4th Assignment: Wind Power modelling: Non-linear regression, Markov-Switching and Adaptive CTSM

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MANGLER: Håndtér NaN globalt

1. Introduction

This report focuses on modelling the wind power production of the wind farm in Klim. It will focus solely on the 1h ahead prediction horizon, but the models can easily be generalized for different horizons (and tied beneficially together using temporal hierarchies). This report sets out to apply many different models (for the sake of learning) instead of going very deep, comprehensivly into few models.

2. Explorative Data Analysis

First, let's load the data and do some explorative data analysis. The variable of particular interest is the power p.

Note, the strong time serial dependance in p. Particularly the AR(1)-term carries a lot of information.

Now let's analyze p in relation to the explanatory variables.

Plot Power vs (normalized) Ttime-of-Year

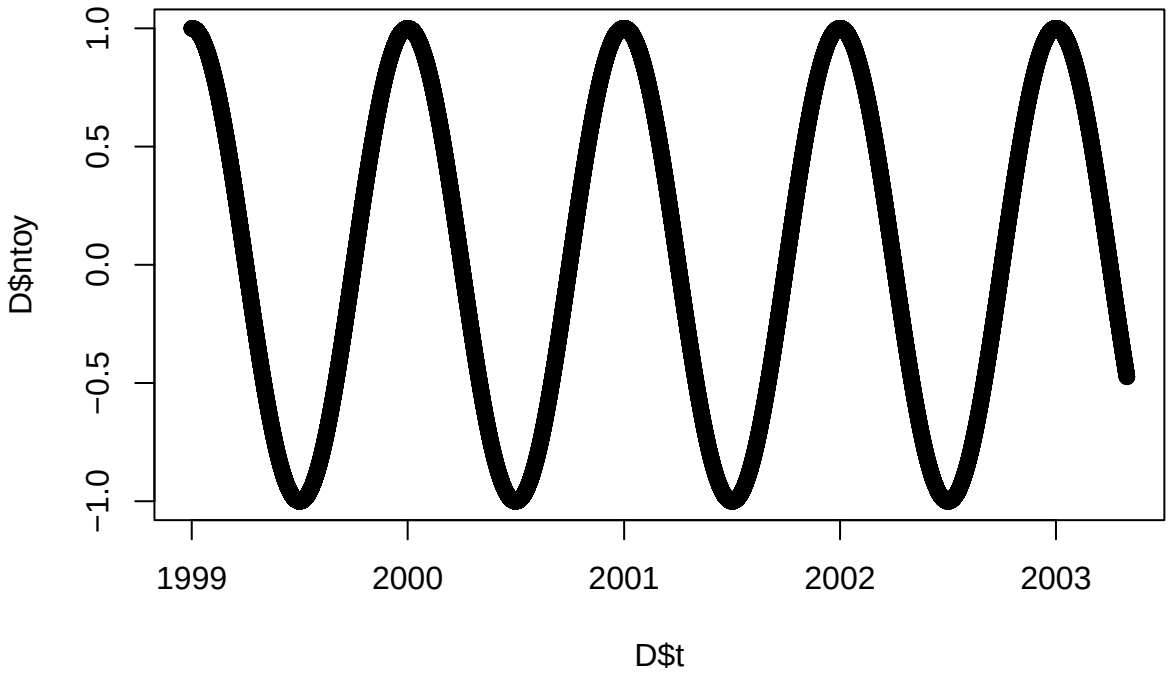
In order to utilize the time-of-year variable in a (linear and non-linear) regression setting, I normalize the time-of-year variable by

$$toy_{norm} = cos(2\pi \cdot \frac{toy}{365})$$

Thereby linking December and January close together at a value near 1, while in the summer time the value is close to -1.

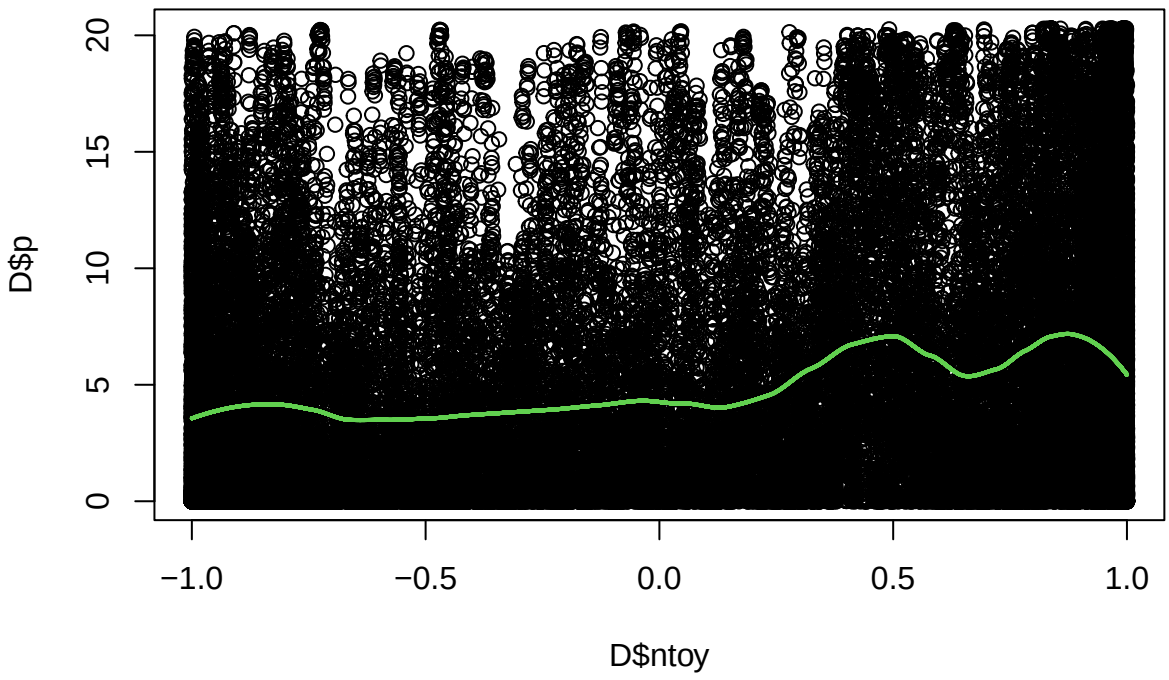
```
# Define normalized time of year: Ratio between 0 and 1 indicating the extent of winter (end of year)
D$ntoy = cos(2*pi*D$toy/365)

plot(D$t, D$ntoy) # Utilise the time-of-year variable as follows, decsribing the degree of winter.
```




```
plot(D$ntoy, D$p)

loessfit_ntoy = loess(p ~ ntoy, dat=D, span=0.3, family='gaussian', degree=2)
lines(D$ntoy[!is.na(D$p)], predict(loessfit_ntoy, ntoy=list(D$ntoy)), col=3, lwd=2)
```



In order to be able to interpret the density of the points, I've added local polynomial regression smoothing using a Gaussian kernel of size 30 %. The figures indicates higher power productions in winter time.

