

ML-Guided IBP Reduction

Neural Network Guided Integration-by-Parts Reduction of
Feynman Integrals

Two-Loop Triangle-Box Topology

The Problem

- ▶ **IBP reduction:** Express complex Feynman integrals as linear combinations of simpler “master” integrals
- ▶ **Challenge:** Exponential search space of IBP identities
- ▶ **Traditional approaches:** Laporta algorithm, Kira, FIRE

The Memory Wall

Traditional IBP codes hit **memory limits** as integrals grow more complex:

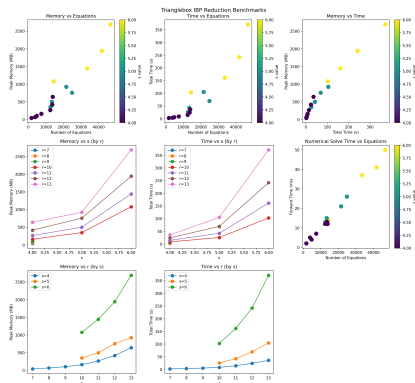


Figure 1: Kira Memory Scaling

Kira benchmarks: Memory grows exponentially with integral weight (r), reaching 2.5+ GB for $r=13$.

Our Solution

- ▶ **ML-guided beam search:** Train a neural network to score IBP actions
- ▶ **Hierarchical reduction:** Process sectors from highest to lowest
- ▶ **Parallel execution:** Distribute across Condor cluster for $\sim 10\times$ speedup
- ▶ **Constant memory:** Each one-step reduction is independent - no system accumulation

The Action Space: Key Innovation

Why Traditional IBP is Hard

- ▶ Infinite possible IBP identities (any seed, any operator)
- ▶ No clear way to choose which identity to apply
- ▶ Laporta: build giant linear system, solve globally

Our Approach: Finite Action Space

- ▶ Restrict to actions that **solve for a specific target integral**
- ▶ Each action = (IBP operator, seed shift)
- ▶ Enumerate only **valid actions** that eliminate the target
- ▶ Enables ML: classification over finite action set

Action Representation

Action = (ibp_op, delta)

- ▶ **ibp_op**: Which IBP/LI operator (0-15)
- ▶ **delta**: Shift from target to seed integral

Example

Target: $I[2,0,1,0,1,1,0]$, Action: $(3, (-1,0,0,0,0,0,0))$

IBP eq (op=3, seed=[1,0,1,0,1,1,0]):

$$c_1 \cdot I[2,0,1,0,1,1,0] + c_2 \cdot I[1,0,1,\dots] + \dots = 0$$

↑ target

Solve for target \rightarrow express in terms of simpler integrals

Direct vs Indirect Actions

Direct Actions

- ▶ Target appears directly in the raw IBP equation
- ▶ Straightforward: apply IBP, solve for target

Indirect Actions (Key Innovation)

- ▶ Target does **not** appear in raw IBP equation
- ▶ But appears **after applying substitutions** from previous steps

Raw IBP: $c_1 \cdot I[A] + c_2 \cdot I[B] = 0$ (no target)

After subs: $c_1 \cdot I[A] + c_2 \cdot (\dots + c_3 \cdot I[\text{target}] + \dots) = 0$
 \uparrow target appears!

Indirect actions leverage reduction history for deeper reductions

Subsector Filtering

The Problem

Arbitrary IBP actions can introduce integrals in **higher sectors** → explosion

Solution: Subsector Constraint

Only allow actions where all resulting integrals are in **subsectors** of target

Sector Hierarchy

Sector 63 [1,1,1,1,1,1] (6 propagators)

↓ subsectors

Sector 62 [0,1,1,1,1,1] (5 propagators)

Sector 61 [1,0,1,1,1,1] (5 propagators)

↓

...lower sectors...

Result: Reductions flow downward through sector hierarchy

Why This Works

Finite + Learnable

- ▶ Typically 10-100 valid actions per state
- ▶ Model learns which actions lead to successful reductions

Hierarchical Structure

- ▶ Subsector filtering ensures monotonic progress
- ▶ Never introduces integrals harder than current target

Substitution Chains

- ▶ Indirect actions enable multi-step reasoning
- ▶ Model implicitly learns useful substitution patterns

