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Business Analytics

Tasks and slide collection

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Chapter 4

Network Design

4.1 Introduction

Framework conditions in practice:

- In most cases, a previously developed infrastructure already exists. a complete redesign of a route network is rather uncommon and therefore the question of a suitable route network extension or reduction is the rule.

- Renewal measures are of particular importance: For safety reasons, deteriorated sections of tracks require that vehicles are only allowed to travel at reduced speeds in such areas. By acceleration measures, the vehicles and the corresponding personnel are available for a return trip sooner, so savings in the operational (personnel, vehicles) area become possible.

- Typically, projects and measures (decisions) are evaluated and then implemented or rejected depending on political expectations. Political expectations influence the price of a decision. Thus a decision can be quantified by comparison with a model-based solution.

- Depending on the needs of customers, decision-makers can make a distinction between long-distance, regional, and local transport networks. The networks are usually planned separately, with regional transport generally aligned with long-distance and local and short-distance traffic to regional traffic.

4.2 Minimal spanning tree

Applications

- Construction of a network with minimum total costs for n locations such that all locations are directly or indirectly connected, with branching points of the network being situated only of locations.

Graphical illustration

- For each location we consider a node and for each possible direct connection an edge is considered weighted by construction costs or operating costs.

Procedure

- Determination of a minimal spanning tree using Kruskal's algorithm.

Minimal spanning tree

- V vertices or nodes
- E edges
- c cost (weight) of an edge
- Let $G = [V, E, c]$ be a contiguous (connected), weighted, undirected graph.
- Tree T^* from G is a minimal spanning tree of G, if there is no other spanning tree T' of G with smaller total cost (=sum of related edge cost).

Kruskal algorithm

Prerequisite: a weighted, contiguous, loop-free, undirected graph $G = [V, E, c]$ with n nodes and m edges; with set $\bar{E}$ edge quantity to be determined with minimal spanning tree $T = [V, \bar{E}]$.

Start: sort or number the edges k_i of G in order $k_1, k_2, \ldots, k_m$ after not decreasing weights $c(k_i)$, so that holds: $c(k_1) \leq c(k_1) \leq \ldots \leq c(k_m)$.

Set $\bar{E} := \emptyset$ and $T := [V, \bar{E}]$. **Iteration** $\mu = 1, 2, \ldots, m$**:**

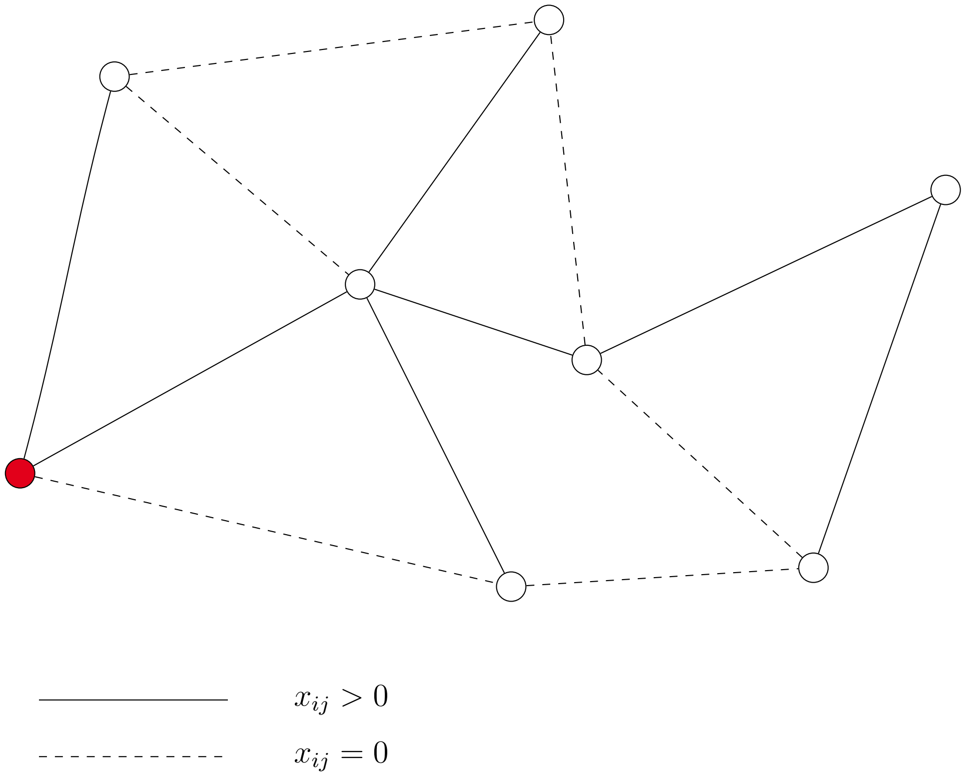
- choose edge k_μ and check if its inclusion in $T = [V, \bar{E}]$ creates a circle.
- if k_μ does not create a circle then set $\bar{E} := \bar{E} \cup \{k_\mu\}$.
- next iteration.

