

Insurance, biases, discrimination & fairness

Arthur Charpentier

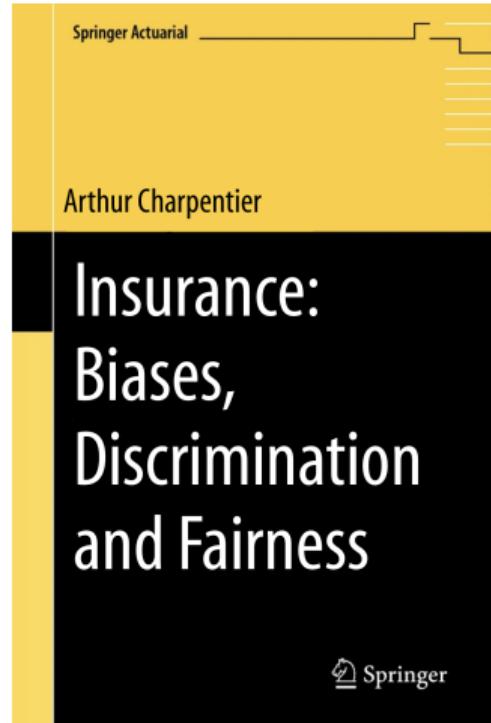
2024

Reference book

Insurance, Biases, Discrimination and Fairness

ISBN : 978-3-031-49782-7

Pitch: Discrimination and fairness of predictive models, in insurance, in the context of data enrichment ("big data") and opaque models ("machine learning", not to say "artificial intelligence").

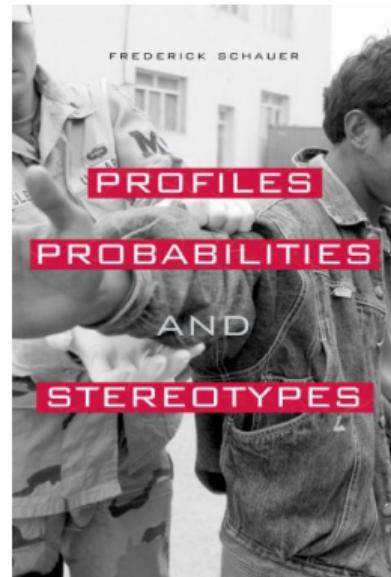


Preliminaries

Definition 1.1: Actuaries, Schauer (2006)

To be an **actuary** is to be a specialist in generalization, and actuaries engage in a form of decision making that is sometimes called actuarial. Actuaries guide insurance companies in making decisions about large categories that have the effect of attributing to the entire category certain characteristics that are probabilistically indicated by membership in the **category**, but that still may not be possessed by a particular member of the category.

See **Barry and Charpentier (2020)** on personalization of insurance prices.



Preliminaries

...

- *Tu la troubles, reprit cette bête cruelle,
Et je sais que de moi tu médis l'an passé.*
- *Comment l'aurais-je fait si je n'étais pas né ?
Reprit l'Agneau, je tette encor ma mère.*
- *Si ce n'est toi, c'est donc ton frère.*
- *Je n'en ai point.*
- *C'est donc quelqu'un des tiens.*

...

de La Fontaine (1668), *Le Loup et l'Agneau*.



Preliminaries

Definition 1.2: Discrimination, Merriam-Webster (2022)

Discrimination is the act, practice, or an instance of separating or distinguishing categorically rather than individually.

Definition 1.3: Prejudice, Merriam-Webster (2022)

Prejudice is (1) preconceived judgment or opinion, or an adverse opinion or leaning formed without just grounds or before sufficient knowledge; (2) an instance of such judgment or opinion; (3) an irrational attitude of hostility directed against an individual, a group, a race, or their supposed characteristics.

Preliminaries

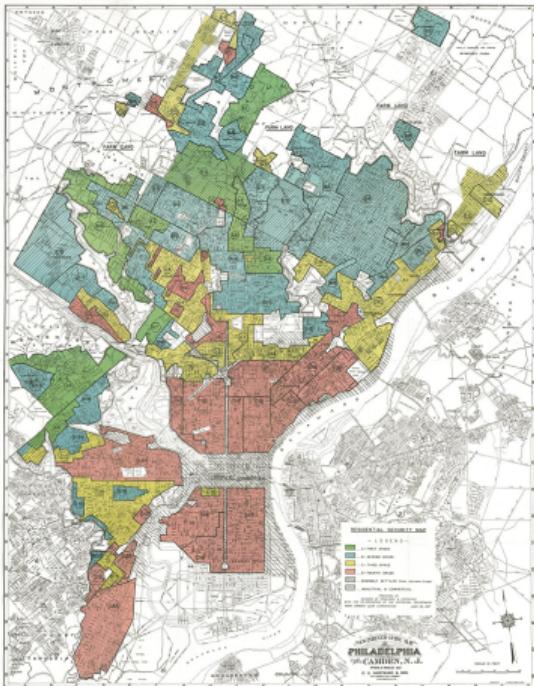
Definition 1.4: Disparate treatment, Merriam-Webster (2022)

Disparate treatment corresponds to the treatment of an individual (as an employee or prospective juror) that is less favorable than treatment of others for discriminatory reasons (as race, religion, national origin, sex, or disability).

Definition 1.5: Disparate impact, Merriam-Webster (2022)

Disparate impact corresponds to an unnecessary discriminatory effect on a protected class caused by a practice or policy (as in employment or housing) that appears to be nondiscriminatory.

Motivation (1. Redlining)



1937 HOLC (Home Owners' Loan Corporation)
"residential security" map of Philadelphia

RESIDENTIAL SECURITY MAP

— L E G E N D —

- [Solid Green Box]A - FIRST GRADE
- [Solid Blue Box]B - SECOND GRADE
- [Yellow Box]C - THIRD GRADE
- [Red Box]D - FOURTH GRADE
- [Diagonal Stripes Box]SPARSELY SETTLED (Color Indicates Grade)
- [Grey Box]INDUSTRIAL & COMMERCIAL

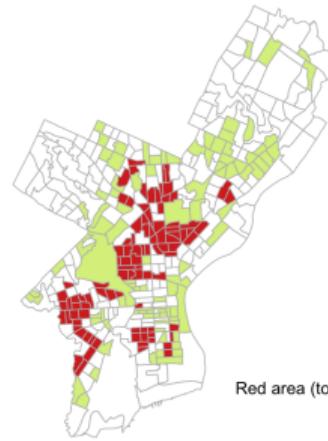
PREPARED BY
DIVISION OF RESEARCH & STATISTICS
WITH THE CO-OPERATION OF THE APPRAISAL DEPARTMENT
HOME OWNERS' LOAN CORPORATION JUNE 25, 1937

HS FORM-B
2-3-37

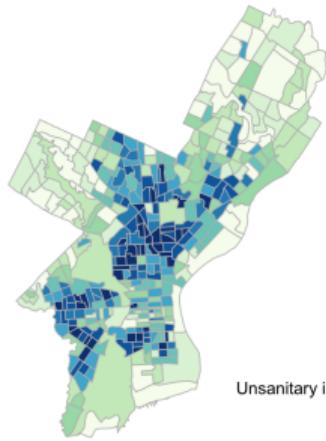
AREA DESCRIPTION
(For Instructions see Reverse Side)

1. NAME OF CITY Philadelphia, Pa. SECURITY GRADE C AREA NO. 6
2. DESCRIPTION OF TERRAIN. Level
3. FAVORABLE INFLUENCES. Good transportation, particularly in eastern part, -Near to industrial plants of major consequence to entire Philadelphia areas.
4. DETRIMENTAL INFLUENCES. Nominal
5. INHABITANTS:
a. Type Skilled labor; b. Estimated annual family income \$1,500 - \$1,800.
c. Foreign-born nominal; d. Negro No (Yes or No); \$
e. Infiltration of No; f. Relief families moderate
g. Population is increasing decreasing; static.
6. BUILDINGS:
a. Type or types predominately; b. Type of construction brick
c. Average age 20 - 40; d. Repair Fair

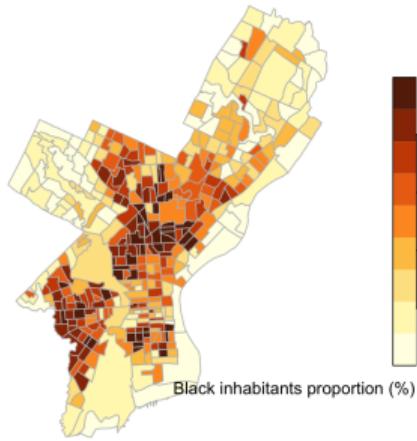
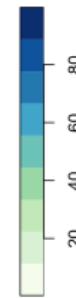
Motivation (1. Redlining)



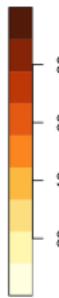
Red area (too risky)



Unsanitary index (0-100)



Black inhabitants proportion (%)



(Fictitious maps, inspired by a Home Owners' Loan Corporation map from 1937)

- ▶ Federal Home Loan Bank Board (FHLBB) "*residential security maps*" (for real-estate investments), [Crossney \(2016\)](#) and [Rhynhart \(2020\)](#)
- ▶ Unsanitary index and proportion of Black inhabitants

Motivation (1. Redlining)

Definition 2.1: Redline, Merriam-Webster (2022)

To **redline** is (1) to withhold home-loan funds or insurance from neighborhoods considered poor economic risks; (2) to discriminate against in housing or insurance.

See <https://evolutionofraceandinsurance.org/> for some historical perspective, Squires and Velez (1988), or more recently Squires (2003)

... but still a concern see, e.g., Li (1996) about homosexuals.

Motivation (2. "Gender directive", 2004/113/EC)

Treaty on European Union (26.10.2012, C326)

– Article 2 –

The Union is founded on the values of respect for human dignity, freedom, democracy, equality, the rule of law and respect for human rights, including the rights of persons belonging to minorities. These values are common to the Member States in a society in which pluralism, non-discrimination, tolerance, justice, solidarity and equality between women and men prevail.

– Article 3 –

(...) It shall combat social exclusion and discrimination, and shall promote social justice and protection, equality between women and men, solidarity between generations and protection of the rights of the child.



Motivation (2. “Gender directive”, 2004/113/EC)

Charter of Fundamental Rights of the European Union (18.12.2000 , C364)

– Article 21 (Non discrimination) –

Any discrimination based on any ground such as sex, race, colour, ethnic or social origin, genetic features, language, religion or belief, political or any other opinion, membership of a national minority, property, birth, disability, age or sexual orientation shall be prohibited.

– Article 23 (Equality between men and women) –

Equality between men and women must be ensured in all areas, including employment, work and pay.

The principle of equality shall not prevent the maintenance or adoption of measures providing for specific advantages in favour of the under-represented sex.

Motivation (2. “Gender directive”, 2004/113/EC)

EU Directive ([2004/113/EC](#)), 2004 version

– Article 5 (Actuarial factors) –

1. Member States shall ensure that in all new contracts concluded after 21 December 2007 at the latest, the use of sex as a factor in the calculation of premiums and benefits for the purposes of insurance and related financial services shall not result in differences in individuals' premiums and benefits.
2. Notwithstanding paragraph 1, Member States may decide before 21 December 2007 to permit proportionate differences in individuals' premiums and benefits where the use of sex is a determining factor in the assessment of risk based on relevant and accurate actuarial and statistical data. The Member States concerned shall inform the Commission and ensure that accurate data relevant to the use of sex as a determining actuarial factor are compiled, published and regularly updated.

Motivation (2. "Gender directive", 2004/113/EC)

- There was initially (2004) an **opt-out clause** (Article 5(2)).
- Where gender is a determining factor in the assessment of risk based on relevant and accurate actuarial and statistical data then proportionate differences in individual premiums or benefits are allowed.
- March 2011, the European Court of Justice issued its judgement into the "Test-Achats case". The ECJ ruled Article 5(2) was invalid.
- Insurers were no longer able to use gender as a risk factor when pricing policies, "**unisex pricing**".

"Machine learning won't give you anything like gender neutrality 'for free' that you didn't explicitly ask for", Kearns and Roth (2019)

Motivation (2. “Gender directive”, 2004/113/EC)

“Ten Oever” judgement (*Gerardus Cornelis Ten Oever v Stichting Bedrijfspensioenfonds voor het Glazenwassers – en Schoonmaakbedrijf*, in April 1993), the Advocate General Van Gerven argued that “*the fact that women generally live longer than men has no significance at all for the life expectancy of a specific individual and it is not acceptable for an individual to be penalized on account of assumptions which are not certain to be true in his specific case*,” as mentioned in [De Baere and Goessens \(2011\)](#). Schanze (2013) used the term “*injustice by generalization*.”



Motivation (2. “Gender directive”, 2004/113/EC)

The Telegraph News Sport Money Business Opinion

Men are still charged more than women for car insurance, despite EU rule change

Car insurers are dodging European equality laws by making gender judgements based on people's jobs, an economist has found

By Kate Palmer
10 April 2015 • 12:33pm



Insurers will price by occupation, and female-dominated jobs tend to attract cheaper premiums | CREDIT: Photo: Rex Features

CAR COSTS: Insurance according to job

Job	Proportion of men	Approximate average premium for a Fiat 500 driver
Dental Nurse	Less than 1pc male	£840
Solicitor	59pc male	£848
Sports and leisure assistants	56pc male	£880
Civil engineer	92pc male	£910
Social worker	21pc male	£920
Plasterer	98pc male	£950

McDonald, 'Indirect Gender Discrimination' (2015); ONS occupation data (2008)

(data source: Mcdonald (2015))

Motivation (3. Colorado)

Andrus et al. (2021), "*What we can't measure, we can't understand*"



First Regular Session | 74th General Assembly

Colorado General Assembly

September 27, 2023, the Colorado Division of Insurance exposed a new proposed regulation entitled **Concerning Quantitative Testing of External Consumer Data and Information Sources, Algorithms, and Predictive Models Used for Life Insurance Underwriting for Unfairly Discriminatory Outcomes**



Motivation (3. Colorado)

– Section 4 (Definitions) –

Bayesian Improved First Name Surname Geocoding, or “BIFSG” means, for the purposes of this regulation, the statistical methodology developed by the RAND corporation for estimating race and ethnicity.

External Consumer Data and Information Source, or “ECDIS” means, for the purposes of this regulation, a data source or an information source that is used by a life insurer to supplement or supplant traditional underwriting factors. This term includes credit scores, credit history, social media habits, purchasing habits, home ownership, educational attainment, licensures, civil judgments, court records, occupation that does not have a direct relationship to mortality, morbidity or longevity risk, consumer-generated Internet of Things data, biometric data, and any insurance risk scores derived by the insurer or third-party from the above listed or similar data and/or information source.

Motivation (3. Colorado)

– Section 5 (Estimating Race and Ethnicity) –

Insurers shall estimate the race or ethnicity of all proposed insureds that have applied for coverage on or after the insurer's initial adoption of the use of ECDIS, or algorithms and predictive models that use ECDIS, including a third party acting on behalf of the insurer that used ECDIS, or algorithms and predictive models that used ECDIS, in the underwriting decision-making process, by utilizing:

1. BIFSG and the insureds' or proposed insureds' name and geolocation (information included in the applications) for life insurance shall be used to estimate the race and ethnicity of each insured or proposed insured.
2. For the purposes of BIFSG, the following racial and ethnic categories shall be used: Hispanic, Black, Asian Pacific Islander (API), and White.

Motivation (3. Colorado)

– Section 6 (Application Approval Decision Testing Requirements) –

Using the BIFSG estimated race and ethnicity of proposed insureds and the following methodology, insurers shall calculate whether Hispanic, Black, and API proposed insureds are disapproved at a statistically significant different rate relative to White applicants for whom the insurer, or a third party acting on behalf of the insurer, used ECDIS, or an algorithm or predictive model that used ECDIS, in the underwriting decision-making process.

1. Logistic regression shall be used to model the binary underwriting outcome of either approved or denied.
2. The following factors may be accounted for as control variables in the regression model: policy type, face amount, age, gender, and tobacco use.
3. The estimated race or ethnicity of the proposed insureds shall be accounted for by including Hispanic, Black, and Asian Pacific Islander (API) as separate dummy variables in the regression model.