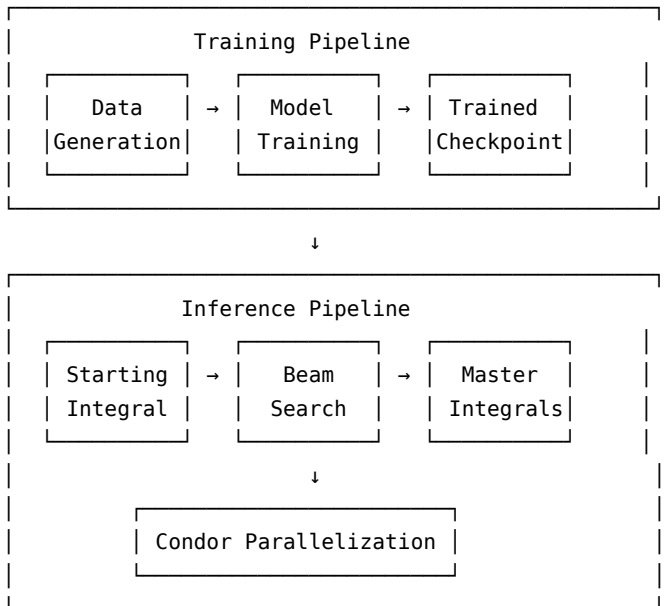
Key Results

Triangle-Box Topology (arXiv:2502.05121)

Integral	Weight	Seq.	Par.	Speedup	Masters
I[2,0,2,0,1,1,0]	(6,0)	5 min	-	-	4
I[1,1,1,1,1,1,-3]	(6,3)	73 min	12 min	6x	16
I[3,2,1,3,2,2,-6]	(13,6)	~ 20 hr	115 min	~ 10x	16

- ▶ Results match Kira exactly
- ▶ Reduces to exact 16 paper masters from arXiv:2502.05121

System Architecture



Part 1: Data Generation

Data Generation: Scrambling Approach

Key Insight

Instead of collecting reduction trajectories (expensive), **reverse the process**: 1. Start from master integrals 2. Apply random IBP identities to increase complexity 3. Record each step - becomes training data when reversed

Constraints During Scrambling

- ▶ Only apply IBPs that don't introduce higher-sector integrals
- ▶ Stay within target sector and subsectors
- ▶ Ensures training data reflects valid reduction paths

Data Generation: Coverage

Sector Coverage

- ▶ All 63 non-trivial sectors covered
- ▶ Uses 16 paper masters for their respective sectors
- ▶ Uses corner integrals for remaining sectors

Dataset Statistics

Split	Samples	Size
Train	946,168	3.8 GB
Validation	118,271	480 MB
Test	~118,000	480 MB

Data Format

Each training sample contains:

```
{
  'sector_mask': [1,0,1,0,1,1], # 6-bit sector encoding
  'expr': [
    ([1,0,2,0,1,1,0], 107), # (integral, coefficient)
    ([1,0,1,0,1,1,0], 303),
    ...
  ],
  'subs': [
    (key_integral, [(repl_int1, coeff1), ...]),
    ...
  ],
  'target_integral': [1,0,2,0,1,1,0], # Integral to eliminate
  'valid_actions': [(ibp_op, delta), ...],
  'label': 3 # Index of correct action
}
```

Part 2: Model Architecture

Model: IBPActionClassifierV5

Overview

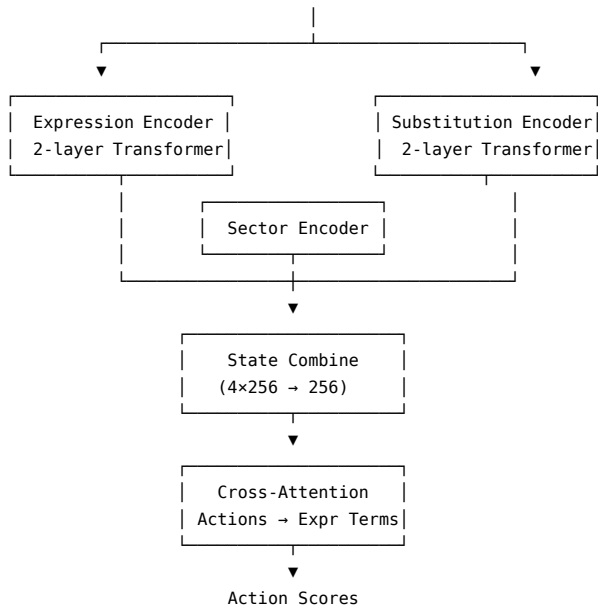
- ▶ **Task:** Score candidate IBP actions given current reduction state
- ▶ **Architecture:** Transformer-based with specialized encoders
- ▶ **Parameters:** 7.7M
- ▶ **Best validation accuracy:** 90.77%