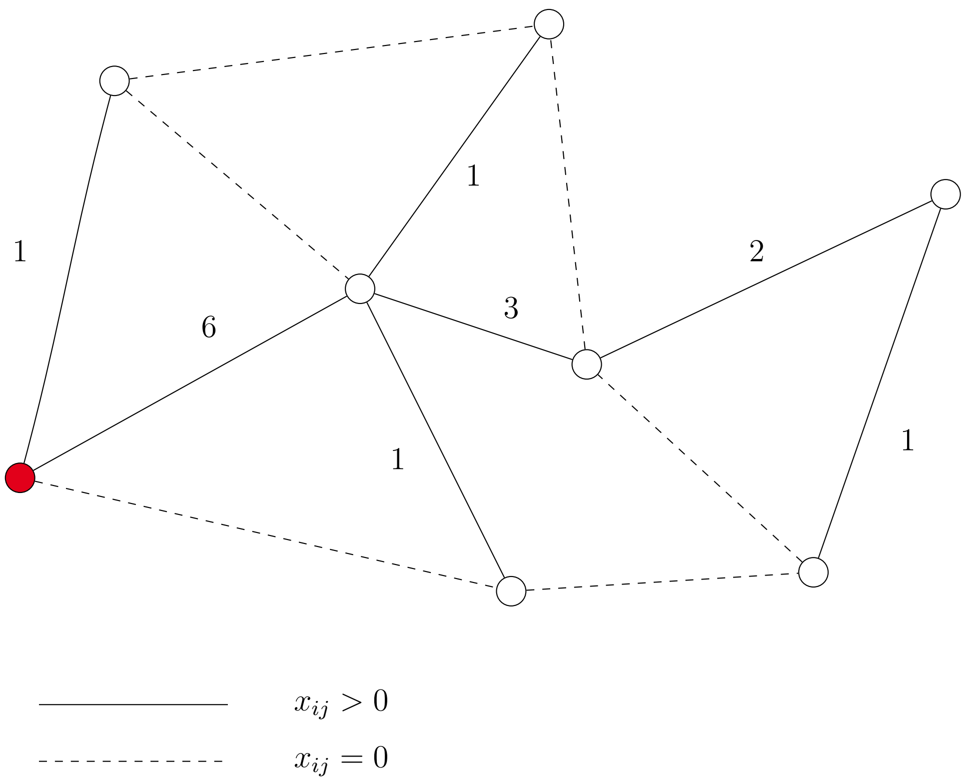
Termination: the procedure terminates as soon as $\bar{E}$ contains $n - 1$ edges.

Result: $T = [V, \bar{E}]$ is a minimal spanning tree of G.

Example 4.1 For an air traffic company, a route network that connects six cities, must be established. The possible flight connections and the associated costs can be seen in the following illustration:

Determine the minimal cost route network.


