Comparative analysis of the line integral and Frankot-Chellapa algorithms for surface normal integration in photometric stereo

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Abstract

Photometric stereo is a 3D image reconstruction technique that utilizes intensity images to compute an object's surface normal and depth. To evaluate their effectiveness in reconstruction, a comparative analysis between two widely used methods–the line integral method and the Frankot-Chellappa method, was conducted. The method was implemented on simulated images of 3D objects of increasing complexity: a spherical surface and a 3D model of a Tamagotchi character. Results demonstrate the capability of the two methods to successfully reconstruct the object's surface geometry. In terms of both accuracy and processing speed, the Frankot-Chellappa method performed better than the line integral method by providing sharper, smoother, and more consistent surface reconstructions for both structures. Its runtime was faster for the sphere reconstruction as well, while the line integral method failed to reconstruct the more complex Tamagotchi model altogether.

Keywords: photometric stereo, surface reconstruction, line integral, Frankot-Chellappa

1 Introduction

Photometric stereo is an imaging technique to recover the shape of an object using multiple images captured under varying lighting conditions. [1, 2] The surface normals and the surface elevation of the object are used to construct the 3D image of the object's surface. The brightness B from a point source decreases proportionally to $\frac{1}{r^2}$, where r is the distance from the source. Consider a point source at infinity represented by the vector $mathbfS_1$. The intensity captured by the camera at a point (x, y) is expressed as

$$I(x, y) = kB(x, y) \tag{1}$$

where k is some constant. The brightness $B(x, y)$ is defined as

$$B = \rho(x, y)\hat{\mathbf{n}}(x, y)\mathbf{S_0} \tag{2}$$

where $\rho(x, y)$ is the surface reflectance, $\hat{\mathbf{n}}(x, y)$ is the normal vector at (x, y), and $\mathbf{S_0}$ represent the vector from (x, y) to the point source. Equation 1 can further be defined in terms of Equation 1, which is given by

$$I(x, y) = k\rho(x, y)\hat{\mathbf{n}}(x, y) \cdot \mathbf{S_1} = \mathbf{g}(x, y) \cdot \mathbf{V_1} \tag{3}$$

For N light sources, the matrix $\mathbf{V}$ is

$$\begin{bmatrix} V_{11} & V_{12} & V_{13} \\ V_{21} & V_{22} & V_{23} \\ ... & ... & ... \\ V_{N1} & V_{N2} & V_{N3} \end{bmatrix} \tag{4}$$

For each point on the surface,

$$I_i(x, y) = V_{i1}g_1 + V_{i2}g_2 + V_{i3}g_3 \tag{5}$$

This yields a system of equations to obtain $\mathbf{g}(x, y)$, which is the normalized surface normals

$$\hat{\mathbf{n}}(x, y) = \frac{\mathbf{g}(x, y)}{|\mathbf{g}(x, y)|} \tag{6}$$

1.1 Line integral method

Line integration in photometric stereo involves integrating the surface normals to reconstruct the shape of the surface. The main objective is to relate the normals at each pixel to the surface height function $z(x, y)$, whose relationship is given by

$$n_x = \frac{\partial z}{\partial x}, \quad n_y = \frac{\partial z}{\partial y} \tag{7}$$

where n_x and n_y are the components of the normal vector [3]. The surface elevation $z(x, y)$ is then given by

$$z(x,y) = \int_0^u \Big(\frac{\partial z}{\partial x'}\Big) dx' + \int_x^y \Big(\frac{\partial z}{\partial y'}\Big) dy'. \tag{8}$$

This method, however, requires the surface to be continuous, as discontinuities brought about by abrupt changes in depth would lead to inconsistencies in the calculated depth.

1.2 Frankot-Chellappa algorithm

The Frankot-Chellappa algorithm reconstructs an object's surface using Fourier transforms to convert the gradient fields into the frequency domain [4]. Computations in the frequency domain ensure integrability because the curl of the gradient field is always zero, effectively resolving the discontinuity issues encountered using the line integral method.

For a surface $z(x, y)$, with partial derivatives $p(x, y)$ and $q(x, y)$ with respect to x and y, respectively. The corresponding Fourier transforms can be denoted by $P(u, v)$ and $Q(u, v)$ where u and v are the frequency domain variables. The depth function should then be:

$$z(x,y) = \frac{-i(u \cdot P(u,v) + v \cdot Q(u,v))}{u^2 + v^2} \tag{9}$$

An inverse Fourier transform is then performed to recover the surface in the spatial domain.