Motivation (3. Colorado)

4. Determine if there is a statistically significant difference in approval rates for each BIFSG estimated race or ethnicity variable as indicated by a *p*-value of less than .05.
 - a. If there is not a statistically significant difference in approval rates, no further testing is required.
 - b. If there is a statistically significant difference in approval rates, the insurer shall determine whether the difference in approval rates is five (5) percentage points or greater as indicated by the marginal effects value of each BIFSG estimated race or ethnicity variable. (...)

Motivation (3. Colorado)

– Section 7 (Premium Rate Testing Requirements) –

Using the insureds' BIFSG estimated race and ethnicity, insurers shall determine if there is a statistically significant difference in the premium rate per \$1,000 of face amount for policies issued to Hispanic, Black, and API insureds relative to White insureds for whom the insurer, or a third party acting on behalf of the insurer, used ECDIS, or an algorithm or predictive model that used ECDIS, in the underwriting decision-making process.

1. Linear regression shall be used to model the continuous numerical outcome of premium rate per \$1,000 of face amount.
2. The following factors may be accounted for as control variables in the regression model: policy type, face amount, age, gender, and tobacco use.
3. The estimated race or ethnicity of the proposed insureds shall be accounted for by including Hispanic, Black, and Asian Pacific Islander (API) as separate dummy variables in the regression model.

Motivation (3. Colorado)

4. Determine if there is a statistically significant difference in the premium rate per \$1,000 of face amount for each BIFSG estimated race or ethnicity variable as indicated by a p-value of less than .05.
 - a. If there is not a statistically significant difference in premium rate per \$1,000 of face amount, no further testing is required.
 - b. If there is a statistically significant difference in premium rate per \$1,000 of face amount, determine whether the premium rate per \$1,000 of face amount is at least 5% more than the average premium rate per \$1,000 for all policies.
 - i. If the difference in premium rate per \$1,000 of face amount is less than 5%, no further testing is required.
 - ii. If the difference in premium rate per \$1,000 of face amount is 5% or greater, further testing is required as described in Section 8.

Motivation (3. Colorado)

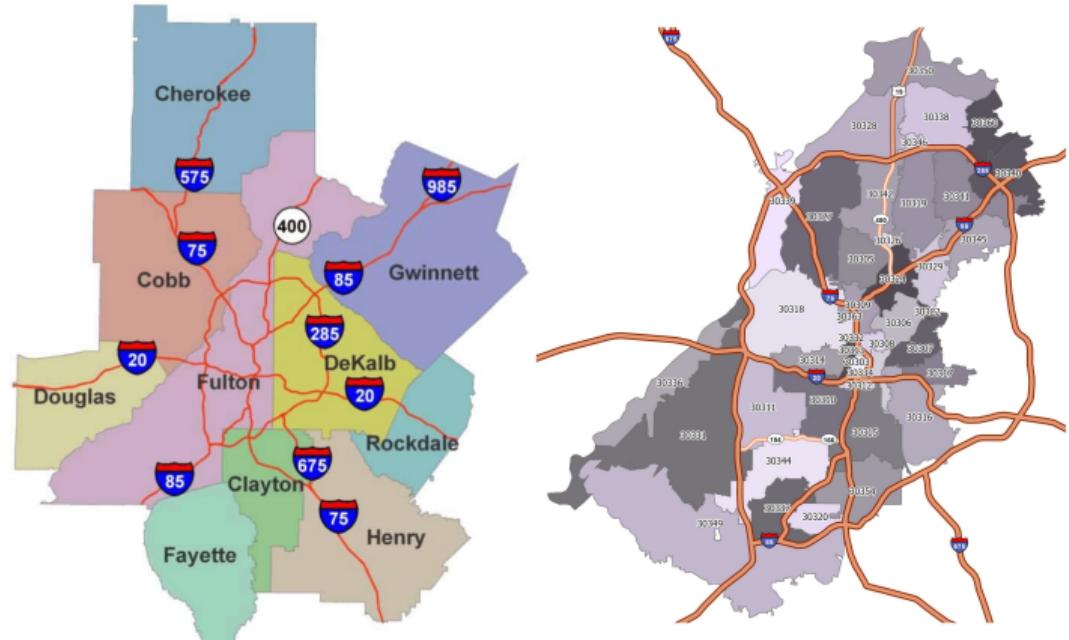
In Elliott et al. (2009), BIFSG¹, `library(eiCompare)`. , consider 12 people living near Atlanta, GA (Fulton & Gwinnett counties), and `eiCompare::wru_predict_race_wrapper`

		last	first	county	city	zipcode	whi	bla	his	asi
1	1	LOCKLER	GABRIELLA	Fulton	Atlanta	30318	0	0	0	0
2	2	RADLEY	OLIVIA	Fulton	Fairburn	30213	14	83	1	0
3	3	BOORSE	KEISHA	Fulton	Atlanta	30331	97	0	3	0
4	4	MAZ	SAVANNAH	Gwinnett	Norcross	30093	5	6	76	13
5	5	GAULE	NATASHIA	Gwinnett	Snellville	30078	67	19	14	0
6	6	MCMELLEN	ISMAEL	Gwinnett	Lilburn	30047	73	15	6	3
7	7	RIDEOUT	LUQMAN	Gwinnett	Snellville	30078	77	18	2	0
8	8	WASHINGTON	BRYN	Gwinnett	Norcross	30093	0	95	3	0
9	9	KULENOVIC	EVELYN	Gwinnett	Buford	30518	100	0	0	0
10	10	HERNANDEZ	SAMANTHA	Gwinnett	Duluth	30096	3	1	94	1
11	11	LONG	BESSIE	Gwinnett	Duluth	30096	53	39	1	1
12	12	HE	JOSE	Gwinnett	Lawrenceville	30045	2	3	4	89

¹Bayesian Improved First Name Surname Geocoding

Motivation (3. Colorado)

We have 12 people,
in two counties near Atlanta
(about 10 zip-codes)



Motivation (3. Colorado)

- Use `eiCompare::wru_predict_race_wrapper` on a revised dataset with the same name “Savannah Maz”

1	last	first	county	city	zipcode	whi	bla	his	asi
2	1	MAZ	SAVANNAH	Fulton	Atlanta	30318	0	0	0 100
3	2	MAZ	SAVANNAH	Fulton	Fairburn	30213	13	61	22 3
4	3	MAZ	SAVANNAH	Fulton	Atlanta	30331	3	77	19 1
5	4	MAZ	SAVANNAH	Gwinnett	Norcross	30093	5	6	76 13
6	5	MAZ	SAVANNAH	Gwinnett	Snellville	30078	13	18	69 0
7	6	MAZ	SAVANNAH	Gwinnett	Lilburn	30047	28	22	34 16
8	7	MAZ	SAVANNAH	Gwinnett	Snellville	30078	53	3	40 3
9	8	MAZ	SAVANNAH	Gwinnett	Norcross	30093	5	6	76 13
10	9	MAZ	SAVANNAH	Gwinnett	Buford	30518	79	4	14 2
11	10	MAZ	SAVANNAH	Gwinnett	Duluth	30096	32	8	38 22
12	11	MAZ	SAVANNAH	Gwinnett	Duluth	30096	55	19	22 5
13	12	MAZ	SAVANNAH	Gwinnett	Lawrenceville	30045	15	19	62 4

Motivation (3. Colorado)

- Use `eiCompare::wru_predict_race_wrapper` on a revised dataset with the same name “Bryn Washington”

1	last	first	county	city	zipcode	whi	bla	his	asi
2 1	WASHINGTON	BRYN	Fulton	Atlanta	30318	0	0	0	100
3 2	WASHINGTON	BRYN	Fulton	Fairburn	30213	0	99	0	0
4 3	WASHINGTON	BRYN	Fulton	Atlanta	30331	0	99	0	0
5 4	WASHINGTON	BRYN	Gwinnett	Norcross	30093	0	95	3	0
6 5	WASHINGTON	BRYN	Gwinnett	Snellville	30078	0	96	1	0
7 6	WASHINGTON	BRYN	Gwinnett	Lilburn	30047	1	98	0	0
8 7	WASHINGTON	BRYN	Gwinnett	Snellville	30078	6	87	2	0
9 8	WASHINGTON	BRYN	Gwinnett	Norcross	30093	0	95	3	0
10 9	WASHINGTON	BRYN	Gwinnett	Buford	30518	7	92	1	0
11 10	WASHINGTON	BRYN	Gwinnett	Duluth	30096	2	96	1	0
12 11	WASHINGTON	BRYN	Gwinnett	Duluth	30096	1	96	0	0
13 12	WASHINGTON	BRYN	Gwinnett	Lawrenceville	30045	0	98	1	0

Motivation (3. Colorado)

- Use `eiCompare::wru_predict_race_wrapper` on a revised dataset with the same name "Samantha Hernandez"

	last	first	county	city	zipcode	whi	bla	his	asi
1	HERNANDEZ	SAMANTHA	Fulton	Atlanta	30318	0	0	0	100
2	HERNANDEZ	SAMANTHA	Fulton	Fairburn	30213	2	12	85	0
3	HERNANDEZ	SAMANTHA	Fulton	Atlanta	30331	0	16	81	0
4	HERNANDEZ	SAMANTHA	Gwinnett	Norcross	30093	0	0	99	0
5	HERNANDEZ	SAMANTHA	Gwinnett	Snellville	30078	1	1	97	0
6	HERNANDEZ	SAMANTHA	Gwinnett	Lilburn	30047	3	3	92	1
7	HERNANDEZ	SAMANTHA	Gwinnett	Snellville	30078	5	0	94	0
8	HERNANDEZ	SAMANTHA	Gwinnett	Norcross	30093	0	0	99	0
9	HERNANDEZ	SAMANTHA	Gwinnett	Buford	30518	17	1	81	0
10	HERNANDEZ	SAMANTHA	Gwinnett	Duluth	30096	3	1	94	1
11	HERNANDEZ	SAMANTHA	Gwinnett	Duluth	30096	8	4	86	0
12	HERNANDEZ	SAMANTHA	Gwinnett	Lawrenceville	30045	1	2	97	0

Motivation (3. Colorado)

- Use `eiCompare::wru_predict_race_wrapper` on a revised dataset with the same name “Jose He”

1	last	first	county	city	zipcode	whi	bla	his	asi
2	1	HE	JOSE	Fulton	Atlanta	30318	0	0	0 100
3	2	HE	JOSE	Fulton	Fairburn	30213	2	9	2 84
4	3	HE	JOSE	Fulton	Atlanta	30331	1	27	3 55
5	4	HE	JOSE	Gwinnett	Norcross	30093	0	0	2 98
6	5	HE	JOSE	Gwinnett	Snellville	30078	13	18	30 0
7	6	HE	JOSE	Gwinnett	Lilburn	30047	1	1	1 97
8	7	HE	JOSE	Gwinnett	Snellville	30078	8	1	3 86
9	8	HE	JOSE	Gwinnett	Norcross	30093	0	0	2 98
10	9	HE	JOSE	Gwinnett	Buford	30518	19	1	2 78
11	10	HE	JOSE	Gwinnett	Duluth	30096	1	0	0 98
12	11	HE	JOSE	Gwinnett	Duluth	30096	6	2	1 85
13	12	HE	JOSE	Gwinnett	Lawrenceville	30045	2	3	4 89

Motivation (4. Motor Insurance in the U.S.)

California

Allowed (with applicable limitations): driving experience, marital status, address/zip code

Prohibited (or effectively prohibited): gender, age, credit history, education, occupation, employment status, residential status, insurance history

Notes & Clarifications: California's insurance commissioner banned gender as of January 2019. Occupation and education are permitted for use in group plans (i.e. for alumni associations and other membership programs).

Georgia

Allowed (with applicable limitations): gender, age, years of driving experience, credit history, marital status, residential status, address/zip code, insurance history

Prohibited (or effectively prohibited): occupation, education, and employment status

Notes & Clarifications: none

Hawaii

Allowed (with applicable limitations): address/zip code, insurance history

Prohibited (or effectively prohibited): gender, age, years of driving experience, credit history, education, occupation, employment status, marital status, residential status

Notes & Clarifications: none

Illinois

Allowed (with applicable limitations): gender, age, years of driving experience, credit history, education, occupation, employment status, marital status, residential status, address/zip code, insurance history

Prohibited (or effectively prohibited): none

Notes & Clarifications: none

Massachusetts

Allowed (with applicable limitations): years of driving experience, address/zip code, insurance history

Prohibited (or effectively prohibited): gender, age, credit history, education, occupation, employment status, marital status, residential status

Notes & Clarifications: none

Michigan

Allowed (with applicable limitations): gender (group-rated policies), age, years of driving experience, credit history, education, occupation, employment status, marital status (group-rated policies), residential status, address/zip code, insurance history

Prohibited (or effectively prohibited): gender (non-group policies), marital status (non-group policies)

Notes & Clarifications: Gender and marital status are permitted only in rate-making for group plans (i.e. for alumni associations and other membership programs). **UPDATE: Michigan lawmakers approved a major insurance reform bill** in May 2019 that will ban insurers in the state from using gender, marital status, address/zipcode, residential status, education and occupation in rate setting. The ban will be enforced starting in July 2020. Insurers will be permitted to use "territory" as approved by the state regulators instead of zip code.

New York

Allowed (with applicable limitations): gender, age, years of driving experience, credit history, marital status, residential status, address/zip code, insurance history

Prohibited (or effectively prohibited): occupation, education, employment status

Notes & Clarifications: none

via **The Zebra (2022)**

Motivation (5. Admission in Graduate Program, UC Berkeley)

	Total	Men	Women	Proportions
Total	5233/12763 ~ 41%	3714/8442 ~ 44%	1512/4321 ~ 35%	66%-34%
Top 6	1745/4526 ~ 39%	1198/2691 ~ 45%	557/1835 ~ 30%	59%-41%
A	597/933 ~ 64%	512/825 ~ 62%	89/108 ~ 82%	88%-12%
B	369/585 ~ 63%	353/560 ~ 63%	17/ 25 ~ 68%	96%- 4%
C	321/918 ~ 35%	120/325 ~ 37%	202/593 ~ 34%	35%-65%
D	269/792 ~ 34%	138/417 ~ 33%	131/375 ~ 35%	53%-47%
E	146/584 ~ 25%	53/191 ~ 28%	94/393 ~ 24%	33%-67%
F	43/714 ~ 6%	22/373 ~ 6%	24/341 ~ 7%	52%-48%

Data from [Bickel et al. \(1975\)](#)

Formalize the later, S is the (binary) genre, Y the admission and X the program (category),

Motivation (5. Admission in Graduate Program, UC Berkeley)

