Definition A **local field** is a field that is locally compact with respect to a non-discrete topology. Every local field is isomorphic (as a topological field) to one of the following:

- Archimedian local fields (characteristic zero): the real numbers R, and the complex numbers C.
- Non-archimedian local fields of characteristic zero: finite extensions of the p-adic numbers Q_p (where p is any prime number).
- Non-archimedian local fields of characteristic p: the field of formal Laurent series $\mathbf{F}_q((T))$ over a finite field $\mathbf{F}_q$ (where q is a power of p).

There is an equivalent definition of a non-archimedean field: it is a field that is complete with respect to a discrete valuation and whose residue field is finite. However, some authors consider a more general notion, requiring only that the residue field be perfect, not necessarily finite.

Definition Let K be a field. A **discrete valuation** on K is a function $\nu : K^\times \to \mathbf{Z}$ satisfying

1. $\nu(ab) = \nu(a) + \nu(b)$ (i.e., ν is a homomorphism from the multiplicative group of nonzero elements of K to $\mathbf{Z}$).
2. ν is surjective, and
3. $\nu(x + y) \geq \min\{\nu(x), \nu(y)\}$ for all $x, y \in K^\times$ with $x + y \neq 0$.

Definition The **ring of integers** of an algebraic number field K is the ring of all integral elements contained in K.

Definition A topological space X is **locally compact** if every point of X has a compact neighborhood.

Definition Let M be an ideal in a ring R. M is said to be **maximal** if $M \neq R$ and for every ideal N such that $M \subset N \subset R$, either $N = M$ or $N = R$.

Definition A **symplectic** matrix is a $2n$ by $2n$ matrix M with entries in a field F that satisfies the condition

$$M^T \Omega M = \Omega$$

where Ω is a nonsingular, skew-symmetric matrix.

Jacobsen Radical and Background Definitions

Definition A ring R is local if it has any one of the following equivalent properties:

- R has a unique maximal left ideal.
- R has a unique maximal right ideal.
- $1 \neq 0$ and the sum of any two non-units in R is a non-unit.
- $1 \neq 0$ and if x is any element in R, then x or $1 - x$ is a unit.
- If a finite sum is a unit, then so are some of its terms.

Definition A module A over a ring R is **simple** (or **irreducible**) provided $RA \neq 0$ and A has no proper submodules. A ring R is **simple** if $R^2 \neq 0$ and R has no proper (two-sided) ideals.

Definition A module A is **faithful** if its annihilator $\mathcal{A}(A)$ is 0. A ring R is **primitive** if there exists a simple faithful left R-module.

Definition And ideal P of a ring R is said to be **left (right) primitive** if the quotient ringt R/P is a left (right) primitive ring.

Definition An element a in a ring R is said to be **left quasi-regular** if there exists $r \in R$ such that $r + a + ra = 0$. The element r is called a **left quasi-inverse** of a. A (right, left, or two-sided) ideal I of R is said to be **left quasi-regular** if every element of I is left quasi-regular. Similarly, $a \in R$ is said to be **right quasi-regular** if there exists $r \in R$ such that $a+r+ar = 0$. Right quasi-inverses and right quasi-regular ideals are defined analogously.