$x_{ij} > 0$
$x_{ij} = 0$


1
1
6
3
2
1
1
$x_{ij} > 0$
$x_{ij} = 0$

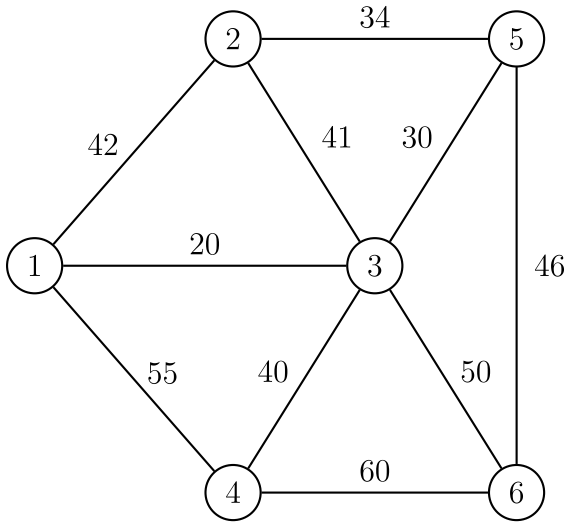


Figure 4.1: Numerical example

4.3 Network design problem

Assumptions

- Railway network for high-speed trains
- Investment costs for direct connection of two cities
- Minimization of travel times (or related cost)
- Graphical illustration
  - Nodes: cities, stations or stops
  - Edges: candidate connections
  - Edge weight: (periodized) fixed costs, (value of) travel time

Sets

V : sorted nodes (indices: i, j, u, v)

E : edges; with edge $[i, j] \in E$, where $i < j$ holds

Parameters

d_{uv} : expected number of passengers traveling from node u to node v, with $v \neq u$.
When $v = u$, d_{uv} represents the expected number of passengers leaving node $u = v$

t_{ij} : value of travel time for the direct trip from node i to node j with $[i,j] \in E$ and $t_{ij} = t_{ji}$

c_{ij} : fixed cost per period for the construction of an edge that directly connects the nodes i and j

B : maximum total cost allowed, where $B \geq F^{\star}{}_1$

Variables

Y_{ij} $= 1$, if edge $[i,j] \in E$ is built (0, otherwise)

X_{uij} $=$ number of passengers starting their trip in node u and traveling from node i to node j with $[i,j] \in E$

Mathematical model

$$\text{Minimize } G = \sum_{[i,j] \in E} t_{ij} \cdot \sum_{u \in V} X_{uij} \tag{4.1}$$

such that

$$\sum_{[i,v] \in E} X_{uiv} - \sum_{[v,j] \in E} X_{uvj} = d_{uv} \qquad \forall u, v \in V \mid u \neq v \tag{4.2}$$

$$X_{uij} + X_{uji} \leq d_{uu} \cdot Y_{ij} \qquad \forall u \in V, [i,j] \in E \mid i < j \tag{4.3}$$

$$\sum_{[i,j] \in E \mid i<j} c_{ij} \cdot Y_{ij} \leq B \tag{4.4}$$

$$X_{uij} \geq 0 \qquad \forall u \in V, [i,j] \in E \tag{4.5}$$

$$Y_{ij} \in \{0,1\} \qquad \forall [i,j] \in E \mid i < j \tag{4.6}$$

$$\tag{4.7}$$

Remarks

- Objective function (4.1) minimizes the periodic infrastructure and travel time-induced opportunity costs.
- Constraints (4.2) ensure conservation of flow between departing and arriving passengers at each station.
- Constraints (4.3) ensures that a transport from i to j or j to i is only possible if the connection (edge) $[i,j]$ is established, i.e. such that $y_{ij} = 1$.

- Constraint (4.4) ensure the cost of the connections to built cannot exceed the available budget.
- Constraints (4.5) and (4.6) defines the domain of the variables.