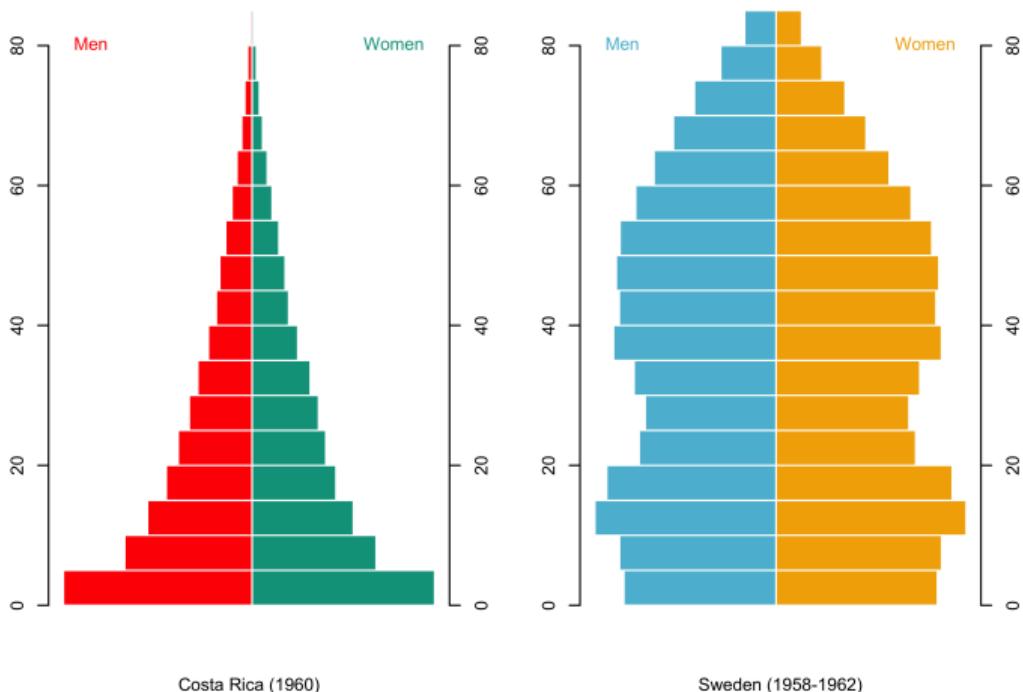
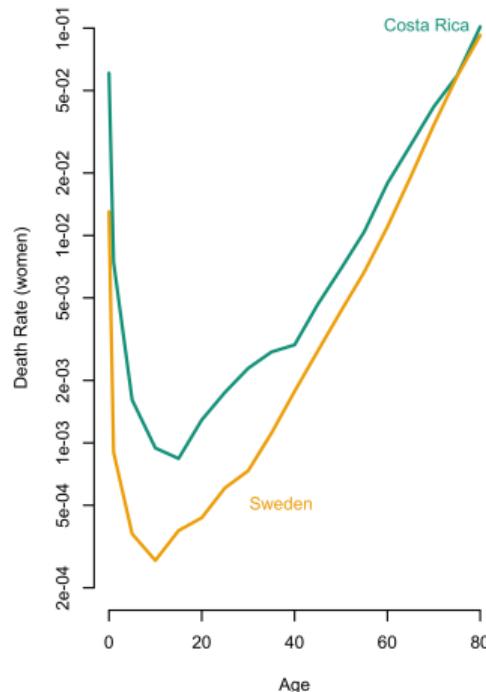
$$\begin{aligned} \mathbb{P}[Y = \text{yes} | S = \text{men}] &\geq \mathbb{P}[Y = \text{yes} | S = \text{women}] \\ \mathbb{P}[Y = \text{yes} | X = x, S = \text{men}] &\leq \mathbb{P}[Y = \text{yes} | X = x, S = \text{women}], \forall x. \end{aligned}$$

overall admission

conditional on program

"the bias in the aggregated data stems not from any pattern of discrimination on the part of admissions committees, which seems quite fair on the whole, but apparently from prior screening at earlier levels of the educational system. Women are shunted by their socialization and education toward fields of graduate study that are generally more crowded, less productive of completed degrees, and less well funded, and that frequently offer poorer professional employment prospects," Bickel et al. (1975)

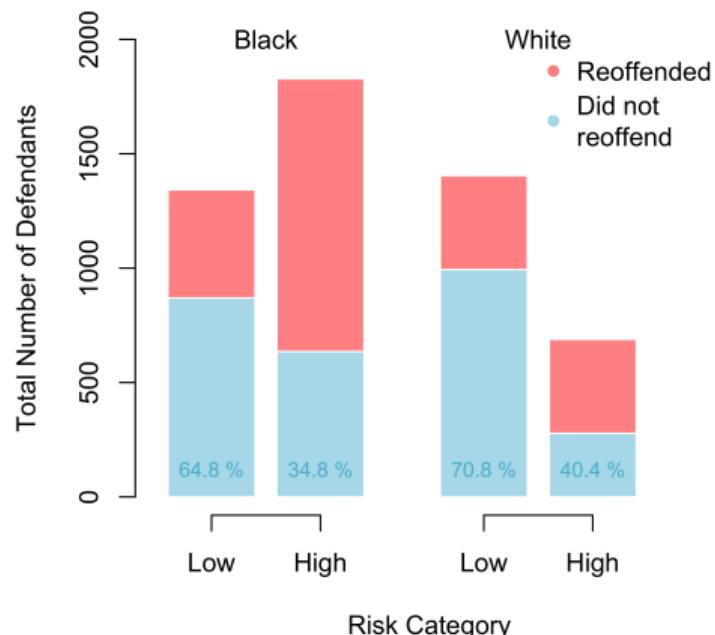
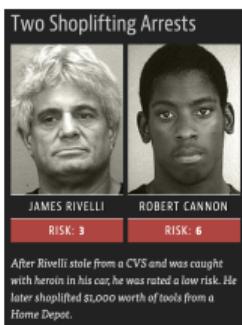
Motivation (5'. Mortality in Costa Rica and Sweden)



Overall mortality rate for women, 8.12% in Costa Rica, against 9.29% in Sweden.

Motivation (6. Propublica, Actuarial Justice)

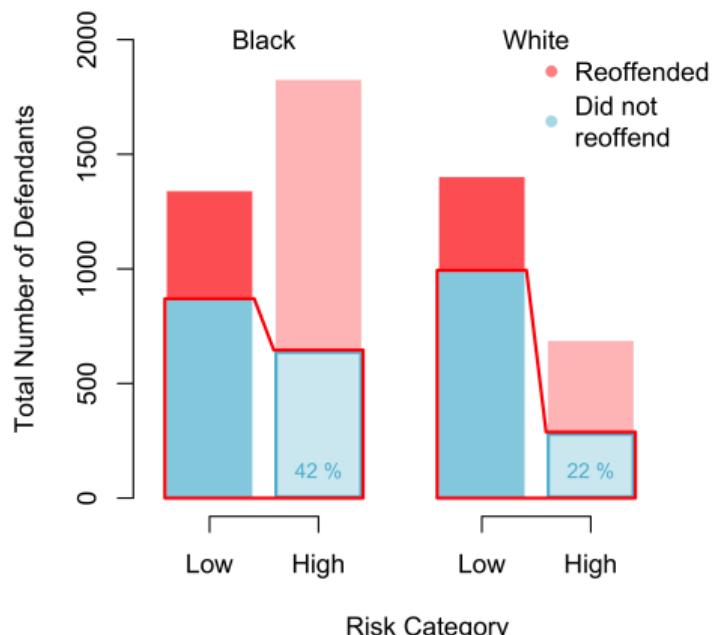
- Concept of "actuarial justice" as coined in Feeley and Simon (1994)
- Correctional Offender Management Profiling for Alternative Sanctions (COMPAS), Perry (2013)



- <https://github.com/propublica/compas-analysis>
- Angwin et al. (2016) Machine Bias
Dressel and Farid (2018)

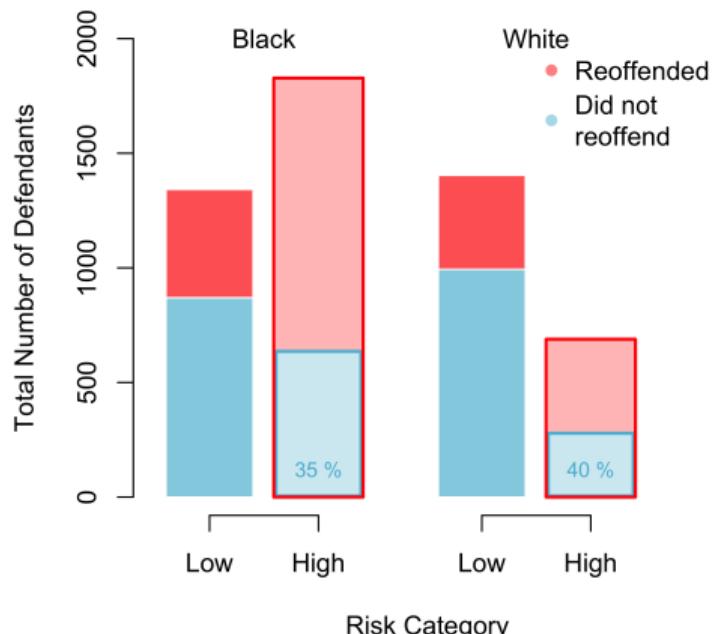
Motivation (6. Propublica, Actuarial Justice)

- From Feller et al. (2016),
 - ▶ for White people, among those who did not re-offend, 22% were wrongly classified,
 - ▶ for Black people, among those who did not re-offend, 42% were wrongly classified,
 - ▶ problem, since $42\% \gg 22\%$



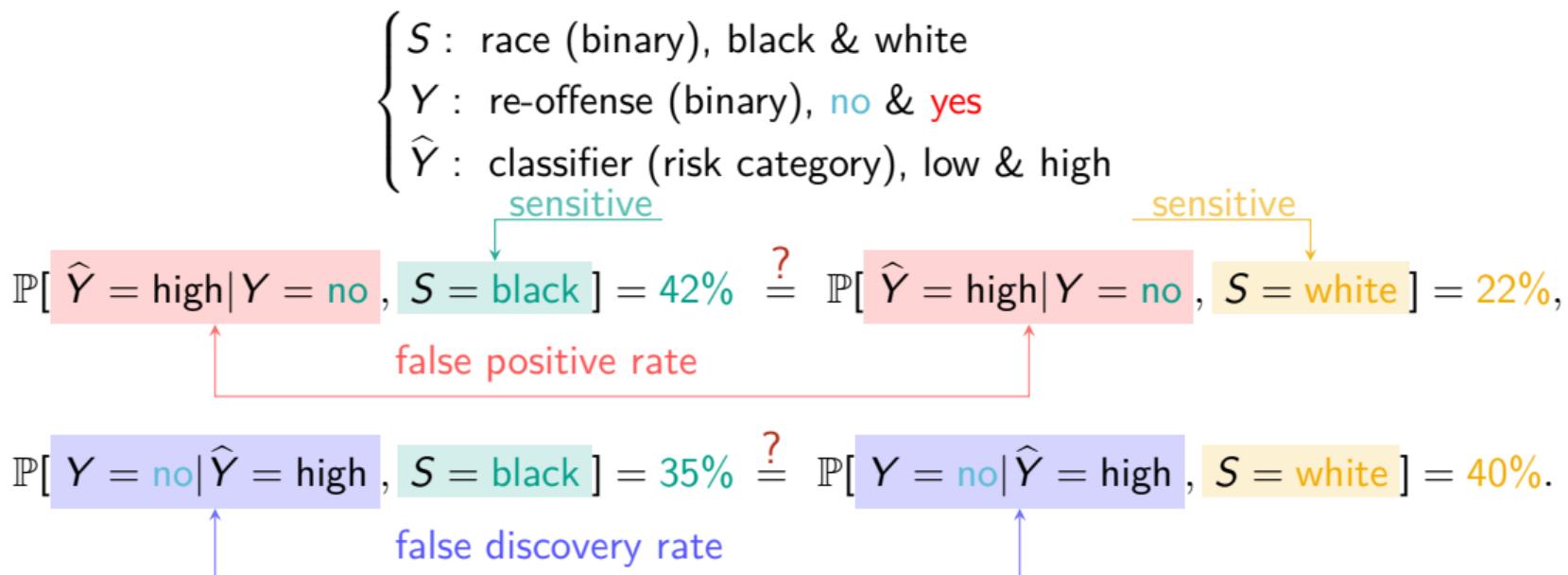
Motivation (6. Propublica, Actuarial Justice)

- From Dieterich et al. (2016),
 - ▶ for White people, among those who were classified as high risk, 40% did not re-offend,
 - ▶ for Black people, among those who were classified as high risk, 35% did not re-offend,
 - ▶ no problem, since $40\% \approx 35\%$

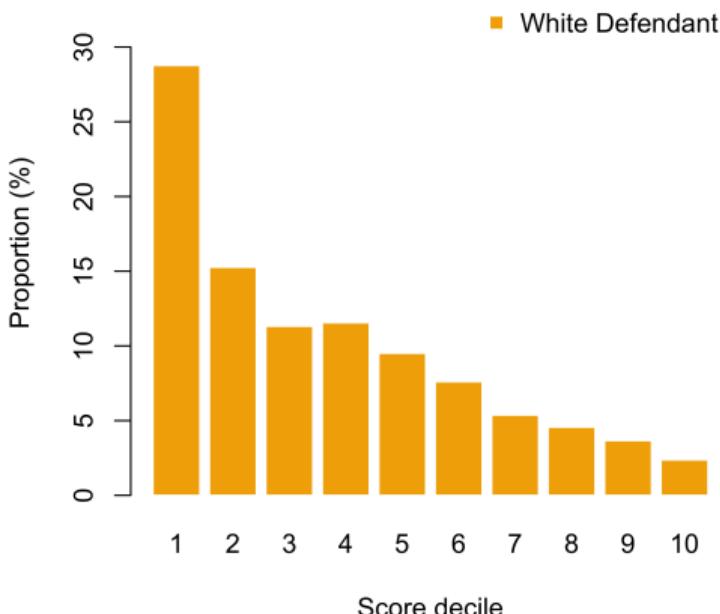
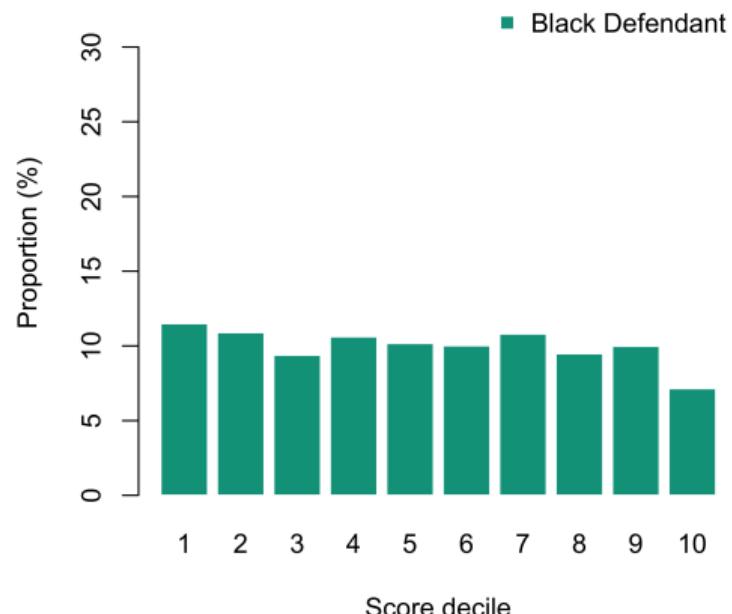


Motivation (6. Propublica, Actuarial Justice)

Formalize the later,

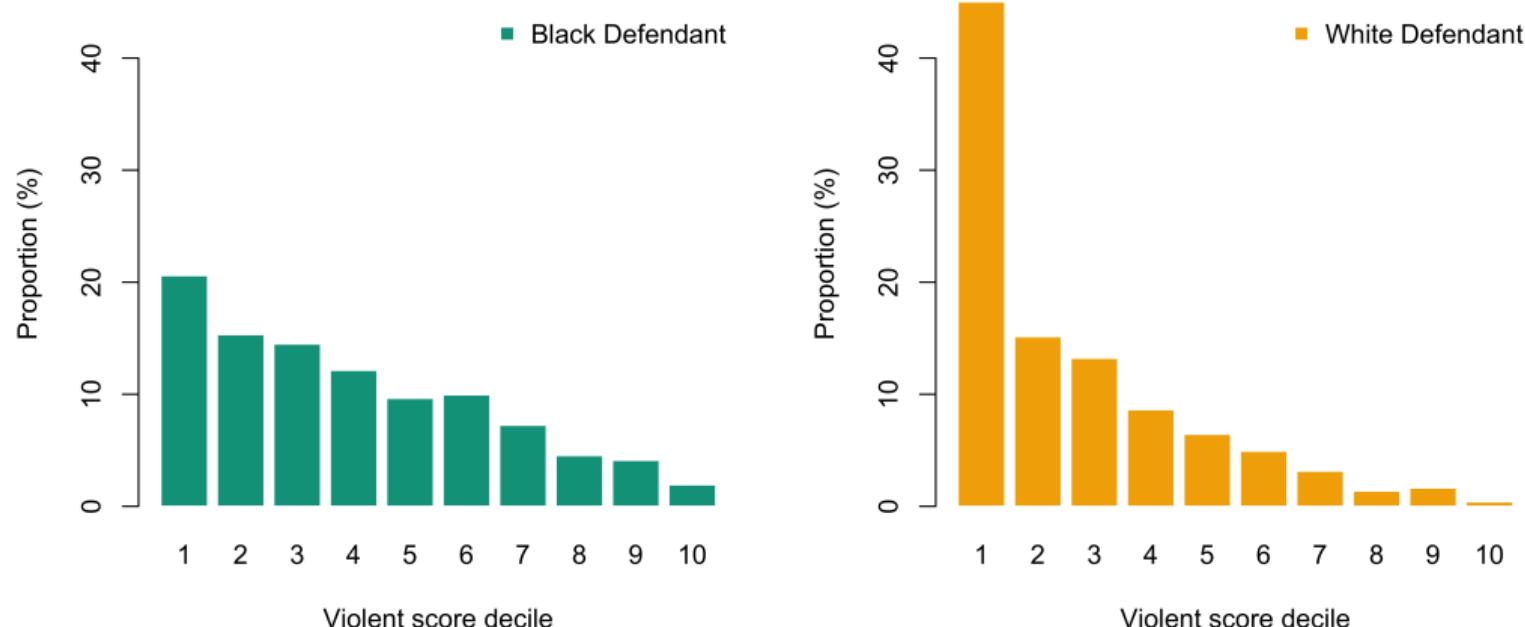


Motivation (6. Propublica, Actuarial Justice)



