Bachelor Thesis

Reinforcement Learning for 3-player Chinese Checkers

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1. Introduction

In recent years the field of machine learning has attracted a lot of attention from researchers as well as the public. Advancements in computational power and the rapidly growing amount of data make lead to new possibilities in software design and problem solving. There are programs that detect patterns in visual, aural and abstract data. The quality of automatic translation between languages has increased. However, one field in machine learning that particularly interests me is the training of programs to make decisions. In the past, board games have been used to test training techniques and demonstrate the state-of-the-art.

The idea of a machine that could defeat humans in a board game, especially in chess, has sparked people's curiosity for several centuries.
In the year 1770 Wolfgang von Kempelen invented a machine he called the "Chess Turk". It consisted of a puppet attached to a wooden desk with a chess board on top of it. Just like a human would, the puppet moved its arm, moving the chess pieces. Claiming that this machine could defeat even strong players, he toured across Europe. The machine was thought of as a mechanical masterpiece and defeated many contenders including Benjamin Franklin and Napoleon Bonaparte. Only after the machine's destruction the hoax was revealed. During the games influential chess players from that time hid inside the wooden desk and controlled the puppet's arms.
The idea of chess playing machines and algorithms was picked up again in the 21st century. Great computer scientists and mathematicians like Konrad Zuse, Alan Turing and Claude Shannon worked on chess programs and thought about ways to solve the game. Lacking the sufficient hardware, their ideas stayed logical constructs without application.
Computer chess is also a common theme in science fiction, like in the 1968 science-fiction movie "2001: A Space Odyssey" where the protagonist, an astronaut, loses a game of chess against his spaceship's board computer HAL 9000 - a scene that convinced the viewer of its super-human intelligence. In the year 1997 these fantasies became reality, as the chess program Deep Blue, made by IBM, beat the former world chess champion Garry Kasparov. The resource of computational power made the already established techniques applicable. The focus of machine learning for games shifted towards the game of Go. Its complexity and popularity made it the next target for many machine learning researchers worldwide. 2016 the program AlphaGo invented by the research company Google DeepMind won four out of five matches against 18-time world champion Lee Sedol. In December 2018 DeepMind published a paper in Science Magazine that describes AlphaZero and its evaluation. It is built upon AlphaGo but was generalized in such a way, that it was applicable to not only Go, but also also two-player games with perfect information. Another difference to its inferior predecessor was that the program learns these games by itself given only the rules but no examples of games played by humans.

The research in this field is not only conducted to gain insights about board games. Rather, these games are a suitable training ground in Machine Learning. In contrast to most other real-life problems, they can easily be defined as a closed system and allow clear evaluation. Neural Networks as well as training methods can be tested on them. The insight in this research field is needed for machine learning concepts that can be applied to real world problems.

The purpose of this thesis is to investigate the possibilities and issues of Reinforcement Learning applied to multiplayer-games and games with recurring states. As domain I chose a game I know from my childhood - Chinese Checkers. The program trains a neural network through self-play and improves its strategies over time. The construct is heavily inspired by AlphaZero. The code is built upon an open-source project by Surag Nair used in his paper "Playing Othello without human knowledge". It was significantly modified to fit a 3-player game with possible repetitions in game states, using a different neural network and a modified tree search. I also enabled it to play several games in self-play silmultaneously.

section 2 will explain the rules of the game. section 3, section 5 and section 4 will present the components of the program and how they are combined. Lastly, the results of the training process will be presented in section 9.

2. Chinese Checkers

Chinese Checkers is a modified version of the board game Halma and dates back to 1892. While it is suited for 2-4 players, this project focuses solely on the 3-player variant. The board size was reduced from the standard 121 field board to 37 fields. To make the game more suitable for the project, the rules were slightly modified.

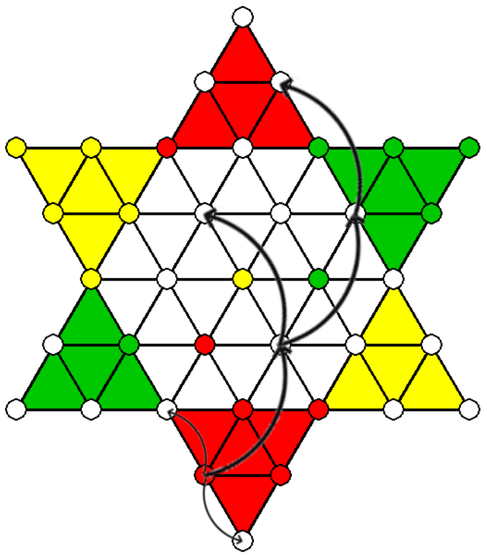


Figure 2.0.0: Board state after two moves each

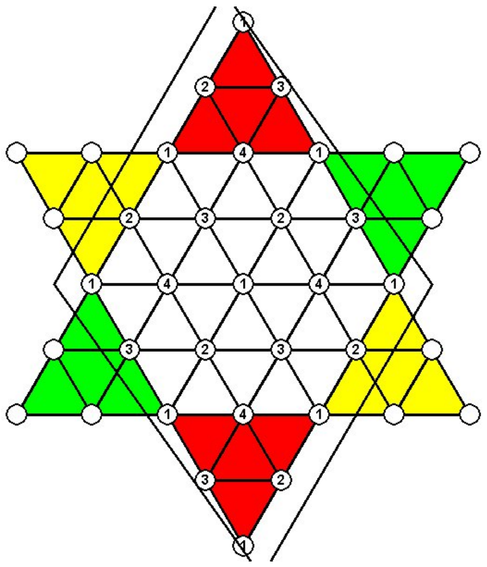


Figure 2.0.0: Grid NOCH KORRIGIEREN

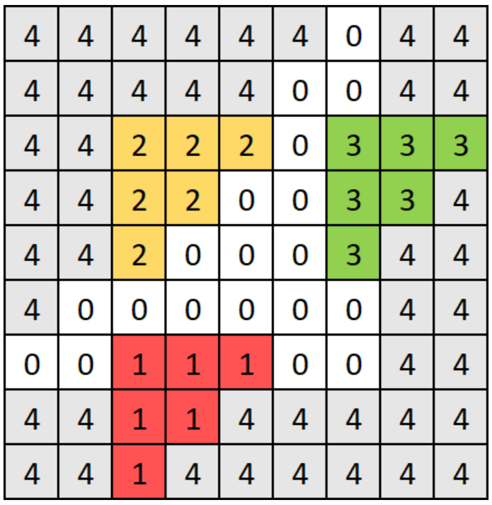
2.1. Rules

Every player starts with 6 pieces, placed in one corner of the star-shaped board. Player 1 has red figures. Player 2 has yellow pieces. Player 3 has green pieces.
Chinese checkers is a race game with 1st, a 2nd and a 3rd place. 1st is whoever manages to move all his pieces from the start zone into his end zone before the others. 2nd is who has all fields in his end zone occupied, after the 1st place is decided. Here it does not matter, which player's pieces occupy the end zone. We call this the second-winner rule. 2nd place is determined, the game is over, and the remaining takes the 3rd place. After the winner is decided, the remaining two players continue to win the second place.
After a player makes a move, it is the next player's turn.
A move can be either a short hop or a jump sequence.
With a short hop a piece can be moved to an adjacent space, provided that it is not occupied by another piece.
A piece can jump over another piece, provided that the field behind it is not occupied. These jumps can be chained together. As long as another jump by the same figure is possible, the player is free to execute it.
Short hops and jumps can not be combined in one move.
A player can not move his pieces on fields of the other player's zone, with the exception of those that intersect with the neutral center board.
Once placed in its own end zone, a piece can no be moved outside of it anymore.

The game embodies offensive as well as defensive tactics. A player can move his pieces forward and build ladders to prepare for further hops. On the other hand one can also block enemy pieces and destroy ladders by occupying the ladder space.

2.2. Mathematical Description

This subsection will provide a mathematical description of our game. Even though a more compact description is possible, this one is used to stay close to the implementation of the game.
$S \subset \{0, 1, 2, 3, 4\}^{9\times 9}$ is the set of all possible board configurations. In a board state $B \in S$ the value of $B_{i,j}$ describes the state of the field in the corresponding position. 0 denotes an empty field. 1,2 and 3 denote a field occupied by the respective player. 4 denotes fields that are unreachable, because the lie outside of the actual board.
There are 37 field on the board and every of the three players has 6 pieces.
We call s a terminal state, if it represents an ended the game. The board is represented by a 9x9 array.



4	4	4	4	4	4	0	4	4
4	4	4	4	4	0	0	4	4
4	4	2	2	2	0	3	3	3
4	4	2	2	0	0	3	3	4
4	4	2	0	0	0	3	4	4
4	0	0	0	0	0	0	4	4
0	0	1	1	1	0	0	4	4
4	4	1	1	4	4	4	4	4
4	4	1	4	4	4	4	4	4

Figure 2.2.0: Board representation

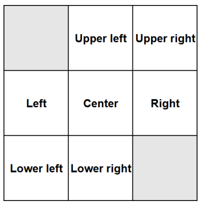


Figure 2.2.0: Directions

$A = \{1, 2, \ldots, 320\}$ is the set of all actions.
We describe the moves out of perspective from player one.
The algorithm requires a vector with each move in the game corresponding to an entry. This action vector is logically split into two parts with the first part representing direct steps and the second one representing (consecutive) jumps.

Every of the 25 accessible gets assigned 6 step directions. This results in 150 moves.
To encode jumps the board is subdivided into four grids of different sizes. A sequence of jumps starting in a grid has to also end in it. Combinations of fields in the same grid are compiled. With grid sizes 9, 6, 6 and 4 this leads to 169 jump sequences.
Lastly there is one passive move that does not change the board. It gets chosen if and only if there are no other possible moves available. This is the case, when a every figure of a player is blocked, or when a player has won the game and is waiting for his opponents to determine the second place. Thus the action vector contains 320 entries. $|A| = 320$.