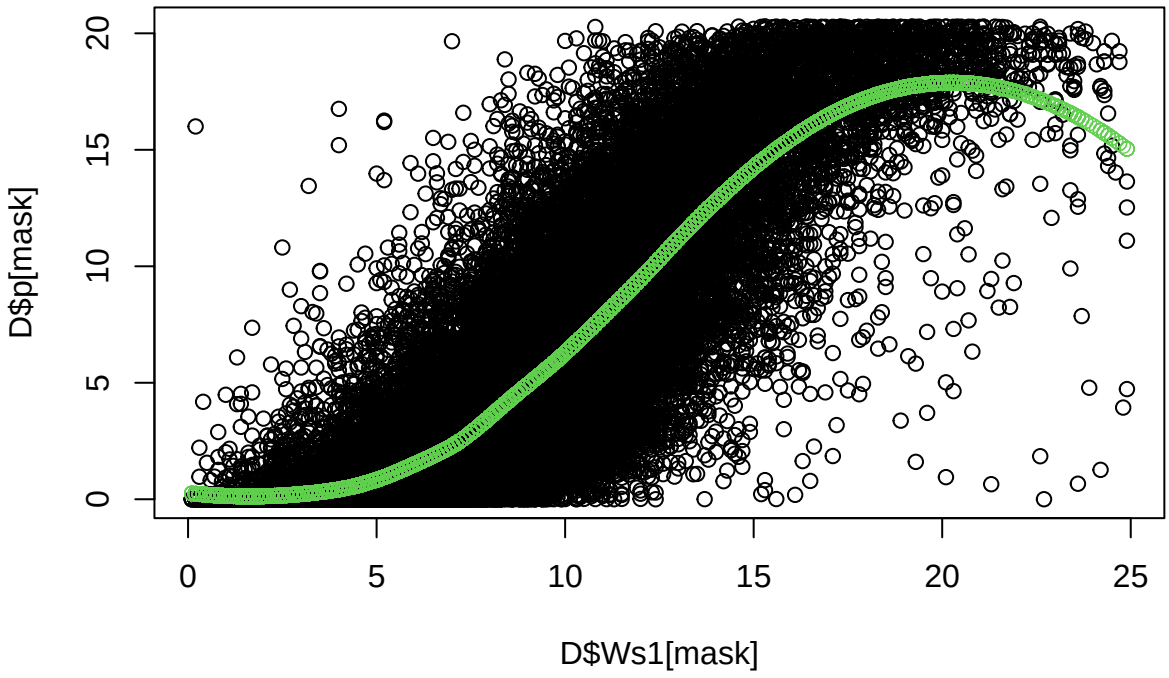
Power vs. Wind Speed

Similarly, let's scatter the data and make a non-parametric fit for the forecasted wind speed (1h ahead).

```
library(ggplot2); library(RColorBrewer)
rf <- colorRampPalette(rev(brewer.pal(11,'Spectral')))
r <- rf(32)

mask = !is.na(D$Ws1) & !is.na(D$p) & D$Ws1<25 # Mask non-NaN

plot(D$Ws1[mask], D$p[mask])
points(D$Ws1[mask], predict(loess(p ~ Ws1, dat=D[mask,], span=0.3, family='gaussian', degree=2), Ws1=lis
```



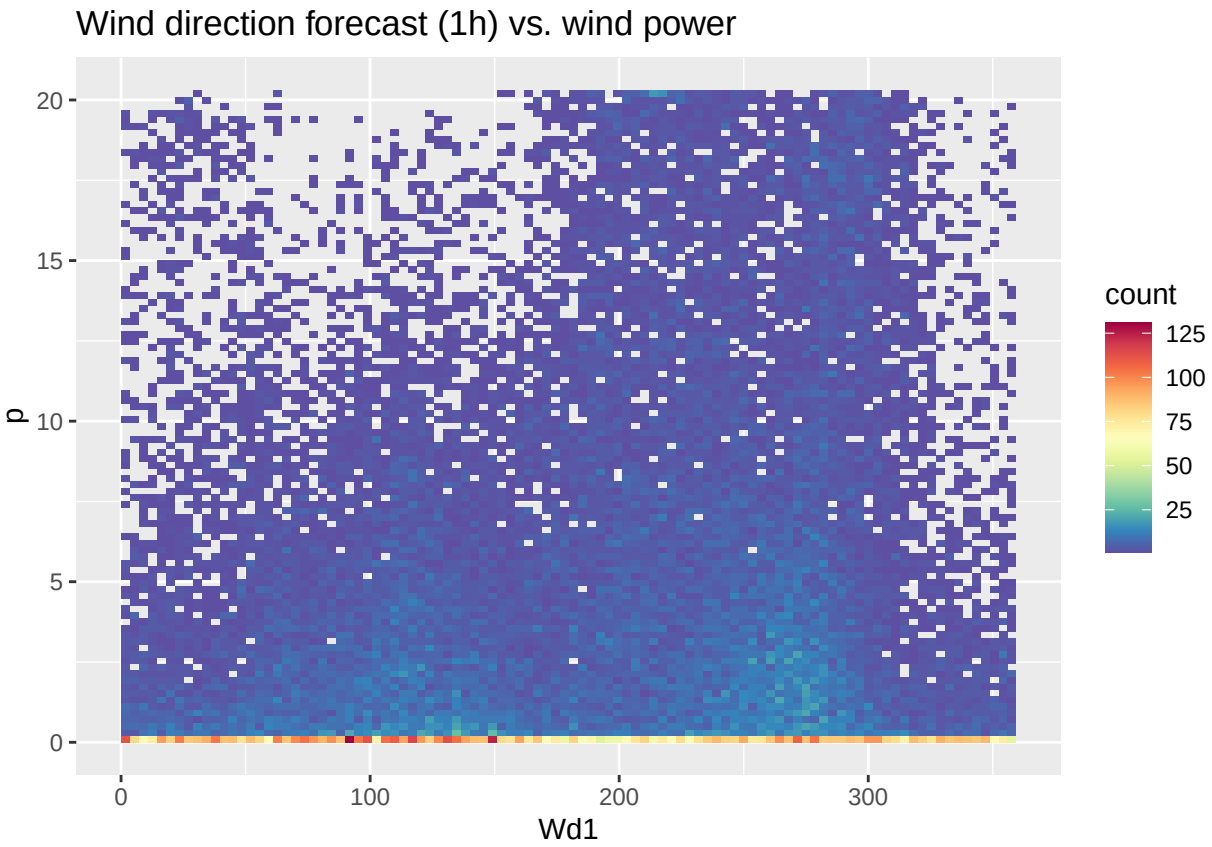

```
#p <- ggplot(D, aes(Ws1,p))
# Add colouring and change bins
#library(RColorBrewer)
#h3 <- p + stat_bin2d(bins=100) + scale_fill_gradientn(colours=r) + ggtitle(sprintf("Wind speed forecas
#h3
```

There seem to be somewhat of an logistic effect of wind speed on the power, i.e. the power increases exponentially in the beginning but becomes rather saturated with more and more wind.

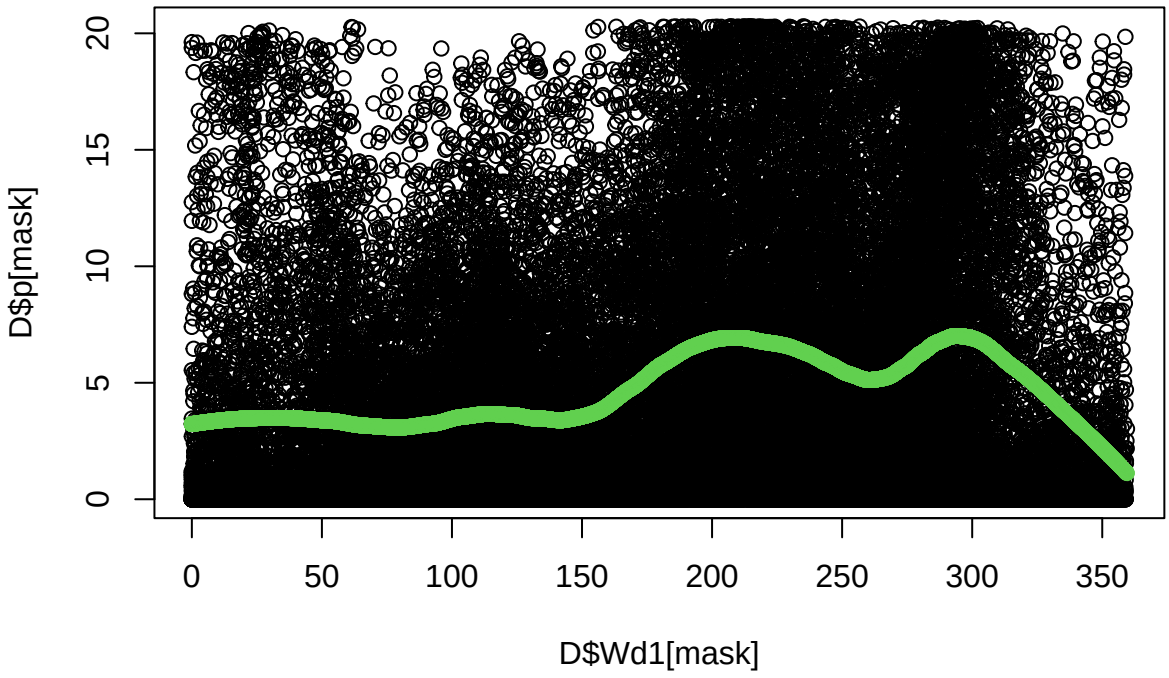
Power vs Wind Direction

```
# Wind direction
p <- ggplot(D, aes(Wd1,p))
h4 <- p + stat_bin2d(bins=100) + scale_fill_gradientn(colours=rf(32)) + ggtitle(sprintf("Wind direction
h4
```

```
## Warning: Removed 1654 rows containing non-finite values (stat_bin2d).
```




```
mask = !is.na(D$Wd1) & !is.na(D$p)# Mask non-NaN
plot(D$Wd1[mask], D$p[mask])
loessfit_Wd1 = loess(p ~ Wd1, dat=D[mask,], span=0.3, family='gaussian', degree=2)
points(D$Wd1[mask], predict(loessfit_Wd1, Ws1=list(D$Wd1[mask])), col=3, lwd=0.7)
```