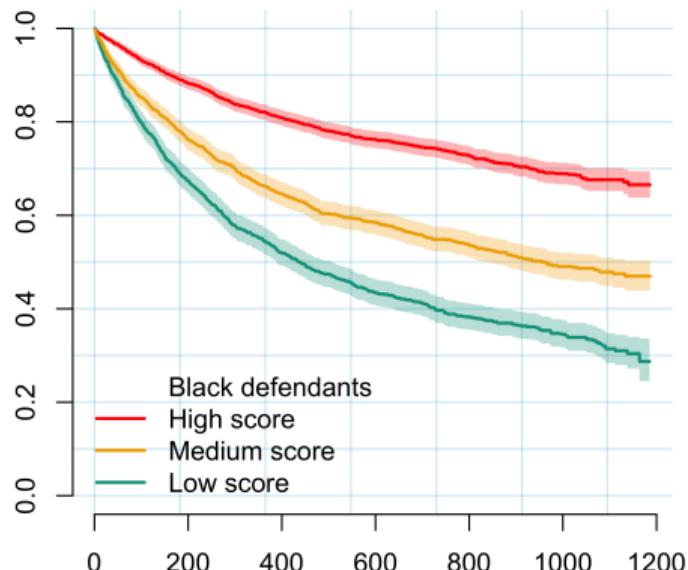
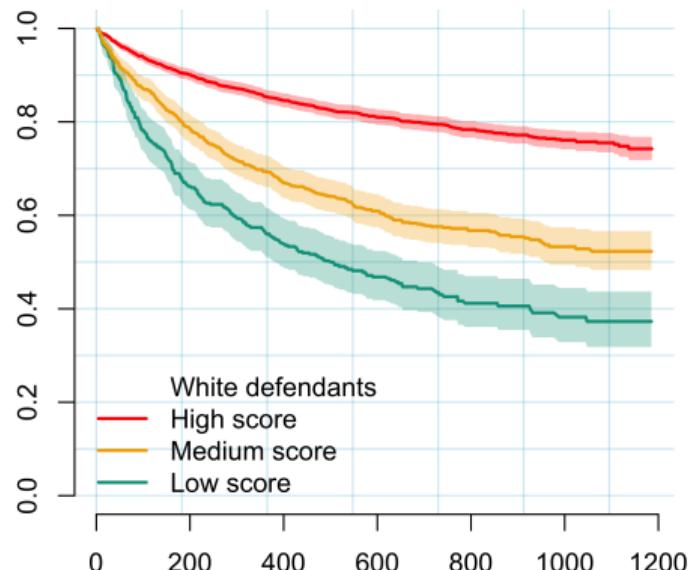
Look at score distributions, black and white defendant, Larson et al. (2016)

Motivation (6. Propublica, Actuarial Justice)



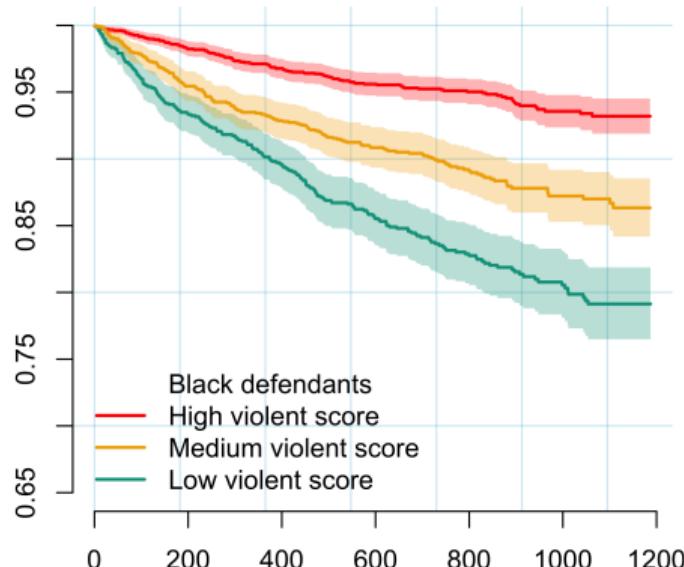
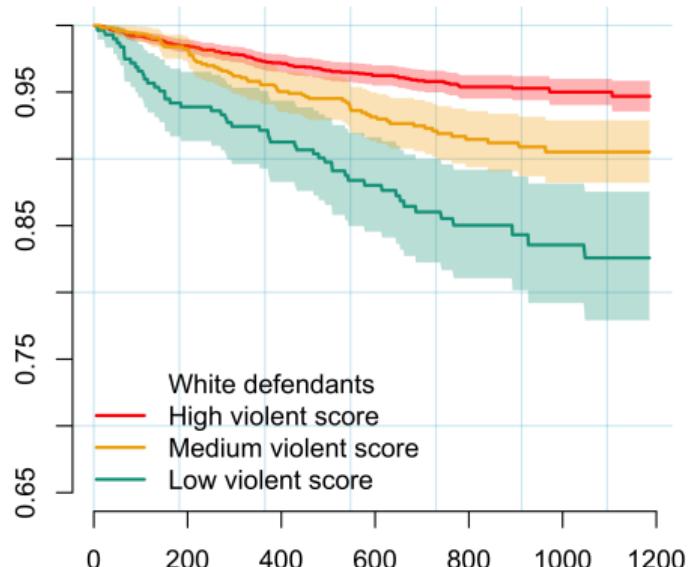
Look at score distributions, black and white defendant, Larson et al. (2016)

Motivation (6. Propublica, Actuarial Justice)



Cox Proportional Hazards model, **black** and **white** defendant, Larson et al. (2016) 

Motivation (6. Propublica, Actuarial Justice)



Cox Proportional Hazards model, **black** and **white** defendant, Larson et al. (2016)

Motivation (7. Québec)

Au Québec, Charte des droits et libertés de la personne ([C-12](#))

– Article 10 –

Toute personne a droit à la reconnaissance et à l'exercice, en pleine égalité, des droits et libertés de la personne, sans distinction, exclusion ou préférence fondée sur la race, la couleur, le sexe, l'identité ou l'expression de genre, la grossesse, l'orientation sexuelle, l'état civil, l'âge sauf dans la mesure prévue par la loi, la religion, les convictions politiques, la langue, l'origine ethnique ou nationale, la condition sociale, le handicap ou l'utilisation d'un moyen pour pallier ce handicap.

Il y a **discrimination** lorsqu'une telle distinction, exclusion ou préférence a pour effet de détruire ou de compromettre ce droit.



Motivation (7. Québec)

Au Québec, Charte des droits et libertés de la personne ([C-12](#))

– Article 20.1 –

Dans un **contrat d'assurance** ou de rente, un régime d'avantages sociaux, de retraite, de rentes ou d'assurance ou un régime universel de rentes ou d'assurance, une distinction, exclusion ou préférence fondée sur l'âge, le sexe ou l'état civil est **réputée non discriminatoire lorsque son utilisation est légitime et que le motif qui la fonde constitue un facteur de détermination de risque, basé sur des données actuarielles.**



Motivation (8. Intention)

En France, Loi n° 2008-496 du 27 mai 2008

– Article 1 –

Constitue une **discrimination indirecte** une disposition, un critère ou une pratique neutre en apparence, mais susceptible d'entraîner, pour l'un des motifs mentionnés au premier alinéa, un désavantage particulier pour des personnes par rapport à d'autres personnes, à moins que cette disposition, ce critère ou cette pratique ne soit objectivement justifié par un but légitime et que les moyens pour réaliser ce but ne soient nécessaires et appropriés.

Extention de la "Loi n° 72-546 du 1 juillet 1972", qui supprima l'exigence de l'intention spécifique.

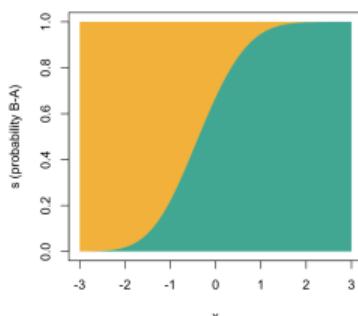
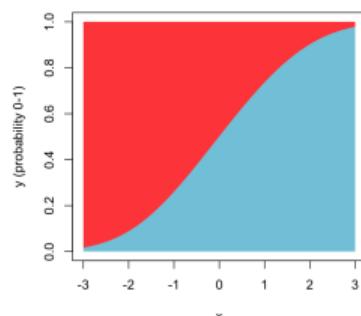
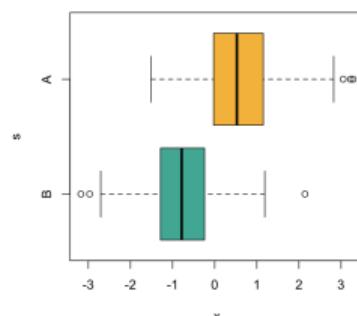
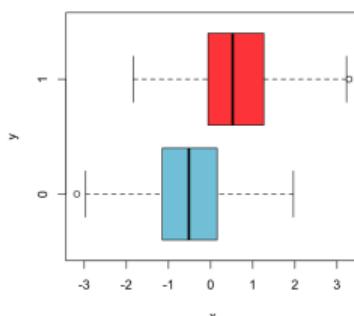
"*Technology is neither good nor bad; nor is it neutral*" , Kranzberg (1986)

Datasets

▶ toydata1

Consider a confounding Gaussian variable X_0 , $X_0 \sim \mathcal{N}(0, 1)$, and

$$\begin{cases} X = X_0 + \epsilon, \quad \epsilon \sim \mathcal{N}(0, 1/2^2), \\ S = \mathbf{1}(X_0 + \eta > 0), \quad \eta \sim \mathcal{N}(0, 1/2^2), \quad s \in \{\textcolor{teal}{A}, \textcolor{orange}{B}\}, \text{ PROBLEME !!!!} \\ Y = \mathbf{1}(X_0 + \nu > 0), \quad \nu \sim \mathcal{N}(0, 1/2^2), \quad y \in \{\textcolor{teal}{0}, \textcolor{red}{1}\}. \end{cases}$$

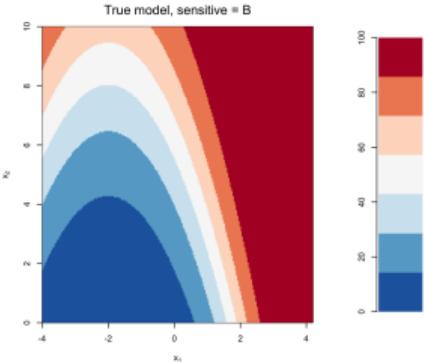
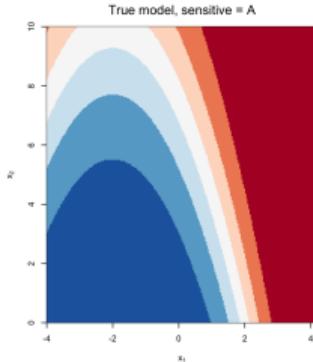
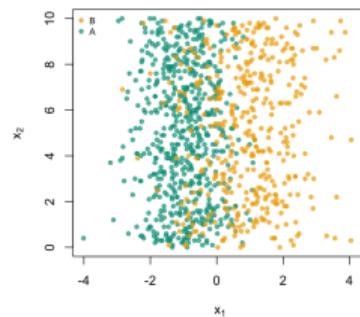
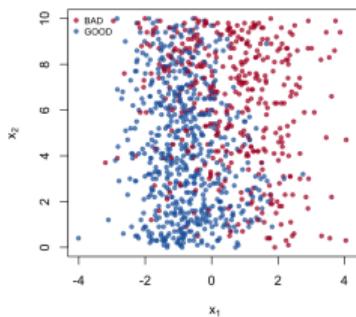


$x \mapsto \mathbb{P}[Y = \textcolor{teal}{0}|X = x]$ (left-hand side) and $x \mapsto \mathbb{P}[S = \textcolor{teal}{A}|X = x]$ (right-hand side)

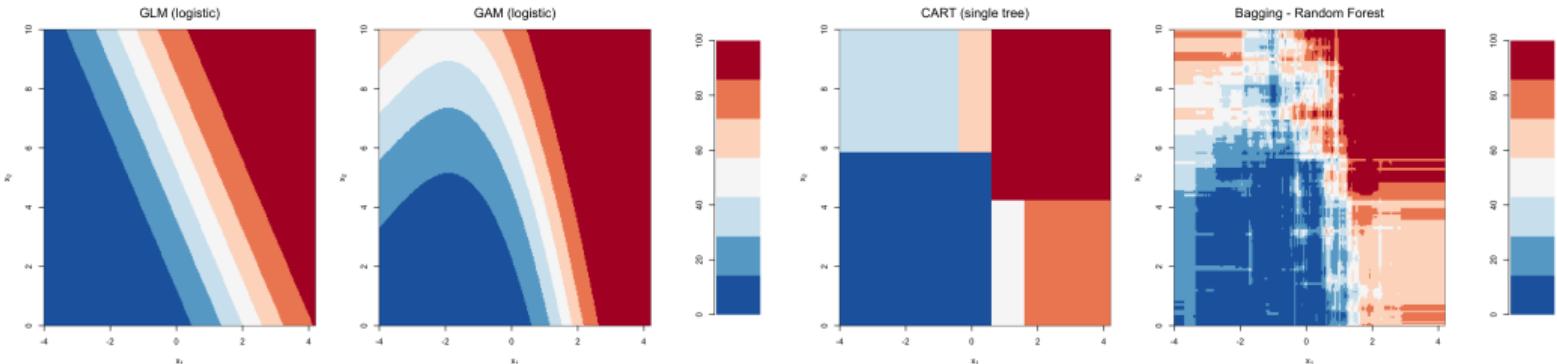
Datasets

▶ toydata2

- ▶ binary sensitive attribute, $s \in \{A, B\}$, (60% and 40%)
- ▶ $(x_1, x_3) \sim \mathcal{N}(\mu_s, \Sigma_s)$, $r_{s=A} = 0.4$ and $r_{s=B} = 0.7$
- ▶ $x_2 \sim \mathcal{U}([0, 10])$, independent of x_1 and x_3
- ▶ $\eta = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1^2 + \beta_4 \mathbf{1}_B(s)$, that does not depend on x_3
- ▶ $y \sim \mathcal{B}(p)$ where $p = \exp(\eta)/[1 + \exp(\eta)] = \mu(x_1, x_2, s)$.