It should be noted that this action vector contains moves that are illegal in every board state. Those are the direct moves leading outside of the board (22) or into the forbidden parts of opponents' zones (16). Also jumping moves that lead to the starting point(25) are included (25). Direct moves (8) as well as jumping sequences that lead out of the end zones ($3{\cdot}6{+}2{\cdot}1{\cdot}5{+}1{\cdot}3 = 31$) are not allowed. Taking into account the intersection of these moves, this sums up to 102 moves or about $31,9\%$ of the total action space. Even though these impossible moves enlarge the action vector, they are included, because it greatly simplifies encoding and decoding.

$V = [0, 3]^3$ is the set of state values. A state value describes the expected scores of each player at a board state.
$P = \{1, 2, 3\}$ is the set of players.
$\alpha : S \times P \to \mathcal{P}(A)$ assigns every board state and current player a set of possible actions.
$m : S \times A \times P \to S \times P$ returns the next board state given a board state and an action by a specific player.
$m_s : S \times A \times P \to S$ returns the next board state given a current board state, an action and the player that executes it.
$m_p : S \times A \times P \to P$ returns the next player given a current board state, a action and the player that executes it.
$\phi : SxP \to V$ assigns board state a situation with board state s and current player p a state value.
$g : V \times P \to \mathbb{R}$ returns the benefit of board a value for a specific player p, For the following implementation, it is assumed that g is linear with respect to its first argument.

3. Tree search

In this section we will introduce two decision making algorithms used for deterministic games with perfect information. At first we will introduce the Minimax algorithm to to convey the general methodology of tree search algorithms for decision making. Afterwards we will take a look at the Monte-Carlo tree search and see, why it used in this project.
For now we will assume that no game loops are possible, meaning that every board state can only occur once in one game iteration. Although this does not hold true for Chinese Checkers, this assumption is required for the game to be represented as a tree. We will further discuss this issue in subsection 3.3

Minimax and the Monte-Carlo tree search gradually build game trees. The tree representation of the game is following. A board state and current player $(s, p) \in S \times P$ is denoted by a tree node. The two nodes (s, p) and (s', p') can be connected by an edge if there is an action $a \in \tilde{a}(s, p)$ such that $m_s(s, a, p) = s'$ and $m_p(s, a, p) = p'$.
Given a board state s and the current player p, these algorithms are applied to determine the best action to take. That is the action that is expected to lead the game towards the terminal game state s^* that maximizes $g(v(s^*), p)$.
It is assumed that the evaluation function g is known for each player, and that every player acts by the same policy.

3.1. Minimax

To illustrate the principles of Minimax, we will demonstrate it on a simpler game than Chinese Checkers. The name Minimax originates from the fact, that the algorithm is generally applied to zero-sum games with two players of which one tries to maximize the state value, while the other one tries to minimize it. $\tilde{V} = \mathbb{Z}$
$\tilde{P} = \{1, 2\}$
$\tilde{A}$ with $|\tilde{A}| = 2$.

$$\tilde{g}(v, p) = \begin{cases} v & \text{if } p = 1 \\ -v & \text{else} \end{cases} \tag{3.1.1}$$

While this assumption enables an intuitive explanation, it should be noted that it can also be applied to an arbitrary amount of players as well as different evaluation functions and value sets.
Minimax builds a game tree with the current game state and player (s, p) as root node. The tree contains every possible node down to a maximal depth. Since both players are taking turns, the decision maker alternates between each level of the tree. Leaf notes are in the deepest layer of the tree or represent terminal game states. We evaluate each of these leaf nodes (s, p). It gets assigned the state value $V_{s,p} := \phi(s)$. These values are being back-propagated. Knowing that depending on the tree level the current decision maker will either maximize or minimize the following state value, his action is deducible. Therefore the values of internal nodes are set to be either the maximum or minimum of the values assigned to its children.
Knowing the opponent's policy a player can optimize not only the state value after his next move but also several moves in advance.

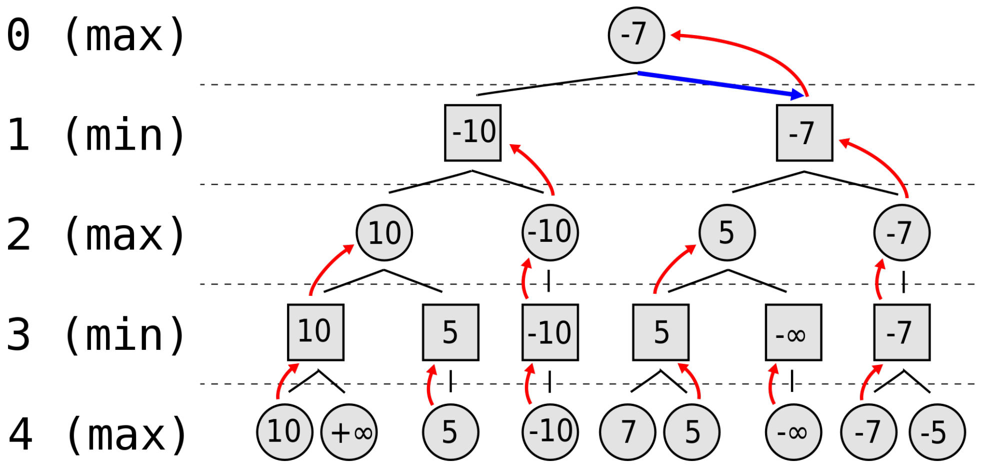


Figure 3.1.2: Minimax Tree
In this tree both players have two available actions for every state. The state values get back-propagated in a Depth First Search. The result is that the max-player will choose the action corresponding to the right edge of the root node.

Algorithm 1 Minimax

```
1: function MINIMAX($s, p, depth, depth_{max}$)
2:     if $s$ is a terminal state or $depth = max_{depth}$ then
3:         $v \leftarrow \phi(s, p)$
4:         return $v, 0$
5:     $g_{best} \leftarrow -\infty$
6:     $a_{best} \leftarrow 0$
7:     for each $a \in \alpha(s, p)$ do
8:         $s' \leftarrow m_s(s, a, p)$
9:         $p' \leftarrow m_p(s, a, p)$
10:        $v', \_ \leftarrow$ MINIMAX($s', p', depth + 1, depth_{max}$)
11:        $g \leftarrow g(v', p')$
12:        if $g > g_{best}$ then
13:            $g_{best} \leftarrow g$
14:            $v_{best} \leftarrow v'$
15:            $a_{best} \leftarrow a$
16:    return $v_{best}, a_{best}$
```

We call the function with the parameter $depth = 0$.

The problem with Minimax algorithm is that the branching factor of its tree becomes fairly large depending on the action space of its application. This makes the tree grow exponentially with every tree layer. The amount of tree nodes N satisfies:

$$N \leq \sum_{i=0}^{max_{depth}} |A|^i = \frac{|A|^{max_{depth}} - 1}{|A| - 1}, \qquad \text{if } |A| > 1 \tag{3.1.3}$$

This makes Minimax unsuitable for our problem. Although there are techniques like Alpha-Beta-pruning to skip the evaluation of unpromising nodes, modern Machine Learning programs mostly work with another algorithm.

3.2. Monte-Carlo tree search

The Monte-Carlo tree search algorithm is designed on the basis of Minimax. In contrast to Minimax, where every node down to a specified depth gets visited, it gradually builds an asymmetric tree. Every iteration of the search expands the tree by one leaf node, possibly turning previous leaf nodes into internal nodes. To do so, it heuristically chooses promising paths to explore, disregarding the others. Because this enables the algorithm to traverse deeper into the tree without exponential growth, it is especially useful for applications with large action spaces. There are several variants of MCTS. The following explanations refer to the variant used in this project.

The procedure can be summarized into three steps:

Selection

Starting at the root node that represents the current game state, a path navigating through the tree is selected until a leaf node is reached.
The selection of actions that define paths through the tree is determined by the following formula:

$$U_{s,a,p} = \overbrace{Q_{s,a,p}}^{\text{Exploitation}} + \overbrace{c_{puct} \cdot P_{s,a,p} \cdot \frac{\sqrt{N_{s,p}}}{1 + N_{s,a,p}}}^{\text{Exploration}} \tag{3.2.4}$$

$Q_{s,a,p}$ is a evaluation of the profitability of a for player p in state s. It is initialized as 0.
$P_{s,a,p}$ is a pre-evaluation of the action a in state s for player p. The values of $P_{s,a,p}P$ are entries of vector $\pi \in [0,1]^{|A|}$, which is the output of following function:

$$\rho : S \times P \mapsto [0,1]^{|A|} \tag{3.2.5}$$

ρ will makes use of the neural network. A more detailed explanation can be found in section 4.
$N_{s,a,p}$ is the number of times action a was chosen by player p in state s.
$N_{s,p}$: The number of times that player p chose an action in state s. It is the sum of values $N_{s,a,p}$ over all $a \in A$.
$U_{s,a,p}$ is the upper confidence bound of action a at state s. It describes how important a exploration in direction of the different edges of a node is.
c_{puct} is the exploration parameter. It will be put into context in the next paragraph.

From all possible actions the one maximizing U is chosen.

$$a^* = \arg\max_{a \in \alpha(s,p)} U_{s,a,p} \tag{3.2.6}$$

Expansion

The leaf node s' gets evaluated by the state evaluation function.

$$s' = m_s(s,a,p) \tag{3.2.7}$$

$$V_{s',p'} = \phi(s',p') \tag{3.2.8}$$

$V_{s,p}$ is a state value for the node corresponding to state s and player p. Just like in Minimax, these values are determined by ϕ at the leaf nodes and by the values of child nodes at the internal nodes.

