Macroeconomic Factors in Oil Futures Markets

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This paper documents new evidence against perfect risk spanning in crude oil futures, and develops an affine futures pricing model that allows for unspanned macroeconomic factors. Compared to previous estimates, the oil spot premium is more volatile and strongly procyclical, which suggests that previous models miss the majority of variation in oil risk premiums. The estimates reveal a dynamic two-way relationship between oil futures and economic activity: productivity shocks are associated with higher oil prices, while oil price shocks affect economic activity by lowering future consumption spending. Unspanned macro factors also affect the valuation of real options.

Key words : commodities, futures, unspanned factors, affine models, real activity, real options

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1. Introduction

Research in finance has yielded models that closely fit the dynamics of commodity futures markets. However, these models explain futures prices and risk premiums in terms of purely latent state variables that are extracted from futures prices.[1] Research in macroeconomics has investigated the time series dynamics of the spot price of oil with economic data, finding that oil shocks forecast recessions.[2] However, oil futures markets are more active than spot markets, and have a time-varying term structure which reflects both risk premiums and the market's forecast of oil prices.

This paper develops an affine futures pricing model with both latent and macroeconomic state variables. The model amounts to a minimal set of cross equation restrictions, and can be applied to any futures market and any set of macroeconomic factors. I focus on oil futures because oil is the single most important commodity in the modern economy. The Energy Information Administration (EIA) estimates that in 2010 expenditures on energy, the vast majority of which was petroleum based, accounted for 8.3% of U.S. GDP.[3] Oil is also the single most important commodity to financial markets: in 2017 crude oil and refined products made up 53% of the benchmark Goldman Sachs Commodity Index.

[1] e.g. Gibson and Schwartz (1990), Schwartz (1997), Casassus and Collin-Dufresne (2005)

[2] e.g. Hamilton (1983), Bernanke et al. (1997), Hamilton (2003), Barsky and Kilian (2004), Kilian (2009)

[3] This figure reflects expenditures on petroleum products for energy and does not include chemical products such as asphalt, tar, wax, coke, lubricants, and petrochemicals that are critical inputs into many industries.

Crucially, the model allows variation in the macroeconomic factors that is unspanned by derivatives prices. Although previous models such as Schwartz (1997), Casassus and Collin-Dufresne (2005) and Trolle and Schwartz (2009) do not explicitly examine how derivatives markets interact with the real economy, they implicitly impose strong restrictions on those interactions. Specifically, they assume perfect spanning, the condition that all relevant information is fully reflected in derivatives prices. I document that this restriction is rejected in the data: forward-looking measures of real economic activity predict oil prices and returns, conditional on the information in the oil futures curve. Previous research finds that unspanned factors have material effects in the Treasury bond market.[4] This paper is the first to show that unspanned macro factors are material in commodity derivatives.

Studies such as Casassus and Collin-Dufresne (2005) and Hamilton and Wu (2014) find that oil risk premiums are relatively stable and do not covary with the business cycle. This conclusion is a consequence of the spanning assumption, plus the fact that the contemporaneous correlation of oil prices with the business cycle is low. Compared to the nested model that enforces perfect spanning, the estimated oil spot premium in this paper is nine times more volatile and strongly procyclical, which suggests that previous models miss the majority of variation in oil risk premiums. The model in this paper also fits the realized returns to oil futures better, more than tripling the R^2 from 1.3% to 4.3% after adjusting for the added degrees of freedom.

Both regressions and model estimates suggest that the oil spot premium is procylical while the oil term premium is countercyclical. Interpreted in light of the structural model of Kilian (2009), these results are consistent with oil risk premiums being driven primarily by demand for hedging oil-specific supply shocks. Higher expected inflation also appears to drive greater hedging demand and a lower spot risk premium, independent of the business cycle.

The model estimates also reveal new dynamics that are not apparent from time-series vector autoregressions (VARs).[5] I find that the strength of an oil shock's effect on real activity varies depending on the market's forecast of oil prices: oil shocks that are forecast by the market to be more persistent have a larger and longer lasting effect on real activity. Conversely, although shocks to real activity dissipate in less than a year the market forecasts that the resulting higher oil price persists for decades, perhaps because oil is a nonrenewable resource. I also document that the effects of oil shocks are stronger when the economy is in expansion; that hedging expected, rather than realized, inflation also appears to drive oil risk premiums; and that economic activity affects oil prices through shocks to industrial production, while oil shocks affect economic activity through changes in consumer spending.

[4] Duffee (2011); Chernov and Mueller (2012); Joslin et al. (2014).

[5] Hamilton (1983), Bernanke et al. (1997), Kilian (2009).

I further explore the importance of unspanned factors for real options valuation. Pindyck (1993) argues that the value of real options should depend on macroeconomic in addition to financial factors. This point has not been emphasized in the literature on commodity real options (e.g., optimal resource extraction) because the implicit spanning assumption rules out any role for macro factors after conditioning on the state of financial markets. In an example calibrated to my empirical estimates, I find that adding real activity as an unspanned factor increases the valuation of a hypothetical oil well by between 35% and 405%. There are two channels by which an unspanned factor affects option value: its unspanned dynamics and its risk premium. In simulations, I find that the dynamics effect is dominant and the risk premium effect is small.

This paper adds to a growing literature that suggests that although derivatives prices and returns can often be summarized by a low dimensional set of state variables, such "spanned factors" need not be the only relevant factors in these markets. More subtly, perfect spanning implies that conditional on current futures prices, macroeconomic data is superfluous in forecasting prices and returns *and* future macroeconomic conditions. The results in this paper suggest that the link between oil markets and the real economy is more complex, and develops a useful framework for further research.

1.1. Related Literature

There are two strands of the literature in commodity futures that this paper builds on. In the first, commodity futures prices are modeled as affine functions of latent (unobserved) state variables. Classic examples are Gibson and Schwartz (1990), Schwartz (1997), and Casassus and Collin-Dufresne (2005); more recent examples include Casassus et al. (2013) and Hamilton and Wu (2014). Models of this type do not incorporate macroeconomic data and the latent variables can often be rotated and translated without changing the likelihood (Dai and Singleton (2000), Collin-Dufresne et al. (2008)), so their economic meaning is unclear. More subtly, they implicitly assume that all relevant information in the economy is reflected (spanned) in current futures prices and no other information can contribute incremental forecasting power. I show that real economic activity (by various measures) has material effects on oil risk premiums and forecasts of oil prices, over and above the information in current futures prices. Recent studies (Diebold et al. (2006), Duffee (2011), Chernov and Mueller (2012), Joslin et al. (2014)) have documented similar apparent violations of the spanning assumption in Treasury bond markets.

The second strand uses vector autoregressions (VARs) to explore the time series relation of oil prices with the real economy; examples include Hamilton (1983, 2003), Kilian (2009), Alquist and Kilian (2010), Kilian and Vega (2011). These studies include a single state variable based on the spot price of oil. This approach does not incorporate the full panel of futures prices of different maturities, and is silent regarding risk premiums and the market's forecast of the spot price. The model in this paper imposes the additional assumption that risk premiums are "essentially affine" (Duffee 2002) in the state variables, which lets us bring the full term structure to bear on returns, forecasts, and risk spanning (Ang and Piazzesi 2003).

Fama and French (1987), Bessembinder and Chan (1992), Singleton (2013) and Hamilton and Wu (2015) run return forecasting regressions for individual returns to futures on individual commodities; Gorton et al. (2012) sort the cross section of commodities into portfolios. Hong and Yogo (2012) find that futures market open interest has robust predictive power for futures returns and bond returns. Szymanowska et al. (2014) decompose futures returns into a spot premium and a term premium. This paper contributes to this literature as well, as it offers a simple and consistent framework to use the full term structure of futures prices and returns to investigate these questions.

Chiang et al. (2015) extract spanned factors from oil futures and an additional volatility factor from oil options, and find that exposure to the volatility factor carries a risk premium in equity markets but not in oil futures. I examine the effects of the real economy on price forecasts and risk premiums in oil futures markets, and the dynamic relationship between the two.

2. Data

2.1. Futures Price Data

I use closing prices for West Texas Intermediate (WTI) oil futures with maturities of one to twelve months, on the last business day of each month from January 1986 to June 2014. The futures price data is denoted

$$f_t^j = \log(F_t^j),\ j = 1 \ldots J,\ t = 1 \ldots T$$

$$f_t = \left[\, f_t^1 \ f_t^2 \ \ldots \ f_t^J \,\right]'$$

where F_t^j is the closing price at end of month t of the future that expires in month $t+j$, $t=1$ corresponds to 1/1986, $T=342$ corresponds to 6/2014, and $J=12$. The maximum maturity of twelve months is because longer dated futures were seldom traded in the early years of the sample. The results do not change if I extend J to longer maturities.

