Homework 5

Problem 1. *1. Determine the coefficient of* x^{50} *in* $(x^7 + x^8 + x^9 + x^{10} + \cdots)^6$

2. Determine the coefficient of x^3 *in* $(2 + x)^{\frac{3}{2}}/(1 - x)$

3. Determine the coefficient of x^4 *in* $(2 + 3x)^5 \sqrt{1 - x}$

Problem 2. *Find generating functions for the following sequences (express them in a closed form, without infinite series!):*

1. $0, 0, 0, 0, -6, 6, -6, 6, -6, \cdots$.

2. $1, 0, 1, 0, 1, 0, \cdots$.

3. $1, 2, 1, 4, 1, 8 \cdots$

Problem 3. *Let* a_n *be the number of ordered triples* $\langle i, j, k\rangle$ *of integer numbers such that* $i \geq 0$, $j \geq 1, k \geq 1$, *and* $i + 3j + 3k = n$. *Find the generating function of the sequence* $(a_0, a_1, a_2, \ldots)$ *and calculate a formula for* a_n.

Problem 4. *If* $a(x)$ *is the generating function of a sequence* $(a_0, a_1, a_2, \ldots)$, *please find the generating function of the sequence of partial sums* $(a_0, a_0 + a_1, a_0 + a_1 + a_2, \ldots)$.