

1 Introduction Workthrough with p -adic Numbers

K is a non-archimedian local field. In this case K will be the p -adic numbers \mathbb{Q}_p , where $p > 1$ is prime. Let \mathcal{O} be the ring of integers (integral elements) of K . These are elements of the form

$$a = \sum_{i \geq 0} a_i p^i.$$

with each $0 \leq a_i \leq p - 1$ for all i . See [2], page 28 for more explanation of this representation. It is important to note that because of the metric imposed on the field of p -adic numbers, adding terms with higher powers of p does not increase the magnitude of the integer. Thus for every p -adic integer a , $|a| \leq 1$. As in the rational numbers (can be shown with rational root theorem) these are the integers. To see that (p) is the maximal ideal, note that for any $a \in \mathcal{O}$, if $|a| = 1$, then a is invertible. Thus the only non-invertible elements of \mathcal{O} have p as a divisor, and (p) is the unique maximal ideal of \mathcal{O} .

Now let $F_q = \mathcal{O}/(p)$. Then this is a field of characteristic p . Let $F = K[\sqrt{p}]$ be a ramified quadratic extension of K with ring of integers R . From Wikipedia [1]: Define

$$\omega = \begin{cases} \sqrt{p} & \text{if } p \equiv 2, 3 \pmod{4} \\ \frac{1+\sqrt{p}}{2} & \text{if } p \equiv 1 \pmod{4} \end{cases}$$

Then $R = \{a + b\omega\} = \mathcal{O}[\sqrt{p}]$ and $R/(\sqrt{p}) \cong F_p$.

References

- [1] Wikipedia contributors. Quadratic integer, July 2013.
- [2] Svetlana Katok. *p-adic Analysis Compared with Real*. American Mathematical Society, Providence, Rhode Island, 2007.