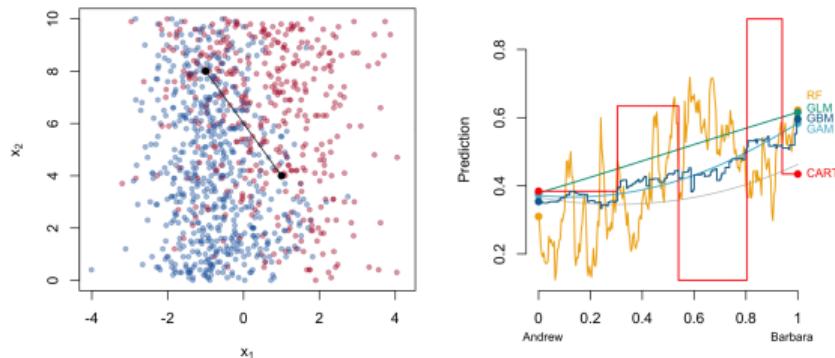


Datasets



Five models are considered

- ▶ plain GLM (logistic)
- ▶ GAM (cubic splines)
- ▶ CART (classification tree)
- ▶ RF (random forest)
- ▶ GBM (gradient boosting)



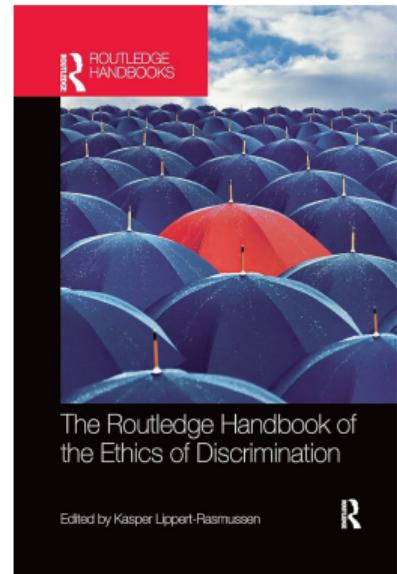
Datasets

- **GermanCredit**, $m = 1,000$
 - ▶ binary sensitive attribute, $s \in \{A, B\}$, (64% and 36%) corresponding to gender
 - ▶ y denotes a default (30%)
 - ▶ x_1, \dots, x_k denote legitimate credit variables (Duration, Purpose, Credit_amount, Age, Housing, Existing_credits, Foreign_worker, Resident_since, etc)
- **FrenchMotor** (policy observe over one year), $n = 12,437$
 - ▶ binary sensitive attribute, $s \in \{A, B\}$, (31% and 69%) corresponding to gender
 - ▶ y denotes the occurrence of a car accident (8.67%, unbalanced data)
 - ▶ x_1, \dots, x_k denote legitimate credit variables (MariStat, VehAge, SocioCateg, DrivAge, VehBody, VehEnergy, VehMaxSpeed, Garage, VehUsage, etc)

– Part 1 –
Insurance

Discrimination and Insurance

"What is unique about insurance is that even statistical discrimination which by definition is absent of any malicious intentions, poses significant moral and legal challenges. Why? Because on the one hand, policy makers would like insurers to treat their insureds equally, without discriminating based on race, gender, age, or other characteristics, even if it makes statistical sense to discriminate (...) On the other hand, at the core of insurance business lies discrimination between risky and non-risky insureds. But riskiness often statistically correlates with the same characteristics policy makers would like to prohibit insurers from taking into account." Avraham (2017)



Discrimination and Insurance

Definition 2.2: Mutuality, Wilkie (1997)

Mutuality is considered as the normal form of commercial private insurance, where participants contribute to the risk pool through a premium that relates to their particular risk at the time of the application, i.e., the higher the risk that they bring to the pool, the higher the premium required.

Definition 2.3: Solidarity, Wilkie (1997)

Solidarity is the basis of most national or social insurance schemes. Participation in such state-run schemes is generally compulsory and individuals have no discretion over their level of cover. All participants normally have the same level of cover. In solidarity schemes the contributions are not based on the expected risk of each participant.

Insurance Pricing and Predictive Modeling

“Humans think in stories rather than facts, numbers or equations - and the simpler the story, the better,” Harari (2018). For insurers, it is often a mixture of both.

For Glenn (2000), insurer's risk selection process has two sides:

- › the one presented to regulators and policyholders (numbers, statistics and objectivity),
- › the other presented to underwriters (stories, character and subjective judgment).

The rhetoric of insurance exclusion – numbers, objectivity and statistics – forms what Brian Glenn calls “*the myth of the actuary,*” “*a powerful rhetorical situation in which decisions appear to be based on objectively determined criteria when they are also largely based on subjective ones*” or “*the subjective nature of a seemingly objective process*”.

Glenn (2003) claimed that there are many ways to rate accurately. Insurers can rate risks in many different ways depending on the stories they tell on which characteristics

Insurance Pricing and Predictive Modeling

are important and which are not. “*The fact that the selection of risk factors is subjective and contingent upon narratives of risk and responsibility has in the past played a far larger role than whether or not someone with a wood stove is charged higher premiums.*” Going further, “*virtually every aspect of the insurance industry is predicated on stories first and then numbers.*”

“*all models are wrong but some models are useful,*” Box et al. (2011) (in other words, any model is at best a useful fable).

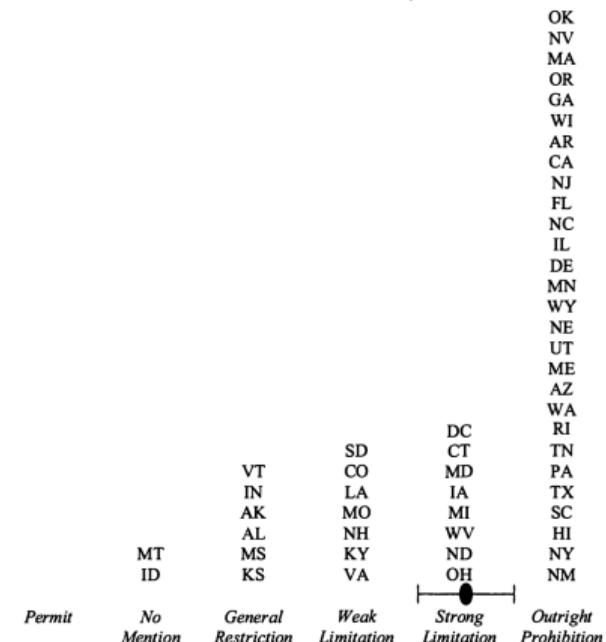
Insurance Pricing and Predictive Modeling

From Avraham et al. (2013),

- Expressly Permit (-1) - The state has a statute expressly or impliedly permitting insurers to take the characteristic into account.
- No Law on Point (0) - The state laws are silent with respect to the particular characteristic.
- General Restriction (1) - The state has a statute that generally prohibits "unfair discrimination," either across all lines of insurance or in some lines of insurance, but that statute does not provide any explanation as to what constitutes unfair discrimination and does not single out any particular trait for limitation.

(...)

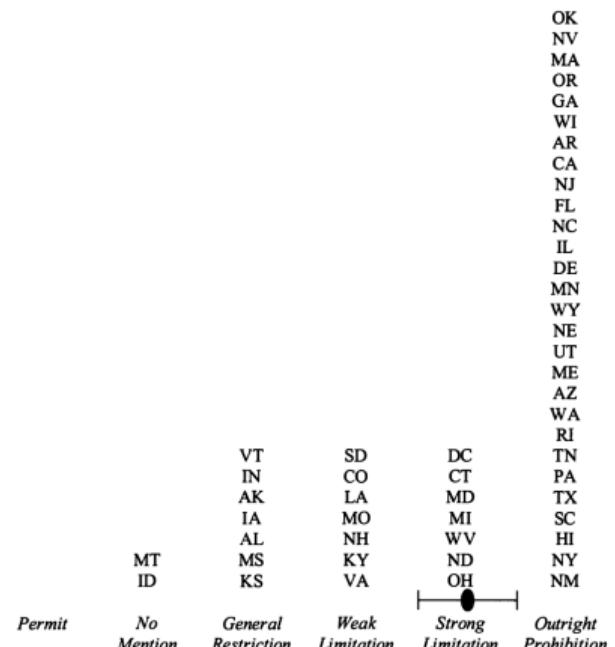
FIGURE 1a. Distribution of States' Scores for Race, in Auto Insurance



Insurance Pricing and Predictive Modeling

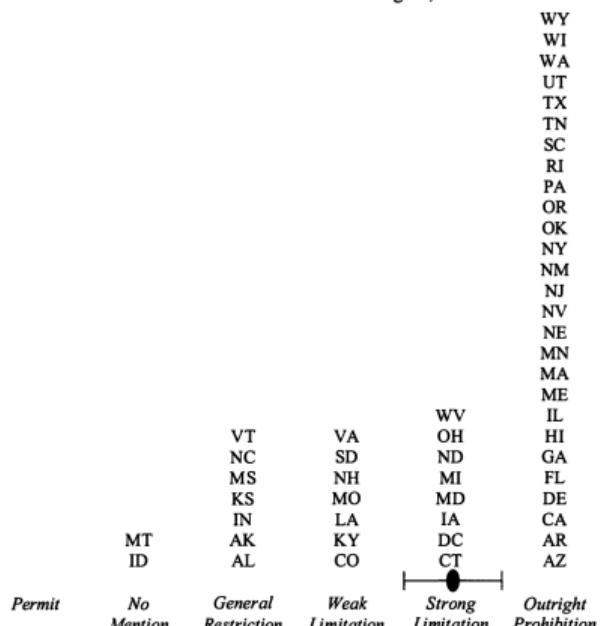
- **Characteristic-Specific Weak Limitation (2)** - The state has a statute that limits the use of a particular characteristic in either issuance, renewal, or cancellation.
- **Characteristic-Specific Strong Limitation (3)** - The state has a statute that prohibits the use of a particular characteristic when the policy is either issued, renewed, or cancelled, or the state has a statute that limits but does not completely prohibit the use of a particular characteristic in rate setting.
- **Characteristic-Specific Prohibition (4)** - The state has a statute that expressly prohibits insurers from taking into account a specific characteristic in setting rates.

FIGURE 1b. Distribution of States' Scores for National Origin, in Auto Insurance



Insurance Pricing and Predictive Modeling

FIGURE 1c. Distribution of States' Scores for Religion, in Auto Insurance

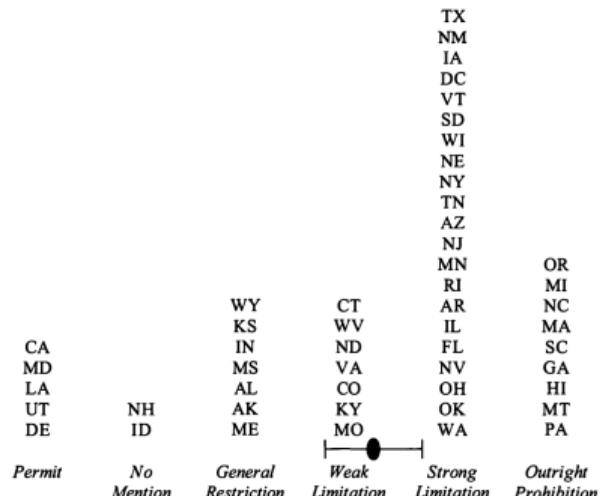


"Race, national origin, and religion have a special place in this country's history; and, as discussed above, discrimination on the basis of these three characteristics has been subject to stricter scrutiny in American law than have other characteristics," Avraham et al. (2013)

Insurance Pricing and Predictive Modeling

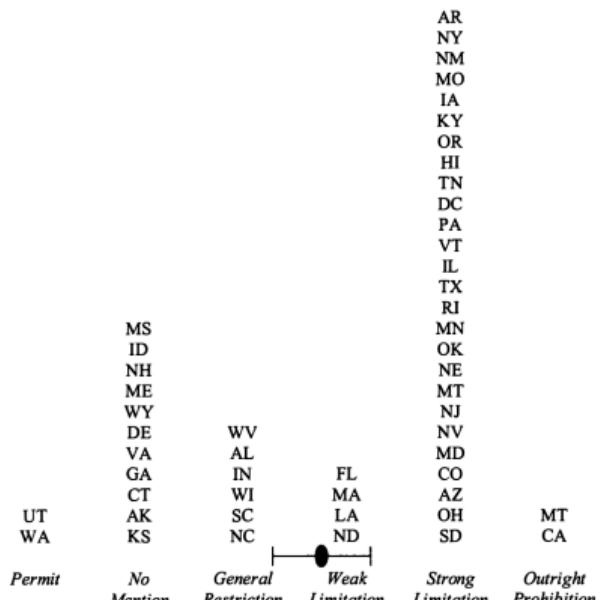
"Gender-based discrimination in insurance has long been controversial. And differential treatment on the basis of gender is, of course, in many contexts widely considered unacceptable or illegal. Nevertheless, there does not seem to be the same level of agreement-as there is for race, religion, and national origin-that drawing gender-based distinctions is always wrong. Federal constitutional law treats gender as only a quasi-suspect classification; as a result, laws that discriminate on the basis of gender are subject to an intermediate level of scrutiny." Avraham et al. (2013)

FIGURE 3a. Distribution of States' Scores for Gender, in Auto Insurance



Insurance Pricing and Predictive Modeling

FIGURE 3c. Distribution of States' Scores for Gender, in Disability Insurance

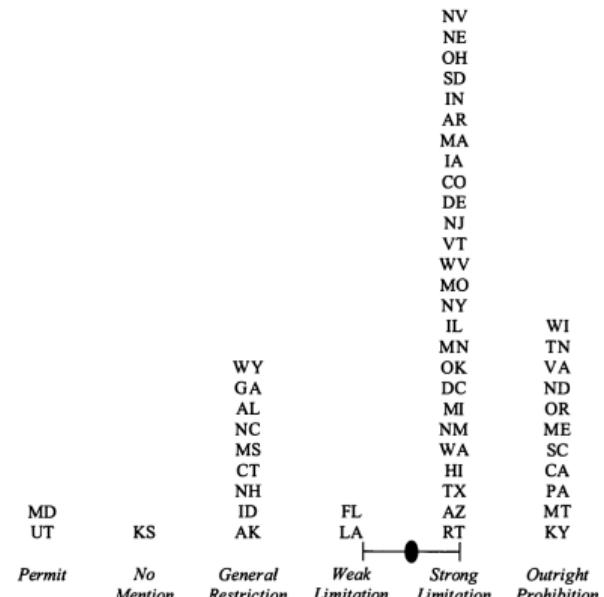


"With respect to life insurance, we predict that the laws regulating gender discrimination will be on average relatively weak, since adverse selection in the life insurance market is especially problematic." Avraham et al. (2013)

Insurance Pricing and Predictive Modeling

"Regarding property/casualty insurance, as there seems to be no conceivable correlation between those risks and gender, we predict either states will cluster around no regulation, or, alternatively, states will cluster around forbidding the use of gender in property/casualty insurance on symbolic or expressive grounds." Avraham et al. (2013)

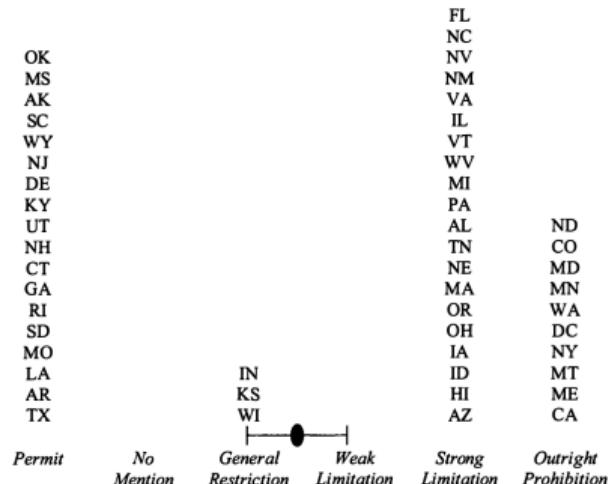
FIGURE 3d. Distribution of States' Scores for Gender, in Property/Casualty Insurance



Insurance Pricing and Predictive Modeling

"The gender discrimination will be more strictly regulated on average for health insurance (where gender-rated policies often result in higher premiums for women) than for auto insurance (where gender-rated policies result in higher premiums for men)." Avraham et al. (2013)

FIGURE 3e. Distribution of States' Scores for Gender, in Health Insurance

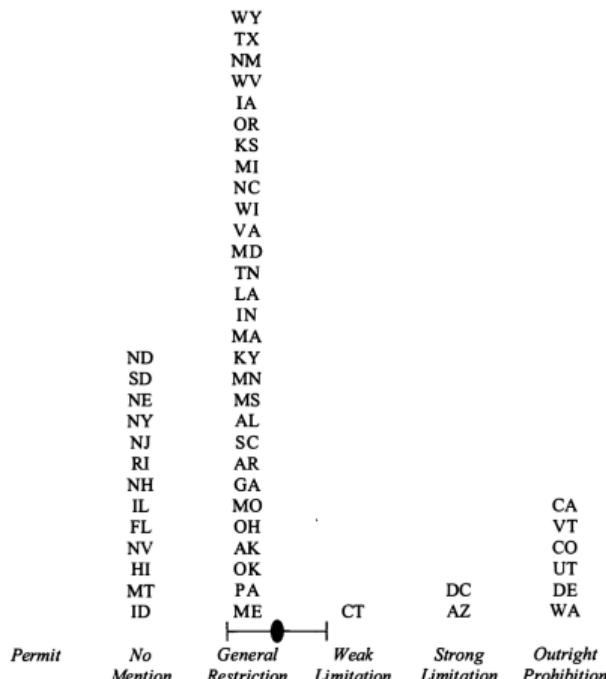


Insurance Pricing and Predictive Modeling

FIGURE 4a. Distribution of States' Scores for Sexual Orientation, in Auto Insurance

"Unlike with race, national origin, religion, and gender, legal classifications on the basis of an individual's sexual orientation have not clearly been identified by the Supreme Court as deserving special scrutiny. In addition, unlike race, national origin, and gender, there are no federal laws forbidding discrimination on the basis of sexual orientation in employment."

