Monetary Unions in a Political Economy Model of Fiscal and Monetary Policy Interaction

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Abstract This paper explores the consequences of a monetary union that pairs multiple independent fiscal authorities microfounded by the political economy model of Battaglini and Coate (2008) with a single monetary authority. The model explains Greece's experience with the Eurozone. The fiscal decision-making units in the monetary union are fiscal authorities controlled by self-interested political coalitions rather than welfare-maximizing countries. Joining a monetary union, a choice by the political coalition, is beneficial for the political coalition, but may not be for the country. Membership in a monetary union allows a fiscal authority to issue excess debt that benefits the coalition but is detrimental to the country. The excess nature of the debt will be evident only after a significant negative shock. Repaying the debt then requires excessively high taxation hence has adverse welfare consequences. A monetary union with heterogenous countries provides extra freedom to the political coalition.

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1 Introduction

Monetary unions suffer from a free rider problem: countries issue debt to maximize their own welfare, ignoring the externality the debt creates for the monetary union as a whole. In the real world, a country's fiscal authority is controlled by a political coalition which may put its own interest above the country's. This paper considers politically controlled fiscal authorities as in the political economy model of Barseghyan et al. (2013) and Miller (2016) to be the base unit inside a monetary union, exposing a new layer to the free rider problem: membership in a monetary union provides cover for the political coalition in charge of a country's fiscal authority to issue debt that is beneficial to the coalition but detrimental to the country. A monetary union composed of heterogenous countries exacerbates this problem by allowing the political fiscal authority more budgetary freedom.

While a monetary union provides a benefit by allowing the self-interested coalition to issue debt to smooth shocks and taxes, it comes with the cost that other debt may be used solely to benefit the coalition. The model predicts that a political coalition who uses a fiscal authority inside a monetary union to issue debt to enrich itself will be exposed only after a negative productivity shock. This ability to hide excess debt explains why Greece's creditors failed to recognize its precarious debt situation until after a recession began. A political party with fiscal control will issue debt solely to reward itself while setting the tax rate to a low value. To outsiders observing debt issuance and the low tax rate, it's not obvious that the revenue from debt is being used for wasteful purposes. If the economy experiences a good productivity shock, there will be enough tax revenue for the politically controlled fiscal authority to repay the bonds. If the economy experiences a bad productivity shock, there won't be enough tax revenue to repay the bonds so the excess borrowing will be revealed as the tax rate will need to rise to a punishing level.

Viewing the political coalition inside a country to be the decision-maker rather than the country as a whole provides new intuition as to why heterogenous countries would join a monetary union, how countries in the union benefit from the heterogeneity, the consequences of imperfect monitoring and the need for fiscal rules. The key is that the goal of the political coalition, to maximize the utility of a subset of the country's citizens, diverge from that of the country as a whole, to maximize the utility of all the country's citizens. Joining a monetary union replaces the domestic monetary authority with the monetary authority of the monetary union. The choice to join is a decision by the coalition in charge of the fiscal authority that must make the political coalition better off in expectation, not necessarily the country. Similarly the benefits of being in a monetary union of heterogenous countries accrue to the political party rather than the country.

The classic reason to join a monetary union is to gain the inflation fighting reputation, and commitment, of the union's monetary authority. If the politically distorted fiscal authority decides to join a monetary union, it outsources the monetary authority. The new monetary authority would be independent from the coalition in power in the country. An independent monetary authority knows that inflating away the entire real value of nominal bonds will give the politically distorted fiscal authority budgetary freedom to spend revenue on wasteful transfers rather than on public goods. Maintaining positive nominal debt will constrain wasteful spending, but if debt is too high it will require high distortionary taxes to pay off. Thus the split between the aims of monetary union's independent monetary authority and the politically distorted aims of the country's fiscal authority anchors inflation expectations: the independent monetary authority will inflate away some of the debt, so that taxes will be lower, but not all of the debt, so that the fiscal authority is still constrained in its spending decisions. The remaining debt will be enough to prevent the country's fiscal authority from spending revenue wastefully but enough to provide some tax smoothing against a productivity shock.

A country which has its own independent monetary authority doesn't need to join a monetary union. The choice to do so highlights the difference between the choice of a country and the choice of the political coalition in the country. On its own, such a country is already able to raise revenue from nominal bonds. In expectation there is no gain to the country's welfare from joining a monetary union in order to replace an already independent monetary authority with another. However, there is a gain for the political coalition in the country: unexpected inflation caused by the monetary union's monetary authority reacting to fiscal decisions in other countries in the monetary union can free revenues the coalition will use to reward itself. Thus the political coalition would want to form a monetary union with other, heterogenous, countries while the country as a whole would be, at best, ambivalent.

If a country's own monetary authority has been captured by the coalition in charge of the fiscal authority it will be unable to raise revenue from nominal debt. This is due to the time inconsistency problem of nominal debt: a captured monetary authority will inflate away the real value of nominal debt at the start of every period, in order to free revenue for the political coalition to use to reward itself. Anticipating this inflation, consumers won't hold nominal bonds whose real value will disappear. Replacing a native captured monetary authority with the monetary union's independent monetary authority allows the government to raise revenue to smooth taxes between periods.

Ideally, the independent monetary authority of the monetary union has full knowledge of the fiscal policies of the countries in the union and will use the threat of inflation to keep the politically controlled fiscal authorities in check. Perfect knowledge of fiscal policies may not be possible: a politically distorted fiscal authority may misreport the amount of nominal debt it has issued, the monetary authority may not pay proper attention to the behavior of the fiscal authority, or the monetary authority may simply not respond to a country's fiscal decisions. If any of these are true, the monetary authority won't properly use inflation to control the country's nominal debt. The fiscal authority will be able to issue debt that appears nominal but the monetary authority won't inflate away. Effectively, the debt is indexed. A politically distorted fiscal authority with the ability to issue indexed debt will do so beyond the amount that is optimal for the country.

The revenue from the excess indexed debt will be used by the coalition to fund transfers to itself today at the cost of higher taxes in the future. As long as the realization of the country's productivity shock is high enough, a continued low tax rate will generate sufficient revenue to hide the excessive debt. If the productivity shock is low, the true nature of the debt will be revealed. The country will have to endure a punishingly high tax rate in order to raise sufficient revenue to repay the indexed bonds since it will be unable to use inflation to lessen its debt burden.

I illustrate the relevance of the model by analyzing Greece's experience with the Eurozone. Prior to joining the Eurozone Greece faced high interest rates on its debt. Joining the Eurozone enabled Greece to issue debt at lower interest rates because consumers were reassured that the monetary authority would not inflate away the nominal debt. The party in power in Greece, through a combination of misreporting debt, obstinacy by the monetary authority, and a lack of enforcement of fiscal rules, was able to issue excess debt whose revenue was used wastefully. While Greece was able to repay the debt during years of expansion, the Great Recession ended its ability to do so.

This paper unites two strands of literature: first, interaction between monetary and fiscal authorities with political economy elements at the individual country level, and second, the problems specifically of monetary unions.

In the first literature, dealing with single countries, the most similar paper is Miller (2016) which uses the same political economy set-up but applied to the interaction between a single monetary and a single fiscal authority. The value of a conservative central banker to overcome time inconsistency in monetary and fiscal interaction as in Rogoff (1985) is central to Adam and Billi (2014) and Niemann (2011) which view the interaction without political economy frictions in a more standard macro model. Adam and Billi (2014) does not feature explicit debt, though inflation expectations serve a similar purpose.

Niemann (2011) does not feature distortionary taxation which is an important limiting force in this paper. These papers have an exogenously conservative central banker, this paper has endogenous political economy frictions that split the aims of the fiscal and monetary authority.

In the second literature, dealing with monetary unions, Dixit and Lambertini (2003) expands the single country model of Dixit and Lambertini (2001) to monetary unions in order to look at output versus inflation tradeoffs without the dynamics of debt and spending present in this paper. Aguiar et al. (2015) shows the value of heterogenous countries in a monetary union without the endogenous spending decisions present here. In that paper, heterogenous countries are valuable because they allow the monetary authority to be irresponsible at times. In this paper, heterogenous countries are valuable because they give the political fiscal authorities more freedom to reward themselves. Cooper et al. (2010) and Cooper et al. (2014) highlight the free rider problem of monetary unions created by the externalities of individual countries issuing their own debt (and other fiscal decisions) without a full political economy model. Chari and Kehoe (2008) attempts to solve these free rider problems through commitment, and analogous fiscal rules. This paper does not study commitment, though finds similar benefits to fiscal rules to constrain the politically controlled fiscal authority.

2 The Model

I first describe a single country in the model. Then I describe a monetary union composed of two such countries, denoted by subscripts a, b. Each country operates independently and will only interact through a common price level for their nominal bonds.

Nominal government debt, when sustainable, links periods. Fiscal policy consists of setting taxes, expenditure on a public good, direct transfers to citizens, and nominal bond issuance. The timing in a period is as follows: a real shock determines wages (and the distortion due to taxes) at the beginning of every period. After the shock, the monetary authority sets the price level then the fiscal authority chooses its policy. Figure 1 illustrates the sequence of decisions in a period.

