Homework 6

Problem 1. *Prove that any natural number* $n \in \mathbb{N}$ *can be written as a sum of mutually distinct Fibonacci numbers.*

Problem 2. *Express the* n^{th} *term of the sequences given by the following recurrence relations*

1. $a_0 = 2, a_1 = 3, a_{n+2} = 3a_n - 2a_{n+1}$ $(n = 0, 1, 2, \ldots)$.
2. $a_0 = 1, a_{n+1} = 2a_n + 3$ $(n = 0, 1, 2, \ldots)$.

Problem 3. *Solve the recurrence relation* $a_{n+2} = \sqrt{a_{n+1}a_n}$ *with initial conditions* $a_0 = 2, a_1 = 8$ *and find* $\lim_{n\to\infty} a_n$.

Problem 4. *Show that for any* $n \geq 1$, *the number* $\frac{1}{2}[(1+\sqrt{2})^n + (1-\sqrt{2})^n]$ *is an integer.*