PytHHOn3D library

David Siedel

20/08/2020

1 Introduction

PytHHOn3D provides a generic implementation of the HHO method for simple elliptic problems (*i.e.* linear elsaticity in the context of solid mechanics). The model problem that PytHHOn3D adresses reads : find $\boldsymbol{u}$ such that

$$\begin{aligned} &\int_\Omega \nabla \boldsymbol{v} : \boldsymbol{A} : \nabla \boldsymbol{u} = \int_\Omega \boldsymbol{v} \cdot \boldsymbol{f} + \int_{\partial_N \Omega} \boldsymbol{v} \cdot \boldsymbol{t} && \text{in } \Omega \\ &\boldsymbol{u} = \boldsymbol{u_D} && \text{on } \partial_D \Omega \end{aligned} \tag{1}$$

The problem then depends on the tangent operator $\boldsymbol{A}$, on the volumetric load $\boldsymbol{f}$, on the Neumann boundary condtion $\boldsymbol{t}$ on $\partial_N \Omega$, on the Dirichlet boundary condition $\boldsymbol{u_D}$ on $\partial_D \Omega$, that are the arguments passed to the method to define the problem to solve.

2 Illustration

Let the unit square $\Omega = [0,1] \times [0,1]$, $\Gamma_0 = \{(x,y) \in \Omega \mid x = 0\}$ and $\Gamma_1 = \{(x,y) \in \Omega \mid x = 1\}$. Let the volumetric load $\boldsymbol{f} : (x,y) \mapsto -10$, the Dirichlet Boundary conditions $\boldsymbol{u_{\Gamma_0}} : x \mapsto 0$ on Γ_0 and $\boldsymbol{u_{\Gamma_1}} : x \mapsto 1$ on Γ_1, and the tangent operator $\boldsymbol{A} = \mathbf{1}$. the problem reads : find $\boldsymbol{u}$ such that

$$\begin{aligned} &\int_\Omega \nabla \boldsymbol{v} : \boldsymbol{A} : \nabla \boldsymbol{u} = \int_\Omega \boldsymbol{v} \cdot \boldsymbol{f} && \text{in } \Omega \\ &\boldsymbol{u} = \boldsymbol{u_{\Gamma_0}} && \text{on } \Gamma_0 \\ &\boldsymbol{u} = \boldsymbol{u_{\Gamma_1}} && \text{on } \Gamma_1 \end{aligned} \tag{2}$$

The solution to (2) is :

$$\boldsymbol{u} : (x,y) \mapsto 6x - 5x^2 \tag{3}$$

Comparison between the exact solution and the computed one are illustrated in Figure 2 and in Figure 3 :

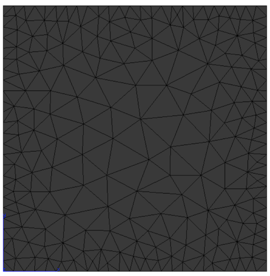

Figure 1: The mesh

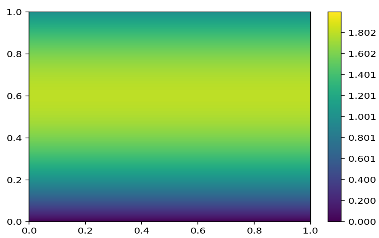

Figure 2: Analytical field

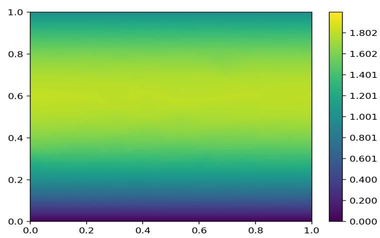

Figure 3: HHO field