It seems to be favourable for the wind power production if the wind direction is around 200 or 300 degrees. These findings will be utilised in the modelling section later.

2. Simple models

For benchmarking, some simple models are fitted.

```
# Mean Model
fit1 = lm(p ~ 1, data=D); summary(fit1); logLik(fit1)
```

```
##
## Call:
## lm(formula = p ~ 1, data = D)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -4.912 -4.632 -2.089  2.938 15.388
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  4.91214    0.02832   173.4   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
```

```
## Residual standard error: 5.491 on 37582 degrees of freedom
##   (358 observations deleted due to missingness)

## 'log Lik.' -117332.2 (df=2)
```

```
# AR(1)
fit2 = lm(p[2:n] ~ 0 + p[1:(n-1)], data=D); summary(fit2); logLik(fit2)
```

```
##
## Call:
## lm(formula = p[2:n] ~ 0 + p[1:(n - 1)], data = D)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -13.0031  -0.4602   0.0004   0.7125  15.2276
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## p[1:(n - 1)] 0.974622   0.001152   846.2   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.645 on 37576 degrees of freedom
##   (363 observations deleted due to missingness)
## Multiple R-squared:  0.9501, Adjusted R-squared:  0.9501
## F-statistic: 7.16e+05 on 1 and 37576 DF,  p-value: < 2.2e-16

## 'log Lik.' -72021.53 (df=2)
```

```
# Linear Regression
fit3a = lm(p ~ Ws1 + Wd1 + T1 + ntoy, data=D) ; summary(fit3a); logLik(fit3a) # Chosen var
```

```
##
## Call:
## lm(formula = p ~ Ws1 + Wd1 + T1 + ntoy, data = D)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -34.765  -1.942  -0.241   1.714  19.901
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.9346027  1.1610573   0.805    0.421
## Ws1          1.0913805  0.0041138 265.299  < 2e-16 ***
## Wd1          0.0014308  0.0001788   8.003 1.25e-15 ***
## T1          -0.0192123  0.0041398  -4.641 3.48e-06 ***
## ntoy        -0.2531055  0.0367966  -6.878 6.15e-12 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.082 on 36282 degrees of freedom
##   (1654 observations deleted due to missingness)
## Multiple R-squared:  0.6856, Adjusted R-squared:  0.6856
## F-statistic: 1.978e+04 on 4 and 36282 DF,  p-value: < 2.2e-16
```

```
## 'log Lik.' -92327.35 (df=6)
```

```
fit3b = lm(p ~ ., data=D[,-1]) ; summary(fit3b); logLik(fit3b) # All var
```

```
##
## Call:
## lm(formula = p ~ ., data = D[, -1])
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -35.207  -1.919  -0.246   1.690  19.844
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) -8.4192875  1.2594497  -6.685 2.34e-11 ***
## toy         -0.0032999  0.0001699 -19.421  < 2e-16 ***
## Ws1          0.5442065  0.0360356  15.102  < 2e-16 ***
## Wd1         -0.0002223  0.0008518  -0.261 0.794153
## T1           0.2627207  0.0755560   3.477 0.000507 ***
## Ws2          0.1061343  0.0510689   2.078 0.037693 *
## Wd2          0.0003175  0.0012187   0.261 0.794464
## T2          -0.0709400  0.1068594  -0.664 0.506782
## Ws3          0.4412830  0.0363411  12.143  < 2e-16 ***
## Wd3          0.0008758  0.0008821   0.993 0.320750
## T3          -0.1755018  0.0760965  -2.306 0.021099 *
## ntoy        -0.0499948  0.0384058  -1.302 0.193010
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.049 on 36271 degrees of freedom
##   (1658 observations deleted due to missingness)
## Multiple R-squared:  0.6924, Adjusted R-squared:  0.6923
## F-statistic:  7423 on 11 and 36271 DF,  p-value: < 2.2e-16
```

```
## 'log Lik.' -91922.78 (df=13)
```

```
# AR(1)X
fit4a = lm(p ~ Ws1 + Wd1 + T1 + ntoy + D$p[-n], data=D[2:n,-1]); summary(fit4a); logLik(fit4a) # Chosen
```

```
##
## Call:
## lm(formula = p ~ Ws1 + Wd1 + T1 + ntoy + D$p[-n], data = D[2:n,
##     -1])
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -14.4771  -0.6775  -0.0123   0.5688  13.1028
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  2.946e-01  6.015e-01   0.490   0.6242
## Ws1          1.652e-01  3.635e-03  45.432   <2e-16 ***
```

```
## Wd1         -2.255e-04  9.276e-05  -2.431   0.0151 *
## T1          -3.220e-03  2.145e-03  -1.501   0.1334
## ntoy        -5.043e-02  1.907e-02  -2.644   0.0082 **
## D$p[-n]      8.522e-01  2.710e-03 314.488   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.596 on 36275 degrees of freedom
##   (1659 observations deleted due to missingness)
## Multiple R-squared:  0.9156, Adjusted R-squared:  0.9156
## F-statistic: 7.874e+04 on 5 and 36275 DF,  p-value: < 2.2e-16

## 'log Lik.' -68446.46 (df=7)
```

```
fit4b = lm(p ~ . + D$p[-n], data=D[2:n,-1]); summary(fit4b); logLik(fit4b) # All var
```

```
##
## Call:
## lm(formula = p ~ . + D$p[-n], data = D[2:n, -1])
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -14.4316  -0.6804  -0.0152   0.5746  13.1566
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) -1.178e+00  6.594e-01  -1.786  0.07408 .
## toy         -5.368e-04  8.937e-05  -6.006 1.92e-09 ***
## Ws1          1.282e-01  1.890e-02   6.782 1.20e-11 ***
## Wd1         -3.268e-04  4.457e-04  -0.733  0.46340
## T1           7.530e-02  3.955e-02   1.904  0.05694 .
## Ws2         -1.901e-02  2.673e-02  -0.711  0.47702
## Wd2          7.975e-05  6.377e-04   0.125  0.90047
## T2          -3.970e-03  5.592e-02  -0.071  0.94341
## Ws3          5.874e-02  1.906e-02   3.082  0.00206 **
## Wd3         -5.155e-05  4.615e-04  -0.112  0.91106
## T3          -6.898e-02  3.982e-02  -1.732  0.08321 .
## ntoy        -1.940e-02  2.010e-02  -0.965  0.33434
## D$p[-n]      8.493e-01  2.739e-03 310.125  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.595 on 36264 degrees of freedom
##   (1663 observations deleted due to missingness)
## Multiple R-squared:  0.9158, Adjusted R-squared:  0.9157
## F-statistic: 3.286e+04 on 12 and 36264 DF,  p-value: < 2.2e-16

## 'log Lik.' -68409.59 (df=14)
```

Clearly, it has a *very* large improvement in log-likelihood when an AR(1)-term is present in the model.