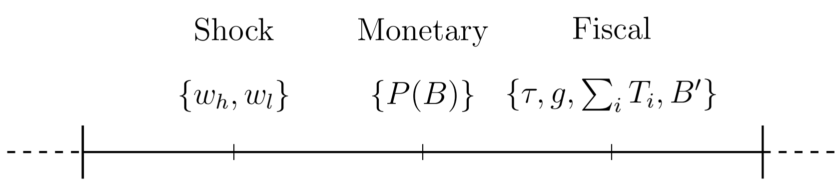

Figure 1: Timing of Monetary and Fiscal Decisions for a Country

2.1 Consumers

There are n identical consumers, indexed by i when necessary. A consumer's per period utility function is

$$u(c, g, l) = c + A\log(g) - \frac{l^{1+1/\epsilon}}{\epsilon + 1} \tag{1}$$

and an individual seeks to maximize $U = \sum_t \beta^t u(c_t, g_t, l_t)$ where c is a consumption good, g is government spending on a public good, l is labor, and β the discount rate. The parameter $\epsilon > 0$ is the Frisch elasticity of labor supply. A is a parameter allowing adjustment of the utility of government spending on the public good. Utility linear in consumption is used to eliminate wealth effects to preserve consumers homogeneity for the political process and simplify the interest rate.[1]

A representative consumer i in period t faces the budget constraint

$$c + qB_i' \leq w_\theta l(1-\tau) + \frac{B_i}{P(B)} + T_i \tag{2}$$

Variables without a prime refer to variables in period t while variables with a prime refer to variables in period $t+1$. The consumer can consume c or purchase nominal bonds B_i' at a price q where each bond pays a nominal unit of income in the next period. $B = \sum_i B_i$ is the total number of bonds in the economy[2].

The consumer's income consists of labor income at wage w_θ that is taxed by the government at the distortionary tax rate $0 \leq \tau \leq 1$ together with direct transfers $T_i \geq 0$ from the government. $P(B)$ is the price level determined by the monetary authority at the start of the period as a function of the number of bonds.

The price level in the current period will generally be abbreviated as $P = P(B)$ and the price level in the next period as $P' = P(B')$. The price level is

[2]With identical consumers, the symmetric equilibrium examined in the paper induces all consumers to hold identical bonds $B_i = \frac{B}{n}$. The choices of fiscal and monetary authorities will only depend on aggregate bonds.

not an intertemporal variable. The ratio of current to next period price level $\frac{P}{P'}$ determines the real return on bonds. For simplicity I normalize this ratio by making bonds pay 1 nominal unit of income in the next period. Thus the real value of bonds will only depend on P' which is set independently every period.

The consumer's budget constraint and linear utility imply the equilibrium bond price

$$q(B') = \beta E_{\theta'} \left[\frac{1}{P'(B')} \right] \tag{3}$$

where the expectation is over possible realizations of the wage shock in the next period.

A consumer's utility is defined entirely by the government's choices of taxation τ and public good spending g. Deriving the optimal amount of labor as a function of the tax rate τ shows

$$l^*_\theta(\tau) = (\epsilon w_\theta (1 - \tau))^\epsilon \tag{4}$$

Plugging this into the consumer's utility function shows the indirect utility function before transfers is

$$W_\theta(\tau, g) = \frac{\epsilon^\epsilon (w_\theta (1 - \tau))^{\epsilon+1}}{\epsilon + 1} + A \log(g) \tag{5}$$

2.2 Firms

The representative firm has a linear production technology

$$z = w_\theta l \tag{6}$$

used to produce an intermediate good z at wage w_θ with labor l. At the beginning of each period an i.i.d. technology shock hits the economy such that wages $w_\theta \in \{w_l, w_h\}$ where $w_l < w_h$. The probability that $w_\theta = w_h$ is π, the probability that $w_\theta = w_l$ is $1 - \pi$.

The intermediate good z is split costlessly between the consumption good c and the public good g such that

$$c + g = z. \tag{7}$$

This defines the per period resource constraint

$$c + g = w_\theta l. \tag{8}$$

2.3 Government

The government controls fiscal policy. Raising revenue is possible via a distortionary labor tax τ and by selling nominal bonds B'. A positive bond level means the government is in debt hence owes revenue to consumers. The government can spend revenue on a public good g that benefits all n citizens or on non-negative transfer payments T_i that benefit individuals. It must also repay nominal bonds $\frac{B}{P}$.

The government's budget constraint is

$$g + \sum_i T_i + \frac{B}{P} \leq \text{Rev}_\theta(\tau) + qB' \tag{9}$$

where

$$\text{Rev}_\theta(\tau) = n\tau w_\theta(\epsilon w_\theta(1-\tau))^\epsilon \tag{10}$$

is the total tax revenue raised by the distortionary labor tax on all n consumers.

Define the budget surplus before transfers as

$$S_\theta(\tau, g, B'; \frac{B}{P}) = \text{Rev}_\theta(\tau) + qB' - g - \frac{B}{P}. \tag{11}$$

The surplus must be large enough to pay for any transfers hence $S_\theta(\tau, g, B'; \frac{B}{P}) \geq \sum_i T_i$. Transfers themselves must be non-negative: $\forall i \; T_i \geq 0$.

There are endogenous limits to the amount of bonds the government can issue. The upper bound on debt is defined as the maximum amount of bonds the government is able to repay in the case of the bad realization of the wage shock w_l if it spends nothing on the public good and transfers. Define the upper bound $\overline{B}$ as $\overline{B} = \frac{\max_\tau \text{Rev}_l(\tau)}{q(\overline{B})}$.

The lower bound on debt is the amount of bonds such that revenue from the bonds would be sufficient to fund optimal public good spending without utilizing the distortionary labor tax. The optimal amount of public good spending is g^s such that $\frac{nA}{g^s} = 1$. This equation equates the declining marginal benefit of providing the public good with the opportunity cost to consumers of consuming the revenue directly with linear utility. Define $\underline{B}$ as $\underline{B} = -\frac{nA}{q(\underline{B})}$ the level of bonds where one more unit of government spending has the same marginal utility as individual consumption. This is the Samuelson level of bonds such that the government can finance g^s directly from the bonds.

The Samuelson level of bonds $\underline{B}$ is the bond level where the government is entirely funded by interest from bonds, all future distortionary labor taxes are zero, and government spending is constant at the optimum.

2.3.1 Self-Interested Fiscal Policy

Fiscal policy decisions will be made either by a benevolent fiscal authority or a self-interested fiscal authority. A benevolent fiscal authority attempts to maximize the welfare of all citizens. A self-interested fiscal authority attempts to maximize the utility of a subgroup of the citizenry. In this section, I provide an overview of the political equilibrium I will be examining the model. A more precise description is included in the analysis of the behavior of the self-interested fiscal authority in Section 3.1.

Following the political system laid out in Battaglini and Coate (2008) who extend the political economy model of Baron and Ferejohn (1989), citizens vote each period to decide that period's fiscal policy $\{\tau, g, B, \{T_i\}_1^n\}$. In each period there are T rounds of voting to determine fiscal policy. Each round of voting starts with one citizen being randomly assigned the power to propose a choice of fiscal policy. The proposer puts forward his policy choices of $\{\tau, g, B, \{T_i\}_1^n\}$. The proposal is enacted if $m \leq n$ citizens vote for it. If enacted, this ends the voting for that period, a new round will begin next period. If the proposal fails the voting round ends and a new round begins with a new randomly selected proposer. There can be a maximum of T proposal rounds after which a dictator is appointed. The dictator chooses policies unilaterally with the constraint that all transfers T_i must be equal. A fiscal policy proposal defines the fiscal policy for a single period. The next period a new proposer is randomly selected and the process begins anew. Fiscal commitment across periods possible due to the design of the political system.

I focus on a symmetric Markov-perfect equilibrium. These are proposals that depend only on current productivity and real debt $\{w_\theta, \frac{B}{P}\}$ where $\theta \in \{h, l\}$. The proposals are independent of both the history of the economy and proposal round. Thus we only need to examine the proposal in the first round.

In order for a proposal to be accepted, the proposal must make the members of the m coalition as well off as the expectation of waiting a round for the next proposal. In practical terms, proposers will propose fiscal instruments to maximize the utility of the m citizens in the coalition without care for non-coalition citizens. This is in contrast to the choices of a benevolent fiscal authority which will maximize the welfare of all n citizens.

2.4 Monetary Authority

The monetary authority chooses the price level P. Inflation is costless. Choosing P directly is equivalent to the monetary authority controlling the interest rate on nominal bonds, as the monetary authority would do in the cashless limit via infinitely small open market operations that result in zero seignorage. Additionally, moving P rather than operating through money holdings simplifies the model by allowing us to consider the simple indirect utility function with a correspondingly simple interest rate, and not split money and bond holdings.

The model utilizes timing akin to a Stackelberg game with the monetary authority as leader and the government as follower. The monetary authority chooses the price level P after the shock in each period. Since P determines the payout to bonds, choosing P is equivalent to choosing the interest rate on those bonds. Thus monetary policy controls the real value of government debt which is equivalent to consumer wealth. After the monetary authority moves, the fiscal authority chooses its fiscal instruments. See Figure 1.

The monetary authority lacks commitment. Each period the monetary authority chooses the price level for that period only and cannot credibly promise what it will do in the future. Specifically, I constrain the monetary authority to choose the price level solely as a function of its information set $\{B, w_\theta\}$ for a single country or $\{B_a, B_b, w_{\theta_a}, w_{\theta_b}\}$ for a monetary union at the beginning of a period. This is a consequence of the monetary authority's lack of commitment, and narrows the space of possible equilibria.

2.5 Monetary Unions

A monetary union is composed of two countries each with its consumers, firm and self-interested fiscal authority. Each fiscal authority will issue its own nominal bonds and will be subject to its own shocks. When discussing variables inside a country I will denote the variables by subscripts a, b respectively. For example, B_a will be the amount of nominal bonds issued by country a and

B_b will be the amount of nominal bonds issued by country b. The shocks w_{θ_a} and w_{θ_b} will be independent of each other though occurring with identical probabilities $\pi, 1-\pi$ for simplicity.