2.2. Macro Factors

I use the Chicago Fed National Activity Index (CFNAI), hereafter labelled *GRO*, as a forward-looking measure of real economic activity. The CFNAI is a weighted average of 85 U.S. macroeconomic time series, published monthly by the Chicago Fed.[6]

[6] https://www.chicagofed.org/publications/cfnai/index

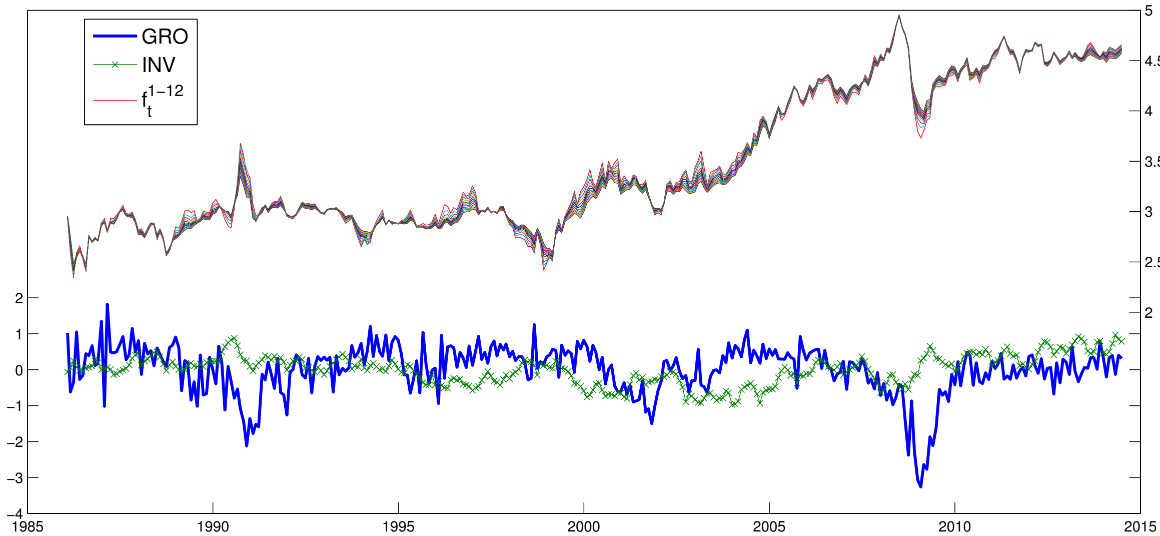


Figure 1 **The figure plots log futures prices for Nymex crude oil f_t^{1-12}, the Chicago Fed National Activity Index GRO, and the log of the EIA's monthly U.S. oil inventory INV from January 1986 to June 2014.**

Forward-looking measures of real activity have been found to forecast inflation (Stock and Watson 1999), bond returns (Ludvigson and Ng 2009, Ang and Piazzesi 2003, Joslin et al. 2014) and equity returns (Cooper and Priestley 2008) and are potentially material to oil markets given oil's importance in the real economy. Importantly, the results in this paper do not depend on using the CFNAI measure specifically but also obtain using other indexes of real activity (see the Internet Appendix).

The second macro factor is the inventory of oil in readily available storage. The Theory of Storage (Working 1949) predicts a natural relation between the cost of carry in the futures market and the level of inventories held. Also, Gorton et al. (2012) find that inventories are associated with returns to futures contracts, using portfolio sorts of the cross-section of commodity futures. I use the log of the Energy Information Administration's "Total Stocks of Commercial Crude Oil excluding the Strategic Petroleum Reserve" as a measure of the available inventory of crude oil,[7] hereafter labelled INV.

Thus, the macro factors are $M_t = [GRO_t, INV_t]'$. Figure 1 plots the time series of log oil futures prices and the macro factors GRO and INV.

3. Model

3.1. Spanned vs Unspanned Factors

The distinction between spanned and unspanned factors in futures markets drives the modelling strategy. Let R_{t+j} be the payoff from going long a $j-$period futures contract at price F_t^j and holding it to maturity:

[7] http://www.eia.gov/petroleum/

$$R_{t+j} = S_{t+j} - F_t^j$$

where S_{t+j} is the spot price at maturity. This accounting identity always holds ex post and thus holds in expectation for any information set X_t :

$$F_t^j = E\left[S_{t+j}|X_t\right] - E\left[R_{t+j}|X_t\right]$$

This is the case for the spanning assumption: any information relevant to forecasting spot prices or returns should be reflected in contemporaneous prices. However, the same argument applies to bond yields, yet recent research finds evidence against the spanning assumption in Treasury bond markets. In particular, real activity (Joslin et al. (2014)) and latent factors extracted by filtering (Duffee (2011)) or from forecasts (Chernov and Mueller (2012)) help forecast bond yields and returns, over and above information in the term structure of bond yields (but see also Bauer and Rudebusch (2017), Bauer and Hamilton (2017)). The first contribution of this paper is to document similar evidence against the spanning assumption in oil futures markets.

3.2. Evidence Against Spanning

Previous affine futures pricing models assume that all relevant factors are spanned by contemporaneous futures prices. As Duffee (2011) and Joslin et al. (2014) observe in the context of bond yields, this assumption has strong implications for the joint behavior of futures prices and the economy. First, any such model with N state variables can be rotated into "reduced form" such that the reduced-form state variables are equal to the prices of N arbitrary linearly independent portfolios of futures contracts (Duffie and Kan (1996)). Second, the N portfolios explain log futures prices up to idiosyncratic errors. Third, conditional on the prices of the N portfolios, no other information can contribute incremental forecasting power.

I document three stylized facts that contradict these restrictions. First, oil futures prices and returns display a low dimensional factor structure. Second, most variation in the macro factors M_t – in particular the real activity factor GRO – is not spanned by variation in oil futures prices. Third and most important, I find that M_t contributes incremental forecasting power for oil prices and returns over and above the information in contemporaneous futures prices.

A) Oil futures prices display a low dimensional factor structure Figure 3.2 plots the loadings of the first three principal components (PCs) for the levels and changes in log oil futures prices by maturity. The first two PCs have the familiar level and slope loadings, and account for 99.9% of variation in log price levels and 99.7% of variation in log price changes.

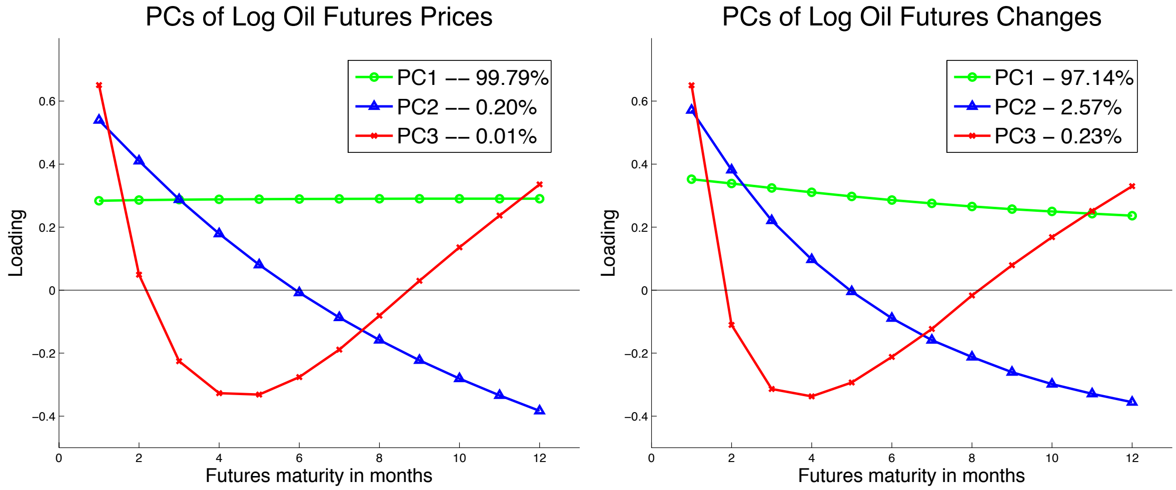


Figure 2 Loadings of the first three principal components (PCs) of the levels (panel A) and changes (panel B) of log oil futures prices monthly from 1/1986 - 7/2014. The legend shows the fraction of total variance explained by each PC.

B) M_t is mostly unspanned by oil futures I project M_t on the first two principal components of log oil futures prices and label the residual UM_t:

$$M_t = \alpha + \gamma_{1,2} PC_t^{1,2} + UM_t$$

The R^2 of the projections for GRO and INV are 6.4% and 27.5% . Projecting on the first five PCs the R^2 are 14.5% and 30.0%, and projecting on all 12 individual futures maturities the R^2 are 18.9% and 30.9%. Thus, most of the monthly variation in M_t – particularly GRO – is unspanned by variation in oil futures prices.