Let's try expanding the model with the non-linearities as found in the explorative data analysis. This means:

1. The wind speed seem to affect the power in a s-curve, i.e. mause a logistic function on the wind speed, i.e. $p = ... + b \cdot \frac{e^{a \cdot WindSpeed}}{1+e^{a \cdot WindSpeed}}$
2. The normalized time-of-year had a non-linear affect on the power. Use the loess-fit linearly to predict power, i.e. $p = ... + c \cdot \hat{f}(ntoy)$, where $\hat{f}(ntoy)$ is the local regression smoother of power as a function of normalized time-of-year.
3. For the wind direction, utilise the loess-fit the same way as for normalized time-of-year.

Let's start with the logistic-fit in order to be able to see the benefit of such.

```
# Initially test nls()
#fit5 = nls(p ~ a0 + a1*Ws1 + a2*Wd1 + a3*T1 + a4*ntoy, start = list(a0=0, a1=0.5, a2=0.5, a3=0.1, a4=1
  # Nice, exact same results as for lm()

# OK let's actually do some non-linear fit: Using findings from the descriptive section
fit6 = nls(p ~ a0  + a11*logit(a1*Ws1) + a2*Wd1 + a3*T1 + a4*ntoy,
           start = list(a0=0, a1=0.1, a11=10, a2=0.5, a3=0.1, a4=1),
           data=D); summary(fit6); logLik(fit6);
```

```
##
## Formula: p ~ a0 + a11 * logit(a1 * Ws1) + a2 * Wd1 + a3 * T1 + a4 * ntoy
##
## Parameters:
##       Estimate Std. Error t value Pr(>|t|)
## a0  -2.088e+02  4.798e+01  -4.351 1.36e-05 ***
## a1   4.943e-03  1.072e-03   4.610 4.04e-06 ***
## a11  2.102e+02  4.787e+01   4.390 1.13e-05 ***
## a2   1.377e-03  1.791e-04   7.687 1.54e-14 ***
## a3  -2.004e-02  4.142e-03  -4.838 1.32e-06 ***
## a4  -2.639e-01  3.685e-02  -7.161 8.17e-13 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.08 on 36281 degrees of freedom
##
## Number of iterations to convergence: 36
## Achieved convergence tolerance: 7.895e-06
##   (1654 observations deleted due to missingness)

## 'log Lik.' -92311.65 (df=7)
```

I.e. only a slight improvement in likelihood from the simple linear regression model (fit3a).

Now let's expand the models further with the non-linear fit of wind direction and time of year.

```
# Let's use the non-parametric fit of Wind Direction
fit7 = nls(p ~ a0  + a11*logit(a1*Ws1) + a2*predict(loessfit_Wd1, Wd1) + a3*T1 + a4*predict(loessfit_nt
           start = list(a0=0, a1=0.1, a11=10, a2=0.5, a3=0.1, a4=0.1),
           data=D); summary(fit7); logLik(fit7);
```

```
##
## Formula: p ~ a0 + a11 * logit(a1 * Ws1) + a2 * predict(loessfit_Wd1, Wd1) +
```

```
##     a3 * T1 + a4 * predict(loessfit_ntoy, ntoy)
##
## Parameters:
##       Estimate Std. Error t value Pr(>|t|)
## a0  -2.161e+02  5.188e+01  -4.165 3.12e-05 ***
## a1   4.725e-03  1.084e-03   4.359 1.31e-05 ***
## a11  2.154e+02  5.178e+01   4.161 3.18e-05 ***
## a2   3.478e-01  1.127e-02  30.866  < 2e-16 ***
## a3  -1.566e-02  3.514e-03  -4.456 8.36e-06 ***
## a4  -8.969e-02  1.768e-02  -5.074 3.91e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.045 on 36281 degrees of freedom
##
## Number of iterations to convergence: 38
## Achieved convergence tolerance: 5.313e-06
##   (1654 observations deleted due to missingness)

## 'log Lik.' -91893.09 (df=7)
```

Again, a small improvement in likelihood. However, this complex with three non-linearities is still beaten by the AR(1)-model. So before moving on to another class of models, let's try adding the AR(1)-term.

```
plag1 = D$p[1:(n-1)]
# Let's use the non-parametric fit of Wind Direction
fit8 = nls(p ~ a0  + a11*logit(a1*Ws1) + a2*predict(loessfit_Wd1, Wd1) + a3*T1 + a4*predict(loessfit_nt
           start = list(a0=0, a1=0.1, a11=10, a2=0.5, a3=0.1, a4=0.1, a5=1),
           data=D[2:n,]); summary(fit8); logLik(fit8);
```

```
##
## Formula: p ~ a0 + a11 * logit(a1 * Ws1) + a2 * predict(loessfit_Wd1, Wd1) +
##     a3 * T1 + a4 * predict(loessfit_ntoy, ntoy) + a5 * plag1
##
## Parameters:
##       Estimate Std. Error t value Pr(>|t|)
## a0  -23.407409  14.111907  -1.659   0.0972 .
## a1    0.006541   0.003661   1.787   0.0740 .
## a11  23.520639  14.054393   1.674   0.0942 .
## a2    0.053883   0.005977   9.016   <2e-16 ***
## a3   -0.003092   0.001841  -1.680   0.0930 .
## a4   -0.018947   0.009260  -2.046   0.0408 *
## a5    0.847923   0.002737 309.804   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.595 on 36274 degrees of freedom
##
## Number of iterations to convergence: 26
## Achieved convergence tolerance: 8.674e-06
##   (1659 observations deleted due to missingness)

## 'log Lik.' -68408.45 (df=8)
```

We now see a large improvement in likelihood. Let's compare it with the corresponding model AR(1)X model with the same variables but without non-linearities:

```
AIC(fit4a)
```

```
## [1] 136906.9
```

```
AIC(fit8)
```

```
## [1] 136832.9
```

I.e. a significant improvement.

3. Autoregressive Markov-Switching Model

Looking at the power over time, it is possible to identify some different states, e.g. periods of constant power production, periods of high volatility etc. This indicates that a state space model might be suitable for the problem. As inspired by (**A Markov-Switching model for building occupant activity estimation**, Wolf, Møller et. al., 2018), I will try a generalization of the Hidden Markov Model where the states determine the autoregressive coefficients on the observations and linear coefficients on the input data specific to each state, i.e. state-specific AR- and linear coefficients. I will utilise the implementation found on **https://cran.r-project.org/web/packages/MSwM/**.

