Pseudocode and Diagrams for Differential Privacy Methods

Contents

1 Introduction

This document presents the pseudocode and conceptual diagrams of the differential privacy methods implemented for quantitative and categorical variables. The included algorithms are:

- **Quantitative Data**:
  - Duchi et al. Mechanism (`duchi_mechanism`)
  - Laplace Mechanism (`laplace_mechanism`)
  - Piecewise Mechanism (`piecewise_mechanism`)
  - Multidimensional Duchi Mechanism (`multidimensional_duchi_mechanism`)
  - Custom Multidimensional Mechanism (`multidimensional_mechanism`)
- **Categorical Data**:
  - Direct Encoding (`direct_encoding`)
  - Optimized Unary Encoding (OUE) (`optimized_unary_encoding`)
  - RAPPOR (`rappor`)

Below, the pseudocode and diagram for each of these methods are presented.

2 Pseudocode and Diagrams for Quantitative Methods

2.1 Duchi et al. Mechanism (`duchi_mechanism`)

Algorithm 1 Duchi et al. Mechanism

Require: Input vector t_i with values in $[-1, 1]$, privacy budget ϵ

Ensure: Privatized vector t_i^*

```
1: for each element t_i in the input vector do
2:     Clone and convert t_i to double precision
3:     Compute tanh(ε/2)
4:     Compute probability p = 0.5 × (1 + t_i × tanh(ε/2))
5:     Generate Bernoulli variable u with probability p
6:     Compute scaling factor w = 1/tanh(ε/2)
7:     Compute t_i^* = (2u − 1) × w
8: end for
9: return Vector t_i^*
```

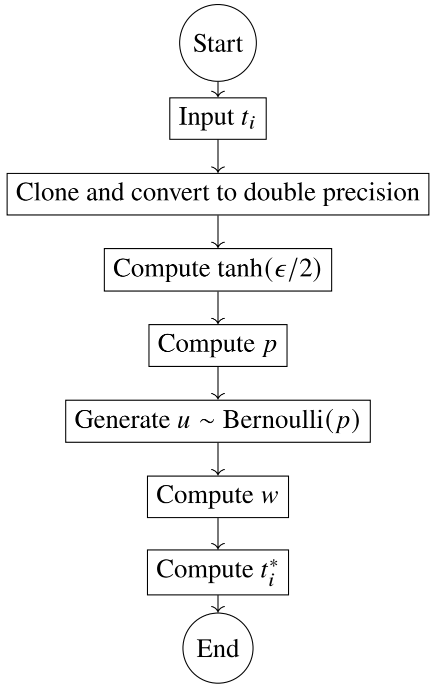

Figure 1: Diagram of the Duchi et al. Mechanism

2.2 Piecewise Mechanism (`piecewise_mechanism`)

Algorithm 2 Piecewise Mechanism

Require: Input vector t_i with values in $[-1, 1]$, privacy budget ϵ

Ensure: Privatized vector t_i^*

Compute $e^{\epsilon/2}$ and $C = \dfrac{e^{\epsilon/2} + 1}{e^{\epsilon/2} - 1}$
for each element t_i in the input vector **do**
    Clone and convert t_i to float
    Compute $l(t_i)$ and $r(t_i)$
    Generate $x \sim \text{Uniform}(0, 1)$
    Compute threshold $u = \dfrac{e^{\epsilon/2}}{e^{\epsilon/2} + 1}$
    if $x < u$ **then**
        Generate t_i^* uniformly in $[l(t_i), r(t_i)]$
    else
        Randomly choose between intervals $[-C, l(t_i)]$ and $[r(t_i), C]$
        Generate t_i^* uniformly in the selected interval
    end if
end for
return Vector t_i^*