Back-propagation

The values of Q get back-propagated up the tree until the root node is reached.

$$V_{s,p} = \begin{cases} \phi(s,p) & \text{if } N_{s,p} = 0 \\ \frac{\sum\limits_{a \in A(s)} V_{m_s(s,a,p)} \cdot N_{s,a,p}}{N_{s,p}} & \text{else} \end{cases} \tag{3.2.9}$$

$$Q_{s,a,p} = \begin{cases} g(V_{m_s(s,a,p)}, p) & \text{if } N_{s,a,p} > 0 \\ 0 & \text{else} \end{cases} \tag{3.2.10}$$

Equation 3.2.4 showcases an important concept in the Monte-Carlo tree search:

Exploitation vs. Exploration
When choosing a path in the tree, promising sub-trees are favored upon those that have a low Q-value. On the other hand, it can be beneficial to explore paths that do not seem promising in the beginning, because they could be superior in the long run.
The first term refers to Exploration. Paths that already have lead to a state with positive reward are investigated further.
The second term describes optimism regarding Exploitation. This term grows everytime, other actions get favored over action a in state s. Therefore seldom chosen actions have a higher Exploration-value.
The exploration parameter c_{puct} balances these two terms out. A low value leads to more focus on exploitation and therefore a deeper tree. When a high value is chosen, more different actions are explored, making the tree wider.

Another difference to Minimax is that the Monte-Carlo tree search does not return a single action a as best choice, but a vector with values for every action in A. Those values are proportional to the number of times that the corresponding edge starting at the root node was traversed.

We split the algorithm into two functions. The first one expands the tree by one more leaf and updates the tree values.

Algorithm 2 search

```
1: function SEARCH($s$)
2:     if $s$ is a terminal state then
3:         return $\phi(s, p)$
4:     if $s$ is a leaf node then
5:         $v = \phi(s, p)$
6:         $\pi = \rho(s, p)$
7:         for each $a$ in $\alpha(s, p)$ do
8:             $P_{s,a,p} \leftarrow \pi_a$
9:             $N_{s,a,p} \leftarrow 0$
10:            $Q_{s,a,p} \leftarrow 0$
11:        $N_{s,p} \leftarrow 0$
12:        return $v$
13:    $u_{best} \leftarrow -\infty$
14:    $a_{best} \leftarrow 0$
15:    for each $a$ in $\alpha(s, p)$ do
16:        $u \leftarrow Q(s, a) + c_{puct} \cdot P(s, a) \cdot \frac{\sqrt{N_s}}{1+N_{s,a,p}}$
17:        if $u > u_{best}$ then
18:            $u_{best} \leftarrow u$
19:            $a_{best} \leftarrow a$
20:    $s' \leftarrow m_s(s, a_{best}, p)$
21:    $p' \leftarrow m_p(s, a_{best}, p)$
22:    $v \leftarrow$ SEARCH($s', p'$)
23:    $Q_{s,a_{best},p} \leftarrow \frac{N_{s,a_{best},p} \cdot Q_{s,a,p} + g(v,p)}{N_{s,a,p}+1}$
24:    $N_{s,a,p} \leftarrow N_{s,a,p} + 1$
25:    $N_{s,p} \leftarrow N_{s,p} + 1$
26:    return $v$
```

The second function calls the search-function multiple times, constructing a tree. Then it counts how often each edge starting at the root node was chosen. It constructs a normalized vector C containing these counts. The higher the value of C_a for $a \in A$ is, the the more often it was chosen by the tree search. In section 4 we will have a more detailed look at how it is utilized to choose an action.

Algorithm 3 mcts

```
1: function GETACTIONCOUNT(board, player, numSimulations)
2:     for $i \leftarrow 1$ to $numSimulation$ do
3:         SEARCH(board, player)
4:     for each $a$ in $A$ do
5:         $C(a) = N(s, a, player)$
6:     $C \leftarrow \frac{C}{len(C)}$
7:     return $C$
```

3.3. Games with Repetition

Until now, we only discussed algorithms for games that have no recurring states. That means that once an action from state s is taken, there is no way for the players to go back to it. This

is necessary for the game to be representable by a tree.
The game Chinese Checkers does not fulfill this condition. Because players can move their pieces back to previous positions, game loops a possible. Therefore we can only represent the game as a directed graph as opposed to a tree.
In order to still use the Monte-Carlo tree search, we make following adjustments. In every iteration we take notes of the path that is traversed in the tree. If the next node (s, p) is one that is already in that trace, it means that our path contains a loop. If we would ignore this problem, the values would be back-propagated in circles. To prevent this, we back-propagate not v, but the actual score $\tilde{v}$ of the state. $\tilde{v}$ does not described the expected scores, but the scores realized in the game state. For all players, that have not completed the game, this will evaluate to 0, making it unpromising to pursue that path further and thus preventing the exploitation of the loop. So we assign the node (s, p) a poor evaluation in the second visit and the back-propagated evaluation when returning to the first visit. We will later refer to this as loop-cutting. Note that this only prevents loops in the tree. Loops in actual game-play can not be prevented this way. subsection 7.1 further addresses this problem and how it was handled.

4. Agents

We call an entity that chooses moves, an agent. Agents are not statically linked to a player. An agent can dictate game-play for any player and even for multiple players in one game.
Just like the other games that AlphaZero was applied to (Go, Chess, Shogi, Othello), Chinese Checkers is a symmetric game. This means that every player has the same action space and same objective.
When the game is being played in real life, every player sees the board from a different perspective, with his own start zone closest to him. Before every move, an agent also sees the board in the current player's perspective. This is achieved by 120° degree turns of the board. Furthermore the game is displayed with the player's pieces as red ones, the next player's pieces as yellow ones and the last player's pieces as green ones. Through this generalization, an agent always plays as the red player with his start zone at the bottom and his end zone at the top side of the board. Therefore we do not have create agents for specific players in the game.
We accomplish this by applying following function to the board state:

$$r_s : S \times P \mapsto S \tag{4.0.11}$$

$$r_p : P \times P \mapsto P \tag{4.0.12}$$

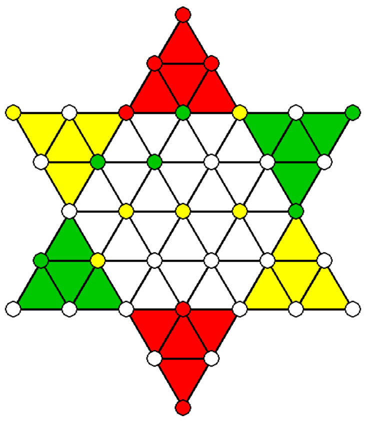


Figure 4.0.12: $r_s(s, 1)$

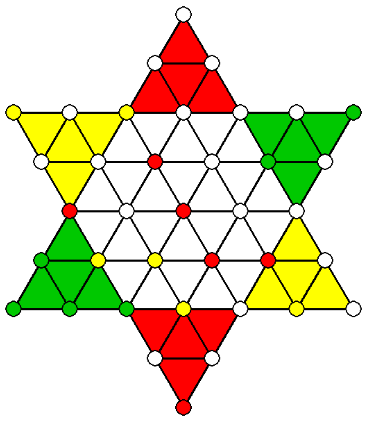


Figure 4.0.12: $r_s(s, 2)$

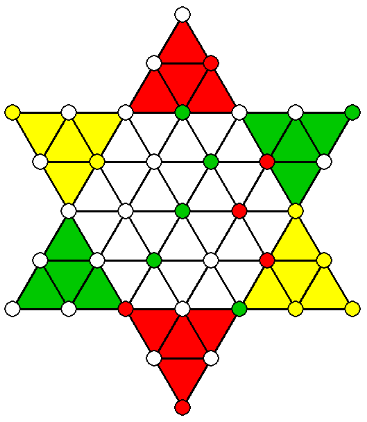


Figure 4.0.12: $r_s(s, 3)$

There are several agents implemented in the project.

4.1. Main Agent

The Main Agent is being trained in the project. The agent chooses moves with the help of a Monte-Carlo tree search. The tree search is entwined with an artificial neural network, which fulfills following two functions:
When a leaf node (s, p) of the tree is reached, it evaluates the board state and returns v. It also provides vector π that contains the the pre-evaluation values $P_{s,a,p}$ for all $a \in A$.
The pre-evaluation values of all possible moves would be enough information to choose one. However, the tree search simulates game-play several moves in advance and provides a look-ahead. In section 6 we will see that the tree search also plays an important role in the training

process of the network.

The function ρ that returns the pre-evaluation values in the tree search, uses the neural network.

$$\rho(s,p) = f_v(r_s(s,p)) \tag{4.1.13}$$

As already explained, the network does not get s as input, but $r_s(s,p)$. It does not have the information, which player's turn it is in the game. Therefore it does not return the state value v, but a permutation v_r of v. In v_r the first entry corresponds not to the expected score of player one, but the expected score of the current player et cetera. Therefore, we need a function, that maps v_r to v:

$$r_v : V \times P \mapsto V \tag{4.1.14}$$

Finally, we can formulate our state evaluation function ϕ.

$$\phi(s,p) = r_v(f_v(r_s(s,p))) \tag{4.1.15}$$

The board is first rotated, then evaluated by the network and lastly, the scores are being re-ordered to match the actual players.

After the execution of the Monte-Carlo tree search, as described in algorithm 3, the vector C is obtained. It describes the number of visits of each edge from the root node.
There are two ways to use this vector.
We can choose the action, whose edges in the last MCTS-iterations has the most count. If there are several edges that have maximal counts, we randomly select one of them.

$$A^* := \arg\max_{a\in A} C_a \tag{4.1.16}$$

$$P(a) = \begin{cases} \frac{1}{|A^*|} & \text{if } a \in A^* \\ 0 & \text{else} \end{cases} \tag{4.1.17}$$

We call this procedure the best selection. If $|A^*|$ equals 1, which is the case most of the time, this leads to a problem. Until the weights of the network are being changed, and thus the probability vector C, action a^* will always be the same for a fixed state s. Given the same group of agents, a game is therefore strictly determined and every replay will be an exact repetition. To collect more data for the training and evaluation process we introduce a second selection method.