In the following section, I will for simplicity only use Ws1, Wd1, T1 and ntoy as input variables (corresponding to fit3a). Additionally, I will only use an AR(1)-term as was found to be of far greatest importance as seen in the pacf. Instead I will vary the number of states. Note that the models are nested, so for model comparison likelihood-ratio test is appropriate.

```
  #install.packages('MSwM')
par(mfrow = c(1,1))
library(parallel)
library(MSwM)

# Simple Linear Regression
fit2b = lm(p ~ Ws1 + Wd1 + T1 + ntoy, data=D) ; summary(fit2b); logLik(fit2b)
```

```
##
## Call:
## lm(formula = p ~ Ws1 + Wd1 + T1 + ntoy, data = D)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -34.765  -1.942  -0.241   1.714  19.901
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.9346027  1.1610573   0.805    0.421
## Ws1          1.0913805  0.0041138 265.299  < 2e-16 ***
## Wd1          0.0014308  0.0001788   8.003 1.25e-15 ***
## T1          -0.0192123  0.0041398  -4.641 3.48e-06 ***
## ntoy        -0.2531055  0.0367966  -6.878 6.15e-12 ***
```

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.082 on 36282 degrees of freedom
##   (1654 observations deleted due to missingness)
## Multiple R-squared:  0.6856, Adjusted R-squared:  0.6856
## F-statistic: 1.978e+04 on 4 and 36282 DF,  p-value: < 2.2e-16

## 'log Lik.' -92327.35 (df=6)
```

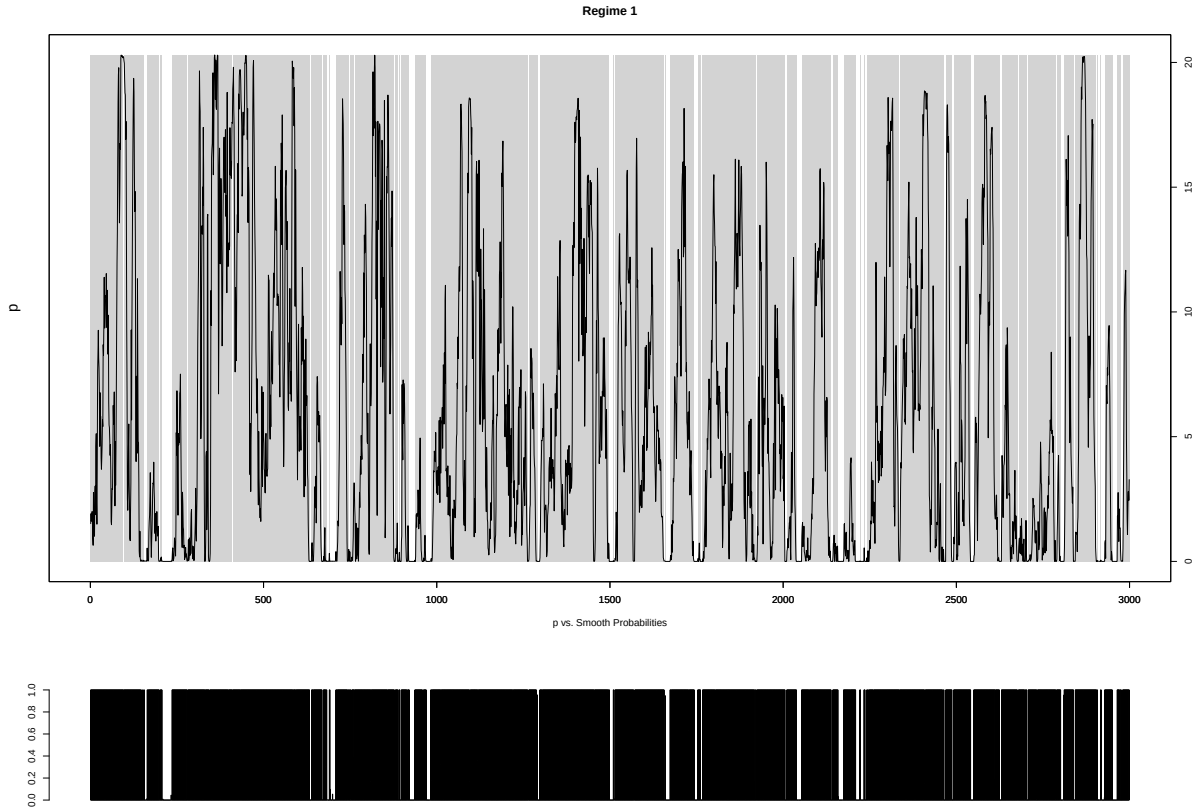
```
# Fit MSM
fit_msm2 = msmFit(fit2b, k=2, p=1, sw=rep(TRUE,7), control = list(parallel = TRUE))
summary(fit_msm2)
```

```
## Markov Switching Model
##
## Call: msmFit(object = fit2b, k = 2, sw = rep(TRUE, 7), p = 1, control = list(parallel = TRUE))
##
##        AIC      BIC    logLik
##   84126.34 84354.32 -42051.17
##
## Coefficients:
##
## Regime 1
## ---------
##                Estimate Std. Error t value Pr(>|t|)
## (Intercept)(S)   0.0052     0.0033  1.5758   0.1151
## Ws1(S)           0.0000     0.0000     NaN      NaN
## Wd1(S)           0.0000     0.0000     NaN      NaN
## T1(S)            0.0000     0.0000     NaN      NaN
## ntoy(S)         -0.0001     0.0001 -1.0000   0.3173
## p_1(S)           1.0000     0.0000     Inf   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.002174313
## Multiple R-squared:     1
##
## Standardized Residuals:
##          Min           Q1          Med           Q3          Max
## -0.008888291  0.000000000  0.000000000  0.000000000  0.008821492
##
## Regime 2
## ---------
##                Estimate Std. Error  t value Pr(>|t|)
## (Intercept)(S)   0.5379     0.8147   0.6602   0.5091
## Ws1(S)           0.1882     0.0033  57.0303   <2e-16 ***
## Wd1(S)          -0.0003     0.0001  -3.0000   0.0027 **
## T1(S)           -0.0046     0.0029  -1.5862   0.1127
## ntoy(S)         -0.0693     0.0031 -22.3548   <2e-16 ***
## p_1(S)           0.8407     0.0030 280.2333   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
##
## Residual standard error: 1.724234
## Multiple R-squared: 0.9017
##
## Standardized Residuals:
##           Min            Q1           Med            Q3           Max
## -1.468619e+01 -6.642497e-01  2.479902e-04  5.807938e-01  1.287982e+01
##
## Transition probabilities:
##           Regime 1   Regime 2
## Regime 1 0.8350077 0.02807167
## Regime 2 0.1649923 0.97192833
```

```
#plotProb(fit_msm, which=1)
#plotProb(fit_msm, which=2)
#plotProb(fit_msm, which=3)
#plot(msmResid(fit_msm))
#plotReg(fit_msm)