However, M_t might be measured with error or some subcomponents of M_t may be irrelevant to oil prices and risk premiums. Thus the main question is not the projection R^2 but whether M_t is relevant to returns and/or price forecasts conditional on the information in futures prices.

C) M_t forecasts returns over and above information in the futures curve Table 1 Panel A shows the results of forecasting returns to oil futures using information from the current futures curve plus the macro factors M_t. We see that real activity GRO helps to forecast both short-roll returns and excess-holding returns, which recover the spot and term premiums in expectation (Szymanowska et al. 2014). Panel B shows that M_t forecasts changes in the level but not the slope of the oil futures curve. In all three specifications the results for returns and the level factor remain after conditioning on the information in futures prices. The coefficients on f^{1-12} (not shown) have different signs on adjacent maturities, a clear sign of overfitting, yet GRO still adds forecasting power.

The unspanned forecasting power of GRO is statistically and economically significant. A one percent increase in GRO, about two standard deviations, forecasts a return to the level portfolio

Table 1 Panel A shows the results of forecasting the returns to the short-roll and 3 month excess-holding strategies in oil futures. Panel B shows the results of forecasting changes in the principal components of log futures prices. GRO is the Chicago Fed National Activity Index and INV is the log of U.S. crude oil stocks. The data are monthly from from 1/1986 to 6/2014. Newey-West standard errors with six lags are in parentheses.

Panel A: Forecasting Returns

	Short Roll Return			Excess Holding Return		
GRO_t	0.0259**	0.0242**	0.0215*	-0.0024**	-0.0019**	-0.0016*
	(0.0109)	(0.0112)	(0.0115)	(0.0010)	(0.0009)	(0.0009)
INV_t	0.040	0.037	0.035	-0.0043	-0.0069	-0.0077
	(0.092)	(0.093)	(0.093)	(0.098)	(0.092)	(0.092)
Spanned Factors $\mathcal{P}_t$	$PC^{1,2}$	PC^{1-5}	f^{1-12}	$PC^{1,2}$	PC^{1-5}	f^{1-12}
T	341	341	341	341	341	341
Adjusted R^2	3.3%	3.0%	6.2%	9.0%	11.5%	11.8%
Adjusted R^2, $\mathcal{P}_t$ only	0.4%	0.7%	4.6%	5.5%	9.4%	10.3%

Panel B: Forecasting PCs

	ΔPC^1			ΔPC^2		
GRO_t	0.0653**	0.0659**	0.0599*	0.0089**	0.0079*	0.0067
	(0.0315)	(0.0326)	(0.0331)	(0.0042)	(0.0045)	(0.0046)
INV_t	0.059	0.027	0.018	0.041	0.047	0.046
	(0.259)	(0.257)	(0.260)	(0.053)	(0.048)	(0.045)
Spanned Factors $\mathcal{P}_t$	$PC^{1,2}$	PC^{1-5}	f^{1-12}	$PC^{1,2}$	PC^{1-5}	f^{1-12}
T	341	341	341	341	341	341
Adjusted R^2	2.0%	1.6%	4.6%	7.5%	8.8%	10.8%
Adjusted R^2, $\mathcal{P}_t$ only	-0.4%	-0.5%	2.9%	6.5%	8.0%	10.3%

that is 1.94% higher over the next month. However, GRO reverts quickly toward its mean and oil futures returns are volatile; the implied Sharpe ratio of a simple market-timing strategy is 0.59 and 0.94 for the short-roll and excess-holding returns respectively. Thus, while real activity is a strong determinant of oil futures returns, the level of predictability is not implausibly large. Robustness checks in the Internet Appendix show that the forecasting power of real activity for oil prices and returns is robust to using two alternative real activity measures, excluding the volatile period after 2007, including several measures of time-varying volatility, and using year-on-year changes to eliminate persistent regressors.

3.3. A Futures Pricing Model with Unspanned Factors

Let X_t be a vector of N state variables that summarize the economy. X_t includes macroeconomic risk factors such as expected economic growth and factors specific to the commodity such as hedging pressure, inventories, and expectations of supply and demand. The state vector follows a Gaussian VAR,

$$X_{t+1} = K^{\mathbb{P}}_{0X} + K^{\mathbb{P}}_{1X} X_t + \Sigma_X \epsilon^{\mathbb{P}}_{t+1} \tag{1}$$

where $\epsilon^{\mathbb{P}}_{t+1} \sim N(0, 1_N)$. Risk premiums are "essentially affine" in the state variables (Duffee 2002) i.e. the stochastic discount factor is given by

$$\mathcal{M}_{t+1} = e^{(\Lambda_0 + \Lambda_1 X_t)' \epsilon_{t+1}} \tag{2}$$

This specification includes previous benchmark models such as Gibson and Schwartz (1990), Schwartz (1997), Casassus and Collin-Dufresne (2005). All of these previous models implicitly assume that X_t is identified (spanned) by contemporaneous futures prices. As is well known for bond yields (Duffie and Kan 1996), this assumption implies that X_t can be replaced by an arbitrary set of linear combinations of log futures prices:

$$\mathcal{P}^N_t = W f_t$$

where W is *any* full rank $N \times J$ matrix. Thus, the spanning assumption implies:

1. Futures prices, and all relevant macroeconomic information, are described up to idiosyncratic errors by the N factors $\mathcal{P}^N_t$.

2. The projection of X_t on $\mathcal{P}^N_t$ has R^2 of one.

3. Conditional on $\mathcal{P}^N_t$, no other information forecasts X_t or futures prices or returns.

Motivated by the stylized facts in Section 3.2, I instead assume that a subspace of X_t is spanned, while its complement is unspanned but observed by the econometrician. Suppose that contemporaneous futures prices are determined by a set of linear combinations $L_t = V f_t$ where V is a real valued $N_L \times J$ matrix and $N_L < N$. That is, the spot price and its evolution under the risk neutral measure are given by:

$$s_t = \delta_0 + \delta_1' L_t \tag{3}$$

$$L_{t+1} = K^{\mathbb{Q}}_{0L} + K^{\mathbb{Q}}_{1L} L_t + \Sigma_L \epsilon^{\mathbb{Q}}_{t+1} \tag{4}$$

where $\epsilon^{\mathbb{Q}}_{t+1} \sim N(0, 1_{N_L})$ and $\Sigma_L = V \Sigma_X$.

By the same rationale as before we can replace L_t with N_L linear combinations of log prices,

$$\mathcal{P}^L_t = W_L f_t$$

where W_L is any full rank $N_L \times J$ matrix, and transform the state space from X_t to $(\mathcal{P}_t^L, UM_t)$ where UM_t are the unspanned components (projection residuals) of the macro factors M_t. In contrast to the perfect-spanning models, this model implies that:

1. Futures prices are described up to idiosyncratic errors by $N_L < N$ factors.
2. The projection of X_t on $\mathcal{P}_t^L$ has R^2 less than one.
3. Conditional on $\mathcal{P}_t^L$, other information may forecast X_t or futures prices or returns.

Motivated by the variance decomposition in the previous section, I assume the number of spanned state variables $N_L = 2$. After estimating the model I rotate and translate so that the state variables correspond to the model implied spot price and cost of carry (s_t, c_t) and the macroeconomic series M_t. [8]

The model can then be described in just two equations:

1) The law of motion for the state variables:

$$\begin{bmatrix} s_{t+1} \\ c_{t+1} \\ M_{t+1} \end{bmatrix} = \begin{bmatrix} K^{\mathbb{P}}_{0sc} \\ K^{\mathbb{P}}_{0M} \end{bmatrix} + \begin{bmatrix} K^{\mathbb{P}}_{sc,sc} & K^{\mathbb{P}}_{sc,M} \\ K^{\mathbb{P}}_{M,sc} & K^{\mathbb{P}}_{MM} \end{bmatrix} \begin{bmatrix} s_t \\ c_t \\ M_t \end{bmatrix} + \Sigma \epsilon^{\mathbb{P}}_{t+1} \tag{5}$$

2) The dynamics of (s_t, c_t) under the risk neutral measure:

$$\begin{bmatrix} s_{t+1} \\ c_{t+1} \end{bmatrix} = K^{\mathbb{Q}}_0 + K^{\mathbb{Q}}_1 \begin{bmatrix} s_t \\ c_t \end{bmatrix} + \Sigma_{sc} \epsilon^{\mathbb{Q}}_{t+1} \tag{6}$$

The model is a canonical form, that is, any affine futures pricing model with two spanned state variables and $N_M \geq 0$ macroeconomic variables can be written in the form above. Extending it to more than two spanned state variables is straightforward.