Key Innovation (V5)

Full substitution encoding - encode not just the key integral but the complete replacement expression

Model Architecture Diagram

Inputs: Expression, Target, Substitutions, Sector, Actions



Component Details

Component	Purpose
Expression Encoder	Encode expression + target
Substitution Encoder	Encode reduction history
Sector Encoder	Condition on sector
Cross-Attention Scorer	Score actions vs expression

All use 2-layer architectures (Transformer or cross-attention)

Model Hyperparameters

- ▶ Embedding dimension: 256, Attention heads: 4
- ▶ Total parameters: 7,696,709

Part 3: Training

Training Configuration

```
epochs = 30  
batch_size = 256  
learning_rate = 0.0004  
weight_decay = 1e-5  
optimizer = AdamW
```

Training Results

- ▶ **Best checkpoint:** Epoch 22
- ▶ **Validation accuracy:** 90.77% (top-1)
- ▶ **Top-5 accuracy:** ~98%
- ▶ **Training time:** ~800s per epoch on GPU

Training Curves

Key Observations

- ▶ Model converges by epoch 20-25
- ▶ Validation accuracy plateaus at $\sim 91\%$
- ▶ No significant overfitting observed
- ▶ Top-5 accuracy very high ($\sim 98\%$) - beam search can recover from top-1 mistakes

Part 4: Beam Search Inference

Beam Search Algorithm

Core Idea

- ▶ Maintain the **beam width** best states (we use beam width = 20)
- ▶ At each step, model ranks actions; only try the **top few**
- ▶ Expand only those actions (not all valid actions)
- ▶ Keep candidates with **lowest max weight** (model score breaks ties)

Termination

- ▶ Stop when best state contains **only master integrals**
- ▶ Return the reduction path from start to masters

Why Beam Search?

- ▶ Greedy ($k=1$) often gets stuck on hard integrals
- ▶ Full search is exponential - beam search is tractable
- ▶ Model guides which actions to try; weight reduction decides what to keep

Beam Search Optimizations

P1: Equation Caching (~3-10x speedup)

- ▶ IBP equation generation is expensive (sympy operations)
- ▶ Cache `get_raw_equation` results
- ▶ Reuse across beam states with shared history

P2: Batched Inference (~50x speedup)

- ▶ Prepare all action candidates as numpy arrays
- ▶ Single batched forward pass through model
- ▶ Eliminates per-action inference overhead

Combined Effect

- ▶ Per-step time: 10-90s \rightarrow 0.1-5s
- ▶ Makes deep reductions feasible

Hierarchical Reduction Strategy

Algorithm

1. Find highest-level sector with non-master integrals
2. Run beam search to eliminate all non-masters in that sector
3. Move to next highest sector
4. Repeat until only masters remain

Example: $I[2,0,2,0,1,1,0]$

Sector	Level	Steps
53	4	53
52	3	17
49	3	4
37	3	42
21	3	46
...
Total	-	176

Beam Restart Strategy (V11+)

Problem

After weight improvement, beam often contains suboptimal states that will never succeed

Solution: Beam Restart

1. Run beam search until weight improves
2. **Stop and restart** with only the best state
3. Prunes dead ends, enables deeper exploration

Impact

- ▶ Essential for high-weight integrals
- ▶ $I[1,1,1,1,1,1,-3]$: 1,416 steps across 45 sectors
- ▶ $I[3,2,1,3,2,2,-6]$: 46,345 steps across 62 sectors

Part 5: Parallelization

Parallelization Motivation

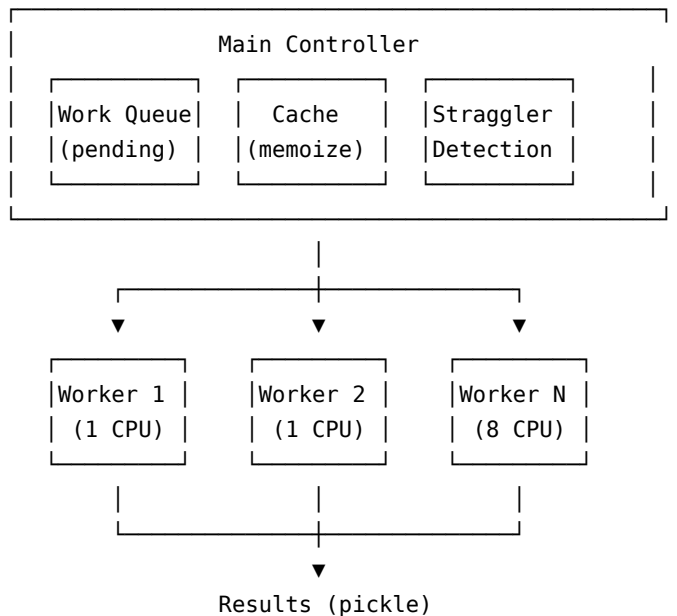
The Bottleneck

- ▶ Sequential reduction of $I[3,2,1,3,2,2,-6]$ takes ~20 hours
- ▶ Single-threaded beam search on CPU
- ▶ Many independent integrals could be processed in parallel