# It's impossible to grasp the fit for the ~40000 observations.
# To assess the fit, fit and visualise the same model on a small subset of the data
Dsub = D[1:3000, ]
fit2bsub = lm(p ~ Ws1 + Wd1 + T1 + ntoy, data=Dsub)
fit_msm2_sub = msmFit(fit2bsub, k=2, p=1, sw=rep(TRUE,7), control = list(parallel = TRUE))
plotProb(fit_msm2_sub, which=2)
```



We see a very large improvement in likelihood and thus also AIC when compared to the best model in the previous section.

Note that in order to visualise the states, I've fitted the same model on a subset of the data. The global decoding, i.e. the most probable regime at time step t as seen above, seem to correspond to periods with constant wind power (regime 1) and periods with changing wind power (regime 2). This corresponds well with what we could expect of a state-dependent AR-coefficient: That one regime would correspond to keeping things constant.

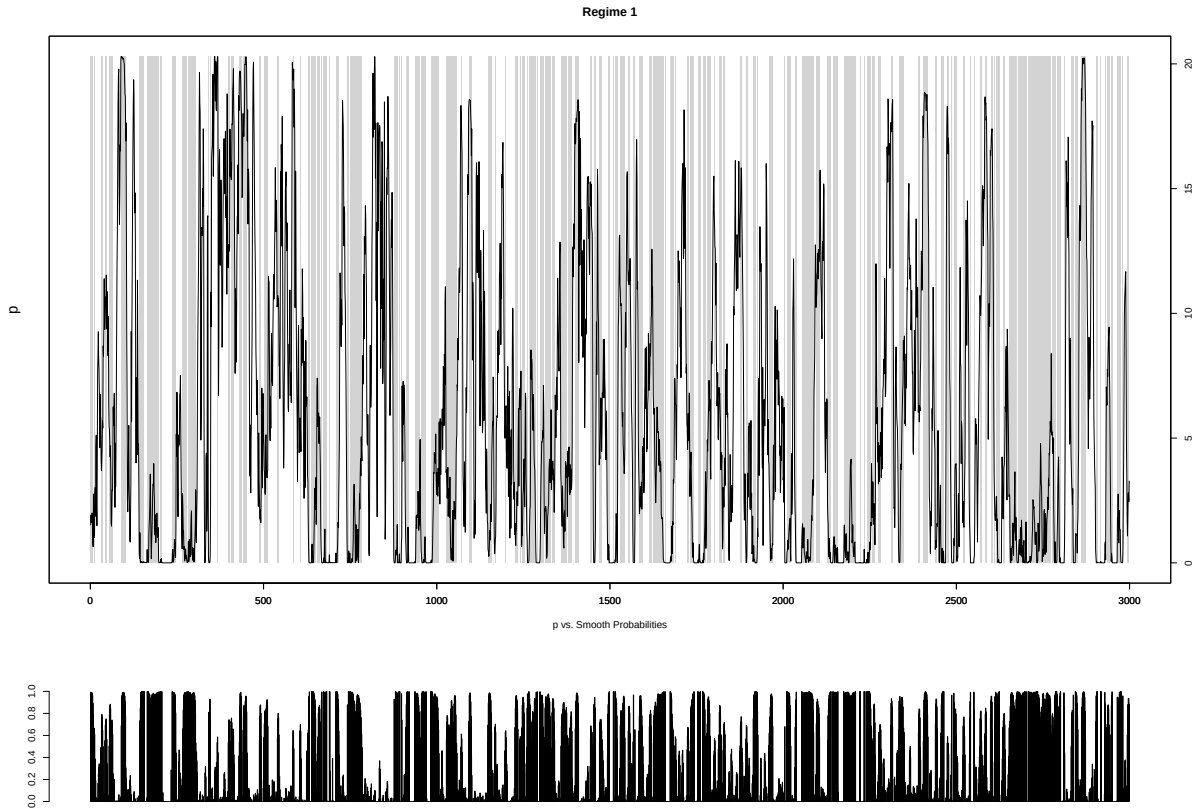
Now it seems obvious to add another state, hopefully separating dynamics of the wind power when in decline / increasing.

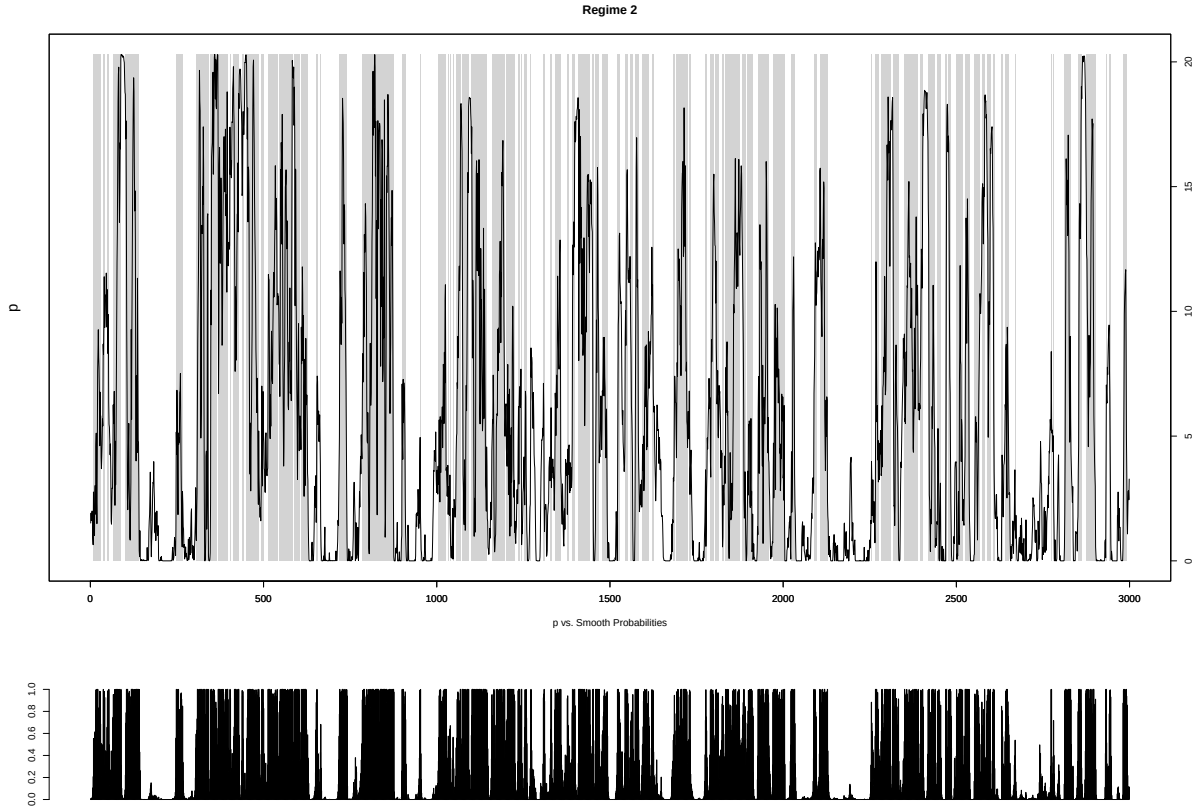
```
# Fit MSM-AR with k=3 states
fit_msm3 = msmFit(fit2b, k=3, p=1, sw=rep(TRUE,7), control = list(parallel = TRUE))
summary(fit_msm3)
```

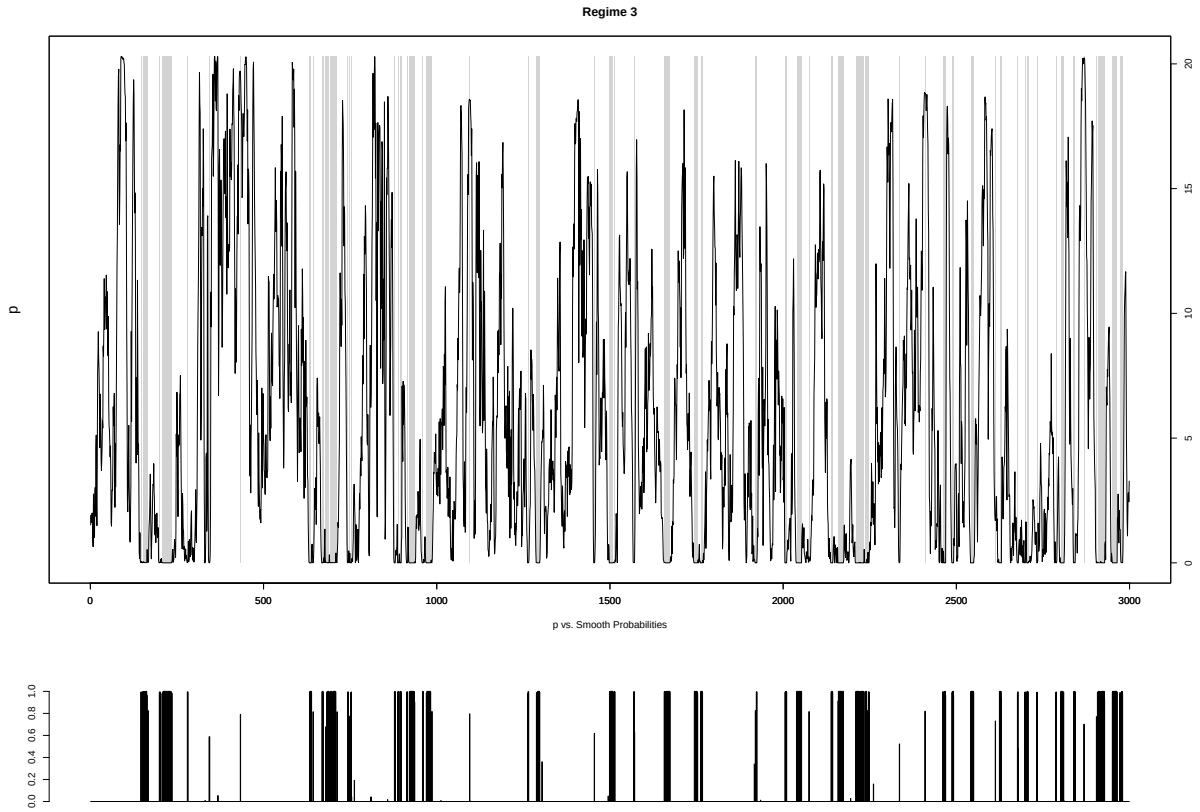
```
## Markov Switching Model
##
## Call: msmFit(object = fit2b, k = 3, sw = rep(TRUE, 7), p = 1, control = list(parallel = TRUE))
##
##        AIC      BIC    logLik
##   72026.76 72368.73 -35995.38
##
## Coefficients:
##
## Regime 1
## ---------
##                Estimate Std. Error  t value  Pr(>|t|)
## (Intercept)(S)   1.2508     0.2703   4.6275 3.701e-06 ***
## Ws1(S)           0.2397     0.0074  32.3919 < 2.2e-16 ***
## Wd1(S)          -0.0006     0.0001  -6.0000 1.973e-09 ***
## T1(S)           -0.0070     0.0009  -7.7778 7.327e-15 ***
## ntoy(S)         -0.0972     0.0223  -4.3587 1.308e-05 ***
## p_1(S)           0.7810     0.0050 156.2000 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.161971
## Multiple R-squared: 0.8136
##
## Standardized Residuals:
##           Min            Q1           Med            Q3           Max
## -15.004969251  -0.412950343  -0.001395152   0.264660886  12.421354699
##
## Regime 2
## ---------
##                Estimate Std. Error t value  Pr(>|t|)
## (Intercept)(S)   0.0043     0.0007  6.1429 8.103e-10 ***
## Ws1(S)           0.0000     0.0000     NaN       NaN
## Wd1(S)           0.0000     0.0000     NaN       NaN
## T1(S)            0.0000     0.0000     NaN       NaN
## ntoy(S)         -0.0001     0.0000    -Inf < 2.2e-16 ***
## p_1(S)           1.0000     0.0000     Inf < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
##
## Residual standard error: 0.001878056
## Multiple R-squared:     1
##
## Standardized Residuals:
##          Min           Q1          Med           Q3          Max
## -0.006950869  0.000000000  0.000000000  0.000000000  0.006884401
##
## Regime 3
## ---------
##                Estimate Std. Error  t value  Pr(>|t|)
## (Intercept)(S)  -0.3908     0.0570  -6.8561 7.077e-12 ***
## Ws1(S)           0.0309     0.0032   9.6562 < 2.2e-16 ***
## Wd1(S)          -0.0001     0.0000     -Inf < 2.2e-16 ***
## T1(S)            0.0009     0.0002   4.5000 6.795e-06 ***
## ntoy(S)          0.0026     0.0004   6.5000 8.032e-11 ***
## p_1(S)           0.9715     0.0025 388.6000 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.5693734
## Multiple R-squared: 0.9887
##
## Standardized Residuals:
##           Min            Q1           Med            Q3           Max
## -2.4621662423 -0.1116031585  0.0003036318  0.1180282771  1.5915755279
##
## Transition probabilities:
##              Regime 1    Regime 2   Regime 3
## Regime 1 0.9153436421 0.002217257 0.11886934
## Regime 2 0.0006219864 0.833494285 0.06500638
## Regime 3 0.0840343715 0.164288458 0.81612428
```