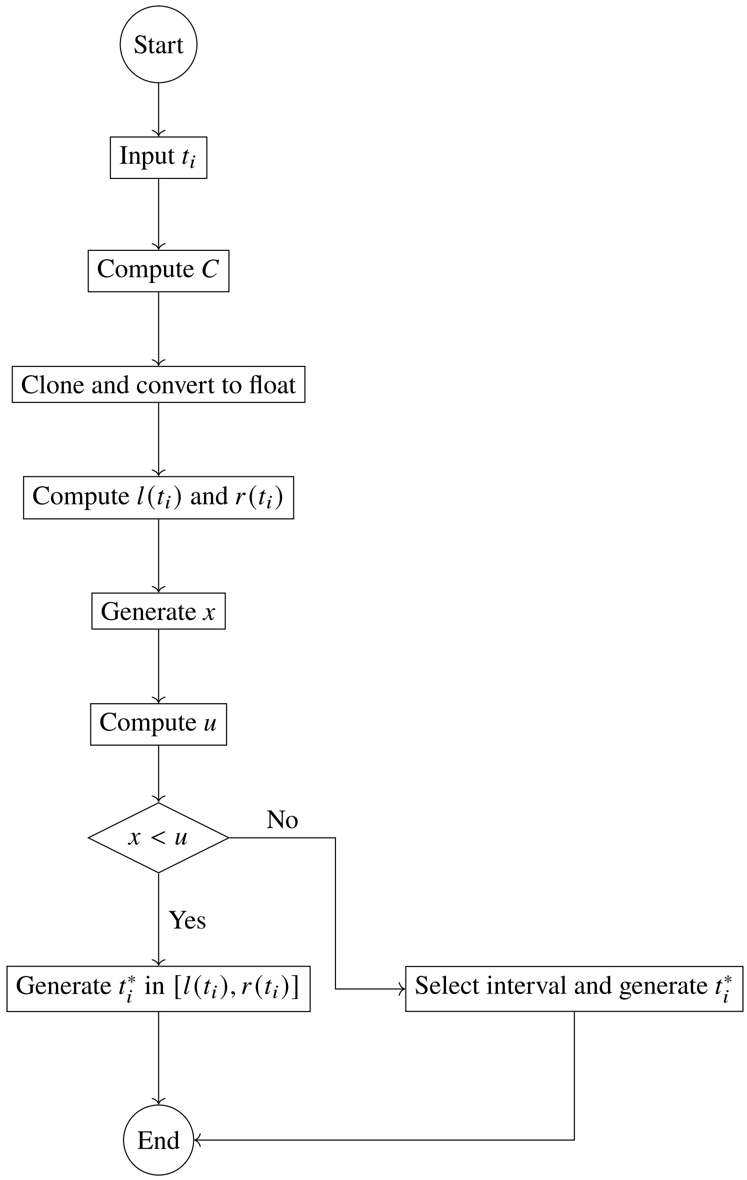

Figure 2: Diagram of the Piecewise Mechanism

2.3 Laplace Mechanism (`laplace_mechanism`)

Algorithm 3 Laplace Mechanism

Require: Input vector t_i, privacy budget ϵ, sensitivity s

Ensure: Privatized vector t_i^*

1: **for** each element t_i in the input vector **do**
2: Clone and convert t_i to float
3: Compute scale $b = s/\epsilon$
4: Generate Laplace noise $n \sim \text{Laplace}(0, b)$
5: Compute $t_i^* = t_i + n$
6: **end for**
7: **return** Vector t_i^*