We choose randomly from the tried actions and assign the relative visits of action a as its probability.

$$P(a) = C_a \quad \text{for all } a \in A \tag{4.1.18}$$

This will be referred to as varied selection.
Both selection methods are being used in the program.

4.2. Greedy Agent

To provide an analytic evaluation of a move, we introduce progress. We divide the board into rows and record how many rows the figure moved upwards.

$$e : A \mapsto \mathbb{Z} \tag{4.2.19}$$

The greedy agent always chooses one of the moves with highest progress. It is the most simple plausible game strategy.

$$A_G = \arg\max_{a \in \tilde{a}(s,1)} e(a) \tag{4.2.20}$$

$$P(a) = \begin{cases} \frac{1}{|A_G|} & \text{if } a \in A_G \\ 0 & \text{else} \end{cases} \tag{4.2.21}$$

The Greedy agent is used as a benchmark for the performance of the Main agent.

4.3. Initialize Agent

The Initialize Agent is used to initialize our neural network.
It chooses random moves, but a has high tendency to move figures towards the end zone. A neutral move to the side is twice as likely as a backward move and a forward move is twice as likely as neutral move to the side. We construct the vector $I \in \mathbb{N}_0^a$ with:

$$I_a = \begin{cases} 1 & \text{if } a \in \tilde{a}(s,1) \text{ and } e(a) < 0 \\ 2 & \text{if } a \in \tilde{a}(s,1) \text{ and } e(a) = 0 \\ 4 & \text{if } a \in \tilde{a}(s,1) \text{ and } e(a) > 0 \\ 0 & \text{else} \end{cases} \tag{4.3.22}$$

$$P(a) = \frac{I_a}{||I||_1} \quad \text{for all } a \in A \tag{4.3.23}$$

5. Neural Net

In this section we will take a look at the neural network and how it is trained.
A neural network is a concatenation of linear and non-linear functions. We describe these functions as layers. The non-linear functions are called activation functions. The activation functions are needed. Without them the linear functions would simplify into one linear function. This would limit the complexity of the network quickly.
A neural network can also contain regularizing functions. These functions improve training.
The functions of the network contain parameters that we denote with the vector $w \in W \subset \mathbb{R}^n$. Therefore the output depends on the function input and those parameters. In the training process we modify these parameters. If this is done correctly, the network will return better outputs than before. What exactly is considered a better output will be measured by labeled data, that consists of an input and the output that is desired by the user. Therefore a neural network can be understood as a complex optimization problem.
Given a board configuration our neural network makes two predictions that are used in the Monte-Carlo tree search. It calculates the pre-estimate probability vector π and the board value v.
We denote these two predictions as following functions:

$$f_\pi : S \times W \to [0,1]^{|A|} \tag{5.0.24}$$

$$f_v : S \times W \to V \tag{5.0.25}$$

Combined they are formulated as following function.

$$f : S \times W \to [0,1]^{|A|} \times V \tag{5.0.26}$$

5.1. Network architecture

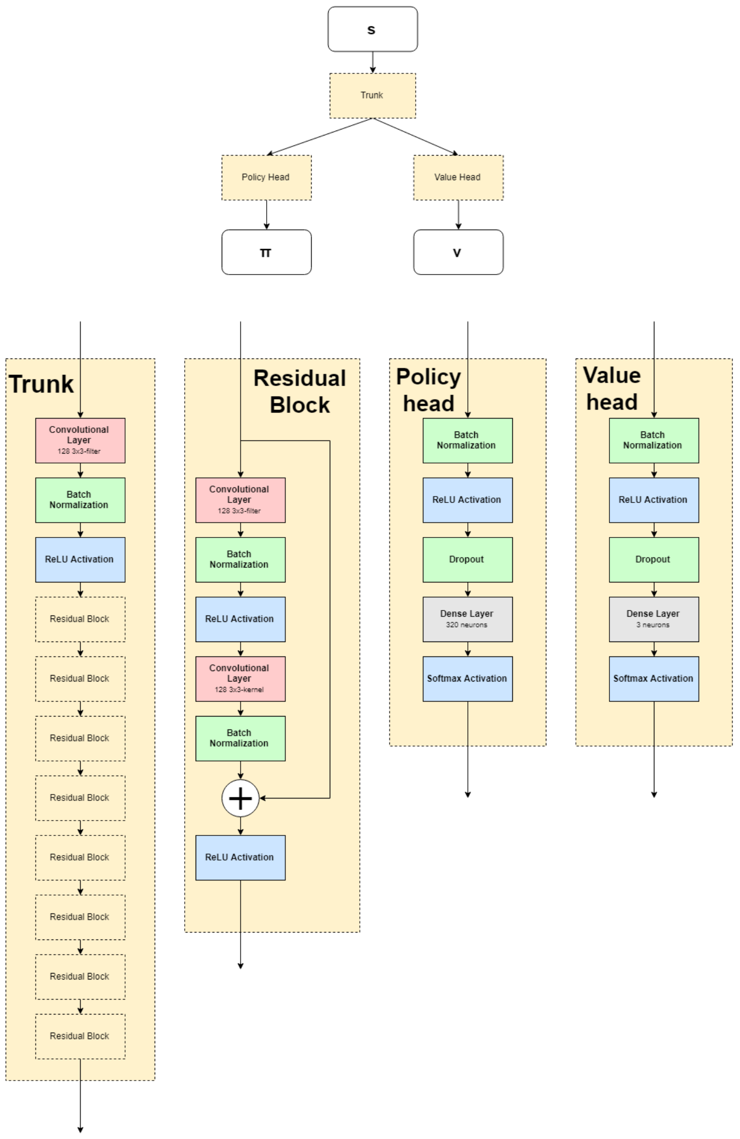


Figure 5.1.27: PLACEHOLDER Neural Network Architecture

We use a residual neural network. It can be divided into three parts. The trunk, the policy head and the value head. As input it takes the 9×9 array representation of the board state. The trunk processes the input and returns a 512-element vector. This vector then is taken as input by the policy head as well as the value head, which return the final outputs.

In the following we will investigate the layers that the network is built of. The network also contains methods that reshape the data. For example, a $n \times m$-matrix will be represented by a $(n \cdot m)$-vector after reshaping. These reshaping methods are not listed, but their placements can be reasoned by the form of outputs and inputs of successive layers.

5.1.1. Dense layer

Dense layers are the most basic layers used in neural networks. They take the signals of a previous layer as input and return linear combinations of them. Inspired by biology the entries of its output are called neurons.

$$\begin{aligned} d : \mathbb{R}^n \times \mathbb{R}^{n \cdot m + m} &\to \mathbb{R}^m \\ (x, w) &\mapsto \begin{pmatrix} x_1 \cdot w_1 + \ldots + x_n \cdot w_n + w_{n \cdot m + 1} \\ \vdots \\ x_1 \cdot w_{n \cdot (m-1)+1} + \ldots + x_1 \cdot w_{n \cdot m} + w_{n \cdot m + m} \end{pmatrix} \end{aligned} \tag{5.1.28}$$

The entries of w are learnable parameters that get modified during training.

5.1.2. Convolutional Layer

Convolutional neural networks are often used for pattern detection. A convolutional layer can, for example, detect edges and corners in an image, and several layers in combination are able to detect complex patterns. We use 2-dimensional convolutional layers to detect abstract patterns in the game, like ladders and blocks of figures.
One layer contains a fixed number of $k \times k$-matrices. Those matrices are called filters. We call k the kernel size. It has to be an uneven number. We slide a filter across the input matrix. The part of the array that is temporarily covered by a filter is called a window. During one iteration every array element is the center of a window once. To make this possible, a frame of the thickness $\frac{k-1}{2}$ consisting of zeros has to be added to the input. We multiply each filter element by the matrix element it is currently covering. Then we sum up the products. This sum is the value for the output matrix corresponding to the center of the window.

For every filter a different matrix is the output. If the input of the layer is one array, like in the first instance used in the project, the input array gets duplicated to match the number of filters. If the input consists of a number of arrays same as the number of filters, every input array will be transformed by a different filter.

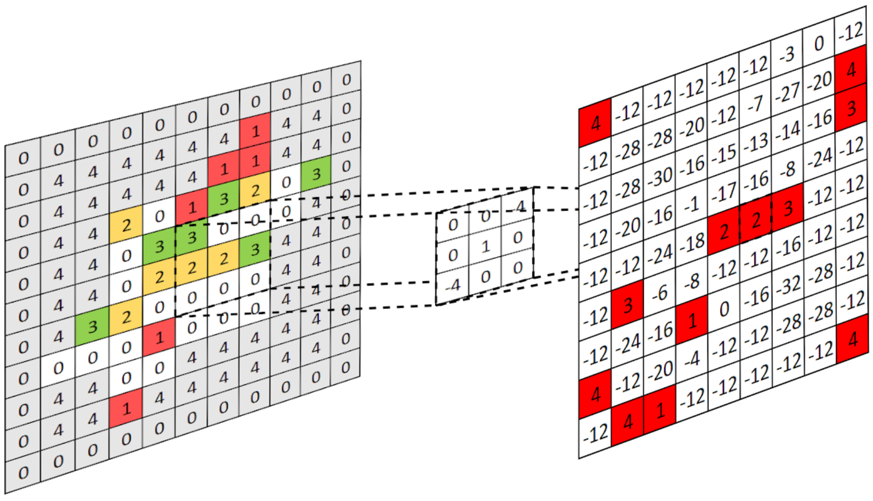


Figure 5.1.29: Convolutional layer application
This image shows an example filter applied to the board state. It detects fields than can be jumped over in the upper right or lower left direction. The positions that have positive indices and do not lie on the border indicate these fields.

$$\tilde{k} := \frac{k-1}{2} \tag{5.1.30}$$

$$\begin{aligned} C : \mathbb{R}^{n_1 \times n_2} \times \mathbb{R}^{k \times k} &\to \mathbb{R}^{n_1 \times n_2} \\ (X, F) &\mapsto Y \end{aligned} \tag{5.1.31}$$

$$\tilde{x}_{i,j} := \begin{cases} X_{i,j} & \text{if } 1 \le i \le n_1 \text{ and } 1 \le j \le n_2 \\ 0 & \text{else} \end{cases} \quad \text{for } i = -\tilde{k}+1, ..., n_1+\tilde{k}; j = -\tilde{k}+1, ..., n_2+\tilde{k} \tag{5.1.32}$$

$$Y_{i,j} = \sum_{l=1}^{k} \sum_{m=1}^{k} F_{k,m} \cdot \tilde{x}_{i-\tilde{k}+l, j-\tilde{k}+m} \tag{5.1.33}$$

The entries of each filter matrix are learnable parameters.