Model critique

- The demand is independent of the actual route network. In general, however, it can be assumed that passengers will be displaced by motorised private transportation if the travel time by PT is significantly longer than the shortest travel time. The effect of passengers migrating will only be recorded as a lump sum by the opportunity costs.
- The maximum number of trips along an edge per time unit (TU) is limited. Therefore, edge-related capacity restrictions must be taken into account. It is conceivable to consider different expansion stages (single or double track).

Example 4.2 Five cities are to be connected by a high-speed rail network. The construction costs f_{ij} and the durations c_{ij} transformed into opportunity costs are shown in Figure 4.2.

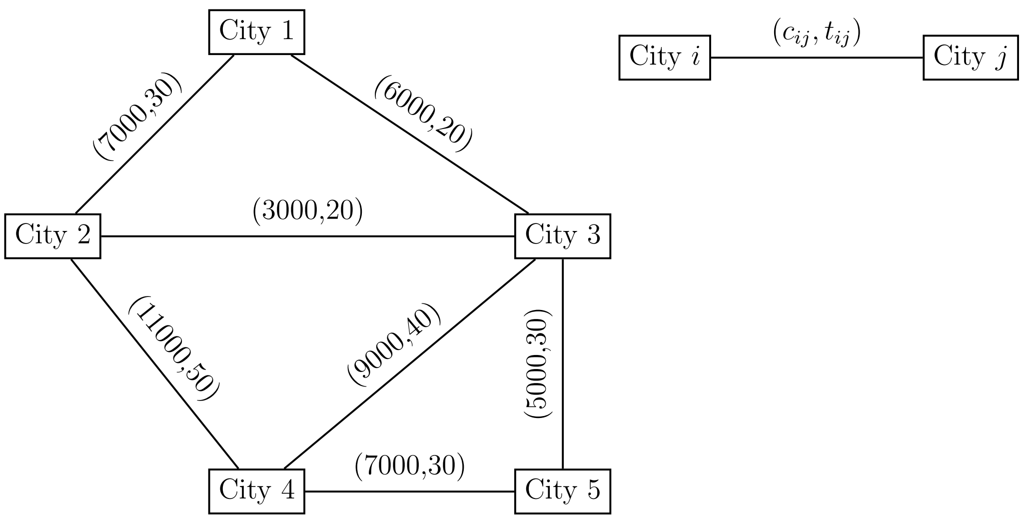


Figure 4.2: Example: Network design problem

A demand analysis shows the following OD matrix. Where, $u, v \in$ {City1, City2, City3, City4, City5 }. Calculation of d_{uu} when $v = u$ is given by:
$d_{uu} = \sum_{v \in V | u \neq v} d_{uv}, \forall u \in V$

$$(d_{uv}) = \begin{pmatrix} 900 & 250 & 400 & 100 & 150 \\ 250 & 400 & 500 & 250 & 400 \\ 400 & 500 & 1300 & 300 & 100 \\ 100 & 250 & 300 & 800 & 150 \\ 150 & 400 & 100 & 150 & 800 \end{pmatrix}.$$

Optimal solutions We solve the problem with GAMS/Cplex (see Appendix A.1) and get the following result:

```
Fixed costs         :  23000

Connections to built
---------------------------------
from city1 to city3   1
from city2 to city3   1
from city3 to city4   1
from city3 to city5   1

Connections loads
-----------------
from city1 to city3 900
from city3 to city1 900
from city2 to city3 1400
from city3 to city2 1400
from city3 to city4 650
from city4 to city3 650
from city3 to city5 650
from city5 to city3 650
from city4 to city5 150
from city5 to city4 150

Origin-related traffic flows
---------------------------------
from city1 from city1 to city3 900
from city1 from city3 to city2 250
from city1 from city3 to city4 100
from city1 from city3 to city5 150
from city2 from city2 to city3 1400
from city2 from city3 to city1  250
from city2 from city3 to city4  250
```

```
from city2 from city3 to city5  400
from city3 from city3 to city1  400
from city3 from city3 to city2  500
from city3 from city3 to city4  300
from city3 from city3 to city5  100
from city4 from city3 to city1  100
from city4 from city3 to city2  250
from city4 from city4 to city3  650
from city4 from city4 to city5  150
from city5 from city3 to city1  150
from city5 from city3 to city2 400
from city5 from city5 to city3 650
from city5 from city5 to city4 150
```