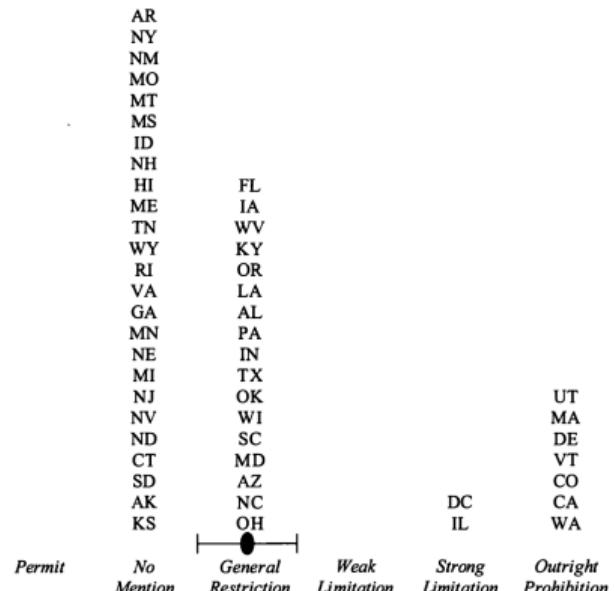
Avraham et al. (2013)



Insurance Pricing and Predictive Modeling

FIGURE 4c. Distribution of States' Scores for Sexual Orientation, in Disability Insurance

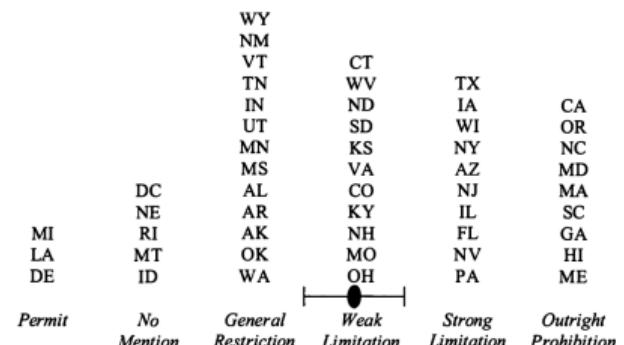
"However, there are state laws that forbid discrimination on the basis of sexual orientation, and some lower courts have held that sexual orientation should be a suspect or quasi-suspect characterisation." Avraham et al. (2013)



Insurance Pricing and Predictive Modeling

"We expect that age will have the lowest average regulatory score of all the risk characteristics we are studying. First, age is not a suspect classification, at least not by constitutional standards. Second, age tends to correlate causally with several important areas of risk (mortality, health, and perhaps disability risks), thereby increasing the perceived fairness of rating on that basis." Avraham et al. (2013)

FIGURE 5a. Distribution of States' Scores for Age, in Auto Insurance

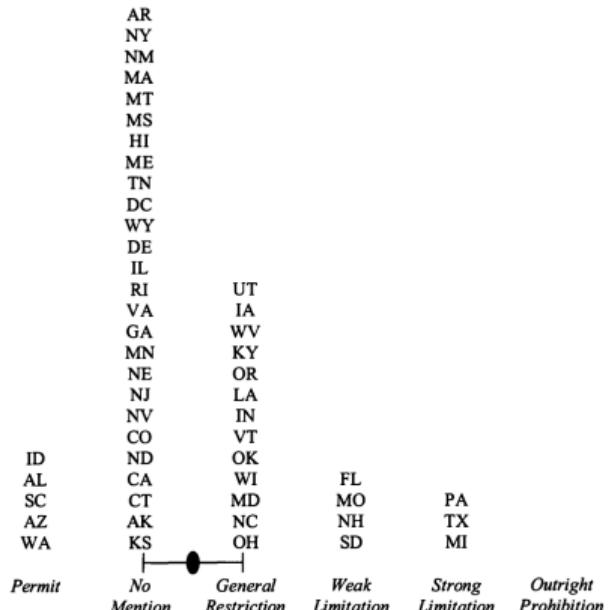


Insurance Pricing and Predictive Modeling

FIGURE 5c. Distribution of States' Scores for Age, in Disability Insurance

"Third, age can present serious adverse selection problems for insurers if they are forbidden from taking it into account, since individual insureds know their own age and the associated risks. Fourth, social solidarity arguments with respect to age are relatively weak, since individuals can spread risk over their lifetime through various income smoothing products."

Avraham et al. (2013)



Insurance Pricing and Predictive Modeling

Avraham et al. (2013) suggested to visualize the distribution of scores (Expressly Permit (-1) / No Law on Point (0) / General Restriction (1) / ... / Characteristic-Specific Prohibition (4))

FIGURE 6. Distribution of States' Scores for Age, by Line of Insurance

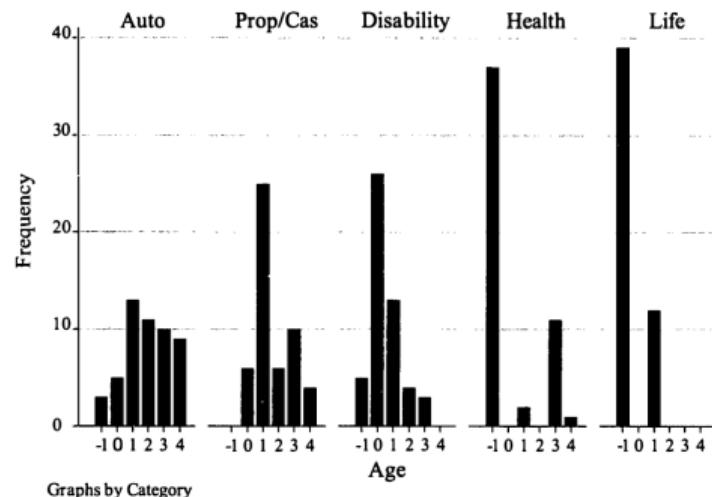
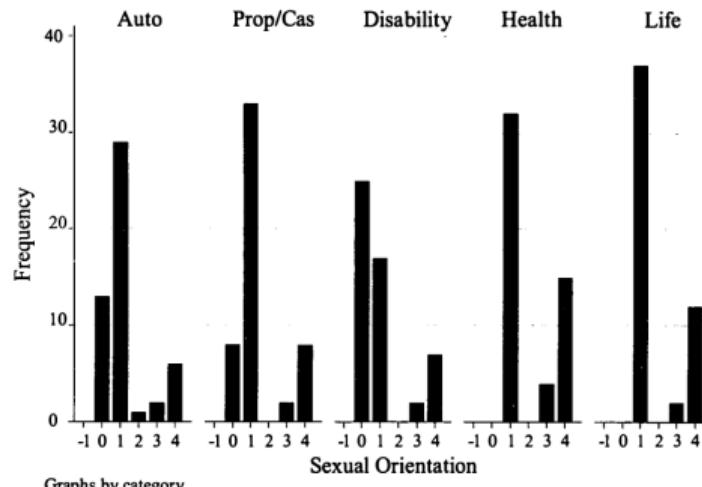


FIGURE 7. Distribution of States' Scores for Sexual Orientation, by Line of Insurance



Insurance Pricing and Predictive Modeling

FIGURE 8. Distribution of States' Scores for Zip Code, by Line of Insurance

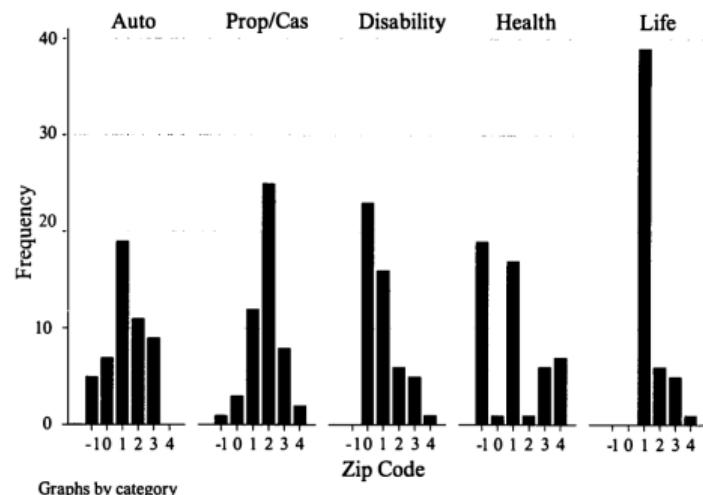
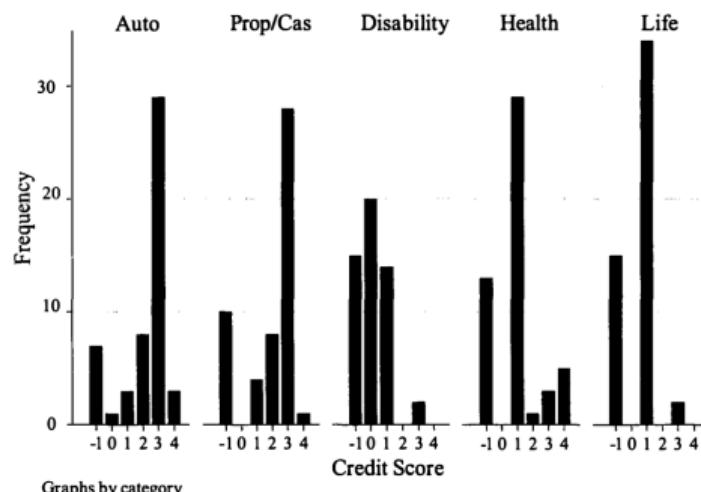


FIGURE 9. Distribution of States' Scores for Credit Score, by Line of Insurance



"Credit score and zip code are not, by themselves, socially suspect characteristics. However, some commentators have argued that credit score and zip code are used by auto and home insurers as proxies for potentially socially suspect characteristics."

Insurance Pricing and Predictive Modeling

Definition 3.1: Pure premium (homogeneous risks)

Let Y be the non-negative random variable corresponding to the total annual loss associated with a given policy, then the **pure premium** is $\mathbb{E}[Y]$.

Proposition 3.1: Law of Large Numbers (2)

Consider an infinite collection of i.i.d. random variables $Y, Y_1, Y_2, \dots, Y_n, \dots$ in a probabilistic space $(\Omega, \mathcal{F}, \mathbb{P})$, with finite expected value, then

$$\underbrace{\frac{1}{n} \sum_{i=1}^n Y_i}_{(\text{empirical}) \text{ average}} \xrightarrow{\text{a.s.}} \underbrace{\mathbb{E}(Y)}_{\text{expected value}}, \text{ as } n \rightarrow \infty.$$

freakonometrics

freakonometrics.hypotheses.org

– Arthur Charpentier, 2024 (UQAM Course)

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Insurance Pricing and Predictive Modeling

More realistically, population is heterogeneous (with respect to risks), with some covariates \mathbf{x} (legitimate, or not).

Definition 3.2: Pure premium (heterogeneous risks)

Let Y be the non-negative random variable corresponding to the total annual loss associated with a given policy, with covariates \mathbf{x} , then the **pure premium** is $\mu(\mathbf{x}) = \mathbb{E}[Y | \mathbf{X} = \mathbf{x}]$.

In this general setting, \mathbf{x} consist in numeric or categorical variables.

Insurance Pricing and Predictive Modeling

Proposition 3.2: Law of Large Numbers (2')

Consider an infinite collection of i.i.d. random pairs (\mathbf{X}, Y) , (\mathbf{X}_1, Y_1) , $(\mathbf{X}_2, Y_2), \dots, (\mathbf{X}_n, Y_n), \dots$ in a probabilistic space $(\Omega, \mathcal{F}, \mathbb{P})$, with finite expected value, then for any $\mathcal{A} \subset \mathcal{X}$ such that $\mathbb{P}[\mathbf{X} \in \mathcal{A}] > 0$,

$$\frac{\sum_{i=1}^n Y_i \mathbf{1}(\mathbf{X}_i \in \mathcal{A})}{\sum_{i=1}^n \mathbf{1}(\mathbf{X}_i \in \mathcal{A})} = \underbrace{\frac{1}{n_{\mathcal{A}}} \sum_{i \in \mathcal{I}_n(\mathcal{A})} Y_i}_{\text{conditional average}} \xrightarrow{\text{a.s.}} \underbrace{\mathbb{E}(Y | \mathbf{X} \in \mathcal{A})}_{\text{conditional expected value}}, \text{ as } n \rightarrow \infty,$$

where $\mathcal{I}_n(\mathcal{A}) = \{i : \mathbf{X}_i \in \mathcal{A}\} \subset \{1, 2, \dots, n\}$ and $n_{\mathcal{A}} = \text{Card}(\mathcal{I}_n(\mathcal{A}))$.

Insurance Pricing and Predictive Modeling

- Excerpt from the Men and Women life tables in 1720 (source: [Struyck \(1912\)](#)). Mortality, as a function of the **age** and the **gender** of the individual.



Table des Hommes.

Années	Per- sonnes								
5	710	20	607	35	474	50	313	65	142
6	697	21	599	36	464	51	301	66	132
7	688	22	591	37	454	52	289	67	123
8	681	23	583	38	444	53	277	68	114
9	675	24	575	39	434	54	265	69	105
10	670	25	567	40	424	55	253	70	97
11	665	26	558	41	414	56	241	71	89
12	660	27	549	42	404	57	229	72	82
13	654	28	540	43	393	58	217	73	75
14	648	29	531	44	382	59	206	74	68
15	642	30	522	45	371	60	195	75	61
16	635	31	513	46	360	61	184	76	54
17	628	32	504	47	349	62	173	77	48
18	621	33	494	48	337	63	162	78	43
19	614	34	484	49	325	64	152	79	38

Table des femmes.