5.1.3. ReLU Activation

The rectified linear unit (ReLU) is an activation function. It introduces non-linearity to the neural network.
We use it as the activation function for output v.

$$\begin{aligned} g : \mathbb{R}^n &\to \mathbb{R}^n \\ x &\mapsto \begin{pmatrix} \max(0, x_1) \\ \vdots \\ \max(0, x_n) \end{pmatrix} \end{aligned} \tag{5.1.34}$$

5.1.4. Softmax Activation

The softmax function is used as activation function for the output layer of π.
The π vector represents a probability distribution with values between 0 and 1 and a sum of 1. The softmax function returns a vector that satisfies this condition. That is the reason why it is usually applied as an activation for outputs that represent probability distributions.

$$\begin{aligned} g : \mathbb{R}^n &\to \mathbb{R}^n \\ x &\mapsto \frac{1}{\sum\limits_{i=1}^{n} \exp(x_i)} \begin{pmatrix} \exp(x_1) \\ \vdots \\ \exp(x_n) \end{pmatrix} \end{aligned} \tag{5.1.35}$$

5.1.5. Dropout

A well-known problem with large neural networks is that they tend to overfit the data. Overfitting occurs, when the network adjusts its predictions too closely to the training data. In this case, the network adapts to outliers, resulting in a loss of generality.
A way to prevent this, would be to train multiple neural networks and take the mean of their predictions. Dropout builds upon this approach. Using it, we do not need to train multiple models, but make a slight variation to the architecture of our network, everytime we train it. When predicting, a dropout layer does not affect the network.
A dropout layer has specified dropout probability $p_d \in (0, 1)$. In our dropout layers it is equal to 0.3.
In every training iteration, every input neuron of a dropout layer has a probability of p_d to be ignored in the training process. The network is temporarily modified such, that these neurons do not exist. Therefore the weights corresponding to the disabled neurons are not being updated. Dropout layers slow down the training of a neural network, but increase stability.

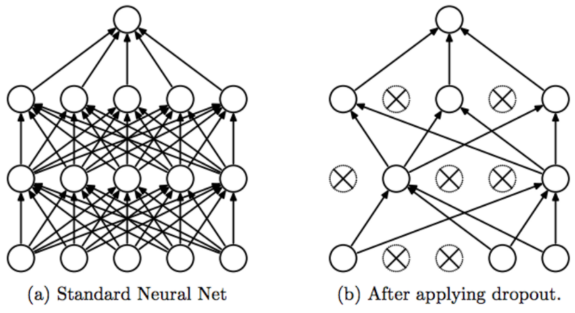


Figure 5.1.36: Network using dropout
This figures shows a network with dense layers and an output. In figure (b) a dropout layer is inserted between each layer. The crossed out neurons represent the ones, that are ignored in the next layer in one specific iteration.

5.1.6. Batch Normalization

Batch normalization is another regularization technique.

$$\mu_B = \frac{1}{m}\sum_{i=1}^{m} x_i \tag{5.1.37}$$

$$\sigma_B^2 = \frac{1}{m}\sum_{i=1}^{m}(x_i - \mu_B)^2 \tag{5.1.38}$$

$$\hat{x}_i = \frac{x_i - \mu_B}{\sqrt{\sigma_B^2 + \epsilon}} \tag{5.1.39}$$

$$y_i = \gamma \hat{x}_i + \beta \tag{5.1.40}$$

5.1.7. Residual Block

The use of residual blocks makes our network a residual neural network. In the residual blocks there is a skip connection implemented. Through this connection the input of the residual block gets added to the processed data just before the activation function of the block. From a certain point on, neural networks without skip connections tend to stagnate or even decline in quality with every layer that is added. This problem is discussed in detail in this article:

!QUELLE!

5.2. Loss Function

The loss function describes the error of a prediction given labeled outputs. For the outputs v and π different loss functions are being used.

Since π represents a probability distribution, the loss is described by Categorical Cross-Entropy.

$$\begin{aligned} l_\pi : S \times A \times W &\to \mathbb{R} \\ (s, \hat{pi}, w) &\mapsto -\sum_{i=1}^{920} \hat{\pi}_i \log(f_\pi(s)_i) \end{aligned} \tag{5.2.41}$$

The difference of score estimations in v and the labeled scores $\hat{v}$ is measured by mean squared error.

$$\begin{aligned} l_v : S \times V \times W &\to \mathbb{R} \\ (s, \hat{v}, w) &\mapsto \frac{1}{3}\sum_{i=1}^{3}(f_v(s,w)_i - \hat{v}_i)^2 \end{aligned} \tag{5.2.42}$$

The overall loss function consists of the sum of both loss functions.

$$\begin{aligned} l : S \times V \times A \times W &\to \mathbb{R} \\ (s, \hat{v}, \hat{\pi}, w) &\mapsto l_\pi(s, \hat{\pi}, w) + l_v(s, \hat{v}, w) \end{aligned} \tag{5.2.43}$$

In the training process s, $\hat{v}$ and $\hat{\pi}$ are fixed. We want to find a $w \in W$ that minimizes the loss. To clarify this we use another formulation of the function that describes the loss of one tuple of training data in respect to w.

$$\begin{aligned} l_w : W &\to \mathbb{R} \\ w &\mapsto l(s, \hat{\pi}, \hat{v}, w) \end{aligned} \tag{5.2.44}$$

We generally don't train on a single tuple, but on a large number of labeled training tuples at the same time. Given the training batch $b \in (S \times A \times V)^{n_t}$ we can formulate the loss function that we want to minimize.

$$\begin{aligned} L_b : W &\to \mathbb{R} \\ w &\mapsto \sum_{i=1}^{n_t} l_w(s_i, \hat{\pi}_i, \hat{v}_i) \end{aligned} \tag{5.2.45}$$

5.3. Optimizers

During the training process an optimizer redefines weights of the model. The loss function gets minimized with respect to the weights. To accomplish this, there are several optimizers that are being frequently used in machine learning. These optimizers are iterative algorithms that start at a starting point $x^{(0)}$ and (not always successfully) go towards the minimum of a function. When training a network, w is that starting point.
In this case the Adam Optimizer was chosen. The following section will explain Gradient Descent as its basis. Afterwards, different enhancements are introduced, leading to Adam Optimizer, which incorporates all of them.
We introduce function F, of which we want to estimate a minimum. We also have to know the gradient J of F.

$$\begin{gathered} \mathbf{F} \colon \mathbb{R}^n \to \mathbb{R} \quad x \longmapsto F(x), \qquad n \in \mathbb{N} \\ \mathbf{J} \colon \mathbb{R}^n \to \mathbb{R} \quad x \longmapsto \begin{pmatrix} \frac{\partial F}{\partial x_1} \\ \vdots \\ \frac{\partial F}{\partial x_n} \end{pmatrix} \end{gathered} \tag{5.3.46}$$

5.3.1. Gradient Descent

Gradient Descent is a numerical iterative algorithm used to find the minimum of a function. Given a arbitrary starting value it successively keeps stepping in the direction of the current negative gradient. The step size is the gradient's magnitude multiplied by the learning rate parameter $\alpha \in \mathbb{R}_{>0}$. The number of iterations that are executed is called n_{epoch} in the context of machine learning.

$$x^{(i)} = x^{(i-1)} - \alpha \cdot J(x^{(i-1)}) \quad \text{for } i = 1, ..., n_{epoch} \tag{5.3.47}$$

Convergence is only guaranteed for convex function with Lipschitz-continuous partial derivatives and a sufficiently small learning rate. Although there usually is no prior knowledge about the often non convex loss functions in machine learning problems, it is widely applied to the loss function with motivation to find a local minimum resulting in loss not much higher than the global minimum.
The right choice for α depends on the problem. A smaller value leads to smaller steps and can slow down the minimization. A value that is too large however can prevent the algorithm from converging or even lead to divergence.
Gradient descent is applied to the loss function L_b to find parameters $w^* \in W$ that minimize the loss.

