date and Time	string	Date, hour, min and sec
`t`	Time since start of data set	Integer	Hour
`yTi1`	Temperature 1	Continuous	C°
`yTi2`	Temperature 2	Continuous	C°
`yTi3`	Temperature 3	Continuous	C°
`yTi4`	Temperature 4	Continuous	C°
`Ta`	Ambient temperature	Continuous	C°
`Gv`	Global Horizontal Solar Radiation	Continuous	W/m^2
`Ph1`	Heating power in northern circuit	Continuous	W
`Ph2`	Heating power in southern circuit	Continuous	W

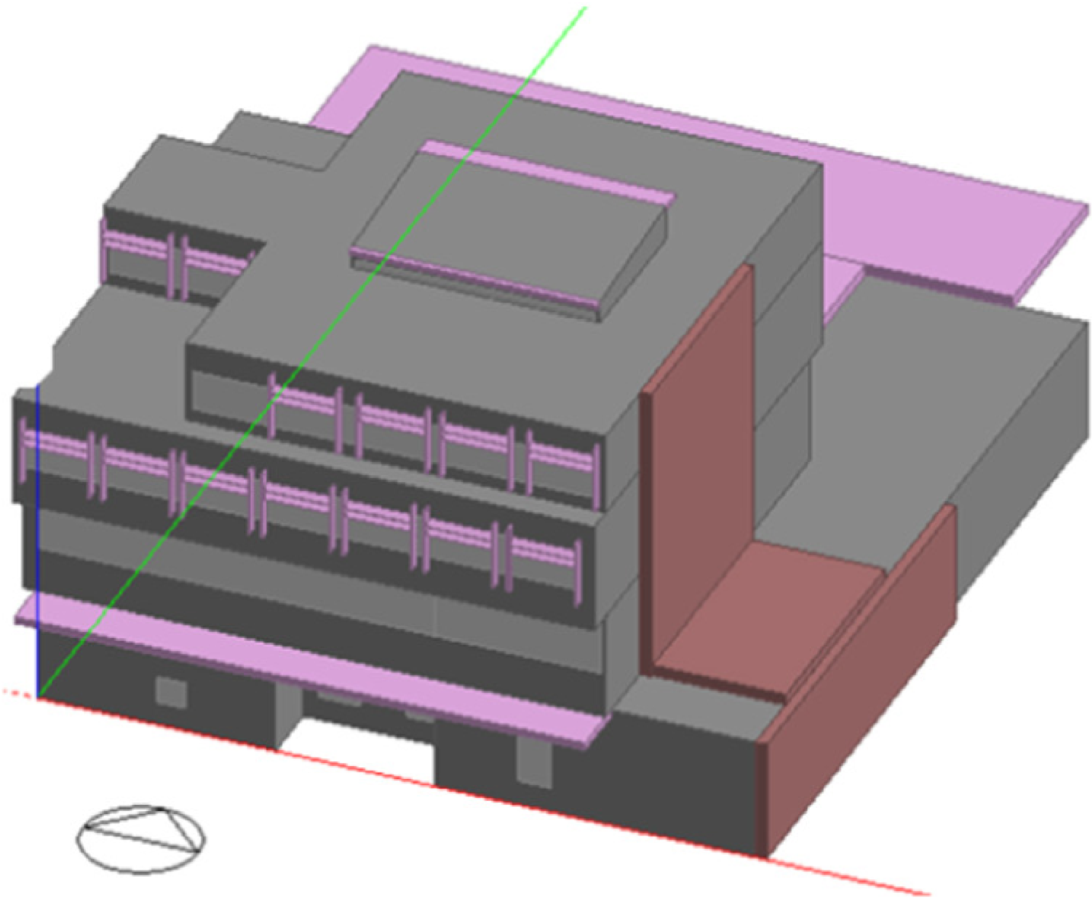

Figure 1: Facade of east-most wing of the building.