Key Insight

Each one-step reduction is independent - distribute across workers!

Async Parallel Architecture



Key Parallelization Features

1. Async Work Distribution

- ▶ Submit all pending non-masters immediately
- ▶ Don't wait for level synchronization
- ▶ Process results as they arrive

2. Memoization Cache

- ▶ Store: `integral` → reduced expression
- ▶ Avoid redundant work (~55,000 cache hits!)
- ▶ Critical when stragglers produce already-cached results

3. Straggler Detection

- ▶ Jobs >30 min are killed and resubmitted
- ▶ Resubmit with 8 CPUs (parallel beam search)
- ▶ Prevents slow integrals from blocking progress

Parallel Performance

I[3,2,1,3,2,2,-6] Results

Metric	Value
Time (sequential)	~20 hours
Time (parallel)	115 minutes
Speedup	~10x
Jobs submitted	21,096
Stragglers resubmitted	24
Cache hits	55,075
Final masters	16

Part 6: Results

Validation Against Kira

$I[2,0,2,0,1,1,0]$ (Sector 53)

Method	Masters	Result
Kira	4	Reference
Our V5	4	Matches exactly

Our reduction produces the **exact same master basis** as professional IBP software.

Scaling Results

Integral	Weight	Time	Steps	Masters
I[2,0,2,0,1,1,0]	(6,0)	5 min	176	4
I[1,1,1,1,1,1,-3]	(6,3)	12 min*	1,416	16
I[3,2,1,3,2,2,-6]	(13,6)	115 min*	131,769	16

*With parallel execution

Final Reduction Example

$I[3,2,1,3,2,2,-6] \rightarrow 16$ Paper Masters

Final expression (mod 1009):

171 * $I[1,0,1,1,1,1,0]$

854 * $I[1,1,0,1,1,1,0]$

377 * $I[1,1,1,1,1,0,0]$

160 * $I[-1,1,1,1,1,0,0]$

100 * $I[0,1,1,1,1,0,0]$

647 * $I[1,-1,1,0,1,1,0]$

9 * $I[1,-1,1,1,1,0,0]$

... (16 masters total)

Matches arXiv:2502.05121 basis exactly!

Part 7: Conclusions

Key Contributions

1. **ML-guided IBP reduction** that matches professional software (Kira)
2. **Constant memory usage** - avoids the memory wall of traditional approaches
3. **Hierarchical beam search** with restart strategy for deep reductions
4. **Async parallel execution** with $\sim 10\times$ speedup
5. **Straggler handling** for robust distributed computing
6. **Paper-masters-only mode** for clean minimal basis

Technical Innovations

Model

- ▶ Full substitution encoding (V5)
- ▶ Cross-attention action scoring
- ▶ Target-aware expression encoding

Inference

- ▶ Equation caching (3-10x speedup)
- ▶ Batched model inference (50x speedup)
- ▶ Beam restart strategy

Parallelization

- ▶ Async one-step distribution
- ▶ Memoization cache (~55k hits)
- ▶ Automatic straggler resubmission

Future Work

1. **GPU workers** for faster beam search
2. **Adaptive timeouts** based on sector statistics
3. **More integrals** - test on other two-loop families
4. **Training on successful paths** - use reduction results to improve model
5. **Path optimization** - shorten saved reduction paths

Code Availability

Repository: github.com/davidshih17/RL_IBPreduction_claude

Key Files

- ▶ `models/classifier_v5.py` - Model architecture
- ▶ `scripts/eval/hierarchical_reduction_async.py` - Parallel reduction
- ▶ `scripts/eval/reduce_integral_onestep_worker.py` - Condor worker
- ▶ `docs/parallelization.md` - Detailed documentation

Thank You

Questions?

Contact

- ▶ Repository:
`github.com/davidshih17/RL_IBPreduction_claude`
- ▶ arXiv: 2502.05121 (paper masters reference)