4. Model Estimates

Table 2 presents the parameters of the maximum likelihood estimate of the model using the panel of crude oil futures prices and macro factors GRO_t, INV_t as described. The coefficient of Δs_{t+1} on GRO_t is positive and statistically significant at the 1% level, echoing the forecasting regressions in Table 1. It is important to note that a higher value of GRO also forecasts a fall in the cost of carry c_{t+1}, whereas Table 1 Panel B shows that higher GRO weakly forecasts a rise in the slope factor. This difference highlights that the state variables (s_t, c_t) are not identical to the level and slope factors and thus a factor-VAR would not deliver the same results.

The spot and cost of carry (s_t, c_t) do a good job of summarizing the term structure of oil futures prices: the model fitted values for f_t explain 99.97% of variation in observed log futures prices and the residuals (pricing errors) explain 0.03%. The root mean squared pricing error (RMSE) of the model is 54 basis points, in line with benchmark futures pricing models.

[8] The detailed derivation and estimation procedure are in the Internet Appendix.

Table 2 Maximum likelihood (ML) estimate of the affine model for Nymex crude oil futures using monthly data from January 1986 to June 2014. s_t, c_t **are the spot price and annualized cost of carry respectively.** GRO **and** INV **are the Chicago Fed National Activity Index and log of U.S. crude oil inventory respectively. The coefficients are over a monthly horizon, and the time series are de-meaned. ML standard errors are in parentheses.**

	$K_0^{\mathbb{P}}$	$K_1^{\mathbb{P}}$			
		s_t	c_t	GRO_t	INV_t
Δs_{t+1}	0.009	-0.007	0.060*	0.025***	0.045
	(0.006)	(0.008)	(0.031)	(0.008)	(0.076)
Δc_{t+1}	-0.007	0.017**	-0.119***	-0.015*	-0.074
	(0.005)	(0.008)	(0.029)	(0.007)	(0.071)
ΔGRO_{t+1}	-0.002	-0.094**	-0.022	-0.380***	0.028
	(0.030)	(0.044)	(0.163)	(0.042)	(0.404)
ΔINV_{t+1}	0.002	0.001	0.029***	-0.001	-0.089***
	(0.002)	(0.002)	(0.008)	(0.002)	(0.020)

	$K_0^{\mathbb{Q}}$	$K_1^{\mathbb{Q}}$	
		s_t	c_t
Δs_{t+1}	-0.003	0.000	0.083***
	(0.007)	(0.003)	(0.011)
Δc_{t+1}	-0.001	-0.004	-0.106***
	(0.011)	(0.009)	(0.030)

	Shock Volatilities & Correlations			
	s	c	GRO	INV
s	0.10			
c	-81%	0.056		
GRO	5%	2%	0.53	
INV	-21%	25%	4%	0.026

4.1. Dynamics of the State Variables

The models of Schwartz (1997) and Schwartz and Smith (2000) impose the restriction a priori that the spot price s_t is a random walk. Without that restriction, using data from 1990 to 2003, Casassus and Collin-Dufresne (2005) estimate that s_t is mean reverting with a half-life of two years, so the expected spot price of oil in ten years' time is effectively constant. By contrast the estimate in Table 2 which adds ten years of subsequent data is consistent with a random-walk process for the spot price. The coefficient of Δs_{t+1} on s_t is -0.007 and is statistically not distinct from zero.

The cost of carry, which drives time variation in the slope of the futures curve, reverts to a slightly negative mean with a half-life of five months. Shocks to the spot price and the cost of carry are strongly negatively correlated ($\rho = -81\%$), so a higher spot price is accompanied by a more downward sloping curve, but shocks to the spot price are essentially permanent while cost of carry shocks decay within a few years. As a result, about half of a typical move in the oil spot price disappears after two to three years, while the other half persists.

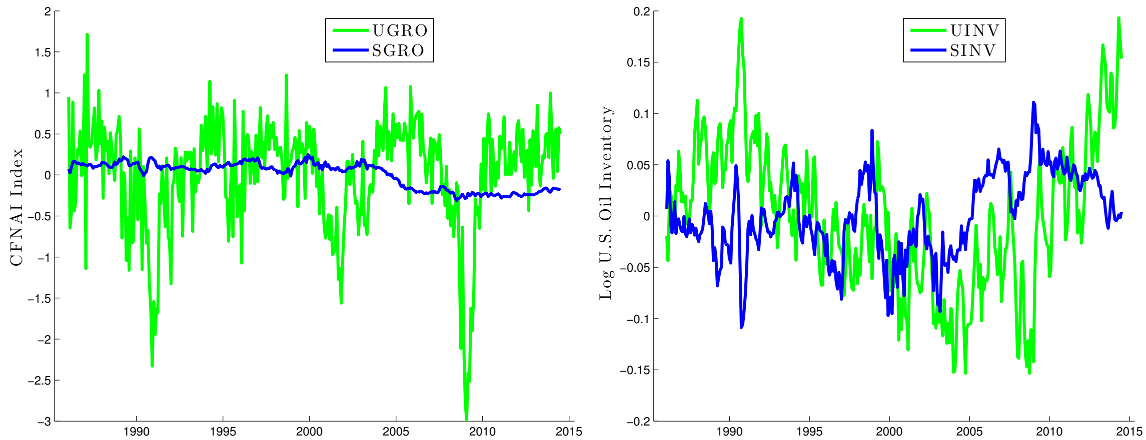


Figure 3 **Panel A plots the components of the monthly Chicago Fed National Activity Index GRO that are spanned (SGRO) and unspanned (UGRO) by the oil futures curve. Panel B plots the components of monthly log U.S. oil inventories INV that are spanned (SINV) and unspanned (UINV) by log oil futures prices.**

Figure 4.1 Panel A plots the components of GRO that are spanned and unspanned by (s_t, c_t). We see that effectively all of the monthly and yearly variation in GRO appears in the unspanned component. Figure 4.1 Panel B plots the spanned and unspanned components of log oil inventories INV. Compared to GRO, much more of the monthly and yearly variation in INV is spanned by futures prices, as INV loads strongly on the cost of carry c_t.

4.1.1. Oil Futures and Real Activity A 1% shock to real activity forecasts a 2.5% higher spot price of oil and a 1.5% lower cost of carry. As a result, the net effect of real activity on oil prices is forecast by the market to be persistent – higher real activity raises both the short run and the expected long run price of oil.

Conversely, a higher spot price of oil forecasts lower real activity. A higher cost of carry – higher expected prices in future – forecasts slightly higher real activity, but c_t naturally forecasts a higher spot price as well. The impulse response functions in Section 4.1.3 make clear that the net effect of c_t on GRO is negative. As a result, a shock to the spot price of oil that the market expects to persist has a more negative effect on growth than a shock that is expected to be transitory.

In sum, the estimates reveal a negative feedback relationship between oil prices and real activity. A positive shock to real activity forecasts persistent higher oil prices, while a positive oil price shock forecasts lower real activity, and the latter effect is stronger for a shock to the spot price of oil that the market expects to persist.

4.1.2. Oil Futures and Inventories Shocks to log inventories are negatively correlated with the spot price and positively correlated with the cost of carry. Both of these observations are consistent with the Theory of Storage – higher inventories reflect movements up the supply-of-storage curve.

The correlation between shocks to inventory and the cost of carry is 27%; in a frictionless storage model (e.g. Working (1949)) INV_t and c_t would be collinear. A higher cost of carry also strongly predicts higher inventories the next month. The less-than-unity correlation and the forecasting power of c_t for inventories suggests significant adjustment costs in production and storage as in, e.g., Carlson et al. (2007). Looking down the last column of the transition matrix, higher inventory has no significant effect on the forecasted spot price or cost of carry. This finding is consistent with the fundamental drivers of oil inventory such as precautionary storage and expected physical supply and demand being fully spanned by the oil futures curve.[9]

4.1.3. Impulse Response Functions Figure 4.1.3 plots the impulse response functions to shocks to oil futures prices and economic activity.[10] Panel A plots the response to a "typical" unit shock to the log spot price, which is accompanied by a negative shock to the cost of carry and a more downward-sloping curve. About half of the increase in s_t decays within two years, while the other half is effectively permanent. The change in oil prices produces a 0.2% fall in real activity that is effectively permanent, a sizeable effect on the real economy. The higher spot price and lower cost of carry also produce a fall in inventories.