Traffic flows starting in City 1 ()$u = 1$):

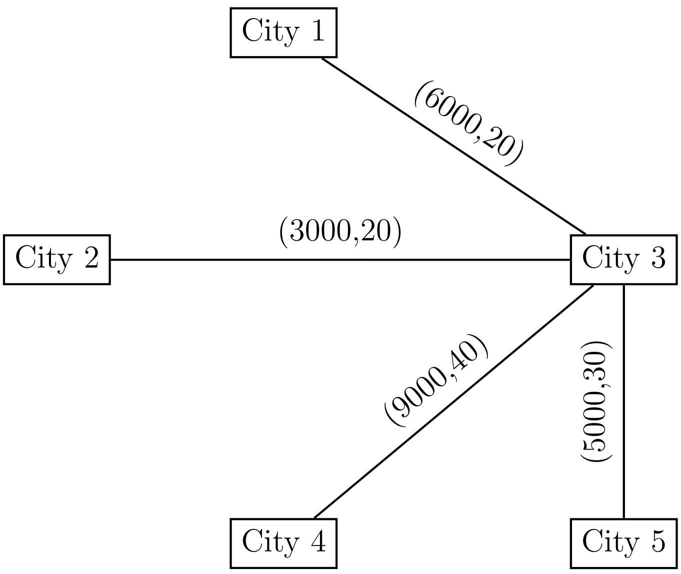


Figure 4.3: Example: Optimal solution

In node $v = 3$, 900 passengers arrive, of which 500 continue. So 400 passengers get off, which is exactly $od_{13} = 400$.

4.4 Network extensions

Assumptions

- Initial conceptual situation: we consider an existing tram network. To improve the public transport network, extension investments are expected to be carried out. An annual budget is available for this purpose. Possible extension investments are already known.
- Each passenger-kilometer provides a benefit (additional revenues, environment, safety, reduced operating costs). Therefore, the network extensions should be made in such a way as to maximize the sum of passenger-kilometers gained.
- Investment funds may be forwarded to the next year.

Sets

I : extension investment, candidate projects (index: i)

T : periods (years) of the planning period (index: t)

Parameters

d_i : duration of the construction period of extension i

k_{ij} : investment provided for extension i in year j ($j = 1, \ldots, d_i$)

B_t : budget in period t

c_{it} : utility of extension i at start of construction in period t

Variables

Y_{ij} = 1, if edge $[i, j] \in E$ is built (0, otherwise)

X_{uij} = number of passengers starting their trip in node u and traveling from node i to node j with $[i, j] \in E$

Mathematical model

$$\text{Maximize } F = \sum_{i \in I} \sum_{t=1}^{|T|-d_i+1} c_{it} y_{it} \tag{4.8}$$

such that

$$\sum_{t \in T} y_{it} \leq 1 \qquad i \in I \tag{4.9}$$

$$u_t + \sum_{i \in I} \sum_{j=1}^{d_i} k_{ij} y_{i,t-j+1} - u_{t-1} = B_t \qquad t \in T \tag{4.10}$$

$$u_t \geq 0 \qquad t \in T \tag{4.11}$$

$$y_{it} \in \{0, 1\} \qquad i \in I, t \in T \tag{4.12}$$

Remarks

- Equations (4.8) maximizes total utility during the planning period
- Constraints (4.9) ensure each extension can be made at most once.
- Constraints (4.10) ensure the total investments of the extensions decided in each period must not exceed the available budget for that period.
- Constraints (4.11) and (4.12) define the domain of the variables

Example 4.3 The following network extensions have to be scheduled for the tram network of the city of Dresden:

1. Schönfeld/Weissig: Ullersdorfer Platz (Bühlau) – Lomnitzer Str. – Possendorfer Str. – Taubenberg – Heinrich-Lange-Str. – Heidestr. – Gasthof Weißig – Radeberger Str. – Zum Hutbergblick – Prießnitzaue (Weißig)
2. Johannstadt: Güntzplatz – Böhnischplatz – Arnoldstr. – Hertelstr. – Fetscherstr.
3. Strehlen-Prohlis-Nickern: H.-Bürkner-Str. – Teplitzer Str. – Dohnaer Str. – Spitzweg – Fritz-Bursch-Str. – M.-Wittig-Str. – Tornaer Str. – Gamigstr. – Fritz-Meinhardt-Str. – Langer Weg – Mechelisstr.
4. Postplatz-Universität: Webergasse – Josephienstr. – Budapester Str. – Schweizer Str. – Arbeitsamt – Chemnitzer Str. – Nürnberger Str. (Nürnberger Ei)
5. Altstadt-Johannstadt: Synagoge – Gerichtsstr. – St. Benno Gymn. – Hans-Grundig-Str. – Stephanienstr. – Fetscherplatz

From studies such as System repräsentativer Verkehrsbefragungen 1998, the following socio-economic data can be derived :

Group		People	PT-users	
Students	12-16 years	46,550	29.20%	13,593
	12-18 years	25,400	37.10%	9,423
Professionals	with car	225,100	6.15%	13,837
	without car	120,000	40.32%	48,387
Non-professionals	with car	79,100	13,84%	10,950
	without car	111,900	34.39%	38,481
Pensioners	with car	141,300	33.50%	47,336
	without car	31,600	12.00%	3,792

Each route segment includes several stops, with each stop having a service area. In combination with socio-demographic data of the service areas, the potential passengers of the route network extensions were forecast. The results and further information on the investment opportunities are shown in the following table:

Candidate	Length (km)	Potential passengers	Length per trip (km)	Construction time (years)
1: Schönfeld/Weißig	3.30 km	2,020	2.5	3
2: Johannstadt	1.50 km	3,571	2.0	1
3: Strehlen-Prohlis-Nickern	4.25 km	4,481	3.1	4
4: Postplatz-Universität	2.45 km	4,400	2.0	2
5: Altstadt-Johannstadt	2.00 km	3,887	2.0	2

The required investment funds per km are estimated at 3 million euros. During the construction phase, the following financial resources [in million euros] are to be made available for the expansion measures:

Candidate	1^{st} year	2^{nd} year	3^{rd} year	4^{th} year
1: Schönfeld/Weißig	4.00	3.50	2.40	-
2: Johannstadt	4.50	-	-	-
3: Strehlen-Prohlis-Nickern	3.50	3.00	3.25	3.00
4: Postplatz-Universität	4.00	3.35	-	-
5: Altstadt-Johannstadt	3.50	2.50	-	-

The benefit per km is identical for all passengers. On average, a Dresden resident takes the OPNV 249 times a year. The transport company has an annual budget of 6 million euros for expansion investments. The planning period covers the years 2006 to 2013. Using the data provided, GAMS/Cplex provides the following optimal solution:

```
MIP Solution:      48421087.800000
Relative gap:             0.000000

Start of construction of an extension
---------------------------
Johann      in  2023
PostUni     in  2024
AltJohann   in  2024
SchoenWeiss in  2027
StreProNick in  2026
```

```
Period surpluses
-------------------
at the end of 2023  1.50 [Mill. Euro]
at the end of 2024  0.00 [Mill. Euro]
at the end of 2025  0.15 [Mill. Euro]
at the end of 2026  2.65 [Mill. Euro]
at the end of 2027  1.65 [Mill. Euro]
at the end of 2028  0.90 [Mill. Euro]
at the end of 2029  1.50 [Mill. Euro]
at the end of 2030  7.50 [Mill. Euro]
at the end of 2031 13.50 [Mill. Euro]
```

4.5 Literature

Ackermann, K., Schöppe, E., & Bradow, A. (1999). System repraesentativer Verkehrsbefragungen 1998-Erhebungsmethode und ausgewaehlte Ergebnisse. Straßenverkehrstechnik, 43(8).