In this paper, we reconstruct two surfaces of increasing detail complexity using the line integral method and the Frankot-Chellappa algorithm for numerical line integration. We then assess the efficiency of these numerical line integration methods by comparing their reconstruction quality and execution time.

2 Methods

Photometric stereo was performed using MATLAB on four synthetically generated images of a spherical surface illuminated by a point source from different positions. As seen in Figure 5, each image represent the XY-plane of the spherical surface illuminated at the corresponding point source positions.

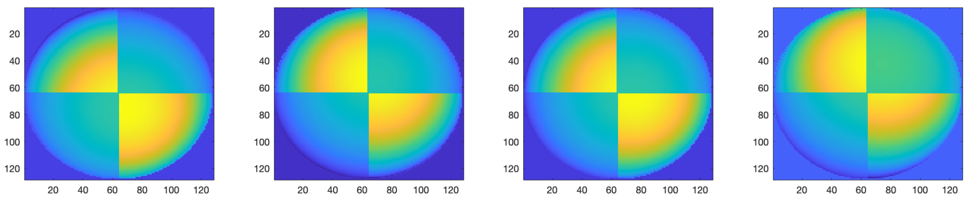


Figure 1: Four synthetic images of a spherical surface each illuminated by a distant point source at varying positions: (from left to right) I_1 to I_4 with corresponding point sources at V_1 to V_4.

The intensity values of each image were combined to obtain matrix $\mathbf{I}$. Meanwhile, the known coordinates of the point source positions:

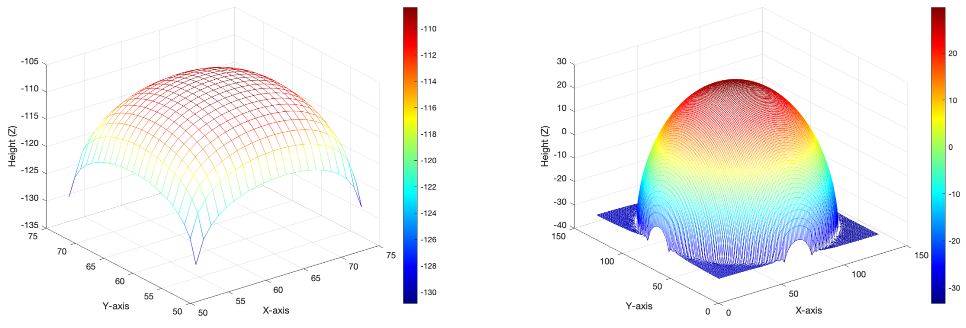
$$\begin{aligned} V_1 &= (0.085832, 0.17365, 0.98106), \\ V_2 &= (0.085832, -0.17365, 0.98106), \\ V_3 &= (0.17365, 0, 0.98481), \\ V_4 &= (0.16318, -0.34202, 0.92542), \end{aligned}$$

were combined, with each row forming the matrix $\mathbf{V}$. After obtaining $\mathbf{I}$ and $\mathbf{V}$, Equations 5 and 6 were used to calculate g and the surface normals, respectively. Then, the line integral method and the Frankot-Chellappa algorithm method were used to compute the surface elevation values needed to reconstruct the 3D model of the sphere. The reconstructed models resulting from numerically solving the integral equation using these two techniques were then compared.

For further experimentation, we applied the aforementioned methods to 3D art generated using the online program, Womp. To obtain the locations of the point sources, we took the lighting values from the program and normalized them before plugging them into the algorithms. The generated 3D models of a Tamagotchi character may be seen in Figure 5 in the Appendix 5.3 where the normalized coordinates of the point source locations are also listed in Appendix 5.1. Again, the results generated using the line integral method and the Frankot-Chellappa algorithm method, as well as the method run times, were compared.

3 Results and Discussion

For this experiment, we investigated the use of two different integration methods for 3D reconstruction – namely, the line integral method and the Frankot-Chellappa method. Figure 2 shows the results we obtained after applying these methods using the synthetic spherical images. As we can see in the figure, both methods were able to capture the physical contour of the sphere. However, it is evident that the Frankot-Chellapa method produced an image with significantly greater detail compared to the line integral method.