Années	Per- sonnes								
5	711	20	624	35	508	50	373	65	205
6	700	21	617	36	500	51	362	66	194
7	692	22	610	37	492	52	351	67	183
8	685	23	603	38	484	53	340	68	172
9	679	24	590	39	476	54	329	69	161
10	674	25	588	40	468	55	318	70	150
11	669	26	580	41	459	56	306	71	140
12	664	27	572	42	450	57	294	72	130
13	660	28	564	43	441	58	282	73	120
14	650	29	556	44	432	59	271	74	110
15	652	30	548	45	423	60	260	75	100
16	647	31	540	46	414	61	249	76	90
17	642	32	532	47	404	62	238	77	81
18	636	33	524	48	394	63	227	78	72
19	630	34	516	49	384	64	216	79	63

Insurance Pricing and Predictive Modeling

- Excerpt from the Men and Women life tables in 1720 (source: [Struyck \(1912\)](#))
Mortality, as a function of the **age** and the **gender** of the individual.

men		
x	L_x	$5p_x$
0	1000	29.0%
5	710	5.6%
10	670	4.2%
15	642	5.5%
20	607	6.6%
25	567	7.9%
30	522	9.2%
35	474	10.5%
40	424	12.5%
45	371	16.6%
50	313	19.2%
55	253	22.9%
60	195	27.2%
65	142	31.7%
70	97	37.1%
75	61	45.9%
80	33	51.5%
85	16	

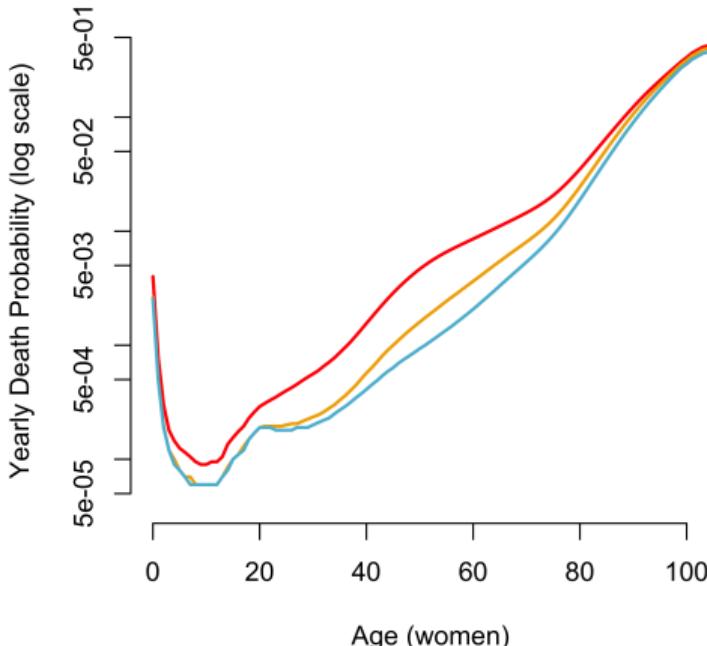
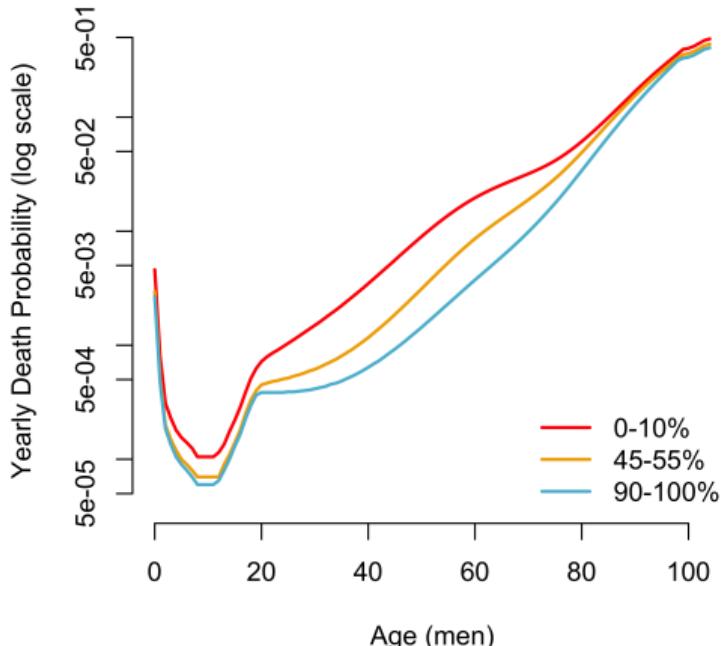
women		
x	L_x	$5p_x$
0	1000	28.9%
5	711	5.2%
10	674	3.3%
15	652	4.3%
20	624	5.8%
25	588	6.8%
30	548	7.3%
35	508	7.9%
40	468	9.6%
45	423	11.8%
50	373	14.7%
55	318	18.2%
60	260	21.2%
65	205	26.8%
70	150	33.3%
75	100	45.0%
80	55	56.4%
85	24	

Insurance Pricing and Predictive Modeling

- Excerpt from the Men and Women life tables in 2016 (source: [Blanpain \(2018\)](#))
Mortality, as a function of the **age**, the **gender** and the **wealth** of the individual.

men				women			
x	0-5%	45-50%	95-100%	x	0-5%	45-50%	95-100%
0	100000	100000	100000	0	100000	100000	100000
10	99299	99566	99619	10	99385	99608	99623
20	99024	99396	99469	20	99227	99506	99526
30	97930	98878	99094	30	98814	99302	99340
40	95595	98058	98627	40	97893	98960	99074
50	90031	96172	97757	50	95021	97959	98472
60	77943	91050	95649	60	88786	95543	97192
70	59824	79805	90399	70	79037	90408	94146
80	38548	59103	76115	80	63224	79117	85825
90	13337	23526	38837	90	31190	45750	55918
100	530	1308	3231	100	2935	5433	8717

Insurance Pricing and Predictive Modeling



Force of mortality (log scale) for various income quantile, in France, [Blanpain \(2018\)](#).

Insurance Pricing and Predictive Modeling

U.S. DECENTNIAL LIFE TABLES FOR 1969-71

Volume I, Number 1



United States Life Tables: 1969-71

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TABLE 10. LIFE TABLE FOR THE NEGRO POPULATION: UNITED STATES, 1969-71

AGE INTERVAL PERIOD OF LIFE BETWEEN TWO AGES (1)	PROPORTION DYING (2)	OF 100,000 BORN ALIVE		STATIONARY POPULATION		AVERAGE REMAINING LIFETIME (7)
		NUMBER LIVING AT BEGINNING OF AGE INTERVAL (3)	NUMBER DYING DURING AGE INTERVAL (4)	IN THE AGE INTERVAL (5)	IN FIVES AND ALL SUBSEQUENT AGE INTERVALS (6)	
					AVERAGE NUMBER OF YEARS OF LIFE REMAINING AT BEGINNING OF AGE INTERVAL (7)	
x to x + t	\bar{d}_x	\bar{I}_x	$\bar{r}^{\bar{d}}_x$	$\bar{r}^{\bar{I}}_x$	T_x	\bar{s}_x
DAYS						
0-1.....	.001348	100,000	1,348	272	6,411,264	64.11
1-7.....	.00648	95,652	659	1,616	6,410,992	64.59
7-14.....	.00248	95,093	248	512	6,410,753	64.51
28-365.....	.01037	97,744	1,013	89,778	6,403,745	65.52

Mortality, gender and “race”

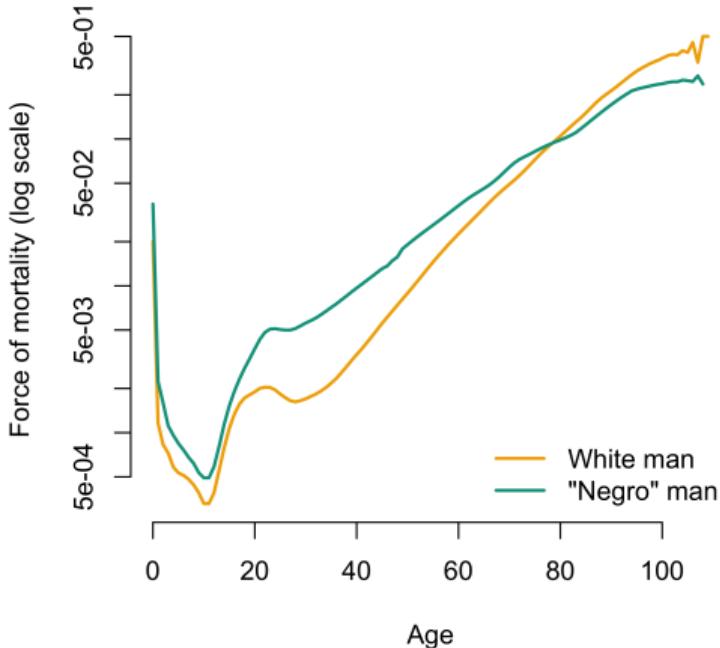
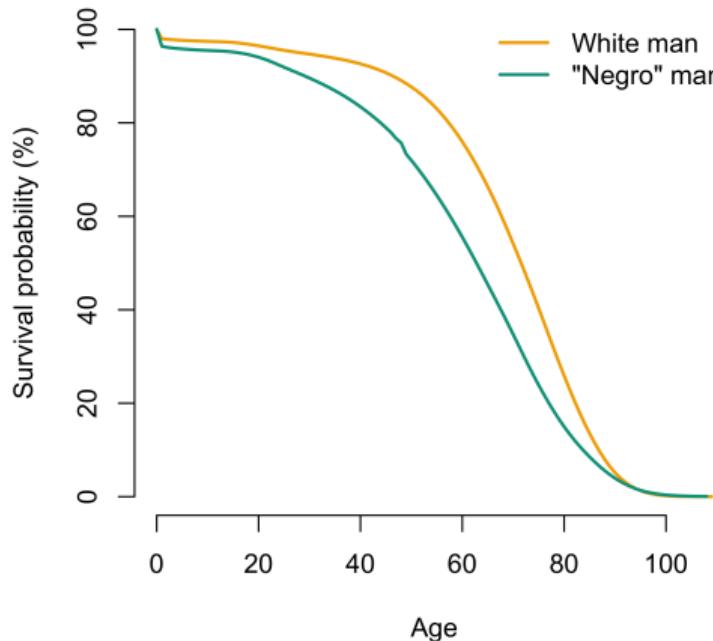


Frederick L. Hoffman
Hoffman (1896, 1918, 1931)

Insurance Pricing and Predictive Modeling

White, men			“Negro”, men		
x	L_x	$5p_x$	x	L_x	$5p_x$
0	100000	2.3%	55	83001	8.5%
5	97671	0.2%	60	75969	12.7%
10	97441	0.2%	65	66343	18.4%
15	97208	0.7%	70	54138	25.5%
20	96480	1.0%	75	40324	35.8%
25	95524	0.8%	80	25885	47.7%
30	94716	0.9%	85	13527	62.1%
35	93843	1.3%	90	5125	75.1%
40	92631	2.1%	95	1274	85.2%
45	90725	3.3%	100	189	90.5%
50	87690	5.3%	105	18	100.0%

Insurance Pricing and Predictive Modeling



Force of mortality (log scale) white men and "Negro" men, 1968-71, U.S.

Insurance Pricing and Predictive Modeling

Definition 3.3: Balance Property

A pricing function m satisfies the **balance property** if $\mathbb{E}_{\mathbf{X}}[m(\mathbf{X})] = \mathbb{E}_Y[Y]$.

Proposition 3.3: Law of total expectations

$$\mathbb{E}_Y[Y] = \mathbb{E}_{\mathbf{X}}[\mathbb{E}_{Y|\mathbf{X}}[Y|\mathbf{X}]] = \mathbb{E}_{\mathbf{X}}[\mu(\mathbf{X})].$$

Proof Since $\mathbb{E}(Y) = \int yf_y(y)dy$ and $\mathbb{E}(Y|\mathbf{X} = \mathbf{x}) = \int yf_{y|\mathbf{x}}(y|\mathbf{x})dy$,

$$\begin{aligned}\mathbb{E}(\mathbb{E}(X|Y)) &= \int \left(\int x \mathbb{P}[X = x | Y = y] dx \right) \mathbb{P}[Y = y] dy = \int \int x \mathbb{P}[X = x, Y = y] dx dy \\ &= \int x \left(\int \mathbb{P}[X = x, Y = y] dy \right) dx = \int x \mathbb{P}[X = x] dx = \mathbb{E}(X).\end{aligned}$$

Insurance Pricing and Predictive Modeling

Homogeneous risk sharing

	Policyholder	Insurer
Loss	$\mathbb{E}[Y]$	$Y - \mathbb{E}[Y]$
Average loss	$\mathbb{E}[Y]$	0
Variance	0	$\text{Var}[Y]$

$\mathbb{E}[Y]$ is the premium paid, and Y the total loss,
from De Wit and Van Eeghen (1984) and Denuit and Charpentier (2004)

Insurance Pricing and Predictive Modeling

Heterogeneous risk sharing, with perfect information

	Policyholder	Insurer
Loss	$\mathbb{E}[Y \Theta]$	$Y - \mathbb{E}[Y \Theta]$
Average loss	$\mathbb{E}[Y]$	0
Variance	$\text{Var}[\mathbb{E}[Y \Theta]]$	$\text{Var}[Y - \mathbb{E}[Y \Theta]]$

where Θ denotes the heterogeneous risk factor.

The term on the bottom right is $\mathbb{E}[\text{Var}[Y|\Theta]]$, corresponding to the standard **variance decomposition** (or Pythagoras theorem)

$$\text{Var}[Y] = \text{Var}[\mathbb{E}[Y|\Theta]] + \mathbb{E}[\text{Var}[Y|\Theta]].$$

to go further → (for more details on Lebesgue spaces, and L^2)



Proposition 3.4: Variance decomposition (1)

For any measurable random variable Y with finite variance

$$\text{Var}[Y] = \underbrace{\mathbb{E}[\text{Var}[Y|\Theta]]}_{\rightarrow \text{insurer}} + \underbrace{\text{Var}[\mathbb{E}[Y|\Theta]]}_{\rightarrow \text{policyholder}}.$$

Proof:

$$\begin{aligned}\text{Var}[Y] &= \mathbb{E}[Y^2] - \mathbb{E}[Y]^2 = \mathbb{E}[\text{Var}[Y|\Theta] + \mathbb{E}[Y|\Theta]^2] - \mathbb{E}[\mathbb{E}[Y|\Theta]]^2 \\ &= (\mathbb{E}[\text{Var}[Y|\Theta]]) + (\mathbb{E}[\mathbb{E}[Y|\Theta]^2] - \mathbb{E}[\mathbb{E}[Y|\Theta]]^2) = \mathbb{E}[\text{Var}[Y|\Theta]] + \text{Var}[\mathbb{E}[Y|\Theta]].\end{aligned}$$

Insurance Pricing and Predictive Modeling

Heterogeneous risk sharing, with imperfect information

	Policyholder	Insurer
Loss	$\mathbb{E}[Y \mathbf{X}]$	$Y - \mathbb{E}[Y \mathbf{X}]$
Average loss	$\mathbb{E}[Y]$	0
Variance	$\text{Var}[\mathbb{E}[Y \mathbf{X}]]$	$\mathbb{E}[\text{Var}[Y \mathbf{X}]]$

$$\mathbb{E}[\text{Var}[Y|\mathbf{X}]] = \underbrace{\mathbb{E}[\text{Var}[Y|\Theta]]}_{\text{perfect ratemaking}} + \underbrace{\mathbb{E}\{\text{Var}[\mathbb{E}[Y|\Theta]|\mathbf{X}]\}}_{\text{misclassification}}$$

This “misclassification” term (on the right) is called “*subsidierende solidariteit*” in De Pril and Dhaene (1996), or “*subsidiary solidarity*”, as opposed to “*kanssolidariteit*” or “*random solidarity*” term (on the left).

Insurance Pricing and Predictive Modeling

Proposition 3.5: Variance decomposition (2)

For any measurable random variable Y with finite variance

$$\text{Var}[Y] = \underbrace{\mathbb{E}[\text{Var}[Y|\mathbf{X}]]}_{\rightarrow \text{insurer}} + \underbrace{\text{Var}[\mathbb{E}[Y|\mathbf{X}]]}_{\rightarrow \text{policyholder}},$$

where

$$\begin{aligned}\mathbb{E}[\text{Var}[Y|\mathbf{X}]] &= \mathbb{E}[\mathbb{E}[\text{Var}[Y|\Theta]|\mathbf{X}]] + \mathbb{E}[\text{Var}[\mathbb{E}[Y|\Theta]|\mathbf{X}]] \\ &= \underbrace{\mathbb{E}[\text{Var}[Y|\Theta]]}_{\text{perfect ratemaking}} + \underbrace{\mathbb{E}\{\text{Var}[\mathbb{E}[Y|\Theta]|\mathbf{X}]\}}_{\text{misclassification}}.\end{aligned}$$

Clubs, Group and Categories

- Groups, or risk classes, are built on the basis of available data, and exist primarily as the product of actuarial models.
- For example, as mentioned in [Bailey and Simon \(1959\)](#), in motor insurance five risk classes can be considered, with rate surcharges relative to the first class (used here as a reference)
 - ▶ “*pleasure, no male operator under 25,*” (reference),
 - ▶ “*pleasure, non-principal male operator under 25,*” +65%,
 - ▶ “*business use,*” +65%,
 - ▶ “*married owner or principal operator under 25,*” +65%,
 - ▶ “*unmarried owner or principal operator under 25,*” +140%.
- There is no “physical basis” for group members to identify other members of *their* group, in the sense that they usually don’t share anything, except some common characteristics, [Gandy \(2016\)](#).