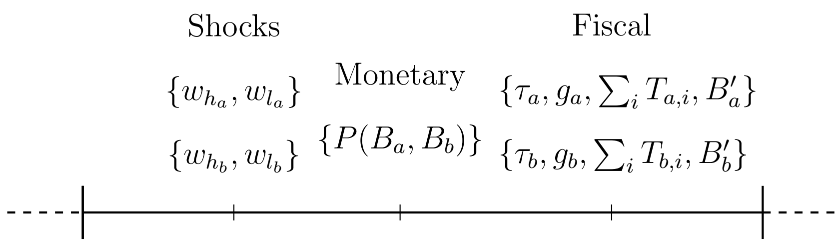

Figure 2: Timing of Monetary and Fiscal Decisions in a Monetary Union

The timing is analogous to the one country case. At the beginning of the period each country faces an independent, idiosyncratic shock that only it knows. The monetary authority observes both shocks, then chooses the price level P based on its information set $\{B_a, B_b, w_{\theta_a}, w_{\theta_b}\}$. After that, the fiscal authority in each country chooses its fiscal policies independently and simultaneously. Importantly, neither fiscal authority has information about the shock and decisions of the other.

3 Model Analysis

I first use the model to illustrate the two cases for a single country prior to joining a monetary union. The self-interested fiscal authority is controlled by politics, and the monetary authority is either independent (benevolent with respect to welfare), or captured (identical goals to the fiscal authority).

Figure 3: Single Country Monetary Authority Analysis

	Monetary Authority	
	Captured	Independent
	Claim 2, Bonds = 0, No tax smoothing	Claim 1, Bonds ≥ 0, Tax smoothing

A self-interested fiscal authority with an independent monetary authority is the optimal state, able to issue nominal bonds and smooth taxes as in Miller (2016). A self-interested fiscal authority with a captured monetary authority is the classic situation of a prospective country that seeks to join a monetary union. The fiscal authority will be unable to issue any nominal bonds due to the time consistency problem of nominal debt. In order to issue nominal debt, the country will need to replace its monetary authority with one that is independent from the fiscal authority. The country, and the political coalition, can do this by joining a monetary union.

After describing the two single country cases, I analyze the possibilities for two countries, a and b, in a monetary union. Each country has its own self-interested fiscal authority and shares a single monetary authority. The monetary authority's decisions will be a function of the amount of nominal bonds B_a issued by both country a and the amount of nominal bonds B_b issued by country b as well as the respective shocks w_{θ_a} and w_{θ_b}

3.1 Single Country Self-Interested Authority's Problem

To condense the description of very similar topics, I illustrate the problem of a single country's self-interested fiscal authority for both cases: when only the fiscal authority is self-interested and when both the fiscal and monetary authority are self-interested and share a coalition because the monetary authority has been captured by the fiscal authority.

Following the outline of Barseghyan et al. (2013) and Miller (2016), I focus on a symmetric Markov-perfect equilibrium for each potential structure. These are proposals that depend on current productivity and debt $\{w_\theta, B\}$. The proposals are independent of both the history of the economy and proposal round. A citizen will vote for a proposal if it makes him at least as well off as waiting for the next proposal round will. Hence a proposer will propose instruments that make citizens indifferent between voting for a proposal and waiting for the next round. I choose an equilibrium where proposals in each round are voted for by the necessary $m-1$ citizens (and the proposer). This means that the equilibrium path consists of a single round with a single proposal that is voted for by the necessary citizens.

The equilibrium is a set of policy proposals for each round $r \in \{1, \ldots, T\}$ of fiscal instruments $\{\tau^r, g^r, B'^r, T_i^r\}_1^T$ if the fiscal authority is self-interested, or $\{P^r, \tau^r, g^r, B'^r, T_i^r\}_1^T$ if both fiscal and monetary authorities are self interested. The transfers will be used by the proposer to convince a random group of $m-1$ other citizens to support the proposal. Revenue not spent on transfers or public good spending is the effective transfer to the proposer. An equilibrium defines a value function $v_\theta^r(B)$ for each round representing the expected continuation payoff value for a citizen. The last value function $v_\theta^{T+1}(B)$ is the result of the default proposal by the dictator appointed after round T.

Given a set of value functions $\{v_\theta^r(B)\}_{r=1}^{T+1}$ the policy proposals must satisfy the proposer's maximization problem. Similarly the policy proposals define the optimal value functions. I start with the first relationship. Since the first proposal in the first round is accepted, I drop the r superscripts for simplicity. Formally, given the value functions, we can write the optimization problem for the policy proposals.

The two possibilities of self-interested authorities will differ in the goal of each authority. The model's economy remains the same. If only the fiscal authority is self-interested, the optimization problem is

$$\max_P \left[\begin{array}{c} \max\limits_{\tau,g,B',T_i} W_\theta(\tau,g) + S_\theta(\tau,g,B';\dfrac{B}{P}) - (m-1)T_i + \beta\left[\pi v_H(B') + (1-\pi)v_L(B')\right] \\ \text{s.t.} \begin{array}{l} W_\theta(\tau,g) + T_i + \beta\left[\pi v_H(B') + (1-\pi)v_L(B')\right] \geq v_\theta^{r+1}(B) \\ T_i \geq 0 \;\forall i, S_\theta(\tau,g,B';\dfrac{B}{P}) \geq (m-1)T_i \end{array} \end{array} \right] \tag{12}$$

The first constraint is the incentive compatibility constraint that states the proposal must make those citizens receiving a transfer at least as well off in expectation as they would be if they waited for the next proposal round. The other constraints force the proposal to be feasible given government's budget constraint.

Similarly, for the situation where the monetary authority is captured, the optimization problem is

$$\max_{P,\tau,g,B',T_i} \left[\begin{array}{cc} \multicolumn{2}{c}{W_\theta(\tau,g) + S_\theta(\tau,g,B';\dfrac{B}{P}) - (m-1)T_i + \beta\left[\pi v_H\left(B'\right) + (1-\pi)v_L\left(B'\right)\right]} \\ \text{s.t.} & \begin{array}{l} W_\theta(\tau,g) + T_i + \beta\left[\pi v_H\left(B'\right) + (1-\pi)v_L\left(B'\right)\right] \geq v_\theta^{r+1}(B) \\ T_i \geq 0\ \forall i, S_\theta(\tau,g,B';\dfrac{B}{P}) \geq (m-1)T_i \end{array} \end{array} \right] \tag{13}$$

Given policy choices, the value functions are determined by

$$v_\theta(B) = \max_{\tau,g,B',T_i} \left[W_\theta(\tau,g) + \frac{S_\theta(\tau,g,B';\frac{B}{P})}{n} + \beta\left[\pi v_H\left(B'\right) + (1-\pi)v_L\left(B'\right)\right] \right] \tag{14}$$

in the case where only the fiscal authority is self-interested and the monetary authority indepedent, and

$$v_\theta(B) = W_\theta(\tau,g) + \frac{S_\theta(\tau,g,B';\frac{B}{P})}{n} + \beta\left[\pi v_H\left(B'\right) + (1-\pi)v_L\left(B'\right)\right] \tag{15}$$

in the case where the monetary authority is captured. These expressions come from the three possibilities for a citizen in a proposal round. With probability $\frac{1}{n}$ a citizen is the proposer and thus receives the surplus after transfers. With probability $\frac{m-1}{n}$ a citizen is not the proposer but is a member of the randomly selected coalition that votes for the proposal and thus receives the transfer T_i. With probability $\frac{n-m}{n}$ a citizen is not in the proposer's coalition and receives no transfer. Since utility is quasilinear, the expected utility in a round is the payoff multiplied by the probability.

Because the first proposal is accepted in each round and all proposals will be identical, the value functions will be identical for every proposal round

$r \in \{1, \ldots, T\}$. If the round T proposal is rejected, the round $T+1$ dictator's proposal will result in the value function

$$v_\theta^{T+1}(B) = \max_P \left[\max_{\tau,g,B'} W_\theta(\tau,g) + \frac{S_\theta(\tau,g,B';\frac{B}{P})}{n} + \beta\left[\pi v_H\left(B'\right) + (1-\pi)v_L\left(B'\right)\right] \right] \tag{16}$$

if only the fiscal authority is self-interested, and

$$v_\theta^{T+1}(B) = \max_{P,\tau,g,B'} \left[W_\theta(\tau,g) + \frac{S_\theta(\tau,g,B';\frac{B}{P})}{n} + \beta\left[\pi v_H\left(B'\right) + (1-\pi)v_L\left(B'\right)\right] \right] \tag{17}$$

if the monetary authority is captured, subject to uniform transfers and the same feasibility constraints as before.

I characterize the equilibrium in each of the structures in the next subsections, and prove their existence in the appendix.

3.2 Characterization of Single Country Problems

The problem of a self-interested fiscal authority and independent monetary authority, as seen in Miller (2016), is

$$v_\theta(B) = \max_P \left[\begin{array}{r} \displaystyle\max_{\tau,g,B',\{T_i\}_1^n} W_\theta(\tau,g) + \frac{\sum_i T_i}{m} + \beta\left[\pi v_H\left(B'\right) + (1-\pi)v_L\left(B'\right)\right] \\ \text{s.t. } T_i \geq 0\ \forall i, S_\theta(\tau,g,B';\dfrac{B}{P}) \geq \displaystyle\sum_i^n T_i \end{array} \right] \tag{18}$$

while the problem of a self-interested fiscal authority and captured monetary authority is

$$v_\theta(B) = \max_{P,\tau,g,B',\{T_i\}_1^n} \begin{bmatrix} W_\theta(\tau,g) + \dfrac{\sum_i T_i}{m} + \beta\left[\pi v_H\left(B'\right) + (1-\pi)v_L\left(B'\right)\right] \\ \text{s.t. } \; T_i \geq 0 \; \forall i, S_\theta(\tau,g,B';\dfrac{B}{P}) \geq \displaystyle\sum_i^n T_i \end{bmatrix} \tag{19}$$

I first describe the optimal choices for fiscal variables $\{\tau_\theta(\frac{B}{P}), g_\theta(\frac{B}{P}), B_\theta(\frac{B}{P})\}$ if transfers are zero and if transfers are positive. Then I explain the cutoff level of real bonds C_θ that determines whether those transfers will be zero or positive. Finally, I derive the optimal choice of the price level P by the two types of monetary authority. I show that an independent monetary authority's choice of the price level depends on the cutoff C_θ while a captured monetary authority's choice doesn't depend on C_θ (though the derivative is kinked there).