(a) Line Integral Reconstruction (b) Frankot-Chellappa Reconstruction

Figure 2: 3D Reconstructions of the four synthetic images

The line integral method generates the 3D image by taking the gradients using the surface normals of the structure. From this, the integral of the gradient is evaluated to the highest surface elevation point, and the result is a 3D digital approximation of the original image [3]. On the other hand, the Frankot-Chellappa method takes things a step further by taking the gradient data and converting it into the frequency domain through Fourier transforms to create a much smoother reconstruction [4]. This is seen in Figure 2b, where, when compared to the plot of Figure 2a, we see much more defined lines and edges. Also of note is how the Frankot-Chellappa method was able to simulate the planar surface the sphere was resting on, which was not captured in the line integral method. Furthermore, we can see at the edges of the plot small parabolic empty segments, which could be attributed to the construction of the synthetic images have the spherical plot strictly cropped within a square, thus, this may have caused a segment of the sphere to be cut off.

In addition to this, we compared the runtime of both methods. Interestingly enough, the line integral method took 0.0060s to generate the plot. On the other hand, the Frankot-Chellappa method only took 0.0030s. Therefore showing how the second method is not only better results wise, but also computationally.

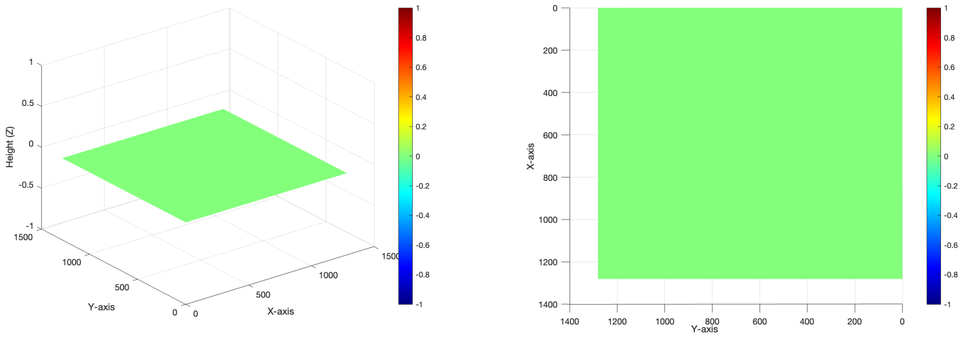
Figure 3 displays the 3D and 2D reconstructed surfaces of the Tamagotchi character using the line integral method. As we can see in the plot, this method was unable to recreate a structure and was limited to generating a planar surface. This expresses the limitations of the line integral method – it is unable to capture the details of much more complicated surfaces.

Figure 4 displayed the result obtained using the Frankot-Chellappa method on the character. Upon looking at the 3D reconstruction plot, the surface generated can mimic the highest and lowest points of the 3D render. This is further confirmed upon looking at the plot along the XY-plane, which displayed an etching of the character. This method was able to capture which points were the highest (the eyes, the lips) as well as which parts were the lowest (the top of the head, in between the character's legs) and display such.

Upon comparing the run times of both methods, we found that the line integral method and the Frankot-Chellappa method took 0.0069s and 0.2511s respectively. Unlike in the first plot, the line integral method was much faster compared to the second method. However, this is expected as the method was unable to display any contours, thus resulting in a much faster run time.

4 Conclusion

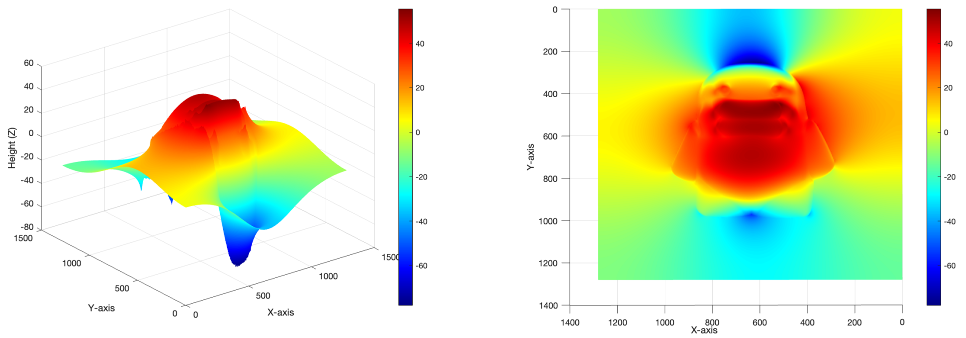
Photometric stereo was successfully implemented on 3D objects of varying detail complexity using the line integral and the Frankot-Chellappa algorithm as numerical integration methods. Results show that for the simpler 3D object, the sphere, both integration methods were able to successfully reconstruct



