

Homework 3

Problem 1. *Prove the formula*

1. $\binom{r}{r} + \binom{r+1}{r} + \binom{r+2}{r} + \cdots + \binom{n}{r} = \binom{n+1}{r+1}$

2. $\sum_{k=0}^n \binom{m+k-1}{k} = \binom{n+m}{n}$

Problem 2. *For natural numbers $m \leq n$ calculate (i.e. express by a simple formula not containing a sum) $\sum_{k=m}^n \binom{k}{m} \binom{n}{k}$.*

Problem 3. *Calculate (i.e. express by a simple formula not containing a sum)*

1. $\sum_{k=1}^n \binom{k}{m} \frac{1}{k}$

2. $\sum_{k=0}^n \binom{k}{m} k$

Problem 4. (a) *Using Problem 1. for $r = 2$, calculate the sum $\sum_{i=2}^n i(i-1)$ and $\sum_{i=1}^n i^2$.*

(b) *Use (a) and Problem 1. for $r = 3$, calculate $\sum_{i=1}^n i^3$.*

Problem 5. *How many functions $f : \{1, 2, \dots, n\} \rightarrow \{1, 2, \dots, n\}$ are there that are monotone; that is, for $i < j$ we have $f(i) \leq f(j)$?*