```r
# Plot subset
fit_msm3_sub = msmFit(fit2bsub, k=3, p=1, sw=rep(TRUE,7), control = list(parallel = TRUE))
for (i in 2:4){
  plotProb(fit_msm3_sub, which=i)
}
```


Regime 1
p
0
500
1000
1500
2000
2500
3000
0
5
10
15
20
p vs. Smooth Probabilities
0.0
0.2
0.4
0.6
0.8
1.0


Regime 2
p
0
500
1000
1500
2000
2500
3000
0
5
10
15
20
p vs. Smooth Probabilities
0.0
0.2
0.4
0.6
0.8
1.0



Again, a large improvement in likelihood with the extra state. Is the likelihood gain worth the extra parameters? I will assess that using likelihood ratio test in the end of this section.

Now regime 1 corresponds to a constant period, while regime 2 and 3 seem to correspond to periods high and low power production respectively.

Let's try add another state

```
# Fit MSM-AR for k=4 states
fit_msm4 = msmFit(fit2b, k=4, p=1, sw=rep(TRUE,7), control = list(parallel = FALSE))


# Plot subset
fit_msm4_sub = msmFit(fit2bsub, k=4, p=1, sw=rep(TRUE,7), control = list(parallel = FALSE))
for (i in 2:5){
  plotProb(fit_msm4_sub, which=i)
}
```

However, this kills the kernel on my computer every time. This might be due to running out of memory (https://www.r-bloggers.com/2019/02/switching-regressions-cluster-time-series-data-and-understand-your-development/). To solve this issue, one might subset the data. However, this makes the likelihood values incompatible. For this assignment, a 3-state Markov-Switching AR model is sufficient.

Likelihood ratio tests

```
L1 = logLik(fit4a)[1]; df1 = 7 # 1-state MSM-AR / AR(1)X
L2 = -42051.17; df2 = 7*2+(2*2)-2; # 2-state
L3 = -35995.38; df3 = 7*(3*3)-3; # 3-state

sprintf('Likelihood ratio test of 2-state MSM-AR vs. non-linear 1-state regression: p-value = %.3f', 1-p
```

```
## [1] "Likelihood ratio test of 2-state MSM-AR vs. non-linear 1-state regression: p-value = 0.000"
```

```
sprintf('Likelihood ratio test of 2-state MSM-AR vs. non-linear 1-state regression: p-value = %.3f', 1-p
```

```
## [1] "Likelihood ratio test of 2-state MSM-AR vs. non-linear 1-state regression: p-value = 0.000"
```

