Homework 1

Problem 1. *Show the Venn-diagram representation for the following sets:*

(a) $(A - B) \cap C$

(b) $\overline{A \oplus (B \cup C)}$

Problem 2. *For any sets A, B and C, prove that*

$$A \cup B = A \cup C, A \cap B = A \cap C \textit{ implies } B = C.$$

Problem 3. *Show that a nonempty set has the same number of odd subsets (i.e., subsets with an odd number of elements) as even subsets.*

Problem 4. A, B, C *are three sets. and two functions* $g : A \to B$, $f : B \to C$

a) *If* $f \circ g$ *is an injective function and* g *is surjective, show that* f *is injective.*

b) *If* $f \circ g$ *is an surjective function and* f *is injective, show that* g *is surjective.*

(Note that $f \circ g(x) = f(g(x))$.*)*

Problem 5. $\mathcal{R}$ *is a binary relation,*

1. *Show that* $\mathcal{R}$ *is symmetric iff* $\mathcal{R}^{-1} \subseteq \mathcal{R}$.
2. *Show that* $\mathcal{R}$ *is transitive iff* $\mathcal{R} \circ \mathcal{R} \subseteq \mathcal{R}$.

Problem 6. *A and B are countable sets. Prove that*

1. $A \cup B$ *is countable*
2. $A \times B$ *is countable*

Problem 7. *Draw the Hasse diagram of the set of all subsets of* $\{1, 2, 3\}$ *ordered by inclusion.*