By comparison, Panel B plots the response to a transitory shock to s_t, in which the market expects s_t to fully revert to the pre-shock baseline. In this scenario the response of economic activity is transient as well, and in fact real activity GRO recovers to the baseline faster than s_t does. Comparing to Panel A, which only differs in the size of the shock to c_t, highlights that oil shocks that are more persistent have a larger and longer lasting effect on real activity. Note that the fact that the forecast of the long-run spot price is unchanged in Panel B does not mean that long-maturity futures prices will be unchanged – the two are equivalent only in the case that risk premiums are constant. Thus, a factor-VAR that includes long-maturity futures prices will not necessarily recover the correct dynamics.

Panel C plots the response to a unit shock to the cost of carry c_t.The higher effective storage cost induces higher inventories, and raising the slope of the term structure of futures naturally raises the forecast of future spot prices s_t, which forecasts lower real activity as well. Panel D plots

[9] The estimates identify both the implied spot price and cost of carry in a relatively agnostic way. c_t is 40% less volatile than the basis spread, which is a standard proxy for the cost of carry. In unreported results I find that c_t is more strongly related to both current and future inventories than the basis. That is, there is considerable variation in the basis which is not reflected in physical storage, and we obtain a stronger link to storage by fitting a pricing model to the futures curve than we do from a single calendar spread.

[10] The ordering of the variables for the impulse response functions is (GRO, s_t, c_t, INV). GRO is first because innovations in the unspanned component of GRO can be thought of as exogenous to contemporaneous oil prices and inventories. We analyze s_t and c_t simultaneously, so their relative ordering is not important. Finally, it is intuitive and also supported by the model estimates that the oil futures curve adjusts to new information before physical inventory does.

the response to a shock to real activity. The index mean reverts rapidly and the shock decays back to the baseline within a year. However, a transient shock to GRO produces a near permanently higher spot price of oil – perhaps because oil is a nonrenewable resource. The magnitude of the effect is large: a 1% shock to real activity produces a spot price of oil that is 5.1% higher than the baseline ten years later.

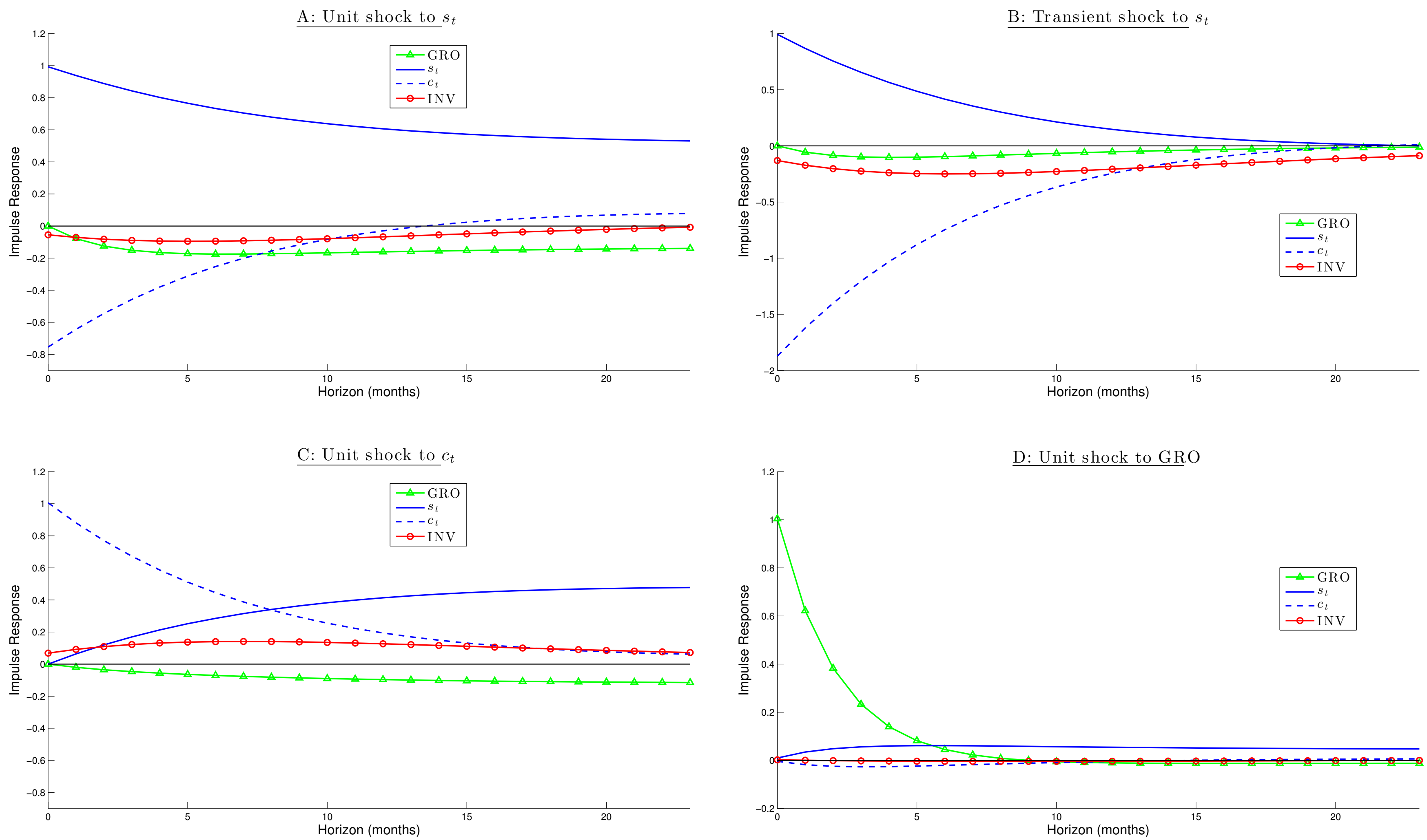


Figure 4 **Panel A shows the response of the four state variables to a typical shock to the log spot price of oil s_t. Panel B shows the response to a transient shock in which s_t fully reverts to the baseline. Panel C shows the response to a unit shock to the cost of carry c_t. Panel D shows the response to a unit shock to economic growth, GRO. The ordering of the variables in the impulse response function is (GRO, s_t, c_t, INV).**

Table 3 Maximum likelihood (ML) estimates of the loadings of risk premiums in the affine model for U.S. crude oil futures using monthly data from January 1986 to June 2014. s_t, c_t are the monthly spot price and annualized cost of carry respectively. GRO and INV are the Chicago Fed National Activity Index and log of U.S. crude oil inventory respectively. The coefficients are standardized to reflect a one standard deviation monthly change, and the time series are de-meaned. ML standard errors are in parentheses.

	Λ_0	Λ_1			
		s_t	c_t	GRO_t	INV_t
Λ_t^s	0.011	-0.001	-0.001	0.013***	0.001
	(0.013)	(0.001)	(0.002)	(0.004)	(0.002)
Λ_t^c	-0.006	0.003	-0.001	-0.008**	-0.002
	(0.016)	(0.002)	(0.003)	(0.004)	(0.002)

4.2. Oil Risk Premiums

Table 3 displays the estimates of the loadings that drive variation in the risk premiums attached to the spot and cost-of-carry state variables (hereafter, the "spot premium" and the "term premium").[11]

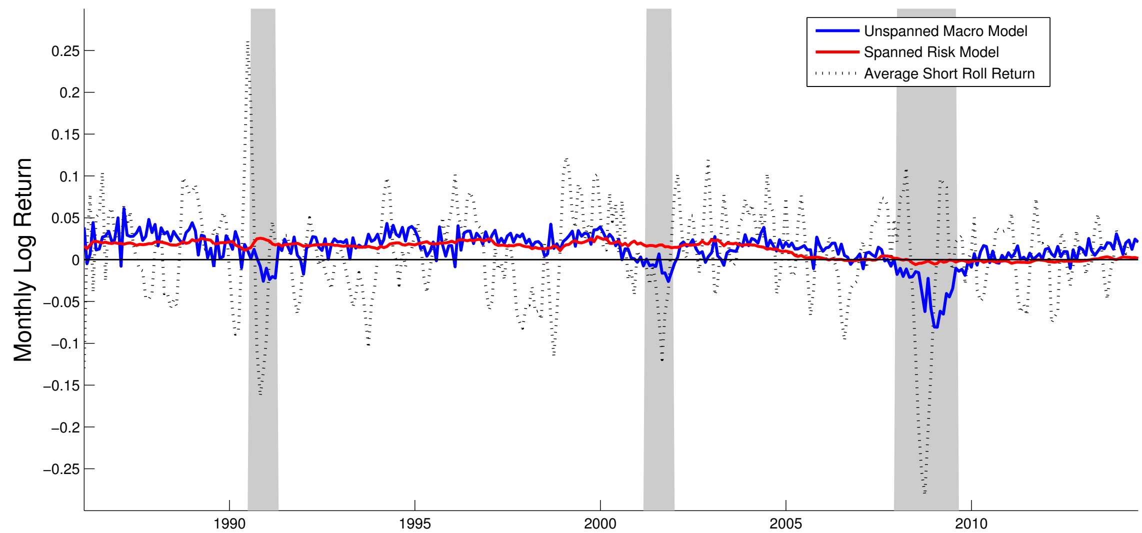
Just two entries in the loadings of risk premiums on the factors are statistically significant: higher real activity GRO is associated with a higher spot premium and a lower term premium in oil futures. As in the forecasting regressions, this finding is inconsistent with the full spanning assumption that is implicit in previous models.