5.3.2. Mini-Batch

When we train a neural network we feed it a large amount of labeled data. Therefore the computation of each gradient is computational expensive. The idea of Mini-Batch is to split the training set into batches of size n_b and execute each gradient descent step optimizing just over one batch. Thus each iteration minimizes w over a slightly different loss function. This speeds up the computation of its gradient, trading off accuracy for each step. This comes from the fact that the randomly selected mini-batch used for one step does not have to be a good representation of the whole training set, resulting in a loss function greatly differing from L_b. In theory however these inaccuracies cancel each other out.
We do not iterate over $L_b(w)$, but over the functions $L_{b_i}(w)$. We take smaller steps, but multiply the number of steps by the number of mini-batches.

$$L_{b_i}(w) := \sum_{j=(i-1)\cdot b+1}^{\min\{i \cdot b, n_t\}} l_w(s_j, \hat{v}_j, \hat{\pi}_j, w) \text{ for } i = 1, ..., \left\lceil \frac{n_t}{n_b} \right\rceil \tag{5.3.48}$$

$$J_{b_i}(x) := \begin{pmatrix} \frac{\partial L_{b_i}}{\partial x_1} \\ \vdots \\ \frac{\partial L_{b_i}}{\partial x_n} \end{pmatrix} \tag{5.3.49}$$

$$x^{(i)} = x^{(i-1)} - \alpha \cdot J_{b_i}(x^{(i-1)}) \quad \text{for } i = 1, ..., \left\lceil \frac{n_t}{n_b} \right\rceil \cdot n_{epoch} \tag{5.3.50}$$

The Batch size has to be set by the user and its efficiency depends on factors like the data size of one training tuple, the physical machine that is used for training and the variance of the data. A Mini-Batch Gradient method using Batch size 1 is called Stochastic Gradient Descent.

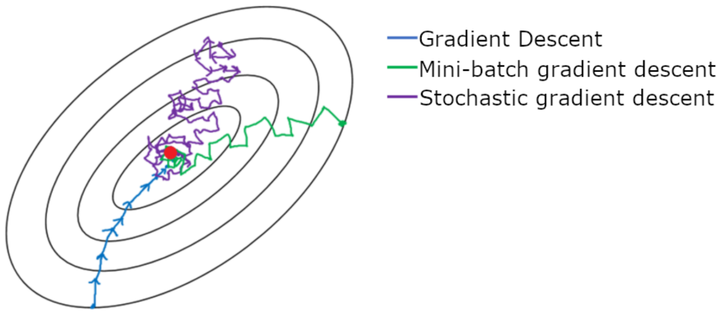


Figure 5.3.51: Mini-batch gradient descent
A 2-dimensional function is minimized with standard gradient descent, Mini-batch gradient descent and stochastic gradient descent. To illustrate this better, different starting points were used.

5.3.3. Momentum

The gradients computed in every step of Mini-batch gradient descend can change directions rapidly, while often alternating around the ideal directional vector leading towards a local minimum. Gradient Descent with momentum takes advantage of that. Just like in physics, where the movement of an object is influenced by its momentum, gradients from previous iterations continue to have an effect on the current step. By taking an exponential average of the past gradients, it dampens the directional changes. This can result in faster convergence.
The parameter $\beta_M \in (0,1)$ controls the influence of momentum.

$$M^{(0)} = 0 \tag{5.3.52}$$

$$M^{(i)} = \beta_M \cdot M^{(i-1)} + (1-\beta_M) \cdot J(x^{(i-1)}) \tag{5.3.53}$$

$$\tilde{M}^{(i)} = \frac{M^{(i)}}{1-\beta_M^i} \tag{5.3.54}$$

$$x^{(i)} = x^{(i-1)} - \tilde{M}^{(i)} \quad \text{for } i = 1, ..., n_{epoch} \tag{5.3.55}$$

Formula 5.3.54 describes a bias correction. Since M_0 is initialized with 0, this low value drags down the moving average, especially in the earlier iterations. To correct this we divide M_i by $(1-\beta_M^i)$. This cancels out the influence of M_0 without interfering with the moving average.

5.3.4. RMSProp

Another method correcting gradient orientation is Root Mean Square Propagation (RMSProp). The magnitude of the partial derivatives can vary to such an extent that the gradient takes huge steps with respect to one weight but neglects some others. Instead of letting only the gradient decide, how far to move in each direction, RMSProp uses a different learning rate for each weight respectively with more respect to the gradient vector entries which have a smaller absolute value. The overall learning rate is divided by the square-root of an exponential average of past squared partial derivatives.

$$s_j^{(i)} = \beta \cdot s_j^{(i-1)} + (1-\beta) \cdot (\frac{\partial F}{\partial x_j}(x^{(i-1)}))^2 \tag{5.3.56}$$

$$x^{(i)} = x^{(i-1)} - \left(\frac{\alpha}{\sqrt{s_1+\epsilon}} \cdots \frac{\alpha}{\sqrt{s_m+\epsilon}}\right) \cdot J(x^{(i-1)}) \tag{5.3.57}$$

5.3.5. Adam

Adaptive Momentum Estimation (Adam) combines the ideas of the past subsections. It is a mini-batch gradient descent algorithm using momentum and Root Mean Squared Propagation.

$$M^{(i)} = \beta_M \cdot M^{(i-1)} + (1-\beta_M) \cdot J_{b_i}(x^{(i-1)}) \tag{5.3.58}$$

$$\tilde{M}^{(i)} = \frac{M^{(i)}}{1-\beta_M^i} \tag{5.3.59}$$

$$s_j^{(i)} = \beta \cdot s_j^{(i-1)} + (1-\beta) \cdot (\frac{\partial F}{\partial x_j}(x^{(i-1)}))^2 \tag{5.3.60}$$

$$x^{(i)} = x^{(i-1)} - \left(\frac{\alpha}{\sqrt{s_1^{(i)}+\epsilon}} \cdots \frac{\alpha}{\sqrt{s_m^{(i)}+\epsilon}}\right) \cdot \tilde{M}^{(i)} \quad \text{for } i = 1, ..., \left\lceil \frac{n_t}{n_b} \right\rceil \cdot n_{epoch} \tag{5.3.61}$$

Adam Optimizer has proven to be a very good off-the-shelf optimizer for neural networks.

6. Project Structure

After examining the building blocks used in the project, we will now look at how they are combined.
The main program consists of a loop that gradually improves the game agent.

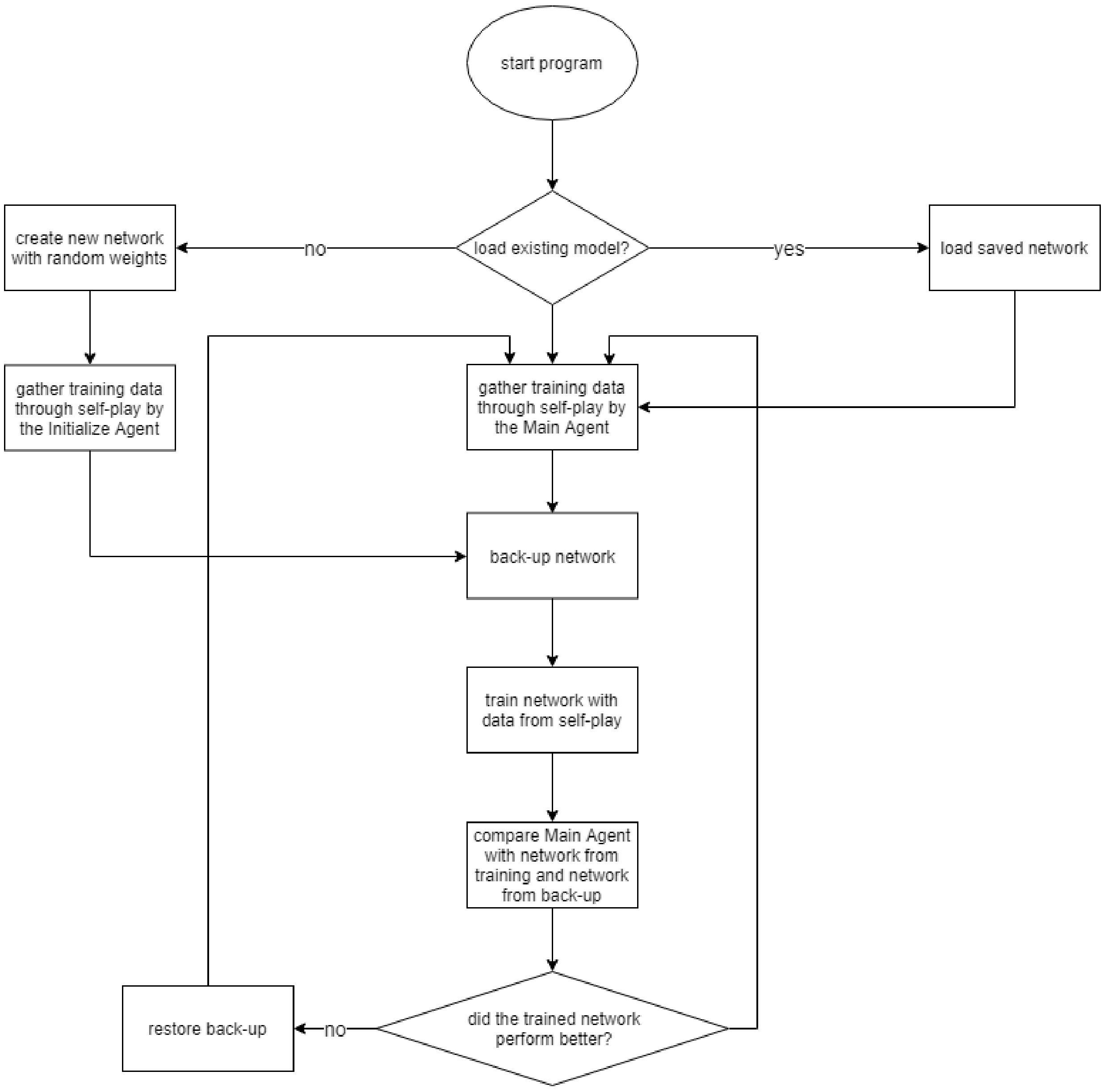

start program
load existing model?
no
yes
create new network with random weights
load saved network
gather training data through self-play by the Initialize Agent
gather training data through self-play by the Main Agent
back-up network
train network with data from self-play
compare Main Agent with network from training and network from back-up
did the trained network perform better?
no
restore back-up

Figure 6.0.62: PLACEHOLDER Flowchart of the training loop

Self-play

In this phase training data for the neural network is created. n_{eps} games are played. Every turn is done by the same agent, trying to optimize the score of the current player. The first n_{varied} steps each are played with varied selection. Afterwards, best selection is used. Before every move the tuple $(s, C, p) \in S \times \mathbb{R}^{|A|} \times P$ is saved, where s is the current board state, C is the vector obtained by the Monte-Carlo tree search and p is the current player. When the game is decided, the final state is evaluated to state value v^*. For every tuple (s, C, p) that we collected, we create a corresponding tuple $(r(s,p), C, r(v^*,p)) \in S \times \mathbb{R}^{|A|} \times V$. They consist of the rotated board, vector C and the final scores out of perspective from player p. These tuples are used as training data for the neural network.