Clubs, Group and Categories

- In ancient Rome, a *collegium* (plural *collegia*) was an association, such as military *collegia*, [Verboven \(2011\)](#).
- As explained in [Ginsburg \(1940\)](#), upon the completion of his service a veteran had the right to join one of the many *collegia veteranorum* in each legion.
- The Government established special savings banks. Half of the cash bonuses, *donativa*, which the emperors attributed to the soldiers on various occasions, was not handed over to the beneficiaries in cash but was deposited to the account of each soldier in his legion's savings bank.
- In case of retirement, upon the completion of his term of service, the soldier would receive a lump sum which helped him somewhat to arrange the rest of his life. The membership in a *collegium* gave him a mutual insurance against “*unforeseen risks*.” These *collegia*, besides being cooperative insurance companies, had other functions.



Clubs, Group and Categories

- › In the early 1660th, the [Pirate's Code](#) was supposedly written by Portuguese buccaneer Bartolomeu Português.
- › A section is explicitly dedicated to insurance and benefits: “*a standard compensation is provided for maimed and mutilated buccaneers. Thus they order for the loss of a right arm six hundred pieces of eight, or six slaves; for the loss of a left arm five hundred pieces of eight, or five slaves; for a right leg five hundred pieces of eight, or five slaves; for the left leg four hundred pieces of eight, or four slaves; for an eye one hundred pieces of eight, or one slave; for a finger of the hand the same reward as for the eye,*” see [Barbour \(1911\)](#) (or more recently [Leeson \(2009\)](#) and [Fox \(2013\)](#) about this piratical schemes).



Clubs, Group and Categories

- In the XIX-th century, in Europe, mutual aid societies involved a group of individuals who made regular payments into a common fund in order to provide for themselves in later, unforeseeable moments of financial hardship or of old age. As mentioned by [Garrioch \(2011\)](#), in 1848, there were in Paris 280 mutual aidsocieties with well over 20,000 members.
- For example, the *Société des Arts Graphiques*, was created in 1808. It admitted only men over twenty and under fifty, and it charged much higher admission and annual fees for those who joined at a more advanced age. In return, they received benefits if they were unable to work, reducing over a period of time, but in case of serious illness the Society would pay the admission fee for a hospice. In England, there were “friendly societies,” as described in [Ismay \(2018\)](#).



Clubs, Group and Categories

- The money collected through contributions came to the rescue of unfortunate workers, who would no longer have any reason to radicalize. It was proposed that insurance should become compulsory (Bismark proposed this in Germany in 1883), but the idea was rejected in favor of giving workers the freedom to contribute, as the only way to moralize the working classes, as [Da Silva \(2023\)](#) explains.
- In 1852, of the 236 mutual funds created, 21 were on a professional basis, while the other 215 were on a territorial basis. And from 1870 onwards, mutual funds diversified the professional profile of contributors beyond blue-collar workers, and expanded to include employees, civil servants, the self-employed and artists.
- The amount of the premium is not linked to the risk.



Clubs, Group and Categories

- › As Da Silva (2023) puts it, “*mutual insurers see in the actuarial figure the programmed end of solidarity.*” For mutual funds, solidarity is essential, with everyone contributing according to their means and receiving according to their needs. Around the same time, in France, the first insurance companies appeared, based on risk selection, and the first mathematical approaches to calculating premiums.
- › Hubbard (1852) advocates the introduction of an “*English-style scientific organization*” in their management. For its members, they had to be able to know “*the probable average of the claims*” that they should cover, like insurance companies. The development of tables should lead insurers to adopt the principle of contributions varying according to the age of entry and the specialization of contributions and funds (health/retirement).
- › For Stone (1993) and Gowri (2014) the defining feature of “modern insurance” is its reliance on **segmenting the risk pool into distinct categories**, each receiving a price

Clubs, Group and Categories

corresponding to the particular risk that the individuals assigned to that category are expected to represent (as accurately as can be estimated by actuaries).

- Once heterogeneity with respect to the risk was observed in portfolios, insurers have operated by categorizing individuals into **risk classes** and assigning corresponding tariffs. This ongoing process of categorization ensures that the sums collected, on average, are sufficient to address the realized risks within specific groups.
- The aim of **risk classification**, as explained in **Wortham (1986)**, is to identify the specific characteristics that are supposed to determine an individual's propensity to suffer an adverse event, forming groups within which the risk is (approximately) equally shared. The problem, of course, is that the characteristics associated with various types of risk are almost infinite; as they cannot all be identified and priced in every risk classification system, there will necessarily be unpriced sources of heterogeneity between individuals in a given risk class.

Clubs, Group and Categories

- In 1915, as mentioned in [Rothstein \(2003\)](#), the president of the Association of Life Insurance Medical Directors of America noted that the question asked almost universally of the Medical Examiner was “*What is your opinion of the risk? Good, bad, first-class, second-class, or not acceptable?*” Historically, insurance prices were a (finite) collection of prices (maybe more than than the two classes mentioned, “first-class” and “second-class”).
- In the early 1920's, Albert Henry Mowbray, who worked for New York Life Insurance Company and later Liberty Mutual (and was also an actuary for state-level insurance commissions in New Carolina and California, and the National Council on Workmen's Insurance) gives his perspective on insurance rate making. See [Mowbray \(1921\)](#).



Clubs, Group and Categories

"Classification of risks in some manner forms the basis of rate making in practically all branches of insurance. It would appear therefore that there should be some fundamental principle to which a correct system of classification in any branch of insurance should conform (...) As long ago as the days of ancient Greece and Rome the gradual transition of natural phenomena was observed and set down in the Latin maxim, 'natura non agit per altum'. If each risk, therefore is to be precisely rated, it would be necessary to recognize very minute differences and precisely measure them. (...) Since we are not capable of covering a large field fully and at the same time recognizing small differences in all parts of the field, it is natural that we resort to subdivision of the field by means of classification, thereby concentrating our attention on a smaller interval which may again be subdivided by further classification, and the system so carried on to the limit to which we find it necessary or desirable to go. But however far we may go in any system of classification, whether in the field of pure or applied science including the business or insurance, we shall always find difficulties presented by the borderline case, difficulties which arise from the continuous character

Clubs, Group and Categories

of natural phenomena which we are attempting to place in more or less arbitrary divisions. While thus acknowledging that classification will never completely solve the problem of recognizing differences between individuals, nevertheless classification seems to be necessary at least as a preliminary step toward such recognition in any field of study. The fact that a complete and final solution cannot be made is, therefore, no justification for completely discarding classification as a method of approach. Since it is insurance hazards that we undertake to measure and classify, the preliminary step in studying classification theory may well be to ask what is an insurance hazard and how it may be determined. It must be evident to the members of this Society that an insurance hazard is what is termed "a mathematical expectation," that is a product of a sum at risk and the probability of loss from the conditions insured against, e.g., the destruction of a piece of property by fire, the death of an individual, etc. If the net premiums collected are so determined on the basis of the true natural probability and there is a sufficient spread then the sums collected will just cover the losses and this is what should be," Mowbray (1921).

Clubs, Group and Categories

- “*1. The classification should bring together risks which have inherent in their operation the same causes of loss.*
- 2. The variation from risk to risk in the strength of each cause or at least of the more important should not be greater than can be handled by the formula by which the classification is subdivided, i.e., the Schedule and / or Experience Rating Plan used.*
- 3. The classification should not cover risks which include, as important elements of their hazard, causes which are not common to all.*
- 4. The classification system and the formula for its extension (Schedule and / or Experience Rating Plans) should be harmonious.*
- 5. The basis throughout should be the outward, recognizable indicia of the presence and potency of the several inherent causes of loss including extent as well as occurrence of loss,” Mowbray (1921).*

Clubs, Group and Categories

- Several articles and textbooks in sociology tried to understand how classification mechanisms establish symbolic boundaries that reinforce group identities, such as [Bourdieu \(2018\)](#), [Massey \(2007\)](#), [Fourcade and Healy \(2013\)](#).
- But here, those “groups” or “classes” do not share any identity, and [Simon \(1988\)](#) or [Harcourt \(2015\)](#) use the term “[actuarial classification](#)” (where “actuarial” designates any decision-making technique that relies on predictive statistical methods, replacing more holistic or subjective forms of judgment). In those class-based systems, based on insurance rating table (or grid), results are determined by assigning individuals to a group in which each person is positioned as “average” or “typical”.
- [Most] “*actuaries cannot think of individuals except as members of groups*” claimed [Brilmayer et al. \(1979\)](#). Each individual is assigned the same value as all other members of the group to which it is assigned.



Clubs, Group and Categories

- Simon (1987, 1988), and then Feeley and Simon (1992), defined “actuarialism,” that designate the use of statistics to guide “*class-based decision-making*,” used to price pensions and insurance. As explained in Harcourt (2015), this “*actuarial classification*” is the constitution of groups with no experienced social significance for the participants. A person classified as a particular risk by an insurance company shares nothing with the other people so classified, apart from a series of formal characteristics (e.g. age, sex, marital status, etc.).
- For Austin (1983) and Simon (1988), categories used by the insurance company when grouping risks are “*singularly sterile*,” resulting in inert, immobile and deactivated communities, corresponding to “*artificial*” groups. These are not groups organized around a shared history, common experiences or active commitment, forming some “*aggregates*” – living only in the imagination of the actuary who calculates and tabulates, not in any lived form of human association.



Clubs, Group and Categories

- If Hacking (1990) observed that standard classes creates coherent group identities (causing possible stereotypes and discrimination, Simon (1988), provocatively suggests that actuarial classifications can in turn “*undo people's identity*.”)
- As mentioned in Abraham (1986), the goal for actuaries is to create groups, or “*classes*” made up of individuals who share a series of common characteristics and are therefore presumed to represent the same risk. Following François (2022), we could claim that actuarial techniques reduce individuals to a series of formal roles that have no “*moral density*” and therefore do not grant an “*identity*” that organizes a coherent sense of self. And the inclusion of nominally “*demoralized categories*,” such as gender, in class-based rating systems makes their total demoralization difficult to achieve – and is in itself an issue of struggle. Heimer (1985) used the term “*community of fate*.”
- Rovroy et al. (2013) and Cheney-Lippold (2017) point out that scoring technologies are continually swapping predictors, “*shuffling the cards*,” so that there is no stable basis for constructing group memberships, or a coherent sense.

Clubs, Group and Categories

"The price which a person pays for automobile insurance depends on age, sex, marital status, place of residence and other factors. This risk classification system produces widely differing prices for the same coverage for different people. Questions have been raised about the fairness of this system, and especially about its reliability as a predictor of risk for a particular individual. While we have not tried to judge the propriety of these groupings, and the resulting price differences, we believe that the questions about them warrant careful consideration by the State insurance departments. In most States the authority to examine classification plans is based on the requirement that insurance rates are neither inadequate, excessive, nor unfairly discriminatory. The only criterion for approving classifications in most States is that the classifications be statistically justified – that is, that they reasonably reflect loss experience. Relative rates with respect to age, sex, and marital status are based on the analysis of national data. A youthful male driver, for example, is charged twice as much as an older driver all over the country (...) It has also been claimed that insurance companies engage in redlining – the arbitrary denial of insurance to everyone

Clubs, Group and Categories

living in a particular neighborhood. Community groups and others have complained that State regulators have not been diligent in preventing redlining and other forms of improper discrimination that make insurance unavailable in certain areas. In addition to outright refusals to insure, geographic discrimination can include such practices as: selective placement of agents to reduce business in some areas, terminating agents and not renewing their book of business, pricing insurance at un-affordable levels, and instructing agents to avoid certain areas. We reviewed what the State insurance departments were doing in response to these problem. To determine if redlining exists, it is necessary to collect data on a geographic oasis. Such data should include current insurance policies, new policies being written, cancellations, and non-renewals. It is also important to examine data on losses by neighborhoods within existing rating territories because marked discrepancies within territories would cast doubt on the validity of territorial boundaries. Yet, not even a fifth of the States collect anything other than loss data, and that data is gathered on a territory-wide basis," Havens (1979)

Clubs, Group and Categories

“On the other hand, the opinion that distinctions based on sex, or any other group variable, necessarily violate individual rights reflects ignorance of the basic rules of logical inference in that it would arbitrarily forbid the use of relevant information. It would be equally fallacious to reject a classification system based on socially acceptable variables because the results appear discriminatory. For example, a classification system may be built on use of car, mileage, merit rating, and other variables, excluding sex. However, when verifying the average rates according to sex one may discover significant differences between males and females. Refusing to allow such differences would be attempting to distort reality by choosing to be selectively blind. The use of rating territories is a case in point. Geographical divisions, however designed, are often correlated with socio-demographic factors such as income level and race because of natural aggregation or forced segregation according to these factors. Again we conclude that insurance companies should be free to delineate territories and assess territorial differences as well as they can. At the same time, insurance companies should recognize that it is in their best interest to be objective and use clearly relevant

Clubs, Group and Categories

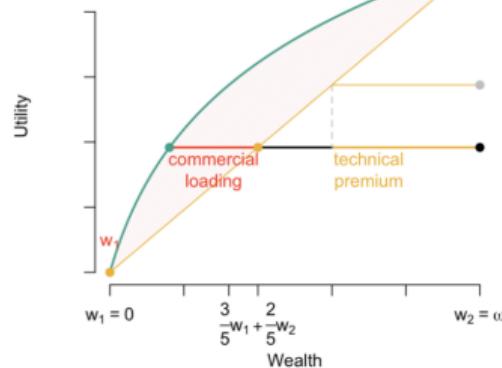
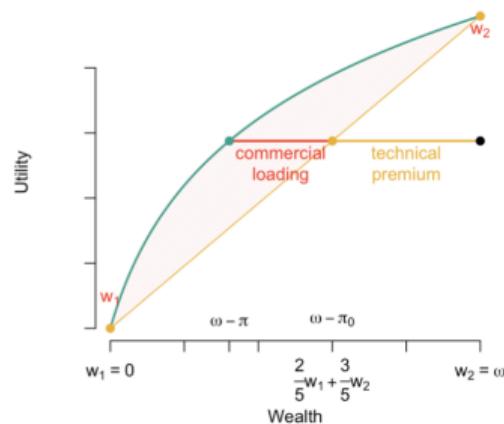
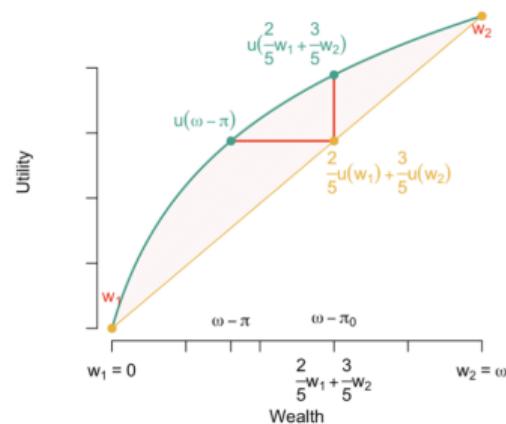
factors to define territories lest they be accused of invidious discrimination by the public. (...) " Casey et al. (1976)

"One possible standard does exist for exception to the counsel that particular rating variables should not be proscribed. What we have called 'equal treatment' standard of fairness may precipitate a societal decision that the process of differentiating among individuals on the basis of certain variables is discriminatory and intolerable. This type of decision should be made on a specific, statutory basis. Once taken, it must be adhered to in private and public transactions alike and enforced by the insurance regulator. This is, in effect, a standard for conduct that by design transcends and preempts economic considerations. Because it is not applied without economic cost, however, insurance regulators and the industry should participate in and inform legislative deliberations that would ban the, use of particular rating variables as discriminatory." Casey et al. (1976)

Price Optimization

Definition 3.4: Indifference utility principle

Let Y be the non-negative random variable corresponding to the total annual loss associated with a given policy, for a policyholder with utility u and wealth w , the **indifference premium** is $\pi = \omega - u^{-1}(\mathbb{E}[u(\omega - Y)])$.



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