Homework 10

Problem 1. *Show that, for constant $p \in (0,1)$, almost no graph in $\mathcal{G}(n,p)$ has a separating complete subgraph.*

[Hint]

1. Recall the property $P_{i,j}$ from the slides
2. You may need to recall the definitions:

- **Separating subgraph**: Given $G = (V, E)$, and some $X \subseteq V \cup E$, we call X a separating subgraph if there exists two vertices $u, v \in V(G - X)$ such that u, v are in the some component of G, while u, v lie in two disconnected components of $G - X$ (i.e., X separates u and v).
- **Separating complete subgraph**: If the above subgraph X is also a complete graph.

Problem 2. *Consider* $\mathbf{G}(n,p)$ *with* $p = \frac{1}{3n}$.

Use the second moment method to show that with high probability there exists a simple path of length 10.

Problem 3. *Prove that 'the disappearance of isolated vertices in* $\mathbf{G}(n,p)$*' has a sharp threshold of* $\frac{\ln n}{n}$.

[Hint: John's book, theorem 8.6]

Problem 4. *(Optional)*

1. *Prove that the threshold for the existence of cycles in $\mathcal{G}(n,p)$ is $p = \frac{1}{n}$.*
2. *Search the* World Wide Web *to find some real world graphs in machine readable form or data bases that could automatically be converted to graphs.*
   (a) *Plot the degree distribution of each graph.*
   (b) *Compute the average degree of each graph.*
   (c) *Count the number of connected components of each size in each graph.*
   (d) *Describe what you find.*
3. *Create a simulation (an animation) to show the evolution of the $\mathcal{G}(n,p)$ (Erdös-Rényi) random graph as its density p is gradually increased. Observe the phase transitions for trees of increasing orders, followed by the emergence of the giant component, etc.*