CUDA 2025 HW3

Problem statement

We solve the 3-D Poisson equation

$$ \nabla^{2}\phi(\mathbf r)= -\rho(\mathbf r) $$

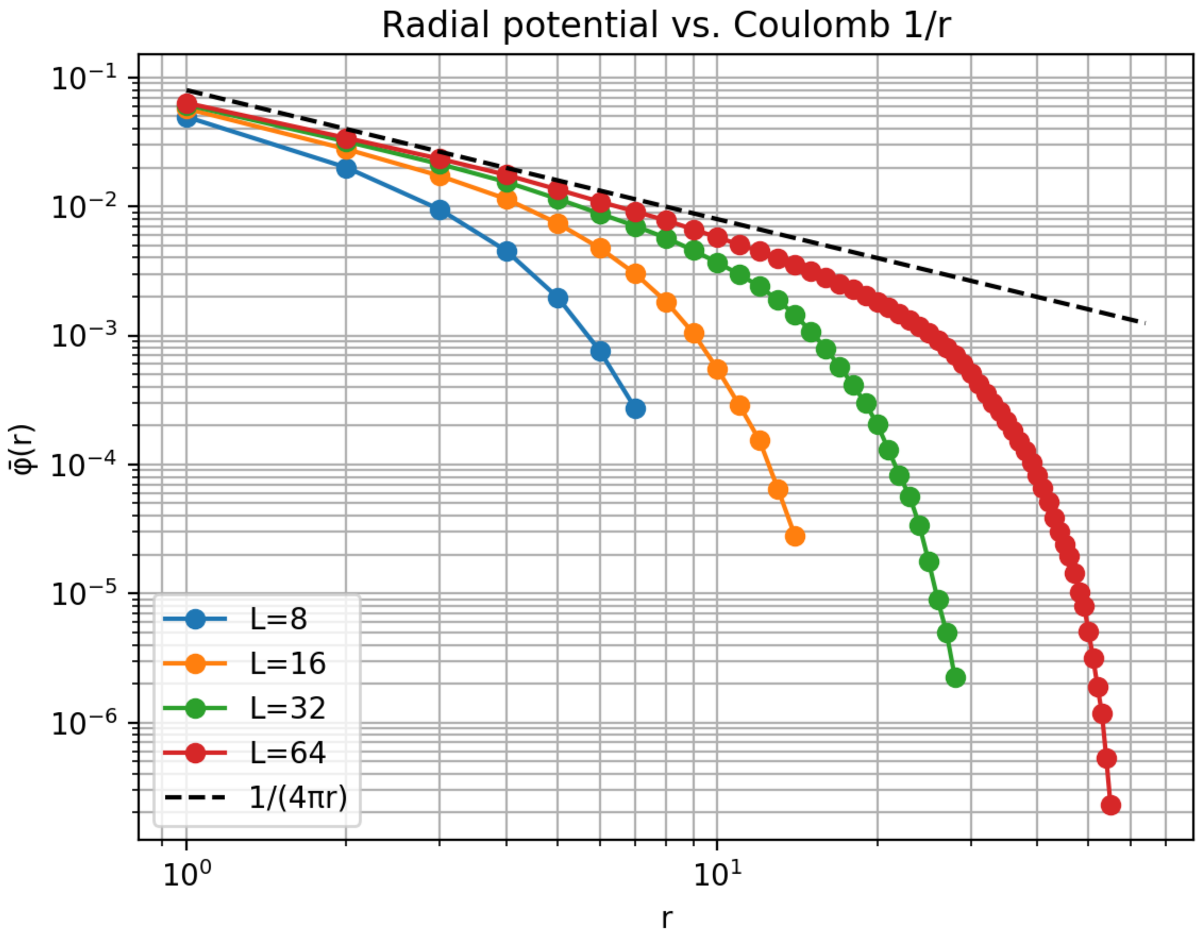
inside a cubic box of edge L (with lattice spacing $h=1$) under boundary conditions $\phi=0$ on every face. A unit point charge is placed at the cube centre:

$$ \rho(\mathbf r)=\delta_{i,i_{c}};\delta_{j,j_{c}};\delta_{k,k_{c}} . $$

Numerical method (CUDA)

Item	Design choice
Discretisation	7-point Jacobi update with a **ghost layer** (array size $(L!+!2)^3$) so no branch is needed at the faces.
Source term	Centre cell adds $+h^{2}\rho/6 = +1/6$ each sweep.
Memory	Two ping-pong buffers (`in` → `out`, then swap).
Kernel launch	Blocks $8\times8\times8$; grid $\lceil L/8\rceil^3$.
Convergence	Fixed sweep counts that were benchmarked once: 800, 1 500, 3 500, 7 500 for $L=8,16,32,64$

Results



Discussion

On log–log axes every numerical curve is parallel to the dashed r^{-1} guide for small r. This confirms the discrete Laplacian reproduces Coulomb's law.

Enviroment

- OS: Ubuntu 22.04.3 LTS
- CPU: Intel(R) Core(TM) i7-9800X CPU @ 3.80GHz
- GPU: NVIDIA GeForce RTX 2080 Ti

Usage

1. Source Files

- Cuda Code: hw3.cu
- Driver Code: driver.py

2. Compile

```
nvcc hw3.cu -o hw3
```

3. Single Run

```
./hw1 <L> <maxiter>
```

4. Poisson experiment

```
python3 driver.py
```