Domschke, W., & Drexl, A. (2014). Logistik: standorte. In Logistik: Standorte. Oldenbourg Wissenschaftsverlag.

4.6 Exercises

Exercise 4.1 Let us look again at the example 4.2. The decision maker wants to know how sensitive the solution is to over- and underestimating opportunity costs. Therefore, determine the solutions that will result if the opportunity costs are 10%, 20% or 30% higher or lower, and interpret the results.

Exercise 4.2 A low-cost carrier would like to serve the cities of Hamburg, Bremen, Hanover, Berlin, Leipzig, Dresden, Mannheim, Saarbrucken, Nuremberg, Stuttgart and Munich. The following data are given:

City	x-Coordinate	y-Coordinate	Population
Hamburg	12.4 cm	15.6 cm	1,734,083
Bremen	9.6 cm	14.4 cm	544,853
Hanover	11.7 cm	12.8 cm	516,160
Berlin	20.4 cm	13.1 cm	3,388,477
Leipzig	18.1 cm	10.3 cm	497,531
Dresden	21.3 cm	9.5 cm	483,632
Mannheim	8.7 cm	5.9 cm	308,353
Saarbrücken	5.2 cm	5.3 cm	181,860
Nuremberg	14.9 cm	5.7 cm	493,553
Stuttgart	10.4 cm	4.2 cm	589,161
Munich	16.0 cm	2.6 cm	1,247,873

A length of 1 cm corresponds to 60 km. All cities should be connected (by changing trains or making stopovers).

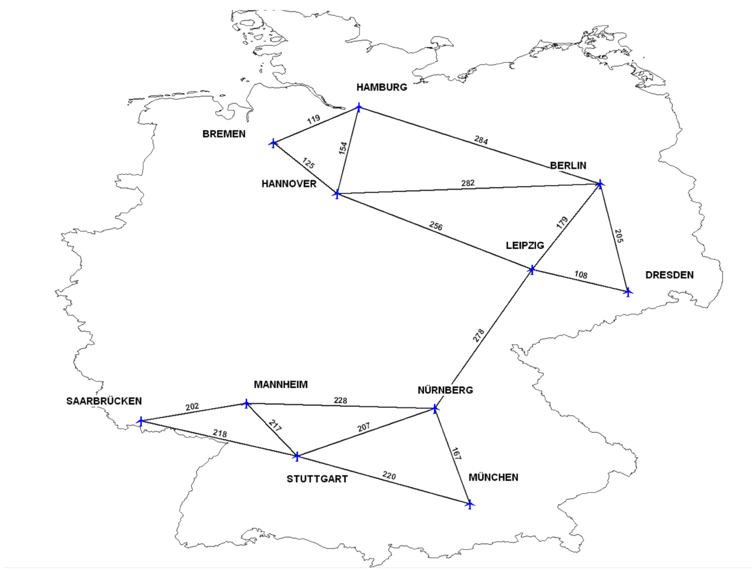


Figure 4.4: Potential connections

(a) It is expected that a resident will fly with the airline 0.05 times per year (outbound and return). The demand is distributed proportional to the number of inhabitants. Determine the OD matrix.

(b) Assume that the aircraft used must be refueled after 300 km at the latest. Determine the corresponding route network with minimum total length.

(c) Assume that the route network should ensure that a passenger makes only one stopover at most (with or without transfer). Formulate a suitable decision model for this purpose. Determine an optimal solution using GAMS.

(d) Formulate a general decision model to determine a route network with of a route network with three hubs. Each airport is to be assigned to exactly one hub and the hubs are to be connected directly.