The unspanned effect of real activity on oil risk premiums is material. Figure 5 Panel A plots the implied spot premium in the full model and the nested model that enforces spanning, as well as the average realized short-roll return over the following three months. The risk premiums differ most noticeably during the recessions of 1990-1991, 2001-2002 and 2008-2009: slumps in real activity forecast falling oil prices and negative returns to the short-roll strategy. This unspanned procyclical component dominates the variation in the spot risk premium; the standard deviation of changes in Λ_t^s in the unspanned macro model is 1.47% per month compared to 0.17% per month in the spanned-risk model, an increase of nearly tenfold.

Figure 5 Panel B plots the implied term premium in the full model and the nested model that enforces spanning, as well as the average realized excess-holding return over the following three months. The unspanned effect of real activity on the oil term premium is smaller and opposite in sign to that on the spot premium. Slumps in real activity forecast a rising cost of carry and higher returns to the excess-holding strategy.

Szymanowska et al. (2014) observe that the spot and term premiums are negatively correlated in a cross-section of commodities. The estimates here strongly support this finding within the term

[11] Szymanowska et al. (2014) decompose futures returns into a spot premium and a term premium based on the returns to different trading strategies. In the Internet Appendix I show that their spot and term premiums correspond exactly to Λ_t^s and Λ_t^c respectively plus a small convexity term.



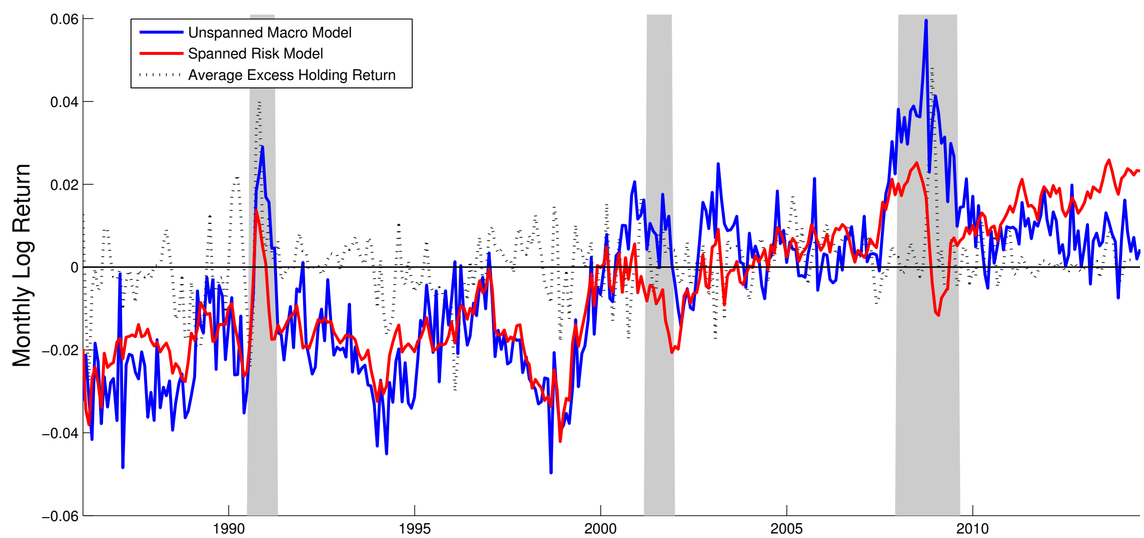


Figure 5 **Panel A plots the spot risk premium from the model with unspanned macro factors versus the nested model that enforces spanning, as well as the average short-roll return over the next three months. Panel B plots the term risk premiums from the two models, as well as the average excess-holding return over the next three months. NBER recessions are shaded in grey.**

structure of the oil futures market; the correlation between the spot and term premiums is -88%. The estimates also suggest a potential explanation, which is that shocks to real activity drive a higher spot premium and a lower term premium. By contrast, in the nested spanned-risk model the correlation between the spot and term premium is strongly *positive* at 74%, which underlines that unspanned real activity is what drives this conclusion.

The model with unspanned factors also better explains the panel of oil futures returns as a whole. Taking the full panel of monthly log returns, the adjusted R^2 of the unspanned-factors model is

4.3% compared to 1.3% for the nested spanned-risks model. Thus, after adjusting for the added degrees of freedom, adding the unspanned factors more than triples the explanatory power of the model for oil futures returns.

4.2.1. What Drives Cyclical Variation in Oil Risk Premiums? Both the regressions and the model estimates indicate that the forecasting power of real activity is not reflected in contemporaneous oil futures prices, which corresponds to real activity having offsetting effects on expected returns and the spot price forecast. We observe directly in the data that higher real activity forecasts a higher spot price of oil. The offsetting pattern in expected returns operates through changing risk premiums. While a full model of oil risk premiums is beyond the scope of this paper,[12] in this section I interpret these results via the model of Kilian (2009).

Kilian (2009) estimates a structural VAR that contains 1) oil supply shocks, 2) oil-specific demand shocks, and 3) aggregate demand shocks. In oil futures markets the spot risk premium will be determined by the covariance of each shock with the pricing kernel, which I assume covaries positively with real activity, and the spot price of oil. The term risk premium will be determined by the covariance of each shock with the pricing kernel and the basis (the spread between the spot price and longer-maturity futures). I assume that the prices of risk (thus the magnitudes of the risk premiums) are higher in economic downturns, as has been documented across many markets (Cochrane 2011).

Kilian finds that oil supply shocks raise the spot price, lower the basis, and lower real activity. This will produce a negative spot risk premium – a long exposure to the spot price is a hedge against oil supply shocks – and a positive term risk premium. To the extent these risk premiums are higher in recessions, exposure to oil supply shocks will produce a procyclical spot premium and a countercyclical term premium and the two will be negatively correlated.

Aggregate demand shocks raise real activity and raise the spot price of oil, and the oil price continues to rise even six months after the shock arrives. Thus, aggregate demand shocks raise the spot price, the basis, and real activity. Oil-specific demand shocks raise the spot price of oil but the increase is partly reversed within a year. They also raise real activity. As a result oil-specific demand shocks raise the spot price, lower the basis and raise real activity.

Table 4 summarizes the predictions for how the three types of structural shocks drive oil risk premiums. In brief, the results suggest that the most important driver of time variation in oil risk premiums is oil supply shocks, for which the model's predictions line up with what we observe in the data.

[12] In recent work Ready (2016) and Baker and Routledge (2017) solve models that generate endogenously time-varying risk premiums in oil futures markets.

	Spot Premium	Term Premium	Correlation
Oil supply shock	Procyclical	Countercyclical	-
Aggregate demand shock	Countercyclical	Countercyclical	+
Oil-specific demand shock	Countercyclical	Procyclical	-
Model Estimates	Procyclical	Countercyclical	-

Table 4 The table shows the predicted effects of the three types of structural shock in Kilian (2009) on risk premiums in oil futures markets.

4.3. Other Macro Factors

4.3.1. Inflation Hedging against inflation risk is an important motivation for trading commodity derivatives (Erb and Harvey 2006, Szymanowska et al. 2014). Oil futures are a strong candidate for inflation hedging as they are highly liquid and the price of oil is a major factor in inflation. To the extent the inflation hedging motive varies over time, we would expect higher inflation risk to be associated with a lower spot risk premium. The effect of inflation risk on the term risk premium is less clear.

Table 5 presents the loadings of oil risk premiums on real activity (GRO), the monthly percent change in the Consumer Price Index (CPI) and on the median one-year forecast of CPI inflation from the Survey of Professional Forecasters (SPF).[13]

Oil futures are indeed an inflation hedge: realized inflation CPI has a correlation of 27% with shocks to the spot price of oil s_t. However, realized inflation does not covary significantly with the spot or term risk premiums. In contrast, *expectations* of future inflation do covary with the spot risk premium: a one standard deviation increase in the inflation forecast SPF is followed by a return to portfolios loading on the spot price factor s_t that is 0.2% lower per month, approximately 2.4% less per year. The magnitude of the inflation forecast's effect on the spot risk premium is around one-sixth that of real activity but it is strongly statistically significant and is not subsumed by the effects of real activity. Thus, the model estimate is consistent with the hypothesis that inflation risk also drives hedging demand that lowers the spot oil risk premium.