If transfers are zero, the fiscal authority's first order conditions are

$$\begin{aligned} \frac{1-\tau}{1-\tau(1+\epsilon)} &= \frac{nA}{g} \\ \frac{1-\tau}{1-\tau(1+\epsilon)} &= \frac{-n\beta}{q(B')}\left[\pi v'_H\left(B'\right) + (1-\pi)v'_L\left(B'\right)\right] \end{aligned} \tag{20}$$

The expression $\frac{1-\tau}{1-\tau(1+\epsilon)}$ is the marginal distortionary cost of taxation. The cost is always greater than amount of revenue raised. The first equation equates the marginal cost of raising an additional unit of revenue via taxation with the marginal benefit of spending that revenue on public goods. The second equation equates the marginal cost of raising an additional unit of revenue via taxation with the expected marginal cost of raising the revenue by issuing bonds at the price q (and thus smoothing taxation by pushing the cost into the future).

If transfers are positive, the optimal choices are

$$\begin{aligned}
\frac{n}{m} &= \frac{1-\tau^*}{1-\tau^*(1+\epsilon)} \\
\frac{n}{m} &= \frac{nA}{g^*} \\
B^{'*} &= \arg\max_{B'} \left[\frac{qB'}{m} + \beta\left[\pi v_H\left(B'\right) + (1-\pi) v_L\left(B'\right)\right]\right]
\end{aligned} \tag{21}$$

The left hand side $\frac{n}{m}$ term represents the amount each individual in the coalition that controls both monetary and fiscal policy will receive as a transfer if an additional unit of revenue is raised. The first equation shows the marginal benefit to coalition members from additional revenue is equal to the marginal cost of raising that additional unit by distortionary taxation. The second equation displays the choice of the government to spend revenue: the marginal benefit of transfers to the governing coalition is equal to the marginal benefit from using that revenue on public good spending. The third equation balances the optimal amount of bond revenue used to fund transfers versus the cost of more bonds in the next period.

When transfers are positive the tax rate, government spending, and level of bonds $\{\tau^*, g^*, B^{'*}\}$ will be constant. The government will raise revenue from taxes τ^* and bonds $B^{'*}$. It will spend g^* on the public good. Whatever revenue is left after that spending will be used to fund transfers. Thus there is a cutoff level of bonds

$$C_\theta = \text{Rev}_\theta(\tau^*) + qB'^* - g^* \tag{22}$$

which is the amount of bonds such that $S_\theta(\tau^*, g^*, B'^*; C_\theta) = 0$. If the level of bonds $\frac{B}{P}$ after the monetary authority's choice of P is above C_θ there will be no revenue for transfers. In this region, the optimization problem will be identical to that of the benevolent fiscal authority. If the level of bonds is below C_θ there will be transfers while taxes, public good spending and bond issuance are $\{\tau^*, g^*, B'^*\}$ respectively.

Claim 1 *The solution for a self-interested fiscal authority with an independent monetary authority is to issue C_h bonds that raise $\beta(\pi C_h + (1-\pi)C_l)$ revenue in every period.*

Using our definition of C_θ, we can rewrite the problem of the self-interested fiscal authority with an independent monetary authority as

$$v_\theta(B) = \max_P \left[\begin{array}{c} \max\limits_{\tau,g,B'} W_\theta(\tau, g) + \dfrac{S_\theta(\tau, g, B'; \frac{B}{P})}{n} + \beta\left[\pi v_H(B') + (1-\pi)v_L(B')\right] \\ \text{s.t. } \ \tau \geq \tau^*, g \leq g^*, B' \geq B'^*, S_\theta(\tau, g, B'; \dfrac{B}{P}) \geq 0 \end{array} \right] \tag{23}$$

The new constraints are limits on the lowest taxes, highest government spending and least bonds. If there are transfers, these three variables will equal their starred values.

The independent monetary authority's choice for the price level is

$$P_\theta(B) = \begin{cases} \frac{B}{C_\theta}, & \text{if } B > C_\theta \\ 1, & \text{if } B \leq C_\theta \end{cases} \tag{24}$$

If the amount of nominal bonds is less than the bond cutoff, there are transfers. If the monetary authority were to increase the price level, the amount of debt the self-interested fiscal authority must repay will go down. Revenue the self-interested fiscal authority had directed to bond repayment will instead go to transfers while taxes and government spending remain constant at τ^* and g^*. The total decrease in the real value of nominal bond holdings equals the total increase in transfers. Because utility is quasilinear, overall welfare is unchanged. If the amount of nominal bonds is greater than or equal to the bond cutoff there are no transfers. Increasing the price level results in lower taxes and higher welfare.

The self-interested fiscal authority issues C_h bonds because that level maximizes the amount of revenue raised while holding taxes and government spending at their constant, starred values. The fiscal authority uses the revenue freed by inflating away bonds to perfectly smooth taxes across periods. For any realization of the wage shock and any bond level, taxes will be constant at τ^* and government spending constant at g^*. These are the lowest possible taxes and highest possible spending.

Claim 2 *The solution for a self-interested fiscal authority with a captured monetary authority is to issue 0 bonds to raise 0 revenue in every period.*

The captured monetary authority's choice for the price level is

$$P(B) = \begin{cases} \infty, & \text{if } B > 0 \\ 1, & \text{if } B = 0 \\ 0, & \text{if } B < 0 \end{cases} \tag{25}$$

A self-interested fiscal authority with a captured monetary authority will be unable to raise any revenue from nominal bonds. The captured monetary authority will set the price level to infinity for any positive level of bonds. For a real value of bonds above the cutoff C_θ, increasing the price level will result in decreased taxes and therefore increased welfare. For a real value of bonds below the cutoff C_θ: the coalition controlling the captured monetary authority will derive a welfare benefit from increasing the price level until the entire real value of the nominal bonds is eliminated even though taxes will remain constant.

The benefit exists because the captured monetary authority only cares about the welfare of its coalition. Increasing the price level so the real value of bonds declines below C_θ decreases the real value of bonds and thus the wealth of all citizens, while taxes and government spending are constant at $\{\tau^*, g^*\}$. However, transfers to the coalition will increase, resulting in a net gain in utility for members of the coalition. For each unit of wealth the coalition loses by raising the price level, the coalition will gain $\frac{n}{m}$ units of transfers. Thus a self-interested monetary authority will eliminate the real value of any positive amount of nominal bonds.

3.2.1 Single Country Equilibrium Existence

Definition 1 *An equilibrium is a collection of value functions* $v_\theta(B)$*, fiscal choices* $\tau_\theta(\frac{B}{P}), g_\theta(\frac{B}{P}), B'_\theta(\frac{B}{P})$*, monetary choice* $P_\theta(B)$*, expectations about the interest rate* $q_\theta(B)$ *for* $\theta \in \{h, l\}$*, such that given* $v_\theta(B)$ *and* $q_\theta(B)$*, the fiscal choices solve the fiscal authority's problem and given* $v_\theta(B)$ *and* $q_\theta(B)$*, the monetary choice* $P_\theta(B)$ *solves the monetary authority's problem where given fiscal choices, monetary choice, and* $q_\theta(B)$*,* $v_\theta(B)$ *are optimal.*

We only need the value functions and debt state variable to establish the equilibrium. Everything can be derived, as above, from those.

Claim 3 *If the value functions* $v_H(B'), v_L(B')$ *satisfy Equation 23, and optimal debt satisfies Equation 21, then there is an equilibrium in which* $v_H(B'), v_L(B')$ *are proposed and accepted in the first round of voting, and fiscal choices* $\tau_\theta(\frac{B}{P}), g_\theta(\frac{B}{P}), B'_\theta(\frac{B}{P})$ *and monetary choice* $P_\theta(B)$ *are optimal. Existence is then established by showing the joint optimality of the value functions and fiscal and monetary choices*

Due to linear utility there may be a non-singleton set of price levels P that result in identical welfare. I impose two price selection criteria. First, absent welfare gains, the monetary authority will set the price level to 1. This default price level is a normalization brought on by specifying that bonds return 1 unit of nominal income.

Second, the monetary authority will deviate from $P = 1$ only for positive welfare gains. When the monetary authority determines the price level, it will minimize $|P - 1|$ while maximizing welfare. For a welfare level k the set $\{P \text{ s.t. } v(B) = k\}$ where $v(B)$ is the welfare function may not be a singleton.

As an equilibrium choice, I assume the government always chooses the element of this set that minimizes $|P-1|$. These requirements mimic an aversion to inflation and deflation.

Similarly, there may be a non-singleton set of bond amounts B' that result in the same bond revenue $qB' = \beta E_{\theta'}\left[\frac{1}{P'(B')}\right] B'$. For a revenue level k the set $\{qB = k\}$ may not be a singleton. Amongst all possible choices in this set, I select the equilibrium where the fiscal authority always chooses the minimum element.

3.3 Monetary Union's Problem

A monetary union combines two countries a and b under a single independent monetary authority. Each country has its own self-interested fiscal authority. Each self-interested fiscal authority knows its own shock, e.g. $\{w_{l_a}, w_{h_a}\}$, and does not know the shock that hits the other country. While a fiscal authority could learn about the other country's fiscal authority's decisions in the next period, the equilibrium examined is Markov and does not take utilize historical information.