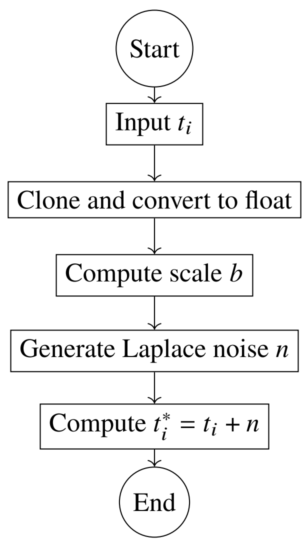

Figure 3: Diagram of the Laplace Mechanism

2.4 Multidimensional Duchi Mechanism (`multidimensional_duchi_mechanism`)

Algorithm 4 Multidimensional Duchi Mechanism

Require: Input vector t_i of dimension d, privacy budget ϵ, number of samples N

Ensure: Privatized vector t_i^*

1: Clone and clamp t_i to the range $[-1, 1]$

2: Generate random vector $v \in \{-1, 1\}^d$ based on t_i

3: Generate sets T_+ and T_- via random sampling

4: Compute probability $p = \dfrac{e^\epsilon}{e^\epsilon + 1}$

5: Generate Bernoulli variable u with probability p

6: **if** $u = 1$ and $T_+ \neq \emptyset$ **then**

7:     Randomly select t_i^* from T_+

8: **else if** $T_- \neq \emptyset$ **then**

9:     Randomly select t_i^* from T_-

10: **else**

11:     Assign $t_i^* = B$ or $-B$ according to u

12: **end if**

13: **return** Vector t_i^*

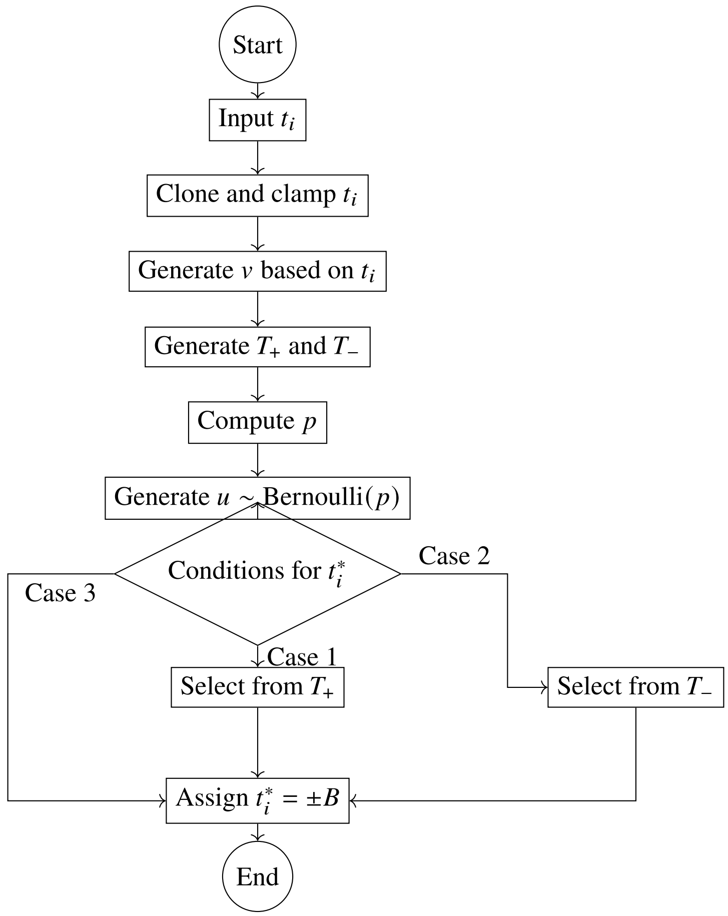

Figure 4: Diagram of the Multidimensional Duchi Mechanism

2.5 Custom Multidimensional Mechanism (`multidimensional_mechanism`)

Algorithm 5 Custom Multidimensional Mechanism

Require: Input vector t_i of dimension n, privacy budget ϵ, unidimensional mechanism M, constant C

Ensure: Privatized vector t_i^*

1: Clone and clamp t_i to the range $[-1, 1]$
2: **for** each index i in n **do**
3: Apply mechanism M to t_i with budget ϵ
4: Scale t_i^* by multiplying with C
5: **end for**
6: Clamp t_i^* to the range $[-C, C]$
7: **return** Vector t_i^*

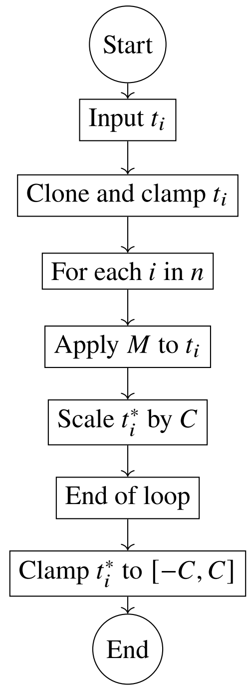

Figure 5: Diagram of the Custom Multidimensional Mechanism

3 Pseudocode and Diagrams for Categorical Methods

3.1 Direct Encoding (`direct_encoding`)

Algorithm 6 Direct Encoding

Require: Categorical data X, privacy budget ϵ

Ensure: Privatized data X^*

Encode categories with numerical values
Obtain the number of categories k
Compute $p = \dfrac{e^\epsilon}{e^\epsilon + k - 1}$
Compute $q = \dfrac{1}{e^\epsilon + k - 1}$
for each element x_i in X **do**
    Assign probabilities: p to the original category, q to the others
    Generate category x_i^* based on the assigned probabilities
end for
return Privatized data X^*