4.3.2. Positive vs Negative Growth Regimes Looking at Figure 5, the effect of real activity on oil prices appears to be concentrated in economic downturns. To investigate this possibility I split GRO into two components: GRO^+ equals GRO in months when the lagged three month average of GRO is positive and equals zero otherwise, and GRO^- equals GRO in months when the lagged three month average of GRO is negative and equals zero otherwise.[14] This split lets the coefficients differ when real activity is in a positive-growth regime versus a negative-growth regime.

[13] The SPF is released quarterly; I use the most recent forecast as of each monthly date.

[14] Because the unspanned factors do not enter into the pricing relation, they need not be Gaussian.

Table 5 Maximum likelihood (ML) estimates of the coefficients of oil risk premiums on latent and macroeconomic factors. Λ^s, Λ^c are the spot and term risk premiums. GRO is the monthly Chicago Fed National Activity Index; CPI is the monthly CPI inflation rate; SPF is the most recent median forecast of one-year CPI inflation from the Survey of Professional Forecasters. The coefficients are standardized to reflect a one standard deviation monthly change. ML standard errors are in parentheses.

	Λ_0	Λ_1				
		s_t	c_t	GRO_t	CPI_t	SPF_t
Λ_t^s	0.011	-0.003	-0.001	0.012***	0.003	-0.002**
	(0.013)	(0.002)	(0.002)	(0.004)	(0.005)	(0.001)
Λ_t^c	-0.006	0.004	-0.002	-0.007*	0.001	0.001
	(0.016)	(0.003)	(0.004)	(0.004)	(0.004)	(0.001)

Table 6 Maximum likelihood (ML) estimate of the transition matrix for the model in which GRO^+ and GRO^- are the monthly Chicago Fed National Activity Index in months when the lagged three month average of GRO is positive and negative respectively. The coefficients are over a monthly horizon, and the time series are de-meaned. ML standard errors are in parentheses.

	$K_0^{\mathbb{P}}$	$K_1^{\mathbb{P}}$			
		s_t	c_t	GRO_t^+	GRO_t^-
Δs_{t+1}	0.009	-0.006	0.069**	0.025	0.025**
	(0.007)	(0.008)	(0.027)	(0.016)	(0.010)
Δc_{t+1}	-0.009	0.016**	-0.133***	-0.009	-0.018*
	(0.006)	(0.008)	(0.026)	(0.015)	(0.010)
ΔGRO_{t+1}^+	0.147***	-0.082***	-0.048	-0.789***	0.076**
	(0.022)	(0.028)	(0.092)	(0.052)	(0.034)
ΔGRO_{t+1}^-	-0.052**	-0.038	0.052	0.036	-0.281***
	(0.025)	(0.031)	(0.102)	(0.058)	(0.038)

Table 6 presents the estimated feedback matrix. The effect of real activity on oil prices is symmetrical in good times versus bad – the point estimates of the coefficients of Δs_{t+1} on GRO^- and GRO^+ are equal. On the other hand, the estimate suggests an asymmetry on the supply side. The forecast of real activity is impaired by oil price shocks in positive growth regimes: the coefficient of ΔGRO_{t+1}^+ on s_t is -0.082 and is statistically significant at the 1% level. By comparison the coefficient of ΔGRO_{t+1}^- on s_t i.e. in negative growth regimes is less than half as large and is not statistically significant. Thus, while the effect of shocks to real activity on the oil price forecast is symmetric in good times and bad, the negative effect of oil price shocks on real activity is stronger in times when real activity is expanding.

4.3.3. Subcomponents of Real Activity The Chicago Fed divides the 85 constituent time series that make up the CFNAI into four subcomponents of the index. The subcomponents are Industrial Production and Income (PI); Employment and Hours (EUH); Personal Consumption and Housing (CH); and Sales Orders and Inventories (SOI). When I estimate the model with

Table 7 Maximum likelihood (ML) estimate of the dynamics for the model in which the macro factors are the subcomponents of the Chicago Fed National Activity Index that measure industrial production and income PI_t and personal consumption and housing PCH_t. The coefficients are over a monthly horizon, and the time series are de-meaned. ML standard errors are in parentheses.

	$K_0^{\mathbb{P}}$	$K_1^{\mathbb{P}}$			
		s_t	c_t	PI_t	PCH_t
Δs_{t+1}	0.009	-0.007	0.069**	0.141**	0.023
	(0.006)	(0.009)	(0.029)	(0.053)	(0.039)
Δc_{t+1}	-0.008	0.015	-0.135***	-0.085	-0.021
	(0.005)	(0.009)	(0.027)	(0.050)	(0.037)
ΔPI_{t+1}^{+}	0.001	-0.001	0.024	-0.728***	0.070*
	(0.006)	(0.009)	(0.028)	(0.053)	(0.039)
ΔPCH_{t+1}^{-}	-0.001	-0.013**	-0.016	-0.047	-0.090***
	(0.003)	(0.006)	(0.017)	(0.032)	(0.023)

$M_t = [PI_t, EH_t, PCH_t, SOI_t]$ I find that employment EUH and sales and inventories SOI do not interact much with oil prices, but that industrial production PI and personal consumption PCH do.

Table 7 presents the results of estimating the model with those two subcomponents of the real activity index, $M_t = [PI_t, PCH_t]$. We see that the "demand" channel in which shocks to real activity affect the oil price forecast runs through industrial production. The coefficient of Δs_{t+1} on PI_t is positive (0.141) and statistically significant while the coefficient of Δs_{t+1} on PCH_t is much smaller and is not statistically significant. Also, shocks to PI are contemporaneously correlated with a higher spot price of oil s_t. By contrast the "supply" channel in which shocks to the oil price affect real activity appears to be driven by personal consumption. The coefficient of ΔPCH_{t+1} on s_t is negative and statistically significant, and the coefficient of ΔPCH_{t+1} on c_t (expected future oil prices) is also negative, while the coefficient of ΔPI_{t+1} on s_t is much smaller and is not statistically significant.

This decomposition suggests another asymmetry in the dynamic between oil prices and the real economy. On one hand, shocks to industrial productivity are associated with higher oil prices both contemporaneously and with a lag, while shocks to consumer spending have little or no effect. On the other hand, oil price shocks affect real activity by lowering future consumption spending.

4.3.4. Hedging Pressure The "hedging pressure" or "normal backwardation" theory which dates back to Keynes (1930) says that risk premiums in commodity futures markets arise due to hedging demand. Empirically, hedging pressure as a factor in futures risk premiums has a mixed track record (e.g. Bessembinder (1992), Gorton et al. (2012)). Table 8 shows the estimated parameters that govern oil risk premiums in the model using $M_t = [GRO_t, HP_t]$ where HP is

Table 8 Maximum likelihood (ML) estimate of the transition matrix in which HP is monthly hedging pressure calculated from the CFTC's Commitment of Traders reports. The coefficients are over a monthly horizon, and the time series are de-meaned. ML standard errors are in parentheses.

	$K_0^{\mathbb{P}}$	$K_1^{\mathbb{P}}$			
		s_t	c_t	GRO_t	HP_t
Δs_{t+1}	0.007	-0.012	0.075***	0.023***	-0.083
	(0.006)	(0.009)	(0.028)	(0.008)	(0.069)
Δc_{t+1}	-0.006	0.022**	-0.139***	-0.013	0.088
	(0.005)	(0.009)	(0.026)	(0.008)	(0.065)
ΔGRO_{t+1}	-0.018	-0.148***	0.040	-0.395***	-0.776**
	(0.030)	(0.050)	(0.146)	(0.043)	(0.364)
ΔHP_{t+1}	-0.010	-0.012**	-0.037**	-0.011**	-0.257***
	(0.003)	(0.005)	(0.016)	(0.005)	(0.039)

the hedging pressure in oil futures computed from the CFTC's Commitment of Traders Report.[15] We see that hedging pressure does not drive oil risk premiums, nor does it subsume the strong relationship of real activity with the oil risk premium. This conclusion from the term structure of oil futures returns is consistent with Gorton et al. (2012)'s conclusion using portfolio sorts across the cross-section of commodity futures.