Due to the information set of each fiscal authority, the problem of a self-interested fiscal authority in a monetary union is very similar to the single country case laid out in Section 3.1. The difference lies in the fiscal authority's expectation of the monetary authority's actions. The monetary union's monetary authority's actions $P_{\{\theta_a,\theta_b\}}(B_a, B_b)$ are a function of the amount of bonds issued by both country a and b, and the shocks in both countries. Thus the corresponding bond price is $q(B'_a, B'_b) = \beta E_{\theta'_a,\theta'_b}\left[\frac{1}{P'(B'_a,B'_b)}\right]$.

3.4 Characterization of Monetary Union Problem

Relying on the model derivation in the single country case, and the simplification in Equation 23, the problem of the monetary authority in a monetary union is

$$v_{\theta_a,\theta_b}(B_a,B_b) = \max_P \left(\begin{bmatrix} \max\limits_{\tau_a,g_a,B'_a} W_{\theta_a}(\tau_a,g_a) + \dfrac{S_{\theta_a}(\tau_a,g_a,B'_a;\frac{B_a}{P})}{n} + \\ \qquad \beta \begin{bmatrix} \pi^2 v_{H,H}(B'_a,B'_b) + \pi(1-\pi)v_{H,L}(B'_a,B'_b) + \\ \pi(1-\pi)v_{L,H}(B'_a,B'_b) + (1-\pi)^2 v_{L,L}(B'_a,B'_b) \end{bmatrix} \\ \qquad \text{s.t.} \begin{array}{l} \tau_a \geq \tau_a^*, g_a \leq g_a^*, B'_a \geq B_a^{'*} \\ S_{\theta_a}(\tau_a,g_a,B'_a;\dfrac{B_a}{P}) \geq 0 \end{array} \end{bmatrix} + \begin{bmatrix} \max\limits_{\tau_b,g_b,B'_b} W_{\theta_b}(\tau_b,g_b) + \dfrac{S_{\theta_b}(\tau_b,g_b,B'_b;\frac{B_b}{P})}{n} + \\ \qquad \beta \begin{bmatrix} \pi^2 v_{H,H}(B'_a,B'_b) + \pi(1-\pi)v_{H,L}(B'_a,B'_b) + \\ \pi(1-\pi)v_{L,H}(B'_a,B'_b) + (1-\pi)^2 v_{L,L}(B'_a,B'_b) \end{bmatrix} \\ \qquad \text{s.t.} \begin{array}{l} \tau_b \geq \tau_b^*, g_b \leq g_b^*, B'_b \geq B_b^{'*} \\ S_{\theta_b}(\tau_b,g_b,B'_b;\dfrac{B_b}{P}) \geq 0 \end{array} \end{bmatrix} \right) \tag{26}$$

The pricing function of the monetary authority will depend on which country deviates more from its respective bond cutoff value $C_{\theta_{\{a,b\}}}$

$$P_{\{\theta_a,\theta_b\}}(B_a,B_b) = \begin{cases} \max\left(\frac{B_a}{C_{\theta_a}}, \frac{B_b}{C_{\theta_b}}\right), & \text{if } B_a > C_{\theta_a} \text{ or } B_b > C_{\theta_b} \\ 1, & \text{if } B_a \leq C_{\theta_a} \text{ and } B_b \leq C_{\theta_b} \end{cases} \tag{27}$$

If the amount of nominal bonds in both countries is less than their respective bond cutoffs ($B_a \leq C_{\theta_a}$ and $B_b \leq C_{\theta_b}$), there will be transfers even if the monetary authority set the price level to 1. If the monetary authority increases the price level, the amount of debt both fiscal authorities must repay goes down. Revenue the fiscal authorities had originally directed to bond repayment will instead go to transfers while taxes and government spending will remain constant at their starred values. The total decrease in the real value of nominal bond holdings is equal to the total increase in transfers in both countries. Because utility is quasilinear it does not matter who gains and who loses: increasing the price level will not change welfare hence the monetary authority will keep set the price level to 1.

If the amount of nominal bonds in one of the countries is greater than or equal to its bond cutoff there will be no transfers in that country and its taxes will be above the minimum τ^*. The monetary authority will erase the real value of bonds by increasing the price level as long as at least one fiscal authority will use the newly freed revenue to decrease taxes rather than increase transfers. By definition this is the case when the amount of nominal bonds in a country is greater than or equal to the bond cutoff in that country. The welfare effect of increasing the price level will be positive in the country where taxes decline and zero in the country where transfers increase (as explained above). Thus the overall welfare effect will be positive though the price level will be set such that one country is below its bond cutoff while the other is equal to its respective bond cutoff.

If both countries have nominal bonds above their respective bond cutoffs, the monetary authority will increase the price level until both countries are at or below their cutoff. The max operator indicates the monetary authority will choose the price level that is necessary to achieve this. If both countries are above their cutoff, increasing the price level decreases taxes in both countries leading to a welfare gain. At some point increasing the price level may drive one country below its cutoff while the other country remains above its own. This situation is discussed above: as long as one of the countries gains lower taxes the monetary authority will increase the price level.

Claim 4 *The self-interested fiscal authority in country a in a monetary union will issue C_{h_a} bonds that will raise*

$$\beta\left[\pi^2 C_{h_a} + \pi(1-\pi)C_{h_a}\left(\frac{C_{l_b}}{C_{h_b}}\right) + (1-\pi)\pi C_{l_a} + (1-\pi)^2 C_{h_a}\left(\min\left[\frac{C_{l_a}}{C_{h_a}}, \frac{C_{l_b}}{C_{h_b}}\right]\right)\right] \tag{28}$$

revenue in every period. Country b is identical, with the obvious corresponding changes.

The amount of revenue is the result of expectations by consumers for the four possible combinations of shocks $\{w_{\theta_a}, w_{\theta_b}\}$, and the monetary authority's reaction to those possiblities.

Consider a one-step deviation where country a issues $B_a > C_{h_a}$ bonds and country b issues $B_b = C_{h_b}$. From the point of view of the monetary authority, increasing the price level will decrease taxes and therefore increase welfare in country a. Since country b issued C_{h_b} bonds, increasing the price level will increase transfers while taxes and public good spending are constant. This means increasing the price level will leave welfare in country b unchanged while overall welfare will be increased. Similarly if each country issues nominal bonds above their respective bond cutoff, the monetary authority will increase the price level until the real value of both countries nominal bonds are at or below their respective cutoffs. Thus, as in Claim 1 for an individual country, for any realization of the wage shock and any bond level, taxes will be constant at τ^* and government spending constant at g^*. These are the lowest possible taxes and highest possible spending.

Claim 5 *Unexpected inflation driven by one country issuing excess bonds results in increased transfers in the other country.*

Imagine country a issues too many bonds so $C_{b_a} > C_{h_a}$ while country b stayed on the equilibrium path by issuing C_{h_b} bonds. Assume both countries experience their respective high wage shock w_{h_a}, w_{h_b}. The monetary authority will raise the price level to eliminate the real value of country a's excess bonds driving the amount country b must repay below the cutoff C_{h_b} value. Country a which originally deviated will have no transfers, country b which stayed on path will have positive transfers.

3.5 Imperfect Monetary Union

An imperfect monetary union is a monetary union in which the monetary authority only responds to the fiscal decisions of one country. Mathematically, if country b is *ignored*, $P_{\{\theta_a,\theta_b\}}(B_a, B_b) = P_{\theta_a}(B_a)$. There are many possible reasons the monetary authority may not respond to one country's fiscal decisions. For example, the monetary authority may put a Pareto weight of zero on country b's welfare or country b may be able to hide its debt from the monetary authority.

I assume that neither fiscal authority, nor consumers, realize the monetary union is imperfect until the monetary authority takes an action different from the actions it would take in a standard monetary union as laid out in Equation 27. Thus the fiscal authorities will continue with their actions described in Claim 4 until the monetary authority deviates, and a country is revealed as ignored.

Claim 6 *An ignored country in an imperfect monetary union is revealed to be ignored only after a bad realization of the ignored country's wage shock.*

Assume country b is the ignored country so $P_{\{\theta_a,\theta_b\}}(B_a, B_b) = P_{\theta_a}(B_a)$. By Claim 4, the ignored country's fiscal authority issues C_{h_b} bonds. If the monetary union were standard, and country b received a good shock w_{h_b}, Equation 27 and Claim 4 say the monetary authority will leave the price level at 1. Conditioning on a good shock means $P_{\{\theta_a,w_{h_b}\}}(B_a, C_{h_b}) = P_{\theta_a}(B_a)$ by Equation 24 so the ignored country will not be revealed. By the same process, conditioning on a bad shock $P_{\{\theta_a,w_{l_b}\}}(B_a, C_{h_b}) \neq P_{\theta_a}(B_a)$ shows the ignored country will be revealed.[3]

3.6 Monetary Union Equilibrium Existence

An equilibrium is defined analogously to the single country definition.

[3]If the two countries in the monetary union are identical with regards to all parameters (shock size, labor elasticity, coalition size, etc.) then country b will only be revealed after a bad realization of its shock and a good realization of country a's shock.

Definition 2 *An equilibrium is a collection of value functions* $v_{\theta_a,\theta_b}(B_a, B_b)$*, fiscal choices* $\tau_{\theta_a}(\frac{B_a}{P}), g_{\theta_a}(\frac{B_a}{P}), B'_{\theta_a}(\frac{B_a}{P})$ *and* $\tau_{\theta_b}(\frac{B_b}{P}), g_{\theta_b}(\frac{B_b}{P}), B'_{\theta_b}(\frac{B_b}{P})$*, monetary choice* $P_{\{\theta_a,\theta_b\}}(B_a, B_b)$*, expectations about the interest rate* $q_{\theta_a,\theta_b}(B_a, B_b)$ *for* $\{\theta_a, \theta_b\} \in \{h, l\}$*, such that given* $v_{\theta_a,\theta_b}(B_a, B_b)$ *and* $q_{\theta_a}(B_a, B_b), q_{\theta_b}(B_a, B_b)$*, the fiscal choices solve the fiscal authority's problem and given* $v_{\theta_a,\theta_b}(B_a, B_b)$ *and* $q_{\theta_a}(B_a, B_b), q_{\theta_b}(B_a, B_b)$*, the monetary choice* $P_\theta(B)$ *solves the monetary authority's problem where given fiscal choices, monetary choice, and* $q_{\theta_a}(B_a, B_b), q_{\theta_b}(B_a, B_b)$*,* $v_{\theta_a,\theta_b}(B_a, B_b)$ *are optimal.*

We only need the value functions and debt state variable to establish the equilibrium. Everything can be derived, as above, from those.