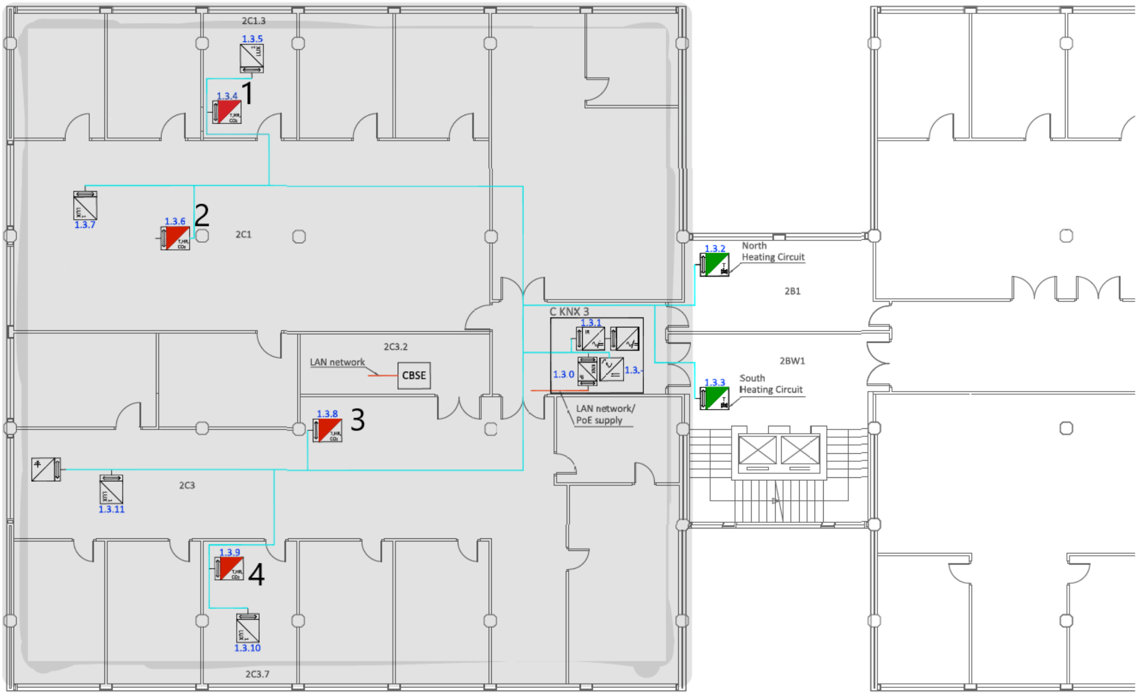

Figure 2: North facing floor plan of the second floor of the east-most wing.

Overall purpose

The following tasks should be seen as guidelines and you are encouraged to try your own ideas and report your findings from these. Even if your ideas do not provide great results you should still include them accompanied by thoughts on what you think went wrong, and what kind of data that could make them work. You should *always* start your analysis by exploring the data through visualisation. This is even more important when working with grey-box models, where often a look into the behaviour of certain variables combined with physical knowledge of the system will enable great improvements of your models. The plots should be included in the report if they provide useful information for the reader, and left out otherwise.

For some of the models it might require substantial amounts of time to estimate the parameters. To reduce this time you might want to only use part of the data for estimation. When choosing your data keep in mind that large temperature differences makes it easier to estimate heat transfer parameters, and furthermore it is not possible to estimate parameters related to the heating, if the heating is not turned on in the period of the data you are using. For these reasons winter periods are usually preferred when modelling buildings. On the other hand parameters related to solar radiation are easier to estimate during the summer when the sun is shining. Another trick to reduce the computational burden is to choose sensible starting guesses for the parameters. Since you will gradually extend your models it makes sense to note the estimated parameters for the simpler models and use these as starting guesses for the more complicated model structures. Sometimes the starting guesses make the difference between finding a fit and the optimizer giving up.

Task 2a: 2-state model of single room

Before modelling all four rooms choose a single room to model. An example for room 1 is provided in the R-script `ExampleRoom1.R`, that estimates the model given by:

$$
\begin{aligned}
dT_i &= \frac{1}{C_i}\left(\frac{1}{R_{ia}}(T_a - T_i) + \frac{1}{R_{im}}(T_m - T_i) + \Phi + A_w G_v\right) dt + \sigma_1 dw_1 \\
dT_m &= \frac{1}{C_m}\left(\frac{1}{R_{im}}(T_i - T_m)\right) dt + \sigma_2 dw_2. \\
yT_i &= T_i + e_1
\end{aligned}
$$

Here T_i and T_m are the temperatures of the air and thermal mass of the room respectively. C_i and C_m are the corresponding heat capacities, while R_{im} is the thermal resistance between the two. R_{ia} is the thermal resistance from the internal air and the outside. Φ is the effect of the heating device and A_w the effective window area, used to apply the solar radiation, G_v. Only the air temperature is measured. Notice how you can easily change the data used, so that instead the model is fitted to one of the other rooms.

This model fits decently, but there are a lot of issues. One of them is that the effective window area is assumed constant, but as the earth and sun circles around each other, the angle with which the radiation hits the windows changes. This can be incorporated in the model by estimating the effective window area as a function of time, for example using splines. Your first task is to **modify `sdeTiTm.R` to estimate the model given by:**

$$
\begin{aligned}
dT_i &= \frac{1}{C_i}\left(\frac{1}{R_{ia}}(T_a - T_i) + \frac{1}{R_{im}}(T_m - T_i) + \Phi + \left(\sum_{k=1}^{N} a_k bs_k(t)\right) G_v\right) dt + \sigma_1 dw_1 \\
dT_m &= \frac{1}{C_m}\left(\frac{1}{R_{im}}(T_i - T_m)\right) dt + \sigma_2 dw_2. \\
yT_i &= T_i + e_1,
\end{aligned}
$$

where bs_i are basis spline functions.

Task 2b: Improving the single-room model

In this section your task is to **Improve the model from Task 2a**. There are many possibilities for doing this, for example including more states describing temperatures of the radiators or wall envelopes. The type and amount of splines can be tweaked as well. You could also include the temperature of the nearby room as a boundary condition. You should also analyse the various ways that each state can be modelled. For example the previous model assumed that none of the heat from radiator and radiation entered directly into the thermal mass, but we could equally well have made a fraction enter the thermal mass directly. Some of your extensions will be useful and others will not, remember that your goal is to improve the model.

Task 3b: Making a multi-room model

In this section your task is to **make a multi-room model that describes the heat dynamics of the 4 rooms that we got measurements from.**

The main difficulties related to this are the heavy computations required to fit large models, and issues of identifying the parameters. To limit these problems you will have to figure out how you can simplify the models, while still being able to describe the heat dynamics adequately. Consider what parameters that that could be assumed constant between rooms. Make sure to consult the floor plan, be creative and use your common sense.