5. Unspanned Factors and Real Options

Firms' capacity to adjust their investment or production ex post make up a substantial part of firm value (Pindyck (1988), Berk et al. (2004)). Previous studies such as Brennan and Schwartz (1985) and Casassus and Collin-Dufresne (2005) have explored what derivatives pricing models imply about the value of real options. As mentioned, these studies make the implicit assumption that all relevant factors are spanned by the futures market. In Trolle and Schwartz (2009) and Chiang et al. (2015), a latent volatility factor is unspanned in oil futures but spanned by oil options.

By contrast, if a macro factor M_t is unspanned in the sense of this paper then it does not affect the price of *any* derivative on the underlying commodity. Unspanned factors of this type are still relevant to real options, however, if the payoff depends on M_t. For example an oil well is often modeled as the right to pump oil out of the ground at a fixed cost per barrel, equivalent to a purely financial option. But in reality the costs of extraction are uncertain. Indeed Moel and Tufano (2002) find that for gold mines, changes in extraction costs over time are a significant predictor of mine openings and closings *after* controlling for factors extracted from gold futures and options prices.

More broadly, commodity prices are only one element of a firm's decision process. For example, in an airline's decision to purchase more fuel efficient planes, the cost savings will vary with oil

[15] Hedging pressure equals the net position among commercial participants divided by the total open interest as of the most recent Commitment of Traders report.

prices while revenues will vary with aggregate economic activity. Pindyck (1993) makes this point and argues that covariance of the payoff with macroeconomic risk factors should affect real option valuation.[16] Such risk factors will *only* affect real options valuation to the extent that they are unspanned i.e. not reflected in contemporaneous derivatives prices.

To examine the potential impact of unspanned factors on real option valuation and exercise, I model an oil well as a ten year strip of European options on an oil field that produces 1000 barrels of oil per month when open. The oil is extracted at unit lifting cost L_t and sold at the spot price S_t. Thus, it is open in any month when $S_t > L_t$. To examine differences in valuation between models I follow a similar approach to that of Carlson et al. (2007): I simulate data for the state variables (s, c, GRO) and the log lifting cost l_t. The Internet Appendix describes the setting in detail. I then estimate the unspanned and spanned-risks models on the simulated data and compute the implied value of the well over a range of current lifting costs L_0.

Figure 6 plots the real option value of wells under the different models across a range of current lifting costs. The lower two lines represent values for the benchmark spanned-risk models in which all relevant risks are assumed to be spanned by oil futures. This means that l_t is restricted to equal a linear combination of s_t and c_t plus an error term. The figure shows that when l_t is restricted to be spanned, whether the error term is modelled as an i.i.d. or AR(1) process is irrelevant to option valuation.

The upper two lines plot real option values from models that incorporate unspanned macro risk. The values are considerably higher; that is, the spanned-risk models miss a large component of real option value. Note that the volatility of l_t in the spanned-risk model is the same as it is in the unspanned-risk models: the difference is that l_t's covariance with GRO adds a persistent, unspanned component to lifting costs which is ignored by the spanned-risk model. This has a large effect on option valuation: Adding the unpriced ($\lambda = 0$) unspanned macro risk raises the real option value by 35% for an 'in the money' well with a current lifting cost of \$20 per barrel and 405% for an 'out of the money' well with a current lifting cost of \$150 per barrel.

The risk premium effect is that the option value is higher when GRO, and hence L_t, carries a positive risk premium ($\lambda > 0$). This effect on valuation is present but smaller, increasing the well value by 1% for the 'in the money' well with $L_0 = 20$ and by 1.3% for the 'out of the money' well with $L_0 = 150$.

[16] *"... this effect is magnified when fluctuations in construction costs are correlated with the economy, or, in the context of the Capital Asset Pricing Model, when the 'beta' of cost is high... [A] higher beta raises the discount rate applied to expected future costs, which raises the value of the investment opportunity as well as the benefit from waiting rather than investing now."*

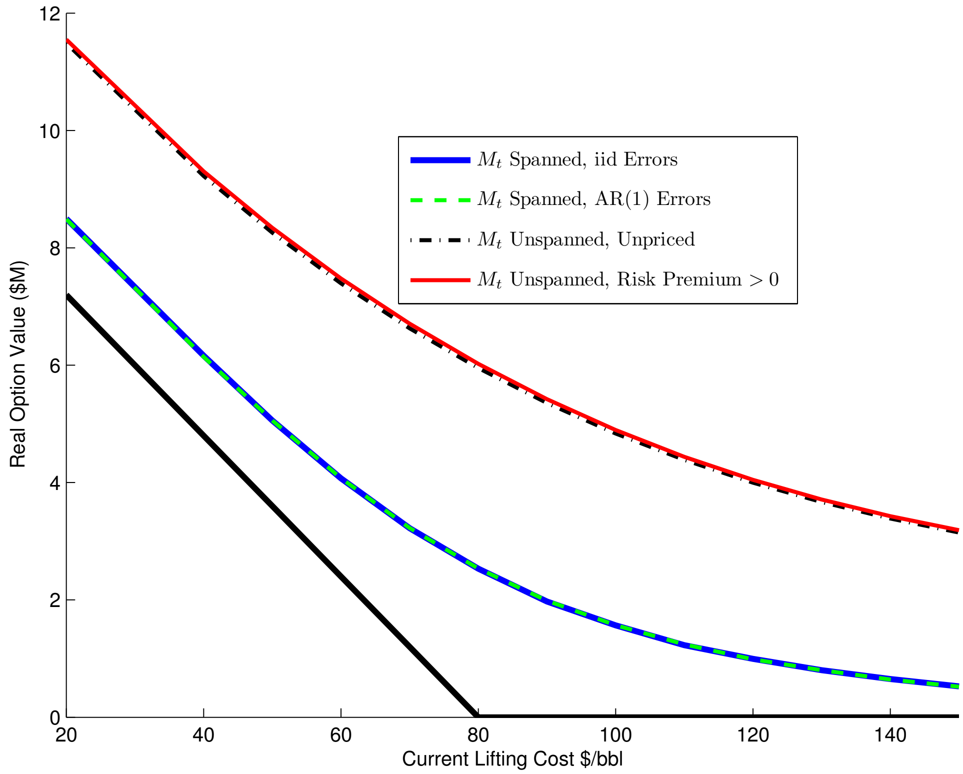


Figure 6 Real options valuation with unspanned macro risk. An oil well is modelled as a strip of European options that are exercised when the stochastic log extraction cost l_t is less than the log spot price s_t. l_t covaries with s_t and the unspanned macro risk GRO_t. The current spot price of oil is $80, and the x-axis indexes the current lifting cost L_0 of different oil wells.

6. Conclusion

This paper investigates the interaction of oil futures markets with the real economy. I develop an affine futures pricing model with potentially unspanned macroeconomic variables. The model generalizes and extends benchmark futures pricing models, and connects them explicitly with macroeconomic studies that use vector autoregressions (VARs).

Both forecasting regressions and model estimates reveal a novel empirical fact: real activity forecasts oil futures prices and returns and the effect is unspanned in contemporaneous oil futures. The estimated oil spot risk premium in the model is strongly procyclical and nine times more volatile than the nested model that assumes complete spanning, which suggests that spanned-risk models miss the majority of variation in oil risk premiums. Both regressions and model estimates suggest that the oil spot premium is procylical, while the oil term premium is countercyclical. Interpreted in light of the structural model of Kilian (2009), these results are consistent with oil risk premiums being driven primarily by demand for hedging oil-specific supply shocks. Higher expected inflation also appears to drive greater hedging demand and a lower spot risk premium, independent of the business cycle.

The model estimates reveal rich two-way dynamics of oil futures markets with the economy. Higher oil prices forecast lower real activity, especially when the price increase is forecast by the market to be persistent, and especially when the economy is in an expansion. Higher real activity forecasts a higher oil price, and although real activity shocks are transient the resulting higher oil price is highly persistent, which may be related to the fact that oil is a nonrenewable resource. The channel from real activity to oil prices flows through industrial production, while the channel from oil prices to real activity flows through consumer spending.

By construction unspanned macro factors do not affect the price of any derivative on the underlying commodity. However, when the value of a real option depends on macroeconomic factors beyond the spot price then unspanned macro factors can have a large effect on real option valuation. In a calibrated example I show that both the dynamics and the risk premiums of unspanned macro risks raise the values of a hypothetical real option significantly relative to a benchmark spanned-risk model.

The estimates presented in this paper are all maximally flexible, meaning that they impose no restrictions beyond what is required to identify the model. Thus, the results are best seen as simply summarizing the data in a consistent and parsimonious way. One direction for future research will be to impose economically motivated restrictions on the dynamics of the state variables, as in the VAR literature (Barsky and Kilian 2004, Kilian 2009), and on risk prices as in the futures pricing literature (Schwartz 1997, Casassus et al. 2013).

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