Claim 7 *If the value functions* $v_{\theta_a,\theta_b}(B_a, B_b)$ *satisfy Equation 26, and optimal debt for each country satisfies Equation 21, then there is an equilibrium in which* $v_{\theta_a,\theta_b}(B_a, B_b)$ *are proposed and accepted in the first round of voting, and fiscal choices* $\tau_{\theta_a}(\frac{B_a}{P}), g_{\theta_a}(\frac{B_a}{P}), B'_{\theta_a}(\frac{B_a}{P})$ *and* $\tau_{\theta_b}(\frac{B_b}{P}), g_{\theta_b}(\frac{B_b}{P}), B'_{\theta_b}(\frac{B_b}{P})$ *and monetary choice* $P_{\{\theta_a,\theta_b\}}(B_a, B_b)$ *are optimal. Existence is then established by showing the joint optimality of the value functions and fiscal and monetary choices*

4 Results

The results describe why a country would want to join a monetary union. The distinction between the welfare of a country, the sum of the utility of all citizens, and the welfare of the political coalition in charge of the monetary authority, the sum of the utility of m out of n citizens, is extremely important.

Figure 4: Who Benefits From Joining A Monetary Union

	Status Quo Ante Monetary Authority	
	Captured	Independent
	Prop. 1, Country Benefits, Coalition Benefits	Prop. 2, Country Identical, Coalition Benefits

A monetary union offers the lure of an independent monetary authority. A country with a captured monetary authority will always want to join a monetary union. The country's political coalition would benefit along with the rest of the country from the ability to use nominal bonds to smooth taxes. A country with an independent monetary authority won't benefit from joining a monetary union, but its political coalition will due to the possibility of transfers caused by shocks in the other country.

Proposition 1 *Compared to the status quo ante of a country with a captured monetary authority,*

- *The welfare of the political coalition in charge of the fiscal authority will be higher if it joins a monetary union.*
- *The welfare of the country as a whole will be higher if it joins a monetary union.*

The comparison of welfare for the country as a whole is clear from comparing Claim 2 and Claim 4. A country with a captured monetary authority will be unable to raise any revenue from nominal bonds. Joining a monetary union allows the fiscal authority to issue nominal bonds that can be used to raise revenue to smooth taxes. The political coalition always benefits when the country as a whole benefits; the citizens in the coalition are also citizens of the country.

The political coalition in a country with a captured monetary authority will always want to join a monetary union. In order to join, it's also necessary for the monetary union to want such a country to join it. Consider a country which has an independent monetary authority, why would that country want to enter into a union with another country?

Proposition 2 *Compared to the status quo ante of a country with an independent monetary authority,*

- *The welfare of the political coalition in charge of the fiscal authority will be higher if it joins a monetary union.*
- *The welfare of the country as a whole will be identical if it joins a monetary union.*

A country with an independent monetary authority is already able to raise revenue from nominal bonds. What the political party in charge of the fiscal coalition in such a country gains is the possibility of transfers when there is a low shock in the other country. If the country were on its own and it received a high shock, the monetary authority would keep the price level constant by Equation 24 and there would be no transfers. In a monetary union, the country can receive a high shock while the other country receives a low shock.

As shown in Claim 5 the monetary authority will raise the price level and there will be transfers to reward the political coalition. Claim 4 shows the amount of revenue the fiscal authority will be able to raise. This is strictly higher than the amount a country can raise with its own independent monetary authority.

This proposition amplifies the distinction between country and political coalition. The political coalition, even in a country with a properly functioning independent monetary authority, will want to form a monetary union with another country. Due to the possible heterogeneity in shocks, the coalition might benefit. However, the country will not benefit.

Proposition 3 *An ignored country in an imperfect monetary union will face extremely high taxes when it is revealed to be ignored following a bad shock.*

Because the ignored country b doesn't realize it's ignored, it issues the usual C_{h_b} bonds. The country experiences a bad shock, and expects the price level to rise to diminish the real value of those bonds to C_{l_b} so that the fiscal authority can set taxes to their minimum τ_b^*. The monetary authority does not raise the price level, hence taxes need to be much higher to pay off the excess bond issuance.

5 Greece

To illustrate the importance of the results above, I analyze Greece's experience with the Eurozone through the lens of the model. Although the model is simplified for tractability the main results hold. Greece suffered from a captured monetary authority prior to joining the Eurozone. Upon joining, Greece operated as if it had an independent monetary authority. The Eurozone proved to be an imperfect monetary union and Greece suffered the consequences of being ignored.

Specifically, joining the Eurozone allowed Greece to raise more revenue from debt than it previously did. Greece used some of this increased revenue on increased transfers. While Greece benefited while times were good, the significant negative shock of the Great Recession revealed that its nominal debt was in fact indexed and would require high taxes to repay.

6 Conclusion

A monetary union is an effective way for a country to gain an independent, and benevolent, monetary authority. A self-interested fiscal authority benefits from an independent monetary authority because the monetary authority's independence allows the fiscal authority to issue nominal debt whose revenue will be used smooth taxes without the debt being obliterated by the time inconsistency problem of nominal debt. This relationship keeps the amount of nominal debt under control and allows the country to effectively buffer shocks.

The relationship can be abused if neither party properly monitors the other. The monetary authority may not weigh the well-being of the country when choosing the price level. The self-interested fiscal authority may not accurately report the amount of debt to the monetary authority. Both sides of the monetary union could be at fault.

If the monetary authority doesn't respond to a country's shocks, the nominal debt that fiscal authority issues will not be inflated away. The self-interested fiscal authority will issue too many nominal bonds that are in fact indexed bonds. When a bad productivity shock hits, the fiscal authority will have to raise taxes to an extremely high level to raise the revenue necessary to pay off those bonds. The country will suffer serious welfare losses.

Heterogeneity in shocks also provides a benefit to the political coalitions in the countries that form a monetary union. A bad shock in another country could induce the monetary authority to provide budgetary freedom in the political coalition's home country as a side effect. The country as a whole would not benefit from this freedom. The possiblity of such freedom provides the reason the political coalition would choose to join a union of heterogenous countries.

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Proof of Claim 1

To prove Claim 1 regarding the optimal choices of a self-interested fiscal authority I need to show the pricing function

$$P(B) = \begin{cases} \frac{B}{C_\theta}, & \text{if } B > C_\theta \\ 1, & \text{if } B \leq C_\theta \end{cases} \tag{29}$$

When $\frac{B}{P} < C_\theta$ the derivative can be taken directly from the definition of $v(B)$. Increasing the price level causes no change in taxes or government spending which are pegged at τ^*, g^* respectively. The explanation for this result is that increasing the price level decreases the real value of debt and thereby decreases the wealth of all n citizens. Revenue previously used to repay the debt is freed and the self-interested fiscal authority redirects it into transfers to the m citizens in the coalition. Because utility is linear taking 1 unit of wealth from n citizens while giving n/m to m citizens results in identical welfare from the perspective of the monetary authority.

When $\frac{B}{P} > C_\theta$, I will show that welfare always increases as the real value of bonds is diminished, thus there is always an incentive for the monetary authority to increase the price level.

To find the appropriate price level, choose $B_0 > C_\theta$. I will build a non-optimal function $\phi_\theta(B)$ that equals $v_\theta(B)$ at B_0 but is less elsewhere (and strictly concave). This will fulfill the conditions of Theorem 4.10 of Stokey et al. (1989) stating that derivatives of $v_\theta(B)$ are equal to the derivatives of $\phi_\theta(B)$ at B_0. For clarity and notational simplicity let $b = \frac{B}{P(B)}$ and $b_0 = \frac{B_0}{P(B_0)}$.

Choose B from a neighborhood of B_0. Define

$$g(b) = \mathrm{Rev}\left(\tau(b_0)\right) + qB'(b_0) - b \tag{30}$$

which is a non-optimal amount of government spending while still fulfilling debt repayment obligations. The amount of transfers will be the residual after paying back b bonds

$$S_\theta(\tau_\theta(b_0), g(b), B'(b_0); b) = \mathrm{Rev}\left(\tau(b_0)\right) + qB'(b_0) - g(b) - b \tag{31}$$

Define the non-optimal utility function to be

$$\phi_\theta(B) = \max_P W\left(\tau(b_0), g(b)\right) + \frac{S_\theta(\tau_\theta(b_0), g(b), B'(b_0); b)}{n} \tag{32}$$

$$+ \beta\left[\pi v_H\left(B'(b_0)\right) + (1-\pi)v_L\left(B'(b_0)\right)\right] \tag{33}$$

$$= \max_P \Gamma_\theta(B) \tag{34}$$

Expand the indirect utility and transfers terms. The terms dependent on P are the direct utility benefit of government spending, the bond holdings of the household in the current period, and transfers. Differentiate the right hand side, noting that the terms dependent on P in transfers will cancel, and find

$$\begin{aligned}
\frac{\partial \Gamma_\theta(B)}{\partial P} &= -\frac{B}{P^2} + \frac{A}{g}\left(\frac{B}{P^2}\right) && (35)\\
&= -\frac{B}{P^2} + \left[\frac{1-\tau\left(\frac{B}{P}\right)}{1-\tau\left(\frac{B}{P}\right)(1+\epsilon)}\right]\left(\frac{B}{nP^2}\right) && (36)\\
&= \left[\frac{\epsilon\tau\left(\frac{B}{P}\right)}{1-\tau\left(\frac{B}{P}\right)(1+\epsilon)}\right]\left(\frac{B}{nP^2}\right) > 0 && (37)
\end{aligned}$$

where I've substituted in the first order condition of the fiscal authority.