```
sprintf('AIC of 3-state MSM-AR:  %2.f', 72026.76)
```

```
## [1] "AIC of 3-state MSM-AR:  72027"
```

```
sprintf('AIC of best benchmark model:  %3.f', AIC(fit8))
```

```
## [1] "AIC of best benchmark model:  136833"
```

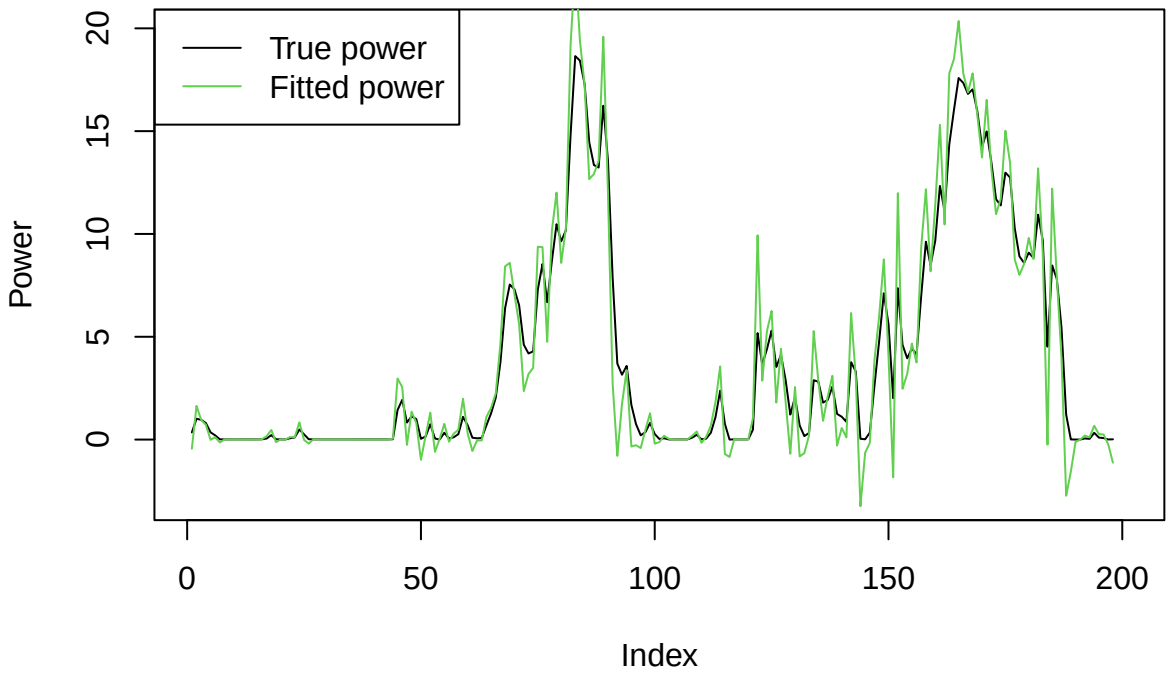
I.e. the Markov-Switching model is a much better fit - despite the extra parameters.

Finally, let's illustrate the last 200 (in-)samples of the fit:

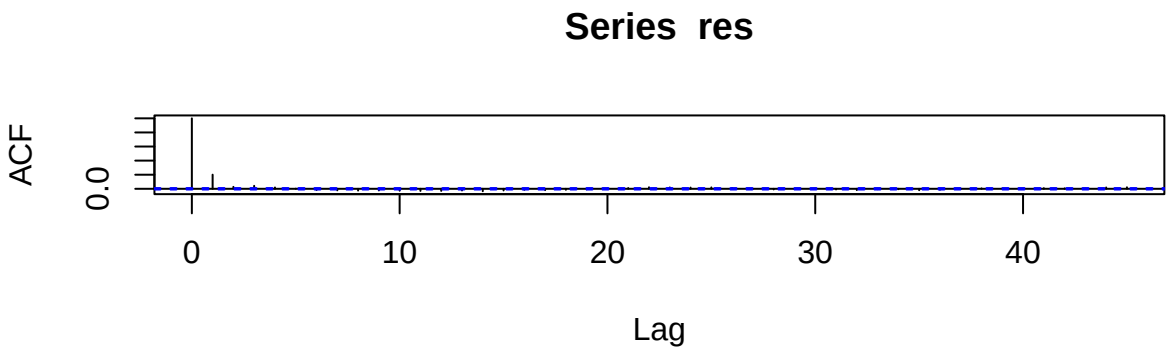
```
res = msmResid(fit_msm3)
n1 = length(res)
fittedp = na.omit(D)$p + res[4:(n1-4)]
```

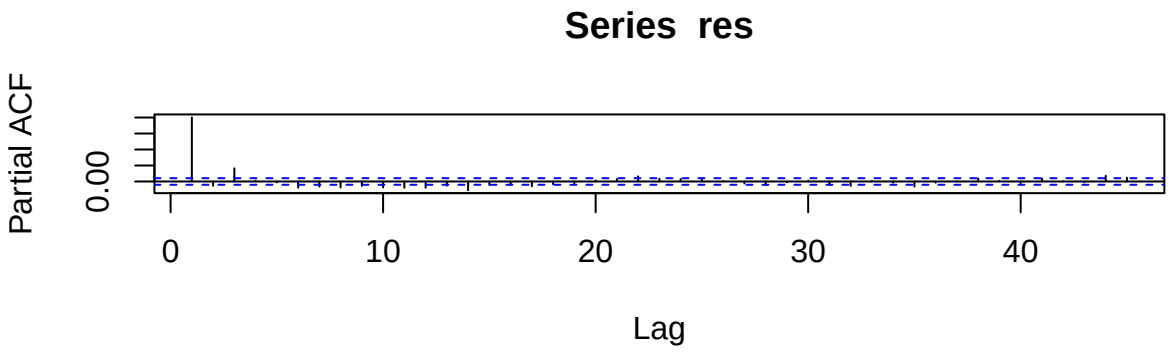
```
## Warning in na.omit(D)$p + res[4:(n1 - 4)]: longer object length is not a
## multiple of shorter object length
```

```
plot(na.omit(D)$p[(n1-200):n1], type='l', ylim=c(-3,20), ylab="Power")
lines(fittedp[(n1-200):n1], type='l', col=3)
legend("topleft",legend=c('True power', 'Fitted power'), lty=c(1,1), col=c(1,3))
```




```
par(mfrow=c(2,1))
acf(res)
pacf(res)
```





We see a reasonable fit, although there is a high variance in the predictions even resulting in negative (!) predictions of power. Also, there is still plenty more information in the residuals that should be modelled.

evt. Discrete State Space Form / Extended Kalman Filter (with adaptivity) (MANGLER)

$X_{t+1} = ...\theta_t X_t ...$ (unobserved) $\theta_{t+1} = \theta_t + ... + \epsilon_{t+1}$

$p_t = ...X + e_t$ (observed)

Continous-Descrete State Space Model (with adaptivity) (MANGLER)

```
#install.packages('pkgbuild')
#install.packages("ctsmr", repo = "http://ctsm.info/repo/dev")
library('ctsmr')
ctsm1 <- function(data){
  # Remove NaN
  mask = !is.na(data$Ws1) & !is.na(data$p)
  data = data[mask,]
```

```
  # Generate a new object of class ctsm
  model = ctsm()

  # Add a system equation and thereby also a state
  model$addSystem(dX ~ (phi*Ws1)*dt + exp(p11)*dw1)
  model$addSystem(dphi ~ 0 * dt + exp(p12)*dw2)

  # Set the names of the inputs
  model$addInput(Ws1)
  # Set the observation equation:
  model$addObs(p ~ X)
  # Set the variance of the measurement error
  model$setVariance(p ~ exp(e11))
  ##-----------------------------------------------------------------
  # Set the initial value (for the optimization) of the value of the state at the starting time point
  model$setParameter(X = c(init = 5, lb = 0, ub = 25))
  model$setParameter(phi = c(init = 0, lb = -5, ub = 5))
  ##-----------------------------------------------------------------
  # Set the initial value for the optimization
  #model$setParameter(Ci = c(init = 1, lb = 1E-5, ub = 1E5))
  #model$setParameter(Cm = c(init = 1000, lb = 1E-5, ub = 1E5))
  #model$setParameter(Ria = c(init = 20, lb = 1E-4, ub = 1E5))
  #model$setParameter(Rim = c(init = 20, lb = 1E-4, ub = 1E5))
  #model$setParameter(Aw = c(init = 6, lb = 1E-2, ub = 7.5+4.8+5))
  model$setParameter(p11 = c(init = 1, lb = -30, ub = 10))
  model$setParameter(p12 = c(init = 1, lb = -30, ub = 10))
  model$setParameter(e11 = c(init = 1, lb = -50, ub = 10))
  #model$setParameter(a1 = c(init = 0.2, lb = -50, ub = 100))
  #model$setParameter(a2 = c(init = 4, lb = -500, ub = 1000))
  #model$setParameter(a3 = c(init = 4, lb = -500, ub = 1000))
  #model$setParameter(a4 = c(init = 4, lb = -500, ub = 1000))
  #model$setParameter(a5 = c(init = 4, lb = -500, ub = 1000))
  ##-----------------------------------------------------------------

  # Run the parameter optimization
  fit = model$estimate(data,firstorder = TRUE)
  return(fit)
}
Dsub = D[1:400,]
fit = ctsm1(Dsub)
summary(fit, extended = F)
fit$loglik
```

Kræver vel-installeret compiler.