(a) Line Integral 3D Reconstruction Result

(b) Line Integral 2D Reconstruction Result

Figure 3: 3D and 2D Reconstructions of the 3D Art created using Womp



(a) Frankot-Chellappa 3D Reconstruction Result

(b) Frankot-Chellappa 2D Reconstruction Result

Figure 4: 3D and 2D Reconstructions of the 3D Art created using Womp

the object. However, the reconstruction using the Frankot-Chellappa algorithm captured the details and depth information of the modeled object more accurately and faster. On the other hand, the line integral method failed to recreate the more complex structure of the 3D Tamagotchi at all. Therefore the Frankot-Chellappa algorithm surpassed the line integral method performance for both simple and complex geometries, in terms of both reconstruction accuracy and processing speed.

For future experiments, additional numerical integration techniques such as the fast marching and cumulative trapezoidal methods can also be implemented and compared to determine their applicability in photometric stereo applications where accurate capture of detail and computational efficiency are both essential.

References

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5 Appendix

5.1 Additional Information

Point source coordinates for the Tamagotchi images:

$$\begin{aligned} V_1 &= (-0.1895909763, -0.5792740547, 0.7927779205), \\ V_2 &= (0.1895909763, -0.5792740547, 0.7927779205), \\ V_3 &= (-0.1895909763, 0.2500865919, 0.7927779205), \\ V_4 &= (0.1895909763, 0.2500865919, 0.7927779205), \end{aligned}$$

5.2 Code Used

The the original images and code used may be found using the following link: Google Drive Folder The code to obtain the gradient and to execute line integral method is sourced from Dr. Maricor Soriano's Photometric stereo tutorial [5]. The code for the Frankot-Chellappa method is sourced from online DfGBox Matlab toolbox [6].

5.3 Additional Images

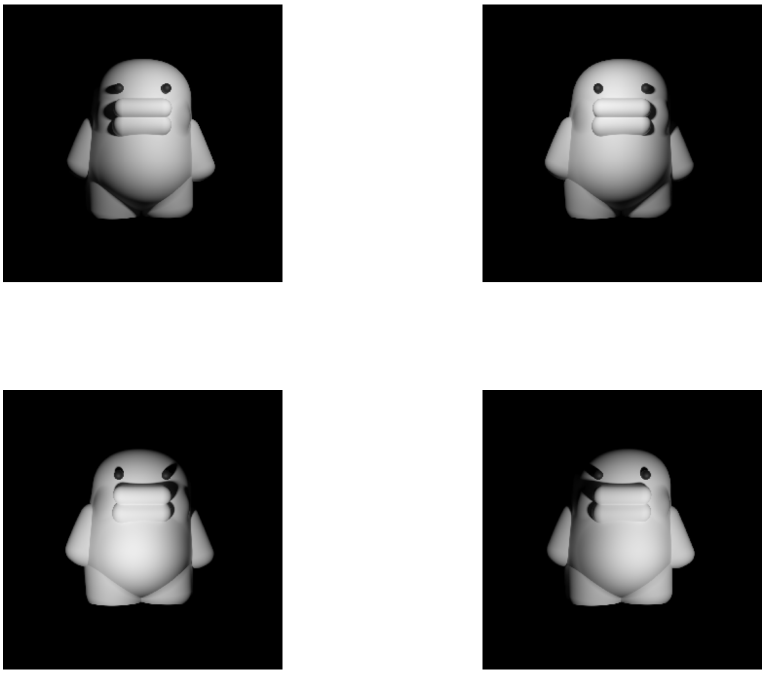

Figure 5: Original images of digital art generated using Womp, exposed to different lighting conditions.