The first order condition for the monetary authority with a self-interested fiscal authority is

$$\frac{\partial \Gamma_\theta(B)}{\partial P} = \begin{cases} \left[\frac{\epsilon\tau_\theta\left(\frac{B}{P}\right)}{1-\tau_\theta\left(\frac{B}{P}\right)(1+\epsilon)}\right]\frac{B}{nP^2}, & \text{if } B > C_\theta \\ 0, & \text{if } B < C_\theta \end{cases} \tag{38}$$

To establish the full pricing function I need to show that the value function is constant for all $B \leq C_\theta$. The derivative establishes that $v(B)$ is maximized on $B < C_\theta$, I need to show the value is identical on the boundary C_θ. By definition at C_θ and below the tax rate, public good spending and bond issuance are (τ^*, g^*, B'^*). As explained above, a lower amount of bonds means less wealth but more transfers. The two effects offset hence the set of maximizers of $v(B)$ includes the bound C_θ.

Claim 1 says that a self-interested fiscal authority will issue C_h bonds. Revenue from issuing bonds is used to either lower the current tax rate or increase transfers. Both of these result in gains for the self-interested fiscal authority. Hence the self-interested fiscal authority will attempt to maximize bond revenue. Claim 1 is equivalent to stating that revenue is maximized by issuing C_h bonds. The core of the argument is that issuing more bonds than C_h will result in no additional revenue due to an offsetting rise in the price level, issuing fewer bonds than C_h will result in foregone revenue if tomorrow has high productivity.

Assume the self-interested fiscal authority issues $B_0 \in (C_l, C_h)$ bonds. There are two possibilities for the next period. If the shock is w_h, The price level P' will be 1 hence the real value of the bonds will be B_0. If the shock is w_l, the price level will be $P' = \frac{B_0}{C_l}$ hence the real amount of bonds will be C_l. Thus issuing B_0 bonds results in revenue of $\beta(\pi B_0 + (1-\pi)C_l)$.

Compare this revenue level to the revenue level that would result if the self-interested fiscal authority issued C_h bonds. If the shock is w_h, the price level P' will be 1 hence the real value of the bonds will be C_h. If the shock is w_l, the price level will be $P' = \frac{C_h}{C_l}$ hence the real amount of bonds will be C_l. Thus issuing C_h bonds results in revenue of $\beta(\pi C_h + (1-\pi)C_l)$.

Issuing C_h bonds raises the maximum possible amount of revenue. From the perspective of the benevolent monetary authority welfare (i.e. the value function at the beginning of next period) is identical at B_0 and C_h. There is no harm to welfare from issuing C_h bonds and there is a gain to the coalition of the self-interested fiscal authority in doing so.

Proof of Claim 2

Assume the self-interested fiscal authority has captured the monetary authority, and thus controls all its actions. Using the same construction as in the proof of Claim 1, the first order condition for the captured monetary authority is

$$\frac{\partial \Gamma_\theta(B)}{\partial P} = \begin{cases} \left[\frac{\epsilon\tau(\frac{B}{P})}{1-\tau(\frac{B}{P})(1+\epsilon)}\right]\frac{B}{nP^2}, & \text{if } B > C_\theta \\ \left(\frac{n}{m}-1\right)\frac{B}{nP^2}, & \text{if } B < C_\theta \end{cases}$$

For $B > C_\theta$, the derivative is identical to the proof of Claim 1. The reasoning is the same. For debt above C_θ, there are no transfers for the self-interested fiscal authority and captured monetary authority to direct. Increasing the price level decreases the real value of debt and thereby decreases taxes and increases public good spending.

The case $B < C_\theta$ arises from the equivalence of government debt and transfers in a consumer's budget constraint: both are income. Receiving a transfer is identical to holding government debt. Increasing the price level decreases the real value of the nominal government bonds every consumer holds. The total decrease in debt will equal the total increase in transfers that benefit the coalition that controls the monetary authority.

A captured monetary authority doesn't average those transfers across the entire population, it only cares about the benefits to the coalition that controls it. Increasing the price level decreases the amount the government has to repay everyone while increasing the transfers to the coalition controlling the monetary authority. Inflation is in effect a lump sum tax on all citizens to fund transfers to the coalition.

The pricing function is

$$P(B) = \begin{cases} \infty, & \text{if } B > 0 \\ 1, & \text{if } B = 0 \\ 0, & \text{if } B < 0 \end{cases}$$

If the government has a negative amount of bonds the self-interested monetary authority will deflate in order to maximize the real amount consumers owe the government. Any amount owed to the government will be directed into transfers to the governing coalition. For every unit of revenue the monetary authority raises from all n consumers it can increase transfers to its coalition by $\frac{n}{m}$. For a member of the coalition the net effect $\frac{n}{m} - 1$ is strictly positive.

Proof of Claim 3

I follow the outline of Barseghyan et al. (2013) (specifically Propositions 1, 2 and 3) and Miller (2016) (specifically Claim 4) to show the value functions are properly defined and converge in the case where both the monetary and fiscal authority are self-interested. The situation where only the monetary authority is self-interested and chooses only P is both simpler and similar.

Define $\tilde{v}_\theta$ as the value function and $\tilde{B}'$ as the bond level such that the value function solves the optimization problem given $\tilde{B}'$ while the bond level solves the appropriate optimality condition given $\tilde{v}_\theta$. Define $\{\tilde{P}(\frac{B}{P}), \tilde{\tau}(\frac{B}{P}), \tilde{g}(\frac{B}{P}), \tilde{B}'(\frac{B}{P})\}$ to be the policy choices conditional on the value function $\tilde{v}_\theta$ and bond level

$\tilde{B}'$.

Because we're looking at an equilibrium where the first proposal is identical and accepted in every proposal round I don't include superscripts to denote the round in which a proposal takes place. Define transfers for each round $r \in \{1, \ldots, T-1\}$ as

$$T_\theta^r = \frac{S_\theta(\tau, g, B'; \frac{B}{P})}{n} \tag{39}$$

and for round T where failure to approve a proposal results in the appointment of a dictator and a default proposal

$$T_\theta^T = v_\theta^{T+1}(B) - W_\theta(\tau, g) - \beta \left[\pi v_H\left(B'\right) + (1-\pi) v_L\left(B'\right)\right] \tag{40}$$

where $v_\theta^{T+1}(B)$ is the default proposal.

By construction the value functions for each proposal round are identical and equal to the proposed $\tilde{v}_\theta(B)$

We can now show that the proposals and value functions describe an equilibrium by showing the joint optimality of the value functions and proposals. In each round, the proposals must solve

$$v_\theta(B) = \max_{P,\tau,g,B'} \left[\begin{array}{l} W_\theta(\tau, g) + \left[S_\theta(\tau, g, B'; \dfrac{B}{P}) - (m-1)T\right] + \beta \left[\pi v_H\left(B'\right) + (1-\pi) v_L\left(B'\right)\right] \\ \text{s.t.} \begin{array}{l} W_\theta(\tau, g) + T + \beta \left[\pi v_H\left(B'\right) + (1-\pi) v_L\left(B'\right)\right] \geq \Gamma_\theta^{r+1}(b) \\ S_\theta(\tau, g, B'; \dfrac{B}{P}) \geq (m-1)T, T \geq 0 \end{array} \end{array} \right] \tag{41}$$

where $\Gamma^r = \tilde{v}_\theta^r(b)$ for $r \in \{1, \ldots, T-1\}$, $\Gamma^{T+1} = v_\theta^{T+1}(B)$ is the set of possible continuation values if a proposal is not approved. The first constraint on the proposal is the incentive compatibility constraint for citizens. It ensures citizens vote for a proposal if it makes them at least as well off as they expect to be if they wait for the next round and next proposal.

For a given proposal round suppose $(\hat{P}, \hat{\tau}, \hat{g}, \hat{B}', \hat{T})$ is the proposal. This means the proposal solves

$$\max_{P,\tau,g,B',\{T_i\}_1^n} W_\theta(\tau, g) + \frac{S_\theta(\tau, g, B'; \frac{B}{P})}{m} + \beta \left[\pi v_H\left(B'\right) + (1-\pi) v_L\left(B'\right)\right] \quad (42)$$

$$\text{s.t. } S_\theta(\tau, g, B'; \frac{B}{P}) \geq 0 \quad (43)$$

which defines the remaining surplus as

$$\hat{T} = \tilde{v}_\theta(B) - W_\theta(\hat{\tau}, \hat{g}) - \beta \left[\pi \tilde{v}_H(\hat{B}) + (1-\pi)\tilde{v}_L(\hat{B})\right] \quad (44)$$

Suppose there is a proposal $(P^\circ, \tau^\circ, g^\circ, B'^\circ, T^\circ)$ that results in a higher value for the proposer. I will construct a contradiction using this new set of proposals and the definition of T°. Since the new proposals result in higher values, we know that

$$T^\circ \geq \tilde{v}_\theta(B) - W_\theta(\tau^\circ, g^\circ) - \beta \left[\pi \tilde{v}_H(B'^\circ) + (1-\pi)\tilde{v}_L(B'^\circ)\right] \quad (45)$$

where the round's continuation value $\tilde{v}_\theta(B)$ has remained the same. Applying this definition to the proposer's problem

$$W_\theta(\tau^\circ, g^\circ) + S_\theta(\tau^\circ, g^\circ, B'^\circ; \frac{B}{P}) - (m-1)T^\circ + \beta\left[\pi\tilde{v}_H(B'^\circ) + (1-\pi)\tilde{v}_L(B'^\circ)\right] \tag{46}$$

$$\leq q\left(W_\theta(\tau^\circ, g^\circ) + \beta\left[\pi\tilde{v}_H(B'^\circ) + (1-\pi)\tilde{v}_L(B'^\circ)\right]\right) + S_\theta(\tau^\circ, g^\circ, B'^\circ; \frac{B}{P}). \tag{47}$$

But the last equation (without the multiplication by q) is the objective function $(\hat{P}, \hat{\tau}, \hat{g}, \hat{B}', \hat{T})$ are defined as solving. To show the equilibrium exists we need to show that v_θ and the proposals are jointly optimal. This means that v_θ is the solution to

$$v_\theta(B) = \max_{P,\tau,g,B'} \left[\begin{array}{r} W_\theta(\tau, g) + \dfrac{S_\theta(\tau, g, B'; \frac{B}{P})}{m} + \beta\left[\pi v_H(B') + (1-\pi)v_L(B')\right] \\ \text{s.t. } T_i \geq 0 \ \forall i, S_\theta(\tau, g, B'; \dfrac{B}{P}) \geq \displaystyle\sum_i^n T_i \end{array} \right] \tag{48}$$

given the proposals (P^*, τ^*, g^*, B'^*) and that given v_θ the proposals solve the requisite optimality conditions.