If this is the first iteration and no existing model was loaded, the Initialize agent is being used. Otherwise the agent corresponds to the current version of the neural network.

Back-up

We create a back-up copy of the current neural network. If the network does not improve through the training, we can restore the version before training.

Training

We use the training data of maximal the last 10 self-play sessions to train the network, provided they were generated by the main agent. The training will result in the following changes in the network: The evaluation of a game state gets shifted towards the evaluation of the final board state s^* that it led to. The pre-evaluation π will shift towards the moves that the tree search favored in past simulations. This can result in problems, as we will discuss in subsection 7.3.

If this is the first iteration and the network is being initialized by the Initialize agent, we use the training examples in this first iteration and discard them afterwards.

Arena

After the training, we have our old back-up version of the neural network and the new one. We let two agents with the respecting neural networks compete against each other.

The agents use varied selection for the first 5 steps each and best selection afterwards. To avoid bias, we play 2 vs. 1 games and alternate which network will play for 2 and which one will play for 1 player. We also rotate between the players, Therefore a set consists of 6 games. We play a fixed number of sets and add up the points scored for each agent. If the agent with the just trained network did significantly better (scored 55% of the points), we take it as the new standard. Otherwise we restore the old version.

This process is repeated with the goal of gradually improving the network's predictions and therefore making the agent a stronger player.

7. Difficulties

During the realization of the project, several problems occured. This chapter will explain them and the measures taken to overcome them.

7.1. Loops

As already discussed, Chinese Checkers is a game, that allows game loops. subsection 3.3 explains, how the tree search is modified to address this problem. However, since the tree is reset and built before every move, this modification does not prevent loops in actual game-play. Similar to the trace, we save inside the tree search, we also list the actual board states of a game and generalize the idea of loop-cutting. We also intervene, if state s was reached in the game and it was the turn of player p This is only done in the self-play to gather training examples.
These counter-measures help against game loops, but not against quasi-loops. Meant by that are series' of board states, in which no state appears twice, but that would likely be perceived as game loop by human players, because no progress in the game is made. An example. These quasi-loops are especially conducted by players, that are steps away from losing the game, preferring to drag the game over losing.

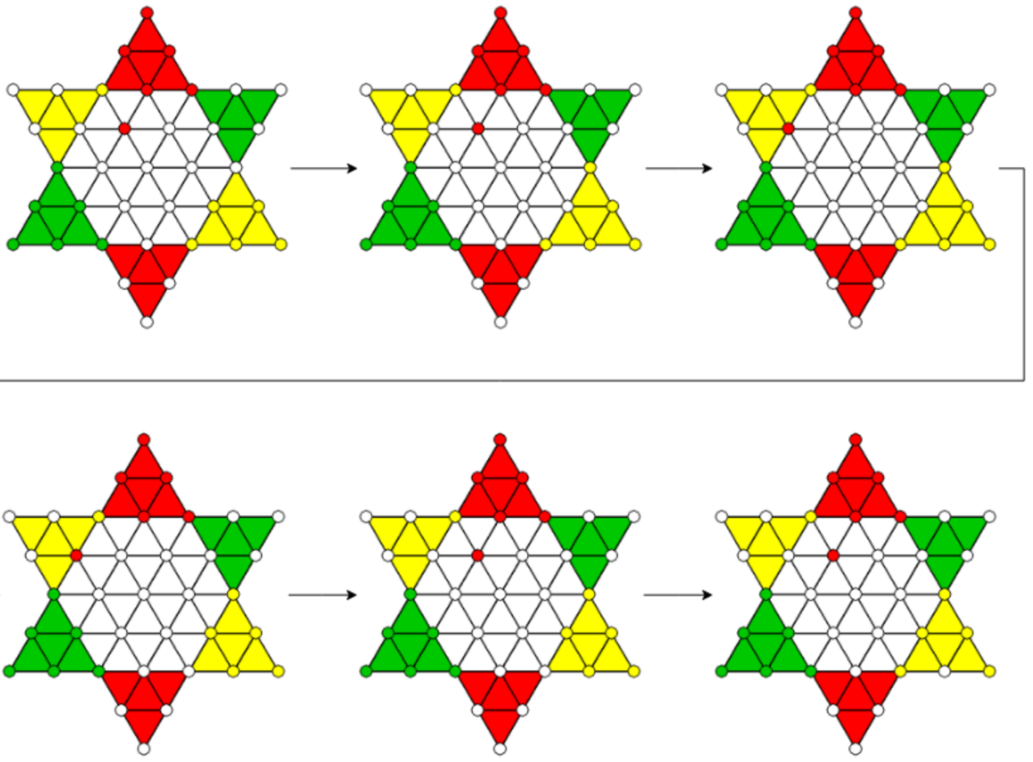


Figure 7.1.63: Example of a quasi-loop before the introduction of the second-winner rule
In the first board state it is the turn of player two. Knowing that a moving his figure out of player one's end zone would result in a loss, he just moves his pieces around.

After the quasi-loop is played a specific number of times, all variations of it have been played. If this situation occurs in a self-play game during training and it's the turn of the player with

the upper hand, he will be forced to prevent an actual game loop and alter the situation to his disadvantage. This way it is possible that he will be overtaken by the player that conducted the quasi-loops.
We do not want this scenario in our training. To prevent it, we introduced the second-winner-rule.
For the case that all these measures fail to prevent an infinite game, we stop the game after 40 moves each. Every player, that has scored points until then gets them awarded, while the others receive a score of zero. The agents do not have knowledge about how many steps were taken in game and therefore will not change their behavior. However, after the training process the board states that led to the situation will be evaluated lower.

7.2. Triple Win

The second-winner-rule makes it possible, that one player wins and the other share the second place. In this case, we award both of them a score of 1.

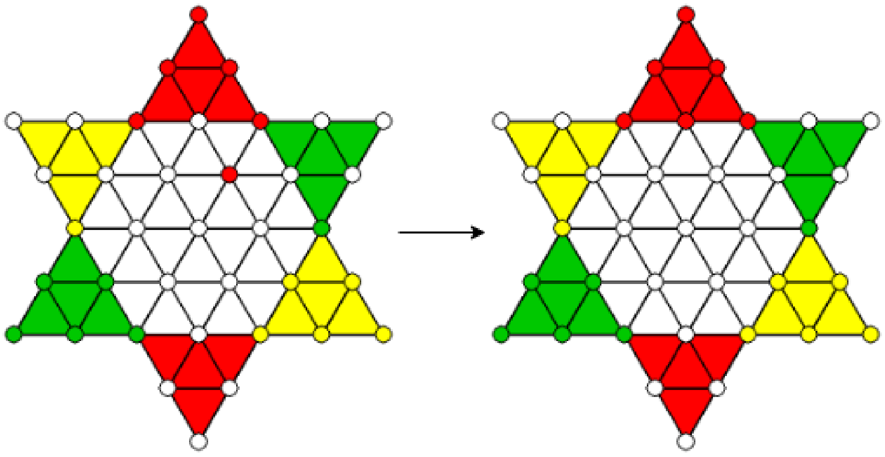


Figure 7.2.64: Triple Win

This scenario occurs very rarely, because both blocking players have no reason to rely on each other and cooperation over multiple games is not possible in the project.

7.3. cpuct

!VERLINKUNG!
The right choice for the exploration parameter c_{puct} is one of the most crucial factors determining the quality of the neural network and therefore the agent. In **??** the mechanics of Monte-Carlo tree search and the role of c_{puct} were already explained. In combination with the training process the influence of c_{puct} is even stronger. The lower c_{puct} is, the more distinct are the values of C. Because we feed C to the network as labeled training data for π, this effect gets amplified over time. When arriving at the same board state, the distribution of π will be more distinctive and the tree search will focus even more on the paths that were previously favored, neglecting the unpromising ones. In a extreme consequence, the distribution of π can become so distinct, that the tree search does not contribute to the decision at all, because from the root node only the path suggested by the network is explored. In the long run, this can stop the improvement of

the neural network.
On the other hand, a value for c_{puct}, that is too high, can shape π towards an uniform distribution, defying its purpose.
If one of these scenarios occur during training , there are several ways to address the problem. If the exploration is too low, but the user is satisfied with the state of the neural network, the exploration parameter can be turned up to an extreme during the use of the agent. One can also change c_{puct} into the opposite extreme for several training iterations with the goal of balancing out the distribution of π. The best solution is to experiment until a exploration parameter is found, and restart the training process using it in all iterations. This however is the most time- and resource-consuming.

7.4. Performance

The training process requires a lot of computational power.
At first a version of the game with larger board size was implemented, but it soon became clear, that the training process would take too long and the board size was reduced. That is also the reason, why the number of possible board states and the action space were reduced with the rule that forbids players to move their pieces deeper into the zones of opponents.

Originally, the network was supposed to be trained with data obtained solely by self-play from previous network versions. Because of the recurring states in Chinese Checkers a game consisting of randomly chosen moves is very long. A figure can be moved just outside the end zones and then back across the board. Therefore the first iterations of self-play would be too time-consuming. The Initialize agent was designed to nudge the network in the right direction without influencing it to much.

