Homework 4

Problem 1. *Count the number of linear extensions for the following partial ordering:*

X is a disjoint union of sets $X_1, X_2, \ldots, X_k$ of sizes $r_1, r_2, \ldots, r_k$, respectively. Each X_i is linearly ordered by $\leq$, and no two elements from the different X are comparable.

Problem 2. *Given a set X with $|X| = n$, determine the number of ordered set pairs $\langle A, B\rangle$ where $A \subseteq B \subseteq X$.*

Problem 3. *There are n married couples attending a dance. How many ways are there to form n pairs for dancing if no wife should dance with their husband.*

Problem 4. *Count the permutations with exactly k fixed points. (Remark: π is a permutation of the set {1,2,..., n}. Call an index i with $\pi(i) = i$ a* fixed point *of the permutation π.)*

Problem 5. *What is wrong with the following inductive proof that $D(n) = (n-1)!$ for all $n \geq 2$? Can you find a false step in it? For $n = 2$, the formula holds, so assume $n \geq 3$. Let π be a permutation of $\{1, 2, ..., n-1\}$ with no fixed point. We want to extend it to a permutation π' of $\{1, 2, ..., n\}$ with no fixed point. We choose a number $i \in \{1, 2, ..., n-1\}$, and we define $\pi'(n) = \pi(i), \pi'(i) = n$, and $\pi'(j) = \pi(j)$ for $j \neq i$, n. This defines a permutation of $\{1, 2, ..., n\}$, and it is easy to check that it has no fixed point. For each of the $D(n-1) = (n-2)!$ possible choices of π, the index i can be chosen in $n-1$ ways. Therefore, $D(n) = (n-2)! \cdot (n-1) = (n-1)!$.*

Problem 6. *How many ways are there to seat n married couples at a round table with $2n$ chairs in such a way that the couples never sit next to each other?*