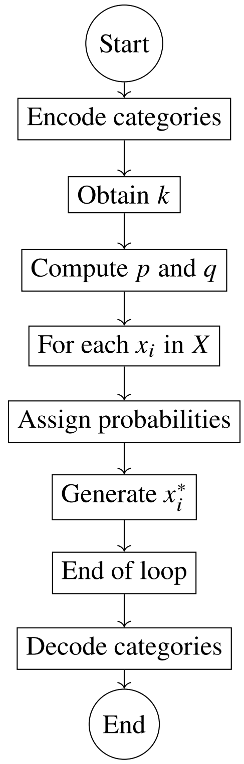

Figure 6: Diagram of Direct Encoding

3.2 Optimized Unary Encoding (OUE) (`optimized_unary_encoding`)

Algorithm 7 Optimized Unary Encoding (OUE)

Require: Categorical data X, privacy budget ϵ

Ensure: Privatized data X^*

Encode categories with numerical values
Obtain the number of categories d
Create original binary matrix (One-Hot Encoding)
Set $p = 0.5$ and $q = \frac{1}{e^{\epsilon} + 1}$
for each binary vector u_i **do**
    for each bit u_{ij} in u_i **do**
        if $u_{ij} = 1$ **then**
            Perturb u_{ij} with probability p
        else
            Perturb u_{ij} with probability q
        end if
    end for
    Reconstruct x_i^* from the perturbed vector
end for
return Privatized data X^*

