

0.1 January 21, 2026

Plan:

- Translate section 2 of Svidzinsky
- Establish a firm ground of accepted facts from which the BdG equations will follow.
- Understand how the approximation of the interaction is made.

Evaluation: I did find a way to translate section 2 of Svidzinsky, but the "accepted facts list" I generated needs improvement, and I did not get to bullet 3.

0.2 January 22, 2026

Plan:

- Do a better job identifying and understanding the necessary foundational material needed to understand the BdG derivation

0.2.1 Fock space and Field operators

Physical meaning. $\mathcal{H}_1^{\wedge N}$ is the Hilbert space describing N identical fermions built from a one-particle Hilbert space \mathcal{H}_1 . A vector in $\mathcal{H}_1^{\wedge N}$ encodes all probability amplitudes for finding the N fermions in arbitrary one-particle states, with the fermionic exchange sign built in. Fermionic Fock space collects all such N -particle sectors into a single Hilbert space.

Mathematical definitions.

1. **Permutation operators.** Let \mathcal{H}_1 be a complex separable Hilbert space and let $\mathcal{H}_1^{\otimes N}$ be its N -fold tensor product. For each permutation $\pi \in S_N$, define a linear operator

$$P_\pi : \mathcal{H}_1^{\otimes N} \rightarrow \mathcal{H}_1^{\otimes N}$$

by its action on simple tensors,

$$P_\pi(\phi_1 \otimes \cdots \otimes \phi_N) = \phi_{\pi^{-1}(1)} \otimes \cdots \otimes \phi_{\pi^{-1}(N)},$$

extended by linearity and continuity.

2. **Antisymmetrizer and its range.** Define the antisymmetrization operator

$$A_N \equiv \frac{1}{N!} \sum_{\pi \in S_N} \text{sgn}(\pi) P_\pi.$$

The range of A_N , denoted $\text{Ran}(A_N)$, is

$$\text{Ran}(A_N) \equiv \{ A_N \Phi \mid \Phi \in \mathcal{H}_1^{\otimes N} \}.$$

Since $A_N^2 = A_N$ and $A_N^\dagger = A_N$, $\text{Ran}(A_N)$ is a closed subspace of $\mathcal{H}_1^{\otimes N}$.

3. **Fermionic N -particle Hilbert space.** The fermionic N -particle Hilbert space is defined as

$$\mathcal{H}_1^{\wedge N} \equiv \text{Ran}(A_N).$$

Equivalently,

$$\mathcal{H}_1^{\wedge N} = \left\{ \Psi \in \mathcal{H}_1^{\otimes N} \mid P_\pi \Psi = \text{sgn}(\pi) \Psi \text{ for all } \pi \in S_N \right\}.$$

4. **Combining one-particle states.** Given $\phi_1, \dots, \phi_N \in \mathcal{H}_1$, the corresponding fermionic N -particle state is

$$\phi_1 \wedge \cdots \wedge \phi_N \equiv A_N(\phi_1 \otimes \cdots \otimes \phi_N).$$

This vector lies in $\mathcal{H}_1^{\wedge N}$ and changes sign under exchange of any two factors.

5. **Meaning of an N -particle state.** For any $\Psi \in \mathcal{H}_1^{\wedge N}$ and any $\chi_1, \dots, \chi_N \in \mathcal{H}_1$, the complex number

$$\langle \chi_1 \wedge \cdots \wedge \chi_N, \Psi \rangle$$

is interpreted as the probability amplitude that the N fermions occupy the one-particle states χ_1, \dots, χ_N .

6. **Algebraic sum and direct sum.** Consider the algebraic sum

$$\sum_{N=0}^{\infty} \mathcal{H}_1^{\wedge N} = \left\{ \sum_{k=1}^m \Psi^{(N_k)} \mid \Psi^{(N_k)} \in \mathcal{H}_1^{\wedge N_k}, m < \infty \right\}.$$

This sum is direct because

$$\mathcal{H}_1^{\wedge N} \cap \mathcal{H}_1^{\wedge M} = \{0\} \quad \text{for } N \neq M.$$

7. **Fermionic Fock space.** The fermionic Fock space is the Hilbert-space completion of this direct sum:

$$\mathcal{F} \equiv \bigoplus_{N=0}^{\infty} \mathcal{H}_1^{\wedge N},$$

with inner product

$$\langle \Psi, \Phi \rangle = \sum_{N=0}^{\infty} \langle \Psi^{(N)}, \Phi^{(N)} \rangle_{\mathcal{H}_1^{\wedge N}}, \quad \Psi = \bigoplus_N \Psi^{(N)}, \quad \Phi = \bigoplus_N \Phi^{(N)}.$$

0.2.2 Understanding initial interaction equation

What is the approximatio

$$H_{\text{int}} = g \int \psi_{\uparrow}^{+}(\mathbf{r}) \psi_{\downarrow}^{+}(\mathbf{r}) \psi_{\downarrow}(\mathbf{r}) \psi_{\uparrow}(\mathbf{r}) d\mathbf{r}. \quad (2.3)$$