The most costly is the generation of training examples via self-play. During a game of self-play, there were three tasks that require a lot of time compared to the other ones. They are the board rotations, the determination of legal moves and the predictions by the neural network. To speed up the predictions, the games are played silmultaneously. A more detailed description can be found in the next chapter.

8. Implementation

The project builds upon a open-source project I found on Github. This project by Surag Nair implements the training process described in the chapter ??? and is a simplification of the structure used in AlphaZero. This project was applied to Othello, but it is generalized in such a way that it can be used for other two-player board as well. The programming language used is Python 3.

The overall project structure was adapted. To be applicable for 3-player games, every class was drastically changed and new classes were written. The Monte-Carlo tree search had to be altered to allow three players. The competition between the old and the just trained network is more complicated, because we have to evaluate two players in a 3-player game. The old project contained neural network templates written with the frameworks pytorch and Tensorflow 1. I wrote a network in Tensorflow 2 using the Keras API. The design is more close to the one used in AlphaZero.

The game Chinese Checkers was written from scratch, including a graphical user interface that is the source of the board figures in this thesis.

To speed up the self-play games, multiple games are played silmultaneously.

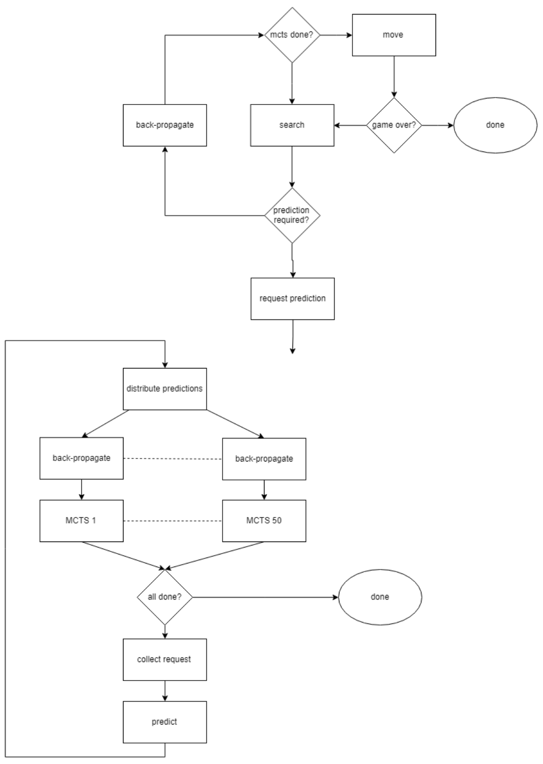


Figure 8.0.65: PLACEHOLDER

In every game instance the agents play and search in the game tree until a prediction by the network is requested. Then the game-play is paused until every instance reaches this point. The game states, of which a prediction is needed, are collected and fed in the network. Then the

outputs get distributed back to the game instances in which they were requested and the values are back-propagated. This process is repeated until every game is over. To realize this, the Monte-Carlo tree search had to be written as an iterative function. The tree exploration and the back-propagation were separated into two functions. Because the path that traversed in the tree can not longer be retrieved from a recursion stack, it has to be saved in a variable. Because this path, the board states and all variables from the Monte-Carlo tree search have to be saved for all games, this solution is very memory-intensive.
The training was executed on Google Cloud Virtual Machine with 4 virtual CPU's, 24GB RAM, SSD hard drive and one Tesla K80 GPU. The training process took over one week.

9. Results

The program was tested with following parameters: The network was initialized by 500 games played by the Initialize Agent.
In one self-play session 200 games were played.
The tree search executes 200 search iterations.
The exploration parameter c_{puct} was set to 15.

The neural network was trained until it reached the 20th version. Because the network was not updated in every training loop, 30 self-play sessions were played.

!GRAFIK SESSIONS PER VERSION!

The different versions were tested with the use of a Monte-Carlo tree search and also without it, relying just on the network. The tree search always uses best selection. In every game, the first two moves of every player are selected randomly, to ensure variation. When we let to two agents play against each other we separate the results of the games, in which the first agent played as one player against two instances of the second agent and vice-versa. In evaluations containing the tree search 30 games were played. In all other evaluations 120 games were played.

!TABLE!

9.1. Evaluation against previous versions

Just like in the training loop, the agents were evaluated against their previous versions.
!PLOTS!

9.2. Evaluation against the Greedy Agent

The agent was also tested against the Greedy Agent.

9.3. Observations

As already explained, the network also suggests invalid moves. We want to investigate, what portion of probabilities π was assigned to illegal moves. We let each version of the network play 30 games against itself without the tree search.
!PLOT!
We also measure the average length of a game.
!PLOT!

9.4. Starting order

9.5. Tactics

Tests:
Graph:
NNetMCTS vs NNetMCTS previous version
NNetMCTS vs Initialize Actor

NNetMCTS vs Greedy Actor

Graph
NNet vs NNet previous version
NNet vs Initialize Actor
NNet vs Greedy Actor

Graph
NNet vs NNet same version step counts
NNet prediction of illegal moves percentage

Scenarios:
Block mit Zugzwang
"Timer" durch Player 3

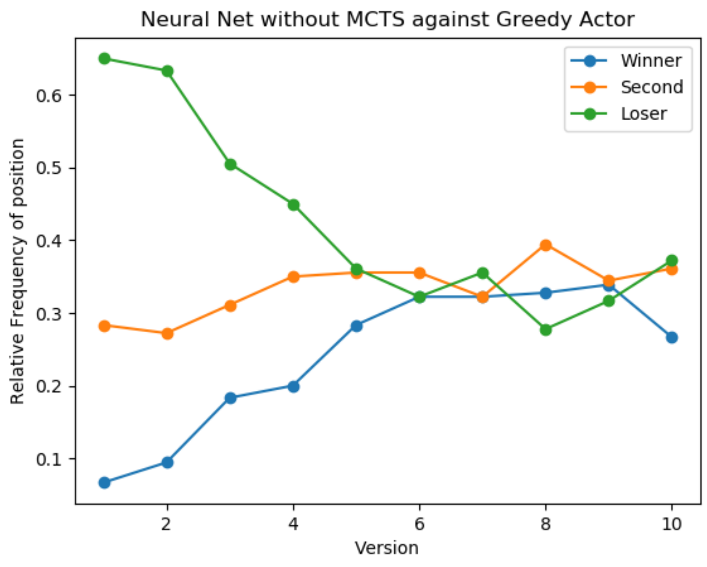


Figure 9.5.66: NNet vs. Greedy

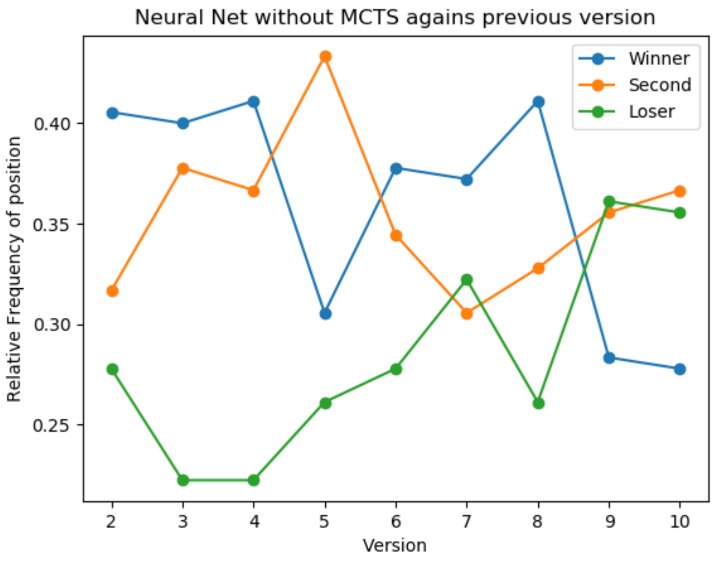


Figure 9.5.67: NNet vs. Previous

10. Conclusions and outlook

The realization of the project was a success. The methodology of AlphaZero was adapted to Chinese Checkers. The results show that the actor significantly improved during training. There are several interesting questions that built upon the findings in this thesis.

In the results we can see that the learning progress does not stagnate in later iterations. Given more time and resources, the training could be continued with the goal of creating a neural network so strong that it reliably defeats human players without the use of a tree search.

The Monte-Carlo tree search was successfully modified for a game with recurring states and loops were prevented. Possibly, a decision algorithm that is designed for graphs from the beginning could improve the results.

Lastly, the project could be enhanced with more focus on cooperation between players. Currently every player tries to optimize his score and ignores those of his opponents. An agent could be trained, that plays in a team and tries to optimize the scores of his team. Another way to approach this question is to train an agent that does not only aims for the maximum score in one game, but in a set of games in which the scores of each game are added together. Games are played until one player's score reach a specific threshold. In a scenario where one player takes the lead, it would be reasonable for the other two players to cooperate, even at the expense of individual scores. This adds a lot of complexity to the game and requires more resources, but the dynamic of temporary cooperation promises an interesting research topic.

A. Notation

A	Action space
a	Action
c_{puct}	Exploration parameter in the Monte-Carlo tree search
f	Neural network function assigning π and v to s
f_v	a
f_π	a
$g(v, p)$	Rating of a board state s for player p
$m_s(s, a, p)$	The board state resulting in the move a by player p in state s
$N_{s,a,p}$	a
$N_{s,p}$	a
n	a
n_b	a
n_t	a
P	Set of players
$P_{s,a,p}$	a
p	Player
$r_s(s, p)$	a
$r_v(v, p)$	a
S	Set of board states
s	Board state
$U_{s,a,p}$	Upper confidence bound for action a by player p in state s
V	Set of state values
$V_{s,p}$	Node value for tree node (s, p)
v	State value
$Q_{s,a,p}$	a
$\alpha(s, p)$	Legal moves for player p in board state s
$\phi(s, p)$	State value of board state s with current player p
$\rho(s, p)$	a

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