Let F be the set of real valued, continuous, concave functions v over the domain of possible bond values $\left[\underline{B}, \overline{B}\right]$.

For $z_\theta \in \left[\text{Rev}_L(\tau^*) - g^*, \overline{B}\right]$ define $N^\theta_{z_\theta}$ from $F \times F \to F$ as

$$N^\theta_{z_\theta}(v_h, v_l)(B) = \max_{P,\tau,g,B'} \left[\begin{array}{r} W_\theta(\tau, g) + \dfrac{S_\theta(\tau, g, B'; \frac{B}{P})}{m} + \beta \left[\pi v_H(B') + (1-\pi) v_L(B')\right] \\ \text{s.t. } T_i \geq 0 \ \forall i, S_\theta(\tau, g, B'; \dfrac{B}{P}) \geq \displaystyle\sum_i^n T_i \end{array} \right] \tag{49}$$

For conciseness let $\mathbf{z} = (z_H, z_L)$ be the vector of possible bond levels for either realization of the productivity shock. Define $N_{\mathbf{z}}(v_H, v_L)(B) = (N^H_{z_H}(v_H, v_L)(B), N^L_{z_L}(v_H, v_L)(B))$ from $F \times F \to F \times F$ as the optimal choices for either realization of the productivity shock. For any $\mathbf{z}$, $N_{\mathbf{z}}$ is a contraction with a unique fixed point $v_{\mathbf{z}}$. We can use this fixed point to define

$$M_\theta(\mathbf{z}) = \arg\max_{B'} \left[\frac{qB'}{m} + \beta \left[\pi v_{H\mathbf{z}}(B') + (1-\pi) v_{L\mathbf{z}}(B')\right] \right] \tag{50}$$

where the fixed point is used for computing the possible continuation values. Let $M(\mathbf{z}) = M_H(\mathbf{z}) \times M_L(\mathbf{z})$. An equilibrium is a fixed point of $M(\mathbf{z})$ that takes a bond level $\mathbf{z}$ to itself given the fixed point $v_{\mathbf{z}}$. Applying Kakutani's Fixed Point Theorem proves the result.

Differentiability of the value function comes from a similar construction as the proof of Claim 1. Using our fixed point $v_{\mathbf{z}}$ we can choose a non-optimal bond level B_0 and show differentiability. In the region with transfers, this is direct, in the region without transfers the construction proceeds identically to Claim 1.

Proof of Claim 4

This proof rests on two points: the two countries in the monetary union face independent constraints and differentiation distributes over addition. The shared price level P enters solely (and identically) through each country's budget constraint. This enables the monetary authority to treat each country's problem independently.

The derivative is the sum of the utility gains from the two countries

$$\frac{\partial v_{\theta_a,\theta_b}(B_a,B_b)}{\partial P} = \begin{cases} \left[\frac{\epsilon\tau_{\theta_a}(\frac{B_a}{P})}{1-\tau_{\theta_a}(\frac{B_a}{P})(1+\epsilon)}\right]\frac{B_a}{nP^2} + \left[\frac{\epsilon\tau_{\theta_b}(\frac{B_b}{P})}{1-\tau_{\theta_b}(\frac{B_b}{P})(1+\epsilon)}\right]\frac{B_b}{nP^2}, & \text{if } B_a > C_{\theta_a} \text{ and } B_b > C_{\theta_b} \\ \left[\frac{\epsilon\tau_{\theta_a}(\frac{B_a}{P})}{1-\tau_{\theta_a}(\frac{B_a}{P})(1+\epsilon)}\right]\frac{B_a}{nP^2}, & \text{if } B_a > C_{\theta_a} \text{ and } B_b < C_{\theta_b} \\ \left[\frac{\epsilon\tau_{\theta_b}(\frac{B_b}{P})}{1-\tau_{\theta_b}(\frac{B_b}{P})(1+\epsilon)}\right]\frac{B_b}{nP^2}, & \text{if } B_a < C_{\theta_a} \text{ and } B_b > C_{\theta_b} \\ 0, & \text{if } B_a < C_{\theta_a} \text{ and } B_b < C_{\theta_b} \end{cases}$$

In the region $B_a > C_{\theta_a}$ and $B_b > C_{\theta_b}$ increasing the price level decreases the real value of nominal bonds in both countries. Since each country is individually above the bond cutoff, decreasing the real value of nominal bonds results in lower taxes hence higher welfare. The two regions ($B_a > C_{\theta_a}$ and $B_b < C_{\theta_b}$) and ($B_a < C_{\theta_a}$ and $B_b > C_{\theta_b}$) are situations when one country, but not the other is above the the bond cutoff. In one country, increasing the price level decreases the real value of bonds and leads to lower taxes and higher welfare. In the other country, increasing the price level decreases the real value of bonds and leads to increased transfers and constant welfare. The last region $B_a < C_{\theta_a}$ and $B_b < C_{\theta_b}$ considers the case that both countries are below their respective bond cutoffs. If this is the case, increasing the price level decreases the real value of bonds and leads to increased transfers and constant welfare in both countries.

All regions except for the last $B_a < C_{\theta_a}$ and $B_b < C_{\theta_b}$ have a positive welfare gain to increasing the price level. Hence the pricing function is

$$P(B_a, B_b) = \begin{cases} \max\left(\frac{B_a}{C_{\theta_a}}, \frac{B_b}{C_{\theta_b}}\right), & \text{if } B_a > C_{\theta_a} \text{ or } B_b > C_{\theta_b} \\ 1, & \text{if } B_a \leq C_{\theta_a} \text{ and } B_b \geq C_{\theta_b} \end{cases}$$

where the max operator is combining the two cases where either ($B_a > C_{\theta_a}$ and $B_b < C_{\theta_b}$) or ($B_a < C_{\theta_a}$ and $B_b > C_{\theta_b}$) holds.

As before, issuing C_h bonds raises the maximum possible amount of revenue for each fiscal authority. From the perspective of the benevolent monetary authority welfare (i.e. the value function at the beginning of next period) is identical at B_0 and C_h. There is no harm to welfare from issuing C_h bonds and there is a gain to the coalition of the self-interested fiscal authority in doing so. Due to the possibility that increases in the price level will be caused by actions of the other country in the monetary union, issuing C_h bonds raises a different amount of revenue than before. For a country a, issuing C_{h_a} bonds raises revenue equal to

$$\beta\left[\pi^2 C_{h_a} + \pi(1-\pi)C_{h_a}\left(\frac{C_{l_b}}{C_{h_b}}\right) + (1-\pi)\pi C_{l_a} + (1-\pi)^2 C_{h_a}\left(\min\left[\frac{C_{l_a}}{C_{h_a}}, \frac{C_{l_b}}{C_{h_b}}\right]\right)\right] \tag{51}$$

This amount of revenue is the result of expectations involving shocks that hit both country a and country b. Each term comes from one of the four possibilities for shock combinations.

- $\pi^2 C_{h_a}$: Both countries have a high shock. There is no inflation
- $\pi(1-\pi)C_{h_a}\left(\frac{C_{l_b}}{C_{h_b}}\right)$: Country a has a high shock while country b has a low shock. The monetary authority increases the price level to bring the real value of bonds in country b from C_{h_b} down to C_{l_b}. This decreases the real value of bonds in country a from C_{h_a} to $C_{h_a}\left(\frac{C_{l_b}}{C_{h_b}}\right)$.
- $(1-\pi)\pi C_{l_a}$: Country a has a low shock while country b has a high shock. The monetary authority increases the price level to bring the real value of bonds in country a from C_{h_a} to C_{l_a}.
- $(1-\pi)^2 C_{h_a}\left(\min\left[\frac{C_{l_a}}{C_{h_a}}, \frac{C_{l_b}}{C_{h_b}}\right]\right)$: Both countries have a low shock. The monetary authority increases the price level so the real value of bonds in both countries is at or below C_{l_a} and C_{l_b}, respectively. The min operator is necessary because the change in the price level necessary to reduce the real value of bonds may be different for countries a and b. The max operator in the pricing function for the monetary authority becomes min because I invert the price P in the formula for the bond price q

Proof of Claim 7

This proof is almost identical to the proof of Claim 3. Because each country is fully independent from the other, both in shocks and information sets, while fully informed about the parameters of the other country (necessary for the amount of bonds in Equation 51) the equilibrium existence is the same on a per-country basis, though the continuation possibilities are now the four possible shock combinations for both countries rather than the two possible shocks for a single country. As shown by Claim 5, there is no benefit to a fiscal authority to deviating from the equilibrium.