Assignment Name

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1 Task

For a given fixed convex polygon $\mathcal{C}$ with vertices $p_0, \ldots p_{n-1}$ in counterclockwise order and a non-vertical query line q, return the area A_q in $\mathcal{C}$ under q in time $\mathcal{O}(\log n)$.

2 Solution

Idea Let us first sketch how we will solve this algorithmic problem efficiently. All the following definitions can be seen in figure ?? which greatly helps understanding the algorithm. We will follow a two step approach.

First, we will have to find out where q intersects the convex polygon. There are three different options for this:

1. The line q is completely above $\mathcal{C}$. In this case we will return the whole area of $\mathcal{C}$. The area calculation will be covered in part two.
2. The line q is completely below $\mathcal{C}$. In this case we will return the an area of 0.
3. The line q intersects $\mathcal{C}$. This will give us two points where the convex polygon $\mathcal{C}$ and line q intersect (one where q "enters" $\mathcal{C}$ and one where q "exits" $\mathcal{C}$). By assumption q is not a vertical line giving the intersection points two different x-coordinates. Let the intersection point with the smaller x-coordinate be denoted by i_l (for intersection-left) and the one with the bigger x-coordinate i_r (for intersection-right).

Using an idea very similar to the *Reporting Points Below a Query Line Problem* as seen in the lecture we will identify the two vertices p_l and p_r. We define p_l (p_r) as the point p_i in the set of points which make up $\mathcal{C}$ which is the lower vertex of the edge which q has pierced while intersecting $\mathcal{C}$, or in other words, the vertex of smaller y-coordinate of the edge of $\mathcal{C}$ on which i_l (i_r) is located on. Note that p_l and p_r can also refer to the same point.

Having located i_l, i_r, p_l, p_r as shown in figure ?? we can move on to finding the area.

Let A_q be the are we are looking for. To find A_q efficiently we split it up into two different subareas $A_{q,a}$ and $A_{q,f}$ with $A_{q,a} \cup A_{q,f} = A_q$.

- $A_{q,f}$ (fixed area under q): The area of the convex polygon formed by including all vertices of C starting at p_l and walking along C in a counter-clockwise rotation and ending with p_r (orange in figure ??).
- $A_{q,a}$ (adaptive area under q): The area of the polygon formed by the points $\{p_l, p_r, i_r, i_l\}$ (blue in figure ??). Note that we can find this area by splitting it up into two triangles (see dashed line in figure ??) and calculating the area of the two individual triangles.

Once we have found the values for $A_{q,f}$ and $A_{q,a}$ we return their sum as the answer.

Having covered the basic idea behind the algorithm we can now develop the algorithm rigorously.

Important Note To make indexing easier we will adopt modular indexing in this paper. This means all indexes p_i should be read as

$$p_i = p_j \quad \text{where} \quad j = i \mod (\text{length of array}).$$

As an example consider $[p_0, p_1, p_2]$, then the point p_{i+1} for $i = 2$ is p_0.

From this point on the notion of modular indexing is implicitly assumed.

References

[1] Some entry here