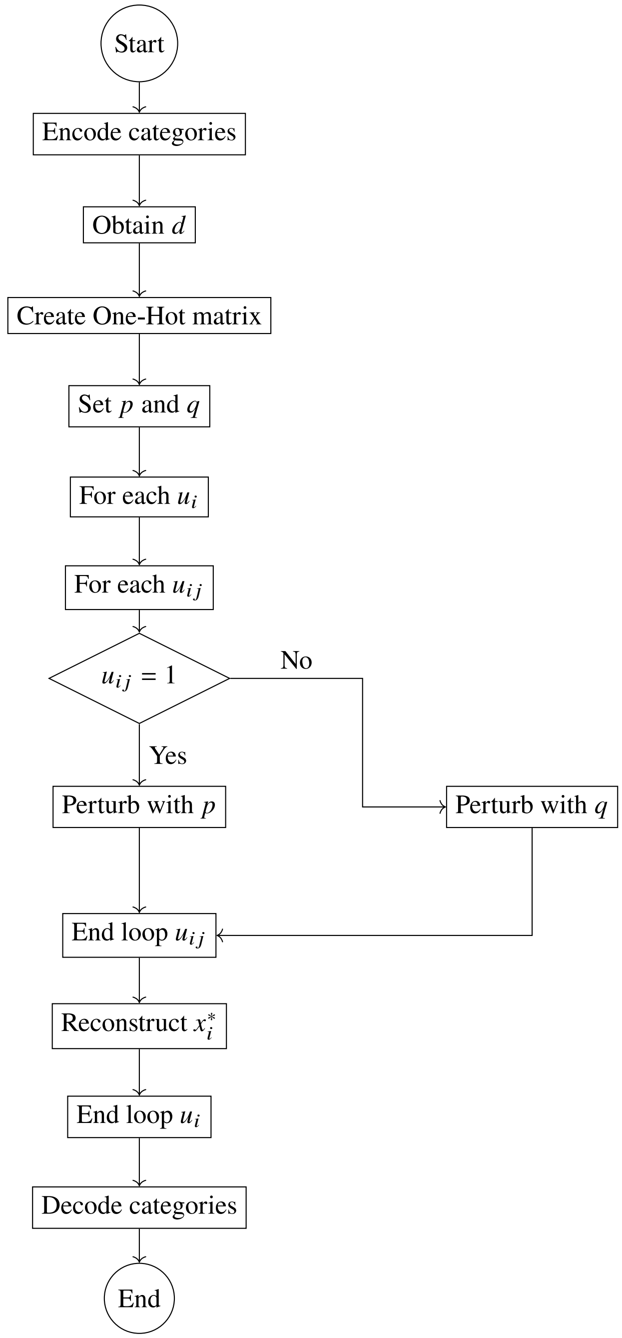

Figure 7: Diagram of Optimized Unary Encoding (OUE)

3.3 RAPPOR (`rappor`)

Algorithm 8 RAPPOR

Require: Categorical data X, privacy budget ϵ

Ensure: Privatized data X^*

Encode categories with numerical values
Obtain the number of categories d
Create original binary matrix (One-Hot Encoding)
Compute $f = \frac{1}{e^{\epsilon} + 1}$
for each binary vector u_i **do**
for each bit u_{ij} in u_i **do**
if $u_{ij} = 1$ **then**
Perturb u_{ij} with probability $1 - f$
else
Perturb u_{ij} with probability f
end if
end for
Reconstruct x_i^* from the perturbed vector
end for
return Privatized data X^*

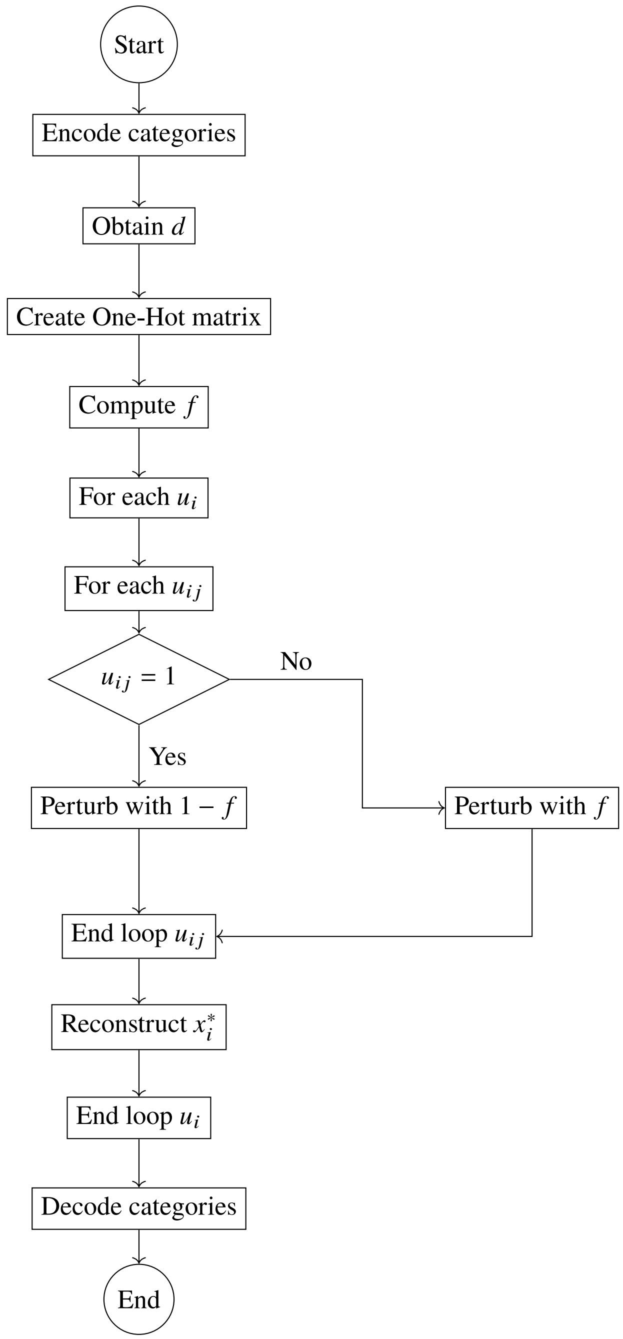

Figure 8: Diagram of RAPPOR